Transient Behaviour of Water Ages in the World Ocean

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Abstract — The transient behaviour of the age of the water and that of the surface water is studied in the World Ocean. At any time and position, the age of the water, the age and the concentration of the surface water are seen to obey a simple algebraic relation. The latter is illustrated by means of results of a three-dimensional World Ocean model and the analytical solution of an idealised, purely-diffusive, one-dimensional problem. © 2002 Elsevier Science Ltd. All rights reserved.

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1. INTRODUCTION

According to England [1] the “World Ocean circulation at its largest scale can be thought of as a gradual renewal or ventilation of the deep ocean by water that was once at the sea surface”. Thus, estimating the age of a particle of seawater as the time that has elapsed since the particle under consideration left the ocean surface layers provides useful insight into the ventilation processes.

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of the World Ocean. This is why the age is a popular diagnostic tool in this domain of interest (e.g., [1-17]).

Several authors [1,6,8,13] estimated a steady-state, position-dependent ventilation timescale, $\theta$, which was also termed "ventilation age" or "water age", as the solution of an equation of which the most general form probably is

$$\nabla \cdot (u \theta - K \cdot \nabla \theta) = 1, \tag{1}$$

$\nabla \cdot$ and $\nabla$ denote the divergence operator and the gradient operator, respectively; $u$ is the velocity vector, of which the divergence is zero, i.e., $\nabla \cdot u = 0$; the diffusivity tensor $K$ is symmetric and positive definite [15,18,19]. The velocity and diffusivity tensor are considered to be time-independent. The boundary of the domain of interest, which consists of the ocean surface, $S$, and the ocean bottom, $B$, is impermeable, implying that $u$ satisfies the boundary conditions

$$[u \cdot n]_{x \in \partial} = 0 = [u \cdot n]_{x \in B} \tag{2},$$

where $x$ denotes the position vector. In accordance with the very concept of ventilation, the ventilation timescale must be zero at the ocean surface

$$[\theta (x)]_{x \in S} = 0. \tag{3}$$

In addition, there must be no diffusive flux of $\theta$ through the ocean bottom. This is expressed by boundary condition

$$[(K \cdot \nabla \theta) \cdot n]_{x \in B} = 0. \tag{4}$$

Recently, new concepts of age [20,21] appropriate for World Ocean models have been introduced [17,22], which are slightly different from that defined by the partial differential problem (1)–(4). In particular, the general theory of the age developed by Delhez et al. [20] allows a particular water mass to be dealt with as a passive tracer, of which the age can subsequently be derived. Accordingly, Deleersnijder et al. [15] simulated numerically the evolution of the concentration, $C_{sw}$, and the age, $a_{sw}$, of a passive tracer tagging surface water. Delhez et al. [20], on the other hand, considered the passive tracer consisting of all the water particles, whatever their origin, and established the equation governing the evolution of its age, $a_w$. Deleersnijder et al. [15] indicated that the limit $t \to \infty$ of $a_{sw}$ and $a_w$ are equal to the ventilation timescale $\theta$. However, no detailed analysis of the transient behaviour of the age of the water and that of the surface water has been carried out so far. Doing so is the objective of the present study.

2. THEORETICAL DEVELOPMENTS

Lagrangian Approach

In Lagrangian terms, the water can be regarded as a large number of similar particles which move as a result of large-scale—advective—and small-scale—diffusive—processes, and are neither produced nor destroyed. As compressibility effects are neglected, the number of water particles contained in a volume delineated by thought is statistically constant, whatever the position of this volume and the instant at which the number of particles present in this volume is evaluated. On the other hand, a particle of surface water is a water particle that has touched at least once the ocean surface [15].

Consider an arbitrarily small region of the domain of interest, of which $x$ is the position vector. This regions contains $N_w$ water particles, of which $N_{sw}$ particles also belong to the surface water category. Clearly, $N_w$ is statistically constant, whereas $N_{sw}$ is likely to increase as time progresses.
If, as in Delhez et al. [20], the concentration of the water, $C_w$, is defined in such a way that it is equal to unity at every time and position, then the concentration of surface water, $C_{sw}$, satisfies

$$C_{sw}(t,x) = \frac{N_{sw}(t,x)}{N_w} \leq 1.$$  

(5)

Furthermore, since the diffusivity tensor is positive definite, all regions of the World Ocean are ventilated—though, for some of them, the ventilation process is very slow. This implies that

$$\lim_{t \to \infty} C_{sw}(t,x) = 1.$$  

(6)

Let the age of a water particle be defined as the time $t$ that has elapsed since the initial instant, $t = 0$, if the particle has not yet touched the ocean surface, or the time elapsed since the particle under consideration last touched the ocean surface. Therefore, by virtue of the age averaging hypothesis put forward in Deleersnijder et al. [15], the age of the water at time $t$ and location $x$ is given by the following arithmetic mean:

$$a_w = \frac{1}{N_w} \left[ \sum_{n=1}^{N_w} \omega^n \tau^n + t \sum_{n=1}^{N_w} (1 - \omega^n) \right],$$  

(7)

where $\tau^n$ is the age of the $n^{th}$ water particle, while $\omega^n$ is equal to unity if the corresponding particle hit the surface at least once, and is equal to zero otherwise, so that

$$\sum_{n=1}^{N_w} \omega^n = N_{sw}.$$  

(8)

Since $\tau^n \leq t$, relations (5) and (7), (8) imply that the age of the water must be smaller than the time elapsed since the initial instant, i.e.,

$$a_w(t,x) \leq t.$$  

(9)

On the other hand, the age of the surface water obviously is

$$a_{sw} = \frac{1}{N_{sw}} \sum_{n=1}^{N_{sw}} \omega^n \tau^n.$$  

(10)

Then, combining relations (5), (7), (8), and (10) yields

$$a_w(t,x) = a_{sw}(t,x) + [t - a_{sw}(t,x)] [1 - C_{sw}(t,x)],$$  

(11)

a relation which is valid at any time $t$ and location $x$. Finally, (5), (6), (9), and (11) imply

$$a_{sw}(t,x) \leq a_w(t,x)$$  

(12)

and

$$\lim_{t \to \infty} a_{sw}(t,x) = \lim_{t \to \infty} a_w(t,x).$$  

(13)

So, both ages are prescribed to be zero at the ocean surface, are equal to zero at $t = 0$ and have the same limit as $t \to \infty$. However, they exhibit different transient behaviours.

The Lagrangian approach, though rather illustrative, is unable to provide any clue to the relationship between the ventilation timescale $\theta$ and the limit $t \to \infty$ of the age of the surface water $a_{sw}$. On the other hand, numerical models of the World Ocean are based on the Eulerian formalism. This is why the Eulerian counterparts of the above developments need to be performed.
Eulerian Approach

According to Delhez et al. [20], the age of a passive tracer, \( a \), is to be obtained as the ratio of the age concentration, \( c_a \), to the concentration, \( C \), of the tracer under consideration, i.e.,

\[
a(t, x) = \frac{\alpha(t, x)}{C(t, x)} ,
\]

where \( C \) and \( \alpha \) obey the partial differential equations

\[
\begin{align*}
\frac{\partial C}{\partial t} &= -\nabla \cdot (u C - K \cdot \nabla C) , \\
\frac{\partial \alpha}{\partial t} &= C - \nabla \cdot (u \alpha - K \cdot \nabla \alpha) .
\end{align*}
\]

The water and the surface water may be treated as passive tracers [15,20]. Therefore, as the concentration \( C_w \) of the water is defined to be equal to unity, it is readily seen from equations (14)–(16) that the age \( a_w \) of the water satisfies

\[
\frac{\partial a_w}{\partial t} = 1 - \nabla \cdot (u a_w - K \cdot \nabla a_w) .
\]

The equations from which the surface water age may be obtained are

\[
\begin{align*}
\frac{\partial C_{sw}}{\partial t} &= -\nabla \cdot (u C_{sw} - K \cdot \nabla C_{sw}) , \\
\frac{\partial \alpha_{sw}}{\partial t} &= C_{sw} - \nabla \cdot (u \alpha_{sw} - K \cdot \nabla \alpha_{sw}) ,
\end{align*}
\]

and

\[
a_{sw}(t, x) = \frac{\alpha_{sw}(t, x)}{C_{sw}(t, x)} .
\]

The initial and boundary conditions that the variables above must satisfy are listed in Table 1.

<table>
<thead>
<tr>
<th>Table 1</th>
<th>Initial and boundary conditions satisfied by the age of the water, ( a_w ), the surface water concentration, ( C_{sw} ), the age concentration of the surface water, ( \alpha_{sw} ), and the age of the surface water, ( a_{sw} ).</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \psi = a_w )</td>
<td>( \psi = C_{sw} )</td>
</tr>
<tr>
<td>Initial Conditions:</td>
<td></td>
</tr>
<tr>
<td>( [\psi]_{t=0} = 0 )</td>
<td>( 0 )</td>
</tr>
<tr>
<td>Surface Boundary Conditions:</td>
<td></td>
</tr>
<tr>
<td>( [\psi]_{x \in S} = 0 )</td>
<td>( 1 )</td>
</tr>
<tr>
<td>Bottom Boundary Conditions:</td>
<td></td>
</tr>
<tr>
<td>( [(K \cdot \nabla \psi) \cdot n]_{x \leq B} = 0 )</td>
<td>( 0 )</td>
</tr>
</tbody>
</table>

Since the velocity, the diffusivity tensor and the boundary conditions are time-independent, the solutions of the differential problem (17)–(20) tend to a steady state as time progresses. It is readily seen that limit (6) holds valid, implying that the age of the water and that of the surface water tend, in the limit \( t \to \infty \), to the ventilation age \( \theta \) defined above [15]:

\[
\begin{align*}
\lim_{t \to \infty} a_w(t, x) &= \theta(x) , \\
\lim_{t \to \infty} a_{sw}(t, x) &= \theta(x) .
\end{align*}
\]
On the other hand, as the Lagrangian and Eulerian approaches are consistent with each other, the solutions to equations (17)-(20) satisfy relations (11). This is readily demonstrated.

While it has been known for some time that the ages dealt with herein are equivalent at a steady state and that the age of the water is larger than that of the surface water for $t < \infty$ [15,17,23], the fact that the transient behaviours of the water and surface water ages are constrained by relation (11) is a new result, which is illustrated below by means of analytical and numerical results.

### 3. ILLUSTRATION

The evolution of the age of the water and that of the surface water is simulated numerically in the World Ocean by means of an off-line version of CLIO, ASTR's coupled large-scale ice-ocean model [24]. This off-line model, named LOCH-CLIO, is somewhat different from its on-line counterpart [25]. Annual mean velocity, diffusivity and convective diagnostic fields are used. In particular, convection is parameterised as an additional diffusive term [26,27] and a small, constant, vertical transport. These terms were calibrated in such a way that the off-line model reproduces the age and $\Delta^{14}C$ fields obtained from the on-line model [28].

In accordance with equation (11), Figure 1 shows that the difference

$$\Delta(t, x) = a_w(t, x) - a_{sw}(t, x),$$

which is initially zero, first increases and, then, decreases toward zero as time progresses. The time evolution of the maximum value of $\Delta$,

$$\Delta_{\text{max}}(t) = \max_x \Delta(t, x),$$

is depicted in Figure 2.

Figure 1. Scatterplot of the age of the surface water, $a_{sw}$, as a function of the age of the water, $a_w$, as simulated numerically at every grid point of the World Ocean model CLIO. The ages are expressed in $10^3$ years, and are displayed at time $t = 300$, $750$, $1500$, and $3000$ years.
It is instructive to compare the evolution of $A_{\max}$ as simulated in the World Ocean with its counterpart obtained from a one-dimensional, purely diffusive problem. In the latter, the domain of interest is the space interval $0 \leq x \leq L$. The point $x = 0$ may be regarded as equivalent to the ocean surface, implying that the ages and the surface water concentration are prescribed to be zero and unity, respectively, at this point. At the other end of the domain, all diffusive fluxes are zero. At $t = 0$, the ages and the surface water concentration are zero. The diffusivity $K$ is assumed to be constant, and the velocity is zero.

Let $\theta_{\max}$ denote the maximum of the time-independent ventilation age $\theta$. In the one-dimensional problem, $\theta_{\max}$ is equal to $L^2/(2K)$, while, in the World Ocean model results, the maximum value of $\theta$ is 1711 years. If $\mu_n = \pi(n + 1/2)$, $\sigma = x/L$, and $\tau = t/\theta_{\max}$, the solutions of the above one-dimensional problem may be written as follows:

\begin{align}
\frac{\theta(\sigma)}{\theta_{\max}} &= 2\sigma - \sigma^2, \quad (25) \\
\frac{a_w(\tau, \sigma)}{\theta_{\max}} &= \theta(\sigma) - 4 \sum_{n=0}^{\infty} \frac{1}{\mu_n^3} \exp \left( \frac{-\mu_n^2 \tau}{2} \right) \sin(\mu_n \sigma), \quad (26) \\
C_{aw}(\tau, \sigma) &= 1 - 2 \sum_{n=0}^{\infty} \frac{1}{\mu_n} \exp \left( \frac{-\mu_n^2 \tau}{2} \right) \sin(\mu_n \sigma), \quad (27) \\
\frac{a_{aw}(\tau, \sigma)}{\theta_{\max}} &= \frac{\theta(\sigma) - 2 \sum_{n=0}^{\infty} \left( \frac{\mu_n t + 2}{\mu_n^2} \right) \exp \left( -\mu_n^2 \tau/2 \right) \sin(\mu_n \sigma)}{1 - 2 \sum_{n=0}^{\infty} \frac{1}{\mu_n} \exp \left( -\mu_n^2 \tau/2 \right) \sin(\mu_n \sigma)}. \quad (28)
\end{align}

In the one-dimensional solutions, the time evolution of the dimensionless ratio $\Delta_{\max}(t)/\theta_{\max}$ is rather similar to its counterpart simulated in the World Ocean (Figure 2). The maximum of $\Delta_{\max}(t)/\theta_{\max}$ is 0.18 and 0.17, and is reached when $t/\theta_{\max}$ is equal to 1.1 and 0.77 for the one-dimensional, purely diffusive solution and the World Ocean model results, respectively. This similarity is rather surprising since transport in the World Ocean is dominated by advection, whereas there is only diffusion in the one-dimensional problem. Clearly, further research is needed to identify the reason why the ratio $\Delta_{\max}(t)/\theta_{\max}$ exhibits similar behaviour in the two models dealt with in the present study.

REFERENCES