LINEAR MANOEUVRING DERIVATIVES IN MUDDY NAVIGATION AREAS

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SUMMARY

An overview of results of systematic captive manoeuvring test series with a container carrier model and a tanker model above a solid bottom as well as above and in simulated mud layers is presented. The effect of under keel clearance on linear manoeuvring coefficients and dynamic stability parameters is discussed, with emphasis on very small and even negative values referred to the water-mud interface. Controllability and manoeuvrability appear to be affected particularly by the influence of water depth on the lateral force due to yaw, as at very small under keel clearance the centrifugal inertia force is completely compensated by a hydrodynamic centripetal force.

NOMENCLATURE

A coefficient of \( \sigma^2 \) (stability indices equation) (kg²·m²)
B ship’s beam (m)
coefficient of \( \sigma^1 \) (stability indices equation) (kg²·m)
C coefficient of \( \sigma^0 \) (stability indices equation) (kg²)
C_B block coefficient (-)
d discriminant (stability indices equation)
h, h_1 water depth (free surface to interface) (m)
I_{zz} moment of inertia about vertical axis (kg·m²)
L, L_{pp} ship’s length between perpendiculars (m)
m ship’s mass (kg)
N yawing moment (>0 clockwise) (Nm)
n propeller rate (rpm)
r yaw rate (>0 clockwise) (rad/s)
t time (s)
T ship’s draft (m)
u longitudinal speed component (>0 forward) (m/s)
U_{crit} critical speed (m/s)
v lateral speed component (>0 to starboard) (m/s)
x_G longitudinal position of centre of gravity (m)
x_{pivot} longitudinal position of pivot point (m)
x_{s}, x_{vr}, x_{p} longitudinal position of application point of \( \tau \)
x_y forces due to yaw, sway, rudder action (m)
Y lateral force (>0 to starboard) (N)
Y_q, N_q hydrodynamic derivative; \( q \equiv \dot{v}, \dot{r}, \dot{u}, \dot{u}, \dot{u}, \dot{u} \)
\( \beta \) drift angle (= - arctan \( v/u \)) (deg)
\( \delta \) rudder angle (>0 to port) (deg)
\( \sigma_{1,2} \) stability index (s⁻¹)
\( \rho, \rho_{1} \) water density (kg/m³)
\( \rho_2 \) mud density (kg/m³)

1. INTRODUCTION

Safe navigation in access channels and harbours requires horizontal and vertical dimensions of the waterway, which are adapted to the design ship’s characteristics. The required depth (h) of a navigation area must exceed the ship’s draft (T) by a gross under keel clearance which is sufficiently large to allow vertical ship motions due to squat effects and the response to waves and inaccurate determination of the bottom level. Moreover, the resulting net under keel clearance should exceed a minimum value in order to allow safe manoeuvring. Indeed, a ship's manoeuvring behaviour depends on the ratio h/T. In this respect, a rather subjective and arbitrary distinction can be made between deep water, medium deep water (where the effect of depth restrictions starts to be noticed), shallow water (where the effect becomes significant) and very shallow water (where the effect is dominating the ship’s behaviour). According to [1] shallow water corresponds with an h/T-range between 1.2 and 1.5, while lower ratios are referred to as very shallow water.

The very shallow water range is quite important for manoeuvring in channels, canals and harbours. As an example, the ICORELS report ([2],[3]) makes following general recommendations on keel clearances: 20% for open sea areas, exposed to strong and long stern or quarter swell; 15% in waiting areas and channels exposed to strong and long swell, 10% for less exposed channels, 10 to 15% for exposed, and 7% for protected manoeuvring and berthing areas.

In some cases, the nature and characteristics of the bottom may influence a ship’s behaviour, too. Many navigational channels have bottoms that are covered with fluid mud suspensions, characterised by low density and weak shear strength. In such conditions, the bottom level and, therefore, the depth are not clearly defined, as traditional survey techniques appear to be inadequate. If the interface between the water and the fluid mud layer is considered as the bottom, it is even possible to navigate with h/T ratios less than 1 and, therefore, with negative under keel clearance. However, in muddy areas it is appropriate to define a "nautical bottom" as "the level where physical characteristics of the bottom reach a critical limit beyond which contact with a ship’s keel causes either damage or unacceptable effects on controllability and manoeuvrability" [4]. This nautical bottom is located at some depth under the water-mud interface, where from rheological point of view a transition between fluid and solid mud can be defined. In harbours where the nautical bottom concept is applied, the operational definition of this level is usually linked to a critical density value, varying between 1150 and 1270 kg/m³.
The choice of a critical density is based on considerations about the rheology of the local mud. In the harbour of Zeebrugge, as a result of a large number of simultaneous point measurements of rheology and density profiles carried out in the 1980s, it could be concluded that the rheological transition was always located below the 1150 kg/m³ level. The latter was accepted to be the nautical bottom and is displayed on nautical charts in areas where echo soundings are inadequate ever since.

Recent rheological measurements in situ, however, have revealed a significant change of the mud characteristics, in particular of the depth-rheology relationship. Two rheological transition levels can be defined: at a first one, occurring at a density between 1080 and 1120 kg/m³, the rheological characteristics increase slightly, while a more severe transition takes place in the range 1180-1250 kg/m³.

Based on these results, suggestions were made to adopt a higher value for the critical density, so that a larger part of the mud layer could be incorporated in the under keel clearance. As at several locations this would cause contact between the ship's keel and the mud layer, the effect of such a decision on a ship's behaviour should be investigated thoroughly and quantitatively. Indeed, a ship's behaviour changes significantly due to the presence of a mud layer, so that safety of shipping traffic requires that the ship's navigator must always be able to compensate for these effects. For this reason, it was decided to start a comprehensive research program, based on model tests and simulations.

The experimental program comprises systematic series of captive manoeuvring tests, carried out with two ship models in the "Towing tank for manoeuvres in shallow water" (co-operation Flanders Hydraulics Research – Ghent University), Antwerp, Belgium, the bottom of which is covered with mud simulating material. Based on the test results, mathematical models are developed for performing fast time simulation runs and full mission bridge simulations in varying bottom and under keel clearance conditions.

In this paper, a selection of results will be presented, focusing on the linear hull derivatives and derived dynamic stability indices. Some preliminary results have already been published in [5]; however, the data discussed in this paper may diverge slightly from the results published in [5] due to the use of another methodology for the calculation of hydrodynamic derivatives.

2. SHIP BEHAVIOUR IN MUDDY AREAS

Even if no contact occurs with the nautical bottom, a ship's behaviour may be affected by the presence of a mud layer, as a result of two kinds of phenomena:

- the rheology of the mud, which is particularly of importance if contact occurs between the mud layer and the ship's keel;
- the presence of a two-layer system, so that undulations are not only generated in the air-water interface, but also in the water-mud interface. This effect also may affect ship behaviour if no contact occurs.

A review of the state-of-the-art concerning the behaviour of ships in muddy navigation areas is given in [4] and [6]. Most information is based on experimental work from full-scale and model tests. So far, model tests have been carried out at MARIN (Wageningen, The Netherlands, 1976, [7]), Flanders Hydraulics Research (Antwerp, Belgium, 1986-1988, [8],[9],[10] and, more recently, 2002-2004) and SOGREAH (Grenoble, France, 1989, [11],[12]). These model tests were carried out under very different conditions, taking account of the type of mud simulating material and the applied test methods. Full scale observations took place in Rotterdam [13], Zeebrugge [14],[10] and the Loire estuary (Nantes – Saint-Nazaire) [11].

The effect of fluid mud on a ship's behaviour is related to the deformation of the interface caused by the pressure field around the moving hull. The vertical interface motions that were observed during both model and full-scale tests appear to depend on the ship's speed. At very low speed, the interface remains undisturbed (first speed range). At an intermediate speed (second speed range), an interface jump is observed under the ship's entrance, which at a certain section changes into an internal hydraulic jump moves aft with increasing speed. At higher speeds, the interface jump occurs abaft the stern (third speed range). This relation between speed and interface undulations is very clear in case of a positive under keel clearance relative to the mud-water interface; if the ship's keel penetrates into the mud, interference takes place with a secondary internal wave pattern. In general, the effect on ship behaviour is most important in the second speed range; the upper limit of this range can be estimated as follows [15], Figure 1:

\[
U_{\text{crit}} = \frac{8}{27} g h \left(1 - \frac{\rho_1}{\rho_2}\right)
\]

The effect of interface deformation on the propulsive properties of a ship is clearly illustrated by the relationship speed - propeller rate. In the second speed range, a given propeller rpm results in a significantly lower speed compared to a solid bottom situation; it appears to be difficult to overcome the critical speed. In the third speed range, the effect of the muddy bottom is practically nil. The transition between second and third speed range is very clear at an under keel clearance of 10 to 20 % of draft relative to the interface, but is smoother with decreasing water depth. This phenomenon should not merely be ascribed to an increased resistance; there
are even indications that contact between the ship's keel and the interface jump even may cause a resistance decrease, due to changes of relative velocity between the ship and the mud and water layers. The effect of the mud layer on the ship's performance is caused by obstruction of the flow to the propeller due to contact between the ship's keel and the risen interface, resulting into a very poor propulsive efficiency.

Concerning manoeuvring behaviour, the MARIN captive model test program [7] led to the conclusion that a ship becomes more sluggish if the under keel clearance is reduced, until the latter is about 4% of draft; further reduction makes the ship less sluggish. The effect of the presence of mud on manoeuvres appeared to be larger at low speed (3 knots) than on higher speeds (7 knots). In general, steady motions are slackened by the mud layers, while dynamic motions are accelerated: drift and rate of turn in a turning circle are lower, but overshoot in a zigzag test is smaller. Another phenomenon, observed during tests with self-propelled ship models at Flanders Hydraulics Research, concerns instability of rudder induced forces that takes place if the ship's keel is in contact with both water and mud. Finally, particular phenomena were observed during full-scale trials carried out in 1988 near Zeebrugge with a suction hopper dredger: the vessel, navigating with the keel in contact with a plastic consolidated mud layer became uncontrollable and tended to follow the 'easiest' way in the mud. At the same time, it was practically impossible to decrease the speed, although the latter was only 1 or 2 knots. It is clear that such behaviour is unacceptable for shipping traffic.

3. EXPERIMENTAL PROGRAM

3.1 TEST FACILITIES

Flanders Hydraulics Research, the hydraulic research station of the Waterways and Maritime Affairs Administration of the Ministry of Flanders, is particularly concerned with investigation of ship hydrodynamics for problems in relation with the concept, adaptation and operation of navigation areas. Therefore, the (very) shallow water range is a main research domain.

For the investigation of nautical aspects of these problems, a ship manoeuvring simulator has been installed. In order to provide the mathematical model of this simulator with realistic data, the availability of experimental facilities was considered as a requirement. At present these facilities consist of a shallow water towing tank (88 m * 7.0 m * 0.6 m), equipped with a planar motion carriage, a wave generator and an auxiliary carriage for ship-ship interaction tests. Thanks to computerised control and data-acquisition, the facilities are operated in a fully automatic mode.

3.2 SHIP MODELS

Two 1/75-scale models, a container carrier (D-model; $L_{pp} = 289.8$ m; $B = 40.25$ m; $T = 13.50$ m; $C_B = 0.59$) and a tanker, (E-model; $L_{pp} = 286.8$ m; $B = 46.77$ m; $T = 15.50$ m; $C_B = 0.82$) were selected. Most experiments have been carried out with the container carrier. Both ship models were equipped with a propeller and a rudder.

3.3 BOTTOM CONDITIONS

The mud is simulated by a mixture of two types of chlorinated paraffin and petrol, so that both density and viscosity can be controlled within certain ranges. For environmental reasons, the tank has been divided into three compartments: a test section, a "mud" reservoir and a water reservoir. Bottom and walls have been covered with a polyethylene coating.

The selected density-viscosity combinations and the tested bottom conditions are represented in Table 1. This selection was based on measurements of density and rheology profiles in situ carried out in the outer harbour of Zeebrugge in 1997-98. A mud layer configuration is defined by two characters: a letter ("b", …, "h"), denoting the material characteristics and a figure ("1", "2", "3"), representing the layer thickness. Tests carried out above a solid bottom are referred to as "S".

For the D-model the gross under keel clearance of the ship relative to the tank bottom was varied between 7 and 32% of draft, yielding an under keel clearance relative to the mud-water interface varying between –12 and +21%. For the E-model the values for the under keel clearance were extended between 10 and 15% of draft referred to the tank bottom, and from –10% to +10% relative to the mud-water interface.

3.4 TEST TYPES

For each combination of bottom type, bottom thickness and under keel clearance, a captive test program has been carried out for determining mathematical manoeuvring models covering a range of forward speeds between 2 knots astern and 10 knots ahead.
The experimental program consists of: bollard pull tests with varying rudder angle and propeller rate; stationary tests with varying forward speed, rudder angle, drift angle and propeller rpm; harmonic sway and yaw tests; multimodal tests with variable speed, rudder angle and/or propeller rpm.

During captive manoeuvring tests, following data are measured: longitudinal force components fore and aft, lateral force component fore and aft, vertical motion at four measuring posts (fore/aft, port/starboard), normal and tangential rudder force components, rudder torque, rudder angle, propeller torque, thrust and rpm. In particular cases, the vertical motion of the mud-water and water-air interfaces was registered as well.

4. LINEAR MANOEUVRING DERIVATIVES

4.1 EQUATIONS OF MOTION

In order to carry out fast and real time simulation runs in varying conditions, a comprehensive mathematical model that is valid for a wide range of forward and lateral speeds, yaw rates, rudder angles and propeller loadings has been developed for each tested situation. In this paper, however, an estimation of the manoeuvrability and controllability of the self-propelled ship at different bottom types is based on a linear set of equations of motions:

\[
\begin{align*}
(Y_v - m)u + (Y_f - mx_G)\dot{v} + Y_{uv}uv + (Y_{ur} - m)ur + Y_{bou}u \delta u^2 &= 0 \\
N_v \dot{v} + (N_f - I_{zz}) \dot{v} + N_{uv}uv + (N_{ur} - mx_G)ur + N_{bou}u \delta u^2 &= 0
\end{align*}
\]

The values for the hull derivatives displayed in the following paragraphs are based on harmonic ("PMM") sway and yaw tests carried out at speeds of 2 to 6 knots (full scale). The control derivatives resulted from multimodal test carried out with constant speed and rpm, but varying rudder angle, and are valid in self-propulsion conditions.

4.2 INERTIA DERIVATIVES

Figure 2 shows a selection of results of harmonic sway tests. The added mass for sway motion increases significantly with decreasing water depth and increasing density and viscosity of the mud layer, and takes very large values (even seven times the ship’s mass for the D-model) in case the ship’s keel penetrates deep into the mud. The mud characteristics and the layer thickness appear to be important parameters, even if no contact occurs with the mud layer: the shallow water effect is smoothened with increasing layer thickness and decreasing mud density and viscosity. Indeed, an abrupt transition cannot be observed at \(h/\lambda = 1\). It should be noted that the results of tests carried out with layers of rather high viscosity and density can be considered as an extrapolation of results above a solid bottom. Similar conclusions can be drawn for the yaw inertia.

4.3 VELOCITY DERIVATIVES

The magnitude of lateral force and yawing moment due to drift increases significantly with decreasing water depth. This is illustrated in Figure 3, displaying the sway velocity derivative \(Y_{uv}\) in function of water depth to draft ratio for several bottom conditions. However, \(Y_{uv}\) appears to reach a maximum for zero under keel clearance relative to the mud-water interface. Especially for the container carrier D, the presence of a mud layer results into an increase of the lateral force due to drift. This is not the case for the drift induced yawing moment, as is shown in Figure 4: the presence of a mud layer results into a decrease of \(N_{uv}\). The latter reaches a maximum if the keel touches the mud layer.

The evolution of the yaw velocity induced lateral force and yawing moment derivatives is of particular interest. The magnitude of the yaw damping moment derivative \(N_{uv}\) gradually increases with decreasing under keel clearance and stagnates once the ship’s keel touches the mud layer, see Figure 5. The hydrodynamic lateral force due to the yaw rate \(Y_{ur}\), which in deep water is practically negligible compared with the centrifugal inertia force \(-mur\), is of increasing importance and counteracts the centrifugal inertia force completely at extremely small positive under keel clearances in this specific case, as shown in Figure 6. For smaller and negative under keel clearances, the resulting lateral force due to yaw is even centripetal. The transition from centrifugal to centripetal action takes place at larger values of the under keel clearance when the density and viscosity of the mud layer increase and the thickness of the layer decreases. Therefore, this effect is not to be considered as a typical characteristic for ship behaviour in muddy areas, but rather as a (very) shallow water effect.

4.4 CONTROL DERIVATIVES

The linear coefficients for the sway force induced by rudder action at self propulsion conditions for the models are given in Figure 7. These characteristics are greatly affected by the resistance and propulsion performance; for this reason, the propeller rate required to reach a forward speed of 6 knots is displayed in Figure 8.

For mud layers with higher density, the control derivatives increase considerably in case of contact between the ship’s keel and the mud layer, due to the higher propeller loading required to overcome the increased resistance. In contact with mud layer g3, maximum propeller loading even appeared to be insufficient for the E-model to reach a speed of 2 knots.

Tests carried out with lower density mud layers resulted into a more gradual transition between positive and
negative under keel clearance. Near \( h_1/T = 1 \), fluctuations can occur, due to the effect of internal wave patterns.

5. CONTROL AND MANOEUVRING

5.1 STRAIGHT-LINE STABILITY

The linear equations of motion (2) lead to following values for the straight-line stability indices:

\[
\sigma_{1,2} = \frac{-B \pm \sqrt{B^2 - 4AC}}{2A}
\]

using following notations:

\[
A = (Y_v - m)(N_v - I_{zz}) - (Y_v - m x_G)N_v
\]
\[
B = Y_{uv}(N_v - I_{zz}) + (Y_v - m)(N_{uv} - m x_G)
\]
\[
C = Y_{uv}(N_{uv} - m x_G) - (Y_{uv} - m)N_{uv}
\]

\( A \) is the part of the stability indices is negative. \( A \) and \( B \) being positive, this is the case if \( C > 0 \). As can be observed in Figure 9, a decrease of under keel clearance results into a significant increase of \( C \). Ship model \( D \), which is slightly unstable and marginally stable at under keel clearance values of 32% and 26% of draft, respectively, appears to be extremely stable if the under keel clearance reaches very small and negative values. If the ship penetrates into the mud, the straight-line stability criterion \( C \) takes the largest values for the mud layers with high density and viscosity. Fluctuations occur near \( h_1/T = 1 \).

Another particularity concerns the sign of the discriminant \( d \): Figure 10 shows that \( d \) is negative at still lower water depth. This transition takes place at lower under keel clearances when the density of the mud layer decreases.

The criterion for straight-line stability can therefore be formulated as follows: \( x_r > x_v \) if \( Y_{uv} - m < 0 \); \( x_r < x_v \) if \( Y_{uv} - m > 0 \). In deep water, the centrifugal inertia force is dominating, so that \( C > 0 \) is fulfilled if \( x_r > x_v \). This is not the case, however, in very shallow water, including negative under keel clearances.

5.2 RESPONSE TO RUDDER ACTION: STEADY STATE

A steady-state solution for the system (2) of the equations of motion can be obtained if \( \dot{\theta} = \dot{r} = 0 \), so that for following values for \( \nu \) and \( r \) can be calculated:

\[
\begin{align*}
\nu & = \delta Y_{\delta uu} Y_{\delta uu} - \nu_{\delta uu} Y_{\delta uu} \nu_{\delta uu} \\
\rho r & = \delta Y_{\delta uu} Y_{\delta uu} - \nu_{\delta uu} Y_{\delta uu} \nu_{\delta uu} \nu_{\delta uu} \\
& = \frac{\delta Y_{\delta uu} x_r - x_v}{Y_{\delta uu} x_r - x_v} \quad (8)
\end{align*}
\]

\( \delta \) is the longitudinal co-ordinate of the application point of the rudder induced lateral force, which is located aft of amidships for all tested conditions, see Figure 11. Indeed, \( Y_{\delta uu} = 0 \) and \( N_{\delta uu} < 0 \), so that \( \delta < 0 \).

The results of (8-9) are displayed in Figure 12. For the yaw rate, it can be concluded that for a ship with straight-line stability, which is the case in all, except one, tested conditions, the resulting steady-state value always has the expected sign, i.e. opposite to the rudder angle. Indeed, \( Y_{\delta uu} = 0 \) (\( \delta = 0 \). At small positive under keel clearance, the yaw rate appears to reach a minimum.

The sway velocity \( (\nu/u = -\tan \beta) \) takes the sign of the rudder angle, implying that the ship’s bow is located within the turning circle – which can be considered as a normal situation – in following cases: \( x_r < x_v \), hence \( \beta \) is positive. At extremely low water depth, however, the application point of the yaw induced lateral force moves forward which leads to a very small, but still positive drift angle. However if the asymmetry of the propeller is taken into account the resulting drift angle can have a sign change in some situations, which means that the ship’s bow is located outside the turning circle.

Figure 13 illustrates the force balance in the different situations. Due to the evolution of yaw rate and drift
angle, the pivoting point moves aft with decreasing water depth, as shown in Figure 14.

5.3 RESPONSE TO RUDDER ACTION: TRANSIENT MOTION

The effect of bottom characteristics and under keel clearance on the hydrodynamic derivatives not only influences the steady-state response; the transient time history is affected as well. Especially the drift angle appears to be subject to oscillations, as is clearly illustrated in Figure 15.

6. CONCLUSION

Captive manoeuvring tests with ship models above and in simulated mud bottoms revealed an important change of controllability and manoeuvring behaviour at very low and negative under keel clearance. The straight-line stability increases very significantly, while the rudder induced yaw rate decreases to reach a minimum at very small positive under keel clearance. Contact with the mud layer yields a slight increase of the yaw rate, due to the higher propeller loading required to overcome the increased resistance.

Extreme under keel clearance conditions clearly influence all linear hydrodynamic derivatives, especially with the container carrier but ship dynamics appears to be affected particularly by the evolution of the yaw induced lateral hydrodynamic force, which appears to counteract and even dominate the centrifugal inertia force. In a steady-state turn, one of the results is a decrease of the drift angle, which may even change sign at negative under keel clearance.

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Figure 1. Transition speed between second and third speed ranges as a function of mud-water density ratio and water depth.

Figure 2: Sway added mass: influence of bottom characteristics and under keel clearance.

Figure 3: Linear sway velocity derivative for lateral force: influence of bottom characteristics and under keel clearance.

Figure 4: Linear sway velocity derivative for yawing moment: influence of bottom characteristics and under keel clearance.
Figure 5: Linear yaw velocity derivative for yawing moment: influence of bottom characteristics and under keel clearance (see Figure 7 for legends).

Figure 6: Linear yaw velocity derivative for lateral force: influence of bottom characteristics and under keel clearance.

Figure 7: Linear control derivative for lateral force: influence of bottom characteristics and under keel clearance.

Figure 8: Propeller rpm required to reach a forward speed of 6 knots.
Figure 9: Straight-line stability criterion "C": influence of bottom characteristics and under keel clearance.

Figure 10: Discriminant of quadratic equation for stability indices: influence of bottom characteristics and under keel clearance.

Figure 13: Steady-state response to rudder action: force balance.

Figure 14: Steady-state response to rudder: position of pivoting point. Influence of bottom characteristics and under keel clearance.
Figure 11: Model D. Application point of lateral force due to sway (x_v/L: △), yaw (x_r/L: O), rudder action (x_δ/L: □) Open symbols: mud; full symbols: solid; Mud layer thickness: 0.75 m (−−−), 1.50 m (----), 3.00 m (−−−).

Figure 12: Steady-state response to rudder action: yaw rate, drift angle. Influence of bottom characteristics and under keel clearance.
Figure 15. Ship model D. Transient response to rudder action: time history of yaw rate (left scale: $\delta / u$, $\text{deg}$, $-$) and drift angle (right scale: $\beta / \delta$, $\text{deg/deg}$, $-$–$-$) for selected bottom characteristics and under keel clearances (figure continuing at following page).