timescale τ_p. Some typical field data are presented in Fig. 2 for a tidal cycle during each field study. Each graph is a circular plot, and the axes have a logarithmic scale with units in milliseconds. The SSC integral timescale data seemed relatively independent of the tidal phase (Fig. 2). They yielded median SSC integral timescales τ_{SSC} of approximately 0.065 and 0.109 s in the middle and upper estuarine zones, respectively (Studies E6 and E7).

A comparison of turbulent and SSC integral timescales showed some key differences, especially during the ebb tide. For example, in the results shown in Figs. 1 and 2, in the middle estuarine zone, the ratio of SSC to turbulent integral timescales was on average $\tau_{SSC}/\tau_t = 0.21$ and 0.14 during the flood and ebb tides, respectively. In the upper estuary, the ratio of τ_{SSC}/τ_t was approximately $\tau_{SSC}/\tau_t = 1$ and 0.18 during the flood and ebb tides, respectively. The ratio of integral timescales τ_{SSC}/τ_t was approximately two to five times lower during the ebb tide periods. The findings showed conclusively the different timescales for the turbulent velocities and suspended sediment concentrations, and they supported the theoretical development presented by the author. Further, the results tended to suggest that the sediment suspension and suspended sediment fluxes were dominated by the turbulent processes in the creek during the flood tide, but not during the ebb tide. The integral timescales for turbulence and SSC were approximately equal during flood tides, but differed significantly during ebb tides. The same pattern might take place with other scalars and be pertinent to the turbulent mixing modeling in shallow-water subtropical estuaries.

References

Chanson, H., Takeuchi, M., and Trevethan, M. (2007). "High-frequency suspended sediment flux measurements in a small estuary." *Proc., 6th Int. Conf. on Multiphase Flow (ICMF)*, Paper No. S7_Mon_C_S7_Mon_C_5, M. Sommefield, ed., Leipzig, Germany.

Chanson, H., Takeuchi, M., and Trevethan, M. (2008). "Using turbidity and acoustic backscatter intensity as surrogate measures of suspended sediment concentration in a small sub-tropical estuary." *J. Environ. Manage.*, 88(4), 1406–1416.

Jackson, R. G. (1976). "Sedimentological and fluid-dynamic implications of the turbulent bursting phenomenon in geophysical flows." *J. Fluid Mech.*, 77(3), 531–560.

Trevethan, M., Chanson, H., and Brown, R. (2008). "Turbulent measurements in a small subtropical estuary with semi-diurnal tides." *J. Hydraul. Eng.*, 134(11), 1665–1670.

Trevethan, M., Chanson, H., and Takeuchi, M. (2007). "Continuous high-frequency turbulence and sediment concentration measurements in an upper estuary." *Estuarine, Coastal Shelf Sci.*, 73(1–2), 341–350.

Closure to "Vertical Mixing in the Fully Developed Turbulent Layer of Sediment-Laden Open-Channel Flow" by Erik A. Toorman

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Erik A. Toorman[1]

[1]Associate Professor and Research Associate, K. U. Leuven Special Research Fund, Hydraulics Laboratory, Dept. of Civil Engineering, Katholieke Univ. Leuven, Kasteelpark Arenberg 40, B-3001 Heverlee, Belgium. E-mail: erik.toorman@bwk.kuleuven.be

Kinetic Theory

In his discussion, Absi compares the original article's Schmidt number closure with one derived from kinetic theory, based on the work of Fu et al. (2005). Comparison with this closure requires a few additional clarifications. The discusser uses the same averaging notation that was used for the Reynolds averaging, but the original work employs ensemble (or Favre) averaging, denoted by $\langle .. \rangle$ (denoted $\hat{}$ in the original paper). Hence, Absi's Eq. (1) corresponds to Eq. (25) of Fu et al. (2005), where the averaging operator $\bar{..}$ should be replaced by $\langle .. \rangle$.

Subsequently, the equivalent form of the particle momentum equation from Fu et al. (2005), their Eq. (22) [or Wang and Fu 2004, Eq. (9)] can be reconstructed starting from Eq. (6) in the original article, assuming this time a hydrostatic pressure distribution. Using furthermore the relationship between Reynolds and Favre averaging, Eqs. (22) and (23), and realizing that the Favre-averaged vertical velocities are zero (hence, no steady drag), the result is

$$T_p \frac{\partial \langle v_z'^2 \rangle \bar{\phi}}{\partial z} + w_s \bar{\phi} = \mathrm{St}\, \overline{v_z' \phi'} = -\mathrm{St} \frac{\nu_t}{\mathsf{Sc}} \frac{\partial \bar{\phi}}{\partial z} \tag{1}$$

Assuming furthermore $\nu_t = \tau_t \langle u_z'^2 \rangle$, following Fu et al. (2005), and rearranging, gives

$$T_p\, \bar{\phi} \frac{\partial \langle v_z'^2 \rangle}{\partial z} + w_s \bar{\phi} = -T_p \left(\langle v_z'^2 \rangle + \frac{\langle u_z'^2 \rangle}{\mathsf{Sc}} \right) \frac{\partial \bar{\phi}}{\partial z} = -\varepsilon_s \frac{\partial \bar{\phi}}{\partial z} \tag{2}$$

Comparison with Fu et al. shows that the diffusion coefficient ε_s has the same form and becomes equal when $\mathsf{Sc} = \mathrm{St}^2(1 + \mathrm{St})$. However, this does not match experimental data. The origin of the factor in Fu et al. is too briefly explained and refers to Chinese literature (Xu and Zhou 2000).

This gives new insight in the structure and physics behind the turbulent diffusion of sediment particles. Contrary to Absi's attempt, ε_s should not be taken equal to ν_t/Sc because only the second part corresponds to the effect of fluid-induced turbulent particle flux that outbalances the sedimentation flux (cf. sediment continuity). The first contribution follows from the particle inertia. This distinction is also made by Fu et al. (2005). Also, comparison of Eq. (2) with Rouse's balance equation [Eq. (1) in the paper under discussion] immediately makes clear that ε_s is not equal to the sediment diffusivity ε. Therefore, Absi should not use ε_s to compute the Schmidt number.

Notice furthermore that the first term in Eq. (1) is negative in the outer layer. Hence, using the vertical mass balance, this first term becomes equal to the fraction $(1 - \mathrm{St})$ of the sedimentation flux, as long as $\mathrm{St} < 1$. Experimental data indeed show that in the outer dilute layer, this condition is fulfilled.

Reynolds versus Favre Averaging

There is a major difference between the Reynolds-averaged (RA) form of the original paper and the Favre-averaged (FA) form, presented in the preceding section. This should be realized when trying to compare or validate the relationships. Nevertheless, the assumptions in the original paper may be questioned. For instance, the assumed relationship between eddy viscosity and vertical turbulence intensity [Eq. (29)] can be expected to be different because the assumption of Fu et al. (2005) would require replacing the RA intensity by the equivalent FA [using the relationships of Eqs. (22) and (23)] $\bar{v}_z'^2 - V_D^2$. Taking the assumption from the original paper that the drift velocity V_D equals w_s, this would modify Eq. (33)

J. Hydraul. Eng., 2011, 137(9): 1097-1100

such that the denominator becomes one and the value of α must be slightly adjusted.

However, the assumption to equal the mean particle velocity and drift velocity to the settling velocity might be questioned. It resulted in the elimination of the pressure gradient term and subsequently the disappearance of the settling flux, which remains present in Fu et al. and in the equivalent form [Eq. (1)] previously. Nevertheless, attempts to introduce these alternative assumptions only result in much worse Sc profiles, compared with the experimental data.

Imagining the real particle movement during equilibrium, during which upward and downward fluctuations on average keep the same concentration of particles at each depth, suggests that the time between opposite fluctuations is too short to establish a steady velocity. Hence, the RA downward movement more likely will be smaller than the settling velocity. This would require revision of the original author's proposed theory. However, without detailed measurements, it will remain difficult to assess this problem. Direct numerical simulation (DNS) modeling of turbulent shear flow around a free-falling cloud of finite-sized particles may become feasible in the future to provide the necessary data.

Timescales

Another major source of uncertainty is the computation of the integral turbulent timescale for the particle fluctuations. In the original paper, it is argued that the timescale for the turbulent particle flux should equal the integral turbulent scale rather than the gravitational particle timescale T_p. Both timescales are reference scales, defined theoretically, which can be computed from (relatively) simple measurements using their definitions.

The discussers Chanson and Trevethan present field data for integral timescales for the fluid and the particle turbulent fluctuations, demonstrating that they are not necessarily equal because of the difference in inertia. In other words, the value of the parameter α [Eq. (19)] most likely is not constant. This may explain why different values match better for the different cases investigated in the original article. Indeed, as the discussers suggest, it can be expected that there may be a relationship to the Stokes number. But this relationship is expected to be different from the one proposed by Liu (1993) because this one reduces the particle integral timescale to the gravitational timescale for small Stokes number particles, which is unlikely, as argued in the original paper and supported by the data analysis presented.

Pattern II Profiles

In a following step, Absi uses simple analytical closures to replace TKE and concentration in Eq. (31). It should be stressed that these exponential closures have limited validity, especially that they are only useful for very shallow channel flows typical for laboratory flumes. For real deep-water scales of rivers, estuaries and coasts, a linear TKE profile and a Rousean concentration profile are closer to reality and preferably should be used for approximation.

Absi rightfully remarks the possibility that the Schmidt number closure can result in negative values. However, it cannot be used to explain the Pattern II shape of concentration profiles, simply because it turns out that the experimental evidence the discusser refers to (e.g., Wang and Ni 1990, and references therein) and others (e.g., Kaushal 2009) show that this primarily occurs in hyperconcentrated flows, whereas the original closure is only valid for dilute conditions. The concentrations in the case of Pattern II profiles are so high that the flow regime actually corresponds to granular flow, rather than dense suspension flow.

In this light it can be understood conceptually why the maximum concentration is no longer measured at the bottom in these hyperconcentrated shear flows. A possible explanation may be a liquefaction near the bottom because of the shear, resulting in increased pore pressure, which allows the particles to be pushed away from each other, while the sand in the low-shear layer moves in a compact plug flow regime (as suggested by the extremely high concentrations, $> 50\%$, not far from the normal packing fraction of sand).

These high concentrations are not expected to occur for usual sediment transport conditions in natural water courses. Nevertheless, the particle image velocimetry (PIV) data of Muste et al. (2005) for a smooth, flat bottom flume do show a sudden decrease of the concentration at the bottom, especially for run NS3. For the lowest concentration runs NS1 and NS2, it still may look more like an artifact from the measuring system and/or data processing because the maximum concentration is located at a distance of less than or approximately the particle size (which is of the same magnitude as the thickness of the laminar sublayer). But for the high concentration case, the Pattern II behavior is distinct (the maximum concentration is reached at a distance of approximately five times the particle size, but still in the transition layer).

In addition, three out of four DNS models in a benchmark comparison exercise obtained negative concentration gradients at the wall ($z_+ < 0.6$, deep in the laminar sublayer) in low concentration suspensions, only for the case St = 1 (Marchioli et al. 2008).

From kinetic theory, the theoretical explanation for this Pattern II behavior is related to the importance of the lift force in the low-turbulence shear layer at the bottom (Ni et al. 2003; Wang and Fu 2004). However, lift can be neglected in the fully developed outer layer, and this is indeed what has been assumed in the derivation of the Schmidt number closures under consideration.

Negative Schmidt Numbers?

Instead of an analysis with Eq. (31), as done by Absi, it is more useful to use Eq. (33), in which the concentration dependence is eliminated (as confirmed by Fig. 1). It is then evident that the turbulent Schmidt number still can become negative, either because of the denominator when $(\alpha/\beta_0 c_\mu)w_s^2/k > 1$, or when the nominator becomes negative, i.e., for large k gradients. For the theoretical

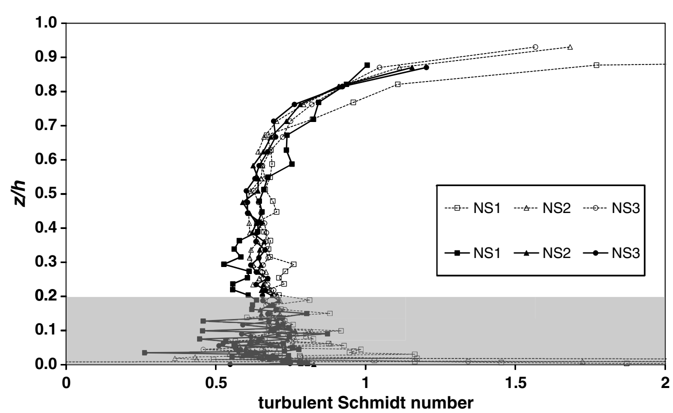

Fig. 1. Turbulent Schmidt number profiles for the three NS runs of Muste et al. (2005) computed with the closure by Eq. (3) (dashed lines) and by Toorman (2009) (full lines); shaded area corresponds to the nondilute bottom layer

J. Hydraul. Eng., 2011, 137(9): 1097-1100

linear approximation of the k-profile [Eq. (28)], based on mixing-length theory, the denominator indeed risks becoming negative near the free surface, but in reality TKE never becomes zero at the free surface because of the wake effect. For the investigated data and for numerically simulated TKE profiles, it was found that the closure [Eq. (33)] remains positive.

Also, from a physical point of view, it does not make sense that the Schmidt number becomes negative because that would imply negative diffusion, which would lead to destabilization, and the equilibrium concentration profile is stably stratified.

Nevertheless, it is very easy to assume heavy particles, such that the critical condition is quickly reached. In such a case it would be expected that bed-load-dominated transport occurs and hardly or no particles are suspended in the outer layer. (A classical numerical sediment transport model, however, would still be able to predict a fraction of particles suspended because it uses a single-fluid approach that cannot recognize discrete particles.) Anyhow, the model would run into trouble.

Looking at sediment transport in nature, it is recognized that natural sediments are graded. Even well-sorted particles show a particle size distribution. Only a few experimental studies have been carried out in which not only concentrations, but also grain size distributions, over the depth have been determined, showing that the average grain size decreases with distance from the bottom (Anderson 1942; Best et al. 1997). This is not unexpected of course if a fractional step approach is applied to polydisperse particles (e.g., Wu et al. 2003). Considering this in an extreme limit, the sediment can be imagined as consisting of finite particles and infinitely small particles. In other words, in the outer layer, where there are no particles, the Schmidt number should be expected to become equal to the neutral value for w_s approaching zero. Eq. (33) fails here because of the settling velocity in the denominator of the second term in the nominator. At present, this problem could not be resolved.

This observation gives additional indication that some of the assumptions in the derivation of Eq. (33) may not be correct or oversimplified. There certainly remains uncertainty about the value or, more likely, the closure of the integral timescale factor α. But also the assumption relating the vertical turbulence intensity (or pseudotemperature) of the particles to the eddy viscosity [Eq. (29)] most likely is too crude. This needs further investigation.

New Validation

The theoretical Schmidt number closure of the original Toorman article [at its errata and addendum, Toorman (2009)] could be validated once more by using the flume data of Muste et al. (2005) for glass beads (the NS runs), using the PIV measurement technique in a shallow shear flow (water depth $h = 2$ cm). The turbulent Schmidt number has been computed in two different ways. The first is the new theoretical closure. The second is obtained from the suspension capacity condition (Celik and Rodi 1991), in which the efficiency factor is defined by the flux Richardson number $\mathsf{Ri}f$ (Toorman 1999, 2002):

$$\mathrm{Sc} = \frac{\mathsf{Ri}f}{C} \frac{\rho_w u_*^4}{(1 - \rho_w/\rho_s) g h w_s^2} \left(\frac{h}{z} - 1 \right) \tag{3}$$

where h = water depth; z = distance from the bottom; u_* = shear velocity; w_s = particle settling velocity; ρ_w = water density; ρ_s = particle density; and g = gravity constant. The flux Richardson number is obtained from

$$\mathsf{Ri}f = -\frac{G}{P} = \frac{(1 - \rho_w/\rho) g w_s}{-\overline{u'v'}\frac{\partial U}{\partial z}} \tag{4}$$

where G = buoyancy destruction of TKE (assuming equilibrium between turbulent and settling flux); P = shear production of TKE; ρ = bulk suspension density; $-\overline{u'v'}$ = Reynolds stress; and U = Reynolds-averaged flow velocity. $\mathsf{Ri}f$ is calculated from Eq. (4) using the experimental data.

Fig. 1 shows the computed profiles of Sc. The profile of Sc does not change for the increasing load of the three NS runs, which differ only in total suspended load, supporting Toorman's (2009) reworked closure, which is independent of the sediment concentration. For this data set, the best fitting value of Sc_0 is found to be 0.7, the most recommended value of the neutral Schmidt number in the literature. Furthermore, for this case the value of α should be 0.164, which corresponds to $c_\mu^{3/4}$, which is a commonly recurring value for the turbulent timescale of the fluid (Elghobashi and Abou-Arab 1983). The near-bottom data ($z/h < 0.2$) should be discarded because dilute conditions are no longer fulfilled in this layer.

Despite the weaknesses, this new validation with the data of Muste et al. is a hopeful indication that the proposed Schmidt number closure is useful for modeling purposes.

Acknowledgments

The writer wishes to thank both discussers for providing the additional material referred to. Dr. Absi (personal communication, 2009) also indicated an overlooked typing error in Eq. (18), in which the drag correction factor should read $(1 + 0.150\mathsf{R}_p^{0.687})^{-1}$. Dr. Muste kindly made available his experimental data from the experiments carried out at Kobe University.

References

Anderson, A. G. (1942). "Distribution of suspended sediment in a natural stream." *Trans., Am. Geophys. Union*, 23(2), 678–683.

Best, J., Bennett, S., Bridge, J., and Leeder, M. (1997). "Turbulence modulation and particle velocities over flat sand beds at low transport rates." *J. Hydraul. Eng.*, 123(12), 1118–1129.

Celik, I., and Rodi, W. (1991). "Suspended sediment-transport capacity for open-channel flow." *J. Hydraul. Eng.*, 117(2), 191–204.

Elghobashi, S. E., and Abou-Arab, T. W. (1983). "A two-equation turbulence model for two-phase flows." *Phys. Fluids*, 26, 931–938.

Fu, X., Wang, G., and Shao, X. (2005). "Vertical dispersion of fine and coarse sediments in turbulent open-channel flows." *J. Hydraul. Eng.*, 131(10), 877–888.

Kaushal, D. R. (2009). "Discussion of 'Analysis of the vertical profile of concentration in sediment-laden flows' by Q. Q. Liu, A. P. Shu, and V. P. Singh." *J. Eng. Mech.*, 135(7), 743–745.

Liu, D. Y. (1993). *Fluid dynamics of two-phase systems*, Junior Educational, Beijing (in Chinese).

Marchioli, C., et al. (2008). "Statistics of particle dispersion in direct numerical simulations of wall-bounded turbulence: Results of an international collaborative benchmark test." *Int. J. Multiphase Flow*, 34(9), 879–893.

Muste, M., Yu, K., Fujita, I., and Ettema, R. (2005). "Two-phase versus mixed-flow perspective on suspended sediment transport in turbulent channel flows." *Water Resour. Res.*, 41(10), W10402.

Ni, J., Huang, X., and Borthwick, A. G. L. (2003). "Characteristics of hyperconcentrated sediment-laden flows." *J. Hydraul. Eng.*, 129(12), 1489–1493.

Toorman, E. A. (1999). "Numerical simulation of turbulence damping in sediment-laden flow. Part 1. The Siltman test case and the concept

J. Hydraul. Eng., 2011, 137(9): 1097-1100

of saturation." *COSINUS Project Rep. HYD/ET/99.2*, Hydraulics Laboratory, Katholieke Universiteit Leuven, Belgium.
Toorman, E. A. (2002). "Modelling of turbulent flow with cohesive sediment." in *Proceedings in marine science: Fine sediment dynamics in the marine environment*, Vol. 5, J. C. Winterwerp and C. Kranenburg, eds., Elsevier Science, Amsterdam, 155–169.
Toorman, E. A. (2009). "Errata and addendum for 'Vertical mixing in the fully developed turbulent layer of sediment-laden open-channel flow' by E. A. Toorman." *J. Hydraul. Eng.*, 135(6), 538.
Wang, G., and Fu, X. (2004). "Mechanisms of particle vertical diffusion in sediment-laden flows." *Chinese Science Bulletin*, 49(2), 1086–1090.
Wang, G., and Ni, J. (1990). "Kinetic theory for particle concentration distribution in two-phase flow." *J. Eng. Mech.*, 116(12), 2738–2748.
Wu, B., Molinas, A., and Shu, A. (2003). "Fractional transport of sediment mixtures." *Int. J. Sediment Res.*, 18(3), 232–247.
Xu, Y., and Zhou, L. X. (2000). "A second-order moment two-phase turbulence model based on the Lagrangian PDF." *Chin. J. Comput. Phys.*, 17(6), 633–640 (in Chinese).

J. Hydraul. Eng., 2011, 137(9): 1097-1100