

A HYBRID NUMERICAL MODEL FOR COASTAL ENGINEERING PROBLEMS

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The implementation and first validation of a hybridization technique between a NLSWE model and a meshless fully Navier-Stokes equation-based model are presented. The scope is to overcome the limitations of each numerical model (different computational costs and capabilities) and to obtain a unique tool capable to represent the whole phenomena of wave propagation, transformation and interaction with coastal structures. The hybrid model has been validated with physical model data of monochromatic waves running over a sandy beach and has provided notably improved predictions of water surface elevation and orbital velocities. Preliminary results for random waves are also reported.

Keywords: hybridization; wave propagation; Lagrangian methods; SWASH; SPH.

INTRODUCTION

Wave modelling is often required to give answers to coastal communities in terms of flood risk analysis for those areas exposed to sea waves and with high environmental and social value. However to represent all the wave processes from the deep ocean to the coastline is really challenging, both in physical and in numerical models. Phenomena such as wave propagation, wave transformation, interaction between sea waves and coastal structures, floodings, etc. occur at different and multiple scales, both in space and in time. Therefore, one single model cannot cope with it.

The present study considers the use of two different numerical models to generate and propagate the wave field from the offshore towards the nearshore and properly represent the wave transformation that occurs at the coastline. The basic idea is to hybridize (i.e. to couple) those two models, namely SWASH and DualSPHysics. The aim is to achieve an accurate modelling of the waves nearshore that will allow to evaluate the interaction between sea waves and coastal defences (sea dikes, breakwaters, embankments) with a particular focusing on the extreme storm conditions propagating from offshore and on the non-linear wave transformation.

The Simulating WAve till Shore (SWASH) model is a time domain model for simulating non-hydrostatic, free-surface and rotational flow. Wave propagation models as SWASH (Zijlema et al. 2011) have been proven to be able to simulate accurately surface wave and velocity field from deep water and with satisfactory results both in the open ocean and in nearshore but they are not suitable to deal with abrupt changes of shape of coastal structures.

DualSPHysics is an open-source code based on the Smoothed Particle Hydrodynamics (SPH) method and can be freely downloaded from www.dual.sphysics.org. The code DualSPHysics (Crespo et al. 2011) has been developed starting from the SPH formulation implemented in SPHysics model. This SPH-based model has been used to study the wave transformation and breaking at detailed scale close to the shoreline. SPH model is used to simulate free-surface flows problems such as dam breaks, landslides, sloshing in tanks and wave impacts on structures (e.g. Lee et al. 2010, Delorme et al. 2009). The expensive computational cost of SPH in comparison with other meshfree methods for CFD problems can be partially alleviated by general-purpose graphics processing unit (GPGPU) where a Graphics Processing Unit (GPU card) is used to perform computations traditionally managed by big cluster machines with thousands of CPU cores. Thereby DualSPHysics was designed from the outset to use SPH for real engineering problems with software that can be run on either CPUs or GPUs and can simulate millions of particles at a reasonable computation time. Nevertheless, this is not enough if the goal is very demanding and if the purpose is to run the whole domain and for the whole duration of storm events.

Previous work using SPHysics code was carried by Narayanaswamy et al. (2010), where a coupling model was developed combining the main advantages of a Boussinesq model (FUNWAVE) and the SPH model. One of the key developments achieved in this model was the algorithm to prescribe the boundary conditions for the individual models in the overlap region. The boundary conditions for the SPH model were implemented in the form of a Boussinesq wavemaker where a

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column of SPH boundary particles moves with a velocity determined from the velocity of the adjacent Boussinesq nodes. A simple working case was used to demonstrate the capability of the model to propagate a solitary wave in a constant depth tank. Nevertheless, numerical data were not compared with experiments.

The hybridization presented here is developed starting from SWASH and DualSPHysics. It aims to heighten the capabilities of both models and to overcome their limitations, in particular:

- Fast computations with large domains can be performed with SWASH, while simulating large domains with SPH implies huge computation times even using hardware acceleration.
- SWASH is suitable for calculation where statistical analysis is necessary such as computing wave height and good accuracy is obtained for wave propagation (results comparable with Mike21 BW), whereas wave decay is observed with standard SPH formulation for large domains.
- SWASH is not suitable for calculation of wave impact, while SPH can easily compute wave impact, pressure load and exerted force onto coastal structures.
- Complex geometries cannot be represented with SWASH and computation stability problems may appear when applied to rapidly changing bathymetry. Using SPH, any complex geometry or varying bathymetry can be simulated.

For all the aforementioned reasons, the development of a hybrid model seems to be appropriate for coastal applications. The implementation of such technique is described in the present work. A comparison with physical model results of waves running over a sandy beach is reported proving the goodness of the hybridization technique. Future works will extend the technique to cases of wave impact on coastal structures, overtopping or highly reflective structures.

NUMERICAL MODELS

A brief description of both numerical models of the hybridisation is reported in this section.

SWASH model

The SWASH model is a time domain model for simulating non-hydrostatic, free-surface and rotational flow. The governing equations are the shallow water equations including a non-hydrostatic pressure term:

$$\frac{\partial \zeta}{\partial t} + \frac{\partial hu}{\partial x} = 0 \quad (1)$$

$$\frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} + g \frac{\partial \zeta}{\partial x} + \frac{1}{2} \frac{\partial q_b}{\partial x} + \frac{1}{2} \frac{q_b}{h} \frac{\partial (\zeta - d)}{\partial x} + c_f \frac{u|u|}{h} = \frac{1}{h} \frac{\partial}{\partial x} \left(h\nu_t \frac{\partial u}{\partial x} \right) \quad (2)$$

$$\frac{\partial w_s}{\partial t} = \frac{2q_b}{h} - \frac{\partial w_b}{\partial t}, w_b = -u \frac{\partial d}{\partial x} \quad (3)$$

$$\frac{\partial u}{\partial x} + \frac{w_s - w_b}{h_{SWASH}} = 0 \quad (4)$$

where t is time, x the horizontal coordinate, u the depth averaged velocity in x-direction, w_s and w_b the velocity in z-direction at the surface and at the bottom, respectively. ζ is the free-surface elevation from still water level, d is the still water depth and h_{SWASH} the total depth (h_{SWASH} is here used to be distinguished from the SPH smoothing length h). q_b is the non-hydrostatic pressure at the bottom, g the gravitational acceleration, c_f the dimensionless bottom friction coefficient and ν_t the eddy viscosity. The SWASH model uses sigma coordinates in the vertical direction and the number of the fluid layer can be changed in the calculation. A full description of the numerical model, boundary conditions, numerical scheme and applications are given in Zijlema et al. (2011). Suzuki et al. (2011) demonstrated that this model produces satisfactory results for both wave transformation and wave overtopping for shallow foreshore topography in their one-dimensional calculation. This numerical model is a strong tool for the estimation of wave transformation since it is not demanding in terms of computation resources due to the depth averaged assumption and parallel computation capability even though it is a time-domain model.

DualSPHysics model

DualSPHysics is a numerical model based on the Smoothed Particle Hydrodynamics (SPH) method. SPH is a Lagrangian and meshless method where the fluid is discretised into a set of particles and each of these particles are nodal points where physical quantities (e.g. position, velocity, density,

pressure) are computed as an interpolation of the values of the neighbouring particles. The contribution of the nearest particles is weighted according to distance between particles and a kernel function (W) is used to measure this contribution depending on the inter-particle distance that is defining using a smoothing length (h). The smoothing length is a characteristic length used to define the area of influence of the kernel and the kernel presents compact support to prevent contributions with other particles beyond the smoothing length. The mathematical fundamental of SPH is based on integral interpolants, therefore any function F can be computed by the integral approximation:

$$F(\mathbf{r}) = \int F(\mathbf{r}')W(\mathbf{r} - \mathbf{r}', h)d\mathbf{r}' \quad (5)$$

Equation (5) can be expressed in a discrete form based on the particles. Thus, the approximation of the function is interpolated at particle a and the summation is performed over all the particles within the region of compact support of the kernel:

$$F(\mathbf{r}_a) \approx \sum_b F(\mathbf{r}_b)W(\mathbf{r}_a - \mathbf{r}_b, h) \frac{m_b}{\rho_b} \quad (6)$$

where the volume associated to the neighbouring particle b is m_b/ρ_b , with m and ρ being the mass and the density, respectively. The kernel functions W must fulfil several properties (Monaghan 1992), such as positivity inside the area of interaction, compact support, normalization and monotonically decreasing with distance. One option is a quintic kernel (Wendland 1995) where the weighting function vanishes for inter-particle distances greater than $2h$.

In the classical SPH formulation, the Navier-Stokes equations are solved and the fluid is treated as weakly compressible (e.g. see Gómez-Gesteira et al. 2012). The conservation laws of continuum fluid dynamics, in the form of differential equations, are transformed into their particle forms by the use of the kernel functions. The momentum equation proposed by Monaghan (1992) has been used to determine the acceleration of a particle (a) as the result of the particle interaction with its neighbours (particles b):

$$\frac{d\mathbf{v}_a}{dt} = -\sum_b m_b \left(\frac{P_b}{\rho_b^2} + \frac{P_a}{\rho_a^2} + \Pi_{ab} \right) \nabla_a W_{ab} + \mathbf{g} \quad (7)$$

being \mathbf{v} velocity, P pressure, ρ density, m mass, $\mathbf{g}=(0,0,-9.81)$ ms^{-2} the gravitational acceleration and W_{ab} the kernel function that depends on the distance between particle a and b . Π_{ab} is the viscous term according to the artificial viscosity proposed in Monaghan (1992). The SPH scheme with numerical diffusive terms, named delta-SPH (Molteni and Colagrossi 2009) is applied:

$$\frac{d\rho_a}{dt} = \sum_b m_b \mathbf{v}_{ab} \cdot \nabla_a W_{ab} + 2\delta h \sum_b m_b \overline{c_{ab}} \left(\frac{\rho_a}{\rho_b} - 1 \right) \frac{1}{\mathbf{r}_{ab}^2 + \eta^2} \cdot \nabla_a W_{ab} \quad (8)$$

where $\overline{c_{ab}} = 0.5(c_a + c_b)$ and $\eta^2 = 0.01h^2$. This term can be expanded into a first order and a second order contributions. The second order corresponds to the diffusive nature of the term, and the first order is approximately zero if the kernel is complete.

In the weakly compressible approach, pressure is calculated starting from density values of the particle using Tait's equation of state. The Symplectic time integration algorithm (Leimkuhler 1996) was used in the present work. A variable time step was calculated, involving the CFL (Courant-Friedrich-Lewy) condition, the force terms and the viscous diffusion term.

DualSPHysics is capable of using the parallel processing power of either CPUs and/or GPUs making the study of real engineering problems possible. Crespo et al. (2011) validated numerical results with experimental data in order to show how the technique combines the accuracy and the efficiency of GPU programming. Thus, this new technology makes the study of real-life engineering problems possible at a reasonable computational cost on a personal computer such as the numerical design of coastal breakwaters with SPH models (Altomare et al. 2014).

HYBRIDIZATION TECHNIQUE

The hybridization between DualSPHysics and SWASH has been obtained through a one-way hybridization at this stage. The idea is to run SWASH for the whole domain to impose some boundary condition on a fictitious wall placed between both media. Each particle on that wall (hereafter called moving boundary or MB) will experience a different movement to mimic the effect of the incoming waves. The time history of the displacement in each point or layer of the propagation model is

reconstructed starting from the velocity information and interpolated along the vertical. The so-calculated movement is passed to the DualSPHysics particles of the MB. Thus, the MB is a set of boundary particles whose displacement is imposed by the wave propagated by SWASH and only exists for DualSPHysics. The hybridization procedure can be summarised in four steps (Figure 1):

- A model domain with SWASH is built to get only incident waves so a flat bottom is added at the location, where the hybridization point is defined, with a sponge layer to absorb all the reflexion.
- SWASH is executed to provide only incident waves in the hybridization point.
- The velocity provided by SWASH in the hybridization point is passed to DualSPHysics and translated in the MB movement.
- DualSPHysics is executed only in the close area to the coast.

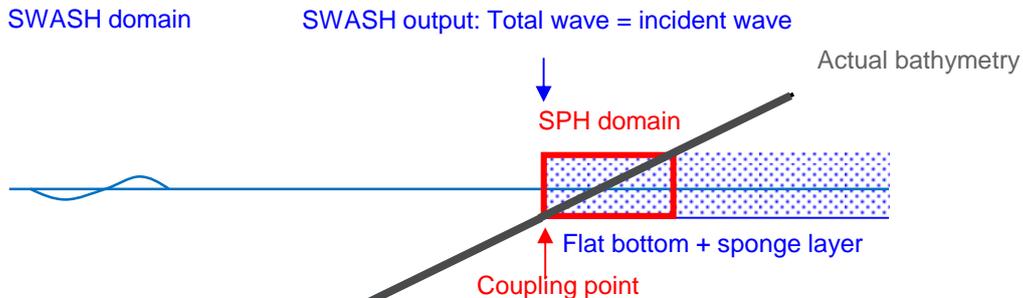


Figure 1. Sketch of the model domain and hybridization point.

SWASH provides values of velocity in different levels of depth. These values are used to move the MB particles. The displacement of each particle can be calculated using a lineal interpolation of velocity in the vertical position of the particle. However, the lineal interpolation is not a good option because a small difference in velocity between two piston particles, which are very close in height, gives rise to an important difference in the accumulated displacement after several seconds of simulation. This problem is aggravated further since the height for the velocity measurements can vary in each instant depending on the height of water, which can result in a broken piston. The solution is a smoothed velocity, so that, the velocity of the particles that form the MB does not change suddenly depending on the height, but a smoothing function is applied in the form of a sort of weighted average for each particle using the information of the neighbour particles.

EXPERIMENTAL DATA

Physical model tests carried out at the Maritime Engineering Laboratory of the Technical University of Catalonia (LIM/UPC) have been used as benchmark case of the hybridization approach between SWASH and DualSPHysics.

Setup

The data refer to the EU-funded projects within the Hydralab III and Hydralab IV frameworks, namely SUSCO (Hydralab III report, 2010), SCANDURA (Hydralab III report, 2009) and WISE (Hydralab IV report, 2012). The flume is 100 m long, 3 m wide and 5 m high and has a wedge type paddle that allows generating waves 1.5 m high. A sketch of the physical flume from SCANDURA is shown in Figure 2. SUSCO and WISE present quite similar configurations, not here reported. The wedge-type wave generator can be distinguished on the left side of the figure, where a 1:15 sloping beach represents the initial condition before erosive and accretive processes triggered by the action of the waves.

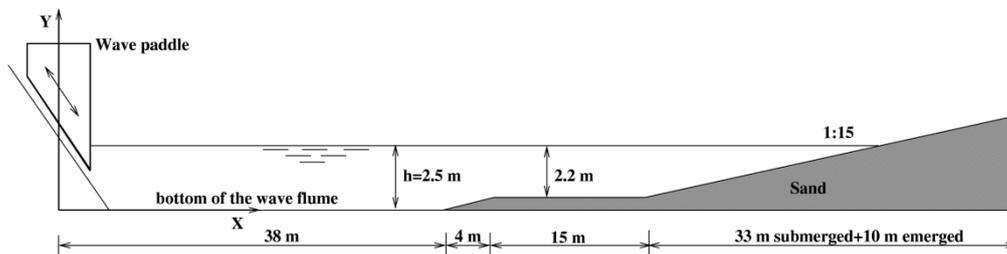


Figure 2. Sketch of the physical flume as in SCANDURA project.

The test cases carried out using the hybridization strategy and the relative wave conditions are summarized in Table 1. The water depth at the wave paddle is 2.5 m for all the tests. Even though the experiments consist in mobile bed tests, the changes of the sand bottom are assumed negligible for the present purposes because only a small part (time window) of the test is reproduced in DualSPHysics and compared with physical data.

Table 1. Test cases from UPC experiments that have been modelled using the hybridization technique.

TEST CASE			WAVE CHARACTERISTICS	
PROJECT	TEST NO.	WAVE CONDITIONS	H [m]	T [s]
SUSCO	115	monochromatic	0.37	3.70
SCANDURA	63	monochromatic	0.60	4.25
SCANDURA	10	monochromatic	0.60	4.25
PROJECT	TEST NO.	WAVE CONDITIONS	H_{m0} [m]	T_p [s]
WISE	7	random	0.47	3.70

Information of water surface elevation and wave particle velocity along the CIEM flume has been collected during the experimental campaign. Resistance type wave gauges (WG) and acoustic wave gauges (AWG) are normally used to measure the free surface elevation and acoustic Doppler velocimeters (ADV) to measure the three components of the velocity field. Wave gauges positions are shown in Table 2 and Table 3, respectively for SUSCO and SCANDURA, where the x-coordinate origin is at the wave paddle at rest conditions (left in Figure 2). The ADV location refers to the test case SCANDURA 63 that is shown later for comparison. The ADV sensors are located at $x = 44.11$ m covering several vertical position as reported on (Table 1). The vertical elevation of the ADV is relative to the still water level.

Table 2. Position of wave gauges in SUSCO tests.

Wave gauge	Distance (m)	Wave gauge	Distance (m)
WG0	7.71	WG5	21.58
WG1	8.72	WG8	43.41
WG2	9.70	WG9	53.28
WG3	10.69	WG13	58.46
WG4	11.69	WG12	63.18

Table 3. Position of wave gauges and ADVs in SCANDURA tests (ADV's position refer to test no. 63).

Wave gauge	Distance (m)	Wave gauge	Distance (m)	Velocimeter	Vertical position (m)
WG0	7.71	AWG0	30.98	ADV0	-0.626
WG1	8.71	AWG1	33.38	ADV1	-1.435
WG2	9.70	AWG2	36.90	ADV2	-1.734
WG3	10.68	AWG3	43.10	ADV3	-1.113
WG4	11.68	AWG4	51.61	ADV5	-2.200
WG5	21.58	AWG5	59.43	ADV6	-0.875
WG12	43.41	AWG6	66.87	ADV7	-1.998

Preliminary validation of SWASH modelling

SWASH has been firstly validated for the entire physical domain, showing high accuracy in terms of wave height, wave period and wave setup. The simulation has been carried out with SWASH using a grid size of 0.5 m in the horizontal direction with an initial time step of 0.05 s in prototype scale (same scale of physical model). Thus, the length of the numerical flume is 100 m long with 200 grid cells. Actually the SWASH domain starts at WG1, so that in 8.71 m. Note that the calculation time step is automatically adjusted in the calculation depending on the Courant-Friedrich-Lewy (CFL) condition. The output time step of the SWASH model is 0.05 s. A weakly-reflective boundary is applied at the wave boundary. A Manning's value of 0.019 is used to calculate the bottom friction, which corresponds to a sandy coast. The time series of the incident waves are prescribed at the wave boundary of the SWASH model. The wave elevations registered in the physical experiment at WG0, WG1 and WG2 are analysed to obtain the incident and reflected wave (Mansard and Funke, 1980) and the signal

of the incident waves is imposed as input of SWASH. A physical time of 44 minutes were executed with SWASH (version 1.10AB) in a CPU device Intel Xeon and the simulation took around 4 minutes.

The number of layers tested in the SWASH was 8. Not like Boussinesq-type models, SWASH uses the number of vertical layers to improve its frequency dispersion rather than increasing the order of derivatives of dependent variables. Note that the result of wave propagation of 1 layer and 8 layers is similar since kd (wave number multiplied by water depth) is less than 1 in this case. Therefore the accuracy of the phase velocity of progressive waves is accurate enough in both cases. However it might be important to simulate velocity profile by using multiple layers since this case is applied to a beach profile. Wave run-down process can affect the velocity distribution when the hybridization point is applied to swash zone.

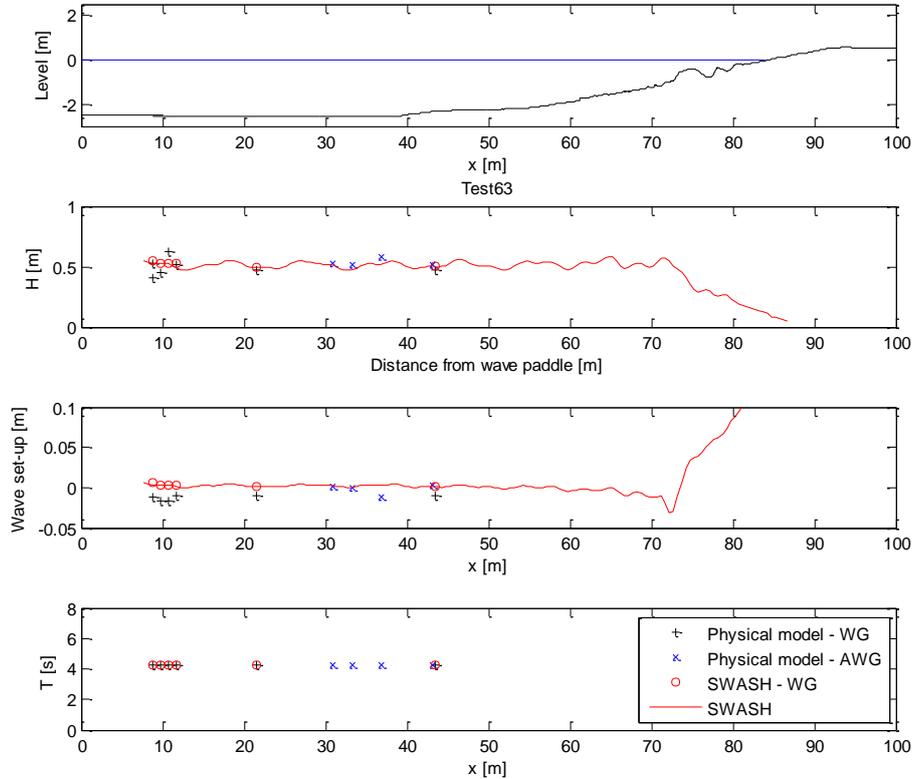


Figure 3. Spatial distribution of wave height, setup and period of the physical results and SWASH measured in the wave gauges WG and AWG.

An example of the results is shown in Figure 3 where the wave height, wave setup and wave period from the SWASH simulation are compared with the physical data. The three wave gauges (WG0, 1 and 2) are used for the wave reflection analysis, in order to get the wave height and period of the incident waves which are successfully reproduced in the SWASH domain. Wave propagation is proven to be accurate in terms of wave height, setup and wave period.

HYBRIDIZATION RESULTS

Once SWASH has been validated and proved to be accurate to model the wave propagation as in the physical flume, the hybridization technique has been applied choosing different hybridization points behind which a flat bottom and sponge layer have been introduced in SWASH to obtain the input wave boundary conditions for DualSPHysics.

Performance analysis

The differences between numerical and physical results have been quantified by the use of ARMAE error (Sutherland et al. 2004, Marzeddu et al. 2013) to evaluate the performance of the hybridization strategy. The ARMAE can be defined as follows and includes all types of error (whether in mean or phase):

$$ARMAE = \frac{\langle |Y - X| - OE \rangle}{\langle |X| \rangle} \quad (9)$$

where X are the observed values (experimental ones), Y the predicted values (numerical ones) and OE the observational error. According to Sutherland et al (2004), this error can be classified in excellent (<0.2), good (0.2-0.4), reasonable, (0.4-0.7), poor (0.7-1.0) and bad (>1.0).

Sensitivity analysis

The influence of the smoothing length and particle boundary viscosity have been investigated. Here we only show the results from Test 63 of SCANDURA project. This test consists of the generation and propagation of regular waves with water height $H=0.6$ m, period $T=4.25$ s, initial depth $h=2.5$ m and wave length $L=19.1$ m. The DualSPHysics domain has been modelled with MB in the location of WG5 (see Table 3); the resulting domain size in DualSPHysics is 78.42 m (from the MB to the end of the beach).

The smoothing length in 2D is normally defined as:

$$h = k \cdot \sqrt{dx^2 + dz^2}$$

Being $dx=dz=dp$, the equation can be written as follows:

$$h = k \cdot \sqrt{2dp^2} = k \cdot dp \cdot \sqrt{2}$$

Thus, the relative smoothing length (h/dp) is proportional to the k coefficient. The k coefficient has to be normally calibrated depending on the engineering problem that is faced. The values chosen in the present analysis vary from 0.86 (usually value referred to dam break problems) to 1.90. It means that the ration h/dp varies between 1.22 and 2.69. Notice that bigger is the smoothing length, bigger is the number of neighbor particles that are supposed to interact with each fluid or boundary particle. A total duration of 100s (physical time) has been run using a GPU GeForce GTX680. The particle size is 0.02 m and the number of fluid particle is equal to 232792.

The influence of the smoothing length on the water surface elevation (Figure 4) and horizontal and vertical velocity results (Figure 5) has been assessed. The results show that bigger the ration h/dp is, more accurate the results are (ARMAE is lower). Small values of h/dp lead to low accuracy already at less than 10m far from the SWASH+SPH moving boundary. In general the wave propagation is improved for high values of h/dp , even though also the highest ratios h/dp give unacceptable results at the farthest distances from the wave generation. The results of orbital horizontal and vertical velocities confirm that high values of the smoothing length improve the modelling performance. However, the results are good (ARMAE < 0.4) and not excellent (ARMAE < 0.2) especially for the vertical velocity and they are even worse if measured closer to the bottom. The Dynamic Boundary Conditions, used in DualSPHysics, might explain such behaviour.

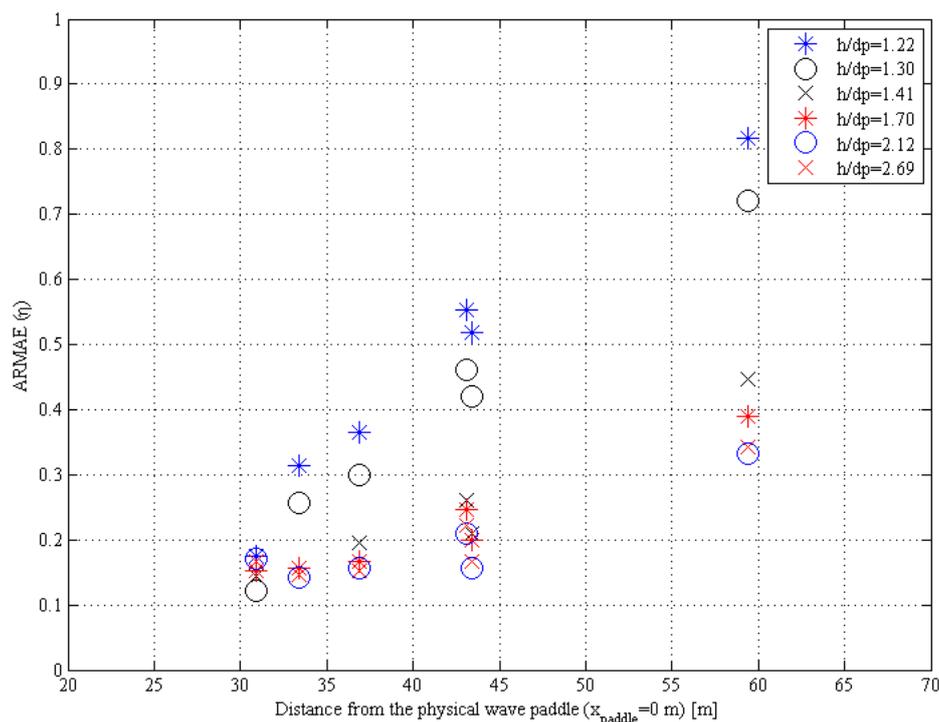


Figure 4. Influence of the smoothing length on the water surface elevation

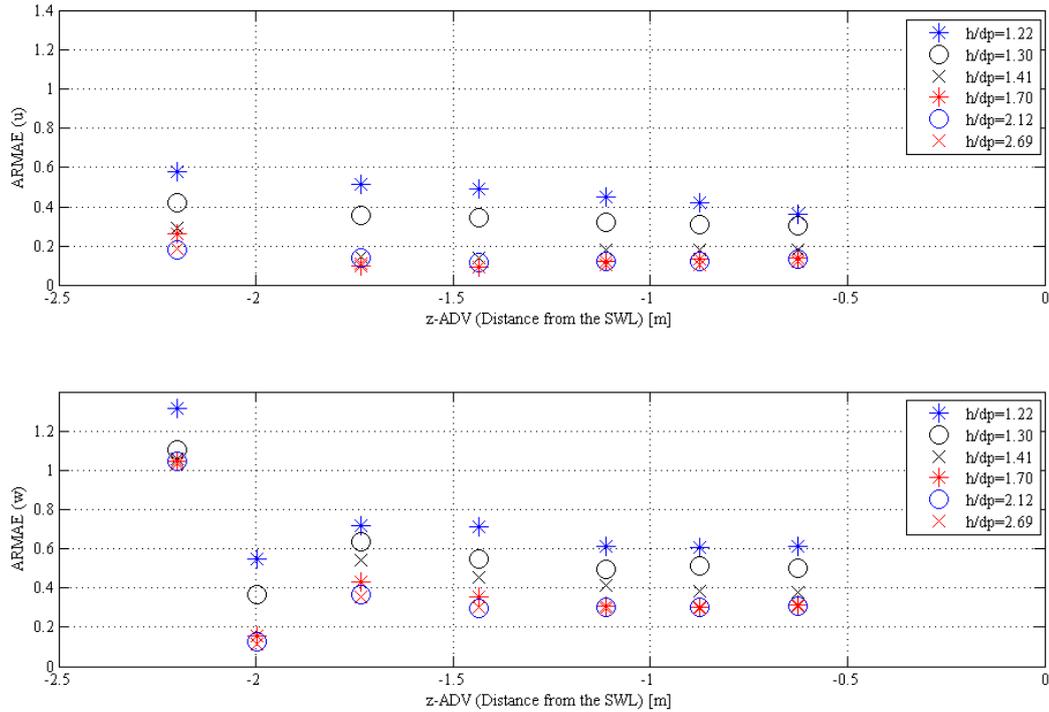


Figure 5. Influence of the smoothing length on the horizontal and vertical orbital velocity.

As previously mentioned, the influence of the boundary particle viscosity has been also investigated. Because of the boundary conditions implemented in DualSPHysics, the particles that form the fix boundaries are treated such fluid particles with the only difference that they cannot move. When a fluid particle is approaching to the boundary particles, the local density is changing (the fluid is weakly compressible) and consequently the local pressure is changing (due to the Equation of State) triggering a repulsive force between the boundary and the fluid particles. Thus, the physics that are governing the fluid particles are as well governing the behaviour of the boundary particles except for the fact that their position is fixed. It means that the artificial viscosity expressed in the Momentum Equation is characterizing the boundary particles and then the interaction between them and the fluid particles. The effects of reducing the artificial viscosity of the boundary particles with respect to the fluid particles are shown. Four different boundary particle viscosity are investigated:

- $\nu b=0$: the coefficient for the artificial viscosity of the boundary particles is set to 0 (no viscosity);
- $\nu b=0.1$: the coefficient for the artificial viscosity of the boundary particles is set to the 10% of the coefficient used for the fluid particles;
- $\nu b=0.5$: the coefficient for the artificial viscosity of the boundary particles is set to the 50% of the coefficient used for the fluid particles;
- $\nu b=1$: the coefficient for the artificial viscosity of the boundary particles is set to the 100% of the coefficient used for the fluid particles.

The results are shown in Figures 6 and 7 respectively for water surface elevation and horizontal and vertical velocity. The differences in water surface elevation are clearly negligible. The boundary particle viscosity is having a significant influence on the horizontal orbital velocities, where less the boundary viscosity is with respect to the fluid one, better are the results. The viscosity seems to interfere with the particle movements, in particular the horizontal one and close to the bottom: it can be seen like a sort of numerical friction due to the boundary conditions and that is advisable to remove.

Velocity profiles

The velocities obtained in SWASH and DualSPHysics have been compared. The results from test case SUSCO 115 are shown, where the MB in DualSPHysics are set at the location corresponding to WG8 in Table 2. It is worthy to remind that SWASH divides the fluid domain along the vertical in 8 different layers and the velocity is a sort of average value for each layer. Furthermore, each time step, the layer has no the same height, because of the passage of the wave (the wet area is changing slightly).

The velocity in DualSPHysics is computed referring to the center of each SWASH layer in still water conditions (beginning of the simulation). SWASH velocities have been also compared with the 2nd order Stoke's theory. Figure 8 shows the comparison between SWASH, DualSPHysics and Stoke's profile under wave crest and wave through.

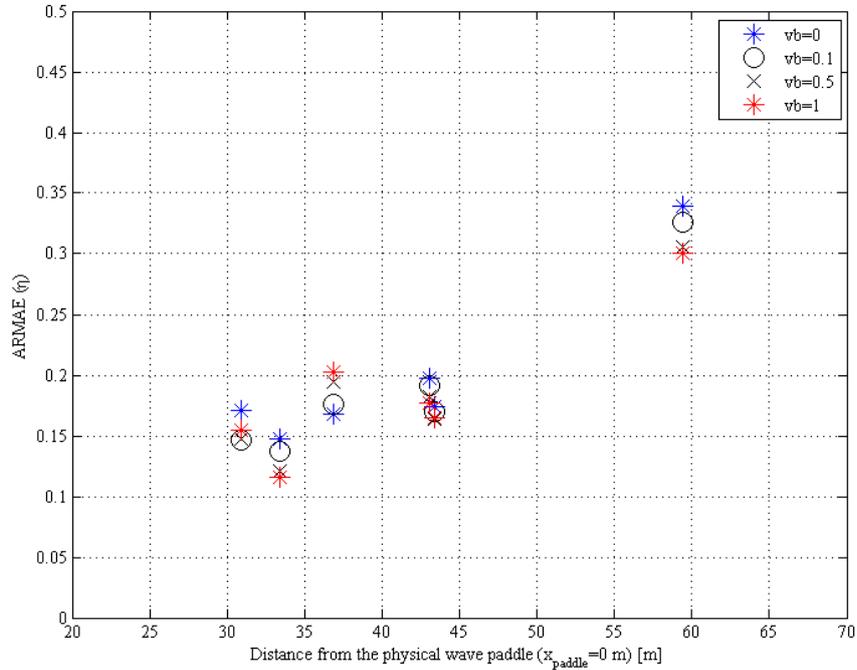


Figure 6. Influence of the boundary particle viscosity on the horizontal orbital velocity.

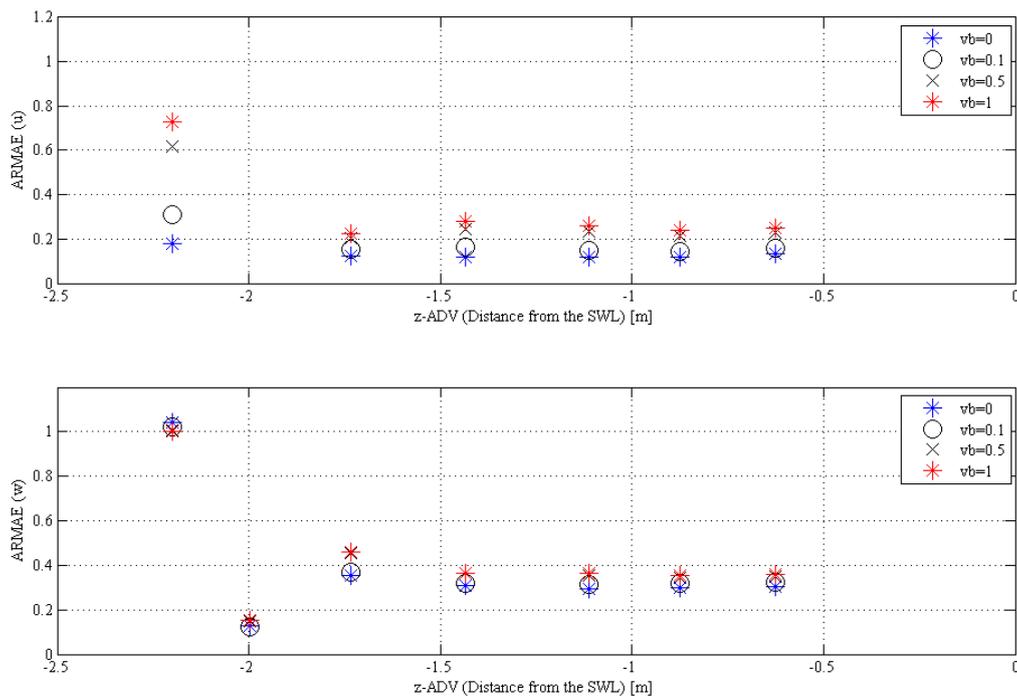


Figure 7. Influence of the boundary particle viscosity on the horizontal and vertical orbital velocity.

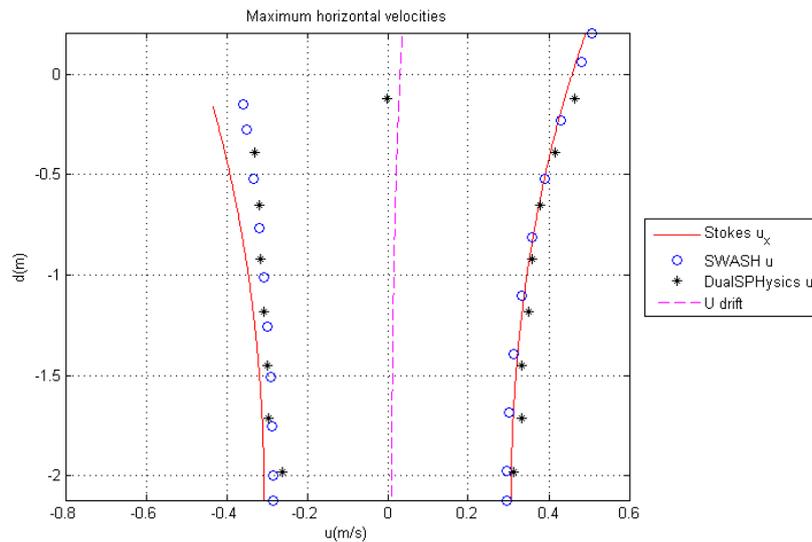


Figure 8. Test case SUSCO 115 – MB in WG8 location: maximum horizontal (wave crest) and minimum (wave through) velocities.

Final results for monochromatic waves

Test case SCANDURA 63 has been used to finally analyze the performance of the hybridization technique with respect to water surface elevation and orbital velocities. The previous section have shown that to remove the artificial viscosity from the boundaries in DualSPHysics and to use large smoothing kernels improve the accuracy of the modelling. Hence the artificial boundary viscosity is set equal to 0 and the relative smoothing length (h/d_p) to 2.12. Initially, the position of WG5 (21.58 m far from the physical wave paddle in CIEM) has been chosen as hybridization point, therefore a numerical domain of 78.42 m is simulated with DualSPHysics. The position of the hybridization point can appear unreasonable if the domain covered by SWASH is compared with the domain covered by SPH: the former one is less than the latter one. However two main reasons justify this initial choice: a) the bottom of the flume is still flat at WG5, so no complications due to a varying bathymetry; b) many wave gauges and all the ADVs are located after WG5, hence the technique can be validated using those data. An initial inter-particle spacing in DualSPHysics, d_p , of 0.02 m has been used resulting in 232792 fluid particles. The simulated physical time corresponds to the first 100 s of the experiment. The simulation runtime is 4.4 hours using an Nvidia GeForce GTX 680 graphic card and the simulation of SWASH to get the input information takes less than 7 seconds using Intel i7.

Figure 9 and 10 present the comparison of numerical and physical measurements for free surface elevation and velocity respectively. Two locations close to the numerical coupling point, hence to the wave generation in SPH, are shown (AWG0 and AWG2): the differences between the physical measurements and the numerical signals appears negligible, hence the wave generation appears reliable as carried out following the hybridization technique. The comparison of the free surface elevation has been extended to two more locations, farther from the hybridization point than the previous ones (AWG3 and AWG5) in order to show good agreement between numerical and physical measurements along the domain (from the hybridization point towards the shoreline). It can be notice that the error in DualSPHysics wave signal is somehow related to the resolution of the model that is function of the kernel length, h . This quantities can be expressed as $h = k \cdot \text{sqrt}(2) \cdot d_p$. The coefficient k is assumed equal to 1.5 in the present calculation, hence h results equal to 0.042 m. Therefore assuming a “numerical resolution” of about 4 cm, the differences between the physical measurements and the numerical signals, as shown in Figure 9, appears of the same order of magnitude of h , confirming once again the goodness of the results.

The comparison between orbital horizontal and vertical velocities measured in the physical flume and those obtained numerically is reported in Figure 10. The ADVs are located at the same distance from the physical wave paddles, hence from the hybridization point: only their location along the vertical, therefore their distance from the bottom, changes. Both cases present good agreement between physical measurements and numerical results, especially on the horizontal velocity u , whereas the

vertical velocity w shows the same trend of the physical one but with a certain underestimation of the peaks in the signal.

Table 4 reports the ARMAE computed for wave elevation and horizontal and vertical velocity when comparing experimental and numerical values. Those sensors whose physical measurements were not reliable or presented a certain noise have been excluded for the error calculation. The error analysis shows finally excellent results for wave surface elevation and horizontal velocity and good results for vertical velocities. The results can be classified as reasonable only for AWG6: actually it can be noticed the error increases with the distance between the measurement location and the hybridization point. However the low accuracy in AWG6 results is due in part to errors of the data acquisition during the experimental campaign: actually the through of most of the waves in the time series of AWG6 seems to be cut-off. In part, the increased ARMAE value calculated for AWG6 is maybe also due to the proper limitation of SPH to represent correctly the wave propagation for long domains (AWG6 is located far from the hybridization point). Further consideration on this problem will be reported in the following section.

So far, results with the hybridization, where the position of WG5 (21.58 m) were chosen as the hybridization point, were presented. The comparison with experimental information showed a very good agreement, which proves the reliability of the hybridization approach and its usability.

Table 4: ARMAE of wave elevation, horizontal and vertical velocity.

Wave elevation	ARMAE	Horizontal velocity	ARMAE	Vertical velocity	ARMAE
WG12	0.16	ADVx0	0.13	ADVz0	0.31
AWG0	0.17	ADVx1	0.11	ADVz1	0.30
AWG1	0.14	ADVx2	0.14	ADVz2	0.36
AWG2	0.16	ADVx3	0.12	ADVz3	0.30
AWG3	0.21	ADVx5	0.18	ADVz6	0.30
AWG5	0.33	ADVx6	0.12		
AWG6	0.56	ADVx7	0.13		

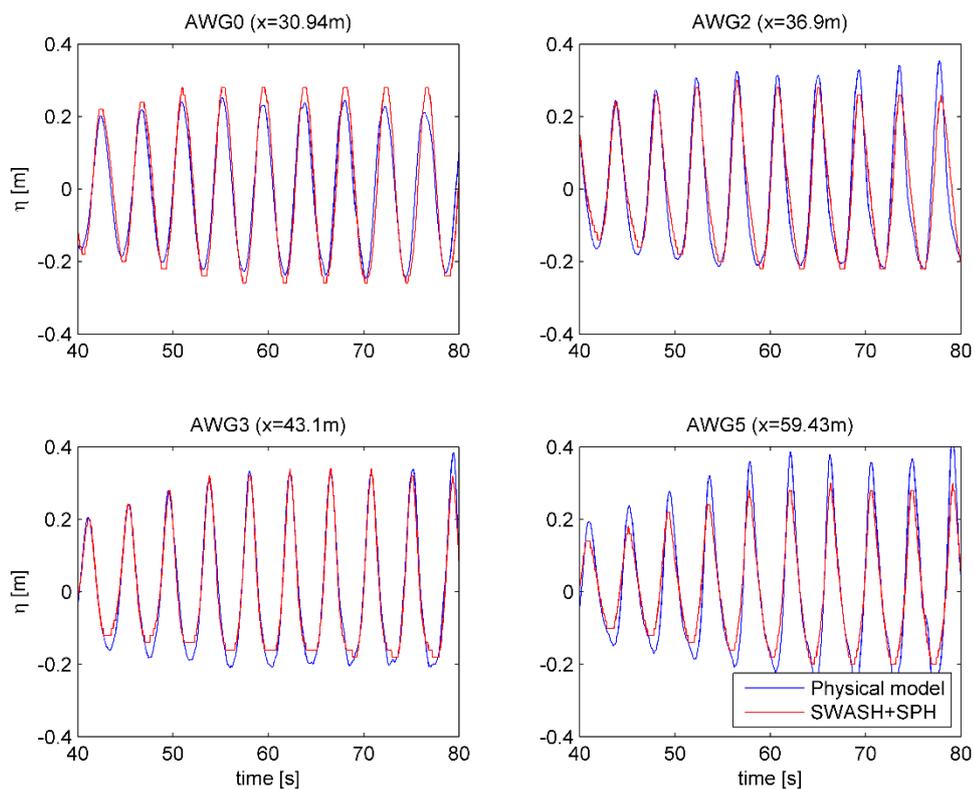


Figure 9: AWG free surface elevation: comparison between numerical and physical results.

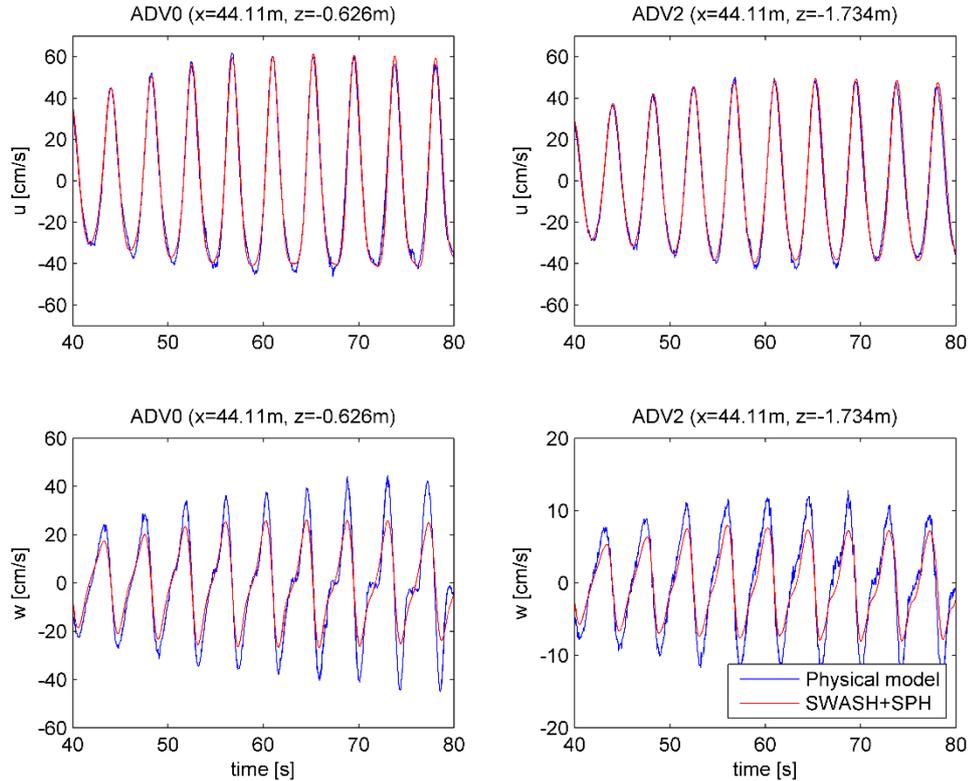


Figure 10: ADVx and ADVz: comparison between numerical and physical results.

In addition, the practical use of the hybridization model may also lie in saving execution time. In this way, different hybridization points leading to different sizes of the DualSPHysics domain can be simulated and compared with the SPH simulation of the entire domain. Therefore, as a reference case, a SPH domain of 100 m was also simulated only with DualSPHysics. The numerical wave paddle mimicked the physical one (wedge type) and used the same time series of displacement. The number of fluid particles was 369346 and the computational time about 6.8 hours (executed on the same graphics card). On the other hand, two different hybridization points (43.41 m and 51.65 m) were chosen to compare runtimes. The ARMAE error in AWG5 (59.43 m) was also computed in the new cases. Table 5 summarises the results of the different SWASH + DualSPHysics simulations of 100 seconds of physical time compared with the case entirely simulated with DualSPHysics. Only runtime of the SPH part is included in the table since the SWASH execution takes several seconds. Results are expressed as percentage in relation to the absolute values of the entire SPH simulation.

Hence, it can be observed how the runtime decreases when decreasing the size of the SPH domain (and increasing the size of the SWASH domain). In addition, the ARMAE error measured at the wave gauge AWG5 decreases following the same criteria.

Table 5: Improvements in time with different hybridization points.

Hybridization point	SPH domain size	Number of fluid particles	Runtime	ARMAE in AWG5
21.58 m (WG5)	78.42%	63.03%	64.71%	0.33
43.41 m (WG12)	56.59%	27.26%	30.88%	0.27
51.61 (AWG4)	48.39%	14.61%	20.59 %	0.22

RANDOM WAVES (PRELIMINARY RESULTS)

The previous sections have shown the results of the validation of the hybridization technique between SWASH model and DualSPHysics model applied to a series of tests with monochromatic waves running over a sandy beach. However, monochromatic or regular waves are mostly used for research purposes. Often physical models and numerical models implement and use time series of random waves, in most of the cases based on standard energy density spectra (e.g. JONSWAP). The main purpose of the hybridization technique is to have a tool that consists of both numerical models

and that can help to assess the interaction between sea waves and coastal structure in the most reliable way. Hence, to model random waves is substantial and the hybridization has to be proven for such a case.

The final goal of implementing an hybridization technique is its possible use to study real engineering problems both in 2D and in 3D. Before that, a preliminary analysis with random waves must be conducted. Test 7 of WISE B (Benchmark) data is used for this purpose. Wave conditions are $H_{m0} = 0.47$ m and $T_p = 3.7$ s with a JONSWAP Spectrum ($\gamma=3.3$).

The DualSPHysics domains has been modelled with MB in the location of WG6 (Table 6): the resulting domain size in DualSPHysics is 49.48m (from the moving boundary to the end of the beach). A total duration of 100 s (physical time) has been run using a GPU GeForce GTX TITAN Black.

Three different initial particle sizes have been modelled, respectively 1 cm, 0.5 cm and 0.25 cm. The number of fluid particles are respectively 188244, 763782 and 3077008 and the computational runtimes 1.41 h, 22 h and 163 h.

Table 6. Position of wave gauges in WISE B 7 tests.

Wave gauge	Distance (m)	Wave gauge	Distance (m)	Wave gauge	Distance (m)
WG0	7.73	WG4	44.52	WG8	56.59
WG1	8.73	WG5	47.52	WG9	59.60
WG2	9.68	WG6	50.52	WG10	62.62
WG3	30.56	WG7	53.52	WG11	65.62

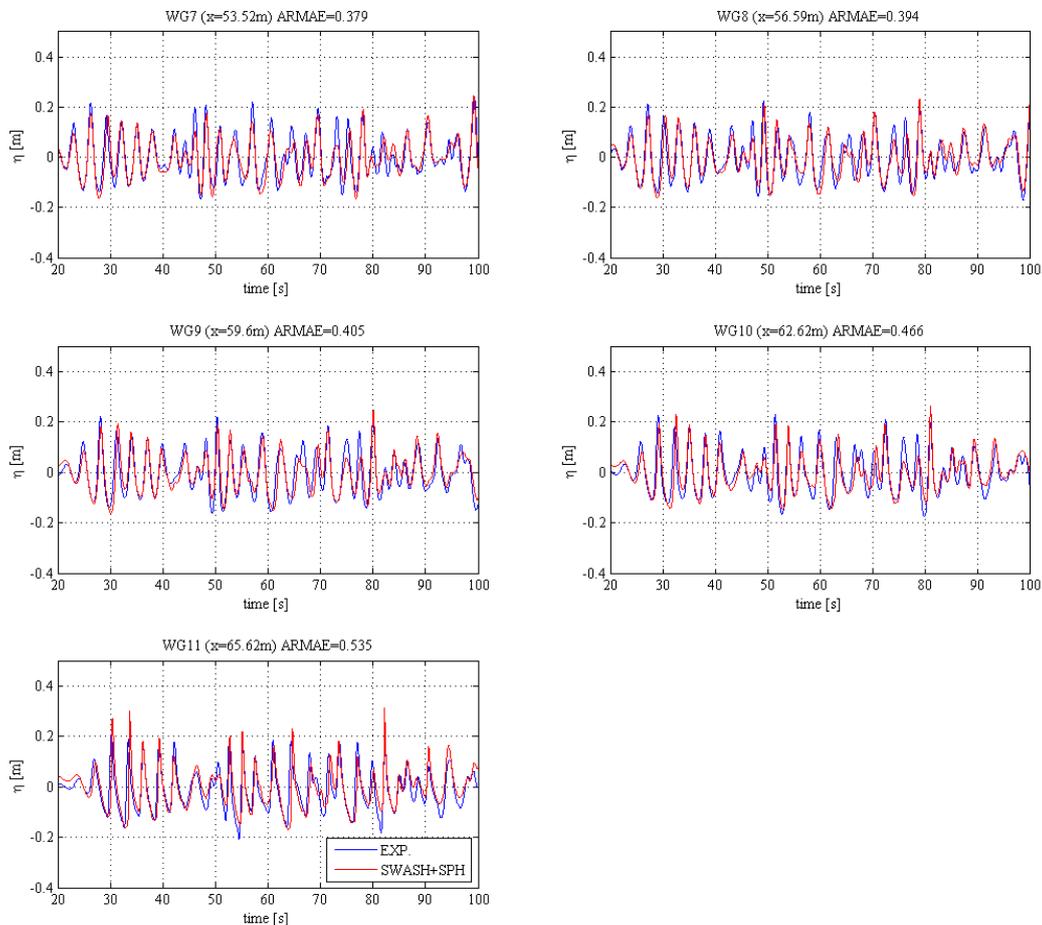


Figure 11. Time series comparison: numerical model with $dp=0.5$ cm (red) vs experimental data (blue)

Examples of the numerical and physical time series are shown in Figure 11. The ARMAE error has been estimated and the results are in the same figure for each time series: the results are good or sufficient and the performance is in general less accurate than the monochromatic wave cases. It has

been already mentioned that the ARMAE estimator contains both magnitude and phase errors. The latter ones can significantly affect the ARMAE calculation especially for irregular wave trains. Further investigation will be carried out on this topic choosing different error estimator and carrying out time-domain or frequency-domain analysis on the numerical time series. Notwithstanding, these preliminary results underline the potential of the hybridization technique to be applied to random waves.

CONCLUSIONS

An hybridization technique is developed starting from a wave propagation model (SWASH) and the SPH-based DualSPHysics model. The basic idea is to use SWASH (less computational expensive) for the biggest part of the domain that wants to be analysed, restricting the analysis with DualSPHysics only to the region very close to the coastline. In this way the capabilities of the two numerical models can be heightened and their limitations can be smoothed.

The approach presented is a one-way hybridization case and its applicability is restricted for a while. SWASH output (horizontal velocity in several layer along the water depth) is used by DualSPHysics as input information that is converted into the displacement of the particles that form the SPH moving boundary. Hence the SWASH output is seen by DualSPHysics like a “frozen information” that is used after that SWASH has completely run. The moving boundary is not a classical wave paddle as in the most usual SPH modelling but it consists of a set of boundary particles that are free to move using the information that SWASH has provide for the water depth where the particle is located. This results in a sort of deformable boundary.

The main aim of the present work is to implement the technique and validate it demonstrating its capabilities to generate sea waves. Hence the hybridization has been validated with experiments from EU-funded projects (SUSCO, SCANDURA, WISE) carried out within HydraLab III and HydraLab IV framework at the Maritime Engineering Laboratory of the Technical University of Catalonia (LIM/UPC). The physical model experiments were conducted at large scale (no scale effects and limited model effects). The experimental campaigns consisted in regular and random waves running over sandy beach profiles. The case are simple in geometry chosen in order to avoid additional complexities making the analysis as simplest and most reliable as possible.

The main part of the conducted analysis is focused on monochromatic wave tests. The data from SUSCO and SCANDURA projects have been for validation and sensitivity analysis. The ARMAE estimator (Sutherland et al. 2004) has been used to quantify the accuracy of the hybridization technique. The errors have been quantified in terms of water surface elevation and orbital velocities. Based on the ARMAE analysis the performance of the modelling by mean of hybridized SWASH+SPH model can be ranked as excellent. Furthermore the differences among three different hybridization sections (hence different DualSPHysics domain sizes) have been evaluated in terms of accuracy of the modelling and computational cost: closer the hybridization point is to the shoreline, smaller the DualSPHysics domain, higher the accuracy results and less expensive the computation. Therefore the objective of reducing computational cost in an SPH model keeping the accuracy of the results is achieved.

Finally the hybridization technique has been applied to random wave test (physical data from WISE B project). The modelling performance has been assess through 3 different analysis: the results for random waves are less accurate than the monochromatic waves, even though they can be still considered reliable. Further investigation on random waves will be carried out.

Concluding, the implementation and application of hybridization technique between two numerical models, namely SWASH and DualSPHysics, to wave propagation and transformation over a sandy beach, based on physical model tests, has proved to be a powerful strategy that overcome the drawbacks of the two models leading to a proper representation of the wave phenomena.

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