

# Water wave-driven seepage in marine sediments

D.S. Jeng<sup>a,\*</sup>, D.A. Barry<sup>b</sup>, L. Li<sup>b</sup>

<sup>a</sup> School of Engineering, Griffith University, Gold Coast Campus, PMB 50 Gold Coast Mail Centre, Qld 9726, Australia

<sup>b</sup> School of Civil and Environmental Engineering and Contaminated Land Assessment and Remediation Research Centre, The University of Edinburgh, Edinburgh EH9 3JN, UK

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## Abstract

The action of water waves moving over a porous seabed drives a seepage flux into and out of the marine sediments. The volume of fluid exchange per wave cycle may affect the rate of contaminant transport in the sediments. In this paper, the dynamic response of the seabed to ocean waves is treated analytically on the basis of poro-elastic theory applied to a porous seabed. The seabed is modelled as a semi-infinite, isotropic, homogeneous material. Most previous investigations on the wave–seabed interaction problem have assumed quasi-static conditions within the seabed, although dynamic behaviour often occurs in natural environments. Furthermore, wave pressures used in the previous approaches were obtained from conventional ocean wave theories, which are based on the assumption of an impermeable rigid seabed. By introducing a complex wave number, we derive a new wave dispersion equation, which includes the seabed characteristics (such as soil permeability, shear modulus, etc.). Based on the new closed-form analytical solution, the relative differences of the wave-induced seabed response under dynamic and quasi-static conditions are examined. The effects of wave and soil parameters on the seepage flux per wave cycle are also discussed in detail. © 2000 Elsevier Science Ltd. All rights reserved.

## 1. Introduction

Sediments in bays, estuaries, and in the seabed near river inlets are often contaminated. Many inorganic contaminants (notably heavy metals) do not decompose. Under certain conditions, these accumulated substances can be released back into the receiving body of water through mass transfer processes at the seabed. The mass transfer rate is largely controlled by the seepage flux exchange between the sediment and the seawater [26]. Increased wave action and higher sediment hydraulic conductivity generally cause larger transfer rates. Clearly, quantification of the mass transfer rate is a key factor in water quality modelling.

Water wave effects on the marine sediment in shallow water have been studied intensively in the last few decades. Most earlier studies [1,18,19,21,23] were based on the assumption that the porous seabed was non-deformable, and that the pore water was incompressible. Numerous investigations have been carried out based on Biot's consolidation equation [2] and the assumptions of

a compressible pore fluid and soil skeleton. The seabed has been modelled as being isotropic [9,20,22,29,30], anisotropic [10] and inhomogeneous [16]. The potential for extreme seabed instability (such as liquefaction) due to a generalised three-dimensional wave system has recently been explored [17]. Limitations and applications of previous investigations have been reviewed also [11]. Two major shortcomings of most previous approximations in the area of wave–seabed interaction are:

- Assumption of a quasi-static state: The dynamic terms generated from the acceleration of soil particles and the movement of pore fluid were not included in most previous solutions. Although the inertia effect generated from the acceleration of soil particles was recently considered in the wave–seabed interaction problem [15], the acceleration due to pore fluid was excluded from the analysis. Thus, it is not a complete dynamic solution.
- Wave pressures at the seabed surface obtained from conventional ocean wave theories: These wave pressures were based on the assumption of an impermeable seabed. However, the wave pressures were then applied to a porous seabed. This contradiction has been removed recently [12], with the assumption of a quasi-static state, not a dynamic state. Recently,

\* Corresponding author. Tel.: +61-7-5594-8683; fax: +61-7-5594-8065.

E-mail address: d.jeng@mailbox.gu.edu.au (D.S. Jeng).

Liu and Wen [19] derived a fully non-linear, diffusion and weakly dispersive wave equation for describing gravity surface wave propagation in a shallow porous medium. However, they only considered a rigid medium (such as rock). Thus, only soil permeability is included in their dispersion relation.

The objective of this paper is to overcome these two shortcomings and investigate water wave-driven seepage flux into and out of the seabed under more realistic conditions. Based on the governing equations presented by Mei [22], which were derived on the basis of poro-elastic theory [3], a closed-form analytical solution for dynamic wave–seabed interactions is derived. The model also includes a new wave–dispersion relation, which accounts for the characteristics (including soil permeability, shear modulus, porosity, etc.). A comparison of the present dynamic and previous quasi-static solution is performed. Then, the effects of wave and seabed characteristics on the seepage flux at the seabed surface are investigated in a parametric study.

## 2. Boundary value problem

In this study, we consider a gravity wave propagating over a porous seabed. The wave crests are assumed to propagate in the positive  $x$ -direction, while the  $z$ -direction is upward from the seabed surface, as shown in Fig. 1.

The proposed model for wave–seabed interaction is based on combining incompressible irrotational flow for the water waves and Biot's poro-elastic theory [3] for flow within the porous soil skeleton.

### 2.1. Governing equations for water waves

The assumption of an incompressible and irrotational flow for water waves in the ocean leads to the Laplace equation for velocity potential,  $\phi$ ,

$$\nabla^2 \phi = 0, \quad (1)$$

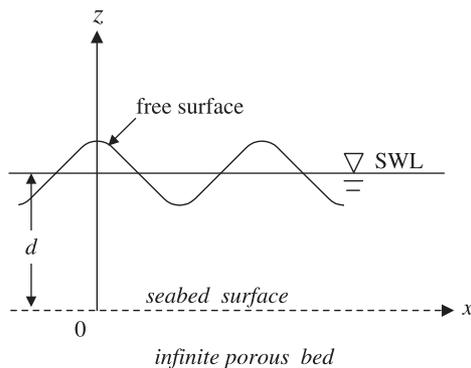


Fig. 1. Definition of wave–seabed interaction problem.

while the free surface fluctuation of the water wave,  $\eta$ , is given as the real part of

$$\eta = a \exp[i(\lambda x - \omega t)], \quad (2)$$

where  $a$  is the wave amplitude and  $\lambda (= \lambda_r + i\lambda_i)$  is the complex wave number. The real part,  $\lambda_r$ , is related to the wavelength by  $\lambda_r = 2\pi/L$  ( $L$  is the wavelength), while  $\lambda_i$  represents the damping of the water waves. Unlike previous investigations for the wave–seabed interaction problem [9,29] where  $\lambda$  is assumed known a priori, here  $\lambda$  is an unknown parameter to be determined. In Eq. (2),  $\omega = 2\pi/T$  is the wave frequency.

### 2.2. Governing equations for porous flow in seabed

Marine sediment is a mixture of up to three phases: a solid phase that forms an interlocking skeletal frame, a liquid phase that occupies a major portion of pore space, and a gas phase that sometimes occupies a small portion of pore space. Thus, it is reasonable to assume both pore fluid (a continuum comprising both liquid and gas) and soil skeleton to be compressible.

Based on the assumption of linear poro-elasticity [3], Mei [22] presented a set of linearised governing equations for dynamic variations from the static equilibrium state, i.e., a storage equation, an equation of motion and a conservation of momentum equation of the soil–skeleton and the pore–water. They are summarised as follows:

- *Storage equation:* This equation is based on conservation of mass, originally derived by Verruijt [28] without dynamic terms generated from acceleration of soil particles and pore fluid. Mei [22] re-derived the storage equation with dynamic terms as

$$\frac{k}{\gamma_w} \nabla^2 p - n\beta \frac{\partial p}{\partial t} = \frac{\partial \nabla \cdot \mathbf{u}_s}{\partial t} - \rho_w \frac{k}{\gamma_w} \frac{\partial^2 \nabla \cdot \mathbf{u}_w}{\partial t^2}. \quad (3)$$

- *Equation of motion:* Again, the accelerations generated by soil particles ( $\partial^2 \mathbf{u}_s / \partial t^2$ ) and pore fluid ( $\partial^2 \mathbf{u}_w / \partial t^2$ ) are also included in the equation of motion, as shown on the right-hand side of equation (4). These terms have not been included in quasi-static models [9,20].

$$G \left\{ \nabla^2 \mathbf{u}_s + \frac{1}{1-2\mu} \nabla (\nabla \cdot \mathbf{u}_s) \right\} - \nabla p = n\rho_w \frac{\partial^2 \mathbf{u}_w}{\partial t^2} + (1-n)\rho_s \frac{\partial^2 \mathbf{u}_s}{\partial t^2}. \quad (4)$$

- *Momentum equation:* In previous quasi-static solutions [9,20], the momentum equation was not included in the governing equations. However, it is necessary to have the momentum equation in dynamic model, because the relative motion between pore fluid and soil particles is considered.

The linearised momentum equation of soil particles and pore fluid can be expressed as [22]

$$\rho_w \frac{\partial^2 \mathbf{u}_w}{\partial t^2} = -\nabla p - \frac{\gamma_w n}{k} \left( \frac{\partial \mathbf{u}_w}{\partial t} - \frac{\partial \mathbf{u}_s}{\partial t} \right). \quad (5)$$

In Eqs. (3)–(5),  $\mathbf{u}_s \equiv (u_s, w_s)$  denotes the displacements of soil particles, while  $\mathbf{u}_w \equiv (u_w, w_w)$  denotes the displacements of pore fluid. The subscripts s and w denote the soil phase and the water phase, respectively. Furthermore,  $\gamma_w$  is the unit weight of pore water,  $p$  is pore water pressure,  $\mu$  is Poisson's ratio,  $k$  is permeability of the seabed,  $n$  is the soil porosity,  $G$  is the shear modulus of the soil,  $\rho$  denotes the material density and  $\beta$  is the compressibility of pore fluid, which is defined as

$$\beta = \frac{1}{K_w} + \frac{1-S}{P_{w0}}, \quad (6)$$

in which  $K_w$  is the true bulk modulus of pore water,  $S$  is the degree of saturation and  $P_{w0}$  is the average absolute pore water pressure in the sediment.

To simplify the rather complicated governing equations (3)–(5), Mei [22] used a boundary-layer approximation. However, this approximation is limited to the case of low permeability [9]. Here, we directly solve (3)–(5) without any further assumptions.

Since all equations are linear, we can further decompose the displacement vectors of the soil and the pore water,  $\mathbf{u}_s$  and  $\mathbf{u}_w$ , into two parts: the irrotational part and the non-divergent part. This decomposition has been used widely for elastic analysis in consolidation problems [4]. By using scalar potentials ( $\Phi_s$  and  $\Phi_w$ ) and vector potentials ( $\Psi_s$  and  $\Psi_w$ ) the decomposition can be expressed in the form

$$\mathbf{u}_s = \nabla \Phi_s + \nabla \times \Psi_s \quad \text{and} \quad \mathbf{u}_w = \nabla \Phi_w + \nabla \times \Psi_w. \quad (7)$$

By substituting Eq. (7) into Eqs. (3)–(5), we obtain, respectively

$$\frac{k}{\gamma_w} \nabla^2 p - \nabla^2 \frac{\partial \Phi_s}{\partial t} - n\beta \frac{\partial p}{\partial t} + \rho_w \frac{k}{\gamma_w} \nabla^2 \frac{\partial^2 \Phi_w}{\partial t^2} = 0, \quad (8)$$

$$\frac{2(1-\mu)}{1-2\mu} G \nabla^2 \Phi_s - p - n\rho_w \frac{\partial^2 \Phi_w}{\partial t^2} - (1-n)\rho_s \frac{\partial^2 \Phi_s}{\partial t^2} = 0, \quad (9)$$

$$\rho_w \frac{\partial^2 \Phi_w}{\partial t^2} + p + \frac{n\gamma_w}{k} \frac{\partial \Phi_w}{\partial t} - \frac{n\gamma_w}{k} \frac{\partial \Phi_s}{\partial t} = 0, \quad (10)$$

$$G \nabla^2 \Psi_s - n\rho_w \frac{\partial^2 \Psi_w}{\partial t^2} - (1-n)\rho_s \frac{\partial^2 \Psi_s}{\partial t^2} = 0 \quad (11)$$

and

$$\rho_w \frac{\partial^2 \Psi_w}{\partial t^2} + \frac{\gamma_w n}{k} \frac{\partial \Psi_w}{\partial t} - \frac{\gamma_w n}{k} \frac{\partial \Psi_s}{\partial t} = 0. \quad (12)$$

Under the condition of plane strain, the stress–strain relationship is given as [5]

$$\sigma'_x = 2G \left( \frac{\partial u_s}{\partial x} + \frac{\mu}{1-2\mu} \nabla \cdot \mathbf{u}_s \right), \quad (13)$$

$$\sigma'_z = 2G \left( \frac{\partial w_s}{\partial z} + \frac{\mu}{1-2\mu} \nabla \cdot \mathbf{u}_s \right), \quad (14)$$

and

$$\tau = G \left( \frac{\partial u_s}{\partial z} + \frac{\partial w_s}{\partial x} \right), \quad (15)$$

where  $u_s$  and  $w_s$  are the components of  $\mathbf{u}_s$  in the  $x$ - and  $z$ -directions, respectively,  $\sigma'_x$  and  $\sigma'_z$  are the effective normal stresses in the  $x$ - and  $z$ -directions, respectively, and  $\tau$  is the shear stress.

### 2.3. Boundary conditions

To solve the governing Eqs. (8)–(12), appropriate boundary conditions are required.

For the water waves, the linearised free surface boundary conditions for a small amplitude wave are given as [6]

$$g\eta + \frac{\partial \phi}{\partial t} = 0 \quad \text{and} \quad \frac{\partial \phi}{\partial z} = \frac{\partial \eta}{\partial t} \quad \text{at} \quad z = d, \quad (16)$$

where  $g$  is the magnitude of gravitational acceleration and  $d$  is the water depth.

For flow in a porous seabed, the boundary conditions must reflect the physical requirement that, for a semi-infinite half-plane, dynamic fluctuations of all physical quantities vanish at negative infinity, i.e.,

$$\mathbf{u}_s \quad \text{and} \quad p \rightarrow 0 \quad \text{as} \quad z \rightarrow -\infty. \quad (17)$$

At the seabed surface, the vertical effective stresses and shear stress vanish, the fluid pressure is transmitted continuously from the sea to the pores in the seabed,

$$\sigma'_z = \tau = 0, \quad p(z=0) = -\rho_w \frac{\partial \phi}{\partial t}. \quad (18)$$

Furthermore, the fluid flux across the seabed should be conserved. Since the wave-induced vertical flux is much larger than the horizontal flux [31], it is reasonable to consider that the vertical flux is dominant, implying that the horizontal flux can be ignored as a first approximation. This assumption is only valid for the seabed under the action of small-amplitude waves. Also, we assume the deformation of seabed surface is small under the action of small-amplitude waves, compared with the fluctuation of water waves ( $\eta$ ). Only the linearised boundary condition of water–sediment interface is used here. Thus, the conservation of vertical flux can be written as [14]

$$\frac{\partial \phi}{\partial z} = -\frac{k}{\gamma_w} \frac{\partial p}{\partial z} + \frac{\partial}{\partial t} (w_w - w_s), \quad z = 0. \quad (19)$$

### 3. General solutions

#### 3.1. Closed-form solution for porous flow

From Eqs. (1), (2) and (16), the velocity potential can be expressed as [6]

$$\phi = -\frac{ig\eta}{\omega} \left[ \cosh \lambda(z-d) + \frac{\omega^2}{g\lambda} \sinh \lambda(z-d) \right]. \quad (20)$$

Note that the wave number  $\lambda$  is an unknown parameter here. Then, the wave pressure,  $p_w$ , can be written as [6]

$$\begin{aligned} p_w(x, z; t) &= -\rho_w \frac{\partial \phi}{\partial t} \\ &= \gamma_w \eta \left[ \cosh \lambda(z-d) + \frac{\omega^2}{g\lambda} \sinh \lambda(z-d) \right]. \end{aligned} \quad (21)$$

In general, the mechanism of the wave-induced seabed response can be classified into two categories, depending upon how the pore water pressure is generated [25]. One is caused by the residual or progressive nature of the excess pore pressure, which appears in the initial stage of cyclic loading. This type of soil response is similar to that induced by earthquakes, caused by the build-up of the excess pore pressure. The other, generated by the transient or oscillatory excess pore pressure, accompanied by the damping of amplitude in the pore pressure, appears as a periodic response to each wave. The residual soil response is normally important for a soft seabed (such as fine sand and clay, etc.), while the transient soil response is important for a non-cohesive sandy bed [27]. We focus on the transient soil response in this paper.

Since the transient soil oscillatory response under harmonic wave loading will be spatially and temporally periodic, the wave-induced soil response can be expressed as [30]

$$\begin{Bmatrix} p \\ \Phi_s \\ \Phi_w \\ \Psi_s \\ \Psi_w \end{Bmatrix} = \gamma_w \eta \begin{Bmatrix} P(z) \\ S_e(z) \\ W_c(z) \\ S_c(z) \\ W_c(z) \end{Bmatrix} \times \text{cshd}, \quad (22)$$

where

$$\text{cshd} = \cosh \lambda d - \frac{\omega^2}{g\lambda} \sinh \lambda d, \quad (23)$$

$\eta$  is defined by Eq. (2), and the subscripts e and c denote the elastic wave and compressive wave components, respectively. The various functions in braces on the right-hand side of Eq. (22) are defined in Appendix A.

Following the procedure proposed by Yuhi and Ishida [30], the governing equations (8)–(12), together with the boundary conditions (17) and (18), are solved for the wave-induced soil displacements, pore pressure and pore fluid displacements (details are given in Appendix A

$$u_s = \gamma_w \eta \{ i\lambda (a_1 e^{\lambda_1 z} + a_2 e^{\lambda_2 z}) - \lambda_3 a_3 e^{\lambda_3 z} \} \times \text{cshd}, \quad (24)$$

$$w_s = \gamma_w \eta \{ \lambda_1 a_1 e^{\lambda_1 z} + \lambda_2 a_2 e^{\lambda_2 z} + i\lambda a_3 e^{\lambda_3 z} \} \times \text{cshd}, \quad (25)$$

$$p = \gamma_w \eta \{ \xi_1 a_1 e^{\lambda_1 z} + \xi_2 a_2 e^{\lambda_2 z} \} \times \text{cshd}, \quad (26)$$

$$u_w = \gamma_w \eta \{ i\lambda (\xi_1 a_1 e^{\lambda_1 z} + \xi_2 a_2 e^{\lambda_2 z}) - \lambda_3 \xi_3 a_3 e^{\lambda_3 z} \} \times \text{cshd}, \quad (27)$$

$$w_w = \gamma_w \eta \{ \xi_1 \lambda_1 a_1 e^{\lambda_1 z} + \xi_2 \lambda_2 a_2 e^{\lambda_2 z} + i\lambda \xi_3 a_3 e^{\lambda_3 z} \} \times \text{cshd}, \quad (28)$$

where

$$\lambda_1^2 = \lambda^2 - \frac{d_2 + \sqrt{d_2^2 - 4d_1 d_3}}{2d_1}, \quad (29)$$

$$\lambda_2^2 = \lambda^2 - \frac{d_2 - \sqrt{d_2^2 - 4d_1 d_3}}{2d_1}, \quad (30)$$

$$\lambda_3^2 = \lambda^2 - \frac{\omega^2}{G(v+in)} \{ in\rho_e + (1-n)v\rho_s \}, \quad (31)$$

$$\rho_e = n\rho_w + (1-n)\rho_s, \quad v = \frac{\rho_w \omega k}{\gamma_w}, \quad m = \frac{Gn\beta}{1-2\mu}, \quad (32)$$

where the constants  $d_i$  ( $i = 1-3$ ) are given in Appendix A.

In Eqs. (24)–(28), the three  $a_i$  coefficients are given as

$$a_1 = -\frac{\delta_2}{\delta_1 \xi_2 - \delta_2 \xi_1}, \quad (33)$$

$$a_2 = \frac{\delta_1}{\delta_1 \xi_2 - \delta_2 \xi_1}, \quad (34)$$

$$a_3 = -\frac{2i\lambda(\lambda_1 \delta_2 - \lambda_2 \delta_1)}{(\lambda_3^2 + \lambda^2)(\delta_1 \xi_2 - \delta_2 \xi_1)}, \quad (35)$$

in which  $\xi_i$  and  $\delta_i$  are coefficients also given in Appendix A.

#### 3.2. Wave dispersion relation

Since the wave number is an unknown in the model, the flux conservation condition (19) is used to determine it. Introducing (20), (25), (26) and (28) into this condition yields the wave dispersion equation

$$\tanh \lambda d = \frac{\omega^2 - g\lambda F}{g\lambda - \omega^2 F}, \quad (36)$$

where

$$F = \frac{i\rho_w k \omega}{\gamma_w \lambda} \{ \lambda_1 (\xi_1 + \lambda \omega) (\lambda_1 a_1 + \lambda_2 a_2) + \omega \lambda a_3 \}. \quad (37)$$

Determination of the wave number  $\lambda = \lambda_r + i\lambda_i$  in the dispersion equation (36) is a simple root-finding problem that can be solved numerically using Newton's method. Once the complex wave number is obtained, all coefficients and the wave-induced soil response can be easily calculated.

As expected, the new wave dispersion relation, Eq. (36), reduces to the conventional wave dispersion

equation if the seabed response is ignored (i.e.,  $F = 0$ ), i.e.,

$$\tanh \lambda_0 d = \frac{\omega^2}{g \lambda_0}, \quad (38)$$

where  $\lambda_0$  denotes the wave number with the assumption of impermeable seabed.

The new wave dispersion relation clearly indicates that the wave characteristics (such as the wavelength and wave profile) are affected by soil parameters appearing in the combined parameter ( $F$ ). However, the effect of soil parameters on the wave characteristics is not the main concern in this paper. Rather, we wish to focus on the seabed response. The effects of the seabed

properties on the wave characteristics are described elsewhere [12,13].

### 3.3. Comparison with the quasi-static solution

Referring to the governing equations for the porous flow, (3)–(5), the major differences between the present dynamic model and previous quasi-static model are the pore fluid and soil skeleton acceleration terms. It is of interest to examine the effects of these additional dynamic terms on the wave-induced soil response, including pore pressure, effective normal stresses, shear stress and soil displacements.

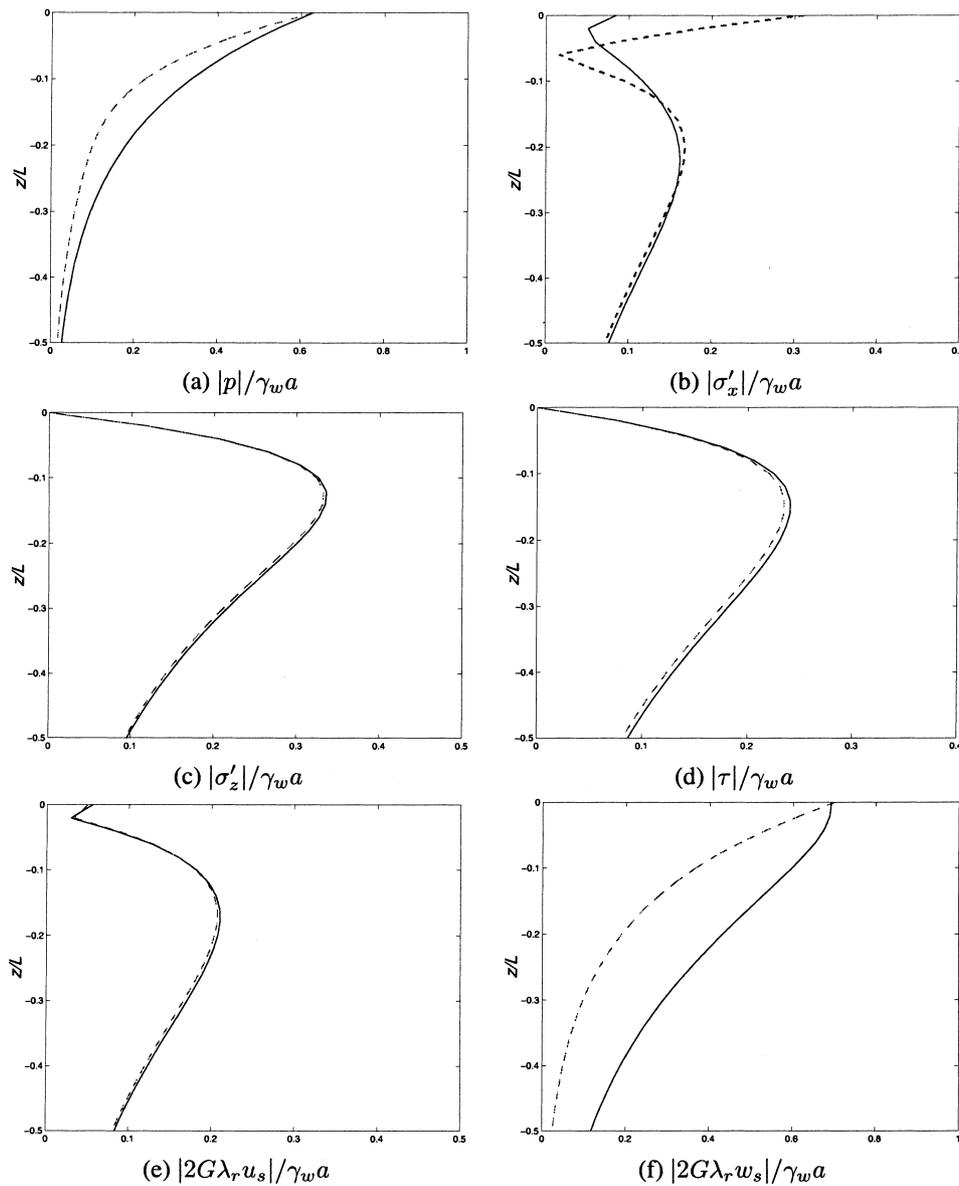


Fig. 2. Distributions of the maximum amplitudes of the wave-induced: (a) pore pressure ( $|p|/\gamma_w a$ ), (b) and (c) effective normal stresses ( $|\sigma'_x|/\gamma_w a$  and  $|\sigma'_z|/\gamma_w a$ ), (d) shear stress ( $|\tau|/\gamma_w a$ ) and (e) and (f) soil displacements ( $|2G\lambda_r u_s|/\gamma_w a$  and  $|2G\lambda_r w_s|/\gamma_w a$ ) in an unsaturated coarse sand ( $S = 99\%$ ). The solid line is the present dynamic solution while the dashes are the quasi-static solution [9].

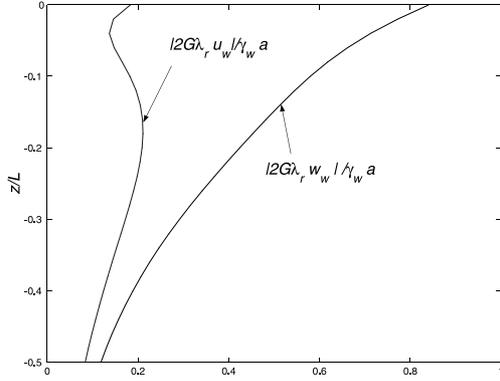


Fig. 3. Distributions of the maximum amplitude of the wave-induced displacements of pore fluid ( $|2G\lambda_r u_w|/\gamma_w a$  and  $|2G\lambda_r w_w|/\gamma_w a$ ) in an unsaturated coarse sand.

Fig. 2 shows a comparison of the wave-induced soil response between the present dynamic solution (solid lines) and the existing quasi-static solution (dashes) [9] in a coarse sand. In the figure, we plot the maximum amplitude of the wave-induced soil response over a wave cycle. The following values for the seabed parameters were used:  $(k, G, S, n, \mu, \rho_s, \rho_w) = (10^{-2} \text{ m/s}, 10^7 \text{ N/m}^2, 99\%, 0.35, 1/3, 2650 \text{ kg/m}^3, 1000 \text{ kg/m}^3)$ . The parameters values related to the water waves were: water wave period  $T = 10 \text{ s}$ , water depth  $d = 20 \text{ m}$  and wave amplitude  $a = 5 \text{ m}$ .

As seen in the figure, the dynamic behaviour of a porous seabed is important in the evaluation of the wave-induced pore pressure ( $|p|/\gamma_w a$ ) and the vertical soil displacement ( $|2G\lambda_r w_s|/\gamma_w a$ ). The other soil response parameters are only slightly affected by the dynamic soil behaviour.

Using the same input data, the vertical distributions of the wave-induced displacements of pore fluid ( $|2G\lambda_r u_w|/\gamma_w a$  and  $|2G\lambda_r w_w|/\gamma_w a$ ) versus the relative burial depth ( $z/L$ ) in an unsaturated ( $S = 99\%$ ) coarse sand are plotted in Fig. 3. These components can be only obtained through the dynamic solution – they are modelled by the previous quasi-static solution. Note that the relative difference between displacement of pore fluid and soil skeleton at the seabed surface is about 10% of  $\gamma_w a$  (Figs. 2(f) and 3).

#### 4. Wave-driven seepage

Water wave over a porous seabed drives a seepage flux into and out of the sediment. As noted above, the volume of fluid exchanged per wave cycle directly relates to the mass transport rate of contaminants in the sediment, an important quantity in water quality modelling.

The net seepage flux over one wave cycle is zero. However, for the mass transport caused by the cyclic wave motion, the relevant quantity is the volume of

Table 1

Parameter values used for the case study

<i>Wave characteristics</i>	
Wave period, $T$	10 s
Water depth, $d$	10 m
Wavelength, $L$	92.356 (the first wave cycle)
Wave amplitude, $a$	5 m
<i>Soil characteristics</i>	
Poisson's ratio, $\mu$	1/3
Porosity, $n$	0.35
Degree of saturation, $S$	99%
Density of soil skeleton, $\rho_s$	2650 kg/m <sup>3</sup>
Density of pore water, $\rho_w$	1000 kg/m <sup>3</sup>
Soil conductivity, $k$	$10^{-2} \text{ m/s}$ (coarse sand) $10^{-4} \text{ m/s}$ (fine sand)
Shear modulus, $G$	$10^7 \text{ N/m}^2$ (coarse sand) $5 \times 10^6 \text{ N/m}^2$ (fine sand)

water pumped into over one-half wave period ( $T$ ) and one-half wavelength ( $L$ ). This volume (per unit width of sea bottom) can be expressed as a function of depth ( $z$ ) [24],

$$V(z) = - \int_{-T/4}^{T/4} \int_{-L/4}^{L/4} \frac{k}{\gamma_w} \frac{\partial p}{\partial z} dx dt = \text{Re} \left\{ \frac{4ka}{\lambda\omega} \frac{\xi_1 \lambda_1 \delta_2 e^{\lambda_1 z} - \xi_2 \lambda_2 \delta_1 e^{\lambda_2 z}}{\delta_1 \xi_2 - \delta_2 \xi_1} \times \text{cshd} \right\}. \quad (39)$$

In this study, we will focus on the volume of seepage exchange between seawater and seabed driven by the water waves, that is, the volume exchange at  $z = 0$ ,

$$V_0 = V(0) = \text{Re} \left\{ \frac{4ka}{\lambda\omega} \frac{\xi_1 \lambda_1 \delta_2 - \xi_2 \lambda_2 \delta_1}{\delta_1 \xi_2 - \delta_2 \xi_1} \times \text{cshd} \right\}. \quad (40)$$

It is noted that  $V_0$  in (40) is the ‘‘volume displacement’’ per unit width at interface between water and sediments.

In the following example, we examine the effects of wave and soil characteristics on  $V_0$ . The input data for the parametric study are tabulated in Table 1.

To have a basic understanding how inertial terms affect the seepage flux at water–sediment interface, the values of  $V_0$  calculated from the present dynamic solution and previous quasi-static solution [9] in coarse sand fine sand are tabulated in Table 2. As shown in Table 2, the inclusion of the inertial terms has significant effects on the seepage rate.

##### 4.1. Effects of wave characteristics

It is well known [8] that the wavelength is determined by wave period ( $T$ ) and water depth ( $d$ ) through the wave dispersion relation. The wave period in ocean environments ranges from 5 to 25 s for gravity ocean waves [6]. For shallow water, the relative water depth ( $d/L_d$ ,  $L_d = 1.56T^2$  is the wavelength in deep water) is less than 0.1, while  $d/L_d \geq 0.5$  defines deep water [6]. In this section, we examine the effects of relative water

Table 2  
Comparison of  $V_0$  calculated from dynamic and quasi-static solutions

	Dynamic solution (1)	Quasi-static solution (2)	Relative difference [(1)–(2)]/(1)
Coarse sand	0.254	0.113	0.555
Fine sand	0.0624	0.023	0.631

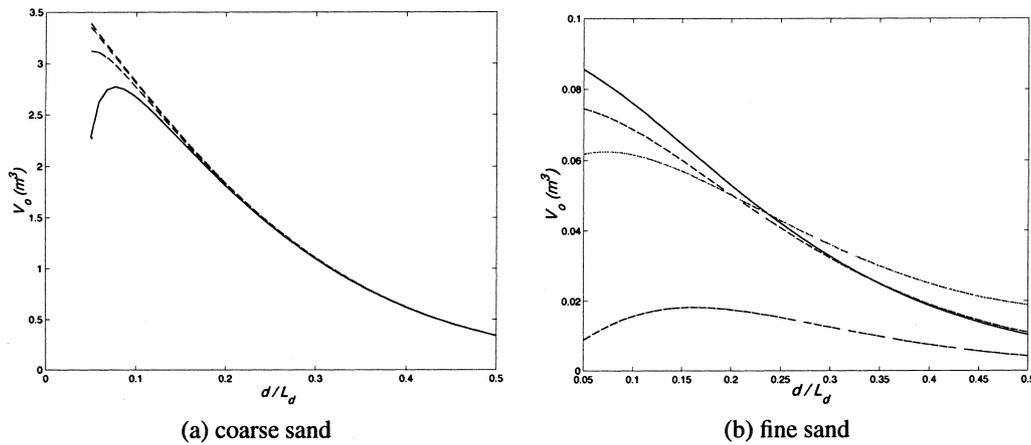


Fig. 4. Distributions of seepage flux ( $V_0$ ) versus relative water depth ( $d/L_d$ ) in (a) coarse sand and (b) fine sand for various degrees of saturation. (–)  $S = 97\%$ , (– –),  $S = 98\%$ , (. . .),  $S = 99\%$ , (-.-.),  $S = 100\%$ .

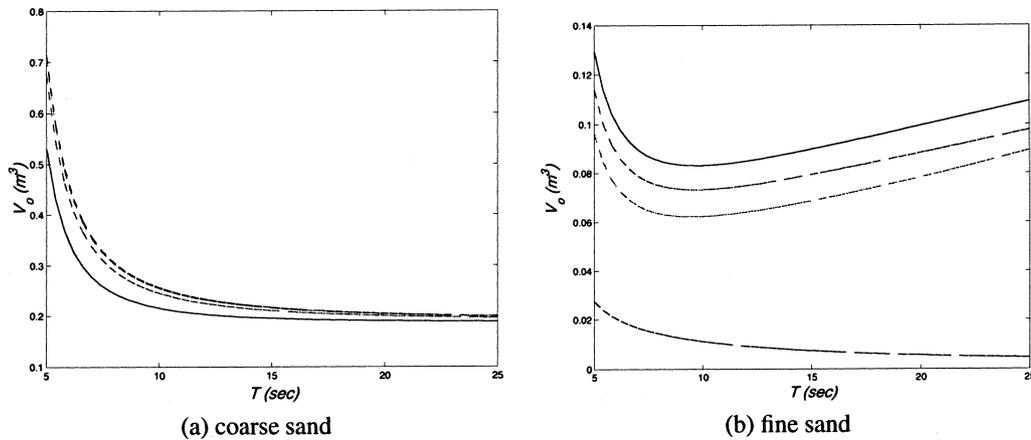


Fig. 5. Distributions of seepage flux ( $V_0$ ) versus wave period ( $T$ ) in (a) coarse sand and (b) fine sand for various degrees of saturation. (–)  $S = 97\%$ , (– –),  $S = 98\%$ , (. . .),  $S = 99\%$ , (-.-.),  $S = 100\%$ .

depth and wave period on the volume of water exchange at the seabed surface ( $V_0$ ) in both coarse and fine sand for various saturations.

Fig. 4 displays the variations of seepage flux ( $V_0$ ) with the relative water depth ( $d/L_d$ ) for both coarse and fine sands under different degrees of saturation. Generally speaking, the seepage flux decreases as the relative water depth ( $d/L_d$ ) increases for most cases. This implies that the volume of seepage exchange is larger in shallow water than in deep water. This confirms that the fluctuations of dynamic wave pressure are insignificant in

deeper water, compared with magnitude of the static wave pressures.

The variations of seepage flux ( $V_0$ ) with wave period ( $T$ ) under different degrees of saturation are illustrated in Fig. 5. Generally speaking, a larger volume of seepage flux occurs under a shorter wave (for example,  $T = 5$  s in this example). The flux decreases as wave period increases, especially in the fine sand under a fully saturated condition. For an unsaturated coarse sand, the seepage flux decreases as the wave period increases when  $T \leq 10$  s, and then increases as  $T$  increases.

Referring to Figs. 4 and 5, it may be concluded that the seepage flux is larger in shallow water and for shorter wave periods.

#### 4.2. Effects of soil characteristics

In reality, the marine sediment might not be completely saturated. For instance, the decomposition of organic materials can produce methane bubbles in the pore water. Since our interest lies in the pollution of estuaries and near-shore water, the existence of gas in the sediment should be considered. When the pore water is not completely saturated, the compressibility of gas–water continuum increases drastically [7], in which case the flux into the sediment will be affected. We will examine this effect here.

Soil type (in terms of soil permeability and shear modulus) is another important parameter in the evaluation of the wave-induced seepage flux. Increasing soil permeability generally enhances the transfer rate of water between seawater and sediments.

Figs. 4 and 5 show the influences of soil characteristics (soil types and degree of saturation) on the volume of seepage flux. It is evident that the seepage flux ( $V_0$ ) increases as the degree of saturation ( $S$ ) increases in a coarse sand while in a fine sand,  $V_0$  decreases as  $S$  increases. Such different behaviours are due to the differences of flows in coarse and fine sands. In the former, the flow is affected predominantly by spatial pressure gradient while in the latter, the flow is mainly a result of water compression and expansion. A comparison of (a) and (b) of Figs. 4 and 5 suggests that a high permeability will lead to a large transfer rate.

## 5. Conclusions

In this paper, we derive a closed-form analytical solution of dynamic flow in a porous seabed coupling the wave motion and porous medium flow. Based on the solution, the wave-driven seepage flux into and out of the marine sediment was investigated. From the numerical examples presented, the following points can be made:

1. A comparison of the wave-induced soil response between dynamic and quasi-static solutions demonstrates that pore fluid and soil skeleton accelerations can significantly increase the pore pressure and soil displacement. The displacement of pore fluid is obtainable with the dynamic solution.
2. The numerical results presented in Figs. 4 and 5 indicate that the seepage flux across the seabed boundary ( $V_0$ ) is larger in shallow water and for shorter waves (e.g.,  $T = 5$  s).
3. The magnitude of the seepage flux,  $V_0$ , is also affected by the soil type and degree of saturation. In general,

$V_0$  is larger in coarse sands than in fine sands. In a coarse sand,  $V_0$  is largest when the bed is fully saturated, while in a fine sand,  $V_0$  may increase as the saturation decreases.

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## Appendix A. Derivation of closed-form solution for porous flow

The derivation of Eqs. (24)–(28) is given in this appendix.

Upon substituting (22) into the governing Eqs. (8)–(10), we obtain a characteristic equation for the compressive waves as

$$d_1 \frac{\partial^4 S_c(z)}{\partial z^4} + d_2 \frac{\partial^2 S_c(z)}{\partial z^2} + d_3 S_c(z) = 0, \quad (\text{A.1})$$

where

$$d_1 = -\frac{2iGk\omega n(1-\mu)}{\gamma_w(1-2\mu)}, \quad (\text{A.2})$$

$$d_2 = n\omega^2[1 + 2m(1-\mu)] - i\nu\omega^2 \left[ 2m(1-\mu) + (1-n)^2 + n(1-n) \frac{\rho_s}{\rho_w} \right],$$

$$d_3 = n\beta\omega^4[n\rho_e - i\nu(1-n)\rho_s], \quad (\text{A.3})$$

in which  $\rho_e$ ,  $m$  and  $\nu$  are given in Eq. (32). In this case, the characteristic equation is in quadratic form, from which two pairs of complex solutions,  $\pm\lambda_1$  and  $\pm\lambda_2$  (see Eqs. (29) and (30)), can be obtained. Then, the general forms of  $p$ ,  $\Phi_s$  and  $\Phi_w$  for a seabed of infinite thickness can be specified as

$$P(z) = P_1 e^{\lambda_1 z} + P_2 e^{\lambda_2 z}, \quad (\text{A.4})$$

$$S_c(z) = S_1 e^{\lambda_1 z} + S_2 e^{\lambda_2 z}, \quad (\text{A.5})$$

$$W_c(z) = W_1 e^{\lambda_1 z} + W_2 e^{\lambda_2 z}, \quad (\text{A.6})$$

where  $\lambda_1$  and  $\lambda_2$  correspond to Biot [3] first and second kind of compressive waves. Similarly, the characteristic equation for shear waves arising from (11) and (12) is

$$\frac{\partial^2 S_c(z)}{\partial z^2} - \lambda_3^2 S_c(z) = 0, \quad (\text{A.7})$$

where the wave number of shear waves  $\lambda_3$  is given by Eq. (31). Thus, we have

$$S_c(z) = S_3 e^{\lambda_3 z}, \quad (\text{A.8})$$

$$W_c(z) = W_3 e^{\lambda_3 z}. \quad (\text{A.9})$$

The coefficients  $P_i$ ,  $S_i$  and  $W_i$  in Eqs. (A.4)–(A.6) and (A.8) and (A.9) are not independent. Inter-relations can

be determined by substituting these equations into (8)–(12). As a result,  $W_i$  and  $P_i$  can be expressed in terms of  $S_i$  as

$$\begin{aligned} P_1 &= \zeta_1 S_1, & P_2 &= \zeta_2 S_2, \\ W_1 &= \zeta_3 S_1, & W_2 &= \zeta_4 S_2, & W_3 &= \zeta_5 S_3, \end{aligned} \quad (\text{A.10})$$

where

$$\zeta_1 = n\rho_w\omega^2\zeta_3 + \frac{2G(1-\mu)}{1-2\mu}(\lambda_1^2 - \lambda^2) + (1-n)\rho_s\omega^2, \quad (\text{A.11})$$

$$\zeta_2 = n\rho_w\omega^2\zeta_4 + \frac{2G(1-\mu)}{1-2\mu}(\lambda_2^2 - \lambda^2) + (1-n)\rho_s\omega^2, \quad (\text{A.12})$$

$$\zeta_3 = \frac{in\omega + \frac{2Gk(1-\mu)}{\gamma_w(1-2\mu)}(\lambda_1^2 - \lambda^2) + (1-n)\omega v \frac{\rho_s}{\rho_w}}{\omega\{in + (1-n)v\}}, \quad (\text{A.13})$$

$$\zeta_4 = \frac{in\omega + \frac{2Gk(1-\mu)}{\gamma_w(1-2\mu)}(\lambda_2^2 - \lambda^2) + (1-n)\omega v \frac{\rho_s}{\rho_w}}{\omega\{in + (1-n)v\}}, \quad (\text{A.14})$$

$$\zeta_5 = \frac{in}{in + v}. \quad (\text{A.15})$$

We have four unknowns (coefficients  $S_i$  and the complex wave number  $\lambda$ ) with four boundary conditions (18) and (19). We solve for three unknown coefficients  $S_i$  with the boundary condition (18) in terms of the complex wave number ( $\lambda$ ). Then, the pore pressure and elastic wave components can be expressed as

$$S_c(z) = a_1 e^{\lambda_1 z} + a_2 e^{\lambda_2 z}, \quad (\text{A.16})$$

$$W_c(z) = \zeta_3 a_1 e^{\lambda_1 z} + \zeta_4 a_2 e^{\lambda_2 z}, \quad (\text{A.17})$$

$$P(z) = \zeta_1 a_1 e^{\lambda_1 z} + \zeta_2 a_2 e^{\lambda_2 z}, \quad (\text{A.18})$$

$$S_c(z) = a_3 e^{\lambda_3 z}, \quad (\text{A.19})$$

$$W_c(z) = \zeta_3 a_3 e^{\lambda_3 z}. \quad (\text{A.20})$$

where  $a_i$  coefficients are given as

$$a_1 = -\frac{\delta_2}{\delta_1 \xi_2 - \delta_2 \xi_1}, \quad (\text{A.21})$$

$$a_2 = \frac{\delta_1}{\delta_1 \xi_2 - \delta_2 \xi_1}, \quad (\text{A.22})$$

$$a_3 = -\frac{2i\lambda(\lambda_1 \delta_2 - \lambda_2 \delta_1)}{(\lambda_3^2 + \lambda^2)(\delta_1 \xi_2 - \delta_2 \xi_1)}, \quad (\text{A.23})$$

in which

$$\delta_1 = \lambda_1^2 - (\lambda_1^2 + \lambda^2)\mu - \frac{2k^2 \lambda_1 \lambda_3 (1-2\mu)}{\lambda_3^2 + \lambda^2}, \quad (\text{A.24})$$

$$\delta_2 = \lambda_2^2 - (\lambda_2^2 + k^2)\mu - \frac{2k^2 \lambda_2 \lambda_3 (1-2\mu)}{\lambda_3^2 + k^2}. \quad (\text{A.25})$$

Substituting (A.16)–(A.20) into (7), we derive the expressions for displacements of pore fluid and soil skeleton as

$$\begin{aligned} u_s &= \frac{\partial \Phi_s}{\partial x} - \frac{\partial \Psi_s}{\partial z} \\ &= \gamma_w \eta \{ i\lambda (a_1 e^{\lambda_1 z} + a_2 e^{\lambda_2 z}) - \lambda_3 a_3 e^{\lambda_3 z} \} \times \text{cshd}, \end{aligned} \quad (\text{A.26})$$

$$\begin{aligned} w_s &= \frac{\partial \Phi_s}{\partial z} + \frac{\partial \Psi_s}{\partial x} \\ &= \gamma_w \eta \{ \lambda_1 a_1 e^{\lambda_1 z} + \lambda_2 a_2 e^{\lambda_2 z} + i\lambda a_3 e^{\lambda_3 z} \} \times \text{cshd}, \end{aligned} \quad (\text{A.27})$$

$$\begin{aligned} u_w &= \frac{\partial \Phi_w}{\partial x} - \frac{\partial \Psi_w}{\partial z} \\ &= \gamma_w \eta \{ i\lambda (\zeta_1 a_1 e^{\lambda_1 z} + \zeta_2 a_2 e^{\lambda_2 z}) - \lambda_3 \zeta_3 a_3 e^{\lambda_3 z} \} \times \text{cshd}, \end{aligned} \quad (\text{A.28})$$

$$\begin{aligned} w_w &= \frac{\partial \Phi_w}{\partial z} + \frac{\partial \Psi_w}{\partial x} \\ &= \gamma_w \eta \{ \zeta_1 \lambda_1 a_1 e^{\lambda_1 z} + \zeta_2 \lambda_2 a_2 e^{\lambda_2 z} + i\lambda \zeta_3 a_3 e^{\lambda_3 z} \} \times \text{cshd}. \end{aligned} \quad (\text{A.29})$$

## References

- [1] Badiy M, Jaya I, Magda W, Ricinine W. Analytical and experimental approach in modeling of water–seabed interaction. In: Proceedings of the Second International Offshore and Polar Engineering Conference, 1992. p. 398–402.
- [2] Biot MA. General theory of three-dimensional consolidation. *J Appl Phys* 1941;12:155–64.
- [3] Biot MA. Theory of propagation of elastic waves in a fluid–saturated porous solid, I. Low frequency range. *J Acoust Soc Amer* 1956;28:168–78.
- [4] Booker JR, Randolph MF. Consolidation of a cross-anisotropic soil medium. *Q J Mech Appl Math* 1984;37(3):479–95.
- [5] Chen WF, Saleeb AF. Constitutive equations for engineering materials vol. 1. New York: Wiley; 1983.
- [6] Dean RG, Dalrymple RA. Water wave mechanics for engineers and scientists. Englewood Cliffs, NJ: Prentice-Hall; 1984.
- [7] Esrig MI, Kirby RC. Implication of gas content for predicting the stability of submarine slopes. *Mar Geotechnol* 1977;2:81–100.
- [8] Fenton JD, McKee WD. On calculating the lengths of water waves. *Coastal Eng* 1990;14:499–513.
- [9] Hsu JRC, Jeng DS. Wave-induced soil instability in an unsaturated anisotropic seabed of finite thickness. *Int J Numer Anal Meth Geomech* 1994;18:785–807.
- [10] Jeng DS. Soil response in cross-anisotropic seabed due to standing waves. *J Geotechnol Geoenviron Encount ASCE* 1997;123:9–19.
- [11] Jeng DS. Mechanism of the wave-induced seabed response in the vicinity of a breakwater: a review. *Ocean Engineering* (in press).
- [12] Jeng DS. On calculating the length of a short-crested wave over a porous seabed. *Appl Ocean Res* 2000;22:63–73.
- [13] Jeng DS. A new wave dispersion equation. In: The 19th International Conference on Offshore Mechanics and Arctic Engineering, ASME. New Orleans, USA, 2000, Available on CD ROM.
- [14] Jeng DS, Lee TL. A new wave dissipation equation: effects of soil response. In: Proceedings of the 1999 Canadian Coastal Conference, Victoria, Canada 1999;1:567–78.

- [15] Jeng DS, Rahman MS, Lee TL. Effects of inertia forces on wave-induced seabed response. *Int J Offshore Polar Eng* 1999;9:307–13.
- [16] Jeng DS, Seymour BR. Response in seabed of finite depth with variable permeability. *J Geotechnol Geoenviron Eng ASCE* 1997;123:902–11.
- [17] Lin YS, Jeng DS. Soil liquefaction due to short-crested wave in a cross-anisotropic non-homogeneous seabed. *J Geotechnol Geoenviron Eng ASCE* 2000;126:481–94.
- [18] Liu PLF. Damping of water waves over porous bed. *J Hydraulic Div ASCE* 1973;99:2263–71.
- [19] Liu PLF, Wen J. Nonlinear diffusive surface waves in porous media. *J Fluid Mech* 347:119–39.
- [20] Madsen OS. Wave-induced pore pressures and effective stresses in a porous bed. *Géotechnique* 1978;28:377–93.
- [21] Massel SR. Gravity waves propagated over permeable bottom. *J Waterway, Harbour, Coastal Eng ASCE* 1976;102:111–21.
- [22] Mei CC. *Applied dynamics of ocean surface waves*. Singapore: World Scientific; 1989.
- [23] Moshagen H, Torum A. Wave-induced pressure in permeable seabeds. *J Waterway Harbour Coastal Eng ASCE* 1975;101:49–57.
- [24] Mu Y, Cheng AHD, Badiy M, Bennett R. Water wave driven seepage in sediment and parameter inversion based on pore pressure data. *Int J Numer Anal Meth Geomech* 1999;23:1655–74.
- [25] Nago H, Maeno S, Matsumoto T, Hachiman Y. Liquefaction and densification of loosely deposited sand bed under water pressure variation. In: *Proceedings of the Third International Offshore and Polar Engineering Conference*, Singapore, 1993. p. 578–84.
- [26] Oldham CE, Lavery PS. Porewater nutrient fluxes in a shallow fetch-limited estuary. *Mar Ecol Progress Ser* 1999;183:39–47.
- [27] Rahman MS. Wave-induced instability of seabed: mechanism and conditions. *Mar Geotechnol* 1991;10:277–99.
- [28] Verruijt A. Elastic storage of aquifers. In: De Wiest RJM, editors. *Flow through porous media*. New York: Academic Press; 1969. p. 331–76.
- [29] Yamamoto T, Koning HL, Sellmeijer H, Hijum EV. On the response of a poro-elastic bed to water waves. *J Fluid Mech* 1978;87:193–206.
- [30] Yuhi M, Ishida H. Analytical solution for wave-induced seabed response in a soil–water two-phase mixture. *Coastal Eng J* 1998;40:367–81.
- [31] Zen K, Jeng DS, Hsu JRC, Ohyama T. Wave-induced seabed instability: difference between liquefaction and shear failure. *Soils Found* 1998;38:37–47.