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Comment on "Internal-tide energy over topography" by S. M. Kelly et al.

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1. Introduction

[1] In a recent paper, *Kelly et al.* [2010, hereinafter *KNK10*] address the important problem of how to determine vertical profiles of horizontal internal-tide energy fluxes, defined as the tidal-average product of baroclinic horizontal velocity and baroclinic pressure. The difficult part of the problem is to obtain the latter.

[2] Observationally, for example, one can determine vertical density and velocity profiles from combined conductivity-temperature-depth (CTD)/lowered acoustic Doppler current profiler (LADCP) profile measurements; repeating such measurements (yo-yoing) then provides the time evolution of these quantities over a tidal period. Subtracting depth-average horizontal velocities, one obtains the baroclinic velocity, but for baroclinic pressure, one has to resort to the vertical momentum equation. From the hydrostatic balance, one can vertically integrate the density perturbation term to obtain baroclinic pressure, but only up to an unknown 'constant' of integration, which in fact is a function of time and the horizontal coordinates. Since the vertical integral of baroclinic velocity is zero, this constant does not feature in the vertically integrated energy flux, but it does affect the vertical *profile* of the energy flux.

[3] *Kunze et al.* [2002] proposed a way to resolve this indeterminacy, claiming (but not proving) that vertically integrated baroclinic pressure can be taken to be zero. *Gerkema and van Haren* [2007] criticized this claim, arguing that it is inconsistent with the notion of baroclinic velocity over a slope. Now, *KNK10* maintain that there is no such inconsistency, and propose a new way of dealing with the problem (see their section 3.4), which, although different from the one proposed by *Kunze et al.* [2002] in that it takes into account surface motion, still rests on the assumption that the vertical average of baroclinic pressure can be taken to be zero. In particular, in the case of a rigid lid, their method leads back to the one proposed by *Kunze et al.* [2002].

[4] To settle this important point of contention, it is desirable to have an exact internal-wave solution over a slope, so that the corresponding pressure field can be examined; this should reveal conclusively whether or not the vertical average of baroclinic pressure is zero. Such solutions are, in general, not available, but one was derived by *Wunsch* [1968] for a wedge-shaped domain. In passing, *KNK10* refer to this solution, but they gloss over its implications. The present note is meant to demonstrate what the solution really implies: namely that the vertical average of baroclinic pressure is *not* zero over a slope.

2. The Basic Equations

[5] *KNK10* introduce no less than twelve different pressure terms. It would seem that one can dispense with some of them. In any case, the discussion gains much in transparency if one introduces just three terms: total dynamic pressure p, its depth average $P(=\langle p\rangle)$, and the remainder $p' = p - P$. By definition, $\langle p'\rangle = 0$.

[6] For the purpose of this *Comment*, we can content ourselves with these pure and simple definitions, without making any claims about their physical meaning; in particular, there is no need here to attempt to ascribe one or the other to barotropic or baroclinic effects. (Note that *Gerkema and van Haren* [2007] defined p' differently and there it *did* denote baroclinic pressure. Here, for the clarity of discussion, we follow instead the notation adopted by *KNK10*.)

[7] Following *Gerkema and van Haren* [2007] and *KNK10*, the starting point is the following set of linearized hydrostatic momentum equations:

$$u_t - fv = -p_x \tag{1}$$

$$v_t + fu = 0 \tag{2}$$

$$p_z = b, \tag{3}$$

where b is buoyancy, u and v horizontal velocity components, and f the Coriolis parameter. For simplicity, it is assumed that the problem is uniform in the y direction. To close the set, one has to add the energy and continuity equations, but they are not needed in the present discussion.

[8] Over a *sloping* bottom $z = -h(x)$, the depth average of p_x can be written

$$\langle p_x\rangle = \langle p\rangle_x + \frac{h_x}{h}\left[\langle p\rangle - p|_{z=-h(x)}\right].$$

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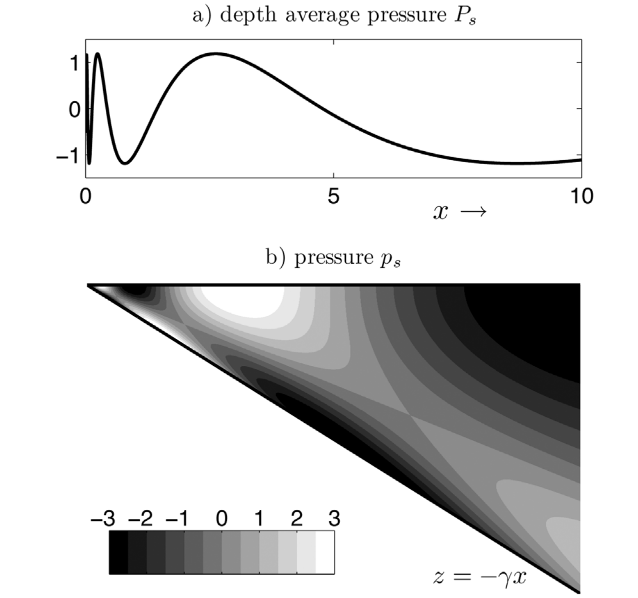

Figure 1. Baroclinic pressure field p_s and its depth average P_s of the internal-wave solution in a wedge-shaped basin. Parameter values are $c = 0.6$, $\gamma = 0.5$, and mode $n = 1$ (the factor σ is left out here).

Taking now the depth average of (1), and using the definitions of P and p', gives

$$U_t - fV = -P_x + \frac{h_x}{h} p'|_{z=-h}, \qquad (4)$$

where $U = \langle u \rangle$ and $V = \langle v \rangle$. Subtracting this from (1), one obtains

$$u'_t - f'_v = -p'_x - \frac{h_x}{h} p'|_{z=-h}. \qquad (5)$$

[9] Equation (4) already illuminates the fundamental difference between the case of a horizontal bottom and that of a sloping one. This is most clearly seen when there are no depth average flows ($U = V = 0$), for this implies

$$P_x = \frac{h_x}{h} p'|_{z=-h}. \qquad (6)$$

On the basis of this equation alone, it follows that $P_x = 0$ over a horizontal bottom (since $h_x = 0$), but $P_x \neq 0$ over a sloping bottom. As a consequence, P must be nonzero in the latter case.

3. Internal-Wave Solution in a Wedge

[10] To refute the assertion put forward by *Kunze et al.* [2002] and *KNK10*, it suffices to give one counterexample. To this end, we examine *Wunsch*'s [1968] solution for internal waves in a wedge, i.e., a half open domain confined by a sloping bottom below, and a rigid lid above. The sloping bottom is described by $z = -\gamma x$. The characteristic coordinates are $\xi_\pm = cx \pm z$; the internal-wave energy propagates along lines of constant $\xi_\pm$. The slope is assumed to be subcritical: $\gamma < c$.

3.1. Pressure

[11] Following *Wunsch* [1968], a leftward propagating mode n can be constructed, which runs into the apex

$$\psi = \sin[q \log(\xi_-) + \sigma t] - \sin[q \log(\xi_+) + \sigma t], \qquad (7)$$

with $q = 2n\pi/\log\Delta$ and $\Delta = (c + \gamma)/(c - \gamma)$. This solution can also be written as

$$\psi = \psi_c \cos(\sigma t) + \psi_s \sin(\sigma t), \qquad (8)$$

where

$$\psi_c = \sin(q \log \xi_-) - \sin(q \log \xi_+)$$

$$\psi_s = \cos(q \log \xi_-) - \cos(q \log \xi_+),$$

each of which satisfies the wave equation $\psi_{xx} - c^2\psi_{zz} = 0$ as well as the boundary conditions (i.e., $\psi = 0$ at $z = 0$ and at $z = -\gamma x$).

[12] The corresponding fields u and p can be written analogously to (8). The components u_c and u_s follow from $u = -\psi_z$:

$$u_c = \frac{q}{\xi_-}\cos(q \log \xi_-) + \frac{q}{\xi_+}\cos(q \log \xi_+)$$
$$u_s = -\frac{q}{\xi_-}\sin(q \log \xi_-) - \frac{q}{\xi_+}\sin(q \log \xi_+).$$

The corresponding components of pressure p follow from (1):

$$p_c = -\frac{\sigma}{c}(\cos(q \log \xi_-) + \cos(q \log \xi_+))$$
$$p_s = \frac{\sigma}{c}(\sin(q \log \xi_-) + \sin(q \log \xi_+)).$$

Here we ignored rotation (i.e., $f = 0$).

[13] A few remarks are in order. First, the vertical averages of u_c and u_s are zero, in accordance with the baroclinic nature of the solution. In terms of pressure, the internal-wave field is given by p_c and p_s; their vertical averages are

$$P_c = 2\sigma(-1)^{n+1}\frac{q\sin\left(\frac{q}{2}\log\chi\right) + \cos\left(\frac{q}{2}\log\chi\right)}{c(1+q^2)}$$
$$P_s = 2\sigma(-1)^{n+1}\frac{q\cos\left(\frac{q}{2}\log\chi\right) - \sin\left(\frac{q}{2}\log\chi\right)}{c(1+q^2)},$$

with $\chi = (c^2 - \gamma^2)x^2$. An example is shown in Figure 1, with p_s (Figure 1b) and P_s (Figure 1a). Clearly, the depth average P_s is nonzero; it attains values nearly as large as one third of the maximum of p_s.

[14] This simple counterexample demonstrates that the depth average of pressure, representing the internal-wave field, can be nonvanishing over a sloping bottom.

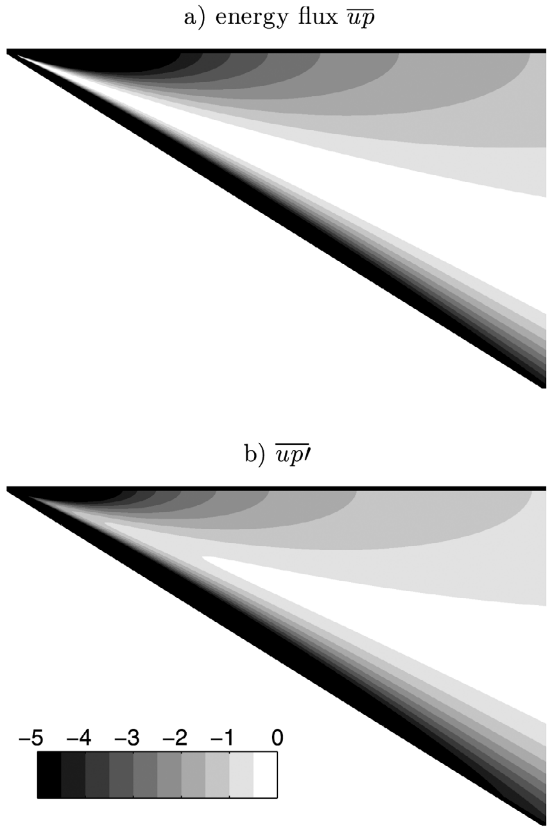

Figure 2. The energy flux. Negative values indicate leftward propagation. In Figure 2a, the energy flux is calculated using (9). In Figure 2b, following *KNK*10, the depth-average part of pressure has been removed ($p' = p - P$). Parameters are as in Figure 1.

3.2. Energy Fluxes

[15] All the constituents of the energy flux have now been gathered; the horizontal flux is given by

$$\overline{up} = \frac{1}{2}(u_c p_c + u_s p_s) = -\sigma \frac{qx(1 + \cos[q \log(\xi_+/\xi_-)])}{\xi_+ \xi_-}, \qquad (9)$$

where the bar stands for time averaging over a wave period. The result is shown in Figure 2a. The vertical integral of this flux can be found analytically and is given by the simple expression $-\sigma n\pi/c$, indicating a negative (i.e., leftward) horizontally uniform energy flux.

[16] Now, *KNK10* would regard the depth-average parts P_c and P_s as *barotropic* tidal components. This interpretation is however incongruous with the nature of the problem. After all, the physical setting considered here precludes barotropic tides; the flow field is purely baroclinic (i.e., $U = V = 0$). The idea of a "shoaling internal tide to induce a surface tide" (see their section 2.6) is impossible here for still another reason: the vertically integrated baroclinic energy flux is horizontally uniform (as noted above) and there is, therefore, no such conversion.

[17] According to *KNK10*, the energy flux should be defined as $\overline{up'}$, the depth-average parts of pressure having been removed ($p' = p - P$). This quantity is shown in Figure 2b. In comparison with Figure 2a, the flux is underestimated in the upper part of the water column, and overestimated in the lower part. For the vertically integrated energy flux, there is, of course, no difference (since $\langle \overline{uP} \rangle = \overline{\langle u \rangle P} = 0$); it is still given by $-\sigma n\pi/c$.

[18] The fact alone that the fluxes in Figures 2a and 2b are different does not necessarily mean that the difference is physically meaningful. *LeBlond and Mysak* [1978, p. 50] pointed out that the energy flux $p\vec{u}$ is defined only up to an arbitrary nondivergent function. This means that the tidal average of $p\vec{u}$ may not coincide with $E\vec{c}_g$ (E energy density, $\vec{c}_g$ group velocity), not even in direction, but that the two can be reconciled by adding an arbitrary nondivergent flux vector to $p\vec{u}$.

[19] In the present setting, this consideration invites the question as to whether the difference between Figures 2a and 2b can be ascribed to an arbitrary nondivergent flux vector, in which case the difference would be physically irrelevant. A simple check shows this not to be the case, in other words, the difference is relevant.

4. Conclusion

[20] An analysis of the internal-wave solution in a wedge has confirmed the earlier finding by *Gerkema and van Haren* [2007]: over a slope, it is inconsistent to assume that the vertical average of baroclinic pressure is zero. Any method based on the contrary, such as those proposed by *Kunze et al.* [2002] and *KNK10*, must therefore be considered inexact.

[21] This also means, unfortunately, that the problem mentioned in the Introduction (how to determine energy flux profiles from CTD/LADCP yo-yoing data?) still stands unresolved. The present *Comment* only refutes a proposed way of fixing the constant of integration, but does not yield an alternative. In fact, *Gerkema and van Haren* [2007] argued that this problem is fundamentally unresolvable, for over a slope, where separation of spatial variables does not apply, one needs to know horizontal gradients to evaluate the constant of integration, but this information is not available from a single yo-yo station.

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