Estimation of Sediment Properties using Frequency Domain Identification and Marine Acoustics

S. Vandenplas, A. B. Temsamani and L. Van Biesen

Vrije Universiteit Brussel, Dept. ELEC
Pleinlaan 2, B-1050 Brussels, Belgium
Email: steve.vandenplas@vub.ac.be

Abstract- In the field of underwater acoustics, the importance of the characterization of the seafloor is well known and of interest for scientific, economical and military applications [1]. An acoustic method would reduce the present needs for sediment sampling, while producing continuous profiles of sediment properties. However, for studying the functional relations between acoustical and geo-physical properties, an accurate determination of the acoustical parameters is necessary first.

In this paper, a global system identification approach is used for the experimental validation of wave propagation models through sediments and for the determination of its acoustical parameters. In order to achieve this goal, laboratory experiments on calibrated degassed sediments with broadband Panametrics piston transducers (300kHz-700kHz) are carried out. For this, two configurations of acoustic measurements are used: a water-Plexiglas-Sand-Plexiglas-water and a water-sediment-Plexiglas configuration.

The Maximum Likelihood Estimator is applied in the frequency domain to retrieve the sediment parameters. The propagation of multiples in the sediment, and the calibration method, which is incorporated in the global system identification approach, are important assets that are not exploited when carrying out direct measurements of dispersion or absorption with transducers buried in the sediment.

Hereby the wave propagation phenomena are represented as Single Input Single Output (SISO) transfer systems and modeled parametrically and with several types of propagation models (viscoelastic models, porous models, Buckingham’s model). A comparison between the estimated parameters is carried out. Furthermore, it is demonstrated that the method can be extended to Multiple Input Multiple Output (MIMO) systems, where now several transfer functions are estimated together parametrically. It is shown how the method can be used in real life experiments.

I. INTRODUCTION

In ocean acoustics, inversion methods are applied to infer parameters which characterize the environment. They do provide a promising alternative for direct measurements of geoaoustic data, which is difficult, expensive and time consuming. However it should be noted that there is no universal optimal method for the inverse problem. Depending on the purpose of the research, the complexity of the model, the geometry and the available measurements, some methods will perform better than others. In this paper, the use of the Maximum Likelihood Estimator in the frequency domain as satisfying solution for the inverse problem in the global framework of a system identification technique for comparison of wave propagation models, and determination of model parameters is demonstrated. Firstly, the laboratory measurements and procedures are explained. Next the system identification approach is elucidated. The theory is then applied to the two experiments considered in this paper. Finally, conclusions are given.

II. LABORATORY MEASUREMENTS AND PROCEDURES

Experiments on the calibrated fine sand “GA39” (See Table I for properties) are carried out. The sand is industrially sifted, washed and dried. Notice that air content in prepared sand samples can alter the acoustical properties of the sediment. Therefore, before the sand is put into the water tank for carrying out the experiments, the air content in the sediment was greatly reduced by slowly stirring the sediment in its preparation phase when the particles were mixed with the water. The mixture is then pulled into the Plexiglas containers placed in the water tank in such a way that the grains themselves are not exposed to air anymore.

The acoustic experiments were carried out with broadband Panametrics immersion piston transducers (500 kHz V389-SU). Such a transducer is a single element longitudinal wave transducer with a ¼ wavelength layer acoustically matched to the water. This technique provides a means of uniform coupling and increases the sound energy output.

Short large-amplitude electric pulses of controlled energy are applied to these transducers which are then converted into short ultrasonic pulses. This is done by the pulser section of

<table>
<thead>
<tr>
<th>Sediment Properties</th>
<th>Value (Uncertainty)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Wet bulk density</td>
<td>1855 (10)</td>
</tr>
<tr>
<td>Grain density</td>
<td>2650</td>
</tr>
<tr>
<td>SiO2 %</td>
<td>&gt;99.1</td>
</tr>
<tr>
<td>Grain size (mean)</td>
<td>77 very well sorted</td>
</tr>
<tr>
<td>Specific Surface</td>
<td>325</td>
</tr>
<tr>
<td>Apparent specific gravity</td>
<td>1.5</td>
</tr>
</tbody>
</table>

TABLE I

PROPERTIES OF “GA39”
the 5055PR Panametrics broadband ultrasonic pulser/receiver. In monostatic experiments (pulse-echo method) the ultrasonic pulses are captured by the transmitting transducer. For the bistatic reflection or transmission experiments, however, a separate receiving transducer is utilized. The receiver section of the pulser/receiver amplifies the voltage signals produced by the transducer representing the received ultrasonic pulses.

Both the generated short electrical pulse and the amplified received signal are digitized. For the setup utilized at dept. ELEC of VUB, this is carried out with a computer in conjunction with the HP E1430A acquisitions cards placed on a VXI-bus platform. This VXI-rack contains also a generator allowing to send a trigger to the pulser-receiver and to synchronize the acquisition.

III. THE SYSTEM IDENTIFICATION APPROACH

System identification globally deals with the selection of a model for a studied process through a limited number of measurements of the input and output [2].

Several steps are necessary in the identification process. Firstly, a specific model needs to be selected. The models that are described in this work are based on a priori knowledge (physical laws); thus a white box modeling method is followed. Once the model has been selected, the model is assumed known and its parameters are determined in the parameter estimation phase. The last phase in the identification process is a validation step, in which the residues and the consistency of the model are checked.

Linear dynamic systems can be represented in the time or in the frequency domain. Both representations have advantages and disadvantages. Working in the time domain is useful in particular if the systems are changing rapidly in time. Time domain identification is not very sensitive to the type of signals and on-line identification can be carried out easily too. However, the time domain representation requires a convolution to solve differential equations, while a simpler multiplication suffices in the frequency domain. Moreover, a broad frequency band can be obtained by adding different frequency bands together. Besides, it is often easier to calculate a model of a system accurately in the frequency domain than in the time domain, and the non-parametric transfer function is easily obtained too.

In this work the frequency domain representation is chosen. Apart from the advantages mentioned above, the calibration asset was of overriding importance. It will be demonstrated how the calibration can be incorporated into the modeling; the system identification method is applied on systems representing the calibrated wave phenomena.

A. The Errors-in-Variables (EV) model

Models where both the input and the output are disturbed by noise or errors are called Errors-in-Variables models. The variables are treated symmetrically. Assume that \( x_m(t) \) and \( y_m(t) \) are the measured time signals for the input and the output of a Linear Time Invariant (LTI) system. The stochastic EV model is defined by the following equations:

\[
\begin{align*}
    y(t) &= h(t) \otimes x(t) \\
    x_m(t) &= x(t) + n_x(t) \\
    y_m(t) &= y(t) + n_y(t)
\end{align*}
\]  

with \( x(t) \) and \( y(t) \) the deterministic part of the I/O signals, \( h(t) \) represents the impulse response of the system, \( n_x(t) \) and \( n_y(t) \) are zero-mean stationary noise (eventually correlated), where the higher order cumulants are absolutely summable. \( \otimes \) represents the convolution operator. If the Fourier transform is applied, the Errors-in-Variable stochastic model can be presented as follows:

\[
\begin{align*}
    Y(\omega) &= H(\omega) X(\omega) \\
    X_m(\omega) &= X(\omega) + N_X(\omega) \\
    Y_m(\omega) &= Y(\omega) + N_Y(\omega)
\end{align*}
\]

Now, the Fourier coefficients for the noise disturbance \( N_X(\omega) \) and \( N_Y(\omega) \) are complex normal distributed and assumed independent across the frequencies [3].

The acoustic laboratory measurements presented in Section II fulfill the conditions for a band limited Errors-in-Variable model. In Fig. 1 a scheme for the monostatic or bistatic acoustic experiments is depicted. The characteristics of the emitter and receiver are described in the frequency domain by the transfer function \( H_T \) and \( H_R \), respectively. For the monostatic experiments physically the same transducer is used as transmitter and receiver, which is not case for the bistatic experiment. \( H_{Amp} \) represents the internal amplifier in the Panametrics 5055PR pulser-receiver. The pulser-receiver is also used for the generation of the excitation signal. The wave propagation itself (the part of the physical process taking part between the transmitter and receiver) is described by the \( H_{prop} \) transfer function. From Fig. 1 the following relationship between the input and output Fourier coefficients is readily deduced:

\[
Y_m(\omega) = H_{Amp}(\omega) \cdot H_{Re}(\omega) \cdot H_{prop}(\omega) \cdot H_T(\omega) \cdot X_m(\omega)
\]

(3.3)

The contribution of the measurement setup in (3.3) is given by:

\[
H^{sys}(\omega) = H_{Re}(\omega) \cdot H_T(\omega) \cdot H_{Amp}(\omega)
\]

(3.4)

Formula (3.3) is now rewritten as:

\[
Y_m(\omega) = H^{sys}(\omega) \cdot H_{prop}(\omega) \cdot X_m(\omega)
\]

(3.5)

![Fig. 1. Monostatic or bistatic experiment represented as EV model.](Image)
Relationship (3.5) is valid both for a monostatic and bistatic experiment. Notice that the contribution of the measurement setup $H^{\text{SYS}}$ is different for both modes. A method to eliminate this transfer function is presented in the next subsection.

B. The calibrated LTI system representing the propagation phenomenon

The modeling is focused on the wave propagation itself. To avoid the modeling of the elements of the measurement circuit, which would complicate the problem unnecessary, a calibration technique is applied. The global method presented here is valid for monostatic and bistatic experiments and is used for the inversions carried out in this work.

Broadly speaking, the calibration technique consists of carrying out another experiment under the same conditions (e.g. same water temperature, same position of transducers,...) with unchanged settings for the instrumentation. For each specific setup, an appropriate calibration experiment is carried out.

The Fourier transform of the generated electrical pulse and the transmitted or reflected signal for the calibration experiment are denoted as $X_c(\omega)$ and $Y_c(\omega)$, respectively. Now, a relationship similar to (3.5) can be written for the calibration experiment, viz.:

$$Y_c(\omega) = H^{\text{SYS}}(\omega) \cdot H^{\text{CAL}}_{\text{PROP}}(\omega) \cdot X_c(\omega)$$  \hspace{1cm} (3.6)

with $H^{\text{CAL}}_{\text{PROP}}$ the transfer function describing the wave propagation phenomenon for the calibration experiment.

In addition, $H_m(\omega)$ and $H_c(\omega)$ are defined as follows:

$$H_m(\omega) = \frac{Y_m(\omega)}{X_m(\omega)}$$  \hspace{1cm} (3.7)

$$H_c(\omega) = \frac{Y_c(\omega)}{X_c(\omega)}$$  \hspace{1cm} (3.8)

In fact, $H_c(\omega)$ and $H_m(\omega)$ can be regarded as the input and output of a LTI Single Input Single Output (SISO) system (Fig. 2.) with transfer function $H(\omega)$:

$$H(\omega) = \frac{H^{\text{PROP}}(\omega)}{H^{\text{CAL}}_{\text{PROP}}(\omega)}$$  \hspace{1cm} (3.9)

The elements of the measurement circuit are eliminated in this system, which represents the calibrated wave propagation phenomena and will be modeled parametrically for several experiments in the next sections.

C. The non-parametric estimate for the input and output

The transfer function depicted in Fig. 2 is modeled parametrically, and the model parameters are retrieved by mean of the Maximum Likelihood Estimator (MLE) in the frequency domain. The MLE requires the measurement of the input and the output spectra of the system under investigation, as well as the knowledge of the perturbing noise variances. Therefore several periods of the input and output are measured for both the calibration and the actual measurement. The transfer functions for the input and output are estimated non-parametrically.

The Errors-in-Variables (EV) estimator is obtained by taking the mean of the input and output Fourier coefficients:

$${\hat H}_{\text{EV}}(\omega) = \frac{1}{M} \sum_{i=1}^{M} X_{c,m}(\omega)$$

with $M$ the number of measured periods. This estimator is unbiased if periodic signals are used, and if the acquisition of the data is synchronized to the generator [4]. Since the generation of acoustic signals is not perfectly synchronized, the $H^{\text{EV}}_{\log}$ technique [5] is used instead to calculate the averaged signals over the different periods:

$${\hat H}_{c,m}(\omega) = \exp \left\{ \frac{1}{M} \sum_{i=1}^{M} \log \left( \frac{Y_{c,m}(\omega)}{X_{c,m}(\omega)} \right) \right\}$$  \hspace{1cm} (3.11)

Notice that the phase of (3.11) is biased. For example, if the following measurements were done, $A \exp(j(\pi - \alpha))$ and $A \exp(j(-\pi + \alpha))$, the logarithmic mean calculated with (3.11) results in $A$. However, if $\alpha < \frac{\pi}{2}$ the most probable solution is $-A$ [5]. Therefore first an estimation of the phase $\phi(\omega)$ is taken, e.g. $\phi(\omega) = \arg(H_{c,m}^\dagger(\omega))$, and the estimator is calculated as follows:

$${\hat H}_{c,m}(\omega) = \exp \left\{ \frac{1}{M} \sum_{i=1}^{M} \log \left( \frac{Y_{c,m}(\omega)}{X_{c,m}(\omega)} \right) \exp(- j\phi(\omega)) \right\} \exp(+ j\phi(\omega))$$  \hspace{1cm} (3.12)

The variances on the non-parametric estimates are determined during the averaging procedure. Furthermore, in [5] is demonstrated that if the SNR of the I/O Fourier coefficients $> 6$ dB, the bias $< 17$ mDB and the bias is $< 18$ μdB for SNR $> 10$ dB. The estimator is called “practically” consistent and may be assumed unbiased.

The high SNR’s of the input and output spectra (typically 40 dB), renders the $H_{\log}$ estimates to be almost complex normally distributed.

The basic noise assumptions required by the MLE on the input and output transfer functions of the defined SISO system are satisfied and can be summarized as follows [1]:

- The noise on input and output has a complex normal distribution;
- The noise on input and output is zero mean valued;
- The measurements at different spectral lines are uncorrelated;
- The noise on input and output are jointly independent;
• The noise on input and output is independent on the deterministic input Fourier coefficients;

D. The Maximum Likelihood Estimator (MLE)

The following error function can be associated to the SISO system of Fig. 2:

\[ e_m(\omega, P) = H(\omega, P) \cdot \hat{H}_c(\omega) - \hat{H}_m(\omega) \]  

(3.13)

with \( H(\omega, P) \) the modeled transfer function for the SISO system depending on the parameter vector \( P \) containing the model parameters. Due to the presence of model errors and noise on the measured input and output transfer functions the error function (3.13) differs from zero. An estimation \( \hat{P} \) of the model parameters is usually based on the minimization of a cost function \( L(P, G_m) \), viz.:

\[ \hat{P} = \arg \min_P L(P, G_m) \]  

(3.14)

where \( t \) is the transpose operator, \( \hat{u} = [\omega_1, \omega_2, \ldots, \omega_F]^{\top} \) the angular frequency vector, and

\[ \hat{H}^{t}_{\text{cm}}(\hat{u}) = \left[ \hat{H}^{t}_{c,m}(\omega_1), \hat{H}^{t}_{c,m}(\omega_2), \ldots, \hat{H}^{t}_{c,m}(\omega_F) \right] \]  

(3.15)

The cost function is constructed as a weighted sum of square residuals, viz.:

\[ L(P, G_m) = \sum_{f=1}^{F} e_m^{H}(P, \omega_f) W(P, \omega_f) e_m(P, \omega_f) \]  

(3.16)

where the superscript \( H \) stands for the complex conjugate-transpose operator. The weighting function \( W \) determines the properties of the estimator. For the Maximum Likelihood Estimator, the weighting function is given by

\[ W(P, \omega) = \left[ \text{var}(H_{c}(\omega_f)), \text{var}(H_{m}(\omega_f)) \right]^{-1} \]  

(3.17)

with \( \text{var}(H_{c}(\omega_f)) \) and \( \text{var}(H_{m}(\omega_f)) \) the “true” variances. In practice these “true” variances are replaced by some estimates, which results in a bias on the estimated parameters. However, in [5] is demonstrated that the bias is approximately a linear function of the second order noise and the covariance term \(-2 \text{Re}[\text{cov}(\hat{H}_c, \hat{H}_m) \cdot \bar{H}]\) appears in the denominator of the weighting function (3.17). If the measurement and calibration are separate experiments, the covariance is zero.

The ML cost function can be written as [1]:

\[ L(P, G_m) = \sum_{f=1}^{F} \frac{1}{\text{var}(H_{c}(\omega_f))} \frac{1}{\text{var}(H_{m}(\omega_f))} \]  

(3.18)

For the MLE estimator the following properties can be shown [2]:

• Consistency for the parameters \( P \) if the noise \( n \) has bounded fourth-order moments;

The parameters are distributed normally with distribution \( N(P, C_p) \) if the noise moments are finite and if the noise is independent over the frequencies;

If the noise is normal distributed and if the SNR is sufficiently high, the MLE will reach asymptotically the Cramér-Rao bound and thus be asymptotically efficient;

The robustness of the asymptotic properties (normality and bias, consistency, efficiency) is presented in [6]. It is demonstrated that the robust estimator reaches its asymptotic properties when the number of harmonics is about twice the number of estimated parameters.

From [2] it follows that in absence of modeling errors, the expected value of the minima of the likelihood cost function equals \( F + \frac{n_c}{2} \) with \( F \) the number of spectral lines and \( n_c \) the number of parameters to be estimated.

E. The Multiple Input Multiple Output (MIMO) representation

The method can be extended to Multiple Input Multiple Output (MIMO) systems. The system identification approach is then adopted to estimate together several transfer functions parametrically. If common parameters are present in the transfer functions, due to the redundancy of information, the uncertainties on the estimated parameters will be smaller than those obtained with SISO estimators. Moreover, it is shown in [7] that by applying the MIMO estimation method, small model errors could be detected. \( N \) SISO experiments are written in a MIMO context as follows:

\[ H_m(\omega) = M_{\text{MIMO}} H_c(\omega) \]  

(3.19)

with \( H_c \) and \( H_m \) the input and output vectors of the MIMO system:

\[ H_{c,m}(\hat{u}) = \begin{bmatrix} H_{c,1}^{1}(\hat{u}), H_{c,2}^{1}(\hat{u}), \ldots, H_{c,N}^{1}(\hat{u}) \\
H_{c,1}^{2}(\hat{u}), H_{c,2}^{2}(\hat{u}), \ldots, H_{c,N}^{2}(\hat{u}) \\
\vdots \\
H_{c,1}^{N}(\hat{u}), H_{c,2}^{N}(\hat{u}), \ldots, H_{c,N}^{N}(\hat{u}) \end{bmatrix} \]  

(3.20)

and \( M_{\text{MIMO}} \) the transfer matrix representing the MIMO system. Since the input and output originate from independent experiments, the complexity of the MIMO estimator is reduced drastically and the MIMO transfer matrix becomes diagonal with main diagonal:

\[ \text{diag}(M_{\text{MIMO}}) = [H^{1}(\hat{u}), H^{2}(\hat{u}), \ldots, H^{N}(\hat{u})] \]  

(3.21)

Now, the ML-cost function for the MIMO system to be minimized towards \( P \), consists of the sum of the SISO system cost functions:

\[ L_{\text{MIMO}}(P, G_m) = \sum_{f=1}^{F} L_{\text{SISO}}^{1}(P, G_m) + \cdots + L_{\text{SISO}}^{N}(P, G_m) \]  

(3.22)

with \( G_m = \begin{bmatrix} H_c^{1}, H_c^{2}, \ldots, H_c^{N} \end{bmatrix} \) and

\[ G_m = \begin{bmatrix} \begin{bmatrix} H_c^{1}, H_c^{2}, \ldots, H_c^{N} \end{bmatrix}^{\top} \end{bmatrix}^{\top} \]  

(3.23)
where \( \hat{H}_{v,m}^c = [H_{e,m}^v(\omega_1), H_{e,m}^v(\omega_2), \ldots, H_{e,m}^v(\omega_F)]' \) \(^{(3.24)}\)

Parameter vector \( \mathbf{P} \) includes all the parameters that have to be estimated. In absence of modeling errors, the expected value of the minima of the likelihood cost function equals now \( NF + \frac{n_c}{2} \) with \( F \) the number of spectral lines and \( n_c \) the number of MIMO parameters to be estimated \(^{[8]}\).

**F. The estimation algorithm**

An estimate for the parameter vector \( \mathbf{P} \) is obtained by minimizing towards \( \mathbf{P} \) cost function (3.18) for the SISO system and cost function (3.22) for the MIMO method, respectively. Due to the construction of the cost function, one deals with a least square optimization problem, viz.:

\[
\text{minimize } F(x) = f(P)' f(P) \quad (3.25)
\]

For the MIMO method, vector \( f(P) \) can be written as

\[
\frac{H(P) \hat{C}_c - \hat{H}_m}{\sqrt{\text{var}(H_c) |H(P)|^2 + \text{var}(\hat{H}_m)}} \quad (3.26)
\]

with

\[
\hat{H}_{c,m}(P) = [\text{Re}(H_{1,m}^c)', \text{Re}(H_{2,m}^c)', \ldots, \text{Im}(H_{1,m}^c)', \ldots, \text{Im}(H_{N,m}^c)']',
\]

and \( H(P) \) a diagonal matrix with main diagonal:

\[
\text{diag}(H(P)) = [\text{Re}(H(P))', \ldots, \text{Im}(H(P))']
\]

\( \hat{H}_{c,m}^v \) is defined in (3.24), \( \hat{H}^v(P) \) is the transfer function vector for the \( v^\text{th} \) SISO system:

\[
\hat{H}^v(P) = [H^v(P,\omega_1), H^v(P,\omega_2), \ldots, H^v(P,\omega_F)]'
\]

(3.29)

For the SISO approach \( N \) equals one. Notice that parameter vector \( \mathbf{P} \) is a real vector, which can in fact contain both the real and imaginary parts of the complex parameters which one has to estimate.

In general, a non-linear programming problem requires an iterative procedure to establish a direction of search at each major iteration. In this work, the method of Levenberg-Marquardt \(^{[2]],[9]} \) is used, which is a combination of the Gauss-Newton method and the gradient method.

**IV. SISO METHOD APPLIED TO THE CONFIGURATION WITH FINE SAND FILLED IN A PLEXIGLAS BOX**

**A. Experimental setup and calibration**

The measurement setup (Fig. 3) consists of a water filled tank in which a smaller Plexiglas tank containing the “GA39” sediment is placed. This provides a Plexiglas-Sediment-Plexiglas configuration, which is used in conjunction with the broadband Panametrics V389 transducers and the Panametrics pulser-receiver 5055PR for the transmission experiment. The transducers are aligned parallel to the

![Fig. 3. Experimental setup for the transmission experiment through the Plexiglas container filled with GA39 Plexiglas surfaces. The thickness of the Plexiglas borders and the sediment layer are 1 cm and 2 cm, respectively. The measured generated pulse and the transmitted time signal (50 periods) are digitized with a sampling frequency of 10 MHz. As calibration experiment, a pitch-catch experiment in the water tank is utilized. The transfer function \( H^T(\omega) \) represents the wave phenomena for the transmitted signal, calibrated with the pitch-catch measurement in the water column.

**B. The plane wave modeling with beamspread correction**

A Debye series expansion \(^{[10]} \) is used in order to introduce the correction for the divergence of the beam. Therefore a closed form expression for \( H^T(\omega) \) is replaced by a finite summation over the multiple propagation paths \(^{[11]} \) viz.:

\[
H_T(\omega, \mathbf{P}) = \sum_{n=1}^{195} D(s_n^T, \omega) T_n
\]

(4.1)

with

\[
s_n^T = \frac{\omega a^2}{\sum_{\text{mult-path}} d_n c_n}
\]

(4.2)

and \( s_n^T \) and is given by

\[
s_n^T = \frac{\omega a^2}{(d_1 + d_2 + d_3 + d_4 + d_5)c_{0\omega}}
\]

(4.3)

The summation in (4.1) is the summation of the products of the distances and velocities over the multiple propagation paths, \( c_{0\omega} \) the velocity in the water, \( a \) the radius of the piston transducer, and function \( D \) represents the beamspread correction function:

\[
D(s(\omega)) = 1 - e^{-js} (J_0(s) + j J_1(s))
\]

(4.4)

In the transmission coefficient \( T_n \) of the propagation path \( n \), interface reflection and transmission coefficients appear
together with exponents representing the propagation through layer $i$: $\exp(\omega t - K_i d_i)$. By expressing the continuity of displacement and stresses at the boundary, the interface reflection and transmission coefficients are calculated.

For the viscoelastic rational form model and the CQ-model, the Plexiglas-Sediment-Plexiglas configuration can be regarded as a linear viscoelastic multilayer medium, where the complex wavenumber $K_i$ of a wave propagating through the structure is written as the ratio of the angular frequency and the complex phase velocity of intergranular friction,

$$K_i(\omega) = \frac{\omega}{V_i(\omega)} = \frac{\omega}{c_i(\omega)} - j\alpha_i(\omega)$$

with $c_i(\omega)$ and $\alpha_i(\omega)$ the longitudinal dispersion and absorption, respectively. For the rational viscoelastic model the complex dilatational phase velocity through layer $i$ is then given by [7]

$$V_i(\omega) = c_i \sqrt{1 + \sum_{l=1}^{N} \alpha_{i,l}(j\omega)^l \frac{P_l(\omega, \alpha_{i,l}, \beta_{l,k})}{1 + \sum_{k=1}^{D} \beta_{l,k}(j\omega)^k}} = c_i P_i(\omega, \alpha_{i,l}, \beta_{l,k})$$

with $c_i$ the frequency-independent phase velocity and $P_i(\omega, \alpha_{i,l}, \beta_{l,k})$ a function describing the frequency dependence of the complex velocity $V_i(\omega)$. $N$ and $D$ stand for the order of the numerator and denominator.

If the Constant Q viscoelastic model of Kjartansson [12] is used for the wave propagation in the sediment, the complex velocity $V_3(\omega)$, in the sediment (Fig. 3-layer 3 ) is then given by:

$$V_3(\omega) = V_{03} \frac{j\omega}{\omega_0} (j\omega)^3 \cdot V_{03} P_3(\omega, \omega_0, \gamma_3) \quad \text{and} \quad \gamma_3 = \frac{1}{\pi} \tan^{-1} \left( \frac{1}{Q_3} \right)$$

with $V_{03}$ the frequency independent velocity for the sediment at a reference angular frequency $\omega_0$.

For Buckingham’s model the complex velocity $V_3(\omega)$ in the sediment is given by [13]

$$V_3(\omega) = c_{03} \left( 1 + (j\omega t_0)^n_{\text{mem}} \chi_f \right)$$

with $c_{03}$ the sediment’s compressional wave speed in absence of intergranular friction, $\chi_f$ the compressional dissipation coefficient, $n_{\text{mem}}$ the memory index and $t_0$ a scaling constant.

The theory of Biot [14] leads to the propagation of a second highly attenuated slower wave in the sediment due to the global relative movement of the fluid towards the frame. Realistic values for 13 or more different parameters are needed to describe the global fluid motion and frequency-dependent frame response. To retrieve the interface reflection and transmission coefficients and the wavenumbers of the dilatational waves, linear systems of equations have to be solved. Notice that the second dilatational wave is too attenuated to be measured. Consequently equation (4.1) is still valid, where now the summation presents a finite sum over the less attenuated propagation paths of the dilatational waves of the first kind. In spite of this, the second kind of dilatational waves act as a supplementary attenuation mechanism through the interface transmission and reflection coefficients appearing in $T_n$.

C. The inverse procedure

The ML cost function to be minimized for the SISO system is given by:

$$L_{\text{SISO}} = \sum_{f=1}^{F} \left| e^T(\omega_f, P) \right|^2$$

with

$$e^T(\omega_f, P) = \frac{\hat{H}_c^T(\omega_f) H_c^T(\omega_f, P) - \hat{H}_m^T(\omega_f)}{\sqrt{V^T(\omega_f, P)}}$$

and

$$V^T(\omega_f, P) = \text{var} H_c^T(\omega_f) \text{var} H_m^T(\omega_f) + \text{var} \hat{H}_c^T(\omega_f)$$

where $\{\omega_f, f = 1, ..., F\}$ denotes the set of angular frequencies to be accounted for, $\hat{H}_c^T$ and $\hat{H}_m^T$ present the estimates for the non-parametric transfer function of the pitch-catch experiment and the transmission experiment through the Plexiglas, respectively. Now, the parameters of the Plexiglas are not estimated, but were determined from transmission experiments, where the Plexiglas container is filled with water [15]. Moreover, it is demonstrated in [7] that the wave propagation through Plexiglas is modeled very well with the viscoelastic rational form model of order $(N = 2, D = 1)$. A priori knowledge is used to initialize the non-linear minimization problem. For the rational modulus of the sediment, the model order is increased progressively, starting with very small new parameters, till the cost function stops descending.

D. Experimental results

An overview of values for the cost function after minimization towards parameter vector P is presented in Table II. The smallest value for the cost function were found with the viscoelastic rational form model of order $(N = 2, D = 1)$, followed with the viscoelastic CQ-model and Buckingham’s model. Much larger values of the cost function were retrieved with Biot’s model. The estimation results are depicted in Fig. 4. It can be seen that modeling with the first three models leads to differences in amplitude and phase of the same order of magnitude. Magnitude errors less than 4% and phase errors less than 0.6% are found. For Biot’s model, the errors rise till 9% and 1.2%. Notice that the phase errors with the viscoelastic CQ-modeling and Buckingham’s modeling are almost identical. Nevertheless, it seems that with the CQ-model, the amplitude matches better with the measurements than with the viscoelastic Buckingham’s model.
The estimated parameters are presented in Table III. With these values, the estimated dispersion and absorption curves of Fig. 4 were calculated. Virtually the same absorption and dispersion curves were found for the viscoelastic CQ-model and Buckingham’s model, while small differences between the viscoelastic CQ-model and the viscoelastic rational model were retrieved.

Notice that because of the experimental setup, which was chosen to avoid possible effects of surface roughness, the global movement of the interstitial fluid towards the frame is quite small. This could also be seen experimentally in Table III where compared to the measured permeabilities of Table I, underestimated values for the permeability were found, which can be physically interpreted as a low fluid motion, relative to the skeletal frame. The larger model errors for the Biot model can be physically explained by the complexity of the model and the many parameters that have to be determined. Only some of these parameters can be estimated by the inverse procedure and most of them need to be fixed. Numerous methods for choosing these parameters have been published [16], [17], [14] but their accuracy is still discussed regularly (e.g. [18], [19]), because different combinations of parameters can lead to similar effects on the measurable acoustical parameters.

### V. A MIMO INVERSION TECHNIQUE APPLIED TO REFLECTED SIGNALS FROM A SIMULATED SEAFLOOR

#### A. Experimental setup, measurements and calibration method

The measurement setup consists of a water-filled tank in which a Plexiglas plate of 5 cm high was placed on the bottom. A 5 cm high frame placed on the Plexiglas plate was then filled with the well sorted fine sand GA39 in such a way that a relatively smooth water-sediment interface was obtained. This provides a Water-Sediment-Plexiglas configuration, which is first used in conjunction with a broadband (300 kHz-700 kHz) Panametrics V389 transducer and a pulser-receiver in order to perform monostatic reflection experiments. The transducer is aligned parallel to the sediment surface (Fig. 5).

Next, bistatic measurements in oblique incidence are carried out. The emitter and receiver (both Panametrics V389 transducers) are placed symmetrical to a vertical plane, perpendicular to the sediment surface (Fig. 6). Measurements are performed at an angle of incidence of 45°. The first reflection is the direct reflection of the beam on the water-sediment surface. The receiver is moved and the second

### TABLE II

<table>
<thead>
<tr>
<th>Model</th>
<th>Viscoelastic rational form model (2,1)</th>
<th>Viscoelastic CQ-model</th>
<th>Buckingham’s model</th>
<th>Biot’s model</th>
</tr>
</thead>
<tbody>
<tr>
<td>GA39</td>
<td>1.5e4</td>
<td>2.6e4</td>
<td>2.9e4</td>
<td>1.5e5</td>
</tr>
</tbody>
</table>

### TABLE III

<table>
<thead>
<tr>
<th>Estimated Parameters</th>
<th>Rational form model</th>
<th>CQ-model $\omega_o = 2\pi \times 500$ kHz</th>
<th>Buckingham’s model $\tau_0 = 1/\omega_o$</th>
<th>Biot’s model</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\rho_1$ (kg/m$^3$)</td>
<td>1.5938e3 (4e-1)</td>
<td>1.5909e3 (4e-1)</td>
<td>1.5905e3 (4e-1)</td>
<td>1.7202e3 (1e-1)</td>
</tr>
<tr>
<td>$c_1$ (m/s)</td>
<td>1.6392e3 (1e-1)</td>
<td>1.66164e3 (3e-2)</td>
<td>1.53508e3</td>
<td></td>
</tr>
<tr>
<td>$V_{os}$ (m/s)</td>
<td>1.045e-6 (5e-9)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$c_{03}$ (m/s)</td>
<td>1.46e-15 (1e-17)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\alpha_{3,1}$</td>
<td>1.013e-6 (1e-9)</td>
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<td></td>
<td></td>
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<tr>
<td>$\gamma_3$</td>
<td>4.104e-3 (2e-6)</td>
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<tr>
<td>$k$ (m$^{-2}$)</td>
<td>1.895e-13 (8e-16)</td>
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<tr>
<td>Apparent mass $m_a$</td>
<td>3.38e3 (2e1)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\chi_f$</td>
<td>1.7223e-1 (5e-5)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\tau_f$ (s)</td>
<td>2.6532e-5 (3e-10)</td>
<td>2.6532e-5 (3e-10)</td>
<td>2.6532e-5 (3e-10)</td>
<td>2.652e-5 (3e-10)</td>
</tr>
</tbody>
</table>

Fig. 4. Estimation results for sand GA39 in transmission. Amplitude (a) and phase (b) of calibrated transfer function (TF), magnitude errors (c) and phase errors (d) between measurements and model, estimated dispersion (e) and estimated absorption (f); Measured TF “—”, viscoelastic rational form model “—”, CQ-model “◊”, Buckingham’s model “o”, Biot’s model “••••”. 

![Fig. 4. Estimation results for sand GA39 in transmission.](image-url)
reflected signal at normal incidence (reflection on the water-sediment surface). In this transfer function, the surface roughness and the reflection on a water-Plexiglas interface at normal incidence, calibrated by the monostatic wave on the sediment-Plexiglas interface is measured. For positions were carried out for the monostatic and bistatic highest frequencies. Averages over 24 and 10 different measurements, 30 realizations of the generated pulse and smoothed away, the surface roughness of the sediment cannot be neglected for wavelengths around 2 mm in the water at the sediment interface and the sediment-Plexiglas interface. The latter is calibrated with the transfer function of the first return on the water-sediment interface at normal incidence. Besides the density and surface roughness, the longitudinal absorption and dispersion of the sediments play an important role.

The third used SISO transfer function $H^v(\omega)$ contains additional information about the shear speed. It represents the at oblique incidence transmitted longitudinal wave (reflected on the Sediment-Plexiglas interface), calibrated with the specular reflection on the water-sediment interface. This is summarized in a MIMO context as follows:

$$
H^v(\omega) = \begin{bmatrix}
H^w(\omega) & 0 & 0 \\
0 & H^l(\omega) & 0 \\
0 & 0 & H^o(\omega)
\end{bmatrix}
\begin{bmatrix}
H^w(\omega) \\
H^l(\omega) \\
H^o(\omega)
\end{bmatrix}
$$

The SISO transfer functions $H^v(\omega)$ (with $v = R, T$ and $O$) are modeled parametrically in the next section.

B. Viscoelastic plane wave modeling

In this section, the viscoelastic rational form transfer function is used for modeling the wave propagation through the sediment.

For describing the interface roughness at the sediment surface, a model developed by Kuperman [20] is used to calculate Kirchhoff’s reflection and transmission coefficients for a randomly rough interface between two liquids. In [21] this model is extended to elastic media. The model uses the perturbation theory and assumes that the product of each vertical wavenumber (compressional and shear) and the RMS roughness $\sigma$ at each interface is small, and makes use of the Kirchhoff’s approximation, which assumes that the field at the rough surface is as if the surface is flat with a slope equal to that of the rough surface at the point in question. Considering the fact that the surface is smoothed away, and that the size of the sand grains themselves are too small to interfere with the wavelength, one can assume that the conditions to apply the model are fulfilled. However, note that the return of the shear wave is not measured and its velocity is assumed to be relatively low. In this case the shear wave cannot become part of the trapped wave field and will, therefore, only operate as an additional attenuation mechanism. In this case a full elastic treatment of the scattering from rough interfaces is not necessary and the sediment can be treated as a fluid to describe the coherent roughness effect, which leads to simplified equations.

The Rayleigh reflection and transmission coefficients at the water-sediment interface need then to be multiplied by a factor $R_s$ and $T_s$, respectively:

$$
R_s = 1 - 2 \left( \frac{\lambda^s}{\lambda^l} \right)^2 \sigma^2
$$

and

$$
T_s = 1 - \frac{1}{2} \left( K^w_{l;z} - K^s_{l;z} \right)^2 \sigma^2
$$

(4.13)

with $K^w_{l;z}$ the vertical wavenumber in medium $u$ ($u=w$ or $u=s$).
transfer functions.

modeled transfer functions rise to 4, 2 and 8 % for amplitude errors in percentage between the measured and divergences of the beam is again compensated by an analytical expressions based on the Lommel’s diffraction integral [1].

The quality of the estimations can be seen in Fig. 7. The amplitude (a) and phase (b) of \( H_R \), amplitude (c) and phase (f) of \( H_T \); measured “—” and modeled “•” transfer functions.

Real transducers do not produce plane waves. The divergence of the beam is again compensated by an analytical expressions based on the Lommel’s diffraction integral [1].

C. The inverse procedure

The ML cost function to be minimized for the MIMO system is given by:

\[
L_{\text{MIMO}}(\mathbf{p}) = L^R(\mathbf{p}) + L^\theta(\mathbf{p}) + L^\phi(\mathbf{p})
\]

\[
= \frac{1}{2} \sum_{f=1}^{F} [c^R(\omega_f, \mathbf{p})]^2 + \sum_{f=1}^{F} |c^\theta(\omega_f, \mathbf{p})|^2 + \sum_{f=1}^{F} |c^\phi(\omega_f, \mathbf{p})|^2
\]

(4.14)

with

\[
e^\psi(\omega_f, \mathbf{p}) = \frac{\hat{H}^R(\omega_f, \mathbf{p}) H^\theta(\omega_f, \mathbf{p}) - \hat{H}^\phi(\omega_f, \mathbf{p})}{\sqrt{V^R(\omega_f, \mathbf{p})}}
\]

(4.15)

and

\[
V^R(\omega_f, \mathbf{p}) = \text{var} H^R(\omega_f, \mathbf{p})\left| H^\theta(\omega_f, \mathbf{p}) \right|^2 + \text{var} H^\phi(\omega_f, \mathbf{p})
\]

(4.16)

where \( \{ \omega_f, l=1, ..., F \} \) denotes the set of angular frequencies to be accounted for.

D. Experimental results

The quality of the estimations can be seen in Fig. 7. The amplitude errors in percentage between the measured and modeled transfer functions rise to 4, 2 and 8 % for \( H^R \), \( H^T \) and \( H^\phi \), respectively. Phase errors of 1, 0.5 and 2% were found, respectively. The estimated parameters are listed in Table IV. The estimated dispersion and absorption of the sediment are depicted in Fig. 8. Notice that for the inversion, the height of the Plexiglas frame was taken as the value for height of the sand. However, considered the method of preparation of the sediment, where the surface was smoothened away with a measurement rod, it is perfectly possible that the real height of the sediment differs a little bit from the fixed value. This explains the difference of the value for the frequency independent phase velocity with the estimated values for GA39 in Table III and leads also to a shifted dispersion curve. However if one compares the estimated density and absorption of the sediment, it can be seen that the differences between the estimates are relatively small.

E. Setup for real-life experiments

Various technologies of acoustics, seismics and coring are combined to study the seafloor in the EC MAST SIGMA project [22]. The acoustical setup is shown in Fig. 9. The following were installed in a towed fish: a parametric array for vertical incidence and a steerable parametric array to perform measurements in oblique incidence. The reflections in oblique incidence are captured by a streamer array of hydrophones. Some hydrophones in the array are used to determine the position of the streamer from the acoustical signals sent from three transducers placed at the back of the towed fish. Specular reflections and returns from a second layer were captured at oblique incidence in the SIGMA June 1999 trial. Note the similarity of symmetry between the real-life setup at sea and the one presented in this paper.

![Fig. 7. MIMO estimation results for the simulated seafloor: amplitude (a) and phase (b) of \( H^R \), amplitude (c) and phase (f) of \( H_T \); measured “—” and modeled “•” transfer functions.](image)
VI. CONCLUSIONS

In this paper the use of a global system identification method is demonstrated. The Maximum Likelihood Estimator is applied in the frequency domain to retrieve the acoustical parameters of a fine sediment. In order to achieve this goal, laboratory experiments on calibrated degassed sediments with broadband Panametrics piston transducers (300kHz-700kHz) were carried out. With a SISO representation several wave propagation models were compared. The smallest model errors were found with the viscoelastic rational form model. With normal incidence, where now also a value for the shear speed could be retrieved by combining the measurements in normal and oblique incidence.

REFERENCES


