A prediction method for squat in restricted and unrestricted rectangular fairways

Evert Lataire a, Marc Vantorre a, Guillaume Delefortrie b

a Ghent University, Maritime Technology Division, Technologiepark 904, 9052 Ghent, Belgium
b Flanders Hydraulics Research, Berchemlei 115, 2140 Antwerp, Belgium

ABSTRACT

The hydrodynamic behaviour of a vessel changes when sailing in shallow and/or confined water. The restricted space underneath and alongside a vessel has a noticeable influence on both the sinkage and trim of a vessel, also known as squat. To assess these influences an extensive model test program has been carried out in the Towing Tank for Manoeuvres in Shallow Water (cooperation Flanders Hydraulics Research — Ghent University) in Antwerp, Belgium with a scale model of the KVLCC2 Moeri tanker. This benchmark vessel was selected for its full hull form, to maximize the effects of the blockage.

To thoroughly investigate the influences of the blockage on the squat of the vessel, tests have been carried out at different water depths, widths of the canal section and forward speeds (2 up to 16 knots full scale whenever possible).

The squat observed during the model tests is compared with the squat predicted with a mathematical model based on mass conservation and the Bernoulli principle. The correlation between measured and modelled squat for each canal width for all tested speeds and water depths is very good, but shows a constant slope deviation. An improved model for the squat is proposed and takes into account the forward speed, propeller action, lateral position in the fairway, total width of the fairway and water depth.

1. Introduction

A sailing ship continuously displaces and accelerates a significant amount of water, which, according to the Bernoulli principle, results in a pressure drop around the vessel. The latter yields a vertical displacement characterized by values of the running sinkage at the forward and the aft perpendiculars, or alternatively, a mean running sinkage and trim. This phenomenon is commonly referred to as squat (Briggs et al., 2010). Principally, the ship does not undergo an increase of its draft or volume displacement, but the level of the free water surface around the vessel changes and, on average, drops. As a result, the ship translates vertically and rotates around its lateral axis.

Squat also occurs in deep and open water but is more pronounced in restricted or confined waters. Moreover, in deep and open water this phenomenon has no major consequences, while in more restricted waters the vessel can run aground. A decrease of the net underkeel clearance as a result of squat may also affect the ship’s manoeuvrability dramatically, resulting in loss of control.

An accurate prediction of the squat that is valid in the complete range from deep and open waters up to narrow and shallow waterways (i.e., channels, canals, docks, locks) is therefore important, to avoid not only groundings but also manoeuvrability issues due to insufficient underkeel clearances. The aim of the present paper is to predict the squat (sinkage and trim) for a wide range of water depths and widths of a canal with rectangular cross section. The forward speed, lateral position of the vessel in the canal, propeller action and hull geometry are taken into account.

In June 2010 systematic model tests were carried out in the Towing Tank for Manoeuvres in Shallow Water (cooperation Flanders Hydraulics Research — Ghent University). The sinkage of a scale model of a very large crude carrier was measured, the lateral position and the forward speed of the model were varied systematically resulting in a database of about 400 different model tests.

The model test results are compared with the semi-empirical calculation method as published by Dand and Ferguson (1973). Based upon this theory a new mathematical model is proposed for the calculation of squat in any rectangular cross section, from a wide and deep fairway, up to underkeel clearances of only a few percent of the vessel’s draft and widths of a fraction more than...
Nomenclature

- $A_c$ [m$^2$]: Canal cross section area
- $A_M$ [m$^2$]: Area of midship section
- $A_W$ [m$^2$]: Waterplane area
- $A(x)$ [m$^2$]: Cross section area of the ship at longitudinal position $x$
- $B(x)$ [m]: Beam of the waterline at longitudinal position $x$
- $B$ [m]: Beam of the ship
- $D$ [m]: Propeller diameter
- $F_{NW}$ [N]: Rudder normal force
- $F_{NR}$ [N]: Rudder tangential force
- $F_{R_0}$ [dimensionless]: Froude number (water depth dependent)
- $F_{RCR}$ [dimensionless]: Critical speed
- $g$ [m/s$^2$]: Gravity of Earth
- $h$ [m]: Water depth
- $I_x$ [m$^4$]: Longitudinal moment of inertia
- $L_{PP}$ [m]: Length between perpendiculars
- $K$ [Nm]: Roll moment
- $M$ [Nm]: Trim moment
- $m$ [dimensionless]: Blockage
- $m_{crit}$ [dimensionless]: Critical blockage
- $N$ [Nm]: Yaw moment
- $n$ [rpm]: Propeller rate
- $Q_{T}$ [Nm]: Torque on the propeller shaft
- $R^2$ [dimensionless]: Coefficient of determination
- $Q_{T_0}$ [Nm]: Torque on the rudder
- $T$ [m]: Draft
- $r$ [m/m]: Trim
- $T_{P}$ [N]: Thrust of the propeller
- $V$ [m/s]: Forward speed of the vessel
- $V_1$ [m/s]: Speed of the water in the disturbed cross section
- $V_{eq}$ [m/s]: Equivalent forward speed

$\gamma(F_{R_0}) = \frac{F_{R_0}^2}{\sqrt{1-F_{R_0}^2}}$  \hspace{1cm} (1)

where $F_{R_0}$ represents the depth-related Froude number

$F_{R_0} = \frac{V}{\sqrt{gh}}$  \hspace{1cm} (2)

Tuck’s theory was later extended to dredged channels by Beck et al. (1975) with a matching of solutions between the deep and shallow regions of the cross section. Naghdi and Rubin (1984) solved Tuck’s problem using a nonlinear steady-state solution of the differential equations of the theory of a thin ship while Cong and Hsiung (1991) combined the thin ship and flat ship theory to solve the same problem for transom stern ships.

Tuck’s theory predicts infinite squat for the critical speed value $F_{R_0}=1$. On the other hand, using methods based on the Bernoulli principle and conservation of mass as proposed by Constantine (1960), Dand and Ferguson (1973) and Gourlay (1999) give a blockage-dependent critical speed range instead of one particular critical speed value. In order to extend their theoretical developments towards random bottom conditions and waterways with large width, Dand and Ferguson (1973) and Gourlay (2000) relied upon experimental research. A correction for the effect of the propeller was proposed by Dand (1972). To cope with large or even infinite canal widths, Dand (1972) applied the effective width parameter $W$ (Eq. (3)) introduced by Tuck (1967) that was based upon model tests in a section width from 7.5 to about 14 times the beam of the vessel.

$W = \frac{W}{L_{PP}} \sqrt{1-F_{R_0}^2}$  \hspace{1cm} (3)

Many empirical methods for estimation of ship squat have been published. It is not the authors’ intention to give an
exhaustive overview of these formulations, but only to mention the methods that explicitly account for the canal geometry. Führer and Römisch (1977) also proposed empirical formulae based on model tests to predict the squat for different cross section parameters of the fairway. Extensive model tests with 13 different ship models and three different canal geometries were carried out and analysed by Guliev (1971). Even though most of the conclusions still stand, Guliev's prediction method is used less as it is a graphical method.

More practical methods based on experimental research are presented by Barrass (1979), who also proposed an equivalent width taking into account the dimensions of the width of the fairway. However, the results obtained by Barrass (1979) were not validated by Seren et al. (1983).


Besides measurements on model scale, interesting full-scale observations were carried out by Anukidinov et al. (2000), Stocks et al. (2004) and Härtling et al. (2002, 2009) among others. Most discussions focus on ships sailing in open water or in rectangular shaped canals without drift angle or propulsion. In some cases the drift angle was considered, as by Von Bovet (1985), Martin and Puls (1986), de Koning Gans and Boonstra (2007) and Eloot et al. (2008). The squat when sailing in a muddy area is investigated by Sellmeijer and van Oortmerssen (1983), Vantorre and Coen (1988), Brossard et al. (1990), Doctors et al. (1996) and most recently by Delefortrie et al. (2010). For an exhaustive overview of publications and calculation methods related to the squat phenomenon, the reader can refer to Briggs et al. (2010).

3. Experimental program

3.1. Test facilities

All the test results discussed in the present paper are obtained by captive model tests carried out in the Towing Tank for Manoeuvres in Shallow Water (cooperation Flanders Hydraulics Research — Ghent University). The main dimensions of the empty towing tank are $88 \times 7 \times 0.5$ m$^3$ (Van Kerkhove et al., 2009), but vertical walls parallel to the longitudinal tank walls were built into the tank to reduce the channel width over a length of 30 m. The tank is equipped with a planar motion carriage that allows both captive and free-running manoeuvring tests.

These model tests are carried out with a model that represents a Very Large Crude Carrier (VLCC) at scale 1:75. The geometric properties of bare hull, propeller and rudder of this vessel are made available and published via (SIMMAN, 2008). This specific model is known as the KVLCC2 Moeri tanker and often used as a benchmark vessel by towing tanks worldwide (Stern and Agdrup, 2008). The main properties of the vessel are summarized in Table 1 and the body plan is shown in Fig. 1.

During the test series the ship model was connected rigidly to the planar motion carriage of the towing tank. The ship model's trajectory was determined in the horizontal plane by the motion of the mechanism but the model was free in the vertical degrees of freedom allowing trim and sinkage.

During every test run the running sinkage of the model was measured at four discrete positions (fore/aft and port/starboard). Based upon these measurements the running sinkage at the forward perpendicular $z_{FP}$ and at the aft perpendicular $z_{VA}$ was derived. Note that the model was not free to roll, the starboard and port measurements should therefore be equal. In addition to the sinkage, forces and moments on hull, propeller and rudder, as listed in Table 2, were also measured.

3.2. Test parameters

The ship model was towed at a constant forward speed along different lateral positions in the rectangular canal with the velocity vector always parallel to the longitudinal walls of the canal (Fig. 2). The canal was open at the inlet and outlet so water from the towing tank could flow in and out the actual canal section during a test run. The width $W$ of the canal section was varied from only 1.05 times the ship's beam $B$ up to the entire width of the towing tank (7.00 m or 9.05 B). Also a range of water depths was tested from 1.05 $T$ to 1.50 $T$, the latter being close to the maximum possible water depth in this shallow water towing tank (Fig. 3). The blockage factor $m$ (or the ratio between the midship section $A_M$ and the cross section of the fairway $A_f$) is summarized in Table 3. In the narrowest canal section, the acceleration of the ship model was initiated in the test section

<table>
<thead>
<tr>
<th>Ship model characteristic</th>
<th>Symbol</th>
<th>Unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>Running sinkage at the FP and AP</td>
<td>$z_{VF}$, $z_{VA}$</td>
<td>[m]</td>
</tr>
<tr>
<td>Longitudinal force</td>
<td>$X$</td>
<td>[N]</td>
</tr>
<tr>
<td>Sway force</td>
<td>$Y$</td>
<td>[N]</td>
</tr>
<tr>
<td>Yaw moment</td>
<td>$N$</td>
<td>[Nm]</td>
</tr>
<tr>
<td>Roll moment</td>
<td>$K$</td>
<td>[Nm]</td>
</tr>
<tr>
<td>Propeller thrust</td>
<td>$T_p$</td>
<td>[N]</td>
</tr>
<tr>
<td>Propeller torque</td>
<td>$Q_p$</td>
<td>[Nm]</td>
</tr>
<tr>
<td>Propeller rate</td>
<td>$n$</td>
<td>[rpm]</td>
</tr>
<tr>
<td>Rudder normal force</td>
<td>$F_{NR}$</td>
<td>[N]</td>
</tr>
<tr>
<td>Rudder tangential force</td>
<td>$F_{TR}$</td>
<td>[N]</td>
</tr>
<tr>
<td>Rudder torque</td>
<td>$Q_R$</td>
<td>[Nm]</td>
</tr>
<tr>
<td>Rudder angle</td>
<td>$\delta_R$</td>
<td>[deg]</td>
</tr>
</tbody>
</table>

Table 1

<table>
<thead>
<tr>
<th>Ship model characteristic</th>
<th>Full scale</th>
<th>Model</th>
<th>Unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>Scale</td>
<td>1</td>
<td>75</td>
<td>–</td>
</tr>
<tr>
<td>$L_{FP}$</td>
<td>320.0</td>
<td>4.267</td>
<td>[m]</td>
</tr>
<tr>
<td>$B$</td>
<td>58.0</td>
<td>0.773</td>
<td>[m]</td>
</tr>
<tr>
<td>$T_F$</td>
<td>20.8</td>
<td>0.277</td>
<td>[m]</td>
</tr>
<tr>
<td>$T_A$</td>
<td>20.8</td>
<td>0.277</td>
<td>[m]</td>
</tr>
<tr>
<td>$V$</td>
<td>312622</td>
<td>0.741</td>
<td>[m$^3$]</td>
</tr>
</tbody>
</table>

Fig. 1. Body plan of the KVLCC2 including the water line at the tested draft (design draft).

Table 2

Measured forces, moments and vertical translations on the ship model.

<table>
<thead>
<tr>
<th>Ship model characteristic</th>
<th>Symbol</th>
<th>Unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>Running sinkage at the FP and AP</td>
<td>$z_{VP}$, $z_{VA}$</td>
<td>[m]</td>
</tr>
<tr>
<td>Longitudinal force</td>
<td>$X$</td>
<td>[N]</td>
</tr>
<tr>
<td>Sway force</td>
<td>$Y$</td>
<td>[N]</td>
</tr>
<tr>
<td>Yaw moment</td>
<td>$N$</td>
<td>[Nm]</td>
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</tr>
<tr>
<td>Rudder torque</td>
<td>$Q_R$</td>
<td>[Nm]</td>
</tr>
<tr>
<td>Rudder angle</td>
<td>$\delta_R$</td>
<td>[deg]</td>
</tr>
</tbody>
</table>
while for all other canal sections the acceleration started with the bow of the model at the entrance of the section. The deceleration started when the vessel was out of the test section for all canal widths. For all model tests it is manually checked that only the running sinkage at the steady state condition was used after all transitional phenomena induced by the acceleration were damped, and before the deceleration or the end of the canal section was reached. This steady-state interval was different depending of the test condition but the model covered a distance, at a constant forward speed, of at least two ship lengths and the used measurements end before the bow was as close as one ship length from the end of the canal.

All tested lateral positions $y$, expressing the distance between the centreline of the vessel and centreline of the fairway, for all canal widths are summarized (in model scale) in Table 4. The distance $y_{wall}$ is defined as:

$$ y_{wall} = \frac{W}{2} - \frac{B}{2} - y $$  \hspace{1cm} (4)

Whenever possible the lateral position of the vessel (Fig. 4) in the canal was varied systematically from sailing at a distance as close as 0.020 m (1.5 m full scale) between the ship’s port side and the wall of the test canal up to sailing at the centreline of the canal.

The forward speed of the towed VLCC model has been varied from 0.2 knots up to 16.0 knots full scale, the higher speeds being only feasible in the deeper and wider canals. An overview of the tested forward speeds in each water depth—canal width combination is given in Fig. 5.

The propeller rate $n$ of the model at each tested forward speed was always at the self propulsion point in open water i.e., without the influence of the water depth or width of the canal. The tests carried out with the entire width of the towing tank available ($W=9.05 \, B$) were also carried out with a fixed propeller ($n=0$).

4. Empirical method by Dand and Ferguson

4.1. Principle

Dand and Ferguson (1973) proposed a semi-empirical theory to calculate the squat as a combination of mean sinkage and trim based on the mass conservation law and the Bernoulli principle. Hereafter, Dand and Ferguson (1973) will be referred to as D&F. This theory takes into account the ship’s beam $B(x)$ on the waterline at a longitudinal position $x$, the cross-sectional area $A(x)$ and the longitudinal distribution of these areas. It supposes

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Table 3
Overview of tested blockage factors $m$.

<table>
<thead>
<tr>
<th>Canal width W</th>
<th>Water depth h</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>1.05 $B$</td>
</tr>
<tr>
<td>1.05 $T$</td>
<td>0.91</td>
</tr>
<tr>
<td>1.10 $T$</td>
<td>0.86</td>
</tr>
<tr>
<td>1.35 $T$</td>
<td>0.70</td>
</tr>
<tr>
<td>1.50 $T$</td>
<td>0.63</td>
</tr>
</tbody>
</table>

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Table 4
Distance between the longitudinal centreline of the model and centreline of the fairway $y$ [m] in model scale.

<table>
<thead>
<tr>
<th>Canal width W [m]</th>
<th>1.05 $B$</th>
<th>1.25 $B$</th>
<th>1.70 $B$</th>
<th>2.50 $B$</th>
<th>5.00 $B$</th>
<th>9.05 $B$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.020</td>
<td>-</td>
<td>0.077</td>
<td>0.251</td>
<td>0.360</td>
<td>1.526</td>
<td>-</td>
</tr>
<tr>
<td>0.050</td>
<td>-</td>
<td>0.047</td>
<td>0.221</td>
<td>0.330</td>
<td>1.496</td>
<td>-</td>
</tr>
<tr>
<td>$B/4$</td>
<td>-</td>
<td>-</td>
<td>0.077</td>
<td>0.386</td>
<td>1.352</td>
<td>-</td>
</tr>
<tr>
<td>$B/2$</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>0.193</td>
<td>1.159</td>
<td>-</td>
</tr>
<tr>
<td>$B$</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>0.773</td>
<td>2.340</td>
</tr>
<tr>
<td>2 $B$</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>1.567</td>
</tr>
<tr>
<td>3 $B$</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>0.794</td>
</tr>
<tr>
<td>Centreline or $W/2$</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
</tr>
</tbody>
</table>

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Fig. 2. The KVLCC2 towed in the most confined canal section ($W=1.05 \, B$).

Fig. 3. Overview of all tested cross sections.

Fig. 4. Graphical interpretation of the lateral position of the vessel in the cross section of the model fairway.
that the vessel sails at a constant speed \( V \) in a waterway with rectangular cross section \( A_c \) with a constant water depth \( h \) (at rest) and width \( W \). When the vessel sails in open water an equivalent width of the section is used and a compensation for the propeller action on the trim of the vessel is proposed. However, these last three parameters are not included initially in the analyses. They are included later with the suggested modifications to the D&F method.

When the vessel sails with a speed over ground \( V \) and a forward speed \( V_1 \) along a straight course along the centreline of this rectangular fairway, a return flow will be initiated at each section \( x \), so that the velocity of the water at the entire section is increased by \( \partial V(x) \). Because of the increased water speed along this section, the pressure will decrease (Bernoulli principle), causing a decrease of the water level over a vertical distance \( \zeta(x) \) proportional to the pressure drop (Fig. 6).

The conservation of mass can be written as

\[
A_c V = V_1(x)(A_c - A(x)) - W \zeta(x)
\]

(5)

with

\[
V_1(x) = V + \partial V(x)
\]

(6)

Eq. (4) assumes that the free surface over the full width of the canal section sinks. Fig. 6 shows a level drop \( \zeta(x) \), while the section of the vessel at this position \( x \) sinks over the same distance. This decrease of the water depth \( \zeta(x) \) can be calculated with Bernoulli's Law:

\[
\zeta(x) = \frac{1}{2g} (V_1(x)^2 - V^2)
\]

(7)

The substitution of Eq. (4) into Eq. (6) results in:

\[
\frac{1}{2g} (V_1(x)^2 - V^2) = \frac{A_c - A(x)}{W} - \frac{A_c - A(x)}{W} \frac{V}{W} V_1(x)
\]

(8)

Because of the rectangular cross section of the fairway \( A_c/W \) equals the initial water depth \( h \) and \( A(x)/A_c \) is known as the blockage ratio \( m \), Eq. (7) can now be written as:

\[
\frac{1}{2g} (V_1(x)^2 - V^2) = \frac{V}{V_1(x)} + m(x) - 1 = 0
\]

(9)

Substituting the water depth dependent Froude number \( Fr_h \) from Eq. (2) and multiplying by \( V_1(x)/V \) in Eq. (9),

\[
\frac{1}{2g} \frac{V_1(x)^2}{V^2} (V_1(x) \frac{V_1(x)}{V}) \frac{V_1(x)}{V} + m(x) - 1 = 0
\]

(10)

Eq. (9) has always three solutions for \( V_1(x) \). One solution is always real and negative, and has no physical meaning. When the blockage factor \( m \) is less than a critical blockage factor \( m_{crit} \), both remaining solutions are real and positive; for greater values of \( m \) these solutions are complex conjugated numbers which means that no stationary solution can be found. For a given value of the depth Froude number, the critical value of the blockage factor is given by:

\[
m_{crit} = 1 - \sin \left( 3\arcsin \left( \frac{Fr_h^{2/3}}{2} \right) \right)
\]

(11)

This equation is valid if \( Fr_h < 1 \), as was shown by Schijf (1949), see also Briggs et al. (2010).

For a given value of the blockage factor \( m \), a physically realistic solution is only possible if \( Fr_h \) is either less than a first critical value, or greater than a second critical value. In this way, subcritical, transcritical and supercritical speed ranges can be distinguished. In the transcritical speed range, which always contains \( Fr_h = 1 \), no stationary solution can be found.

The first critical depth Froude number is always less than 1 and can be calculated as follows as was shown by Schijf (1949) and more recently by Briggs et al. (2010):

\[
Fr_{h, crit1} = \left( 2 \sin \left( \arcsin \left( 1 - m \right) \right) \right)^{3/2}
\]

(12)

The second critical Froude number equals:

\[
Fr_{h, crit2} = \left( 2 \sin \left( \frac{\pi}{3} - \arcsin \left( 1 - m \right) \right) \right)^{3/2}
\]

(13)

Because of a physical maximum blockage factor of one the second critical Froude number \( Fr_{h, crit2} \) is limited to \( 3^{3/4} \). In the subcritical speed range, the critical blockage \( m_{crit} \) decreases for an increasing Froude number \( Fr_h \). For example, for a blockage \( m \) of 0.01 the critical Froude number \( Fr_{h, crit} \) decreases from 1.0 to 0.88. The subcritical area in Fig. 7 indicates the combinations of the depth Froude number and the blockage factor for which a valid physical real solution for Eq. (9) exists. An equilibrium is reached by a water level decrease because the velocity \( V_1 \) at the cross section is higher than the undisturbed velocity \( V \).

At a (high) ship's speed \( V \) the velocity \( V_1 \) cannot increase further and the water in front of the vessel accumulates and induces a pressure wave. This pressure wave travels at a higher speed than the vessel. This causes an increasing water level \( \zeta \) in front of the vessel. Because of this high water volume in front of the vessel, a decrease of the water level occurs at the stern as not enough water can flow to the back of the vessel (Fig. 8). In this transcritical speed range a stationary equilibrium is not reached.

When the ship sails at a speed higher than the critical speed (supercritical speed range) the velocity \( V_1 \) will be lower than the forward speed of the vessel \( V \) and a new equilibrium will be
reached when the forward speed of the pressure wave $V_{PW}$ equals the forward speed of the vessel. This paper, however, focuses only on the subcritical speed range.

When Eq. (9) is solved for $V_1(x)/V$, the local speed difference $dV$ can be calculated for each longitudinal position $x$ along the vessel. With this speed difference the local water level decrease $z(x)$ and the local pressure drop $p(x)$ are known. The integration of the pressure drop over the entire hull results in a vertical force $Z$ and trim moment $M$.

$$ Z = \rho g \int_{x_0}^{x} \zeta(x) B(x) dx $$  \hspace{1cm} (14)

$$ M = \rho g I_{LM} $$ \hspace{1cm} (15)

The vertical force $Z$ and trim moment $M$ equal the hydrostatic force and moment:

$$ Z = \rho g A_W z_{VM} $$ \hspace{1cm} (16)

$$ M = \rho g I_{LM} $$ \hspace{1cm} (17)

This results in a running trim and mean sinkage of the vessel:

$$ z_{VM} = \int_{x_0}^{x} \frac{\zeta(x) B(x)}{B(x)} dx $$ \hspace{1cm} (18)

$$ t_M = \int_{x_0}^{x} \frac{\zeta(x) B(x)}{I_{LM}} B(x) x^2 dx $$ \hspace{1cm} (19)

The running sinkage at the FP (forward perpendicular) $z_{VF}$ and at the AP (aft perpendicular) $z_{VA}$ can now be calculated:

$$ z_{VF} = z_{VM} - \frac{L_{pp}}{2} t_M $$ \hspace{1cm} (20)

$$ z_{VA} = z_{VM} + \frac{L_{pp}}{2} t_M $$ \hspace{1cm} (21)

The trim and sinkage of the vessel (or the changing water level) will change the cross section $A_c(x, \zeta)$ and the blockage ratio. An iteration can be started to calculate the new trim and sinkage based upon this new cross section $A_c(x, \zeta)$ until a final equilibrium is reached. The final trim and sinkage, however, change little after the first iteration.

4.2. Comparison with model tests

The running sinkage at the FP ($z_{VF}$) and at the AP ($z_{VA}$) determined with the D&F-method can now be compared with the measurements of the squat when the ship model was towed at the centreline of the installed fairway. As an example, in Fig. 9 and Fig. 10 the values of the running sinkage at the FP and at the AP, respectively, during model tests performed in a rectangular canal with a width $W=5.0 B$ are plotted versus the sinkage values calculated with the D&F-method, for all tested underkeel clearances and forward speeds. Although a strong correlation (Table 5) between the calculated and measured sinkage is found for this particular fairway, a large absolute deviation between calculated and measured values is observed. The influence of water depth and forward speed was minimal on the ratio between the values calculated with the D&F-method and the measured sinkage at the FP or AP (e.g., Figs. 9 and 10).

The ratio between the measured running sinkage and calculated running sinkage appears to be different for each width of the test section. In Figs. 11 and 12 this ratio (and deviation) is plotted versus the ratio of the ship’s beam and width of the test section. The correlation for the sinkage at the FP is almost linear.
The coefficient of determination \( R^2 \) for the sinkage at the FP is 0.977, except for the extremely narrow test section. For a test section of 1.70 times the beam of the ship, the D&F-method and measurements for the sinkage at the FP almost coincide. When the test section is narrower, the D&F-method overestimates the sinkage. A plausible explanation for this phenomenon is that the test section is open at the outlet, allowing the vessel to act as a piston and push water out of the test section; in this way, not all the displaced water has to flow around the vessel, resulting in a measured sinkage lower than the calculated sinkage. For cross sections wider than about 1.70 times the ship’s beam, the measured sinkage at the FP increases with decreasing distance to the vertical wall. The influence of this eccentricity cannot be neglected and will be captured in the newly proposed mathematical model.

Because the D&F-method does not result in an accurate absolute prediction of the sinkage at the FP and AP, and also the influence of the lateral position of the vessel in the fairway nor the propeller action were taken into account, a new mathematical model that addresses these shortcomings is proposed.

### 5. New mathematical model

#### 5.1. Running sinkage at the FP \( z_{VF} \)

Based upon the measured running sinkage at the FP \( z_{VF} \) and the D&F-method for every ship speed, eccentricity \( y \) and water depth \( h \), a new width can be calculated which results in the same running sinkage at the FP as the measured running sinkage during the model tests. This results in an equivalent width \( W_{eq} \) based upon the measured sinkage at the FP \( z_{VF} \). As discussed in Section 4, this new canal width \( W_{eq} \) is smaller than the real canal width when the canal is wider than about 1.7 the ship’s beam and vice versa. The influence of the eccentricity \( y \) also differs for different

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**Table 5**

<table>
<thead>
<tr>
<th>Canal width ( W )</th>
<th>1.05 ( B )</th>
<th>1.25 ( B )</th>
<th>1.70 ( B )</th>
<th>2.50 ( B )</th>
<th>5.00 ( B )</th>
<th>9.05 ( B )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Coefficient of determination ( R^2 ) for sinkage at the FP D&amp;F/measured</td>
<td>0.9188</td>
<td>0.9556</td>
<td>0.9415</td>
<td>0.9991</td>
<td>0.9996</td>
<td>0.9989</td>
</tr>
<tr>
<td>Coefficient of determination ( R^2 ) for sinkage at the AP D&amp;F/measured</td>
<td>0.8925</td>
<td>0.9809</td>
<td>0.9434</td>
<td>0.9947</td>
<td>0.9923</td>
<td>0.9763</td>
</tr>
</tbody>
</table>
canal widths but remains consistent with \((1/2)B - (1/2)B - y = -(1/2)B - (1/2)W + y\) for all speeds and water depths.

A function dependent on the original canal width \(W\) and eccentricity \(y\) that results in an accurate prediction of the equivalent width \(W_{eq}\) can be derived. The model will be split into a part that takes into account the total width of the cross section \((W_0)\), and a part that takes into account the influence of the eccentric position in the rectangular canal \((W_y)\):

\[
W_{eq}(W,y) = W_0 + W_y
\]  

(22)

For ships navigating in the centreline of the rectangular canal, the following expression appears to give acceptable results:

\[
W_0 = W \frac{\xi_1 B}{\xi_2 B + W}
\]  

(23)

which implies that the equivalent width \(W_0\) is greater than the actual width \(W\) for \(W/B > \xi_1 - \xi_2 = 1.80 \pm 0.25\).

The contribution of eccentricity to the equivalent width is expressed by:

\[
W_y = W \left( \frac{\xi_1 B}{\xi_2 B + W} + \xi_4 \right) \left[ \frac{B}{2} - y - \frac{B}{2} - y \right]
\]  

(24)

or

\[
W_y = W \left( \frac{\xi_1 W_0}{\xi_2 W + y} + \xi_4 \right) \left[ \frac{B}{2} - y - \frac{B}{2} + y \right]
\]  

(25)

The last factor in Eqs. (23) and (24) takes a value between 0 and 1, the latter occurring if the ship sails with zero lateral clearance in an infinitely wide canal and 0 when the ship sails on the centreline \((y = 0)\). For the range of \(W/B\) values considered in the test program (1.05–9.05), the maximum values of the factor within the absolute value marks are physically limited to a value between 0.375 for the narrowest canal and 0.94 for the widest canal. As the sinkage increases due to eccentricity, \(W_y\) is expected to be negative.

Based upon the results of all model tests, but excluding the tests at a canal with width 1.05 \(B\), the \(\xi\)-values listed in Table 6 were obtained.

This new proposed canal width \(W_{eq}\) can be used as input to recalculate the sinkage at the FP with the D&F-method. Fig. 14 is similar to Fig. 13 for the sinkage at the FP, but also includes the new calculated sinkage at the FP \((W_{eq})\) that includes the relative lateral position effects \((W_y \neq 0)\).

Fig. 15 shows the measured sinkage at the FP versus the new calculated running sinkage. A satisfying correlation \((R^2=0.965)\) is found. It should be taken into account that the range of the canal width is very extreme from more than 9 times the ship’s beam up to only 1.05 times the ship’s beam (similar as in the current Panama locks). Some of the discrepancies occur at this smallest canal width because the regression model does not take account of the experiments carried out in this canal.

### Table 6

Regression results for the equivalent width.

<table>
<thead>
<tr>
<th>Coefficient</th>
<th>Value</th>
<th>Std. error</th>
<th>95% Confidence interval</th>
<th>Lower bound</th>
<th>Upper bound</th>
</tr>
</thead>
<tbody>
<tr>
<td>(\xi_1)</td>
<td>8.260</td>
<td>0.134</td>
<td>7.997</td>
<td>8.524</td>
<td></td>
</tr>
<tr>
<td>(\xi_2)</td>
<td>6.458</td>
<td>0.211</td>
<td>6.043</td>
<td>6.873</td>
<td></td>
</tr>
<tr>
<td>(\xi_3)</td>
<td>0.151</td>
<td>0.075</td>
<td>0.004</td>
<td>0.298</td>
<td></td>
</tr>
<tr>
<td>(\xi_4)</td>
<td>−0.317</td>
<td>0.051</td>
<td>−0.418</td>
<td>−0.216</td>
<td></td>
</tr>
</tbody>
</table>

5.2 Running sinkage at the AP \(z_{VA}\)

The trim according to D&F does not correspond with the trim measured during the model tests. Since the ratio \(z_{VT}/z_{VA}\) with the D&F-method is almost constant, no combination of width of the canal and forward speed can be found with a similar running sinkage and trim combination as measured during the model tests. Therefore, an equivalent width is sought based on the running sinkage at the FP \(z_{VF}\) that can be used subsequently with a new forward speed \(V_{eq}\) that produces the same running sinkage at the AP \(z_{VA}\) as measured during the model tests.

Only during model tests in the widest channel \((W=9.05 B)\) was the ship model systematically tested with a propeller rate according to self propulsion in open water (at each tested speed) and with a fixed propeller (propeller rate = 0 rpm). Fig. 16 presents the measured sinkage at the FP with a fixed propeller versus the measured sinkage at the FP with a propeller with self propulsion. All points are very close to the first bisector (dotted line in Fig. 16) indicating a negligible influence of the propeller action on the sinkage at the FP. Fig. 17 presents similar results for the sinkage at the AP. As expected, the propeller rate has an increasing effect on the running sinkage at the AP.

To take the effect of the propulsion into account in the mathematical model, the speed \(V_T\) is introduced based upon the propeller thrust and propeller diameter. This speed represents in a simplified way the axial speed in the flow field behind the propeller induced by the propeller. The flow around the stern will
increase due to the forward propeller action and will decrease when the propeller is fixed. A fraction of the speed \( V_T \) is added (or subtracted) to (from) the forward speed of the vessel to take into account this propeller effect.

\[
V_T = \text{sign}(T_P) \sqrt{\frac{8T_P}{\mu n D^2}}
\]  

(26)

\[
V_{\text{eq}} = V + \xi_5 V_T
\]  

(27)

Based upon the sinkage at the AP, the equivalent speed can be calculated as in eq. (27) with the forward speed of the vessel \( V \), width of the fairway \( W \), \( V_T \) and beam of the vessel as variables.

\[
V_{\text{eq}}(B,V,V_T,W) = (V + \xi_5 V_T) \left( \frac{\xi_6 B}{W} + \xi_7 \right)
\]  

(28)

The following \( \xi \)-values are determined with the measured sinkage at the AP during the model tests, but excluding the tests at a canal with width 1.05 \( B \) (Table 7).

<table>
<thead>
<tr>
<th>Coefficient</th>
<th>Value</th>
<th>Std. error</th>
<th>95% Confidence interval Lower bound</th>
<th>Upper bound</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \xi_5 )</td>
<td>0.053</td>
<td>0.009</td>
<td>0.036</td>
<td>0.070</td>
</tr>
<tr>
<td>( \xi_6 )</td>
<td>0.603</td>
<td>0.022</td>
<td>0.561</td>
<td>0.646</td>
</tr>
<tr>
<td>( \xi_7 )</td>
<td>0.696</td>
<td>0.008</td>
<td>0.680</td>
<td>0.712</td>
</tr>
</tbody>
</table>

Fig. 16. The sinkage at the FP with a fixed propeller versus the sinkage at the FP with the same test parameters but with the propeller rate according to open water self propulsion for all tested speeds, water depths and lateral positions in a canal section of 9.05 \( B \) wide.

Fig. 17. The sinkage at the AP with a fixed propeller versus the sinkage at the AP with the same test parameters but with the propeller rate according to open water self propulsion for all tested speeds, water depths and lateral positions in a canal section of 9.05 \( B \) wide.

Fig. 18. The measured sinkage at the AP vs. the calculated sinkage based upon the newly proposed model with the equivalent width \( W_{eq} \) and forward speed \( V_{eq} \). The correlation is satisfactory \( (R^2=0.974) \), except for the smallest canal widths, which were excluded from the regression analysis.

6. Future research opportunities

The newly proposed mathematical model will be validated against the sinkage of other ship types. The independence of the coefficients relative to the ship’s geometry will also be tested. The cross section of a natural river or a manmade canal is seldom rectangular, therefore other cross sections than rectangular cross sections of the fairway will be investigated. As a consequence a new formulation for the influence of the lateral position of the vessel on the equivalent width will be inevitable. Since this squat model can cope with high blockage ratios, it might be useful to investigate the hydrodynamics when sailing into or out of a lock. However, during this research project the inlet and outlet of the test section was always open, which can cause an important difference between these tests and a real lock.

7. Conclusions

Model tests have been carried out with a scale model of a very large crude carrier in canals of different widths. The sinkage of the ship has been measured at FP and AP and the measurements were compared with the empirical model determined by Dand and Ferguson (D&F).

The original D&F method for the sinkage at the FP corresponds to the measurements when \( W \approx 1.8 \ B \). When the section is wider the original D&F method underestimates the sinkage at the FP, while an overestimation is found when the section is smaller. Overall the original D&F method underestimates the running sinkage at the AP within 25% and could not take account of the effect eccentric sailing has on the running sinkage.
A new mathematical model was introduced based on an equivalent width that provides a better input for the D&F method. The sinkage at the FP and at the AP of a vessel sailing in a rectangular fairway parallel to the wall at any depth, width, subcritical speed, lateral position and propeller rate can then be determined with seven coefficients (Tables 6 and 7). The mathematical model takes account of the magnitude and distribution of the cross-sectional areas of the vessel as well as the longitudinal distribution of the beam on the water line of the vessel. The fairway can vary from an infinite open ocean (water depth and width infinite) to an extremely restricted canal in width and water depth with a rectangular cross section. In the near future the model will be validated against non-rectangular cross sections and other ship types.

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