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A stacking method and its applications to
Lanzarote tide gauge records

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Abstract

A time-period analysis tool based on stacking is introduced in this paper. The original idea comes from the classical tidal analysis method. It is assumed that the period of each major tidal component is precisely determined based on the astronomical constants and it is unchangeable with time at a given point in the Earth. We sum the tidal records at a fixed tidal component center period $T$ then take the mean of it. The stacking could significantly increase the signal-to-noise ratio (SNR) if a certain number of stacking circles is reached. The stacking results were fitted using a sinusoidal function, the amplitude and phase of the fitting curve is computed by the least squares methods. The advantage of the method is that: (1) An individual periodical signal could be isolated by stacking; (2) One can construct a linear Stacking-Spectrum (SSP) by changing the stacking period $T_s$; (3) The time-period distribution of the singularity component could be approximated by a sliding-stacking approach. The shortcoming of the method is that in order to isolate a low energy frequency or separate the nearby frequencies, we need a long enough series with high sampling rate. The method was tested with a numeric series and
then it was applied to 1788 days Lanzarote tide gauge records as an example.

**Key words:** Stacking period, Singularity, Tides

1 **Introduction**

One of the most interesting fields for geophysical studies is to extract the different periodical signals from the observations [Van Ruymbeke et al. (2007); Guo et al. (2004)]. There are many choices to meet this requirement taking the advantage of the rapidly developed mathematical methods accompanied with high speed computers. Among them, the most intensively used method is the Discrete Fourier Transform (DFT). In order to locate certain periodical signals, there exists some similar ways such as Prony Analysis [Hauer et al. (1990)], Phase-Walkout method [Zürn and Rydelek (1994)] and the Folding-Averaging Algorithm [Guo et al. (2007, 2004)]. The stacking tool proposed here could be explained as a simplified procedure the afore-mentioned procedures because it assumed that the signal’s period $T$ is precisely known. The tool also could be viewed as a special case of Prony Analysis (PA). PA analyses signal by directly estimating the frequency, damping, and relative phase of modal components present in a given signal [Hauer et al. (1990)]. In our case, the condition is that the signal is mainly consisting of different periodical harmonic components and noise. We study the individual singularity by summing the time series at a stacking period $T_s$ ($T_s = T/\Delta t$), with $\Delta t$ sampling interval. We average the stacking results and fit it using a sinusoidal function. The amplitudes and phases of fitting curve were computed by the least squares

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method. The precision of phases and amplitude determinations are dependant on two factors. One is the way we assign the initial phase of the first stacking. For instance, when we use the stacking procedure to separate tidal waves, the initial phase of selected wave must be calculated from astronomical parameters, otherwise we will lose the physical meaning of the phase. The way to compute the phase of the tidal component could be found in the earth tide theory textbooks [Melchior (1983)]. The second factor is finding the best stacking period $T_s$ which is not always the integral times of sampling rate. To find the nearest $T_s$ to signal’s true period $T$, sometimes we need to search $T_s$ in several points ($T_s = T_s \pm \delta$) until the minimum differences between stacking results and fitting curves is reached. Beyond its application to singular component analysis, the stacking function also can be used to analyze a time series at a given period range by a linear Stacking-Spectrum (SSP). Another property of the tool is that when the stacking period and the initial phase were selected, we can model the space and time distribution of one singularity by shifting the stacking windows with a constant step. There are several techniques which could be used in time-frequency analysis, such as Short-Term Fourier Transform (STFT) and Continuous Wavelets Transform (CWT). Both of them are focused on overcoming the shortage of FFT in which time information is lost. The CWT are more effective than STFT [Daubechies (1992)]. In order to study the time-period localization of one singularity but not a frequency band signals like STFT and CWT, we developed the Sliding-Stacking approach. Since each classical tidal analysis method like Eterna by [Wenzel (1996)],VAV by [Venedikov et al. (1997)],and Baytap-G by [Tamura et al. (1991)] already meets the requirement of separating the tidal component with high accuracy from tidal records [Dierks and Neumeyer (2002)]. The tool proposed here could be summarized as a simplified approach to study periodic signals and estimate
the response of any signal to a selected period. For example, the isolated tidal constituent from continuous P wave velocity records, could be severed as references for in-situ seismic velocity monitoring [Yamamura et al. (2003)]. It is also possible to study the correlations between different tidal cycles and seismic activities by the stacking approach [Cadicheanu et al. (2007)]. Another promising application field is that the tidal waves can be utilized to calibrate some arbitrary records in-situ since the earth tidal model is the most reliable one [Westerhaus and Zürn (2001)]. Recently published works announce that the precision of calculated theoretical tidal potential $V$ over years 1-3000 C.E. reached $\pm 0.1$ mm [Ray and Cartwright (2007)].

2 Algorithm of stacking

The base function of stacking is:

$$f(t) = \frac{1}{N_S} \sum_{i=1}^{N_S} \sum_{j=1}^{T_s} y(t_j) + \varepsilon$$

(1)

$i = 1, 2, 3, ..., N_s$  $j = 1, 2, 3, ..., T_s$ where $f(t)$ represents averaging stacking results, $t$ the time, $y(t)$ the observed data. $T_s$, stacking period $T_s = T/\Delta t$, $T$ the signal’s period, $\Delta t$ sampling interval, $N_s$ the stacking number of times, $N_s = \tau/T_s$, $\tau$ data length, $\varepsilon$ the uncertainties and errors. We use sinusoidal function to fit the stacking results $f(t)$

$$f(t) = \sum_{i=1}^{T_s} (a\cos(\omega_i + \phi) + a\sin(\omega_i + \phi))$$

(2)

The standard deviation is given by:

$$\sigma = \sqrt{\frac{1}{T_s} \sum_{i=1}^{T_s} (f(t_i) - \hat{f}(t_i))^2}$$

(3)
So the amplitude $a$ and initial phase $\phi$ are determined by minimizing $\sigma$ using the least squares method. For a given $Ts$, we get one solution $(a, \phi)$. If we select a series of stacking periods $(Ts_1, Ts_2, ..., Ts_n)$, we have $n$ solutions $((a_1, \phi_1), (a_2, \phi_2), ..., (a_n, \phi_n))$. Then the linear stacking spectrum (SSP) are constructed by:

$$SSP = \begin{pmatrix} T_1 & a_1 & \phi_1 \\ T_2 & a_2 & \phi_2 \\ ... \\ T_n & a_n & \phi_n \end{pmatrix}$$  

In fact, it is not necessary to stack complete time series by one stacking period $Ts$. From the numerical experiment, it shows that the $Ns$ depends on the signal-to-noise ratio. For high SNR series, a smaller number of stacking times can reach a certain level of accuracy. If the minimum required stacking times $Ns$ are much shorter than the data length $\tau$, one can use a rectangular window $w$ to separate the data into equal length segments. Then, the amplitude and phase was computed by equation (1) to (3) for each segment.

$$w(n) = 1 \quad n = Ns * Ts$$  \hspace{1cm} (5)

When the windows are overlapped with each other by a constant length $(c\Delta t, \quad c > 0)$ and moved in one direction, it is possible to approximate the time-period localization with time resolution $c\Delta t$ by a Sliding-Stacking approach.
Fig. 1. Results obtained from Stacking and DFT with SNR=0.01. The left column shows the original signal, white noise, noise+signal; the middle column shows the amplitude Fourier transforms of left records and the last column shows the stacking results of noise polluted signals. The stacking results (gray), sinusoidal fit (blue) and original signal (red) were plotted together.

3 Numerical test

We firstly tested the method with a synthetic series. A time series was constructed by the addition of three periodical signals, \( s_1(a = 10, T = 2 \text{ secs}, \phi = 0) \), \( s_2(a = 5, T = 3 \text{ secs}, \phi = 0) \), \( s_3(a = 2, T = 5 \text{ secs}, \phi = 0) \), and white noise \( \varepsilon \). The sampling interval (\( \Delta t \)) was 0.01 second. The length of the series were 100,000 points. The \( Ts \)s is 200 for s1, 300 for s2, and 500 for s3. Different cases were computed referring to the noise level, Signal-to-Noise Ratio (\( SNR = \frac{s_{max}^2}{\varepsilon_{max}^2} \)). Eleven series were generated (\( SNR = 0.1 - 0.5 \) step by 0.1, 1.0 - 7.0 step by 1.0). We compared the amplitudes computed by the DFT and the Stacking methods. The accuracy of the amplitudes determination was influenced by the noise level. The results showed that when the SNR was low (\( SNR = 0.1 \)) the DFT gave better amplitude determination for s1 and s3 than the stacking. But when the signal-to-noise ratio was high (\( SNR = 7.0 \),
the stacking results were obviously better than the DFT results (table1). This is true for all the cases of signal s2 because the period of the frequency of s2 was a repeating decimal which contaminated the precision of the DFT results.

Table 1 amplitude determination \( \delta a = \left| \frac{a - a_0}{a_0} \right| \%

<table>
<thead>
<tr>
<th>SNR = 0.1</th>
<th>( a_1 = 10 )</th>
<th>( \delta a_1 )</th>
<th>( a_2 = 5 )</th>
<th>( \delta a_2 )</th>
<th>( a_3 = 2 )</th>
<th>( \delta a_3 )</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>DFT</strong></td>
<td>9.804</td>
<td>1.96%</td>
<td>6.689</td>
<td>33.78%</td>
<td>2.712</td>
<td>35.60%</td>
</tr>
<tr>
<td><strong>Stacking</strong></td>
<td>10.890</td>
<td>8.90%</td>
<td>4.471</td>
<td>10.58%</td>
<td>3.914</td>
<td>95.70%</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>SNR = 7.0</th>
<th>( a_1 = 10 )</th>
<th>( \delta a_1 )</th>
<th>( a_2 = 5 )</th>
<th>( \delta a_2 )</th>
<th>( a_3 = 2 )</th>
<th>( \delta a_3 )</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>DFT</strong></td>
<td>9.075</td>
<td>9.25%</td>
<td>7.077</td>
<td>41.54%</td>
<td>2.132</td>
<td>6.60%</td>
</tr>
<tr>
<td><strong>Stacking</strong></td>
<td>9.984</td>
<td>1.60%</td>
<td>5.071</td>
<td>1.42%</td>
<td>2.099</td>
<td>4.95%</td>
</tr>
</tbody>
</table>

In general, for all eleven test cases, the accuracy of amplitudes determination less than 10% was 67% for stacking and 33% for DFT. Furthermore, we evaluated the influence of the signal-to-noise ratio and the stacking number of times on the results. It should be tested separately because both parameters will directly influence the final results. To test the influence of Ns, the SNR was assigned as 0.1. To compare the effects of different noise levels, the Ns were set as 120. We separated the singularity \((T = 2secs)\) from the synthetic time series by equation (1) and (2). The standard deviation \(\sigma\) was computed by equation (3). The stacking number of times \(N_s\) was increased from 5 until 450 increased by steps of 5 with \(SNR = 0.1\). The \(\sigma\) was oscillating around 0.5% when the \(N_s\) was larger than 60, then the trench became more stable with \(\sigma < 0.5%\) after 120 times stacking (Fig. 2 left). After that, we took the same series, but added different level of noise (SNR from 0.01 to 5 increasing by 0.01 with \(N_s = 120\)). The signal \((T = 2s)\) was isolated again by equation...
Fig. 2. (Left), Plot $\sigma$ against Ns, the Ns increased from 5 until 450 times increasing by steps of 5. (Right), Plot $\sigma$ against SNR, 500 noise levels were compared from 0.01 to 5. (1) and (2) from different level noise contaminated series. The distribution of $\sigma$ was more scattered (Figure 2 right). In fact, even for the very high noise level (SNR=0.01), the $\sigma$ was around 0.8% after 120 times stacking. This again confirms the stacking is an efficient way to reduce the random white noise. If the signal-to-noise level is sufficiently high ($SNR > 0.1$), with small number of stacking times ($Ns > 20$), we can easily isolate the harmonic components with 1% accuracy (Fig. 2 left). The synthetic test proved again that one can use the stacking approach to study known periodical signals behind a long time series. The tidal records are one of the most suitable cases for such an application due to the periods of main tidal constituents which are precisely determined based on astronomical constants.

4 Lanzarote tide gauge station

The landscape of Lanzarote is dominated by numerous volcanoes. The observation site named ”Jameos del Agua” is located in a lava tunnel of the quaternary volcano ”La Corona”. The last periods of volcanic activity at Lanzarote were during the 18th and 20th century. The most special eruption took
Fig. 3. The field site of tidal gauge station, the sensor was installed under an open lake inside a lava tunnel. The only connection to the sea is a crack perpendicular to a sand pyramid located 750 meters away which was discovered by a diving survey in 1985.

The volcanic tunnel where the tide gauge meter is installed was formed since the original eruption.

The tide gauge sensor was set up under an open lake inside a lava tunnel at Lanzarote island (Fig.3). The climatic effects on the instrument are partly reduced by the unique natural environment. It produces a very homogeneous data bank. In this paper, we selected 1788 days minute sampling data since July 3, 2002. The gaps and few spikes were manually cleaned using Tsoft [Van Camp and Vauterin (2005)]. All gaps were filled with zeros since it would not introduce any weight on the stacking results but keep the continuity of the whole series (Fig.4).

5 Application to Lanzarote tide gauge records

From the stacking function (1) to (5), we can get three types of solutions for any given time series: the amplitude and phase of single harmonic wave, the Stacking-Spectrum (SSP), and the time-period distribution of one singularity.

The immediate objective is to access the stacking method as a tool for real ap-
Fig. 4. Zero mean of tide gauge records after eliminating the spikes and filling gaps, the subplot figures show the detail of rectangular marked records.

For instance, we selected four tidal components (O1, T=1548 mins., K1, T=1436 mins., M2, T=745 mins., S2, T=720 mins.). The sampling interval is one minute so that the stacking period $T_s = T$. First, four tidal waves were separated by the stacking method (Fig. 5, left). Second, the SSP were computed from one year’s tidal gauge record with minute sampling rate. The starting stacking period was 500 which was linearly increased by 1 point steps until 2000. The solutions of SSP were computed by equation (1) to (4). The majority tidal components were detached (Fig. 5, right).

The origin of M2 and S2 are lunar and solar principal waves so that it is quite a pure sinusoidal curve. This is not the case for the diurnal waves K1 and O1. The K1 is generated by a combination effect of solar and lunar attraction. The O1 is beating with K1 to produce the M1 modulation. It is a minor component in oceanic tides [Melchior (1983)]. The isolated waves can be used to study the transfer functions between different physical parameters. For instance, the barometric effect on gravitational tidal components can be estimated by comparison of stacking results from gravity and barometric pressure records.
Fig. 5. (left), Four tidal components stacked at their center period $T$, the original phases were computed refer to Julian epoch. (right), The SSP of tide gauge records, $Ts$ was started from 500 and linearly increased by 1 point minute until 2000.

[Van Ruymbeke et al. (2007)].

Suppose that an equally sampled time series $y(t)$, the length of $y(t)$ is $\tau$, sampling interval is $\Delta t$. If one want to obtain the time-period localization of single harmonic component with period $T$, it need to first find the minimum stacking number of times $Ns$ which must be much shorter than $\tau$. From the equations (1),(2), (3) and (5), one can get a Sliding-Stacking result. Now, we select two tidal components K1 and S2 to illustrate the tool. It is assumed that the minimum stacking number of times for K1 is ($Ns = 90$) and S2 is 60 ($Ns = 60$). Then both window functions were moved with a constant step ($c\Delta t = 1440 mins$) which was equal to one day length. The final results, a time-period distribution of K1 and S2, were plotted in Fig.6. S2 amplitude is modulated by long period wave which originates from the declination and ellipticity of the earth orbit cycling the Sun. This effect is clearly visible from the Sliding-Stacking results as variations of the envelope of the S2 wave. Lack of data produced four gaps in both cases.

The Sliding-Stacking on K1 shows the combination effect of the common pe-
Fig. 6. (upper), sliding stacking on S2 component, the window length is 30 days after it is moved by 1 day step, (lower) sliding stacking on K1 component the window’s length is 90 days with 1 day moving step.

The period of K1 is exactly two times that of its harmonic waves K2. In this case, it clearly shows that we can not separate both by 90 times stacking on K1 period, results in two maximum in the Sliding-Stacking results (Fig. 6).

6 Conclusion

A stacking method was introduced in this paper. The tool was firstly tested with a numeric series which were consisted of three harmonic components and random white noise. The amplitude of each harmonic wave was computed by the stacking tool and DFT for different levels noisy contaminated signal. The stacking tool gave better results than the DFT for high SNR series, espe-
cially for the singularity whose frequency was a recurring decimal. Thus the stacking method was reliable when the period of a harmonic wave was well defined. The period of each tidal component is precisely constrained by astro-

dnomic constants which specially meets the basic requirement of the stacking.

Starting from the stacking function, a linear Stacking-Spectrum (SSP) and Sliding-Stacking approach, were developed. They were applied to the Lanzarote tide gauge records. Four tidal components (O1, K1, M2, S2) were selected to illustrate the interesting of the method. Three types of preliminary results were obtained from the tide gauge records: the K1, O1, M2 and S2 singularities were separated from the data, the harmonic waves with periods between 500 and 2000 mins were isolated by the SSP, the amplitude of K1 and S2 time-period distribution were separately demonstrated by Sliding-Stacking approach. But the solutions of Sliding-Stacking were strongly dependent on the stacking number of times, it can be used only when the data length $\tau$ are much longer the $N_s$. The Sliding-Stacking of the K1 constituent showed the such effect in which the result included both the K1 wave and its first harmonic wave K2. The stacking tools are applied to estimate the effects of barometric on gravitational tidal constituents and also intensively utilized to the design of geophysical instruments [Van Ruymbeke et al. (2007)]. It is also possible to test the correlations between some quasi random signals with the secular earth tide when the statistical tests are introduced to evaluate the stacking results. For instance, the correlations between seismic activities and the earth tide at Vrancea seismic zones, have been investigated by the stacking approach [Cadicheanu et al. (2007)].
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