Abstract: Our capability for predicting beach and dune erosion has improved for the last three decades but the recovery of an eroded beach above the mean sea level (MSL) cannot be predicted at present. The cycle of beach erosion and recovery will need to be predicted for the long-term maintenance of a sand beach with a dune for coastal flooding reduction. The cross-shore numerical model CSHORE is extended and evaluated using natural beach erosion and recovery data along 16 cross-shore lines spanning 5 km alongshore for the duration of 272 days. CSHORE predicts beach and dune erosion fairly well as has been shown in the previous comparisons. The bed load formula used in CSHORE is adjusted to predict the accreted beach profile with a berm. The computed beach profile evolutions are shown to be affected little by the alongshore gradient of the longshore sediment transport rate along the straight beach. The extended CSHORE predicts both erosion and accretion above MSL within a factor of about 2.

DOI: 10.1061/(ASCE)WW.1943-5460.

CE Database subject headings: Beach; Sand; Dune; Erosion; Storm; Recovery.

Introduction

Beach nourishment has been widely adopted in the U.S. in order to maintain a wide beach with a high dune and provide coastal storm protection and flooding damage reduction. Numerical models such as SBEACH (e.g., Wise et al. 1996) and XBeach (Roelvink et al. 2009) have been developed to predict storm-induced beach and dune erosion and design the beach and dune profile required for storm protection. XBeach is a horizontally two-dimensional model and has been used to assess storm impacts only. XBeach may not predict shoreline accretion under moderate and calm wave conditions (van Thiel de Vries 2009). The amount and frequency of the periodic beach nourishment may
increase due to the combined effects of sea level rise and storm intensification due to global warming. The cycle of beach erosion and recovery needs to be predicted to assess the long-term performance of a nourished beach in a changing climate. Presently, the long-term shoreline change is predicted using one-line models such as GENESIS (e.g., Hanson 1989) without regard to beach profile evolution.

The cross-shore numerical model CSHORE developed by Kobayashi et al. (2010) consists of the combined wave and current model based on the time-averaged continuity, momentum and wave energy equations coupled with the transparent formulas for suspended sand and bed load transport rates which have been shown to synthesize available sediment transport data and formulas. CSHORE includes a probabilistic model for the wet and dry zone above the shoreline to predict dune erosion and overwash as well as longshore sediment transport in the swash zone. CSHORE has been compared with a number of small-scale and large-scale laboratory data as well as field data. However, these comparisons do not include the recovery above MSL of eroded beaches after storms. The computations for field data on beach and dune erosion were limited to the storm duration only even though the beach profiles were measured well before the storm and about a week after the storm. In this study, CSHORE is compared with the entire duration of beach erosion and recovery data from the Atlantic coast in Delaware. In addition, CSHORE is extended to multiple cross-shore lines so as to include the alongshore gradient of the longshore sediment transport rate in the beach profile evolution prediction.

In the following, field data are presented first. The existing model CSHORE is described concisely and the modifications of CSHORE are presented. The modified CSHORE is compared with the field data and the sensitivities of the computed profiles to the modifications are assessed. Finally, the findings of this study are summarized. This paper presents the essential parts of the report by Jung and Kobayashi (2011).

**Field Data**

The Atlantic sandy coast of 40 km length in Delaware is suffering from chronic beach erosion and has been maintained by periodic beach nourishment for the last two decades (Figlus and Kobayashi 2008).
This study is limited to 16 beach profiles at Rehoboth and Dewey beaches as depicted in Figure 1. These 16 profiles were measured on 29 October 1992, 18 December 1992 and 27 July 1993 before the major nourishment of these beaches in 1994 (Garriga and Dalrymple 2002). These profile data indicate the cycle of erosion and recovery of the natural beaches. The beach sand was fairly well sorted and its median diameter was 0.33 mm. Data from the tide gauge at Lewes, Delaware in Fig. 1 is used to specify the hourly variation of the water level. A wave gauge (DE001 in Fig. 1) was located at a depth of about 9 m off the coast of Dewey Beach. The time series of the root-mean-square wave height $H_{rms}$, spectral peak period $T_p$, and peak spectral wave direction $\theta$ are available every four hours. The available time series are interpolated to obtain the hourly time series corresponding to the hourly water level data.

A storm attacked Rehoboth and Dewey beaches on 10 December 1992. The comparison of the beach profiles measured on 29 October 1992 and 18 December 1992 indicated considerable erosion above the mean sea level (MSL) for the 16 profiles. The eroded beaches recovered almost completely when the 16 profiles were measured on 27 July 1993. The erosion and accretion periods analyzed in this study are summarized in Table 1 which lists the average values of the water level above MSL, $H_{rms}$, $T_p$ and $\theta$ during each period. The wave angle is taken to be positive clockwise from the normal to the straight shoreline inclined at an angle of 8.86° counterclockwise from the north. The net longshore sediment transport along these beaches is northward.

Fig. 2 shows the measured time series of the water level, $H_{rms}$, $T_p$ and $\theta$ during the erosion period where the abscissa is the number of days since 29 October 1992. The horizontal line in each panel is the average value listed in Table 1. The storm of 10 December 1992 lasted about 4 days with a peak water level of almost 2 m, a peak wave height of almost 3 m, wave periods of about 15 s, and wave directions of about 20°. The measured time series of the water level, $H_{rms}$, $T_p$ and $\theta$ for the accretion period indicated several minor storms with a peak water level of about 1 m, a peak wave height of about 2 m, and a storm duration of 1 to 2 days. The water level, wave height and duration of these minor storms were clearly less than those of the major storm during the erosion period. The entire time series of the water level, $H_{rms}$, $T_p$ and $\theta$ are specified as input at the seaward boundary of
CSHORE in the water depth of about 9 m in order to simulate the beach profile evolution for the entire erosion and accretion periods. The alongshore variation of the wave conditions is assumed to be negligible.

The three beach profiles along each of the 16 cross-shore lines measured on 29 October 1992, 18 December 1992, and 27 July 1993 are plotted together to determine the cross-shore overlapping zone of the three profiles along each cross-shore line. The onshore coordinate $x$ along each cross-shore line is taken as $x = 0$ at the seaward boundary of CSHORE and $x = x_m$ at the landward boundary of the overlapping zone above MSL. The values of $x_m$ among the 16 profiles varied in the range of 309 to 634 m and the average value of $x_m$ was 417 m. During the erosion period, erosion and deposition occurred above and below MSL, respectively, and the shoreline location at MSL did not vary much. During the accretion period, accretion and erosion occurred above and below MSL, respectively, and the beach profile at the beginning of the erosion period was similar to the corresponding profile at the end of the accretion period. The measured profiles are presented later when they are compared with the computed profiles using CSHORE. The shoreline location did not represent the observed erosion and accretion on these beaches.

Table 2 summarizes the profile data for Rehoboth (R) and Dewey (D) beaches where N and S indicate the northern and southern segments of each beach. The numeral after N or S is related to the alongshore distance from the middle (numeral = 0) of each beach. The alongshore coordinate $y$ is taken to be positive northward in the direction of the net longshore sediment transport with $y = 0$ at the most southern line DS40. The landward extent of some of the overlapped profiles was too short to resolve the dune profile change adequately. The sand volume per unit alongshore length above MSL is calculated using the beach profile measured on 29 October 1992 for each line to indicate the relatively large alongshore variation of the beach profile above MSL. The erosion volume per unit length above MSL is calculated using the measured profiles at the beginning and end of the erosion period. The accretion volume per unit length above MSL is calculated using the measured profiles at the beginning and end of the accretion period. The erosion and accretion volumes were similar and
did not vary alongshore as much as the initial sand volume. The erosion and accretion above MSL were more uniform alongshore than the initial 16 beach profiles.

**Numerical Model**

The cross-shore numerical model CSHORE assumes unidirectional irregular waves and alongshore uniformity along each cross-shore line. The hydrodynamic model in CSHORE predicts the mean and standard deviation of the free surface elevation above the still water level (SWL) and depth-averaged cross-shore and longshore velocities. The time-averaged continuity, cross-shore momentum, longshore momentum and wave energy equations together with Snell’s law are used in the wet zone seaward of the still water shoreline (Kobayashi et al. 2007) where the roller effect is found to be negligible in the present profile evolution comparison and is not included in the following computed results. The breaker ratio parameter $\gamma$ is taken as its typical value of $\gamma = 0.7$ where the computed profile evolution is not sensitive to $\gamma = 0.7$ or 0.8. The bottom friction factor $f_b$ is taken as $f_b = 0.02$ calibrated for longshore current and sediment transport by Kobayashi et al. (2007). In the wet and dry zone, the wave angle is assumed to be small and remain the same as the wave angle at the still water shoreline. The time-averaged continuity and cross-shore momentum equations are used together with the wet probability for the presence of water (Kobayashi et al. 2010).

The sediment transport model in CSHORE consists of separate formulas for bed load and suspended load. The volume of suspended sediment per unit bottom area is expressed in terms of the wave energy dissipation rates due to wave breaking and bottom friction (Kobayashi et al. 2008). The suspended sediment is assumed to be transported by the cross-shore and longshore currents (Kobayashi et al. 2007). The effect of onshore flow due to wave overtopping is included to estimate the cross-shore suspended sediment transport rate $q_{sx}$ (Figlus et al. 2011) where the wave overwash parameter $a_o$ is taken as $a_o = 0.1$ but the computed profile evolution is not very sensitive to $a_o$ in the range of 0.1 to 1.0. The longshore suspended sediment transport rate $q_{sy}$ is proportional to the longshore current which is sensitive to the bottom friction factor $f_b = 0.02$ assumed in the following.

The cross-shore and longshore bed load transport rates $q_{bx}$ and $q_{by}$ are expressed as a function
of the mean and standard deviation of the cross-shore and longshore velocities (Kobayashi et al. 2009). The effect of the cross-shore bottom slope on \( q_{bx} \) is included to reduce the onshore bed load transport rate \( q_{bx} \) on a steep upward slope. The rates \( q_{bx} \) and \( q_{by} \) are proportional to the empirical bed load parameter \( b \). Kobayashi et al. (2008) calibrated \( b \) using 20 water tunnel tests, 4 large-scale wave flume tests, and 24 sheet flow tests. The calibrated range for these tests with nonbreaking waves on horizontal bottoms was \( b = 0.001 – 0.004 \) and the typical value of \( b = 0.002 \) has been used to predict beach and dune erosion for which offshore suspended sediment transport is dominant. The parameter \( b \) needs to be calibrated for beach accretion in the surf and swash zones. For the present comparison with the beach erosion and recovery data, \( b = 0.002 \) is found to reproduce the 16 eroded profiles as well as for the previous comparisons (e.g., Kobayashi et al. 2010) but could not reproduce the 16 accreted profiles because of the deposition near the shoreline at MSL unlike the observed upward berm reconstruction above MSL. To cause the landward increase of the onshore bed load transport rate \( q_{bx} \), use is made of \( b = B (0.5 + Q) \) with \( B = 0.002 \) where the computed fraction \( Q \) of irregular breaking waves is zero for no wave breaking and unity when all waves break. Consequently, \( b \) increases from 0.001 outside the surf zone to 0.003 near the still water shoreline in the wet zone and in the swash zone. The sensitivity of the computed profile change to the input value of \( B \) is presented after the comparison using \( B = 0.002 \).

The cross-shore beach profile evolution along each of the 16 lines is predicted using the continuity equation of bottom sediment

\[
\left(1 - n_p \right) \frac{\partial z_b}{\partial t} + \frac{\partial q_x}{\partial x} + \frac{\partial q_y}{\partial y} = 0 \tag{1}
\]

where \( n_p \) = porosity of bottom sediment assumed as \( n_p = 0.4 \); \( t \) = morphological time; \( z_b \) = bottom elevation with \( z_b = 0 \) at MSL; \( q_x \) = total cross-shore sediment transport rate per unit width given by \( q_x = (q_{sx} + q_{bx}) \); and \( q_y \) = total longshore sediment transport rate per unit width given by \( q_y = (q_{sy} + q_{by}) \). The alongshore gradient of \( q_y \) is included in Eq. (1) to allow the gradual alongshore variation of longshore sediment transport although the alongshore uniformity is assumed for each cross-shore line. CSHORE is modified to allow the simultaneous computation of the multiple cross-shore lines and
include the effect of the alongshore gradient of \( q_y \) on the temporal variation of \( z_b \) along each line in approximate but computationally efficient manners.

To solve Eq. (1) numerically, the bottom elevation \( z_b \) is expressed as \( z_b = (z_x + z_y) \) and Eq. (1) is rewritten as

\[
\left(1 - n_p\right) \frac{\partial z_x}{\partial t} + \frac{\partial q_x}{\partial x} = 0 \tag{2}
\]

\[
\left(1 - n_p\right) \frac{\partial z_x}{\partial t} + \frac{\partial q_y}{\partial y} = 0 \tag{3}
\]

Eq. (2) is solved using the same numerical method as that used for the case of no alongshore gradient of \( q_y \) in Eq. (1) (Kobayashi et al. 2007). Eq. (3) is integrated with respect to time \( t \) for the duration of constant water level and wave conditions where \( \Delta t = 1 \) \( h \) in the present computation. The bottom elevation change \( \Delta z_y \) is expressed as

\[
\left(1 - n_p\right) \Delta z_y + \frac{\partial v_y}{\partial y} = 0 \quad ; \quad v_y = \int_{t^\Delta}^{t^\Delta+\Delta t} q_y \, dt \tag{4}
\]

where \( v_y \) = longshore sediment transport volume per unit width for the duration of \( \Delta t \) which is obtained during the computation of \( z_x \). The value of \( \Delta z_y \) based on Eq. (4) is added to the bottom elevation \( z_y \) computed using Eq. (2) at the interval of \( \Delta t \). This numerical procedure assumes that the time scale of \( z_x \) in Eq. (2) is smaller than that of \( z_y \) in Eq. (3). This procedure is also convenient because the time step size used to solve Eq. (2) is constrained by the numerical stability criterion and can vary among the cross-shore lines.

The alongshore length scale is assumed to be larger than the cross-shore length scale to be consistent with the profile layout in Table 2 where the spacing of the cross-shore lines is as large as the cross-shore distance \( x_m \). No additional cross-shore line is added to estimate the alongshore gradient of \( v_y \) in Eq. (4) which is expressed as

\[
\frac{\partial v_y}{\partial y} = \frac{1}{L_y} \frac{\partial V_y}{\partial y} \quad ; \quad V_y = \int_{x_m}^{x_m} v_y \, dx \quad ; \quad \int_{x_m}^{x_m} \frac{dx}{L_y} = 1 \tag{5}
\]

where \( V_y = longshore \) sediment transport volume across the entire cross-shore line; and \( L_y = cross- \)
shore length related to the cross-shore variation of $\Delta z_y$ in view of Eq. (4). If $L_x = x_m$ is assumed, the alongshore gradient of $V_y$ causes the uniform change of $\Delta z_x$ across the cross-shore line.

The cross-shore length scale $L_x$ is assumed to be related to the cross-shore profile change

$$L_x = \frac{A_x}{|\Delta z_x|} ; \quad A_x = \int_0^{x_m} |\Delta z_x| \, dx$$

(6)

where $\Delta z_x = \text{bottom elevation change based on Eq. (2) during time } t \text{ to } (t + \Delta t)$. Eq. (6) satisfies the requirement of $L_x$ in Eq. (5). Substitution of Eqs. (5) and (6) into Eq. (4) yields

$$\Delta z_y = -\frac{|\Delta z_x|}{(1-n_p)A_x} \frac{\partial V_y}{\partial y}$$

(7)

which shows that $\Delta z_y$ is proportional to the magnitude of the bottom elevation change $\Delta z_x$ due to the cross-shore sediment transport. This eliminates the need to specify the seaward and landward limit of the profile change for one-line models. The sign (accretion or erosion) of $\Delta z_y$ in Eq. (7) depends on the alongshore gradient of $V_y$ and remains the same along the cross-shore line. The alongshore gradient of $V_y$ is approximated by an upstream differencing method (e.g., Anderson et al. 1984) for its numerical stability where the upstream direction is determined by the direction of $V_y$. Eq. (6) may not be rigorous but allows the use of a large alongshore spacing of two adjacent cross-shore lines.

The input to CSHORE includes the hourly time series of the water level above MSL and the incident wave conditions represented by $H_{rms}$, $T_p$ and $\theta$ at the seaward boundary $x = 0$. Wave setup is assumed to be zero at $x = 0$. The beach sand is characterized by the median diameter of 0.33 mm and the fall velocity of 5 cm/s. The initial beach profile at time = 0 is the measured profile along each of the 16 cross-shore lines on 29 October 1992 for the erosion period and that on 18 December 1992 for the accretion period to assess the capability and limitation of CSHORE for predicting beach erosion and recovery separately. The cross-shore nodal spacing is 3 m for each cross-shore line. The computation time was 9 min for the erosion period of 50.8 days and 30 min for the accretion period of 220.8 days. The short computation time facilitated the above modifications of CSHORE which required a number of trial computations.
Comparison of Numerical Model with Field Data

The measured and computed profiles at the end of each period are compared for each of the 16 lines. The comparisons at RN38 (northern end), DS10 (dune overwash zone), and DS40 (southern end) in Table 2 are presented in the following to represent the alongshore variation of the agreement among the 16 lines. The computed rates \(q_{bx}, q_{sx}, q_{by}, q_{sy}\), and \(q_y\) are integrated with time to obtain the cumulative sediment volumes per unit width transported during each period and to examine the computed cross-shore and longshore sediment transport.

The measured initial profile and the measured and computed profiles at the end of the erosion period are presented in the top panel of Figs. 3 – 5. The elevation \(z\) is zero at the mean sea level (MSL). The offshore zone of the negligible profile change is omitted in these figures. The eroded berm and dune profile is predicted well for RN38 in Fig. 3. Dune overwash occurred at DS10 in Fig. 4 but the measured profile did not extend sufficiently landward. Dune overwash also occurred at DN0 located 305 m north of DS10. The eroded profile above MSL at DS10 is underpredicted partly because of the boundary condition of zero cross-shore gradient of \(q_x\) at \(x = x_m\) used to solve Eq. (2). The accreted profile below MSL may be related to the erosion above MSL but its cause is not certain for lack of the landward profile data because the eroded sand must have also been transported landward. The erosion of the berm and dune at DS40 in Fig. 5 is underpredicted and the nearly horizontal eroded profile near MSL cannot be predicted by CSHORE.

The cumulative cross-shore sediment volumes per unit width transported during the erosion period are plotted in the middle panel of Figs. 3 – 5. CSHORE predicts the onshore (positive) bed load transport and the offshore (negative) suspended load transport for the case of negligible wave overwash. The total load is the sum of bed load and suspended load. The onshore bed load and offshore suspended load are computed to be of similar magnitude and the maximum near MSL. The small total load is plotted in Fig. 6 where small fluctuations occur near and above MSL. For RN38 and DS40 with no or minor dune overwash, the offshore total load is the maximum near MSL and approaches zero landward. CSHORE predicts the small onshore sediment transport outside the surf zone where the onshore bed load transport is predicted to exceed the offshore suspended sediment transport.
transport. This onshore sediment transport rate varies gradually in the cross-shore direction and causes very small profile changes. For DS10 with major dune overwash, the total load is small seaward of MSL and increases landward of MSL because of the onshore flow associated with dune overwash.

On the other hand, the cumulative longshore sediment volumes per unit width transported during the erosion period are plotted in the bottom panel of Figs. 3 – 5. The net longshore transport of bed load and suspended load is positive and northward along the Delaware Atlantic coast. The suspended load is dominant for the longshore sediment transport and the maximum near MSL. It is noted that CSHORE has been shown to predict the cross-shore distribution of longshore sediment transport under constant water level and wave conditions (Kobayashi et al. 2007). The large longshore sediment transport in the swash zone explains the maximum cumulative load near MSL.

An option is provided in CSHORE to compute the beach profile change without (IQYDY=0) and with (IQYDY=1) the correction term given by Eq. (7). The computed bottom elevation $z_b$ at the end of the erosion period for IQYDY=0 is subtracted from that for IQYDY=1. The elevation difference indicates the effect of the alongshore gradient of the longshore sediment transport on the cross-shore profile change. The computed elevation difference shown in Fig. 7 turns out to be of the order of 10 cm or less in the zone of the profile changes in Figs. 3 - 5. The positive (negative) elevation difference implies accretion (erosion) due to the longshore sediment transport gradient. The cross-shore variation of the elevation difference depends on the length scale $L_x$ which is assumed to be given by Eq. (6). The computed elevation difference is much smaller than the profile changes of the order of 1 m or more in Figs. 3 – 5.

Figs. 8 – 10 show the comparison at the end of the accretion period for RN38, DS10 and DS40, respectively. CSHORE cannot reproduce the berm sufficiently for RN38 and DS10 but predicts the accreted profile at DS40 well. The effect of the alongshore gradient of the longshore sediment transport is quantified by computing the elevation difference in the same way as in Fig. 7. The computed elevation difference is less than 5 cm and small for the accretion period as well. The dune at DS10 (dune overwash zone) on 27 July 1993 may be related to the emergency nourishment of
4,400 m$^3$ in July 1993 (Garriga and Dalrymple 2002) which is not included in the computed profile for lack of the fill placement information. Assuming the placement of 4,400 m$^3$ between DN0 and DS10 with the alongshore distance of 305 m, the placement volume per unit length is 14 m$^3$/m which is small in comparison to the accretion volume per unit length of 86 and 70 m$^3$/m at DN0 and DS10, respectively, as listed in Table 2. It is also noted that wind-blown sediment transport (e.g., U.S. Army Engineer Research and Development Center 2002) may not be negligible for the accretion period of 220.8 days.

The cumulative cross-shore bed load and suspended load in the middle panel of Figs. 8 – 10 are larger than those in Figs. 3 – 5 for the erosion period of 50.8 days. The cumulative cross-shore total load appears to be almost zero in these figures and is plotted separately in Fig. 11. The total load is positive (onshore) and the maximum near MSL even at DS10. The magnitude of the onshore total load in Fig. 11 is similar to the magnitude of the offshore total load in Fig. 6. This explains the recovery of the eroded profile. The sum of the onshore total load at $x = 0$ (about 9 m depth) in Figs. 6 and 11 is about 15 m$^3$/m for the duration of 272 days and of the same order of magnitude as the onshore sediment transport rate of 5 – 10 m$^3$/m per year in 20 m depth in Holland estimated by van Rijn (1997). On the other hand, the cumulative longshore bed load, suspended load and total load in the bottom panel in Figs. 8 – 10 are similar in their magnitudes to those in Figs. 3 – 5 for the erosion period but the cross-shore extent is narrower for the accretion period with smaller wave heights as indicated in Table 1.

The computed results shown in Figs. 3 – 11 are based on the bed load parameter $b = B(0.5 + Q)$ with $B = 0.002$. The onshore bed load transport rate is proportional to the value of $B$ specified as input to CSHORE. Fig. 5 for DS40 indicates that $B = 0.002$ does not produce sufficient erosion above MSL. The computed results with $B = 0.001$ are shown in Fig. 12. The reduction of $B$ by the factor of 2 reproduces the eroded profile above MSL but the offshore deposition is overpredicted. The cumulative cross-shore and longshore bed load volumes per unit width are reduced by the factor of more than 2 but the corresponding suspended load volumes are reduced as well. This is because the modified bed load changes the beach profile which affects the computed hydrodynamics and the
offshore transport of suspended sediment by undertow current. The hydrodynamics, sediment dynamics and beach profile changes are closely interconnected for the beach profile evolution.

Fig. 13 shows the computed results with $B = 0.003$ for RN38 in comparison to those with $B = 0.002$ shown in Fig. 8. The increase of $B$ by the factor of 1.5 increases accretion above MSL and the cumulative bed load and suspended load. The value of $B = 0.003$ improves the agreement for the accretion volume per unit width above MSL but the accretion occurs on the seaward side of the shoreline at MSL instead of the upward berm reconstruction. This computed profile looks similar to the accreted profile predicted using $b=B$ with $B = 0.002$ before the present modification of $b=B (0.5 + Q)$. The reproduction of the accreted profile above MSL is found to be more difficult than that of the eroded profile above MSL.

Fig. 14 compares the computed erosion and accretion volumes per unit alongshore length above MSL for the 16 lines with the measured values listed in Table 2 where RN38 and DS40 are located at the alongshore coordinate $y = 4975$ m and $y = 0$, respectively. First, the computed volumes for $B = 0.002$ are compared with the measured volumes. The erosion volume is predicted well in the northern (N) segment of Rehoboth Beach. The underprediction of the erosion volume increases southward. On the other hand, the alongshore variation of the agreement for the accretion volume is opposite. The agreement is good at the southern (S) end of Dewey Beach but the underprediction persists northward. The computed results in this study account for the initial profile differences among the 16 lines but the offshore wave conditions at $x = 0$ are assumed to be invariant alongshore.

The erosion volume is predicted better for $B = 0.001$ which cannot predict accretion. The accretion volume is predicted better for $B = 0.003$ which cannot predict erosion. The use of $B = 0.002$ predicts the erosion and accretion volumes within a factor of about 2. The accurate prediction of both erosion and accretion is difficult because of the small difference between the onshore bed load transport and the offshore suspended sediment transport.

The cumulative total sediment volume per unit width transported alongshore and northward is integrated from $x = 0$ to $x = x_m$ to obtain the sediment volume (no void) transported during the erosion and accretion periods. Fig. 15 shows the computed sediment volumes for the 16 lines for $B =$
0.001, 0.002 and 0.003. The longshore sediment transport is less sensitive to the parameter \( B \) for bed load. The computed volume of sediment transported alongshore increases somewhat in the southern segment of Dewey Beach and fluctuates slightly northward during the erosion period. The erosion and accretion in the computed elevation differences in Fig. 7 are consistent with this alongshore variation. The computed sediment volume is almost constant during the accretion period. For \( B = 0.002 \), the average sediment volume transported alongshore is \( 5.4 \times 10^4 \) m\(^3\) during the erosion period of 50.8 days and \( 2.6 \times 10^4 \) m\(^3\) during the accretion period of 220.8 days. The annual net longshore sediment (no void) transport rate has been estimated to be of the order of \( 6 \times 10^4 \) m\(^3\)/year to the north (e.g., Puleo 2010) where the sand porosity is assumed to be 0.4 in this study. The longshore sediment transport rate predicted by CHSORE is of the same order as the previous estimate partly because of the use of the bottom friction factor \( f_b = 0.02 \) calibrated using two data sets obtained in the Large-scale Sediment Transport Facility of the U.S. Army Engineer Research and Development Center (Kobayashi et al. 2007). The bottom friction factor affects longshore current and suspended sediment transport.

**Conclusions**

Beach profile data from Rehoboth and Dewey beaches are analyzed to examine the cycle of beach erosion and recovery during 1992 to 1993 before major beach nourishment was initiated on these beaches in 1994. The field data are used to assess the capability and limitation of the cross-shore numerical model CSHORE for predicting both beach erosion and accretion. CSHORE has been extended to multiple cross-shore lines so as to include the alongshore gradient of the longshore sediment transport rate in the beach profile evolution. The bed load parameter \( b \) in CSHORE is adjusted to increase \( b \) with the fraction of irregular breaking waves in order to reproduce the berm reconstruction above MSL during the accretion period. The eroded and accreted profiles at 16 cross-shore lines spanning 5 km alongshore are predicted within a factor of about 2. The computed beach profile evolution is found to be affected little by the alongshore gradient of the longshore sediment transport. The total longshore sediment transport volumes computed during the erosion and accretion
periods do not vary much alongshore for the present field data. The extended CSHORE will need to be compared to data with larger alongshore variations in order to assess the accuracy of the approximate method proposed for the alongshore gradient of the longshore sediment transport. Furthermore, CSHORE will need to be verified using nourished beach data before it may be applied to improve the beach nourishment design.

Acknowledgments

This study was supported by the U.S. Army Corps of Engineers, Coastal and Hydraulics Laboratory under Contract Nos. W912HZ-10-P-0234 and W912BU-09-C-0023 as well as by the EU THESEUS Project (Innovative Technologies for Safer European Coasts in a Changing Climate). The writers thank Randy Wise for providing the beach profile data used in this study.

References


List of Figures

Fig. 1. Rehoboth Beach, Dewey Beach, 16 cross-shore survey lines, Lewes Tide Gauge, and Wave Gauge DE001.

Fig. 2. Water level, root-mean-square wave height, spectral peak period, and peak spectral wave direction during erosion period.

Fig. 3. Measured and computed beach profiles (top), cumulative cross-shore (middle) and longshore (bottom) sand transport volume per unit width during erosion period for RN38.

Fig. 4. Measured and computed beach profiles (top), cumulative cross-shore (middle) and longshore (bottom) sand transport volume per unit width during erosion period for DS10.

Fig. 5. Measured and computed beach profiles (top), cumulative cross-shore (middle) and longshore (bottom) sand transport volume per unit width during erosion period for DS40.

Fig. 6. Cumulative total cross-shore sand transport volume per unit width during erosion period for RN38 (top), DS10 (middle), and DS40 (bottom).

Fig. 7. Bottom elevation difference on 18 December 1992 caused by alongshore gradient of longshore sediment transport rate for RN38 (top), DS10 (middle), and DS40 (bottom).

Fig. 8. Measured and computed beach profiles (top), cumulative cross-shore (middle) and longshore (bottom) sand transport volume per unit width during accretion period for RN38.

Fig. 9. Measured and computed beach profiles (top), cumulative cross-shore (middle) and longshore (bottom) sand transport volume per unit width during accretion period for DS10.

Fig. 10. Measured and computed beach profiles (top), cumulative cross-shore (middle) and longshore (bottom) sand transport volume per unit width during accretion period for DS40.

Fig. 11. Cumulative total cross-shore sand transport volume per unit width during accretion period for RN38 (top), DS10 (middle), DS40 (bottom).

Fig. 12. Effects of reduced bed load parameter $B = 0.001$ in Fig. 5 based on $B = 0.002$.

Fig. 13. Effects of increased bed load parameter $B = 0.003$ in Fig. 8 based on $B = 0.002$.

Fig. 14. Measured and computed erosion (top) and accretion (bottom) volumes above MSL per unit alongshore length.
Fig. 15. Computed sand (no void) volumes transported alongshore for $B = 0.001$, $0.002$, and $0.003$ during erosion period (top 3 lines) and accretion period (bottom 3 lines).
Table 1. Average Water Level and Wave Conditions during Erosion and Accretion Periods

<table>
<thead>
<tr>
<th>Period</th>
<th>Erosion</th>
<th>Accretion</th>
</tr>
</thead>
<tbody>
<tr>
<td>Start Date</td>
<td>29 October 1992</td>
<td>18 December 1992</td>
</tr>
<tr>
<td>End Date</td>
<td>18 December 1992</td>
<td>27 July 1993</td>
</tr>
<tr>
<td>Number of Days</td>
<td>50.8</td>
<td>220.8</td>
</tr>
<tr>
<td>Average Water Level (m)</td>
<td>0.17</td>
<td>0.13</td>
</tr>
<tr>
<td>Average $H_{rms}$ (m)</td>
<td>0.81</td>
<td>0.59</td>
</tr>
<tr>
<td>Average $T_p$ (s)</td>
<td>9.4</td>
<td>8.3</td>
</tr>
<tr>
<td>Average Wave Angle $\theta$</td>
<td>8.5°</td>
<td>8.5°</td>
</tr>
</tbody>
</table>
Table 2. Sand Volume per Unit Alongshore Length above MSL on 29 October 1992, Erosion Volume above MSL during Erosion Period, and Accretion Volume above MSL during Accretion Period where Sand Volume Include Voids

<table>
<thead>
<tr>
<th>Line</th>
<th>( y ) (m)</th>
<th>( x_m ) (m)</th>
<th>Sand Volume ( (\text{m}^3/\text{m}) )</th>
<th>Erosion Volume ( (\text{m}^3/\text{m}) )</th>
<th>Accretion Volume ( (\text{m}^3/\text{m}) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>RN38</td>
<td>4975</td>
<td>479</td>
<td>162</td>
<td>49</td>
<td>43</td>
</tr>
<tr>
<td>RN31</td>
<td>4750</td>
<td>466</td>
<td>200</td>
<td>59</td>
<td>59</td>
</tr>
<tr>
<td>RN22</td>
<td>4504</td>
<td>379</td>
<td>73</td>
<td>37</td>
<td>38</td>
</tr>
<tr>
<td>RN17</td>
<td>4345</td>
<td>355</td>
<td>80</td>
<td>42</td>
<td>49</td>
</tr>
<tr>
<td>RS0</td>
<td>3830</td>
<td>309</td>
<td>102</td>
<td>50</td>
<td>56</td>
</tr>
<tr>
<td>RS14</td>
<td>3389</td>
<td>363</td>
<td>142</td>
<td>66</td>
<td>77</td>
</tr>
<tr>
<td>RS24</td>
<td>3112</td>
<td>400</td>
<td>166</td>
<td>65</td>
<td>64</td>
</tr>
<tr>
<td>RS36</td>
<td>2744</td>
<td>575</td>
<td>302</td>
<td>91</td>
<td>77</td>
</tr>
<tr>
<td>DN40</td>
<td>2439</td>
<td>391</td>
<td>227</td>
<td>63</td>
<td>64</td>
</tr>
<tr>
<td>DN25</td>
<td>1982</td>
<td>331</td>
<td>93</td>
<td>68</td>
<td>77</td>
</tr>
<tr>
<td>DN15</td>
<td>1677</td>
<td>326</td>
<td>80</td>
<td>56</td>
<td>52</td>
</tr>
<tr>
<td>DN0</td>
<td>1220</td>
<td>359</td>
<td>89</td>
<td>67</td>
<td>86</td>
</tr>
<tr>
<td>DS10</td>
<td>915</td>
<td>374</td>
<td>87</td>
<td>68</td>
<td>70</td>
</tr>
<tr>
<td>DS20</td>
<td>610</td>
<td>411</td>
<td>173</td>
<td>72</td>
<td>62</td>
</tr>
<tr>
<td>DS30</td>
<td>305</td>
<td>520</td>
<td>138</td>
<td>72</td>
<td>58</td>
</tr>
<tr>
<td>DS40</td>
<td>0</td>
<td>634</td>
<td>194</td>
<td>75</td>
<td>56</td>
</tr>
</tbody>
</table>
Erosion: DS10

Initial

Computed

Measured

Cross-shore Volume [m$^3$/m]

Bedload

Suspend

Total

Longshore Volume [m$^3$/m]

Bedload

Suspend

Total

B = 0.002

X [m]

Z [m]

X [m]

150 200 250 300 350

0 300 600
Initial | Computed | Measured

Cross-shore Volume [m$^3$/m]

- Bedload
- Suspend
- Total

Erosion: DS40

Longshore Volume [m$^3$/m]

- Bedload
- Suspend
- Total

B = 0.002
Accretion: RN38

Initial
Computed
Measured

Cross-shore Volume [m$^3$/m]

Bedload
Suspend
Total

B = 0.002

Longshore Volume [m$^3$/m]

Bedload
Suspend
Total

x [m]
Accretion: DS10

Initial | Computed | Measured

Cross-shore Volume $[\text{m}^3/\text{m}]$

Bedload | Suspend | Total

Longshore Volume $[\text{m}^3/\text{m}]$

Bedload | Suspend | Total

$B=0.002$
Accretion: RN38

- Initial
- Computed
- Measured

Longshore
Volume \( m^3/m \)

- Bedload
- Suspend
- Total

B = 0.003

Cross-shore
Volume \( m^3/m \)

- Bedload
- Suspend
- Total

\[ \text{Initial} \quad \text{Computed} \quad \text{Measured} \]
The graph shows the total sediment volume in $m^3$ x 10^-4 as a function of $y$ in meters for different erosion periods and accretion periods. The legend indicates three erosion periods: $B=0.003$, $B=0.002$, and $B=0.001$. The graph is divided into two sections: Erosion period and Accretion period.