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What is the most cost-effective procedure for sampling landings from a commercial fishery ?

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ABSTRACT

With a limited budget for sampling landings from commercial fisheries, the most cost-effective procedure has to be used. Given the cost for taking a length sample and the cost for taking and processing a otolith sample, what is an optimal balance of length measurements and otoliths under the constraint of a limited budget in terms of man-hours? Does there exist any fancy sampling strategy for taking the length-measurements and the otoliths?

A simulation model is presented which can be used to analyse questions like these. Some sampling schemes have been simulated. Random sampling of both length measurements and otoliths has many advantages, especially when combined with smoothing of age-length-key. An example of a cost-precision graph is presented.

Introduction.

This paper presents some of the procedures used in our investigation of the sampling of landings from the Faroese commercial fishery. The aim of this excersise is to see if changes in sampling schemes or smoothing of ALK could improve the catch-at-age estimates. In our opinion, the only tractable way to do this is by simulating the sampling process, because we will never know what the true values for the catch-at-age. This would require that we read the otolith of every single fish in the catch, and that we merely able to read them correctly. In a computer simulation model we define the true values for the catch-at-age and can investigate the consequence of using a certain sampling scheme, different scaling factors, smoothing of ALK's, etc. On the other hand, a computer simulation model can never include every aspect involved in taking samples from landings of a real commercial fishery.

Why bother to investigate a problem that has been thoroughly investigated by so many scientist, Johnston et al. (1975), Armstrong & Ilardia (1986) and Guðmundsdóttir et al. (1988), just to mention a few. Though simulation techniques have been applied to this problem before they have not been well documented, they have not presented the results as relative deviation and not show the cost-precision relationship with regards to the two main parameters to be decided on *viz.* number of length measurements and otoliths. It is extremely important to have accurate catch-at-age data and to know the accuracy of them, because the precision of any analytical assessment is very much determined by this accuracy and in our opinion, every scientist involved in assessing a fish stock must know the accuracy of the catch-at-age data in order to evaluate the results of the assessment. However, we are not saying that simulation is the only way to get this accuracy. There are many mathematical/statistical approaches. Kutkuhn (1963), Southward (1976), Kimura (1978), Lai (1987) and Jinn et al. (1987).

When simulating a stochastic experiment, like sampling landings from a commercial fishery, the experiment has to be repeated several times. The outcome of a single experiment is not so interesting in itself, but the series of outcomes can be used to derive valuable information about the experiment. This poses a dilemma, on one hand we want to make the experiment as realistic as possible and on the other hand a realistic experiment takes more time than a simplified and not necessarily realistic one. As we need to do the experiment several hundreds, even thousand times to get the information we want, a "realistic" sampling simulation can easily take weeks or even months on a modern computer executing several million instructions per second. One way to confine ourself is to simulate the sampling of a single fishload of only one species in stead of the entire landings of a commercial fishery and so we don't have to bother about sampling a fish stock which changes with time and is fished with different gears by different fleets in different areas. Furthermore we will assume that the species is "ideal" in a sense which will become evident in the next section.

Our intention is to give some guidelines on how this type of simulations can be made on a computer, rather than to make a complex statistical model. We will not be presenting complete programs that perform the simulations, but merely give some algorithms in an informal mathematical notation and leave it to the reader to implement them in a programming language.

We have deliberately placed greater emphasis on the simulation model and less the results, as we believe that the reader is more interested in doing his own experiments, rather than in results from simulations of sampling from an artificial fish stock.

Sampling Model and Simulation.

The aim of the simulation is to investigate how the choice of length- and otolith samples taken from a fishload affects the accuracy of the age distribution estimates. The basic structure is a *fish*. But we are only interested in some specific characteristic of a fish viz. its length l, its age a and its weight w. If we assume isometric relationship between fishes of different sizes and a constant density of flesh and bone for all fish of the same species, its weight is fully determined by the length l and two other constants α and β specific for the species:

$$w(l) = \alpha \cdot l^{\beta} \tag{1}$$

Given this, we can represent a fish by the pair (a, l). In the simulations we need to be able to generate a fishload with some given age and length composition. The parameters necessary to generate this will be combined in the structure *population*. If we assume that the length of the a'th age group is normally distributed, we can make the following definition of a population P:

$$P = \begin{pmatrix} \pi_1, \pi_2, \dots \pi_A \\ \mu_1, \mu_2, \dots \mu_A \\ \sigma_1, \sigma_2, \dots \sigma_A \end{pmatrix}, \quad 0 \le \pi_a \le 1, \quad \sum_a \pi_a = 1 \tag{2}$$

where π_a, μ_a and σ_a is the frequency, the mean length and the standard deviation of length of the a'th age group, respectively. A fish (a', l') is generated from P in a two step procedure: first the age a' is generated from the age frequencies $(\pi_1, \pi_2, \dots \pi_A)$ in P and then the length l' is generated using the inverse normal distribution with mean $\mu_{a'}$ and standard deviation $\sigma_{a'}$. Details can be found in algorithm 1. A *fishload* R is simply a collection of fish. One practical way of representing R is by an $A \times L$ matrix:

$$R_{al} = \textit{number of a years old fish with length } l$$

Depending on the context we will use the terms *fishload*, *sample* or *subsample* interchangeably for such a matrix. Below is the algorithm used for generating a fishload R with n fish from P :

$$\forall a, l : R_{al} := 0;$$

$$\textit{while } n > 0 \textit{ do}$$

$$a' := \min\{ x : \sum_{a \le x} \pi_a \ge \rho_1 \};$$

$$l' := \textit{round}(\sigma_{a'} \cdot \Phi^{-1}(\rho_2) + \mu_{a'});$$

$$R_{a'l'} := R_{a'l'} + 1;$$

$$n := n - 1;$$

$$\textit{end};$$

Algorithm 1: for generation of a fishload R with n fish from the population P.

where Φ^{-1} is the inverse standard normal distribution function and ρ_1, ρ_2 two uniform random number on [0; 1]. We could in stead have used following approximation to a normally distributed random generator to generate the length

$$l' := round(\sigma_{a'} \cdot (\sum_{i=1}^{12} \rho_i - 6) + \mu_{a'}) \tag{3}$$

where ρ_i, $i = 1, 2, \ldots 12$, are 12 uniform random numbers on [0, 1]. In the rest of this paper we will adopt the convention of letting a dot in a subscript index denote a summation over that index. It is easy to check that the fish are generated correctly from P simply by generating a very large fishload R with n fish and computing the frequency p_a, the mean length m_a and the length standard deviation s_a for each age group a:

$$p_a = \frac{R_{a\cdot}}{R_{\cdot\cdot}} \tag{4}$$

$$m_a = \frac{\sum_l l \cdot R_{al}}{R_{a\cdot}} \tag{5}$$

$$s_a = \sqrt{\frac{\sum_l l^2 \cdot R_{al}}{R_{a\cdot}} - \left(\frac{\sum_l l \cdot R_{al}}{R_{a\cdot}}\right)^2} \tag{6}$$

and verifying that for each age group a.

$$\left.\begin{array}{lcl} p_a & \rightarrow & \pi_a \\ m_a & \rightarrow & \mu_a \\ s_a & \rightarrow & \sigma_a \end{array}\right\} \; for \quad n \rightarrow \infty .$$

The weight $W(R)$, the age distribution $A(R)$ and the length distribution $L(R)$ of a sample R can be computed in a straight forward manner:

$$W(R) = \sum_{a,l} R_{al} \cdot \alpha \cdot l^{\beta} \tag{7}$$

$$A(R) = (R_{1\cdot}, R_{2\cdot}, \ldots R_{A\cdot}) \tag{8}$$

$$L(R) = (R_{\cdot 1}, R_{\cdot 2}, \ldots R_{\cdot L}) \tag{9}$$

What we want to do is to obtain an estimate $\hat{A}(R)$ of the age distribution $A(R)$ of a fishload R by taking two samples from it, one length sample S and one otolith sample T and use the usual ALK-transformation combined with an appropriate weight factor scaling. For this purpose we need an algorithm to take a random sample S from R. We might just as well make the algorithm more general, such that it also can handle stratified sampling and sampling without replacement.

The following algorithm will take a random sample S of n fish with lengths between l_1 and l_2 from the sample R. If the sample is to be taken without replacement, there might not be n fish available in R with lengths between l_1 and l_2. In this case the algorithm will pick as many as possible:

$$\forall a,l \ : \ S_{al} := 0;$$

$$\forall l \in \{l_1, \ldots l_2\} \ : \ R'_{.l} := \sum_a R_{al};$$

$$R'_{..} := \sum_{l=l_1}^{l_2} R'_{.l};$$

while $(n > 0)$ *and* $(R'_{..} > 0)$ *do*

$$l' := \min\{\, x : \sum_{l \le x} R'_{.l} \ \ge \ \rho_1 \cdot R'_{..} \,\};$$

$$a' := \min\{\, x : \sum_{a \le x} R_{al'} \ge \rho_2 \cdot R'_{.l'} \,\};$$

$$S_{a'l'} := S_{a'l'} + 1;$$

if not Replace then

$$R_{a'l'} := R_{a'l'} - 1;$$

$$R'_{..} := R'_{..} - 1;$$

$$R'_{.l'} := R'_{.l'} - 1;$$

end;

$$n := n - 1;$$

end;

Algorithm 2: for taking a random sample S of n fish with lengths between l_1 and l_2 from the sample R.

as earlier ρ_1 and ρ_2 are to uniform random numbers on [0; 1]. Note that the matrix R' is a copy of R between l_1 and l_2 and zero elsewhere. Although R' in not directly used in the algorithm, we use its total sum $R'_{..}$ and its row sum $R'_{.l}$ for $l \in \{l_1, \ldots l_2\}$. If we want to make a stratified sampling with N strata $I_i =]a_i\,;\, b_i]$ and n_i fish in each stratum, $i = 1, 2, \ldots$ N, this can be accomplished by generating N random subsamples S_i from R using algorithm 2 and adding them together $S = S_1 + S_2 + \ldots + S_N$.

We are now able to generate a fishload R from the population P and to pick a length sample S and an otolith sample T from R with or without replacement. We can also let the otolith sample T be a subsample of the length sample S by calling algorithm 2 with S and T in stead of R and T. The weights of these samples can be computed from formula (7). The age distribution of S will be estimated from

$$\hat{A}(R) = ALK(T) \bullet L(S)\,\frac{W(R)}{W(S)}, \qquad where\ ALK(T)_{al} = \frac{T_{al}}{T_{.l}} \tag{10}$$

$$i.e. \quad \hat{A}(R)_a = \frac{W(R)}{W(S)} \sum_l \frac{T_{al}}{T_{.l}} S_{.l}$$

where the dot (•) denotes the usual multiplication of an $A \times L$ matrix with a L-dimensional vector. In the sum in formula (10) we multiply $S_{\cdot l}$ with $T_{\bullet l} / T_{\cdot l}$, but this will only work if there are any fish with length l in the otolith sample T. One way out of this is to make sure that there is at least one fish of every length. If T is to be random sample, it must very large to cover every length-group and it would be expensive to read all the otoliths in T. We could also take a smaller otolith sample T and by hand add fish with the appropriate age and length to it, such that every length-group has at least one fish in it. But it is not trivial how this can be done and by doing so we also change its age distribution. We could also do nothing and set

$$ALK(T)_{al} = \begin{cases} \dfrac{T_{al}}{T_{\cdot l}} & \text{if } T_{\cdot l} > 0 \\ 0 & \text{otherwise} \end{cases} \tag{11}$$

If there are fish in the length sample S with some length l and no fish of this length in the otolith sample T, these fish will simply be lost in the length-to-age conversion. Because of this some age groups in $\hat{A}(R)$ might be under estimated, usually age groups which are poorly represented. One way to account for this is to use the weight of the fish which are properly length-to-age transformed $W'(S)$ in stead of the $W(S)$ in the scaling factor, where

$$W'(S) = \sum_{l \in \{x : T_{\cdot x} > 0\}} \alpha \cdot S_{\cdot l} \cdot l^{\beta} \tag{12}$$

However, this approach does not help the problem with under estimation of poorly represented age groups. By scaling with $W(R)/W'(S)$ we introduce a new problem: the well represented age groups might be over estimated. We will use the term *raising* for this type of scaling.

A completely different approach is to use a smoothed version of T in stead in the length-to-age conversion. Now there are many ways of smoothing an ALK, but the one most obvious is to use the assumption made earlier, namely that the length distribution of age group a is normally distributed with mean length μ_a and standard deviation σ_a. So we will use equations (5) and (6) to estimate the mean length m_a and the standard deviation s_a for each age group a in T, and use them to generate a "smoothed" version $\hat{T}$ of T, in which the length distribution for each age group has a nice bell-shaped appearance. However, we will not use the standard deviation s_a directly, but make the assumption that there is a linear relationship between the mean length and the standard deviation. The standard deviation for age group a will be set to

$$d_a = m_a \cdot \overline{cv}, \quad \text{where } \overline{cv} = \exp\left(\frac{1}{\#a} \sum_a \ln\left(\frac{s_a}{m_a}\right)\right) \tag{13}$$

where $\#a$ is the number of age groups with $s_a > 0$. The matrix $\hat{T}$ is computed as follows:

$$\hat{T}_{al} = \hat{T}_{a\cdot} \cdot \left(\Phi\left(\frac{l + \frac{1}{2} + m_a}{d_a}\right) - \Phi\left(\frac{l - \frac{1}{2} + m_a}{d_a}\right) \right) \tag{14}$$

where Φ is the standard normal distribution function. For all practical purposes it is enough to compute $\hat{T}_{a\,l}$ with l ranging from $m_a - 3d_a$ to $m_a + 3d_a$ and set it to zero elsewhere. $\hat{T}_{\cdot l}$ will usually be greater than zero for all lengths where $S_{\cdot l}$ is greater than zero, so we don't loose any fish in the length-to-age conversion. If not, we might try computing $\hat{T}_{a\,l}$ for a wider range, say from $m_a - 4d_a$ to $m_a + 4d_a$. Note however, that this procedure relies heavily on the assumption of normally distributed lengths for each age group a.

The question is now how do we measure the accuracy of the estimate $\hat{A}(R)$? There are indeed many ways to do this. One could for instance simply say that the accuracy is measured by the distance between $A(R)$ and $\hat{A}(R)$ in $\mathbb{R}^A$, *i.e.* $|\hat{A}(R) - A(R)|$ or the relative distance $|\hat{A}(R) - A(R)| \,/\, |A(R)|$, but both of these give a single number which is an overall measure of the accuracy of $\hat{A}(R)$ and therefore they say nothing about the accuracy of the estimate for an individual age group. Since it is desirable to have a simple and comprehensible measure for the accuracy of the estimate $\hat{A}(R)$ for each age group a, we will use the *relative deviation*

$$\varepsilon = (\varepsilon_1, \varepsilon_2, \ldots \varepsilon_A), \qquad where\ \varepsilon_a = \frac{\hat{A}(R)_a - A(R)_a}{A(R)_a} \tag{15}$$

The sign of ε_a tells us whether the estimated number of a years old fish in R has been under- or overestimated.

Now it is time to present the algorithm of the simulations. Actually there is more than one algorithm, since there are many ways to take the length and otolith samples and each has its own algorithm. Therefore we will only present the algorithm in which the fishload R of size n_R is generated from the population P and the length sample S of size n_S is taken randomly without replacement from R and otolith sample T of size n_T is taken randomly without replacement from S and where a smoothed version of T is used in the calculation of $\hat{A}(R)$. The simulation is done *NumSimulations* times (usually 1000) and for each simulation i, the relative deviation ε^i is calculated. After the simulations have finish we compute the mean M and standard deviation D of the ε^i's. M_a tells us if age group a is systematically over- or underestimated, while D_a tells us the relative accuracy of the estimated number of a years old fish in a fishload R with n_R fish, which can be expected when taking a length sample S with n_S fish from R and taking an otolith sample T with n_T fish from S.

for i := 1 to NumSimulations do

generate a fishload R^i from the population P with n_R fish in it using algorithm 1;

compute the age distribution $A(R^i)$, using formula (8);

compute the weight $W(R^i)$, using formula (7);

take a length sample S^i without replacement from R^i with n_S fish and lengths between $l_1 = 0$ and $l_2 = 10^6$, using algorithm 2;

compute the weight $W(S^i)$, using formula (7);

compute the length distribution $L(S^i)$, using formula (9);

take an otolith sample T^i without replacement from S^i with n_T fish and lengths between $l_1 = 0$ and $l_2 = 10^6$, using algorithm 2;

compute the smoothed T^i from T^i, using formulae (13) and (14);

compute the estimated age distribution $\hat{A}(R^i)$ from T^i, $L(S^i)$, $W(R^i)$ and $W(S^i)$, using formulae (10) and (11);

compute the relative deviation ε^i, using formula (15);

end;

compute the mean M and the standard deviation D of the relative deviations ε^i, $i = 1, 2, \ldots$ NumSimulations. **Algorithm 3.**

Results.

The simulation models were implemented in Turbo Pascal on a PC. Six different sampling strategies have been simulated. The simulations differ in the way the otolith sample is taken, which scaling factor is used and whether smoothing is done on the ALK. Common to all simulations is that a fishload with 100000 fish is generated from the Faroe saithe test population (Tab. 1 and Figs. 1 and 2). For simulation second to sixth a length sample with *#lengths* fish is taken without replacement from the fishload and from the length sample an otolith sample with *#otoliths* fish is taken without replacement. From the length- and otolith sample an estimate of the age distribution of the fishload is calculated from formulae **(10)** and **(11)**. In cases where *#lengths* = 0, the otolith sample is used directly to estimate the age distribution of the fishload

$$\hat{A}(R) = A(T) \cdot \frac{W(R)}{W(T)} \tag{16}$$

where $A(T)$, $W(R)$ and $W(T)$ are the age distribution of the otolith sample, the weight of the fishload and the weight of the otolith sample, respectively. All simulations were run 1000 times for each combination of *#lengths* and *#otoliths*.

Simulation 1: "Bootstrap" with smoothing.

The first simulation differs form the others in that neither the length sample nor the otolith sample is taken form the fishload. Instead they are both generated directly form the population using algorithm 1. We have called this simulation "Bootstrap" because it resembles the bootstrap method described by Efron (1979) and Diaconis & Efron (1983). The main reason for making this simulation was to have some results which could be compared with the simulations carried out by Guðmundsdóttir et al. (1988).

If we are only using the otolith sample (*#lengths* = 0) the simulation results (Tab. 2) show that by increasing the number of otoliths with a factor 10 the D_a's, which can be regarded as the uncertainty of age group a in $\hat{A}(R)$, will be reduced by approximately a factor 1/3. If we on the other hand decide to use length samples, we must have enough of them (preferably 5 times as many lengths as otoliths). Using fewer length samples than otolith samples is worse than only using the otolith sample.

With an average sampling budget of 5000 - 10000 lengths and 500 - 1000 otoliths, the D_a's are about 5 - 8 percent for the well represented age groups and about 20 - 50 percent for the poorly represented age groups.

Smoothing is done on the ALK and it seems as if this tends to overestimate age groups 4, 5, 8, 9 and 10 and underestimate age groups 2, 3, 6 and 7, although it is not very much. It is difficult to say why.

Simulation 2: Random sampling without replacement, $T \subseteq S \subseteq R$, no smoothing, no raising.

The algorithm used in this simulation is similar to algorithm 3 on page 7, except that no smoothing is done on T.

The results (Tab. 3) show a clear underestimation of poorly represented age groups, especially when using few otoliths (< 5000). This does not come as a surprise, since the otolith sample will contain lengths with zero frequency when there are too few fish in it. It looks as if the D_a's are a little higher than for the "Bootstrap" simulation, especially for the poorly represented age groups, otherwise the results are fairly much the same. As with the other simulations it appears that a factor 10 in increment in the number of otoliths will produce a factor 1/3 reduction in D_a's.

With an average sampling budget of 5000 - 10000 lengths and 500 - 1000 otoliths, the D_a's are about 5 - 8 percent for well represented age groups and about 20 - 50 percent for poorly represented age groups.

Simulation 3: Random sampling without replacement, $T \subseteq S \subseteq R$, no smoothing, with raising.

The algorithm used in this simulation is identical to the algorithm for simulation 2 except that the scaling factor $W(R)/W'(S)$ is used in computing the age distribution estimate $\hat{A}(R)$.

The results (Tab. 4) show that the poorly represented age groups still are underestimated. When using 1000 otoliths or more, the results are pretty much identical to those from simulation 2, but below this the M_a's show that well represented age groups will tend to be overestimated. This does not come as a surprise, since all age groups are raised by the same factor and not just the poorly represented ones which were underestimated. If raising is to be used, some more intelligent way of scaling must be applied.

Simulation 4: Random sampling without replacement, $T \subseteq S \subseteq R$, with smoothing and no raising.

The algorithm for this simulation is identical to algorithm 3 on page 7. The results (Tab. 5) show some underestimation of age groups 6 and 7 and some overestimation of age groups 8, 9 and 10, although not very much. This is apparently an effect of the smoothing. The same effect was seen in simulation 1. As a whole the estimates are less biased than in the other simulations, especially when the number of otoliths is small and the D_a's are about 1 - 3 percent lower.

With an average sampling budget of 5000 - 10000 lengths and 500 - 1000 otoliths, the D_a's are about 4.5 - 6.5 percent for well represented age groups and about 20 - 50 percent for poorly represented age groups and there is no systematic underestimation of poorly represented age groups.

Simulation 5: Stratified sampling without replacement, $T \subseteq S \subseteq R$, no smoothing and no raising.

The algorithm used in this simulation is slightly different from algotithm 3 on page 7. The length sample is still taken randomly, but stratified sampling with 8 - 10 strata, each covering a 10 cm interval starting with a length divisible by 10 and containing *#otoliths* fish, was used to take the otolith sample from the length sample. If the length sample contained less than *#otoliths* fish in a stratum, all fish in that stratum were taken. It was decided to run the simulations with 13, 25, 63, 125, 625 and 1250 otoliths in each stratum, as these numbers approximately correspond to a total of 100, 200, 500, 1000, 5000 and 10000 otoliths, thereby being able to compare random and stratified sampling.

The results (Tab. 6) show an improvement in the estimation of the poorly represented age groups at the cost of a deterioration of the estimates for the well represented age groups. However, there is a tendency to underestimate all age groups especially when the number of otoliths is small. This is not surprising since a stratification with the same small number of otoliths in all strata introduces holes all over the length distribution of the otolith sample.

With an average sampling budget of 5000 - 10000 lengths and 500 - 1000 otoliths the D_a's are about 7 - 11 percent for the well represented age groups and about 15 - 30 percent for the poorly represented age groups.

Simulation 6: Stratified sampling without replacement, $T \subseteq S \subseteq R$, no smoothing, with raising.

The algorithm used in this simulation is identical to the algorithm for simulation 5 except that the scaling factor $W(R)/W'(S)$ is used in computing the age distribution estimate $\hat{A}(R)$.

The results (Tab. 7) show that the systematic underestimation of all age groups found in simulation 5 have vanished, but the D_a's have not improved, on the contrary they are significantly higher, especially when using few otoliths.

With an average sampling budget of 5000 - 10000 lengths and 500 - 1000 otoliths the D_a's are about 7 - 11 percent for the well represented age groups and about 20 - 35 percent for the poorly represented age groups.

Cost versus precision.

A usefull way of looking at the question of cost-effectiveness is by drawing isolines for cost and precision with respect to #lenghts and #otoliths.

Based on actual man-hours used per length measurement and otolith, a cost function has been defined for the Faroese sampling scheme, Tab. 8, and its isolines drawn, Fig. 3. Looking only at the result from simulation 1, which covers the largest number of #length and #otolith combinations, isolines of precision of age group 5 have also been drawn, Fig. 3. For a given budget the maximum precision can be read from the graph. Say, if the budget grants the use of 20 man-hours a precision of about 0.12 to 0.15 can be achieved. Using information in Tabs. 2-7 similar figures may be drawn for other age groups and other sampling strategies.

It is obvious from Fig. 3 that precision increases faster when a low budget scheme is, say, doubled, compared to a high budget. For instance, increasing the manpower from 10 to 20 man-hours decreases D_a from about 0.30 to 0.15, but going from 100 to 200 man-hours only decreases precision from about 0.06 to about 0.045. In the latter case this is hardly worth the effort as we are already in a favorable range of precision and the additional costs are very high.

For other laboratories using other routines and techniques, the cost function will of course be different. As population parameters differ, this probably applies to the precision curve as well.

Discussion.

From the comments on the results of the simulations it appears that if stratified sampling is to be used for taking the otolith sample, a fixed portion of otoliths per strata is not the right way to do it. Perhaps an analysis of the variance of the length distribution in each stratum could yield a better stratification if the number of otoliths in a stratum was proportional to the variance of the stratum. But most likely such an analysis would show that the best way would be to let the number of otolith in each stratum be proportional to the number of lengths in the stratum and then we would be back to a procedure that resembles random sampling of otoliths. It is clear that smoothing a stratified otolith sample is not appropriate, since there is no way to guarantee that the length distribution for an age group is normally distributed when the otoliths have been picked according to some stratification scheme. One further advantage of taking randomized otolith samples is that the otoliths then can be used directly to obtain an estimate of the fishload without the use of length samples like in equation (16). From the comments above it also appears that scaling with $W(R)/W'(S)$ in stead of $W(R)/W(S)$ does not make the estimates any better. On the contrary. But smoothing of the ALK gives better estimates, especially when the number of otoliths is small, thou smoothing too had disadvantages - it may introduce a small bias in the estimates and it is difficult to say which age groups are affected and in which direction. All in all the best results are obtained by taking random length- and otolith samples, where the number of length is about

5 times as large as the number of otoliths and by smoothing the ALK if we have less than about 100 otoliths. Finally, if we try to improve the precision of the poorly represented age groups for example by stratifying, it will be at the cost of loosing some precision of the well represented age groups. The moral should be to "let marginal age groups play a marginal role".

The question is now, can this model be used to improve the sampling of landings from a commercial fishery ? As the model stands, it can only be used to analyze the sampling from a single fishload of only one species. Usually the fish is sorted into boxes when it is caught, so when analyzing the landing of a species from one boat, there is a problem in taking samples from several boxes, especially if the fish in the boxes is sorted by size, but that is a relatively isolated problem which easily can be dealt with. If we want to analyze sampling from a commercial fishery, we must take into account that the fish stock changes with time and that it is taken by different fleet using different gear and in different areas. One way to do this is to parameterize our population

$$P = P(t, s, f) = \begin{pmatrix} \pi_a(t, s, f) \\ \mu_a(t, s, f) \\ \sigma_a(t, s, f) \end{pmatrix}_{a = 1, 2 \dots A}$$

where t is a time effect, s an area effect and f a fleet and gear effect. By generating a number of fishloads with different sizes and different values of t, s and f, we could imitate a "real" commercial fishery. However, simulating sampling from such a fishery could create a problem, the time to do it may easily be measured in years.

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α = 1.7529e-5		β = 2.8022e+0	
a	$100 \cdot \pi_a$	μ_a	σ_a
2	2.56	36	4.0
3	12.14	46	4.5
4	25.88	55	5.0
5	22.36	60	5.5
6	8.95	69	7.5
7	14.06	75	8.0
8	9.58	85	8.5
9	2.56	93	9.0
10	0.64	99	8.0
11	1.28	105	8.5

Table 1. The Faroe Saithe test population.

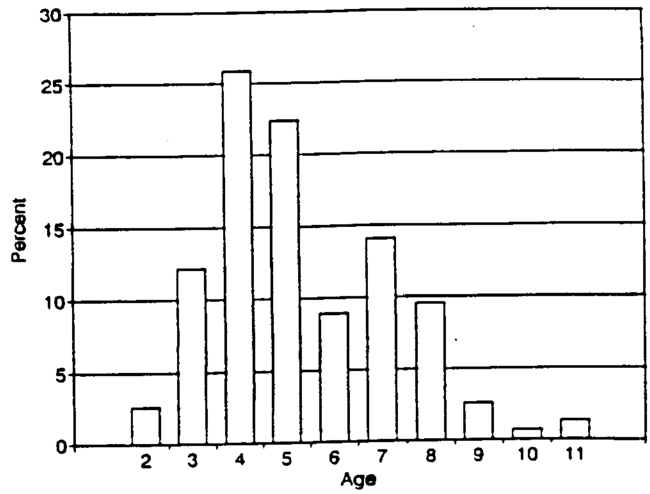

Figure 1. Age distribution of the test population.

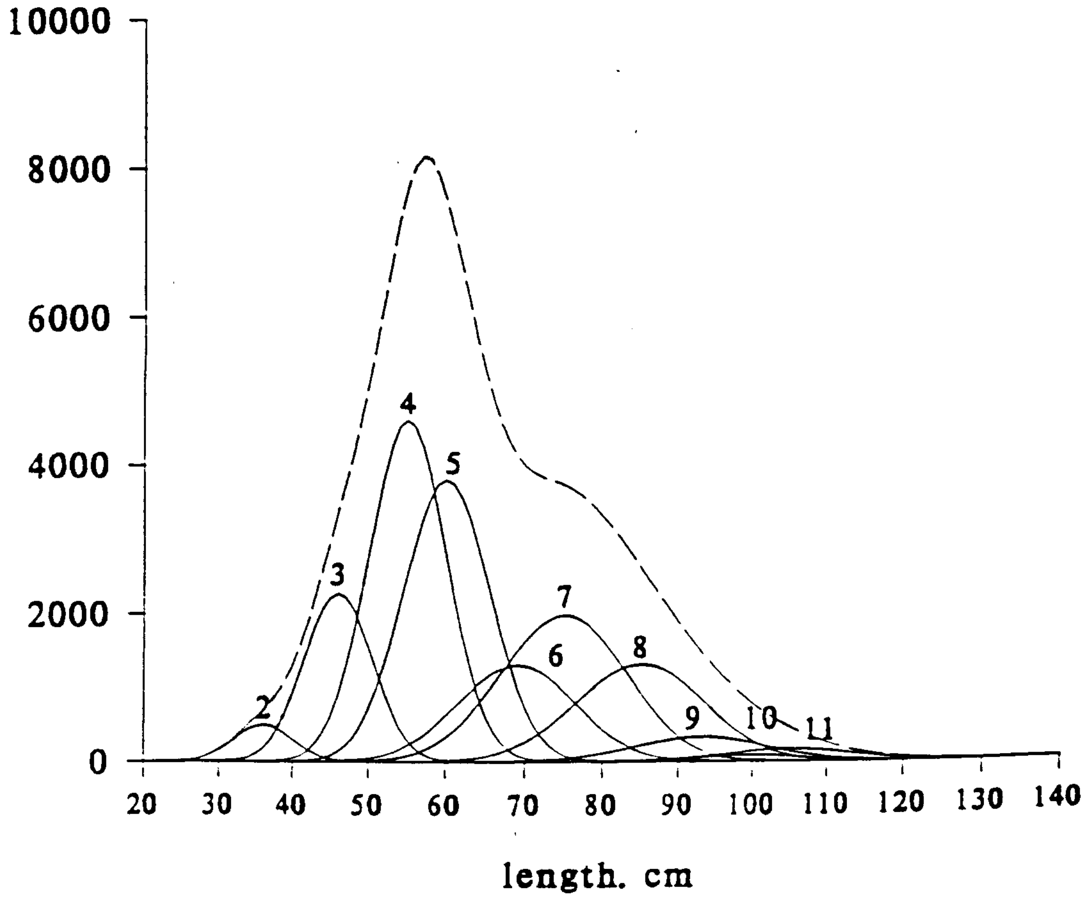

Figure 2. Length distribution in total and for each agegroup of a random sample with 200000 fish generated from the Faroe Saithe test population.

TABLE 2. Results form simulation 1 : "Bootstrap" with smoothing, 1000 simulations.

	Agegroup	2		3		4		5		6		7		8		9		10		11	
#Lengths	#Otoliths	M	D	M	D	M	D	M	D	M	D	M	D	M	D	M	D	M	D	M	D
0	100	-0.0100	0.6317	0.0221	0.2938	0.0214	0.1994	0.0174	0.2105	0.0135	0.3236	-0.0024	0.2412	-0.0173	0.2839	-0.0182	0.5931	-0.0723	1.2114	-0.0128	0.8220
	200	0.0209	0.4689	0.0226	0.2151	0.0105	0.1434	0.0031	0.1555	0.0192	0.2312	-0.0012	0.1691	-0.0062	0.1970	-0.0219	0.4240	-0.0133	0.8750	-0.0211	0.6093
	500	0.0066	0.2881	0.0119	0.1312	-0.0021	0.0888	0.0053	0.0986	0.0007	0.1457	-0.0042	0.1070	0.0004	0.1284	0.0107	0.2821	-0.0094	0.5602	-0.0120	0.3882
	1000	0.0038	0.1992	0.0067	0.0950	0.0019	0.0683	0.0039	0.0675	0.0008	0.1053	0.0007	0.0775	-0.0025	0.0882	0.0022	0.1916	-0.0131	0.3832	-0.0092	0.2654
	5000	0.0009	0.0892	0.0012	0.0422	-0.0013	0.0307	0.0010	0.0313	0.0033	0.0458	-0.0003	0.0346	-0.0029	0.0421	0.0102	0.0895	0.0089	0.1741	-0.0040	0.1246
	10000	0.0012	0.0662	0.0022	0.0311	-0.0003	0.0227	0.0002	0.0219	0.0013	0.0335	0.0002	0.0256	-0.0034	0.0291	0.0063	0.0620	0.0071	0.1251	-0.0007	0.0914
100	100	-0.0613	0.6535	0.0482	0.3150	0.0338	0.2101	0.0191	0.2225	-0.0031	0.3172	-0.0456	0.2330	0.0148	0.2888	0.0295	0.6749	0.1118	1.5596	-0.0916	0.8845
	200	0.0261	0.6398	0.0392	0.2875	0.0244	0.1809	0.0247	0.1871	-0.0089	0.2630	-0.0467	0.1837	0.0099	0.2234	0.0646	0.5040	0.0307	1.0294	-0.0560	0.7177
	500	0.0193	0.5623	0.0013	0.2557	0.0177	0.1654	0.0238	0.1512	-0.0231	0.1866	-0.0410	0.1495	0.0051	0.1765	0.0621	0.3528	0.0596	0.7020	0.0233	0.6088
	1000	0.0260	0.5359	0.0076	0.2464	0.0264	0.1619	0.0188	0.1511	-0.0277	0.1616	-0.0356	0.1402	0.0025	0.1658	0.0488	0.2970	0.0590	0.5583	0.0007	0.5311
	5000	-0.0066	0.5156	-0.0112	0.2322	0.0306	0.1509	0.0215	0.1367	-0.0266	0.1362	-0.0343	0.1258	0.0042	0.1462	0.0461	0.2363	0.0453	0.3806	-0.0174	0.4890
	10000	-0.0105	0.5681	0.0034	0.2473	0.0281	0.1581	0.0241	0.1390	-0.0178	0.1331	-0.0268	0.1270	-0.0061	0.1457	0.0279	0.2335	0.0356	0.3549	-0.0089	0.4919
500	100	-0.0808	0.4530	0.0207	0.2129	0.0034	0.1529	0.0105	0.1704	-0.0215	0.2886	-0.0402	0.2061	0.0237	0.2516	0.0916	0.6528	0.0828	1.4269	-0.0766	0.8424
	200	-0.0054	0.3470	0.0133	0.1682	0.0070	0.1206	0.0100	0.1328	-0.0127	0.2104	-0.0380	0.1415	0.0087	0.1768	0.0808	0.4234	0.0064	0.9395	0.0057	0.5924
	500	0.0209	0.2887	-0.0014	0.1341	0.0179	0.0952	0.0116	0.0912	-0.0312	0.1396	-0.0432	0.1032	0.0104	0.1229	0.0694	0.2860	0.0587	0.6020	0.0299	0.3934
	1000	-0.0122	0.2588	-0.0076	0.1174	0.0162	0.0763	0.0141	0.0785	-0.0317	0.1059	-0.0371	0.0839	0.0039	0.1003	0.0640	0.2143	0.0594	0.4486	0.0375	0.3242
	5000	-0.0022	0.2374	-0.0079	0.1029	0.0222	0.0691	0.0150	0.0635	-0.0362	0.0706	-0.0388	0.0625	0.0083	0.0728	0.0566	0.1305	0.0680	0.2406	0.0075	0.2346
	10000	-0.0130	0.2422	-0.0200	0.0988	0.0214	0.0658	0.0162	0.0612	-0.0331	0.0643	-0.0412	0.0586	0.0038	0.0683	0.0559	0.1097	0.0760	0.1951	0.0221	0.2222
1000	100	-0.0787	0.4297	0.0112	0.1974	0.0053	0.1506	0.0100	0.1628	-0.0212	0.2853	-0.0404	0.2040	0.0183	0.2470	0.0912	0.6374	0.1552	1.4340	-0.0554	0.7969
	200	-0.0094	0.2969	0.0006	0.1465	0.0125	0.1075	0.0055	0.1166	-0.0372	0.1972	-0.0403	0.1401	0.0257	0.1809	0.0861	0.4347	0.0967	1.0277	-0.0062	0.5795
	500	-0.0027	0.2188	-0.0056	0.1098	0.0162	0.0774	0.0141	0.0830	-0.0297	0.1307	-0.0401	0.0941	0.0059	0.1163	0.0721	0.2754	0.0400	0.5900	0.0396	0.3725
	1000	0.0035	0.1954	-0.0103	0.0919	0.0189	0.0650	0.0138	0.0656	-0.0311	0.0981	-0.0427	0.0737	0.0052	0.0865	0.0654	0.2091	0.0781	0.4280	0.0310	0.2750
	5000	-0.0142	0.1715	-0.0175	0.0794	0.0192	0.0531	0.0132	0.0486	-0.0350	0.0575	-0.0399	0.0512	0.0078	0.0580	0.0585	0.1092	0.0734	0.2212	0.0242	0.1923
	10000	-0.0015	0.1792	-0.0196	0.0740	0.0222	0.0521	0.0145	0.0473	-0.0379	0.0520	-0.0404	0.0456	0.0073	0.0524	0.0597	0.0959	0.0683	0.1719	0.0191	0.1726
5000	100	-0.0721	0.4224	0.0154	0.1830	-0.0009	0.1374	0.0123	0.1640	-0.0317	0.2884	-0.0454	0.1971	0.0305	0.2494	0.1057	0.6266	0.1183	1.4133	-0.0672	0.7765
	200	0.0048	0.2592	0.0055	0.1315	0.0077	0.1038	0.0100	0.1141	-0.0266	0.1996	-0.0451	0.1354	0.0145	0.1702	0.0823	0.4265	0.1025	0.9744	0.0121	0.5425
	500	0.0069	0.1651	-0.0086	0.0856	0.0167	0.0669	0.0071	0.0763	-0.0357	0.1271	-0.0362	0.0884	0.0011	0.1061	0.0915	0.2689	0.0820	0.5932	0.0438	0.3413
	1000	0.0045	0.1308	-0.0104	0.0636	0.0159	0.0494	0.0125	0.0549	-0.0356	0.0943	-0.0393	0.0640	0.0058	0.0786	0.0733	0.1952	0.0613	0.4135	0.0359	0.2478
	5000	-0.0079	0.0881	-0.0178	0.0406	0.0216	0.0303	0.0129	0.0306	-0.0352	0.0452	-0.0381	0.0331	0.0066	0.0404	0.0544	0.0914	0.0687	0.1790	0.0222	0.1307
	10000	-0.0101	0.0824	-0.0195	0.0388	0.0213	0.0270	0.0142	0.0253	-0.0356	0.0334	-0.0384	0.0266	0.0042	0.0331	0.0550	0.0704	0.0742	0.1396	0.0278	0.1037
10000	100	-0.0424	0.4123	0.0089	0.1743	0.0061	0.1334	-0.0002	0.1538	-0.0159	0.2836	-0.0457	0.1904	0.0225	0.2469	0.0963	0.6411	0.1075	1.4067	-0.0096	0.8602
	200	-0.0053	0.2596	0.0040	0.1268	0.0111	0.0972	0.0036	0.1108	-0.0392	0.1904	-0.0365	0.1348	0.0178	0.1764	0.0930	0.4439	0.0372	0.9631	0.0093	0.5762
	500	0.0083	0.1592	-0.0059	0.0837	0.0144	0.0638	0.0086	0.0710	-0.0364	0.1267	-0.0393	0.0888	0.0023	0.1106	0.0867	0.2663	0.0879	0.6016	0.0504	0.3305
	1000	0.0083	0.1245	-0.0131	0.0622	0.0168	0.0476	0.0108	0.0539	-0.0371	0.0912	-0.0413	0.0635	0.0108	0.0787	0.0642	0.1851	0.0799	0.4306	0.0307	0.2419
	5000	-0.0058	0.0740	-0.0182	0.0349	0.0204	0.0257	0.0146	0.0268	-0.0350	0.0427	-0.0381	0.0306	0.0045	0.0375	0.0544	0.0888	0.0767	0.1873	0.0257	0.1166
	10000	-0.0092	0.0637	-0.0166	0.0297	0.0210	0.0225	0.0144	0.0227	-0.0343	0.0311	-0.0384	0.0247	0.0049	0.0286	0.0567	0.0657	0.0737	0.1378	0.0146	0.0893
50000	100	-0.0526	0.4007	0.0121	0.1766	0.0086	0.1389	0.0078	0.1605	-0.0290	0.2881	-0.0549	0.1928	0.0286	0.2410	0.0762	0.6281	0.1002	1.4103	-0.0192	0.8210
	200	-0.0025	0.2566	-0.0008	0.1262	0.0124	0.1045	0.0091	0.1197	-0.0317	0.2075	-0.0458	0.1342	0.0118	0.1669	0.0782	0.4348	0.1550	1.0155	0.0038	0.5436
	500	0.0129	0.1474	-0.0039	0.0796	0.0143	0.0610	0.0092	0.0702	-0.0338	0.1287	-0.0460	0.0878	0.0103	0.1041	0.0831	0.2772	0.1101	0.6033	0.0307	0.3520
	1000	0.0105	0.1074	-0.0122	0.0595	0.0173	0.0444	0.0101	0.0522	-0.0331	0.0914	-0.0407	0.0622	0.0078	0.0782	0.0613	0.1917	0.0945	0.4201	0.0298	0.2312
	5000	-0.0076	0.0561	-0.0182	0.0283	0.0215	0.0226	0.0124	0.0247	-0.0366	0.0401	-0.0398	0.0280	0.0066	0.0355	0.0590	0.0831	0.0799	0.1855	0.0217	0.1060
	10000	-0.0075	0.0429	-0.0178	0.0231	0.0206	0.0173	0.0140	0.0193	-0.0372	0.0304	-0.0392	0.0209	0.0072	0.0272	0.0563	0.0608	0.0705	0.1333	0.0196	0.0805

TABLE 3. Results form simulation 2 : Random sampling without replacement, 1000 simulations, $T \subseteq S \subseteq R$, no smoothing, no raising.

	Agegroup	2		3		4		5		6		7		8		9		10		11	
#Lengths	#Otoliths	M	D	M	D	M	D	M	D	M	D	M	D	M	D	M	D	M	D	M	D
0	100	0.0126	0.6404	0.0131	0.3000	0.0059	0.1957	0.0037	0.2223	0.0093	0.3429	0.0022	0.2355	0.0094	0.2853	-0.0321	0.5985	-0.0358	1.1737	-0.0252	0.8321
	200	-0.0122	0.4377	0.0160	0.2176	0.0072	0.1412	-0.0018	0.1513	-0.0069	0.2304	0.0080	0.1734	0.0032	0.1976	-0.0135	0.4311	0.0059	0.8548	-0.0252	0.5974
	500	0.0041	0.2880	0.0011	0.1376	0.0007	0.0902	-0.0004	0.0948	-0.0010	0.1499	-0.0006	0.1053	0.0064	0.1235	0.0034	0.2715	-0.0184	0.5633	-0.0047	0.3770
	1000	0.0072	0.2016	0.0047	0.0958	-0.0002	0.0639	0.0021	0.0654	-0.0054	0.1021	-0.0018	0.0753	0.0059	0.0894	0.0050	0.1897	-0.0182	0.3983	-0.0048	0.2634
	5000	0.0004	0.0890	-0.0004	0.0417	0.0005	0.0282	0.0002	0.0290	0.0012	0.0446	-0.0000	0.0334	0.0018	0.0386	-0.0007	0.0841	-0.0084	0.1688	-0.0063	0.1148
	10000	0.0001	0.0614	0.0007	0.0295	-0.0001	0.0189	0.0001	0.0201	0.0007	0.0302	0.0006	0.0237	-0.0001	0.0274	0.0024	0.0546	0.0010	0.1170	-0.0064	0.0810
500	100	-0.5450	0.3185	-0.2017	0.2305	-0.0478	0.1640	-0.0636	0.1902	-0.1650	0.3043	-0.2186	0.2065	-0.3308	0.2191	-0.4739	0.3815	-0.5911	0.5967	-0.6622	0.3400
	200	-0.2925	0.3253	-0.0493	0.1818	0.0020	0.1298	-0.0072	0.1373	-0.0367	0.2358	-0.0470	0.1689	-0.1269	0.1952	-0.2200	0.3766	-0.3327	0.6964	-0.4266	0.4040
1000	100	-0.6194	0.2701	-0.2391	0.2147	-0.0561	0.1668	-0.0599	0.1837	-0.1716	0.3135	-0.2521	0.1997	-0.3825	0.1983	-0.5335	0.3348	-0.6623	0.5082	-0.7332	0.2719
	200	-0.3884	0.2925	-0.0770	0.1665	-0.0053	0.1166	-0.0023	0.1341	-0.0519	0.2349	-0.0667	0.1720	-0.1678	0.1905	-0.3093	0.3508	-0.4510	0.5736	-0.5520	0.3198
	500	-0.1119	0.2497	-0.0103	0.1134	0.0016	0.0801	0.0012	0.0884	-0.0064	0.1527	-0.0043	0.1073	-0.0181	0.1251	-0.0742	0.2570	-0.1252	0.5063	-0.2355	0.3156
5000	100	-0.6782	0.2203	-0.2471	0.2135	-0.0600	0.1604	-0.0773	0.1791	-0.1744	0.2936	-0.2723	0.1891	-0.4280	0.1950	-0.5808	0.3031	-0.7101	0.4550	-0.8088	0.2125
	200	-0.4858	0.2505	-0.0904	0.1542	-0.0045	0.1097	-0.0088	0.1293	-0.0438	0.2415	-0.0826	0.1618	-0.2101	0.1823	-0.3811	0.3148	-0.5216	0.5139	-0.6714	0.2564
	500	-0.2049	0.2134	-0.0114	0.0951	0.0016	0.0687	-0.0027	0.0767	-0.0051	0.1442	-0.0030	0.1022	-0.0355	0.1305	-0.1332	0.2613	-0.2722	0.4599	-0.4331	0.2674
	1000	-0.0809	0.1591	-0.0013	0.0700	0.0001	0.0499	-0.0013	0.0543	-0.0008	0.0968	0.0010	0.0709	-0.0051	0.0864	-0.0443	0.1880	-0.1227	0.3893	-0.2482	0.2287
10000	100	-0.6669	0.2447	-0.2481	0.2008	-0.0652	0.1602	-0.0767	0.1789	-0.1787	0.2975	-0.2712	0.1956	-0.4326	0.1928	-0.6221	0.2693	-0.7393	0.4031	-0.8140	0.2101
	200	-0.5008	0.2270	-0.0935	0.1608	-0.0074	0.1138	-0.0089	0.1333	-0.0355	0.2393	-0.0828	0.1669	-0.2103	0.1815	-0.3701	0.3062	-0.5393	0.4959	-0.6927	0.2451
	500	-0.2255	0.2088	-0.0128	0.0943	0.0011	0.0674	-0.0011	0.0778	0.0043	0.1451	-0.0032	0.1029	-0.0490	0.1243	-0.1420	0.2599	-0.2627	0.4824	-0.4367	0.2567
	1000	-0.0919	0.1472	0.0005	0.0653	-0.0020	0.0474	0.0009	0.0519	0.0016	0.0990	-0.0001	0.0704	-0.0094	0.0846	-0.0495	0.1828	-0.0998	0.4094	-0.2699	0.2375
	5000	-0.0061	0.0724	0.0007	0.0348	0.0003	0.0239	0.0006	0.0259	0.0009	0.0446	0.0012	0.0325	-0.0009	0.0364	0.0015	0.0812	-0.0153	0.1635	-0.0349	0.1136
50000	100	-0.6891	0.2276	-0.2426	0.2027	-0.0680	0.1572	-0.0793	0.1802	-0.1823	0.2908	-0.2693	0.1978	-0.4284	0.1957	-0.5813	0.2959	-0.7124	0.4321	-0.8325	0.1932
	200	-0.5082	0.2307	-0.0927	0.1594	-0.0128	0.1124	-0.0082	0.1279	-0.0343	0.2371	-0.0829	0.1634	-0.2123	0.1843	-0.3892	0.3100	-0.5504	0.4963	-0.6886	0.2464
	500	-0.2289	0.2049	-0.0150	0.0925	-0.0023	0.0644	0.0015	0.0771	0.0005	0.1480	-0.0052	0.1013	-0.0440	0.1217	-0.1581	0.2623	-0.2873	0.4867	-0.4495	0.2541
	1000	-0.1057	0.1485	-0.0014	0.0640	0.0013	0.0454	-0.0007	0.0514	0.0003	0.0971	-0.0020	0.0678	-0.0054	0.0862	-0.0599	0.1943	-0.1451	0.3876	-0.2768	0.2230
	5000	-0.0115	0.0542	0.0019	0.0271	0.0002	0.0204	-0.0010	0.0232	-0.0001	0.0410	0.0013	0.0287	-0.0009	0.0355	-0.0015	0.0826	-0.0025	0.1701	-0.0550	0.1074
	10000	-0.0004	0.0384	0.0011	0.0189	0.0002	0.0143	-0.0012	0.0163	0.0001	0.0290	0.0002	0.0200	0.0003	0.0247	0.0029	0.0541	-0.0018	0.1198	-0.0249	0.0704

TABLE 4. Results form simulation 3 : Random sampling without replacement, 1000 simulations, $T \subseteq S \subseteq R$. no smoothing, but raising.

	Agegroup	2		3		4		5		6		7		8		9		10		11	
#Lengths	#Otoliths	M	D	M	D	M	D	M	D	M	D	M	D	M	D	M	D	M	D	M	D
0	100	0.0126	0.6404	0.0131	0.3000	0.0059	0.1957	0.0037	0.2223	0.0093	0.3429	0.0022	0.2355	0.0094	0.2853	-0.0321	0.5985	-0.0358	1.1737	-0.0252	0.8321
	200	-0.0122	0.4377	0.0160	0.2176	0.0072	0.1412	-0.0018	0.1513	-0.0069	0.2304	0.0080	0.1734	0.0032	0.1976	-0.0135	0.4311	0.0059	0.8548	-0.0252	0.5974
	500	0.0041	0.2880	0.0011	0.1376	0.0007	0.0902	-0.0004	0.0948	-0.0010	0.1499	-0.0006	0.1053	0.0064	0.1235	0.0034	0.2715	-0.0184	0.5633	-0.0047	0.3770
	1000	0.0072	0.2016	0.0047	0.0958	-0.0002	0.0639	0.0021	0.0654	-0.0054	0.1021	-0.0018	0.0753	0.0059	0.0894	0.0050	0.1897	-0.0182	0.3983	-0.0048	0.2634
	5000	0.0004	0.0890	-0.0004	0.0417	0.0005	0.0282	0.0002	0.0290	0.0012	0.0446	-0.0000	0.0334	0.0018	0.0386	-0.0007	0.0841	-0.0084	0.1688	-0.0063	0.1148
	10000	0.0001	0.0614	0.0007	0.0295	-0.0001	0.0189	0.0001	0.0201	0.0007	0.0302	0.0006	0.0237	-0.0001	0.0274	0.0024	0.0546	0.0010	0.1170	-0.0064	0.0810
500	100	-0.4538	0.3805	-0.0413	0.2721	0.1446	0.1944	0.1251	0.2234	0.0028	0.3617	-0.0621	0.2413	-0.1962	0.2612	-0.3686	0.4544	-0.5092	0.7142	-0.5928	0.4109
	200	-0.2540	0.3423	0.0026	0.1905	0.0569	0.1367	0.0474	0.1455	0.0161	0.2482	0.0051	0.1765	-0.0796	0.2028	-0.1776	0.3955	-0.2961	0.7351	-0.3949	0.4266
1000	100	-0.5311	0.3324	-0.0640	0.2590	0.1622	0.2009	0.1582	0.2256	0.0193	0.3815	-0.0800	0.2394	-0.2402	0.2409	-0.4251	0.4128	-0.5839	0.6269	-0.6708	0.3363
	200	-0.3429	0.3130	-0.0078	0.1761	0.0698	0.1264	0.0729	0.1432	0.0195	0.2517	0.0030	0.1809	-0.1055	0.2032	-0.2579	0.3755	-0.4104	0.6157	-0.5184	0.3434
	500	-0.1010	0.2523	0.0021	0.1147	0.0141	0.0811	0.0136	0.0894	0.0060	0.1549	0.0082	0.1088	-0.0060	0.1261	-0.0629	0.2597	-0.1146	0.5120	-0.2262	0.3188
5000	100	-0.5949	0.2760	-0.0533	0.2641	0.1833	0.2000	0.1610	0.2211	0.0384	0.3653	-0.0851	0.2314	-0.2810	0.2414	-0.4730	0.3785	-0.6360	0.5682	-0.7596	0.2671
	200	-0.4389	0.2718	-0.0071	0.1646	0.0875	0.1208	0.0827	0.1427	0.0439	0.2618	0.0016	0.1743	-0.1382	0.1950	-0.3245	0.3423	-0.4787	0.5586	-0.6415	0.2792
	500	-0.1874	0.2168	0.0108	0.0969	0.0242	0.0709	0.0198	0.0784	0.0174	0.1479	0.0195	0.1044	-0.0139	0.1323	-0.1138	0.2666	-0.2562	0.4698	-0.4205	0.2732
	1000	-0.0735	0.1600	0.0068	0.0704	0.0082	0.0504	0.0068	0.0550	0.0073	0.0977	0.0091	0.0717	0.0029	0.0868	-0.0366	0.1892	-0.1158	0.3921	-0.2422	0.2301
10000	100	-0.5791	0.3094	-0.0508	0.2506	0.1805	0.1990	0.1658	0.2219	0.0354	0.3690	-0.0803	0.2426	-0.2843	0.2396	-0.5234	0.3391	-0.6702	0.5104	-0.7654	0.2640
	200	-0.4549	0.2473	-0.0096	0.1734	0.0851	0.1253	0.0833	0.1459	0.0544	0.2619	0.0019	0.1790	-0.1378	0.1942	-0.3123	0.3328	-0.4974	0.5401	-0.6646	0.2668
	500	-0.2074	0.2125	0.0106	0.0962	0.0249	0.0696	0.0227	0.0798	0.0282	0.1490	0.0204	0.1047	-0.0265	0.1265	-0.1219	0.2651	-0.2456	0.4929	-0.4236	0.2623
	1000	-0.0840	0.1480	0.0093	0.0655	0.0069	0.0483	0.0097	0.0524	0.0104	0.0998	0.0088	0.0711	-0.0007	0.0852	-0.0411	0.1841	-0.0919	0.4127	-0.2636	0.2391
	5000	-0.0056	0.0724	0.0012	0.0348	0.0009	0.0239	0.0012	0.0259	0.0014	0.0446	0.0018	0.0325	-0.0004	0.0365	0.0021	0.0813	-0.0147	0.1636	-0.0344	0.1136
50000	100	-0.6080	0.2869	-0.0439	0.2530	0.1768	0.1944	0.1622	0.2222	0.0319	0.3629	-0.0781	0.2450	-0.2791	0.2424	-0.4714	0.3739	-0.6371	0.5449	-0.7887	0.2424
	200	-0.4621	0.2513	-0.0067	0.1721	0.0813	0.1246	0.0863	0.1403	0.0572	0.2576	0.0039	0.1756	-0.1382	0.1985	-0.3314	0.3385	-0.5084	0.5418	-0.6592	0.2694
	500	-0.2099	0.2086	0.0098	0.0943	0.0228	0.0662	0.0267	0.0794	0.0257	0.1520	0.0199	0.1036	-0.0200	0.1240	-0.1372	0.2679	-0.2696	0.4983	-0.4360	0.2597
	1000	-0.0972	0.1493	0.0082	0.0647	0.0109	0.0460	0.0089	0.0519	0.0099	0.0978	0.0076	0.0687	0.0041	0.0868	-0.0509	0.1960	-0.1370	0.3910	-0.2700	0.2247
	5000	-0.0105	0.0542	0.0029	0.0271	0.0012	0.0204	-0.0000	0.0232	0.0009	0.0410	0.0023	0.0288	0.0001	0.0356	-0.0005	0.0827	-0.0015	0.1703	-0.0540	0.1074
	10000	-0.0001	0.0384	0.0014	0.0189	0.0006	0.0143	-0.0009	0.0163	0.0004	0.0290	0.0005	0.0200	0.0007	0.0247	0.0032	0.0541	-0.0015	0.1198	-0.0246	0.0704

TABLE 5. Results form simulation 4 : Random sampling without replacement, 1000 simulations, $T \subseteq S \subseteq R$, smoothing, no raising.

	Agegroup	2		3		4		5		6		7		8		9		10		11	
#Lengths	#Otoliths	M	D	M	D	M	D	M	D	M	D	M	D	M	D	M	D	M	D	M	D
0	100	0.0321	0.6752	0.0121	0.3054	0.0023	0.2046	0.0043	0.2133	-0.0013	0.3276	-0.0135	0.2258	0.0041	0.2829	0.0111	0.5965	0.0147	1.2034	-0.0277	0.8306
	200	0.0130	0.4449	0.0086	0.2154	0.0096	0.1485	0.0043	0.1523	-0.0025	0.2328	0.0052	0.1715	-0.0099	0.2037	0.0157	0.4043	-0.0255	0.8527	-0.0236	0.6017
	500	0.0041	0.2880	0.0011	0.1376	0.0007	0.0902	-0.0004	0.0948	-0.0010	0.1499	-0.0006	0.1053	0.0064	0.1235	0.0034	0.2715	-0.0184	0.5633	-0.0047	0.3770
	1000	0.0072	0.2016	0.0047	0.0958	-0.0002	0.0639	0.0021	0.0654	-0.0054	0.1021	-0.0018	0.0753	0.0059	0.0894	0.0050	0.1897	-0.0182	0.3983	-0.0048	0.2634
	5000	0.0004	0.0890	-0.0004	0.0417	0.0005	0.0282	0.0002	0.0290	0.0012	0.0446	-0.0000	0.0334	0.0018	0.0386	-0.0007	0.0841	-0.0084	0.1688	-0.0063	0.1148
	10000	0.0001	0.0614	0.0007	0.0295	-0.0001	0.0189	0.0001	0.0201	0.0007	0.0302	0.0006	0.0237	-0.0001	0.0274	0.0024	0.0546	0.0010	0.1170	-0.0064	0.0810
500	100	-0.0333	0.4709	0.0101	0.2115	0.0117	0.1509	0.0028	0.1679	-0.0402	0.2954	-0.0458	0.1983	0.0395	0.2542	0.0878	0.6360	0.0855	1.3852	-0.0279	0.8176
	200	0.0230	0.3475	0.0021	0.1679	0.0178	0.1234	0.0045	0.1284	-0.0394	0.2119	-0.0401	0.1475	0.0176	0.1834	0.0956	0.4335	0.0578	0.9834	0.0218	0.6083
1000	100	-0.0378	0.4432	0.0009	0.1868	0.0128	0.1440	0.0056	0.1668	-0.0308	0.2973	-0.0487	0.1893	0.0280	0.2439	0.0950	0.6298	0.0391	1.3488	-0.0249	0.7978
	200	0.0204	0.3065	-0.0030	0.1497	0.0140	0.1091	0.0098	0.1214	-0.0502	0.2060	-0.0398	0.1457	0.0227	0.1805	0.0712	0.4188	0.0829	1.0065	0.0226	0.5627
	500	0.0194	0.2343	-0.0101	0.1106	0.0148	0.0784	0.0098	0.0869	-0.0394	0.1416	-0.0436	0.0977	0.0181	0.1171	0.0717	0.2698	0.0926	0.5763	0.0330	0.3546
5000	100	-0.0485	0.4034	0.0086	0.1790	0.0109	0.1398	0.0016	0.1598	-0.0285	0.2813	-0.0428	0.1876	0.0254	0.2501	0.0993	0.6329	0.0467	1.4150	-0.0319	0.8192
	200	-0.0109	0.2601	0.0061	0.1264	0.0150	0.0953	0.0047	0.1128	-0.0320	0.2062	-0.0478	0.1358	0.0174	0.1782	0.0813	0.4344	0.0665	0.9572	0.0157	0.5509
	500	0.0117	0.1681	-0.0053	0.0845	0.0154	0.0653	0.0077	0.0734	-0.0378	0.1296	-0.0438	0.0861	0.0184	0.1123	0.0711	0.2692	0.0803	0.5699	0.0241	0.3411
	1000	0.0022	0.1280	-0.0134	0.0657	0.0181	0.0491	0.0102	0.0533	-0.0369	0.0890	-0.0392	0.0640	0.0131	0.0801	0.0638	0.1850	0.0457	0.4061	0.0210	0.2408
10000	100	-0.0464	0.4304	0.0107	0.1807	0.0060	0.1320	0.0016	0.1515	-0.0217	0.2813	-0.0376	0.1931	0.0293	0.2493	0.0463	0.6350	0.0424	1.3784	-0.0151	0.8063
	200	0.0023	0.2431	0.0007	0.1273	0.0088	0.0967	0.0077	0.1140	-0.0262	0.2030	-0.0439	0.1350	0.0143	0.1671	0.0985	0.4190	0.0920	0.9541	-0.0120	0.5508
	500	0.0024	0.1580	-0.0070	0.0835	0.0157	0.0651	0.0086	0.0731	-0.0327	0.1258	-0.0399	0.0876	0.0072	0.1071	0.0704	0.2634	0.0859	0.5900	0.0377	0.3177
	1000	0.0023	0.1148	-0.0122	0.0598	0.0160	0.0463	0.0122	0.0513	-0.0352	0.0902	-0.0403	0.0626	0.0101	0.0762	0.0614	0.1826	0.0933	0.4363	0.0186	0.2439
	5000	-0.0084	0.0698	-0.0192	0.0343	0.0218	0.0243	0.0136	0.0264	-0.0359	0.0427	-0.0379	0.0312	0.0093	0.0362	0.0535	0.0837	0.0486	0.1726	0.0158	0.1118
50000	100	-0.0586	0.4042	0.0163	0.1735	0.0158	0.1372	-0.0057	0.1542	-0.0292	0.2890	-0.0394	0.1945	0.0130	0.2464	0.1448	0.6337	0.0765	1.3564	-0.1313	0.7954
	200	-0.0062	0.2506	0.0033	0.1277	0.0078	0.0973	0.0081	0.1135	-0.0269	0.1986	-0.0426	0.1377	0.0136	0.1703	0.0746	0.4112	0.0745	0.9696	0.0289	0.5491
	500	0.0152	0.1484	-0.0069	0.0807	0.0119	0.0605	0.0099	0.0735	-0.0323	0.1300	-0.0408	0.0832	0.0107	0.1092	0.0627	0.2681	0.0793	0.6013	0.0523	0.3261
	1000	0.0029	0.1047	-0.0127	0.0550	0.0188	0.0443	0.0097	0.0503	-0.0354	0.0878	-0.0429	0.0593	0.0138	0.0755	0.0599	0.1872	0.0483	0.4097	0.0383	0.2369
	5000	-0.0089	0.0482	-0.0188	0.0253	0.0220	0.0204	0.0119	0.0230	-0.0372	0.0383	-0.0376	0.0261	0.0095	0.0340	0.0513	0.0825	0.0632	0.1700	0.0185	0.1042
	10000	-0.0084	0.0351	-0.0195	0.0184	0.0221	0.0146	0.0120	0.0163	-0.0363	0.0272	-0.0383	0.0188	0.0100	0.0238	0.0520	0.0551	0.0574	0.1240	0.0179	0.0707

TABLE 6. Results form simulation 5 : Stratified sampling without replacement, 1000 simulations, $T \subseteq S \subseteq R$, no smoothing, no raising.

	Agegroup	2		3		4		5		6		7		8		9		10		11	
#Lengths	#Otoliths per 10 cm	M	D	M	D	M	D	M	D	M	D	M	D	M	D	M	D	M	D	M	D
0	13	0.0188	2.0126	0.0719	0.9323	0.0685	0.6098	0.0442	0.6099	0.0720	0.9456	-0.0121	0.6774	-0.0060	0.7637	-0.1015	1.5910	-0.0748	3.1832	-0.1995	2.0406
	25	0.0697	1.3439	0.0646	0.6676	0.0453	0.4412	0.0294	0.4613	0.0121	0.6530	0.0040	0.4783	-0.0193	0.5602	-0.0433	1.1555	-0.0876	2.2640	-0.0587	1.5675
	63	-0.0318	0.8009	0.0031	0.4043	0.0055	0.2630	-0.0012	0.2694	-0.0092	0.4137	-0.0119	0.2963	0.0273	0.3657	0.0011	0.7491	-0.0132	1.6067	-0.0454	1.0418
	125	-0.0022	0.5979	-0.0008	0.2709	-0.0006	0.1790	0.0045	0.1999	-0.0126	0.2770	-0.0032	0.2162	0.0059	0.2491	-0.0090	0.5289	0.0349	1.1034	-0.0112	0.7236
	625	0.0028	0.2625	-0.0049	0.1216	-0.0007	0.0822	-0.0024	0.0830	0.0081	0.1316	0.0009	0.0970	0.0007	0.1121	0.0051	0.2366	-0.0039	0.4932	-0.0085	0.3378
	1250	0.0085	0.1806	-0.0039	0.0817	-0.0016	0.0569	-0.0028	0.0592	0.0065	0.0909	-0.0008	0.0682	0.0035	0.0776	0.0037	0.1632	0.0068	0.3267	-0.0094	0.2377
500	13	-0.0943	0.3030	-0.2088	0.2282	-0.2108	0.2009	-0.2042	0.2235	-0.2277	0.3239	-0.1983	0.2157	-0.1692	0.1984	-0.1192	0.3781	-0.1067	0.6449	-0.0359	0.4189
	25	-0.0036	0.3298	-0.0451	0.2042	-0.0626	0.1761	-0.0570	0.1928	-0.0574	0.2846	-0.0474	0.1866	-0.0258	0.1716	-0.0219	0.3165	-0.0070	0.5891	-0.0061	0.3732
1000	13	-0.1993	0.2572	-0.2416	0.2143	-0.2232	0.1961	-0.2215	0.2091	-0.2235	0.3331	-0.2381	0.2017	-0.2082	0.2011	-0.1579	0.3601	-0.1641	0.5674	-0.0927	0.3338
	25	-0.0428	0.2579	-0.0845	0.1969	-0.0556	0.1675	-0.0699	0.1785	-0.0606	0.2871	-0.0691	0.1808	-0.0527	0.1728	-0.0386	0.3113	-0.0054	0.5120	-0.0106	0.3118
	63	0.0025	0.2272	-0.0054	0.1423	-0.0029	0.1089	0.0036	0.1257	0.0010	0.1757	-0.0062	0.1171	0.0009	0.1098	0.0093	0.2142	-0.0019	0.3862	-0.0087	0.2592
5000	13	-0.2579	0.2349	-0.2557	0.2044	-0.2454	0.1911	-0.2212	0.2189	-0.2671	0.3036	-0.2469	0.2002	-0.2402	0.1876	-0.2475	0.3176	-0.2210	0.5338	-0.2106	0.2907
	25	-0.1129	0.2160	-0.0888	0.1910	-0.0649	0.1625	-0.0753	0.1877	-0.0793	0.2759	-0.0694	0.1820	-0.0719	0.1739	-0.0757	0.3011	-0.0404	0.4648	-0.0568	0.2604
	63	-0.0019	0.1604	-0.0093	0.1202	-0.0081	0.1033	0.0065	0.1190	-0.0079	0.1742	-0.0040	0.1128	-0.0009	0.1081	0.0063	0.1985	-0.0096	0.3095	-0.0038	0.1771
	125	-0.0030	0.1263	-0.0021	0.0867	-0.0025	0.0714	0.0009	0.0793	0.0045	0.1221	0.0010	0.0743	-0.0019	0.0749	-0.0019	0.1315	-0.0010	0.2219	0.0037	0.1369
10000	13	-0.2887	0.2151	-0.2403	0.2060	-0.2327	0.1893	-0.2233	0.2116	-0.2524	0.3122	-0.2639	0.1924	-0.2566	0.1838	-0.2402	0.3322	-0.2287	0.5682	-0.2326	0.2865
	25	-0.1178	0.2173	-0.0860	0.1929	-0.0695	0.1679	-0.0589	0.1811	-0.0763	0.2878	-0.0716	0.1850	-0.0698	0.1723	-0.0831	0.2998	-0.0966	0.4732	-0.0876	0.2583
	63	-0.0130	0.1464	-0.0069	0.1197	-0.0037	0.1025	0.0017	0.1165	-0.0120	0.1799	0.0015	0.1148	-0.0067	0.1075	0.0082	0.1935	0.0145	0.3178	-0.0064	0.1681
	125	0.0036	0.1093	-0.0028	0.0790	-0.0001	0.0688	0.0008	0.0798	0.0003	0.1182	0.0007	0.0769	0.0006	0.0734	-0.0053	0.1322	0.0038	0.2122	-0.0010	0.1170
	625	-0.0012	0.0640	0.0011	0.0402	-0.0030	0.0326	0.0006	0.0362	0.0018	0.0535	0.0008	0.0360	0.0010	0.0350	-0.0012	0.0634	-0.0007	0.1251	0.0037	0.0805
50000	13	-0.3033	0.2173	-0.2589	0.2125	-0.2273	0.1860	-0.2350	0.2082	-0.2792	0.3064	-0.2484	0.2095	-0.2607	0.1861	-0.2514	0.3228	-0.2476	0.5207	-0.2524	0.2759
	25	-0.1276	0.2135	-0.0801	0.1835	-0.0635	0.1594	-0.0753	0.1890	-0.0700	0.2764	-0.0761	0.1825	-0.0813	0.1764	-0.0823	0.2966	-0.0629	0.4911	-0.0937	0.2516
	63	-0.0133	0.1446	-0.0057	0.1174	-0.0053	0.1002	0.0042	0.1106	-0.0053	0.1806	-0.0035	0.1128	-0.0013	0.1081	-0.0088	0.1874	-0.0142	0.3107	-0.0119	0.1618
	125	0.0005	0.0982	-0.0014	0.0803	-0.0004	0.0686	0.0001	0.0774	-0.0042	0.1212	0.0034	0.0765	-0.0008	0.0761	-0.0015	0.1310	0.0196	0.2140	-0.0022	0.1132
	625	0.0018	0.0442	-0.0016	0.0348	-0.0011	0.0297	0.0003	0.0337	0.0007	0.0506	0.0008	0.0319	0.0005	0.0308	0.0022	0.0559	0.0037	0.0861	-0.0020	0.0466
	1250	-0.0010	0.0305	-0.0019	0.0242	0.0011	0.0206	-0.0013	0.0232	0.0014	0.0354	-0.0001	0.0218	0.0003	0.0213	-0.0015	0.0367	-0.0027	0.0590	0.0016	0.0355

TABLE 7. Results form simulation 6 : Stratified sampling without replacement, 1000 simulations, $T \leq S \leq R$, no smoothing, but raising.

	Agegroup	2		3		4		5		6		7		8		9		10		11	
#Lengths	#Otoliths per 10 cm	M	D	M	D	M	D	M	D	M	D	M	D	M	D	M	D	M	D	M	D
0	13	0.0188	2.0126	0.0719	0.9323	0.0685	0.6098	0.0442	0.6099	0.0720	0.9456	-0.0121	0.6774	-0.0060	0.7637	-0.1015	1.5910	-0.0748	3.1832	-0.1995	2.0406
	25	0.0697	1.3439	0.0646	0.6676	0.0453	0.4412	0.0294	0.4613	0.0121	0.6530	0.0040	0.4783	-0.0193	0.5602	-0.0433	1.1555	-0.0876	2.2640	-0.0587	1.5675
	63	-0.0318	0.8009	0.0031	0.4043	0.0055	0.2630	-0.0012	0.2694	-0.0092	0.4137	-0.0119	0.2963	0.0273	0.3657	0.0011	0.7491	-0.0132	1.6067	-0.0454	1.0418
	125	-0.0022	0.5979	-0.0008	0.2709	-0.0006	0.1790	0.0045	0.1999	-0.0126	0.2770	-0.0032	0.2162	0.0059	0.2491	-0.0090	0.5289	0.0349	1.1034	-0.0112	0.7236
	625	0.0028	0.2625	-0.0049	0.1216	-0.0007	0.0822	-0.0024	0.0830	0.0081	0.1316	0.0009	0.0970	0.0007	0.1121	0.0051	0.2366	-0.0039	0.4932	-0.0085	0.3378
	1250	0.0085	0.1806	-0.0039	0.0817	-0.0016	0.0569	-0.0028	0.0592	0.0065	0.0909	-0.0008	0.0682	0.0035	0.0776	0.0037	0.1632	0.0068	0.3267	-0.0094	0.2377
500	13	0.1304	0.3812	-0.0138	0.2826	-0.0183	0.2402	-0.0103	0.2689	-0.0381	0.4014	-0.0013	0.2649	0.0363	0.2492	0.1006	0.4783	0.1165	0.8092	0.2019	0.5210
	25	0.0498	0.3494	0.0053	0.2142	-0.0136	0.1812	-0.0077	0.1994	-0.0083	0.2967	0.0029	0.1951	0.0264	0.1844	0.0302	0.3341	0.0464	0.6219	0.0470	0.3945
1000	13	0.0310	0.3343	-0.0255	0.2730	-0.0046	0.2378	-0.0017	0.2573	-0.0032	0.4227	-0.0204	0.2564	0.0182	0.2593	0.0853	0.4703	0.0737	0.7235	0.1694	0.4371
	25	0.0211	0.2778	-0.0248	0.2062	0.0056	0.1724	-0.0092	0.1864	0.0015	0.3063	-0.0080	0.1910	0.0099	0.1848	0.0252	0.3348	0.0604	0.5457	0.0549	0.3331
	63	0.0042	0.2275	-0.0038	0.1423	-0.0013	0.1090	0.0053	0.1259	0.0025	0.1756	-0.0046	0.1175	0.0025	0.1101	0.0109	0.2145	-0.0003	0.3869	-0.0071	0.2594
5000	13	-0.0170	0.3167	-0.0156	0.2716	-0.0056	0.2377	0.0260	0.2747	-0.0317	0.3988	-0.0050	0.2615	0.0052	0.2513	-0.0044	0.4221	0.0349	0.7182	0.0442	0.3875
	25	-0.0417	0.2344	-0.0159	0.2066	0.0088	0.1698	-0.0026	0.1966	-0.0061	0.2958	0.0046	0.1942	0.0024	0.1885	-0.0014	0.3259	0.0365	0.5031	0.0191	0.2837
	63	0.0006	0.1601	-0.0069	0.1200	-0.0056	0.1035	0.0090	0.1194	-0.0055	0.1744	-0.0015	0.1129	0.0016	0.1082	0.0089	0.1993	-0.0071	0.3105	-0.0013	0.1774
	125	-0.0030	0.1263	-0.0021	0.0866	-0.0024	0.0714	0.0009	0.0793	0.0046	0.1221	0.0011	0.0743	-0.0019	0.0749	-0.0019	0.1315	-0.0010	0.2219	0.0037	0.1369
10000	13	-0.0589	0.2914	0.0014	0.2637	0.0102	0.2358	0.0230	0.2677	-0.0145	0.4073	-0.0290	0.2495	-0.0181	0.2440	0.0039	0.4432	0.0197	0.7521	0.0144	0.3819
	25	-0.0486	0.2356	-0.0148	0.2072	0.0022	0.1748	0.0141	0.1923	-0.0052	0.3061	0.0002	0.1959	0.0030	0.1862	-0.0119	0.3212	-0.0263	0.5095	-0.0155	0.2814
	63	-0.0102	0.1463	-0.0041	0.1197	-0.0008	0.1025	0.0046	0.1166	-0.0092	0.1802	0.0044	0.1151	-0.0038	0.1080	0.0111	0.1940	0.0175	0.3187	-0.0036	0.1688
	125	0.0037	0.1093	-0.0028	0.0790	-0.0001	0.0688	0.0009	0.0798	0.0004	0.1182	0.0008	0.0769	0.0007	0.0735	-0.0053	0.1322	0.0038	0.2121	-0.0010	0.1170
	625	-0.0012	0.0640	0.0011	0.0402	-0.0030	0.0326	0.0006	0.0362	0.0018	0.0535	0.0008	0.0360	0.0010	0.0350	-0.0012	0.0634	-0.0007	0.1251	0.0037	0.0805
50000	13	-0.0727	0.2900	-0.0158	0.2757	0.0244	0.2328	0.0143	0.2639	-0.0430	0.4021	-0.0019	0.2741	-0.0158	0.2516	-0.0048	0.4296	-0.0004	0.6909	-0.0042	0.3711
	25	-0.0566	0.2316	-0.0060	0.1954	0.0119	0.1679	-0.0011	0.1992	0.0049	0.2972	-0.0016	0.1936	-0.0065	0.1909	-0.0077	0.3200	0.0142	0.5338	-0.0195	0.2745
	63	-0.0089	0.1451	-0.0039	0.1173	-0.0021	0.1011	0.0073	0.1097	-0.0040	0.1788	0.0003	0.1124	0.0017	0.1084	-0.0045	0.1892	-0.0136	0.3049	-0.0110	0.1617
	125	0.0006	0.0982	-0.0013	0.0802	-0.0003	0.0686	0.0002	0.0775	-0.0041	0.1212	0.0034	0.0765	-0.0008	0.0761	-0.0014	0.1310	0.0197	0.2141	-0.0021	0.1132
	625	0.0018	0.0442	-0.0016	0.0348	-0.0011	0.0297	0.0003	0.0337	0.0007	0.0506	0.0008	0.0319	0.0005	0.0308	0.0022	0.0559	0.0037	0.0861	-0.0020	0.0466
	1250	-0.0010	0.0305	-0.0019	0.0242	0.0011	0.0206	-0.0013	0.0232	0.0014	0.0354	-0.0001	0.0218	0.0003	0.0213	-0.0015	0.0367	-0.0027	0.0590	0.0016	0.0355

Table 8. Cost of taking and processing length measurements and otoliths.

	Hours/unit
Length measurement	0.01496
Otolith	0.08297

Cost function:

$$cost = 0.01496 * \#lengths + 0.08297 * \#otoliths$$

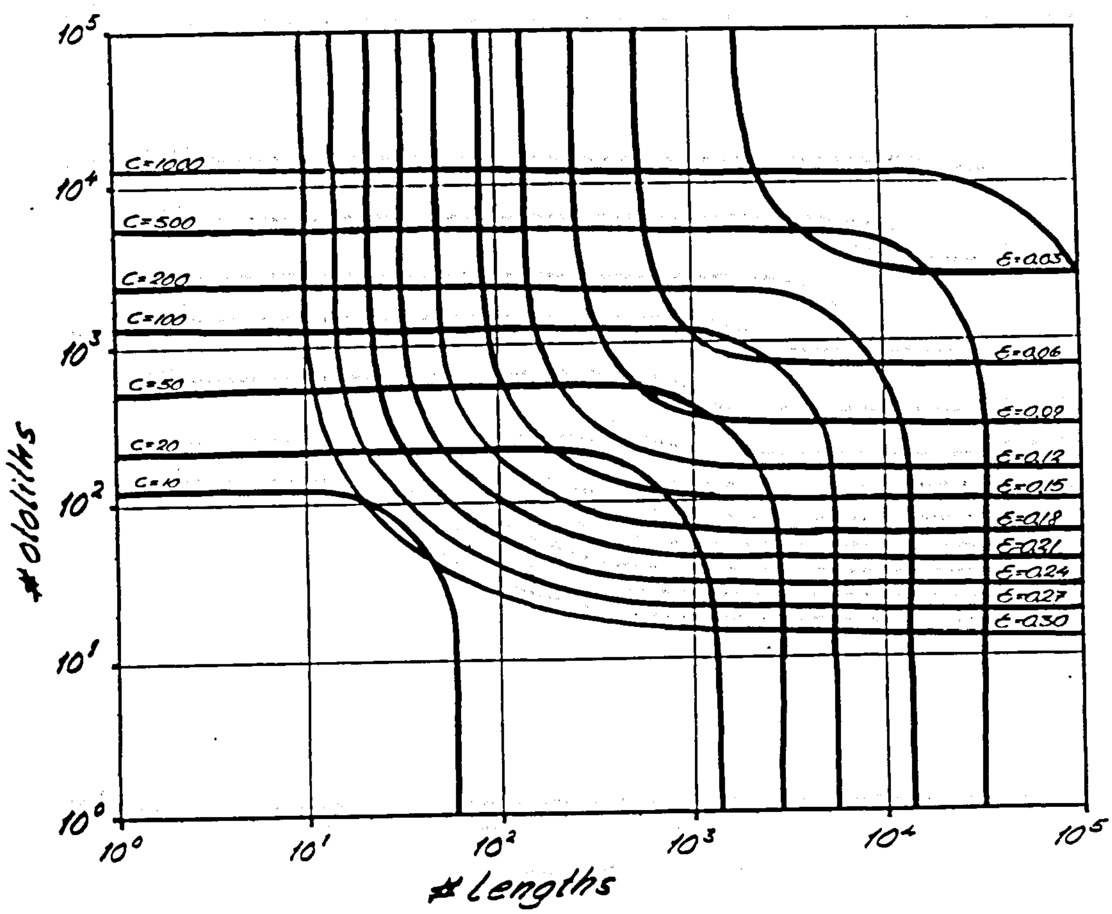

Figure 3. Contour plot showing cost and precision (for age group 5) as function of #lengths and #otoliths.