Measurement and modelling of the properties of cohesive sediment deposits

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Research studies undertaken as part of the “Bed Dynamics” Task D of the EC funded COSINUS project are described. The studies undertaken involve the reformulation of sediment exchange equations, in situ field measurements of bed strength, laboratory settling column experiments, bed consolidation modelling, the development of a model of bed dynamics based on generalised Biot theory and the testing of an integrated erosion/entrainment model against laboratory experiments. The results of the various studies are synthesized and overall conclusions drawn.

KEY WORDS
cohesive sediment, erosion, bed strength, consolidation, flocculation, numerical modelling.

1. INTRODUCTION

This paper summarises work undertaken as part of Task D of the European MAST3-COSINUS Project. The aim of Task D on “Bed Dynamics” was to provide a greater understanding of the development of cohesive sediment beds through various laboratory and field based measurements and numerical model development. The work was carried out by a number of organisations – Delft University of Technology, Delft Hydraulics, Katholieke Universiteit Leuven, HR Wallingford, Oxford University and the University of Wales, Bangor. This paper briefly describes the key points of each of the separate studies and attempts to synthesise the results of the various studies by drawing overall conclusions from the results.

2. BACKGROUND

The modelling of cohesive sediment transport in estuaries and coastal areas requires a description of the sediment exchange with the bed through the processes of deposition and erosion. The sediment transport (or sediment mass balance) equation can be written as:
\[ \frac{\partial \phi}{\partial t} + U \frac{\partial \phi}{\partial x_j} = \frac{\partial}{\partial x_j} \left( \varepsilon \frac{\partial \phi}{\partial x_j} \right) - \frac{\partial}{\partial z} \left( w \phi \right) \]  

(1)

where: \( \phi \) = concentration by volume of the sediment particles, \( \varepsilon \) = the eddy diffusivity, \( w \) = the settling velocity, \( U \) = flow velocity. At the bottom the boundary condition reads:

\[ \varepsilon \frac{\partial \phi}{\partial z} - w \phi = S_0 = S_e - S_d \]  

(2)

i.e. the sediment exchange flux \( S_0 \) consists of two contributions: the upward flux \( S_e \) and the deposition flux \( S_d \), which is (proportionally) related to the settling flux, i.e. \( S_d = p_D w \phi \), where \( p_D \) = the fraction which effectively becomes part of the bed. Both fluxes are generally described by an empirical relationship which evaluates the bottom shear stress of the flow \( \tau_0 \) against a critical shear stress (e.g. Teisson, 1997).

In practice, the bed exchange module is the weakest part in cohesive sediment transport modelling. There are many reasons for this. One of the fundamental problems is the lack of a general relationship between erosion strength and bed shear strength. But even if the fundamental questions regarding the modelling of sediment exchange between water column and bed were resolved, the fact remains that the bed properties vary in time and space (both horizontally and in depth), for instance due to variations in sediment composition and biological activity.

Although the change of bed level as a result of consolidation is of interest, it is usually less important than the effect of consolidation on increasing the resistance of the bed to erosion. Consolidation, especially over short periods (hours to days), can have an important influence on the behaviour of applied models as this influences whether deposits at slack water or on neap tides can then be re-eroded during periods of faster currents. Effects of consolidation can be taken into account in a number of ways. Many approaches involve a simplified representation of the vertical structure of the bed. At its simplest, this may be only two layers: a surface, unconsolidated layer, with a weak resistance to erosion, and a more consolidated layer below. The time variation in bed properties as a result of consolidation is represented by introducing fresh deposits into the weak top layer, and then gradually transferring that material to the consolidated layer (e.g. Odd & Cooper, 1989; Kusuda & Futawatari, 1992). Different values of the erosion parameters and density are assigned to each layer. A characteristic consolidation time must be specified to determine the rate of transfer of material from one layer to the next.

This general approach can be extended to a multi-layer representation of the vertical bed structure and consolidation effects. This is simple and quick to calculate. A weakness is the discontinuous representation of bed properties and of course the representation of the consolidation process is very crude, in particular with regard to the relationship between density and strength, which are directly related to the bed structure and its history.

Another weakness in these bed models is their inability to account for weakening of the bed by wave induced forces, which under extreme conditions (e.g. storms) may even cause complete loss of erosion strength by liquefaction (shear-induced structural break-up) or fluidisation (pore pressure induced break-up) of part of the bed.
3. FORMULATION OF SEDIMENT EXCHANGE AT THE BED

The manner in which exchange of sediment at the bed is currently characterised in sediment transport models has been investigated by the Katholieke Universiteit Leuven (KUL). These investigations have identified ways in which both the deposition and erosion processes can be more effectively represented. A brief summary of this work is given below but the work is presented in more detail in Toorman (2000).

In practice, the critical shear stress for deposition is a tuning parameter of the model. According to Sanford & Halka (1993), numerical models perform better when no threshold is considered for deposition. This makes even more sense if one considers the fraction of the settling flux \( w_s \phi \) which does not stick to the bed as a part that is immediately "eroded" (Toorman, 2000). It is then possible to include the fraction that does not stick to the bottom in the erosion flux, i.e.:

\[
S_e = S_e - S_b = [S_e + (1 - p_b)w_s \phi] - w_s \phi = S'_e - w_s \phi
\]

A typical erosion rate equation (for surface erosion) is of the form:

\[
S_e = E_0 \left( \frac{\tau_s}{\tau_e} \right)^n - 1 \quad (\tau_s > \tau_e)
\]

(4)

with \( \tau_s \) the critical stress for erosion. The erosion rate parameter \( E_0 \) is expected to be proportional to the bed surface (e.g. volumetric) concentration, as the amount that can be eroded cannot exceed the available amount. The critical stress for erosion is empirically related to a measure of the shear strength of the bed, often the vane shear strength. Toorman (1995) proposed an alternative formulation for erosion strength:

\[
\tau_e = a \left( e^{e^{C_e}} - 1 \right) \quad (C > C_s)
\]

(5)

which accounts for the fact that there is no structure below the space-filling concentration \( C_s \), which makes this form physically more realistic. If the non-sticking fraction of depositing particles should be included into the erosion law, a contribution without critical erosion stress should be added.

Besides the difficulty in determining the critical stress for erosion, the calculation of the correct bottom shear stress is also crucial, because of buoyancy induced drag reduction. The traditional method in numerical models, which is based on wall functions for homogeneous flow, in the case of a fixed shear velocity overestimates the bed shear stress with increasing sediment load up to a factor of 3 at the saturation condition, whereas a consistent approach, which corrects the near-wall boundary conditions for buoyancy effects, yields the correct value (Toorman, 2002).

4. FIELD MEASUREMENTS OF BED STRENGTH

A field measurement campaign was carried out in September 1998 at Calstock in the Tamar Estuary. The measurements concentrated on suspended sediment properties and hydrodynamics, but HR Wallingford (HRW) and University of Wales, Bangor (UWB) were also involved in measuring properties of the sediment deposits. HRW measured particle size distributions and the resistance to erosion of the sediment deposits exposed on the inter-tidal banks at low water. The sediment was predominantly mud, with a median grain size in the range 10-20 microns and loss on ignition measurements were
between 8% and 14%. The water content of the deposits was high and the deposits were accordingly very weak, often fluid. The slope of the inter-tidal banks was steep, at around 10-20°.

Because of the weakness of the sediments and the site conditions the SEDERODE instrument deployed by HRW was unsuccessful at all but 2 of the 16 sites. The critical erosion shear stresses measured were 0.1 Pa and 0.21 Pa for sediments with surface bulk densities of 1230 kg/m³. The sand content of the surface sediment was about 13% in both cases and the organic content (by loss on ignition) also about 13%.

UWB measured acoustic shear wave velocity in situ at a series of 5 intertidal locations near Sites A and B, on both neaps and springs, using paddle-shaped piezoelectric transducers embedded in the surface sediment (Jones et al., 1993). Acoustic shear wave velocity is a measure of bulk sediment rigidity. No significant difference was found between locations or between springs and neaps. The mean over all sites was 48 m/s. The high degree of variability (31-69 m/s) obtained is indicative of high porosity, low rigidity muds.

UWB also deployed a multi-corer to retrieve five 100 mm diameter cores of up to 210 mm in length from Site A. These were transferred into an instrumented column for measurement of acoustic shear wave velocity and electrical formation factor (Wren, 1996). The electrical formation factor (defined as the ratio of electrical resistivity of the bulk sediment to the electrical resistivity of the pore fluid) is a measure, for a given packing configuration, of the sediment porosity (Jones et al., 1993). In addition, some of the cores were sectioned into 20 mm slices to examine the vertical variation in bulk density.

All five cores exhibited a bioturbated surface layer 60 – 70 mm in thickness overlying a more uniform subsurface layer. Beneath this, bulk density was found to increase down each core and this corresponded with increases in electrical Formation Factor and shear wave velocity. Bulk density and organic matter were similar to those determined at nearby inter-tidal sites although shear wave velocities were significantly higher (107-157 m/s). This may be explained by the fact that transducers of higher resonant frequency were used for the cores.

The main conclusion from the UWB measurements is that physical properties of surficial sediments were not found to vary significantly between Sites A and B or between neap and spring tides, and that depth variation was negligible within the mobile surficial layer. So erosion rate parameters used in the models, which depend on physical properties of the deposits, can be assumed to be constant.

5. SETTLING COLUMN EXPERIMENTS

Laboratory settling column experiments were carried out at both the Delft University of Technology (DUT) and the University of Oxford (UOX).

The DUT experiments concentrated on the consolidation process and examined the variations of density and vane shear strength with time and depth below the sediment surface. Two types of natural mud were used: Caland-Beer mud (from the entrance channel of the Port of Rotterdam) and Dollard mud. The mud beds were allowed to consolidate in short (0.3m) and tall (1.5m) settling columns. The measured parameters for the consolidation process were density and pore water pressure. In order to make accurate strength measurements, segmented settling columns were designed and built.
The segmented columns provided well-defined samples of the bed that were suitable for strength measurement by shear vane testing. Three types of shear vane test were carried out, namely rate controlled, stress controlled and oscillatory rate controlled. Since the shear vane tests are destructive, each series of experiments consisted of multiple settling columns that were set up identically, so that the strength development with time could be monitored.

Data from settling column tests are generally processed in order to obtain empirical relationships of effective stress and of permeability as a function of density. It is now qualitatively understood why these relationships are not unique, but show time-dependence, which is related to the histories of floc and bed structures, depending on the forcing (Sills, 1995; Toorman, 1999). Besides this physical aspect, the accumulation of experimental error in the traditional data processing method contributes to the difficulty in interpretation of the data. A new data processing method has therefore been developed at the Katholieke Universiteit Leuven, based on filtering of errors by using analytical smoothing functions. Simple analytical functions have been derived which give a good approximation of the excess pore water pressure profiles and the constant mass contours in the settling curve plot. The resultant mass gradients are used for the determination of the permeability. This method allows a significant reduction in the error involved in the calculation of the pore pressure gradient and the filtration rate. The method has been applied to experimental data of DUT. Further details can be found in Toorman & Leurer (2000a).

The UOX settling column experiments concentrated on the relationship between the density and strength of the bed and the way in which the deposits are formed. Deposits formed from a slurry were compared with those formed by slow, steady deposition. In the steady deposition experiments a flocculation chamber was used to control the properties of the settling flocs and the effect of floc size and density on the properties of the deposit were investigated.

### Table 1
Tamar sediment property tests measured in the UOX tests

<table>
<thead>
<tr>
<th>Sample</th>
<th>Particle density (Mg/m³)</th>
<th>Organic content (% by mass)</th>
<th>Median diameter, D₅₀ (µm)</th>
<th>Clay content (% conc.)</th>
<th>Silt content (% conc.)</th>
<th>Liquid limit</th>
<th>Plastic limit</th>
</tr>
</thead>
<tbody>
<tr>
<td>top</td>
<td>24.6</td>
<td>23</td>
<td>5</td>
<td>81</td>
<td></td>
<td>63.3</td>
<td></td>
</tr>
<tr>
<td>mid</td>
<td>19.7</td>
<td>20</td>
<td>5</td>
<td>85</td>
<td></td>
<td>65.2</td>
<td></td>
</tr>
<tr>
<td>bottom</td>
<td>20</td>
<td>22</td>
<td>5</td>
<td>83</td>
<td></td>
<td>63.7</td>
<td></td>
</tr>
<tr>
<td>average</td>
<td>2.570</td>
<td>21</td>
<td>21.6</td>
<td>83</td>
<td>88</td>
<td>64.1</td>
<td></td>
</tr>
</tbody>
</table>

Sediment collected from Calstock in the Tamar estuary during the field measurement campaign was used in the UOX experiments. Repeated grain size measurements using a CILAS 920 laser particle sizer indicated that the collected sample had a median grain diameter of 22 µm. This is slightly higher than the 10-20 µm median grain diameter reported in the field. Organic content analyses were conducted according to Head (1992), using a hydrogen peroxide decomposition method, and revealed a mean of 21% organic content by mass. Again, this is higher than the 8 to 14% reported in the field.
using a loss on ignition technique. Other parameters measured are reported in Table 1, including particle density, clay and silt content and liquid and plastic limits. The sediment has a British Standards classification as a high plasticity clayey silt.

Details of the instrumentation are given in Lintern (2000). Methods include the use of floc video imaging technology with a laser light source, and indirect density measurement using a non-destructive X-ray technique described in numerous reports (Been, 1980; Been 1981; Sills, 1997; Sills, 1998). Pore water pressures were measured using a technique and apparatus originally developed by Bowden (1988).

Figure 1 shows results of the floc measurements for the experiment COS6. Due to variation in floc shape, the floc size is reported as equivalent spherical diameter (ESD)—the diameter of a sphere that occupies the same volume as the imaged floc. The median floc size (100 μm) is significantly larger than the mean primary particle diameter for Tamar sediment. Floc velocities are calculated from a sequence of images, and using these velocities in a modified form of Stokes' velocity equation the effective floc densities can be calculated.

The experiments demonstrated significant differences in the properties of a bed formed by settlement from a slurry by comparison with one formed by steady sedimentation. The latter show a much higher degree of aggregation than those sedimented from slurries. Furthermore, the flocculated beds contain larger aggregates than the flocs in the water column which formed them, thus indicating aggregation is continuing during bed development.

![Figure 1. Effective density vs. equivalent spherical diameter for experiment COS6 flocs.](image)

Under self weight consolidation, bed densities range from 1.15 to 1.20 Mg/m³ at the surface to values above 1.3 Mg/m³ at depths of 10 cm or more. The flocculation conditions clearly affect the density of the beds. Figure 2 shows density profiles for experiment COS1, in which the sedimentation concentration was varied throughout the experiment. The peaks in density correspond to stages of high sedimentation rates.
(above 3 g/l), and the troughs are formed during low sedimentation rates (down to 0 g/l). The profiles show that the sedimentation conditions lead to higher variations than either consolidation time, or depth of burial for these self weight experiments. Shear wave velocities ranged from 2 to 30 m/s, increasing with consolidation time. Rigidity moduli calculated from these range between approximately 1 and 5 kPa.

![Density profile graph](image)

Figure 2. Density profile at the end of consolidation in experiment COS1.

The experiments were designed to simulate mud in its natural state from the Tamar Estuary. The biological components of the mud were not removed for most of the experiments. Within days after the end of sedimentation the bed surfaces became covered with a biological layer. Microscopic analysis showed an abundance of diatoms (many dormant) and other mobile organisms. In one experiment worm burrows became apparent from the outset. Observations show that these worm burrows greatly enhance the settling by providing channels for water to escape upward, and for particles to move downward. The worms then feed on the surface, where they alter the surface properties by pelletizing the sediment. Other biological factors are also at work in the surface sediment. Density profiles often show a layer of low density 4-15 mm below the surface sediment. Such a layer is thought to arise due to biological activity, and most probably gas production. This layer is found to be rich in diatoms and other algae.

Nine experiments were carried out using the in-situ erosion device ISIS developed by HR Wallingford (Williamson & Ockenden, 1993). The ISIS work and the properties of the biological surface layers are discussed further in a separate paper (Lintern et al., this volume). The ISIS measurements have been made on beds settled from slurries of Tamar mud. The bulk densities of surface of the beds tested were typically about 1.2 Mg/m³, which is similar to that measured by HRW during the field measurements. Erosion of the bed in the laboratory appears to commence at a similar applied bed shear stress as that in the field, generally around 0.01 Pa.
6. BED DYNAMICS MODELLING

6.1. Consolidation and strength modelling

Delft University of Technology (DUT) modelled the process of consolidation as a one-dimensional process using the Gibson equation (Gibson et al., 1967) written in an Eulerian reference frame and with the particle volume fraction as the dependent variable:

\[
\frac{\partial \phi_v}{\partial t} = \frac{\rho_s - \rho_w}{\rho_w} \frac{\partial}{\partial z} \left( k \phi_v^2 \right) - \frac{1}{\rho_w g} \frac{\partial}{\partial z} \left( k \phi_v \frac{\partial \sigma'}{\partial z} \right) = 0
\]

where \( \phi_v \) is the solids volume fraction, \( \rho_w \) the density of water, \( \rho_s \) the density of solids, \( g \) acceleration due to gravity, \( k \) the permeability, \( \sigma' \) the effective stress, \( z \) the vertical coordinate (positive in upward direction) and \( t \) time.

New constitutive equations for effective stress and permeability are derived on the basis of the concept of a scale-invariant bed structure (Merckelbach & Kranenburg, 2000). It is assumed that the volume filling network structure is built by aggregates that consist of clay and silt particles. The structure of the aggregates is assumed scale-invariant. This assumption may be regarded as a generalisation of Krone's concept of orders of aggregation (Krone, 1963). The following relationship between the length scale of an aggregate and the solids volume fraction can be established:

\[
R_a^{n_f/3} \propto \phi_v
\]

where \( R_a \) is the length scale of the aggregates and \( n_f \) the fractal dimension. During consolidation, excess pore water pressure is transferred to effective stress. The effective stress is assumed to relate to the number of critical bonds within an aggregate. In accordance with the concept of scale invariance, the number of critical bonds per aggregate is independent of the size of the aggregate. Consolidation may be regarded as a condition in which the effective stress is the maximum effective stress that can exist in a network structure. Hence, an increase of effective stress must result in an increase of the number of bonds per unit area. This is achieved by a break-up of aggregates and a reduction in the length scale of the aggregates.

Assuming a linear relationship between the effective stress and the number of bonds as suggested by experimental data presented by Mitchell (1976) and a constant number of bonds per aggregate results in the constitutive equation for effective stress:

\[
\sigma' = K_o \phi_v^{\frac{2}{3-n_f}}
\]

where \( K_o \) is an empirical parameter which includes shape effects and the size of clay particles, for example. This relationship, however, does not include time dependency effects, which may play a significant role (Toorman, 1999).

Assuming that the pore water can be modelled as Poiseuille flow and that the size of the virtual tubes is proportionally related to the size of the aggregates, the following relationship for the permeability can be obtained:
\[ k = K_k \phi_r^{3-n_r} \]  \hspace{1cm} (9)

where \( K_k \) is an empirical parameter which includes shape effects and the size of clay particles, and also the viscosity of water.

These new constitutive equations relate the effective stress and permeability to the volume fraction of solids. Effective stress and permeability turn out to be related through the fractal dimension.

Strength may be regarded as resistance against failure. It is assumed that the bed strength is generated by intra- and inter-aggregate particle bonds. The concept of scale invariance implies that the number of intra-aggregate bonds per aggregate is independent of the aggregate size and that the number of inter-aggregate bonds per aggregate is proportional to \( R_n^{\alpha-1} \). Similar to the procedure followed for the effective stress, the critical shear stress can be expressed in terms of the critical shear stress generated by intra- and inter-aggregate particle bonds, which gives the failure criterion:

\[ \tau_c = k_1 \phi_r + k_2 \sigma' \]  \hspace{1cm} (10)

![Figure 3. Measured and calculated density profiles for experiment CT.](image)

The empirical coefficients \( k_1 \) and \( k_2 \) account for inter- and intra-aggregate bond strengths and number of bonds per aggregate. This criterion resembles the Mohr-Coulomb criterion,

\[ \tau_c = c' + \tan(\phi') \sigma' \]  \hspace{1cm} (11)

where the true cohesion \( c' \) is given by \( c' = k_1 \phi_r \), and the angle of internal friction \( \phi' \) by

\[ \tan(\phi') = k_2. \]  

The failure criterion can be used in strength modelling. In an accompanying paper (Merckelbach et al. 2000) this is elaborated further by applying the failure criterion to a shear vane test model.
One of the DUT settling column consolidation experiments was simulated using the consolidation equation with the new constitutive equations. The experiment was carried out using Caland-Beer mud. The initial conditions were $\rho_i = 1070 \text{ kg/m}^3$ and $h_i = 1.53 \text{ m}$. The simulation was carried out using $n_f = 2.75$, $K_o = 3.2 \text{ MPa}$ and $K_k = 2.9 \times 10^{-5} \text{ m/s}$. The results are shown in Figure 3 for 9, 24, 58 and 95 days of consolidation. Figure 3 shows a good correspondence between the measured and computed density profiles, which enhances the confidence in the newly derived constitutive equations.

The results of the shear vane test model (Merckelbach et al. 2000) are shown in Figure 4. A good agreement between the measured and modelled strength profiles is observed. The absolute deviation between measured and modelled yield stresses can be as large as 50 Pa for the lower part of the bed on day 9 and day 24, but with respect to the absolute values of the yield stresses, this is an error of maximally 33 %.

![Figure 4. Measured and calculated yield strength profiles for experiment CT.](image)

### 6.2 Generalised bed dynamics modelling

Traditional bed models, such as the previous one, only describe the strengthening of the sediment bed by self-weight consolidation. Sediment beds in nature are also subjected to shear forces by currents and waves and oscillating pore water pressures due to waves. At KUL, a model of bed dynamics has been developed based on the generalised Biot theory for saturated porous media, which offers a holistic framework to simulate the combination of all the processes in the bed, i.e. consolidation, liquefaction and fluidisation. It addresses the development in time of density and strength of the bed and can include effects of thixotropy and creep. The major difference with the previous geotechnical model is the replacement of the empirical normal effective stress-void ratio relationship by a rheological model which relates stresses to strains and strain rates. The model can simulate extremely large deformations of fresh mud deposits by implementation of the arbitrary Euler-Lagrange method. Subsequent changes in density and permeability are accounted for.

The model actually combines numerical methods applied in the generalized Biot theory for saturated porous media and in creeping non-Newtonian fluid mechanics, implemented within a mixed Eulerian-Lagrangean co-ordinate system. The model solves the sediment mass balance, stress balance, pore water continuity and a
rheological closure relationship. The equations are solved iteratively in three uncoupled groups with unknowns being the bulk density of the bed, solids displacement, pore water pressure and stresses, using the finite element method. In principle, various rheological models can be incorporated, from pure elastic to generalized elasto-plastic and creep, including non-linear material properties. The large deformations and highly non-linear material behaviour pose severe problems regarding numerical stability which are still to be overcome. Thus far, only the idealised case of consolidation of a pure visco-elastic porous material could be simulated without numerical instabilities, already showing that the relationship between effective stress and density is not unique, but time-dependent as expected (Figure 5).

![Figure 5](image_url)

Figure 5. Simulated time evolution of the density (left) for the consolidation of an idealised saturated visco-elastic soil skeleton with initial bulk density $\Delta_0 = 1110 \text{ kg/m}^3$, shear modulus $G = 1000 \text{ Pa}$ and viscosity $\theta = 100 \text{ Pa.s}$. Right: corresponding effective stress versus excess density $\rho - \rho_w$ (full lines; dotted lines = pure elastic case, $\theta = \infty$).

The model is not suitable for large-scale 3D applications, but is intended for use as a research tool to better understand the dynamic behaviour of cohesive sediment beds. Further details on this model can be found in a separate report by Toorman et al. (2000).

7. INTEGRAL EROSION/ENTRAINMENT MODEL

Modelling work was carried out at Delft Hydraulics, where a 1DV model was used to simulate flume experiments carried out at DUT (Winterwerp and Kranenburg, 1997). These experiments simulated the chain of processes through a tidal cycle consisting of settling, hindered settling, fluid mud formation, consolidation and re-entrainment. Consolidation of the mud layer was modelled with Equation (6) and its strength as a Bingham plastic, its parameters being derived from a fractal description of the mud flocs (as described in the section on consolidation and strength modelling by DUT). It appears that the prediction of the vertical concentration profile in the consolidating mud layer is at present the weak link in simulating this chain of processes. One probable cause is segregation of the fine and coarser fraction during the settling process.
One of the advantages of the aforementioned modelling of the consolidation process in Eulerian co-ordinates using the fractal theory (power law model of material functions) is that the Gibson equation evolves into an advection-diffusion equation for the sediment concentration. This concept was implemented in the 1DV POINT MODEL of Delft Hydraulics to describe consolidation around slack water as part of the settling and mixing processes during successive tidal cycles (Winterwerp, 1999). The resulting mass balance equation accounts for the effects of molecular diffusion ($D_s$), turbulent mixing (eddy diffusivity $F_T$), hindered settling ($f_{hs}$-function) and consolidation ($f_c$ and $\Gamma_c$-function) and reads:

$$\frac{\partial c}{\partial t} - \frac{\partial}{\partial z} \left( X_c \frac{\partial c}{\partial z} \right) - \frac{\partial}{\partial z} \left( D_s + \Gamma_c + \Gamma_r \right) \frac{\partial c}{\partial z} = 0$$

where:

$$X_c = f_n + \frac{f_c}{1 + \eta f_c}, \quad \text{with} \quad f_n = \frac{1 - \phi}{1 + 2.5 \phi} \quad \text{and} \quad f_c = k \frac{\rho_s - \rho_c}{\rho_w} \phi_p,$$

$$\Gamma_c = \frac{2 K_s K_p}{(3 - n_f) \phi_k}, \quad k = K_k \phi_p \left( \frac{\phi_c}{\phi_k} \right)^{\frac{3 - n_f}{2}},$$

and where $c$ is the sediment concentration by mass, $\phi$ the volumetric concentration of the flocs ($\phi = c/c_{gel}$), $c_{gel}$ is the gelling concentration at which a space-filling network forms (also referred to as the structural density), $\phi_p$ the volumetric concentration of the primary mud particles, $\phi_c = \min \{1, \phi\}$, $w_{s,t}$ is a reference settling velocity, and $\eta$ a parameter ($\eta = 10^5 \text{ s/m}$).

This model was used to simulate settling, consolidation and remixing measured in a rotating annular flume (Winterwerp and Kranenburg, 1997). After some trial and error, the best results were obtained for $n_f = 2.71$, $c_{gel} = 100 \text{ g/l}$, $K_k = 1 \cdot 10^{-14} \text{ m/s}$ and $K_\sigma = 1 \cdot 10^9 \text{ Pa}$, the results of which are presented in Figure 6. This figure shows a reasonable agreement between the measured and computed concentration profiles; however the large concentrations near the base of the profile are not properly predicted, which also affects the concentrations higher in the profile. This deviation is probably caused by the segregation of fine and coarse material that was observed during the experiments.
Further improvement can be obtained by including a second (sand) fraction in Equation (12). This is elaborated in Winterwerp (2002). Note that the consolidating sediment of Figure 6 does not contain a coarse sand fraction, hence no segregation occurs here.

Next, the soft mud layer is subject to erosion by a turbulent flow entraining the sediment. This effect is modelled by an additional stress term in the momentum equation in the 1DV POINT MODEL, an approach very similar to the one deployed by Le Hir (2001):

\[
\frac{\partial u}{\partial t} + \frac{1}{\rho} \frac{\partial p}{\partial x} = \frac{\partial}{\partial z} \left[ (\nu_\tau + \nu_\gamma) \frac{\partial u}{\partial z} + \frac{\tau_{xz}}{\rho} \right]
\]

(13)

where the stress term \( \tau_{xz} \) is described as a Bingham-like plastic model:

\[
\tau_{xz} = \mu_{mud} \frac{\partial u}{\partial z} \quad \text{with} \quad \mu_{mud} = \frac{a_\gamma \tau_\gamma}{1 + a_\gamma |\partial u/\partial z|} + \mu^s
\]

(14)

in which:

- \( a_\gamma = \) coefficient (\( a_\gamma = 0.02 \) implies \( \tau_{xz} = 0.95 \tau_\gamma \) for \( \partial u/\partial z = 10^{-3} \, \text{s}^{-1} \)),
- \( \mu^s = K_p \varphi^{n_r} \cdot n \) ranges between 2 and 6 for various kinds of mud, and
- \( \tau_\gamma = K_\gamma \varphi^{1/n_r} \)

Note that the computational domain covers the entire water depth including the mud layer; hence entrainment at the interface is not explicitly modelled, and the mud properties change from solid at the flume bottom to liquid at the water surface.

The results of the simulations are presented in Figure 7 and 8 showing the measured and computed increase in suspended sediment concentration in the water column above the soft mud bed. The various parameter settings are given in Table 2.

Table 2

<table>
<thead>
<tr>
<th>yield strength parameter ( K_\gamma ) [Pa]</th>
<th>comments</th>
</tr>
</thead>
<tbody>
<tr>
<td>measured bed density 1.0 \cdot 10^8</td>
<td>after Merckelbach with c.o.l.s. = 0.5</td>
</tr>
<tr>
<td>measured bed density 1.0 \cdot 10^8</td>
<td>after calibration</td>
</tr>
<tr>
<td>computed bed density 5.0 \cdot 10^8</td>
<td>from consolidation with c.o.l.s. = 0.5</td>
</tr>
<tr>
<td>computed bed density 5.0 \cdot 10^6</td>
<td>after calibration</td>
</tr>
</tbody>
</table>

Figure 7 shows that the suspended sediment concentration measured in the water column of the flume can properly be predicted using the measured density profile of the mud layer, using the proper parameters for the bed strength. A proper simulation using the computed density distribution of the bed is only possible for unrealistic strength parameters, e.g. Figure 8. From these results it can be concluded that a proper simulation of the entrainment process requires a proper vertical density profile at a
coefficient of lateral stress of 0.5 (Van Kessel, 1997). This sensitivity is of course the result of the high sensitivity of the mud strength \( \tau_c \) to the mud concentration \( c \): \( \tau_c \propto c^7 \) for \( n_r = 2.71 \).

![Diagram](attachment:diagram.png)

Figure 7: Effect of strength module on entrainment rate; measured initial density. Figure 8: Effect of strength module on entrainment rate; consolidation model.

It is concluded that the correct computation of the vertical density profile is a necessary, though probably not sufficient, condition for a proper prediction of the strength profile within a soft mud layer.

8. DISCUSSION & CONCLUSIONS

Traditional bed models, if present, coupled to cohesive sediment transport, allow updating the bed surface erosion strength as a function of density and time, by solving a simplified point-consolidation model and assuming a certain empirical relationship between density and shear strength.

In reality, the bed is not only subjected to gravity forces, which result in compaction and subsequent strengthening of the bed structure, but also to weakening shear and oscillating pressure forces due to currents and waves. Particularly in relatively shallow areas, such as estuaries and coasts, these forces may become significant, especially during storms, resulting in liquefaction and fluidisation of the bed, generating fluid mud layers, which can flow, or which are easily entrained. A new general bed dynamics model is developed which can be used to study these processes.

Settling column tests presently provide the only reliable means of examining the development of strength in a consolidating bed. However, such tests themselves may not be sufficiently representative of most natural conditions under which siltation occurs. Certainly biological effects have been demonstrated to significantly affect the bed sediment processes but biological activity in the laboratory may be different from that occurring in nature under perhaps more balanced ecosystem conditions.

The interesting observations of biological activity within the UOX experiments demonstrate that caution is necessary when using laboratory results as the basis for model validation. The models developed during this research project are unable to include the
effects of biological activity. Comparison of the model against test results with no biological activity is required for rigour but then application of these models to situations where biological activity occurs will be erroneous.

The settling column work at UOX has provided a link between the field observations of floc behaviour (Dyer et al., 2000) and bed processes. The work has also demonstrated clear differences in the properties of a bed formed by settlement from a slurry by comparison with one formed by to steady sedimentation.

From the modelling undertaken within this project the question of whether flocs can be described as self-similar arises (i.e. whether is reasonable to assume a constant fractal dimension, as in the Merckelbach/Winterwerp model). The bed structure is not self-similar at every scale, and this assumption may not fully describe the changes in sediment structure that occur with aggregation, but the assumption has been used successfully in modelling work undertaken within this project.

Another important question is what bed surface density is obtained after deposition. Deposited aggregates form a space-filling structure. Assuming a certain averaged floc shape, i.e. spherical, a simple relationship between floc density and bed surface density is found, showing that the bed density will be smaller than the floc density (Toorman, 2000). This also implies that the density of eroded aggregates will be larger than the bed surface density. In addition, observations in the UOX experiments indicate that eroding aggregates are larger than those settling, which suggests that additional inter-aggregate bonds are formed on the bed surface.

It is traditionally assumed that eroded cohesive sediments are all entrained and take part in the suspension transport. However, if bulk erosion of the bed is important, it may be possible that the transport mode might be dominated by true bed load transport of relatively large mud chunks, as has been observed in previous laboratory erosion experiments (e.g. Migniot, 1968; Toorman, 2000). This is a subject which requires further study, i.e. the possible need for the derivation of a bed load function for cohesive sediment should be investigated.

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