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Stability parameters of
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STABILITY PARAMETERS OF WESTERN SCHELDT ESTUARY

by

H. de Jong[1] and F. Gerritsen[2]

1. INTRODUCTION

The Western Scheldt is a major estuary in the Southern part of The Netherlands and the Northern part of Belgium. It is an important navigational route connecting the city of Antwerp with the North Sea. At the entrance Vlissingen is a major Dutch port. (Figure 1)
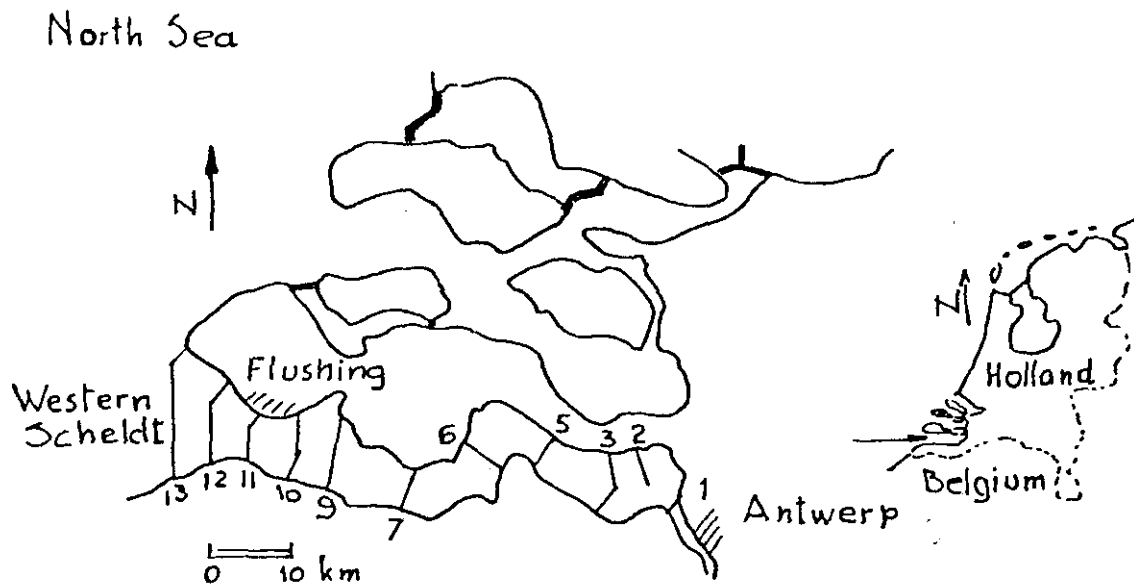

Figure 1: Western Scheldt Estuary

During recent years much dredging work has been carried out by the Belgian Government (16 million m^3 in 1977) to remove shoals and improve channel alignments in the estuary, in view of increasing vessel drafts and traffic density to the port of Antwerp (Belgium).

It is of great importance to predict the effects of future dredging operations, carried out in the outer shoals and in major navigation channels on tidal regime (tides and currents) and on morphological changes in other parts of the estuary.

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Calculations based on detailed hydrographic surveys have indicated that where the estuary used to be subject to sedimentation, the situation may now have changed to one of dominant erosion.

2. MORPHOLOGICAL STABILITY OF TIDAL CHANNELS

The stability equation

The morphological stability of tidal channels has two major components:

(a) location stability
(b) cross-sectional stability

The location stability has to do with the meandering of channels, where the channel migrates gradually within the physical boundaries of the estuary. The nature of this process is strongly affected by the type of estuary, by the nature of bottom conditions (presence of silty layers) and by the type of coastal protection works (if present) along the estuary's boundaries.

Migration of tidal channels in the outer part of the estuary is often a cyclic process with a period of a number of years which is governed by the external boundary conditions related to tide, waves and littoral drift. The process of cyclic changes can be arrested or slowed down by high capacity dredging operations.

The development and migration of tidal channels in an estuary is connected with the system of flood and ebb channels in an estuary. Flood channels are characterized by dominating flood flow; they have a shoal at their inner end. In ebb channels the ebb flow dominates and a shoal is usually present at their seaward ends. Tidal channels are neutral when flood and ebb flow are of equal magnitude.

Cross-sectional stability relates to the variability of the cross-sectional area of tidal channel and to its dependency on tidal flow characteristics.

Tidal inlets are often characterized by a gorge, in which the tidal flow is well organized and where channel migration does not play a major role.

In this paper we will particularly be concerned about the cross-sectional stability of tidal channels whereby we will examine the total channel, as well as the individual flood or ebb channels, if they can be clearly identified.

The cross-sectional stability of a tidal channel is in general governed by the equation:

$$\frac{\partial Q_s}{\partial x} = 0 \qquad (1)$$

if Q_s represents the sediment transport rate (m³/sec) and x the mean direction of the flow through a channel.

For (1) to be valid it is necessary that the mean sediment concentration over the channel cross-section does not vary with time.

For channels subject to tidal flow the stability equation must be intergrated over time and gives the condition (T = tidal period):

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$$\frac{\partial}{\partial x} \int_0^T Q_s \, dt = 0 \qquad (2)$$

under the assumption that the transport process repeats itself in equal manner during subsequent tidal cycles.

In case a strong difference exists between neap tide and spring tide cycles the integration is to be performed over a series of cycles to cover a sequency which repeats itself:

$$\frac{\partial}{\partial x} \int_0^t Q_s \, dt = 0 \qquad (3)$$

where $t \gg T$.

Unfortunately the detailed knowledge about the sediment transport process is not adequate enough at the present time to use equation (3) with confidence to predict changes in bottom configuration.

Solutions to the stability problem

For many years the relation between the cross-section of a tidal channel and the tidal flow characteristics has been recognized.
A classical paper is that of M.P. O'Brien (9) who arrived at an empirical relationship between cross-sectional area and tidal prism for inlets on the U.S. West Coast. The tidal prism was calculated as the product of tidal storage area and tidal range. For the latter the difference between MHHW and MLLW was assumed.
The relation obtained was purely empirical because no consideration was given to relevant physical parameters such as:

- grain size
- littoral drift transport in relation to tidal transport,
- river discharge
- relative depth of channel (Chézy)
- presence of jetties
- presence of shoals, etc.

Because of the complexity of the problem it may be expected that certain empirical relationships would not be universally valid for other situations.
In more recent papers O'Brien and several other authors recognized this problem and certain modifications of the relationship were proposed.

In 1960 and in subsequent years Bruun and Gerritsen (3) attempted to evaluate the effect of various physical parameters on the empirical stability equations. They introduced the concept of stability shear stress τ_s and found that this parameter was a useful parameter to define inlet stability. The variability of τ_s with a number of physical parameters affecting inlet stability was thereby investigated. A comprehensive analysis is presented by Bruun (4).

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A satisfactory solution to the stability equation (3) can only be obtained if the process of sediment transport through the channels can be accurately described by a sediment transport formula.

At present existing knowledge does allow the description and calculation of tides and currents in an estuary in a satisfactory manner. Work of fundamental importance was done by Dronkers (6) for one-dimensional flow and by Leendertse (8) for two-dimensional tidal problems. However the state of the art regarding the description of the sediment transport phenomenon in an estuary is not accurate enough to predict erosion and sedimentation rates according to equation (3).

The Kalinske and Morra formulation has been used in combination with a one-dimensional tidal model to predict morphological changes in the form of erosion and sedimentation rates for the Western Scheldt. The one-dimensional tidal model does not adequately distinquish between transport through channels and over shoals. Results of this approach can at best be qualitative.

The problem is magnified in the outer estuary where a formulation is required describing the sediment transport as a function of waves and currents combined. Such a formulation was proposed by Bijker (2).

Until a sufficiently accurate formulation for the sediment transport processes has been found an interim solution may be utilized, in which a mathematical model for flow is utilized to calculate the changes in water levels and currents, and where the changes in bottom configuration are predicted from empirical relationships between flow parameters and channel dimensions. For this, however, it is necessary that these empirical relationships, which have a statistical nature, are sufficiently accurate to use them as a predictive tool. The method to be applied is an iterative scheme: the changes in tide conditions are predicted using the numerical tidal hydraulics model; then channel profiles are adjusted based on the empirical relationship between tide parameters and channel dimensions. The process is repeated until stability conditions are fulfilled. It is the purpose of this paper to investigate a number of empirical relationships for the whole of the Western Scheldt, whereby the channel cross-section is related to a variety of tidal parameters such as velocity, tidal discharge, tidal prism, bottom shear stress, Chézy coefficient, etc.

The assumption is made that certain empirical relationships are not only valid for the entrance area but are applicable to the inner tidal channels as well.
It is furthermore assumed and verified that river discharge is negligible compared to tidal discharge and that density differences have no appreciable effect on the sediment transport equation.

The usefulness of various relationships will be evaluated from the calculated values of correlation coefficients obtained from the regression analysis.

In the study the cross-sectional area is treated as the independent variable, plotted on the horizontal axis of the regression diagrams.

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Flow parameters are then considered as a function of the cross-sectional area.

Furthermore it must be noted that all tidal characteristics refer to mean tidal conditions, unless otherwise stated.

3. METHOD OF INVESTIGATION

Data used for analysis

The data have been obtained from extensive field measurements carried out by the Hydraulic Advisory section of the Rijkswaterstaat at Vlissingen over a number of years.

The basis is the set of velocity measurements, simultaneously carried out in different stations, at depths ranging from a little below the surface to a short distance above the bottom. Most of the measurements were carried out with the Ott current meter, and included current directions. The measurements were always done twice on successive days. Velocity measurements were integrated to flow values and flow values to tidal volumes. All flow data were reduced to mean tide conditions valid for the year in which the measurements were carried out. For this reduction the tidal range of the nearest tidal station was used as a reference. The reduction was done in a linear fashion, which has been shown to be an adequate procedure. Simultaneous measurements were usually carried out in 5-7 stations in one profile in cross-sections numbered 1-11 in Figure 1. Simultaneous measurements were carried out in a reference station. The vertical tide was measured (continuously) at 5 fixed tidal stations along the estuary. A discussion on the tidal regime for the Western Scheldt is presented by Daamen (5). The bottom of the Western Scheldt consists of fine alluvial sands, diameter d_{50} = 40-450μ. For the profiles 12 and 13, situated in the outer part of the estuary (see Figure 1) insufficient velocity measurements were available to form a reliable base for the calculation of tidal flow and tidal prism. Therefore these values were calculated from the flow values for Profile 11, using the continuity equation. The possible error in the flow data is estimated at ± 10%.

The cross-sectional area was used in two different ways in the analysis of regression relationships:

(1) the area A_c measured between bottom and the standard reference level N.A.P. (approximately equal to Mean Sea level)

(2) the area A_c' measured between the bottom and the water level at the time the flow (Q) has its maximum value (Q_{max})

Reference is made to Figure 2. For the ebb situation A_c and A_c' are not much different. For the flood, however, there is a considerable difference between A_c and A_c'.
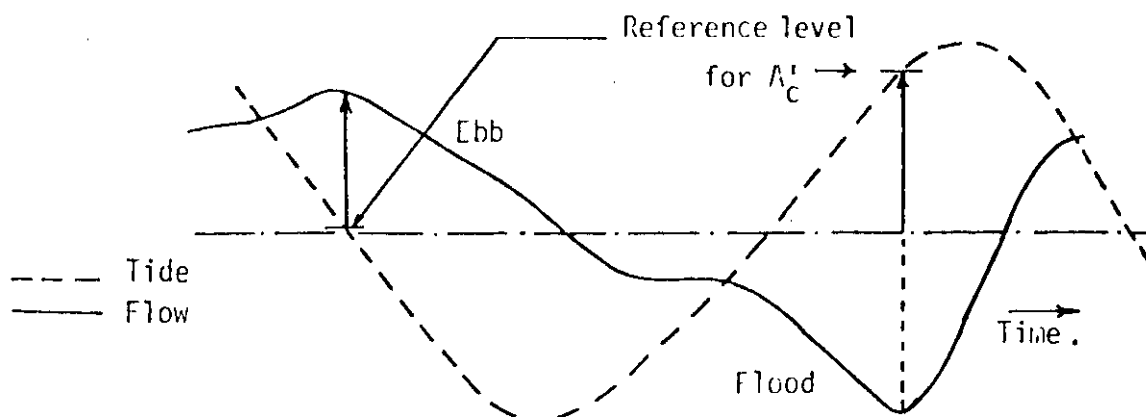
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Figure 2: Tide and Flow Characteristics

Method of Analysis

The following hydraulic and morphological parameters were included in the analysis:

$\bar{v}$: mean velocity over tidal period (m sec^{-1})

v_{max}: maximum velocity $\dfrac{Q_{max}}{A}$ (m sec^{-1})

Q_{max}: maximum tidal flow through cross-section (m^3/sec)

Ω_{flood}: tidal volume ($\int Q$ dt) during flood (m^3)

Ω_{ebb}: tidal volume during ebb (m^3)

A_c: cross-sectional area below NAP ($\sim$MSL) (m^2)

A_c': cross sectional area below level at which Q_{max} occurs (m^2)

τ_s: stability shear stress (N/m^2)

C: Chézy coefficient (m$^{1/2}$ sec^{-1})

A regression analysis between morphological and hydraulic parameters has been carried out for various cross-sections (Fig. 1). The statistical significance of the relationship tested is expressed by the value of the calculated correlation coefficient.

In the regression analysis the cross-section is used as the independent parameter, and is plotted horizontally in various graphs.

In Bruun and Gerritsen (3) the stability shear stress was used as a parameter of tidal inlet cross-sectional stability. It was defined as the mean bottom shear stress along a cross-sectional profile at maximum flow conditions during spring tide.

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The use of springtide was selected because bedforming flow conditions occur during that phase of the tide. To some degree this corresponds with the calculation of the tidal prism in O'Brien (9) where for the U.S. West Coast the difference between MHHW and MLLW is used as basis for the calculations.

In this study all data have been referenced to mean tide conditions; regarding the stability shear stress values with reference to both mean tide and springtide were calculated.

The general formulation of the mean shear stress in a channel cross-section may be given by:

$$\tau = \rho g R S \qquad (4)$$

where R is the hydraulic radius of the channel and S the slope of the energy gradient line.

Using the Chézy expression for the average channel velocity v:

$$v = C \sqrt{RS} \qquad (5)$$

The mean shear stress can be expressed in terms of v:

$$\tau = \rho g \frac{v^2}{C^2} \qquad (6)$$

and since $v = \frac{Q}{A}$, also in terms of Q:

$$\tau = \rho g \frac{Q^2}{C^2 A^2} \qquad (7)$$

From equation (7) the stability shear stress is defined by:

$$\tau_s = \rho g \frac{Q_{max}^2}{C^2 A^2} \qquad (8)$$

where Q_{max} is the maximum tidal flow during springtide for either ebb or flood conditions.

From equation (8) the cross-sectiona area A can be expressed in terms of stability shear stress by the equation:

$$A = \frac{Q_{max}}{C \sqrt{\tau_s / \rho g}} \qquad (9)$$

In a correlation diagram between $\dfrac{Q_{max}}{C \sqrt{\tau_s / \rho g}}$ and A a line under 45°

will be obtained if proper values for C and τ_s are introduced.

For the cross-section value in equation (9) both the quantities A_c and A_c' have been utilized in the regression analysis.

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Stability shear stress for a combined effect of current and wave conditions.

In the outer part of the estuary the sediment transport by tidal currents is affected by waves.

As a first attempt to evaluate the effect of wave action on the stability shear stress parameter the formula developed by Bijker (2) was used to describe the bedload transport as a function of waves and currents. It is realized that the bottom sediment in this area is comparatively fine and therefore transport in suspension dominates. The use of the Bijker formula may still be valid because of the relationship between bedload transport and transport in suspension.

The Bijker formula can be written in the form:

$$Q_b = BD \frac{v}{C} \sqrt{\frac{\tau_c}{\rho}} \, e^{- \dfrac{0.27 \Delta D \rho g}{\mu \tau_c \left\{ 1 + \frac{1}{2} \left(\xi \frac{u_0}{v} \right)^2 \right\}}} \tag{10}$$

where Q_b = bedload transport

B = dimensionless coefficient

C = Chézy coefficient

τ_c = bottom shear stress from currents only

v = mean velocity over profile

u_0 = maximum orbital velocity near the bottom induced by waves

Δ = relative sediment density = $\dfrac{\rho_s - \rho}{\rho}$

ρ = density of sea water

ρ_s = density of sediment

g = acceleration of gravity

μ = ripple coefficient

ξ = factor to increase shear stress in a combination of current and waves

An equivalent shear stress τ' is now defined in such a manner that transport by currents and waves and shear stress τ'_s is equal to the transport without waves and shear stress τ_s.
This gives the relationship:

$$\tau_s \, e^{- \dfrac{0.27 \Delta D \rho g}{\mu \tau_s}} = \tau'_s \, e^{- \dfrac{0.27 \Delta D \rho g}{\mu \tau'_s \left\{ 1 + \frac{1}{2} \left(\xi \frac{u_0}{v} \right)^2 \right\}}} \tag{11}$$

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The values of τ_s can be calculated using the above relationship and can be used in stability relationships for the outer estuary (profiles 12, 13, Figure 1). The value of u_0 is related to wave height, wave period and depth.

4. RESULTS OF INVESTIGATIONS

Mean tidal velocity ($\bar{v}$)

In past studies (v.d. Kreeke and Haring, (7)), the mean tidal velocity has been used as a reference velocity for stability. Its value is defined by:

$$\bar{v} = \frac{\Omega_{flood} + \Omega_{ebb}}{AT}$$

where Ω_{flood}, Ω_{ebb} are respectively the flood and ebb volumes of a tidal cycle, also called tidal prism.

Figure 3 shows the value of this velocity along the axis of the estuary, as well as the cross-sectional areas at the same locations.
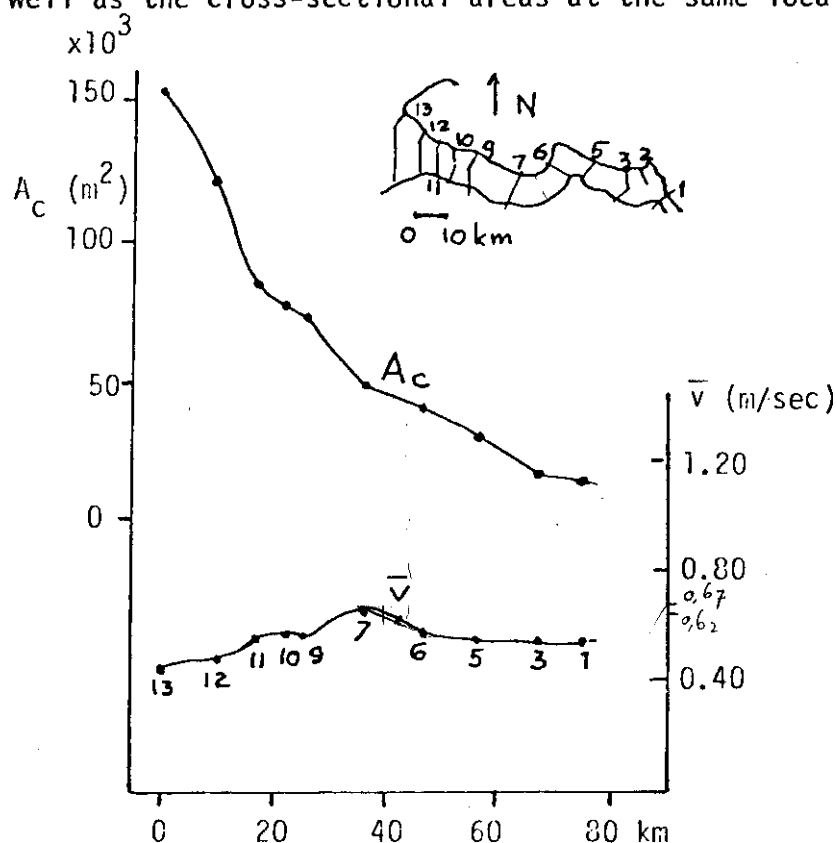

Figure 3: Variation in cross-sectional area and tidal velocity, $\bar{v}$, along longitudinal axis of the estuary.

The geometry of the estuary shows profiles at stations 7 and 11 to be smaller than would be expected from the general trend due to restrictions in the natural width at these locations. The graph for the mean tidal velocities show bumps in these locations (7,11) accordingly.

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The general trend is a fairly constant value of 0.55 m/sec for $\bar{v}$ for the upper 30 km and a decrease in value for the outer portion of the estuary (profiles 12, 13) to 0.45 m/sec.

Maximum tidal velocities (V_{max})

The trend in maximum velocities is shown in Figure 4. Maximum flood velocities are considerably higher than maximum ebb velocities and show greater modulations. Part of this is due to the width restrictions in profiles 7 and 11. In profile 7 the maximum flood velocity (averaged over profile) is 1.25 m/sec.

Seaward of profile 11 maximum velocities also decrease significantly in seaward direction.
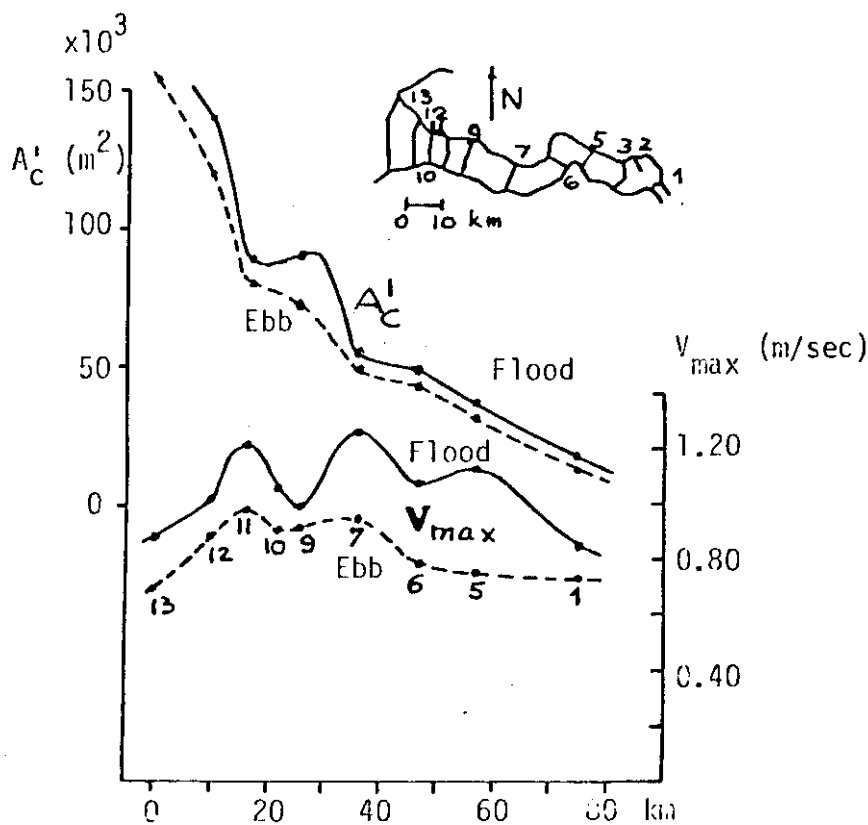

Figure 4: Variation in cross-sectional area and maximum tidal velocity, V_{max}, along longitudinal axis of the estuary.

Maximum flow (Q_{max})

In earlier studies (Bruun and Gerritsen, (3)), it was found that Q_{max} was a useful parameter to characterize a stable channel. This was confirmed in this study.

It appears from the analysis that the use of A_c gives a slightly better result than that of A_c' (contrary to expectations).

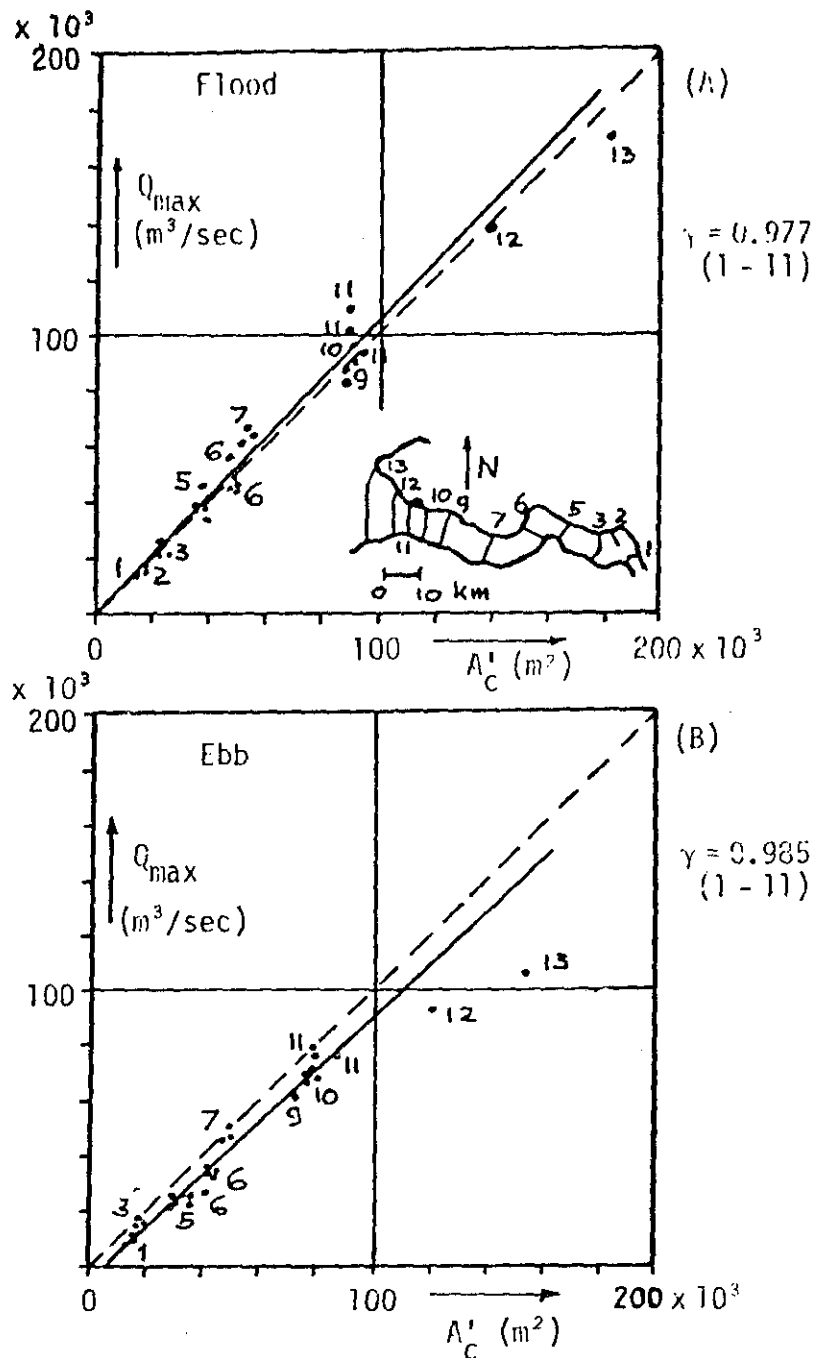Figure 5 gives the regression using the A_c' for flood and ebb.

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Figure 5:   Regression Q_{max} versus A_C^1
            (A) Flood; (B) Ebb.

The dotted line is for $Q_{max} = A_C^1$ (in metric units). Flood flows are somewhat higher and ebb flows somewhat lower than this dotted line relationship.

This corresponds to higher maximum flood currents compared to the maximum ebb current as shown in Figure 4. The profiles 12 and 13 do fit the general relationship for flood but they do not fit the relationship for ebb.

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The sinusoidal maximum flow ($\hat{Q}$)

Another useful parameter for regression analysis is the sinusoidal maximum flow, $\hat{Q}$, defined by

$$\hat{Q} = \pi \frac{(\Omega_{flood} + \Omega_{ebb})}{2T}$$

The correlation $\hat{Q}$ versus A_c, has a correlation coefficient of 0.990, for profiles 1-11.

Van de Kreeke and Haring (7) found for the relation between $\hat{Q}$ and A_c (for inlet portion only):

$$A_c = 1.117 \, \hat{Q}$$

This study gives (for profiles 1-11 only)

$$A_c = 1.08 \, \hat{Q}$$

with profiles 12 and 13 deviating from this relationship.

Tidal prism (Ω)

The correlation between tidal prism and cross-sectional area A_c is shown in Figure 6 for flood and ebb.

The solid lines represent the average regression for profiles 1-11. The difference betweeen the lines for Ω_{flood} and for Ω_{ebb} is very small.

Again the profiles 12 and 13 do not fit the average relationship for the profiles 1-11. The deviations are almost equal for flood and for ebb.

Stability shear stress (τ_s)

In order to satisfy the identity expressed in equation (9) a certain value for the stability shear stress must be selected. Again it was found that the behavior of profiles 12 and 13 differed from the other profiles (1-11). The values of the stability shear stress which had to be introduced to obtain a satisfactory identity are presented in table 1. Values are given for mean tide as well as for spring tide.
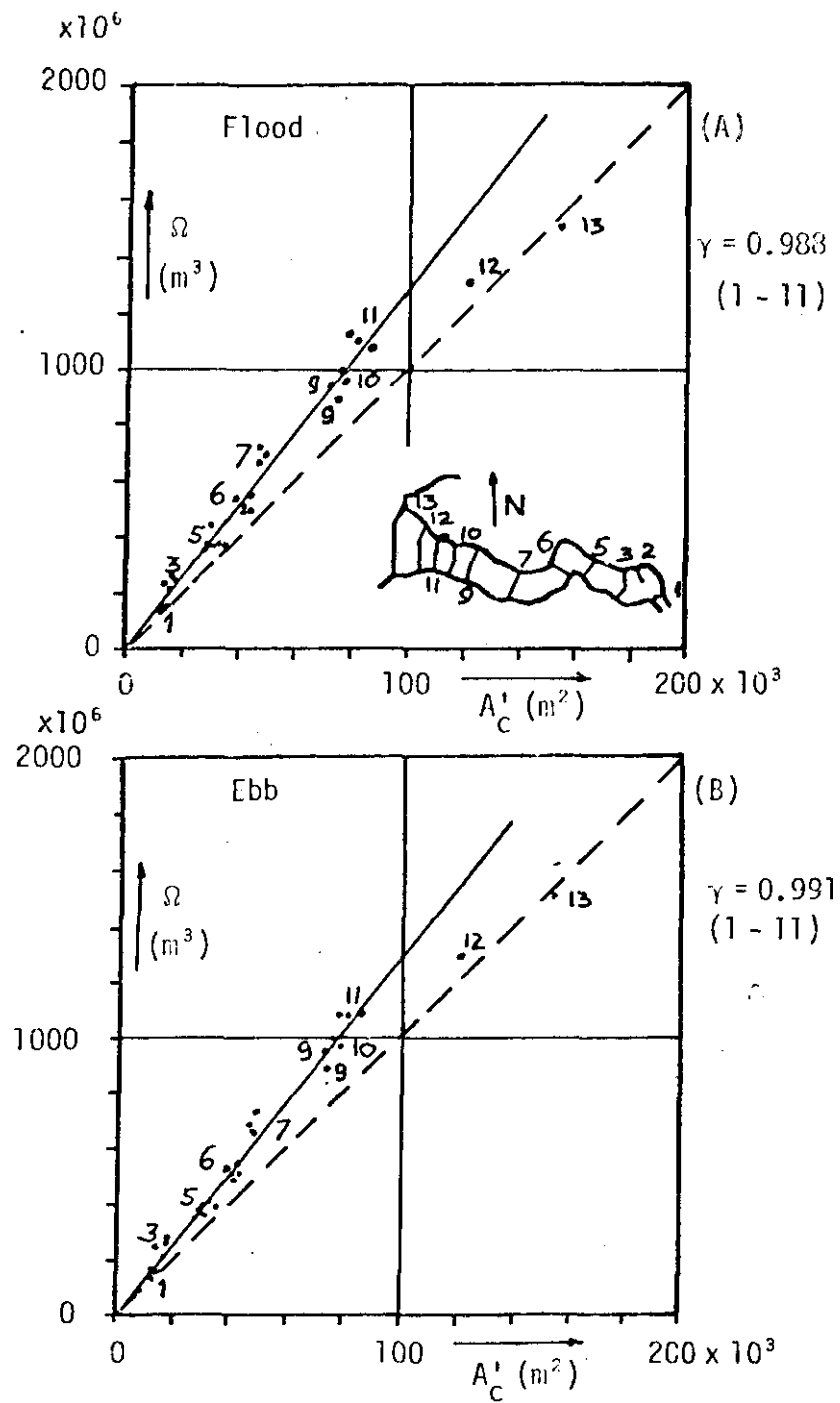
Figure 6: Regression Ω versus A_c
(A) Flood; (B) Ebb.

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Table 1. Values of stability shear stress, profiles 1-11 (N/m^2)

Tide	Average τ_s (1-11)	τ_s (ebb)		τ_s (flood) (1-11)
		(1-5)	(6-11)	
Mean tide	3.90	4.00	4.90	3.30
Spring tide	5.00	5.00	6.50	4.30

For the calculation of the required values of τ_s, values for the Chézy coefficient must be introduced. Based on earlier studies in the Western Scheldt (Beyl (1)), the following relationships were used to calculate C:

flood: $C = 56.9 \ v^{0.48} \quad m^{1/2}/sec.$

ebb: $C = 42.9 \ v^{0.28} \quad m^{1/2}/sec.$

The results of the regression analysis (based on A'_c) are shown in Figure 7, indicating good agreement for profiles 1-11. In section 3 it was proposed to adjust the stability shear stress in areas where currents and waves affect the sediment transport. A method to calculate a representative shear stress, based on the Bijker formula, was suggested.

Introducing the modified stability shear stress τ'_s as proposed in section 3, corrections for the profiles 12 and 13 can be made. These are shown as 12[a] and 13[a] in Figures 7A and B. Regarding Figure 7A which describes flood conditions, the corrections for profiles 12 and 13 to points 12[a] and 13[a] bring the latter very well in agreement with the other profiles. For the ebb, (Figure 7B) there is considerable improvement but the fitting of 12[a] and 13[a] into the general curve is not quite satisfactory. A possible reason for the difference between flood and ebb may lie in the way the Chézy coefficients were calculated. The C-values used in the calculations may deviate from the actual C-values for those profiles.

Overall, however, the method suggested seems to be promising to bring all data of the entire estuary into one regression relationship.

Flood and ebb channels

For various flow parameters correlations have been determined for a number of well defined flood and ebb channels.

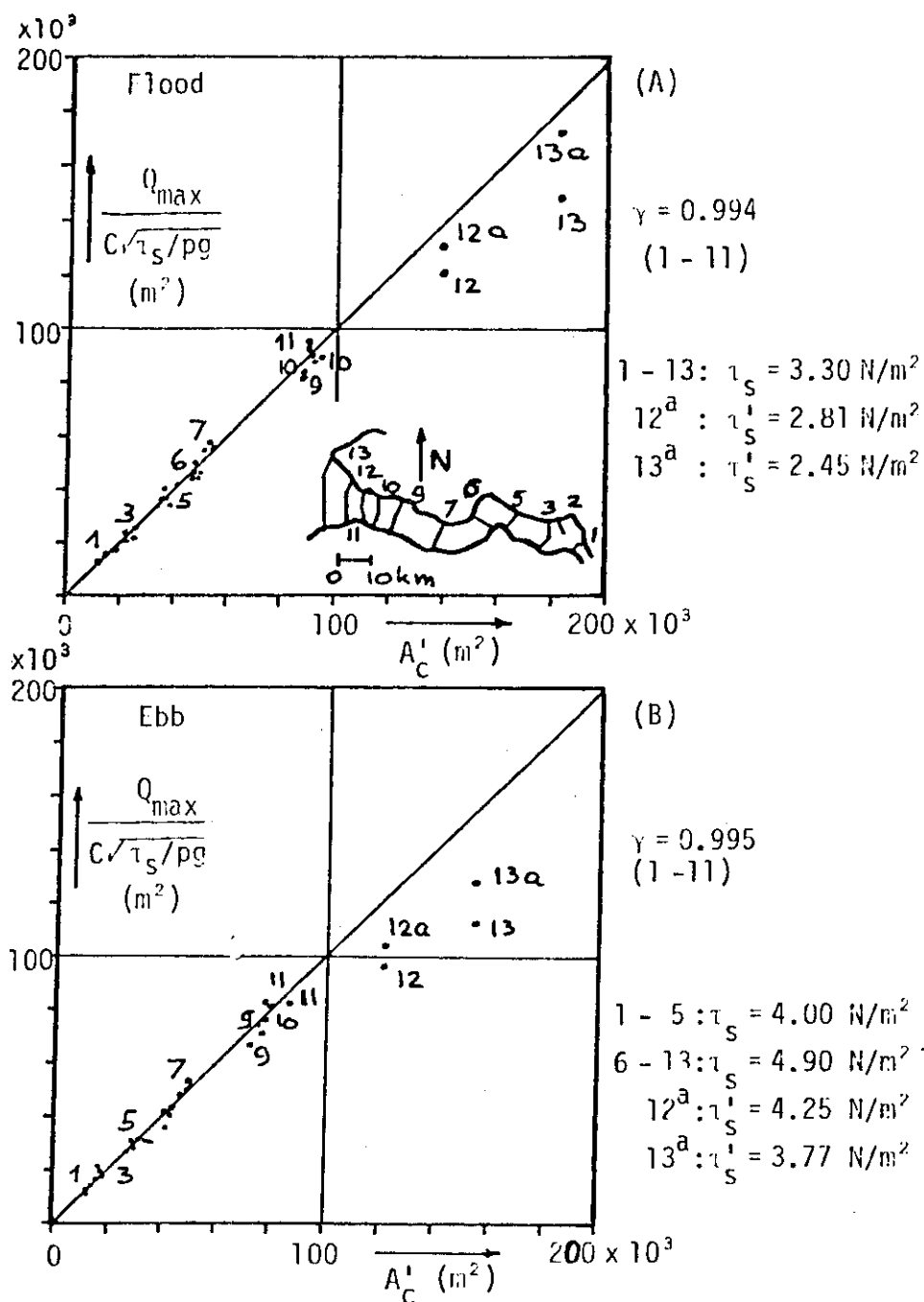In general correlation coefficients of the same order of magnitude were found.

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Figure 7: Regression $\dfrac{Q_{max}}{C\sqrt{\tau_s/\rho g}}$ versus A'_C

(A) Flood; (B) Ebb.

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5. CONCLUSIONS

The study of the stability relationships for the Western Scheldt Estuary has led to the following conclusions:

(1) The correlation coefficient (γ) for the regression between various flow parameters and the cross-sectional area, for profiles 1-11, decreases in the following order:

1. $\dfrac{Q_{max}}{C\sqrt{\tau_s/\rho g}}$ versus A'_c : $\quad \gamma_{flood} = 0.994$

   $\gamma_{ebb} = 0.995$

2. Ω versus A_c : $\quad \gamma_{flood} = 0.988$

   $\gamma_{ebb} = 0.991$

3. Q_{max} versus A_c : $\quad \gamma_{flood} = 0.978$

   $\gamma_{ebb} = 0.985$

(2) For the separate tidal channels the same sequence is found for the flood channels. For the ebb channels the sequence is 1-3-2.

(3) Profiles 12 and 13 generally deviate from the regression relationships established for profiles 1-11. Exceptions are the relationships Q_{max} versus A_c and Q_{max} versus A'_c for flood.

(4) Profiles 12 and 13 also fit in the relationship $\dfrac{Q_{max}}{C\sqrt{\tau_s/\rho g}}$ versus A'_c, for flood if τ_s is adjusted to τ'_s accounting for wave action.

(5) The values for τ_s found for the Western Scheldt corresponds very well with the values established by Bruun and Gerritsen (3).

(6) The established relationships have correlation coefficients between 0.95 and 0.99. It appears that these relationships may be utilized in one-dimensional tidal models for the calculation of changes in profiles as a result of changes in external boundary conditions such as in dredging operations.

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6. REFERENCES

1. Beyl, P.R. (1977): Determination of the accuracy of sandtransport calculations using the Bijker -Einstein method, in the Western Scheldt. Report as part of graduation requirements for the Degree of Civil Engineering, Delft University of Technology.

2. Bijker, E.W. (1967): Some considerations about scales for coastal models with movable bed. Doctoral Dissertation, Delft University of Technology.

3. Bruun, P. and F. Gerritsen, (1960): Stability of coastal inlets. North Holland Publishing Company, Amsterdam.

4. Bruun, P. (1978): Stability of Tidal inlets, theory and engineering. Elsevier Scientific Publishing Company, Amsterdam-Oxford-New York.

5. Daamen, J.W. (1983): Tidal regime of Western Scheldt. Report Adviesdienst Vlissingen, Rijkswaterstaat, WWKZ-83 V297.

6. Dronkers, J.J. (1964): Tidal computations North Holland Publishing Co. - Amsterdam.

7. Kreeke, J.v.d. and Jac. Haring (1979): Equilibrium flow areas in the Rhine-Meuse Delta. J. Coastal Engineering, 3, 97-111.

8. Leendertse, J.J. (1967): Aspects of a computational model for long-period water-wave propagation. Doctoral Dissertation, Delft University of Technology.

9. O'Brien, M.P. (1931): Estuary tidal prisms related to entrance areas, Civil Engineering, May, 1931.

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