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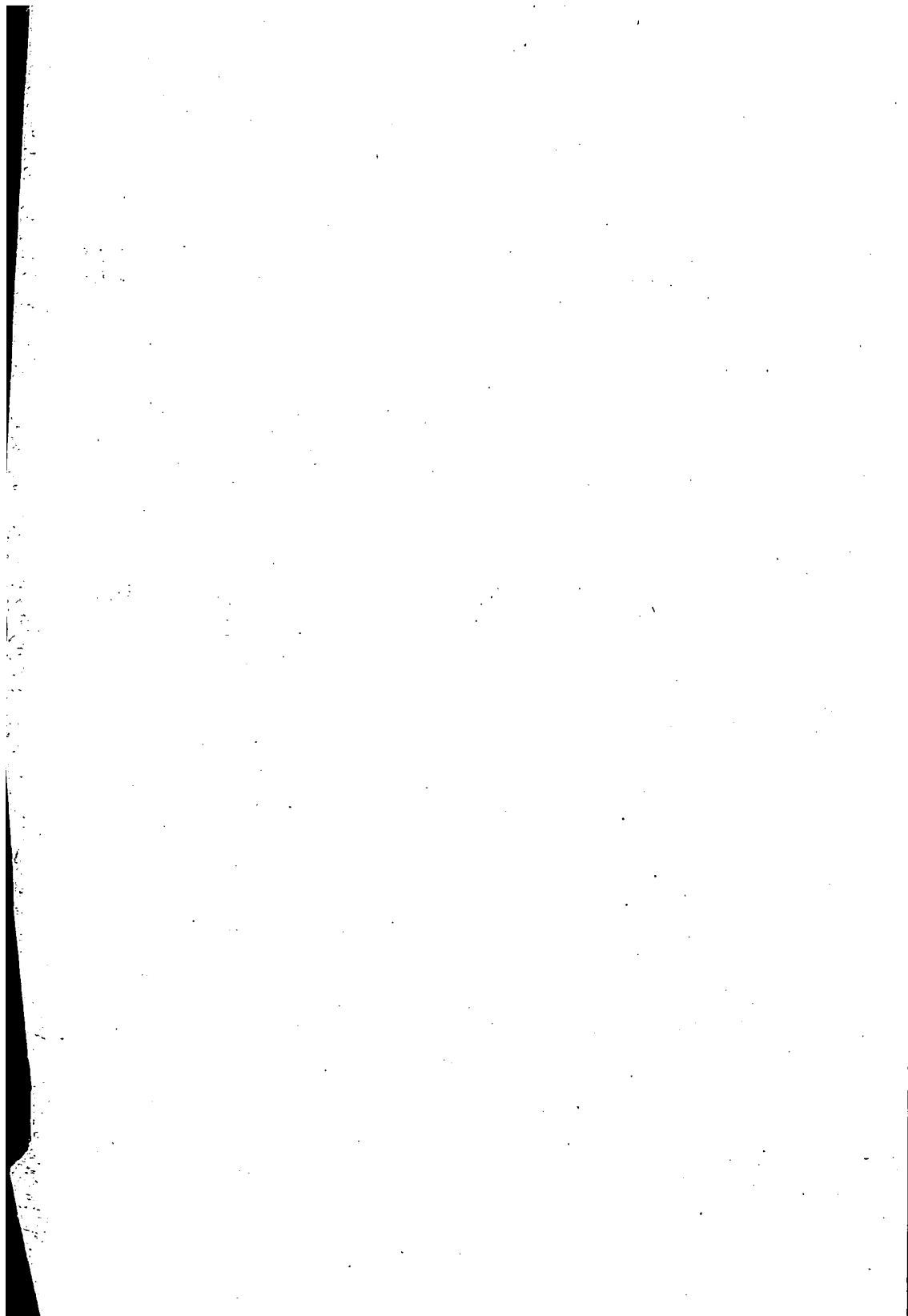
THE PRINCIPAL WORKS
OF
SIMON STEVIN

EDITED BY

ERNST CRONE, E. J. DIJKSTERHUIS, R. J. FORBES,
M. G. J. MINNAERT, A. PANNEKOEK

AMSTERDAM
C.V. SWETS & ZEITLINGER





III

THE PRINCIPAL WORKS
OF
SIMON STEVIN

VOLUME I

GENERAL INTRODUCTION
MECHANICS

EDITED BY

E. J. DIJKSTERHUIS

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1955

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SIMON STEVIN

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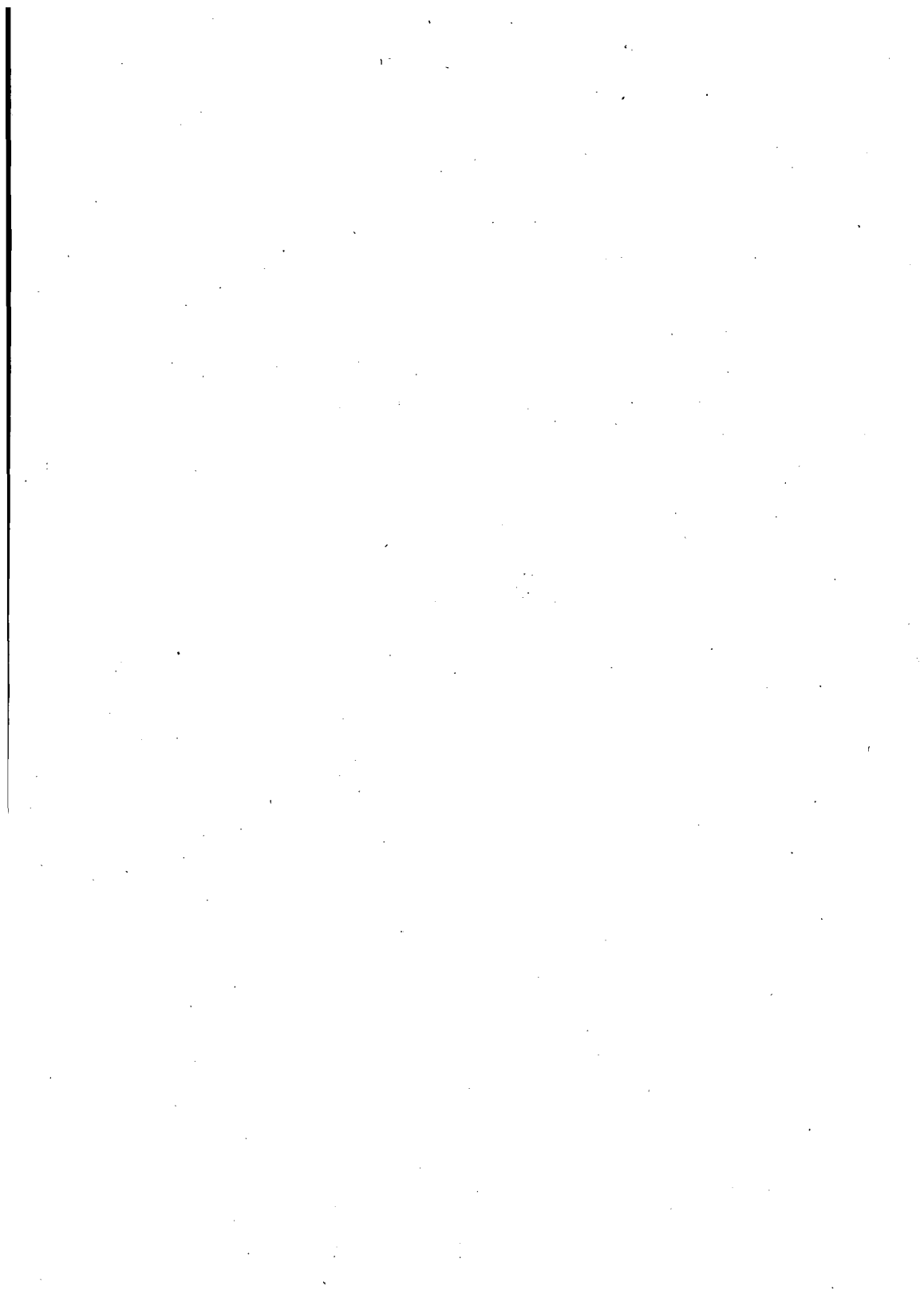
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GENERAL INTRODUCTION

THE LIFE AND WORKS
OF
SIMON STEVIN



1. INTRODUCTION

Modern science was born in the period beginning with Copernicus' *De Revolutionibus Orbium Coelestium* (1543) and ending with Newton's *Philosophiae Naturalis Principia Mathematica* (1687).

For reasons we shall not enter into here medieval Scholasticism had not succeeded in finding an effective method for the investigation of natural phenomena. Nor had Humanism been able to find the new paths science had to follow, though it was indirectly instrumental in promoting natural science by fostering the study of Greek originals on mathematics, mechanics and astronomy. The conviction shared by both movements that science was something mankind had once possessed but lost since, led to the conviction that it had to be rediscovered in ancient books, and so turned men's eyes toward the past instead of to the future, where it was to be found.

The creation of modern science required a different mental attitude. Men had to realize that, if science were to grow, each generation had to make its own contribution and that all the wisdom of Antiquity was useful only as a starting point for new research.

In the development of this view the universities, which had always been the bulwarks of medieval science, could play none but a minor part. Naturally inclined to conservatism, they on the whole exerted a retarding influence. For the greater part the revival of learning was the work of individual scholars who, in full possession of traditional science, took the initiative of transcending its boundaries and venturing into unexplored realms of scientific thought.

During the sixteenth century these pioneers of modern science are to be found all over Europe. The Italians Tartaglia, Cardano, Benedetti took the lead in the domains of mathematics and mechanics. A new era in astronomy was opened by Copernicus in Prussia and by Tycho Brahe in Denmark. In France the mathematician Vieta prepared the way for the great progress in algebra that was to be accomplished in the seventeenth century.

The work of these prominent scholars was supplemented by the activity of numerous craftsmen who, urged on by economic necessity, tried to put science to practical use. Some of them were well-known artists (like Leonardo da Vinci and Albrecht Dürer), who at the same time worked as engineers planning or constructing canals, locks, dikes and fortifications. For the greater part, however, their names do not survive; they were the numerous makers of clockworks and nautical or astronomical instruments, the cartographers and, somewhat later, the grinders of lenses and the makers of telescopes and microscopes.

Simon Stevin acquired his honourable position in the history of civilization by working both in theoretical science and in engineering. This combination of faculties was prophetic: modern science truly required the cooperation of theory and practice. It could only come into being by theoretical speculation on data furnished by experience. Matter, being obstinate and unwilling to yield its secrets to pure reasoning, can only be forced to disclose its properties if submitted to experimental

research. However, to perform experiments, technical skill in constructing and using instruments is wanted. On the other hand the accumulation of empirical data is not in itself sufficient. Only mathematical formulation of quantitative relations leads to theories, the consequences of which can be put to the test in newly devised experiments. Thus the evolution of science can only proceed by a constant interaction between theory and practice.

The role of the technician is by no means exhausted with his contributions to the experimental side of science. His help is needed again when the results achieved are to be applied for the benefit of humanity.

In later centuries the various departments of scientific work were as a rule separated, most scientists concentrating either on theoretical or on experimental research or on the application of science in technical inventions. In the age of pioneers, however, their concentration in one person was not yet uncommon. Stevin was an example of this, and he appears to have been fully aware of the significance the combination had for the growth of natural science.

We shall not endeavour to depict in this biography the intellectual atmosphere in which he accomplished his work as a scientist and an engineer; this will be done, as far as necessary, in the introductions to the works that will be published in this edition. A few words on the political background of his career in the Low Countries will be given in the description of his life, to which we will now pass on.

2. ORIGIN AND YOUTH

Stevin seldom published a work without mentioning on its title-page that he was a native of Bruges (Flanders) ¹⁾. In doing so he provided us with one of the very few facts on his origin which are beyond all doubt. Another datum which is at least fairly certain was his birth in 1548 ²⁾. We derive it from the legend of a portrait in oils which is the property of the Library of Leyden University. It is confirmed (at least not contradicted) by one of four documents ³⁾ which contain all the further information available on his birth and parentage. These are four deeds of the year 1577, in which his majority is declared and certain financial affairs are settled ⁴⁾. They reveal that he was the natural child of one Antheunis Stevin by Cathelyne van der Poort. It is rather perplexing that the only particulars which have become known about his parents refer to other natural children

¹⁾ As a rule he writes: Simon Stevin van Brugge; only in Work I: Bruggheleinck. The mention of his origin is lacking in Works VII and X, in the latter of which the name of the author is not given at all.

²⁾ In older biographies (Valerius Andreas 719; Sweertius 677; Foppens 1102) no year of birth is given. Voorduin 28, on the authority of the *Encyclopédie d'Yverdon*, has 1555.

³⁾ Schouteet 140-142.

⁴⁾ These deeds were discovered by Schouteet in the municipal archives of Bruges; they bear the date Oct. 30, 1577 and their contents, which Schouteet gives in full, may be summarized as follows: Simon Stevin, natural son of Antheunis Stevin by Cathelyne van der Poort, is relieved from guardianship and declared independent, so that he can use and administer his own goods. He further promises to indemnify certain persons for giving bail on his behalf to Jan de Brunc, a tax-official in the „Vrije van Brugge“, now that he is going to occupy a position in his office. In one of the documents his age is given as twenty-eight or thereabouts. This may tally with the year of his birth as given above, though it leaves room for 1549 also. The „Vrije van Brugge“ was a rural district surrounding the town; it was one of the four „members“ of Flanders whose representatives, together with those of the nobility and the clergy, constituted the States.

of both the one and the other ⁵⁾. There are reasons to suppose that young Stevin was reared by his mother. All the anecdotes told by biographers on the traits he displayed as a child, and on the scholarly education he received are the fruit of pure fancy.

In 1577 we find him occupying a position in the financial administration in the "Vrije van Brugge" ⁶⁾. From a casual remark in one of his books we gather, further, that earlier he had worked as a bookkeeper and cashier in Antwerp ⁷⁾.

Most of Stevin's biographers tell us a good deal about the motives which are supposed to have prompted him to leave Bruges some time after 1571 and to set out on extensive travels in Poland, Prussia and Norway in the period between 1571 and 1581 ⁸⁾. However, it has proved impossible to check any of these statements. In particular there is no ground for the assertion that his departure had anything to do with the religious persecution becoming more intensive under the Duke of Alva ⁹⁾. Unless new documents be discovered, we shall have to put up with the deplorable lack of facts on the first three decades of his life.

This situation, though far from satisfactory, is somewhat ameliorated from the moment of his settlement in the Northern Netherlands. It is certain that he established himself at Leyden in the year 1581 ¹⁰⁾, and that he was matriculated at the University on February 16, 1583, under the name of Simon Stevinus Brugen-sis ¹¹⁾.

3. IN THE NORTHERN NETHERLANDS

At that moment he had already written some of the long series of works he was to publish, some of which were to win him immortal fame. In 1582 the renowned printer and publisher Plantijn of Antwerp had published his *Tafelen van Interest* (*Tables of Interest*) (I), ¹²⁾, to be followed in the next year by a geometrical work, *Problemata Geometrica* (*Geometrical Problems*) (II), published by Joannes Bellerus, also of Antwerp. Henceforth all his works were to be published in the Northern Netherlands. In 1585 Plantijn, who in the meantime (1583) had transferred his business to Leyden, published his works *Dialektike ofte Bewysconst* (*Dialectics or the Art of Demonstration*) (III), *De Thiende* (*The Disme*) (IV) and *l'Arithmétique* (*Arithmetic*) (V). In the next year Plantijn's son-in-law Frans van Ravelingen continued the series with Stevin's most famous works: *De Beghin-*

⁵⁾ For further particulars cf. Dijksterhuis 3.

⁶⁾ See Note 4 and a remark made by Stevin himself in Work XI (v; 2; dedication to M. de Bethune 6).

⁷⁾ *ibidem*.

⁸⁾ It is certain that he visited these countries; in his works he refers to experiences gained in Poland, Prussia and Norway, and the elaborate plans for the improvement of the harbour works and waterways and for the drainage system of such Prussian towns as Danzig, Braunsberg and Elbing testify to his having been there.

⁹⁾ J. J. van Hercke; reprint 2-3.

¹⁰⁾ The municipal registers of Leyden for the year 1581 contain the enrolment of Symon Stephani van Brueg as a student having taken residence at Nicolaas Stochius' at the Pieterskerkgracht. Stochius was the head-master of the Latin school.

¹¹⁾ In the registers of the University his name is found up to 1590 with the addition *stud. art. apud Stochium*. It may be mentioned that his name is given by Andreas Valerius 719 as Simon Stevinus, sive Stephanus.

¹²⁾ The Roman numerals between parentheses refer to the list of Stevin's works given below.

selen der Weegbconst (*The Principles of the Art of Weighing*) (VI), *De Weeghdaet* (*The Practice of Weighing*) (VIa) and *De Beghinselen des Waterwichts* (*The Elements of Hydrostatics*) (VIb).

As apparent from this survey, the first years of Stevin's residence at Leyden must have been crowded with scientific work. This, however, did not prevent him from being active in technical science also. As early as 1584 we find him starting negotiations with the municipality of the town of Delft on the application of one of his inventions in the field of drainage. In the same year he is granted patents by the States General and the States of the province of Holland for various inventions, which are followed by several others during the ensuing years¹³). The majority of these patents refer to dredging and drainage; in particular he applies himself to improve the marshmill (i.e. a wind-driven scoop wheel used for pumping out water), a very important form of engine in a country the greater part of which had been literally wrested from the sea. There would be hardly any reason to mention, besides these highly practical inventions, his mechanically driven spit, which was no more than one of the countless mechanical toys the period revelled in, were it not that he marked this piece of work with the sign of the *clootcrans* (wreath of spheres), which was to become famous later on¹⁴).

In 1588 in order to apply his hydraulic inventions in practice he entered into partnership with his friend Johan Cornets de Groot¹⁵), the father of the boy who was to become the world-famous jurist Grotius. Together they built watermills in several places or improved existing ones by applying their new system.

It is characteristic of Stevin's wide range of interest that in this same year, in which all his energy seemed to be concentrated on technical problems, he published a book on quite a different subject: *Het Burgherlick Leven* (*Vita Politica*) (*The life of the burgher*) (VII).

Four years later, in 1594, the pamphlet *Appendice Algébrique* (VIII) proved that he still occupied himself with things mathematical. In the same year his *Sterckenbouwing* (*The building of fortresses*) (IX) saw the light, a work that ensured him a prominent position in the history of the art of fortification.

4. STEVIN'S IDEAS ON LANGUAGE

Though it is not uncommon to find mathematicians interested in linguistics, it is rather unusual to meet with one who as a scientist exerted a powerful influence on the language of his people and became no less famous in this respect than by his scientific achievements. Of this rare phenomenon Stevin is an example. Being strongly convinced that the Dutch language was singularly suitable for the rendering of scientific reasoning, and endowed with a peculiar gift of finding or coining words fitting this purpose, he became the founder of scientific and technical Dutch. Without being aware of the fact, everyone in the Netherlands

¹³) The texts of these patents are given by Doorman (1) 82, 86-88, 274; (2) 17-19.

¹⁴) See below: *Art of Weighing* Prop. 19.

¹⁵) Johan Hugo Cornets de Groot (Janus Grotius; March 8, 1554-May 3, 1640), burgo-master of Delft (1591-1595), was a close friend of Stevin, who speaks of him with great admiration and gratitude (Work V; Dedication). De Groot wrote in Work V a Latin poem and in Work VI a Latin and a Greek one. They collaborated in an experiment on falling bodies (Work VIIb 66).

daily uses terms and expressions which, if not introduced by Stevin, were at least brought into vogue by him.

It cannot be denied that in his digressions on the history and the qualities of the Dutch language he often exceeds the boundaries his intellectual soberness and scientific turn of mind should have imposed on him, and that his action for purity of language sometimes degenerates into fanatical purism. On the whole, however, his influence on the Dutch language must be considered beneficial.

His ideas on the superiority of Dutch as a scientific language are developed at length in the Memoir *Uytspraeck van de Weerdicheyt der Duytsche*¹⁶⁾ *Tael* (*Discourse on the Worth of the Dutch Language*), which serves as an introduction to the *Weeghconst* (VI). They will be summarized in the Introduction to this Memoir. It will then be seen that they are of biographical rather than scientific interest. One of them, however, deserves general attention. It is directed against the exclusive use of Latin for scientific purposes, entailing that those who in their youth lacked the opportunity of a scholarly education are for ever prevented from participating in scientific activity. In order to promote science, Stevin argues, all available forces should be released, and this is possible only if nobody is unnecessarily hampered by linguistic difficulties.

In using this argument Stevin associates himself with a number of similarly minded authors in various countries who during the sixteenth century were advocating the dethronement of Latin by the vernacular. So what at first view seems to be no more than a chauvinistic overestimation of Dutch, turns out to be a particular instance of a general plea for the good rights of the vernacular as the language of science, now applied to the special case of the Low Countries.

In accordance with his conviction of the superiority of his own language as a medium for scientific reasoning, Stevin after the publication of *l'Arithmétique* wrote all his works in Dutch. In doing so he deprived himself of the chance of being read in other countries. Indeed, his ideas and achievements were only made known there through Latin and French translations of some of his works¹⁷⁾. The same circumstance has made it necessary to add an English translation to each of the works published in this edition.

Stevin's extensive digressions in the field of language can only be fully understood if considered in connection with his phantastical theory of the *Wijssentijt* (Age of the Sages). Indulging in the old dream of mankind that in a remote past all things, which we now only know in a deficient and incomplete state, were in perfect order, he wanders away into an elaborate discussion of the means by which this primordial Golden Age might be brought back again. One of these means consists in a systematic cultivation of natural science, which requires the ordered collaboration of all persons able to do scientific work, regardless of their previous training and social status. However, this will be possible only if all scientific ideas and reasonings are expressed in the vernacular. And this will meet with greater success according as the vernacular itself proves more fit for the purpose. Here the matter of the superiority of Dutch turns up again, culminating in the argument that the language of the Sages in the *Wijssentijt* can have been none other than Dutch.

¹⁶⁾ In Stevin's time this was the name of the language in use in the Low Countries. It has to be translated by Dutch, not by German.

¹⁷⁾ Works XIa, XIb, XIII.

The whole theory forms a typical example of how the most rational and scientific of minds may at the same time foster the most irrational and phantastical ideas on topics lying outside the sphere of his specific competence.

5. THE POLITICAL BACKGROUND OF STEVIN'S LIFE ^{17a)}

In the period dealt with above Stevin must have come into contact with Maurice, Count of Nassau, later Prince of Orange and Stadtholder of the United Provinces ¹⁸⁾. As his activity now shifts from that of a private scholar and engineer to that of a person of importance in the young Republic, we must interrupt his life-history for a moment to give a short survey of the political background.

In the sixteenth century the so-called Low Countries, out of which in the long run the present states of Belgium and the Netherlands were to emerge, consisted of seventeen provinces, owing allegiance to the descendant of the House of Habsburg-Burgundy, Philip II, who was also king of Spain. In the sixties of the sixteenth century a movement of opposition to the absolutist, centralizing and alien tendencies of Philip's government sprang up, to which the activities of the small groups of Calvinists, scattered over all the provinces, soon imparted a revolutionary character. In 1567 William, called the Silent, Count of Nassau and Prince of Orange, had to emigrate with many others in view of the arrival of the Duke of Alva. In 1572 the latter's repressive policy led to a second revolutionary attempt. At first only the provinces of Holland and Zealand managed to free themselves under the leadership of the determined Calvinist minority and of the Prince of Orange. In 1576 the other provinces joined in (Pacification of Ghent), but soon afterwards a war of reconquest was undertaken by the Spaniards and conducted by the Duke of Parma. The Walloon (French-speaking) provinces made their peace with the King in 1579. Parma's success was greatly favoured by dissension between the Catholics, to whom the aims of the revolution were primarily political, and the Calvinists, but the determining factor in the campaigns, which swayed forwards and backwards for a number of years, was the strategic barrier of the rivers traversing the Netherlands from East to West. In 1581, when Flanders and Brabant were already gravely threatened, but still represented on the States-General, the latter solemnly deposed Philip II. However, owing allegiance to the throne of Habsburg-Burgundy, they did not yet venture to take the sovereignty into their own hands. The feeling prevailed that they could not do without foreign help. An attempt to enlist the help of France by investing the Duke of Anjou, brother to the French King, with the sovereignty, led to failure. A movement set up in Holland to invest William the Silent with the sovereign power and the title of Count, was frustrated by the murder of the Prince on June 10, 1584.

^{17a)} We are indebted for this paragraph to Prof. Dr P. Geyl, Professor of History at the University of Utrecht.

¹⁸⁾ The Stadtholder, originally the Central Ruler's representative in a province, since the rebellion was appointed by the States of the province. Nevertheless, the tradition of a sovereign position still clung to the office, and the Stadtholder had a say in the appointment of town magistrates. Maurice was born in 1567 and died in 1625. He was Stadtholder from 1585 to the year of his death. There is no ground for the wide-spread belief that he was a pupil of Stevin at Leyden University. Indeed, there is no evidence that Stevin ever taught at this University.

At this moment the whole territory of the States consisted of no more than the Provinces of Holland, Zealand, Utrecht, parts of Guelders, Overijsel and Friesland and a few towns in Brabant and Flanders. In 1579 the famous Union of Utrecht had been concluded, and all the provinces still holding out had acceded to it. That in the end seven provinces only should remain, was decided by the fortunes of war. In 1585 Antwerp, the largest and wealthiest city of all the Low Countries, always a bulwark of the rebellion, had to surrender to Parma. Realizing the danger that the Low Countries might be entirely subdued by Spain, Queen Elizabeth of England now declared herself willing to send an auxiliary force under the command of the Earl of Leicester. However, Leicester did not succeed in improving the situation. When he resigned in 1587, even the provinces of Holland and Zealand, the real stronghold of the Netherlands, were in danger.

In these provinces, however, the spirit of resistance remained unbroken, and especially the great statesman Oldenbarneveldt, the advocate of the States of Holland and in that capacity their virtual leader¹⁹⁾, did not waver one moment in his resolution to continue the struggle for independence. Full of confidence in the magic power the name of Orange had over the people, he made the States of Holland and Zealand invest William the Silent's son Maurice with the Stadtholderate of these provinces in 1585. In 1589 he persuaded the States of Utrecht, Guelders and Overijsel to follow this example and entrust the military power to the young prince.

The choice proved to be an excellent one. Together with his cousin, brother-in-law and intimate friend, William Louis of Nassau, who held the Stadtholderate of Friesland and Groningen, Maurice set about reorganizing the States Army and soon revealed himself as a military genius in using it as an instrument in the struggle for independence. Several successful sieges of towns occupied by the Spaniards, conducted as it were according to scientific methods unknown before that date, made his name as a commander famous all over Europe. He succeeded in liberating the whole territory of the remaining seven of the United Provinces. After ten years of hard fighting the "fence" of the Republic (to use a popular expression of the time) was closed and its domain was extended even beyond the boundaries. Thanks to his military achievements and the energetic politics pursued by Oldenbarneveldt the international position of the Dutch Republic underwent a radical change in the course of the same ten years. The States General, in 1585 still in quest of foreign help, in 1596 concluded an alliance with France and England, in which they were recognized on a footing of equality with these European powers.

6. STEVIN AND MAURICE

Since Stevin served as an engineer in the States Army and acted as a tutor to Prince Maurice, he doubtlessly played some part in this formidable change of things. Unfortunately we do not know exactly the nature, the extent and the

¹⁹⁾ The Advocate of Holland, later called the Grand Pensionary, was an official appointed on an instruction by the States of the Province. He may be described as the leading minister of the province; he presided over the meeting of its States and was a permanent member of its delegation in the States-General. At the same time he acted in effect as the foreign secretary of the Union.

relative importance of the role he played, but we may surmise that it was by no means negligible. Outwardly, it is true, his position always remained rather modest. Up to 1603 his title was no other than that of "engineer". It was only then that, upon the recommendation of Maurice, he was appointed Quartermaster of the States Army with the special commission of planning the army camps. There is no evidence that he ever held the position of Quartermaster-General, assigned to him by his son Hendrick on the title-page of an edition of posthumous papers, the *Materiae Politicae* or *Burgherlicke Stoffen* (XIV). In historical works on the military operations of Maurice his name is not mentioned, and only a few documents give any particulars of his activity²⁰).

Nevertheless he must doubtlessly have exercised a certain influence on the course of events in the United Provinces because of his intimate relation to the Prince as his tutor in mathematics and natural science and, later, as superintendent of his financial affairs. It was generally known that the Prince held him in great respect, and his reputation grew with Maurice's fame. He frequently sat on committees investigating matters of defence and navigation, and he was entrusted with the organization of a school of engineers to be incorporated into the University of Leyden²¹).

Whatever influence Stevin's cooperation with Maurice may have had on the latter's achievements, the effect of his activity as a tutor on his own scientific development is manifest enough. He had to compose textbooks on all the subjects the Prince wanted to study, and he was too original a thinker ever to confine himself to mere reproduction of what he found in existing works. He always managed to add some invention of his own or at least to improve the method of treatment.

After having compiled a considerable number of textbooks for the instruction of the Prince, Stevin took the initiative of publishing the whole corpus in a comprehensive edition. Thus between 1605 and 1608 the immense volume of his *Wiskonstighe Ghedachtenissen* (*Mathematical Memoirs*) was formed (XI), to be followed by a partial French translation, the *Mémoires Mathématiques* (XIa), and a complete Latin one, the *Hypomnemata Mathematica* (XIb).

The version presented here of the origin of these magnificent editions shows a marked deviation from the current story²²). According to this version the publication was entirely due to the initiative of Maurice, who, being accustomed to carry the manuscripts with him in his campaigns and afraid of losing them, decided to have them published. It is suggested that he paid for the publication, too. There is, however, no evidence to support the legend of this noble gesture. Stevin tells us, it is true, that he had sometimes seen the Prince anxious lest he should lose the manuscripts, but he leaves no doubt that he acted entirely on his own initiative when he undertook the publication²³). And Snellius, who made the Latin translation, explicitly states that the idea of this translation occurred to him spontaneously and that the publishing firm had to meet all the expenses²⁴).

It appears that Stevin's intimate relation to Maurice was not always regarded without some misgivings. To this Stevin himself alludes in a passage of the

²⁰) The documentary evidence for the above: Dijksterhuis 10-16.

²¹) Dijksterhuis 14.

²²) See e.g. Sarton 256. For further particulars: Dijksterhuis 330.

²³) Work XI. Preface.

²⁴) Work XIb. Dedication.

1 5 1 5
 Je Mamitrendray
 Mammuelle nasyon.

De gramme is in ofers
 wisthe deijnster
 Emv Stevin

Autographs of Count Maurice and Stevin.
 (From *Overzicht ener verzameling Alba Amicorum uit de XVIe en XVIIe eeuw* door
 Jhr F. A. Ridder van Rappard. Album B, p. 86).
 The text of Stevin's autograph means: A man in anger is no clever dissembler.

*Wisconstighe Ghedachtenissen*²⁵). Another hint is furnished by a letter written by the Dutch theologian Ubbo Emmius²⁶) after the publication of this work. In the treatise on Astronomy, which forms part of it, Stevin frankly and wholeheartedly adopted the Copernican doctrine of the mobility of the earth, which in the opinion of many scholars of the time had a taint of heterodoxy. Emmius qualifies the astronomical theories held by Stevin as worse than absurd and preposterous, and seriously regrets that the name of the Prince should be stained by this "dirt".

After accomplishing the *Wisconstighe Ghedachtenissen*, Stevin published only two more works, which appeared in one volume in 1617: *Castrametatio* (*Marking out of army camps*) and *Nieuwe Maniere van Sterctebou door Spilsluysen* (*A new manner of fortification with the help of pivoted locks*) (XII). In the dedication to the first-mentioned work he styles himself a Castrametator (Measurer of Camps). This has given rise to the opinion that he here refers to a new post, instituted on his behalf. However, this opinion is unfounded. There is no evidence that his official position had undergone any change since his appointment as quartermaster in 1604; the term of Castrametator is no more than a personal way of describing the special duties he had to perform.

It appears²⁷) that in the long run he felt dissatisfied by the lack of opportunity to show his capacities in a more important function than that of a Castrametator. His son Hendrick tells us that he petitioned the States General, advocating the institution of the office of Superintendent of the fortifications and recommending himself for this post. This petition, however, seems to have met with no more success than another, in which he asked for an increase of his salary as quartermaster; that, too, was refused by the States in 1620.

7. MARRIAGE, OFFSPRING AND DEATH

Returning now to Stevin's personal life, we have to relate some facts about his late marriage and his offspring. The data, however, are again disconcertingly scarce. It has been established²⁸) that in the second decade of the seventeenth century he married a young woman from Leyden, called Catherine Cray (day and year of birth unknown), that she bore him four children, and that he bought a house at The Hague. The dates of these events, however, do not, as given, tally with one another. The house was bought on March 24, 1612 (it is 47, Raamstraat, The Hague, which in 1897 was adorned with a bust of Stevin, made after the Leyden portrait²⁹); the eldest son, Frederick, was probably born in 1612, the second, Hendrick, probably in 1613, the eldest daughter, Susanna, on April 19, 1615 and the youngest, Levina, at some unknown date. Notification of the marriage, however, was not given until April 10, 1616. No date of the ceremony could be traced. If all these dates are correct (all of them have been derived from

²⁵) Work XI; i, 21; p. 40. There are now many, he tells us, who cannot believe that the scientific occupations of the Prince are not detrimental to many affairs which might otherwise have been done more efficiently.

²⁶) Ubbo Emmius (Dec. 5, 1547-Dec. 5, 1625) was afterwards the first Rector of Groningen University (founded in 1614).

²⁷) Dijksterhuis 16.

²⁸) Dijksterhuis 18-20.

²⁹) Dijksterhuis 30.

authentic documents), we must conclude that at least three out of the four children were born out of wedlock.

Stevin's marriage was not to last long, as he died in 1620. Again we know no particulars; we can only prove³⁰⁾ that he was still alive on February 20, and that his death occurred before April 18. No single further detail on the exact dates of the decease and the funeral is available. His widow remarried on March 14, 1621, her second husband being Maurice de Viry (or de Virieu), bailiff of Hazerswoude near Leyden. She died on January 5, 1672.

Out of the four children only the second son, Hendrick³¹⁾, is of interest to the reader of a biography of Stevin. He followed a career which outwardly resembled his father's, but he lacked the latter's genius. He studied mathematics at Leyden University. After having travelled through Europe, he became an engineer in the army and later held the post of quartermaster. A wound having obliged him to retire from military service, he married the widow of the lord of a manor at Alphen-on-Rhine (in Southern Holland)³²⁾ in 1642, and obtained the manorial title for himself after the death of his wife. He died without issue in January 1670.

It is greatly to Hendrick's credit that he considered it a debt of honour to his father's memory to edit the latter's posthumous papers, which had been very carelessly dealt with by the widow. Thanks to his pious care we possess the volume of the *Materiae Politicae*, in which some thirty years after Stevin's death several of his unpublished treatises saw the light, whilst others were published some eighteen years later in Hendrick's own work, *Wiskonstich Filosofisch Bedrijf* (*Mathematico-Philosophical Activity*)³³⁾.

8. STEVIN'S LEGACY

Stevin had to wait long for recognition of his real value to the history of civilization. In the Netherlands he was remembered for a long time almost exclusively as the tutor of Maurice and as the builder of two sailing-chariots, with which the Prince would occasionally amuse himself and his guests. Works on the history of mathematics and natural science, it is true, mentioned his name, but did not do full justice to his achievements. When on the occasion of the tercentenary of the year of his birth a movement was started in his native city to erect a monument to his memory (the statue which now actually adorns the "Simon Stevin-plaats" at Bruges), he became the subject of a heated controversy³⁴⁾, in which, through a curious twist of historic perspective, his loyalty to his native country was impugned (he was represented as a "Belgian", serving his country's enemy, Maurice). In answer to doubts which were expressed in the debate as to his merits as a scientist, one of his defenders collected all the testimonies on his achievements he had been able to lay hands upon in encyclopedias

³⁰⁾ Dijksterhuis 20.

³¹⁾ Dijksterhuis 23.

³²⁾ Not Alphen near Breda, as Sarton (1) 246 supposes.

³³⁾ This work would deserve a closer study. As Van Zutphen points out, Hendrick had remarkable ideas on the drainage of the Zuider Zee.

³⁴⁾ Dijksterhuis 26-29.

and historical works ³⁵). This miscellaneous collection appears to have been impressive enough to contemporaries. Nevertheless it now strikes us as singularly inadequate.

It was only in the first decades of the twentieth century that the study of Stevin as a scientist was undertaken in a thorough and systematic way, the leader of this movement being the meritorious Belgian historian of mathematics, Father Henry Bosmans S.J.

We conclude this short introductory sketch of Stevin's life with a discussion of the correct pronunciation of his name: Stévin or Stevín. In Holland people are generally inclined to stress the second syllable, sometimes even to pronounce it as a French ending. Yet there is no doubt that the correct pronunciation is Stévin: the Flemish have always pronounced it so, and they still do so to this day.

³⁵) van de Weyer (1).

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STEVIN'S ACHIEVEMENTS

The present edition does not attempt to embrace the totality of Stevin's writings. The bulk of the *Wisconstighe Ghedachtenissen* made it practically impossible to include this work as a whole. Moreover, several of his works are too much in the nature of a textbook to justify re-publication. Consequently the editors were obliged to make a selection and to publish only such works as contain mainly original contributions to science. This policy, however, entails that the image of Stevin evoked in the forthcoming volumes must needs be slightly distorted.

In order to remedy this unavoidable defect, we give as a prelude a concise survey of all Stevin's achievements, hoping thereby to convey a general impression of the wide range of his activities. We classify them under the following headings.

I. MATHEMATICS

A. *Arithmetic and Algebra.*

From the very beginning Stevin's desire is evidently to put science to practical use in daily life. Long before his time, tables of interest were used by bankers, but they were kept secret as tools of trade. In his *Tafelen van Interest* (I) Stevin now published a complete set of these tables and supplemented them with a textbook showing their application to problems of interest.

De Thiende (IV) contains a systematic treatise on decimal fractions and their application, with which this highly important improvement was formally introduced into arithmetic. Neither the fact that the idea of these fractions had been applied before him in goniometrical tables nor the circumstance that his so-called "thiendetalen" or "ghetalen van den tienden voortganck" (numbers of the tenth progress) were, properly speaking, no fractions at all, but integers introduced to avoid fractions, need prevent us from associating with the invention Stevin's name before all others. Undoubtedly the new technique was marred in the beginning by a cumbersome notation, which, however, was substantially improved in later works. In a series of examples the practical value of the "thiendetalen" for various categories of craftsmen is demonstrated. In an appendix Stevin advocates the introduction of the decimal principle in all human accounts and measurements, thereby anticipating the (partial) realization of this simple idea by two centuries.

The work *l'Arithmétique* (V) with its appendix, *La Pratique d'Arithmétique*, in which French translations of Works I and IV were incorporated, mainly deals with widely known subjects, leaving little room for originality. Stevin, however, succeeds in improving the symbolism in many respects and in contributing to the formulation of general rules for the solution of equations. The mainly practical character of the work does not prevent him from delving rather deeply into some highly theoretical topics and taking part in some fundamental

controversies concerning the principles of arithmetic. Algebra, which is still considered as one of the numerous rules taught in arithmetic, is advanced by the above-mentioned rules on equations, by the solution of the problem how to find the greatest common divisor of two polynomials and, in the *Appendice Algebrique* (VIII), by a method for the approximation of a numerical root of an equation of any degree. Moreover, the application of the "rule of algebra" is illustrated in a translation (or rather paraphrase) of the first four books of Diophantus, the first to appear in any European vernacular.

B. Geometry

The work *Problemata Geometrica* (II) deals with problems in pure mathematics, such as division of a polygon, construction of regular and semi-regular polyhedra, construction of solids to satisfy certain conditions. The subject is resumed in a less strict form with many practical applications in *De Meetdaet* (XI; ii) (forming part of the *Wisconstighe Ghedachtenissen*). In this work various geometrical instruments are described in detail. The construction of an ellipse by lengthening the ordinates of a circle in the same proportion, which is taught here, seems to be Stevin's invention.

C. Trigonometry

The *Wisconstighe Ghedachtenissen* contain a very elaborate systematic treatise on plane and spherical trigonometry under the title *Van den Driehouckbandel* (XI; i, 1). It is shown that the trigonometrical formulae relative to the right-angled spherical triangle can be reduced to six, and how the various practical problems may be solved by means of them.

D. Perspective

The desire, expressed by Maurice, to become familiar with perspective drawing induced Stevin to compose a textbook on the subject (*Van de Verschaeuwing* XI; iii, 1), which is remarkable for the care with which all the terms to be used are defined, for the writer's inventiveness in coining Dutch scientific words, and for some personal contributions (construction of a perspective drawing on a picture plane not perpendicular to the ground plane, and solution of the so-called inverse problem of Perspective: given an object and a perspective drawing of it, to determine the place of the observer's eye).

II. MECHANICS

With one single exception all Stevin's contributions to this branch of science, which are to be found in the *Weeghconst* (VI) and the *Weeghdaet* (VIa), refer to statics. In the first book of the *Weeghconst* Stevin continues the work of Archimedes in the latter's *De Planorum Aequilibriis* by giving a mathematical demonstration of the condition of equilibrium of a horizontal lever. In the same book he proves in a most ingenious and interesting way the law of equilibrium on an inclined plane, basing himself on the conviction of the impossibility of perpetual motion. From this theorem the rule for composition and decomposition of a force acting on a point is deduced, by which the study of the equilibrium

of a rigid body with one fixed point is made possible. It should be noted that Stevin for reasons of principle rejects the method of virtual displacements. The second book is devoted to the determination of centres of gravity. Here, too, Stevin applies the Archimedean method; he succeeds, however, in simplifying it in a way which is of some importance in the history of the Calculus. The *Weeghdaet* contains practical applications of the theorems of the *Weeghconst* in various instruments. In an appendix to the latter work we find Stevin's only contribution to dynamics: an experiment performed in collaboration with J. de Groot, in which falling bodies are proved to traverse the same distance in the same time regardless of their weight. A *Byvough der Weeghconst*, included in the *Wisconstighe Ghedachtenissen* (XI; iv, 7), brings new applications of theoretical statics in problems on cords and pulleys and in an investigation on the horsebit.

III. HYDROSTATICS

Here again Stevin acts as the immediate successor to Archimedes. He proves the latter's theorem on the force exerted by a fluid on a solid immersed in it in a more satisfactory way, and evaluates the forces which by its weight a liquid exercises on the bottom and the walls of the enclosing vessel. This leads him to the hydrostatic paradox, which is tested by experiment. The *Waterwicht* (VI b) with its appendix, the *Waterwichtdaet*, in which his hydrostatic theories are developed, must be considered as a valuable step towards the complete systematization of hydrostatics given by Pascal. The theory is applied in the problem of the diver and in a discussion on the stability of a floating vessel (*Van de vlietende Topswaerheyt*; XI; iv, 73).

IV. ASTRONOMY

In the section of the *Wisconstighe Ghedachtenissen* which bears the title *Van den Hemelloop* (XI; i, 3) Stevin first expounds the classical Ptolemaic theory of the structure of the universe and then shows how it can be transformed into the modern Copernican theory by a shift of the observer's standpoint. It should be borne in mind that his aim is no other than to bring home to his pupil as clearly as possible the two existing theories without pretending at all to enrich them with findings of his own. That he treats the Copernican system on a footing of equality with the Ptolemaic is in itself remarkable enough, when one considers that at the time the *Hemelloop* was composed this innovation in astronomy was still far from being generally accepted, that the authority of the greatest astronomer of the second half of the sixteenth century, Tycho Brahe, was against it, and that none of the leading scholars of the time had pronounced himself in favour of it. More remarkable still, Stevin not only explains the Copernican system, but also states his intimate conviction that this theory represents the real structure of the world, and endeavours to make this acceptable. In doing so, he shows his independence from the founder of the theory by rejecting the latter's hypothesis of a third motion of the earth besides the daily rotation about its axis and the annual revolution round the sun.

A second astronomical topic is dealt with in the *Eertclootschrift* under the title

Van de Spiegeling der Ebbenvloet (XI; i, 26). Here Stevin develops a theory of the tides, based on the assumption of an attraction exercised by the moon on the water, which for simplicity's sake is supposed to cover the whole of the earth's surface.

V. GEOGRAPHY

In the whole section *Eertclootschrift* of the *Wisconstighe Gbedachtenissen* there is only one subsection which can be brought under the heading of this paragraph, and this treatise, entitled *Vant stofroersel des Eertcloots* (XI; i, 22), deals neither with topography nor with cartography (as might have been expected), but with the gradual changes in the materials constituting the earth's surface. It discusses the accretion of land by sedimentation, the formation of dunes, the origin of mountains, and the gradual changes in the courses of sinuous rivers. It is supplemented by a number of hydrographical considerations in the memoir *Van de Waterschuyring* in Hendrick Stevin's *Wisconstich Filosofisch Bedryf* (XVIB 3).

VI. NAVIGATION

Stevin's nautical treatises refer to two different subjects. In the *Eertclootschrift* is found a treatise *Van de Zeylstreken* (XI; i, 24), in which the doctrine of courses and distances is taught. The only two methods of navigation susceptible of scientific treatment, viz. great circle sailing and loxodromic sailing, are explained both on mathematical lines and with the aid of a globe.

In *De Havenvinding* (X; abridged version XI; i, 25) Stevin discusses the possibility of making a landing on the basis of the known latitude and the magnetic declination of the harbour. The longitude can then be dispensed with.

VII. TECHNOLOGY

On Stevin's work as an engineer his own writings give none but the scantiest information, which is only partially supplemented by the study of various patents granted or applied for. The subjects of major interest are:

A. *Mills*, especially marshmills. He tried to make various improvements in the mechanism of these engines, and in his treatise *Van de Molens* (XV) corroborated them by a theoretical study based on his statical and hydrostatical theories. This seems to be the oldest scientific work on the subject, anticipating Smeaton's famous researches by some 150 years.

B. *Sluices and Locks*. This subject is treated in the first of the two chapters of the work *Nieuwe Maniere van Sterctebou, door Spilsluysen* (XIIB), which, as far as we know, constitutes the oldest extant printed treatise on sluices. The various types of sluices and possible improvements in their construction are discussed.

C. *Hydraulic Engineering*. In a posthumous paper on waterscouring, published by his son Hendrick (XVIB; 3), Stevin develops detailed plans for the im-

provement of the waterways of the town of Danzig and other towns in Prussia and the Netherlands.

D. *Sailing Chariots*. For years and years Stevin's fame has rested entirely on this invention, which plays, however, only a minor part among his technological inventions and is nowhere mentioned in his works. The reports of contemporaries are incomplete and bear obvious features of exaggeration and phantasy.

VIII. MILITARY SCIENCE

A. *Art of Fortification*. As in the history of mathematics, here again Stevin carries out the dual task of systematizing the existing knowledge and enriching it with personal contributions. Because of his work *Sterckenbouwing* (IX) his name ought to be associated before all others with the so-called old Dutch method of fortification, which is here explained systematically for the first time. The work *Nieuwe Maniere van Sterctebou, door Spilsluysen* (XIIB) deals with the use of sluices for defensive purposes and gives plans for the fortification of various towns in the Netherlands by these means.

B. *Castrametatio*. In the work of this title (XIIA) Stevin describes in detail the method of laying out camps and their internal organization, which was in use in the States Army. Several other writings on military science, destined for a *Crijchsconst*, were published in Work XIV.

XI. BOOK-KEEPING

Stevin's works on this subject (XI; v, 2; XIV B) can be brought under the two headings of mercantile and princely financial administration. In order to persuade Maurice to have his affairs as a prince and an army-commander organized by the Italian method of double-entry book-keeping, which had been in use in commerce for many years, he first explains this method in a textbook on commercial administration (XI; v, 21) and subsequently argues why it is desirable and in how far it is possible to apply it to the Prince's own affairs (XI; v, 22). He does this in the form of an extremely lively dialogue between Maurice and himself, which he reports to be a faithful record of a real conversation. Another system of administering the princely domains is explained in the Work *Verrechting van Domeine mette Contrerolle* (XIVB).

X. ARCHITECTURE

The work *Materiae Politicae* (XIV) contains a number of posthumous papers on town planning and house building, and a treatise on the aesthetic aspects of architecture.

XI. MUSIC

In Stevin's time the traditional connection between music and arithmetic, a heritage of the medieval quadrivium, still survived. It was by no means necessary

for a writer to be musical in the emotional sense of the word to feel interested in the theory of intervals. The important question of characterizing the various intervals by ratios in order to obtain a practicable temperament for keyed instruments is solved by Stevin in his *Spiegelung der Singconst* (XV). He boldly asserts that an octave consists of six equal intervals of a tone and twelve equal intervals of a semi-tone. In doing so, he anticipates the system of equal temperament, which was to be introduced about a century later.

XII. CIVIC MATTERS

The work *Het Burgerlick Leven* (VII) gives rules for the conduct of a citizen in cases where the divine and natural laws that determine his status are no longer mutually concordant or clash with his personal conviction. Moreover, it discusses the position of a prince (the highest citizen in the state) and sets the limits of his competence.

The work *Materiae Politicae* (XIV) contains an appendix to this treatise and develops plans for the organization of the various Councils which are to assist the prince of a great empire in his government.

This is but one of the numerous examples of Stevin's predilection for the organization of all kinds of military and civic matters.

XIII. LOGIC

This subject is dealt with in the booklet *Dialectike ofte Bewysconst* (III). Though it does not enrich this classical branch of science with any substantial innovation, it is remarkable for two reasons: it is one of the two oldest treatises on logic written in Dutch, and the method of exposition deviates greatly from that followed in the traditional textbooks. By both means Stevin hoped to make logic accessible to all readers, regardless of their previous training. The work is concluded by a *Tsamespraek*, a dialogue, in which one of the interlocutors defends Stevin's views on the superiority of the Dutch language.

CONCLUSION

The description of Stevin's life and the survey of his achievements given above can still be supplemented with some general considerations on his personality. In the history of civilization Stevin figures as the prototype of the engineer, of the perfect technologist, who deals with practical problems in a scientific way. Being well acquainted with the work already done by others, he freely applies their results wherever possible, but in fields where nobody preceded him he seeks and finds his own paths. His scientific turn of mind is strong enough to make him pay full attention to purely theoretical problems without feeling unduly anxious about their useful effect in practical life; but he is too practically minded to let himself be entirely absorbed by theory. Thus he continually oscillates between what he calls *spiegelung* (speculation, i.e. theoretical investigation) and *daet* (practical activity).

This two-sidedness of his natural bent does not prevent him from appreciating one-sidedness in one special case; *daet* without *spiegelung* is, he thinks, absolutely worthless, but he is ready to accept *spiegelung* without *daet*, if only other men's activities are promoted by it, as was the case with the work of great mathematicians like Euclid, Archimedes and Apollonius.

The combination of theoretical interest and sense of the practical, though complete in itself, may exhibit itself in a single domain only and thus leave a man one-sided in another sense. This, however, is not the case with Stevin. It may be true that his preference and natural ability draw him particularly towards things mathematical, but in each case presenting itself he does everything that has to be done with the same rational deliberation. His is the precious gift of always considering the subject actually dealt with as the most important in the world. The foregoing survey will have demonstrated clearly to the reader how numerous these subjects were.

Stevin's versatility indeed is astonishing as long as one looks at the range of his achievements, not if one considers their nature and pays attention to the method, the style of his thinking. Here the mathematical character predominates. During long periods of his life he did not occupy himself with mathematical problems, but there was not a single moment at which he ceased thinking like a mathematician. To define carefully all terms to be used; to pay the utmost attention to the choice of words; to enounce exactly all assumptions to be accepted without demonstration; after having done so, to take for granted all that has been logically derived from these principles, and nothing else; this in a nutshell is the style of thought he never abandoned.

It is a method which in some fields is indispensable and in others highly useful, but like every method, limited in its applicability. On reading Stevin's works, we realize that he does not always abstain from transcending its natural boundaries; his inability to see in religion anything else but a slyly contrived means for making men behave decently (VII) may serve as one example; his lack of appreciation of all architectural beauty that does not consist in mathematical symmetry (XIV, i) as an other. To him music appears to have been no more than that hidden problem in arithmetic Leibniz speaks of, in which the human soul—Heaven knows why — is said to take so intense a pleasure (XV). He determines the value of a language by statistical methods (VI). It would be interesting to know what his lost work on poetry (XIV D; ix) may have contained.

However this may be, the domain in which his mathematical way of tackling problems is correctly applied is wide enough for us not to over-emphasize the cases in which he failed to see its limitations. The more so, because it worked so admirably and always safeguarded him from its inherent natural dangers.

From the most serious of these dangers, dogmatism, to which his younger contemporary Descartes fell a victim in such a notorious way, Stevin was safeguarded by the equilibrium he purposely maintained between theory and practice. If an assertion proves to be at variance with the facts, it has to be rejected as deliberately as when it is contrary to reason.

Just as he does not value practical ability without a theoretical foundation, he has no use for experience which does not stand the test of rational reflection. Having demonstrated in the *Toomprang* (XI; iv, 74) that, contrary to an opinion current among horsemen, the curves in the cheeks of a bit cannot have any in-

fluence on its action but that of increasing its weight, he rejects in advance the argument that all horsemen, grooms and bridlemakers hold the contrary on the ground of experience only.

As a natural consequence of this attitude he repeatedly opposes the appeal to authority which, after having been of the utmost importance in the Middle Ages, was still widely current in his age. This does not, however, prevent him from appealing to authority himself in cases in which he was specially interested. This happens e.g. in the *Huysbou* (XIVC, i), where a whole chapter on the building of houses in Antiquity is inserted solely for the reason that the authority of the classics helps him to combat those who opposed his exclusive appreciation of symmetry.

But here indeed one of his most deeply rooted convictions is at stake. Though he is generally willing to recognize the good right of an opinion different from his own, on this point he is apt to grow intolerant. There are a few more of these issues: one should not try to maintain that irrational numbers are in the least absurd or inexplicable; or that it is possible to build a good fortress not all the parts of which can be exposed to flanking fire; nor that scientific views could be expressed in any foreign language as clearly and concisely as in Dutch.

Anyone carefully restraining his personal opinions on these special points was likely to encounter a very reasonable, reliable and benevolent man, ready to give everybody credit for his own merits and striving after the advancement of the public welfare rather than personal honours and privileges. He repeatedly voices his opinion that science should be cultivated for the sake of the commonwealth only, and more particularly so with a view to the speedy restoration of the *Wijsentijt* era. Accordingly, when he feels obliged to point out mistakes in the works of others, he does so exclusively because of the retardation of the return of this Golden Age caused by any imperfection in human science. On the same grounds he urgently requests the reader in several passages of his works not to spare him his criticisms and to correct him wherever this is possible.

The advancement of learning for the sake of the commonwealth being his highest aim, nothing is more alien to his habits than letting another man's intellectual property pass for his own. This does not mean that his references are always as complete as we should like them to be. Here, however, it should be remembered, firstly, that the custom of the age did not impose on a writer the stringent obligation to mention all his sources, and, secondly, that this obligation does not even now apply to authors of textbooks; and, as we have seen, Stevin's works were textbooks to a high degree. However this may be, our curiosity as to his sources is by no means always satisfied.

The unselfishness with which Stevin devotes his scientific activity to the service of the community, and his efforts to release all available spiritual forces for the same purpose, regardless of social class, are signs of a strong social conscience. It is in accordance with all we know about him that his opinions in this field are of an intellectual rather than an emotional character. The same applies to various passages in which he advocates the interests of the poor (XIVA; i and iii); what at first sight appears to be a symptom of humanity not infrequently turns out to be a mere piece of rather commonplace utilitarianism.

As pointed out before, there is one element in the charm exercised upon the reader of Stevin's works which cannot possibly be brought home to anyone who

has not mastered the Dutch language; it is the lucidity of his style and the peculiar flavour of the words of his own making in which he expresses his views. Like Galileo in Italy, he is one of the classical authors of Dutch national literature; as such he should be read in the schools, but this idea is still far from being realized in the Netherlands.

Here, however, we have to leave aside the national traits of his character and to consider him from an international point of view only. Undoubtedly he does not belong to the limited group of those great scientific geniuses who ring in a new era of human thought. But among the historical figures of the second rank his name as a mathematician and an engineer may be mentioned with honour, and his personality is always sure to arouse human interest in any reader making his acquaintance.

A BIBLIOGRAPHY OF STEVIN'S WORKS

The following bibliography contains the titles of all Stevin's works, with reprints and translations. For readability's sake no attempt at bibliographical correctness has been made. The reader interested in typographical details should consult the bibliography of Stevin's works in the *Bibliotheca Belgica*¹⁾, which also gives an account of pagination and foliation.

The Roman numerals preceding the titles are used throughout this edition in quotations of Stevin's works.

The numbers between [] refer to the libraries in the Netherlands and Belgium that possess the original works. Frequently occurring vignettes are indicated by capital letters. The meaning of these numbers and letters is explained in the footnote 2).

¹⁾ *Bibliotheca Belgica. Biographie générale des Pays-Bas par le bibliothécaire en chef et les conservateurs de la bibliothèque de l'université de Gand.*

Première Série. Tome XXIII. — Gand-La Haye 1880—1890.

Libraries:

- 1) 1. Library of the Royal Academy of Sciences, Amsterdam.
2. Royal Library, The Hague.
3. Library of the Netherlands Society of Sciences, Haarlem.
4. Library of the University, Amsterdam.
5. Library of the University, Groningen.
6. Library of the University, Leyden.
7. Library of the University, Utrecht.
8. Library of the Technical University, Delft.
9. Library of the Province of Zealand, Middelburg.
10. Library of the Military Academy, Breda.
11. Library of the War Ministry, The Hague.
12. Athenaeum Library, Deventer.
13. Municipal Library, Rotterdam.
14. Municipal Library, Arnhem.
15. Historical Museum of Navigation, Amsterdam.
16. Royal Museum for the History of Natural Science, Leyden.
- I. Royal Library of Belgium, Brussels.
- II. Library of the University, Ghent.
- III. Library of the University, Liège.
- IV. Municipal Library, Antwerp.
- V. Municipal Library, Bruges.
- VI. Library of the Museum Plantin-Moretus, Antwerp.

Vignettes

A. Hand with a pair of compasses.	Legend: Labore et Constantia.
B. Idem.	Ribbon with Legend: Labore et Constantia.
C. Idem.	The figure of a man with a spade on the left and that of a woman with a cross-staff on the right hold a ribbon with legend: Labore et Constantia.
D. Wreath of spheres.	Legend: Wonder en is gheen Wonder.
E. A pair of compasses	The Patroness of the Netherlands in an enclosure with four arms. Legend: Labore et Constantia.
F. Idem.	A laurel wreath. Legend: Labore et Constantia.

- I. *Tafelen van Interest, midtsgaders de constructie derselver, ghecalculeert door SIMON STEVIN Bruggelinck.* — T'Antwerpen. By Christoffel Plantijn in den gulden Passer. 1582. 92 pp. Vignette A.
[13.I, II, VI]
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- II. *Problematum geometricorum in gratiam D. Maximiliani, Domini Cruningen etc. editorum, Libri V. Auctore SIMONE STEVINIO Brugense.* — Antverpiae, Apud Ioannem Bellerum ad insigne Aquilae aureae. 118 pp. Vignette: Commerce in a vessel. Legend: In Dies Arte ac Fortuna.
[6, I, II, III, IV, V]
- III. *Dialectike ofte Bewysconst. Leerende van allen saecken recht ende constelic oordeelen; oock openende den wech tot de alderdiepste verborghentheden der Natueren. Beschreven int Neerduytsch door SIMON STEVIN van Brugghe.*
— Tot Leyden, By Christoffel Plantijn. 1585. 172 pp. Vignette C.
[3, 4, 5, 13, IV, VI]
Reprint: Rotterdam 1621. [2, 6, 13, I, II].
- IV. *De Thiende leerende door onghehoorde lichticheyt allen rekeningen onder den menschen noodich vallende afveerdighen door heele ghetalen sonder ghebrokenen. Beschreven door SIMON STEVIN van Brugghe.* — Tot Leyden, By Christoffel Plantijn. 36 pp. Vignette C.
[13, IV, VI]
Reprints:
Gouda 1626, as an appendix to Ezechiel de Dekker, *Eerste Deel van de Nieuwe Telkonst.*
Gouda 1630, as an appendix to Ezechiel de Dekker, *Nieuwe Rabattafels.*
Anvers—La Haye 1924. Facsimile. With an introduction by H. J. Bosmans.
French translations:
La disme in V, XIII. Facsimile-reprint in G. Sarton, *The first explanation of decimal fractions and measures* (1585). Together with a history of the decimal idea and a facsimile of Stevin's *Disme*.—*Isis* 65. Vol. 23, 1 (1935) Nr. 65 153-244.
English translations:
Robert Norton, *Disme, the Art of Tenths, or Decimall Arithmetike. Invented by Simon Stevin.* — London 1608.
Henry Lyte, *The Art of tens, or decimall arithmeticke.* — London 1619.
Vera Sanford, *The Disme of Simon Stevin* — *The Mathematics Teacher* 14 (1921) 321—333.
- V. *L'Arithmetique de SIMON STEVIN de Bruges: Contenant les computations des nombres arithmetiques ou vulgaires: Aussi l'Algebre, avec les equations de cinq quantitez. Ensemble les quatre premiers livres d'Algebre de DIOPHANTE d'Alexandrie, maintenant premierement traduits en François.*

Encore un livre particulier de la Pratique d'Arithmetique, Contenant entre autres, Les Tables d'Interest, La Disme; Et un traité des Incommensurables grandeurs: Avec l'Explication du Dixiesme Livre d'EUCLIDE.

— A Leyde, De l'Imprimerie de Christophe Plantin. 1585. 642 + 203 pp. Vignette B.

[3, 5, 6, I, III, V, VI]

Reprint, augmented and corrected by Albert Girard. Leiden 1625. [9, I, IV, VI]. This edition contains also Work VIII and a translation of the books V and VI of Diophantus by Albert Girard. Reprint of this edition in XIII.

- VI. *De Beghinselen der Weeghconst beschreven duer SIMON STEVIN van Brugghe.* — Tot Leyden, In de Druckerye van Christoffel Plantijn. By François van Raphelingen. 1586. 34 + 95 pp. Vignette D.

[1, 2, 3, 4, 6, 7, 9, 10, 15, I, III, IV, V, VI]

- VIa. *De Weeghdaet beschreven duer SIMON STEVIN van Brugghe.* Title-page as in VI.

- VIIb. *De Beghinselen des Waterwichts beschreven duer SIMON STEVIN van Brugghe.* Title-page as in VI.

The works VI, VIa and VIIb, which are always found bound together, are reprinted in XI. A Latin translation is contained in XIIb, a French one in XIII.

Partial English translation of VIIb by A. Barry in J. H. B. and A. G. H. Spiers, *The Physical Treatises of Pascal.* — New York 1937. 133—158.

- VII. *Vita Politica, Het Burgherlick Leven, beschreven duer SIMON STEVIN* — Tot Leyden, By François van Ravelenghien. 1590. 56 pp. Vignette A.

[5, 6, I, II]

Reprints:

Delft 1611 [1, 2, 6, I, II, IV, VI]; Amsterdam 1646 [1, 4, I]; In XIV, with Appendix; Haarlem 1649 [1]; Middelburg 1658; Harlingen 1668 [V]; Amsterdam 1684 [2].

Amsterdam 1939 (with an introduction by A. Romein-Verschoor and G. S. Overdiep).

- VIII. *Appendice Algebrique, de SIMON STEVIN de Bruges, contenant regle generale de toutes equations.* 1594. (Leiden, Frans van Ravelingen). 6 pp.

The only extant copy of this booklet, which was the property of the Library of the University at Louvain, was lost when the library was burnt in 1914. The contents appear as a corollary to Prop. LXXVII of *L'Arithmétique* in the edition of 1625 and its reprint.

- IX. *De Stercktenbouwing, beschreven door SIMON STEVIN van Brugge.* — Tot Leyden, By François van Ravelenghien. 1594. 91 pp. Vignette C.

[2, 6, I, VI]

Reprint: Amsterdam 1624 [6, 10, I]

German translation:

Festung-Bawung. Das ist, kurtze und eygentliche Beschreibung, wie man Festungen bawen, und sich wider allen gewaltsamen Anlauff der Feinde zu Kriegszeiten auffhalten sichern und verwahren möge: Auff jetziger Zeit Zustand und Gelegenheit gerichtet, und auss Niederländischer Verzeichnusz SIMONIS STEVINI Brugensis, Unserm geliebten Vatterland Teutscher Nation zu besondern Nutzen in hochteutscher Sprach beschrieben durch Gothardum Arthus von Dantzig. — Getruckt zu Frankfort am Mayn, durch Wolffgang Richtern, In Verlegung Levini Hulsii Wittib. 1608. 8 + 132 pp. [2, II].

Reprint of this translation: Frankfort am Main 1623.

French translation in XIII.

- X. *De Havenvindung.* — Tot Leyden, In de druckerye van Plantijn, By Christoffel van Ravelenghien, Gesworen drucker der Universiteyt tot Leyden. 1599. Vignette E.

[2, 4]

Reprint: a shortened version in XI; i, 25

Latin translations:

LIMENEVPETIKH, *sive, Portuum investigandorum ratio. Metaphraste* Hug. Grotio Batavo. — Ex Officina Plantiniana. Apud Christophorum Raphelengium, Academiae Lugduno-Batavae Typographum. 1599.

21 pp. Vignette F. [2, 6]

Limenheuretica; in XIIb, translation of the version XI; i, 25 likewise by Grotius.

French translation:

Le Trouve-Port; in XIII.

English translation by Edward Wright: *The Haven-finding Art.* — London 1599. Inserted in the translator's work: *Errors in navigation detected.* — London 1657.

Partial reprint of X in *Rara Magnetica. Neudrucke von Schriften und Karten über Meteorologie und Erdmagnetismus.* No. 10. — Berlin 1898.

- XI. *Wisconstighe Ghedachtenissen, inhoudende t'ghene daer hem in gheoeffent heeft den Doorluchtichsten Hoochgebornen Vorst ende Heere, Maurits, Prince van Oraengien, Grave van Nassau, Catzenellenbogen, Vianden, Moers &c. Marckgraef Van der Vere, ende Vlissinghen, &c. Heere der Stadt Grave ende S'landts van Cuyc, St. Vyt, Daesburgh &c. Gouverneur van Gelderland, Hollant, Zeelant, Westvrieslant, Zutphen, Utrecht, Overysseel &c. Opperste Veltheer vande vereenichde Nederlanden, Admiraal generael van der Zee &c. Beschreven duer SIMON STEVIN van Brugghe.*

Tot Leyden, In de Druckerye van Jan Bouwensz. Int Jaer 1608. Vignette D.

[1, 2, 3, 4, 6, 7, 8, 9, 11, 15, I, II, V, VI]

In folio with a very complicated division and pagination.

The principal division is into five parts:

- i *Vant Weereltschrift*
- ii *Van de Meetdaet*
- iii *Van de Deursichtighe*
- iv *Van de Weeghconst*
- v *Van de Ghemengde Stoffen*

Part i. *Vant Weereltschrift*

- i, 1 *Van den Driebouckhandel.*
 - i, 11 *Vant maecksel der tafels der Houckmaten.*
 - i, 12 *Van de platte drieboucken.*
 - i, 13 *Van de clootsche drieboucken.*
 - i, 14 *Van de hemelclootsche werckstucken duer rekeninghen der clootsche drieboucken ghewrocht.*
- i, 2 *Vant Eertclootschrift.*
 - i, 21 *Van syn bepalinghen int ghemeen.*
 - i, 22 *Vant stofroersel des Eertcloots.*
 - i, 23 *Van de Eertclootsche Damphooghde.*
 - i, 24 *Van de Zeylstreken.*
 - i, 25 *Van de Havenvinding.*
 - i, 26 *Van de Spiegeling der Ebbenvloet.*

The treatises i, 24—26 constitute the *Zeeschrift*.
- i, 3 *Van den Hemelloop.*
 - i, 31 *Van de vinding der Dwaelderloopen en der vaste sterren deur ervaringsdachtafels met stelling eens vasten Eertcloots.*
 - i, 32 *Van de Dwaelderloop deur wisconstighe wercking ghegront op de oneyghen stelling eens vasten Eertcloots.*
 - i, 33 *Van de vinding der Dwaelderloopen deur wisconstighe wercking ghegront op de wesentlicke stelling des roerenden Eertcloots.*

Part ii. *Van de Meetdaet*

- ii, 1 *Van het teyckenen der grootheden.*
- ii, 2 *Van het meten der grootheden.*
- ii, 3 *Van de vier afcomsten, als vergaring, aftrecking, menichvuldiging en deeling der grootheden.*
- ii, 4 *Van de everedenheysregbel der grootheden.*
- ii, 5 *Van de everedelicke snyding der grootheden.*
- ii, 6 *Van 'tverkeeren der grootheden in ander formen.*

Part iii. *Van de Deursichtighe.*

- iii, 1 *Van de Verschaeuwing.*
- iii, 2 *Van de beghinselen der Spiegelschaeuwen.*
- iii, 3 *Van de Wanschaeuwing.* (lacking)

Part iv. *Van de Weeghconst.*

- iv, 1 *Van de beghinselen der Weeghconst.*
- iv, 2 *Van de vinding der swaerheysmiddelpunten.*
- iv, 3 *Van de Weeghdaet.*

- iv, 4 *Van de beginselen des Waterwichts.*
- iv, 5 *Van den anvang der Waterwichtdaet.*
- iv, 6 *Anhang der Weeghconst.*
- iv, 7 *Byvough der Weeghconst.*
- iv, 71 *Van het Tauwicht.*
- iv, 72 *Vant Catrolwicht.*
- iv, 73 *Van de Vlietende Topswaerheyt.*
- iv, 74 *Van de Toomprang.*
- iv, 75 *Van de Waterrecking.* (lacking)
- iv, 76 *Vant Lochtwicht.* (lacking)

Part v. *Van de Ghemengde Stoffen.*

- v, 1 *Van de Telconstighe Anteyckeningen.*
- v, 2 *Van de Vorstelicke Bouckhouding in Domeine en Finance Extraordinaire.*
- v, 21 *Coopmans Bouckhouding op de Italiaensche Wijse.*
- v, 22 *Vorstelicke Bouckhouding op de Italiaensche Wijse.*
- v, 221 *Bouckhouding in Domeine op de Italiaensche Wijse.*
- v, 222 *Bouckhouding in Vorstelicke Dispense op de Italiaensche Wijse.*
- v, 223 *Bouckhouding in Finance Extraordinaire op de Italiaensche Wijse.*
- v, 3 *Van de Spiegbeling der Singconst.* (lacking)
- v, 4 *Van den Huysbou.* (lacking)
- v, 5 *Van den Crijchsbandel.* (lacking)
- v, 6 *Van verscheyden Anteyckeningen.* (lacking)

XIa. *Memoires Mathematiques, contenant ce en quoy s'est exercé..... Maurice, Prince d'Orange..... décrit premierement en Bas Alleman par SIMON STEVIN de Bruges, translaté en François par Jean Tuning, Licentié és Loix, & Secrétaire de Monseigneur le Prince Henry, Comte de Nassau &c.* — A Leyde, Chez Jan Paedts Jacobsz. Marchand Libraire, & Maistre Imprimeur de l'Université de la dite Ville. L'An 1608. Vignette: An angel with a book and a scythe.

[6, I]

French translation of XI, except the works i, 2; i, 3; ii, 5; ii, 6; iv.

XIb. *Hypomnemata Mathematica, hoc est eruditus ille pulvis, in quo se exercuit..... Mauritius Princeps Auraisus..... SIMONE STEVINO conscripta & è Belgico in Latinum à Wil.Sn. conversa.* — Lugduni Batavorum, Ex Officina Ioannis Patii, Academiae Typographi. Anno 1608. Vignette D.

[4, 5, 6, I, II, VI]

Complete Latin translation of XI by Willebrord Snellius.

- XII. A. *Castrametatio, Dat is legermeting, Beschreven door SIMON STEVIN van Brugghe. Na d'oordening en 't ghebruyc van..... Maurits, Prince van Oraengien.....* — Tot Rotterdam, By Jan van Waesberghe, in de Fame. Anno 1617. Legend: Literae immortalitatem pariunt. Vignette: Fame. 4 + 55 pp.

- B. *Nieuwe Maniere van Sterctebou, door Spilsluysen. Beschreven door SIMON STEVIN van Brugghe.* 4 + 59 + 2 pp.

Title-page as above.

The two works are always found bound together.

[1, 2, 4, 6, 7, 9, 10, 11, 13, I, II, IV, V]

Reprint Leiden 1633. [4, 6, 9, V]

French translations:

La Castramétation. Nouvelle Maniere de Fortification par Escluses.

Leiden 1618 [2, 4, 5, 6, 7, I, IV, V]

Idem. Rotterdam 1618. [2, 4, 10, 13, III]

Reprint of this translation in XIII.

German translations:

Castrametatio Aurica-Nassovica, das ist: Gründlicher und auszfürlicher Bericht, welcher Gestalt ein vollkommenes Feldtläger abzumesen..... seye: Erslich in Niderländischer Sprach beschrieben durch Simonem Stevinum: Anjetzo aber durch einen Liebhaber ins Hoch Teutsch übersetzt. Franckfurt, Frid. Hulsii. 1631.

Wasser-Baw, das ist Eygentlicher und vollkommener Bericht von Befestigung der Stätte durch Spindel-Schleussen. — Frankfurt, in Verlegung Friderici Hulsii. Im Jahr 1631.

[2, I]

POSTHUMOUS EDITIONS

- XIII. *Les Oeuvres Mathematiques de SIMON STEVIN de Bruges. Ou sont insérées les Memoires Mathematiques Esquelles s'est exercé le Tres-Haut & Tres-illustre Prince Maurice de Nassau, Prince d'Aurenge, Gouverneur des Provinces des Pais-bas unis, General par Mer & par Terre, &c. Le tout reveu, corrigé, & augmenté par Albert Girard Samiellois, Mathe-maticien.*

— A Leyde Chez Bonaventure & Abraham Elzevier, Imprimeurs ordinaires de l'Université, Anno 1634. Vignette: Le solitaire. Legend: Non Solus.

[2, 3, 4, 5, 6, 7, 8, 9, 11, I, II, III, V]

The work contains six parts.

- i. *l'Arithmétique* (reprint of the edition of 1625).
- ii. *Cosmographie*.
- ii, 1 *Doctrine des Triangles* (translation of XI; i, 1 with corrections and additions by A. Girard; the tables have been omitted).
- ii, 2 *Geographie* (translation of XI; i, 2).
- ii, 3 *Astronomie* (translation of XI; i, 3).
- iii. *La Practique de Geometrie* (translation of XI; ii).

- iv. *L'Art Pondéraire ou La Statique* (translation of XI, iv).
- v. *L'Optique* (translation of XI; iii).
- vi, 1 *La Castramétation* (reprint of the translation of XIIA, 1618).
- vi, 2 *La Fortification par Escluses* (idem of XIIB).
- vi, 3 *La Fortification* (translation of IX by A. Girard).

XIV A. *Materiae Politicae. Burgherlicke Stoffen. Vervanghende Ghedachtenissen der Oeffeninghen des Doorluchtichsten Hoogstgebooren Heere Maurits by Gods Genade Prince van Oraengie &c. Ho:LO: Ghedachtenisse. Beschreven deur zal. SIMON STEVIN van Brugghe, desselfs Heeren Princen Superintendent van de Finance &c. En uyt zijn naghelaten Hantschriften bij een ghestelt deur Sijn Soon HENDRICK STEVIN Ambachtsbeere van Alphen.*

Tot Leyden, Ter Druckerye van Iustus Livius, tegen over d'Academie. Vignette D.

B. *Verrechting van Domeine mette Contrerolle en ander behouften van dien. 't Welck is Verclaring van ghemeene Regel, waer deur verhoet worden alle abuysen mette swarichheden uytte selve spruytende, die men tot noch toe uyt geen Rekencamers van Domeine en Finance heeft connen weren. Wesende Oeffeningen &c. as above. — Tot Leyden, Ter Druckerye van Iustus Livius, In 't tweede Iaer des Vredes. Vignette D.*

[1, 2, 4, 6, 7, 8, I, II]

Reprint 1660, preceded by
Loochening van een Ewich Roersel, gesecht Perpetuum Mobile, by
HENDRIK STEVIN.

A contains eight memoirs on administrative and military matters.

- i *Van de oirdening der steden. Van de oirdening der deelen eens huys met 't gheene daer ancleeft.*
- ii *Het Burgherlick Leven, vermeerderd met een Anhang van de Regiering des Vorsten, tegen Machiavel. Mitsgaders des Keyzers Octaviaens gevoelen en ander getuygenissen angaende Phalaris. (Augmented reprint of VII).*
- iii *Van der Raden oirden.*
- iv *Van de amptienkiesing en ghemeene anclevinghen der ampten.*
- v *Ghemeene Regel op Gesaniterie.*
- vi *Van de Verdrucking.*
- vii *Van de geduerige verlegghing des Crijchsvolcx.*
- viii *Van de Crijchspiegeling.*

In B parts of XI; v have been reprinted.

Reprint of XIV: 's-Gravenhage 1686.

C. In some copies of XIV one finds a list of titles of treatises, which were destined for XI, but were not inserted in this work:

- i *Van de Crijchconst.*

- 1 *Van de Crijch te Lande.*
- 2 *Van de Crijch te Water.*
Parts of this work were inserted in XIVA; iii, iv, viii and in XIIA.
- ii *Van den Huysbau... waerby noch gevoucht is Weechdadelicken Handel van Cammen en Staven in Watermolens en Cleytrecking.*
Parts of this work were inserted in XIVA and XVIB.
- iii *Spiegeling der Singconst. Byvough der Singconst.*
Published in XV.
- iv *Van de tweede oneventheyt na myn gevoelen.*
Supplement to XI; i, 3.
- v *Van de metael-prouf.*
- vi *Van ettelicke wisconstige voorstellen en aenteyckeningen.*
- vii *Nederduytsche Dialectica dats Bewysconst, anders geseyt Redenstryt.*
New Version of III.
- viii *Nederduytsche Retorica dats Redenconst, anders geseyt Welsprekenheyt.*
 - 1 *Van de eygenheyt des spraecx.*
 - 2 *Van duysterheyt en claerheyt.*
 - 3 *Van d'oirden des uytspiraecx.*
 - 4 *Vant cieraet.*
 - 5 *Vant wesen.*
- ix *Nederduytsche Dichtconst ghegront op de Françoysche Dichtconst, die daerom eerst beschreven wort. En hier is by gevoucht een verhael van Letterconstige geschillen.*
 - 1 *Van de spelling.*
 - 2 *Vant geslacht der namen.*
 - 3 *Op seker E, EN en DER.*
 - 4 *Van de buyging en vervouging.*

- XV. „*Van de Spiegeling der Singconst*” et „*Van de Molens*”. Deux traités inédits. Réimpression par Dr. D. Bierens de Haan. L.L.D. — Amsterdam 1884.

FRAGMENTS IN WORKS OF OTHERS

- XVI A. *Journal tenu par Isaac Beeckman de 1604 à 1634 publié avec une introduction et des notes par C. de Waard. Tome II. — La Haye 1942.*
Appendice I (394—438) contains fragments on the following subjects:
- 1 *Huysbou.*
 - 2 *Spiegeling der Singconst.*
 - 3 *Cammen ende Staven, Watermolens ende Cleytrecking.*
 - 4 *Waterschueringh.*
 - 5 *Van de Crijchconst.*
- 33
- B. *Wisconstich Filosofisch Bedryf, van HENDRIC STEVIN, Heer van Alphen, van Schrevelsrecht, &c. Begrepen in veertien Boeken.*

— Tot Leyden, Gedruet by Philips de Cro-y, in 't Jaer 1667.

*Plaetboec. Vervangende de figuren of formen gehorig tottet Wiscon-
stich Filosofisch Bedryf van HENDRIC STEVIN, Heer van Alphen,
van Schrevelsrecht &c.*

Gedruet in 't Jaer 1668.

1 Boek VI, Prop. 2	<i>Van den handel der cammen en staven onses Vaders als bewegende oirsaec van dese.</i>
2 Boek X	<i>Van den handel der Watermolens onses Va- ders SIMON STEVIN.</i>
3 Boek XI	<i>Van den handel der Waterschuyring onses Vaders SIMON STEVIN.</i>

DE BEGHINSELEN DER
WEEGHCONST

THE ELEMENTS OF
THE ART OF WEIGHING

INTRODUCTION

§ 1. HISTORICAL INTRODUCTION

Just as all other branches of mathematics and natural science, theoretical mechanics is rooted in Greek antiquity. Its roots are twofold, and of quite different origin. They are associated with the names of two great ancient thinkers, Aristotle and Archimedes.

In the former's Work, *Mechanica Problemata*, the statical problem of the equilibrium of a balance is dealt with from a dynamical point of view, a seemingly paradoxical idea, which was, however, to prove extremely fruitful. Archimedes on the other hand treated mechanics as a branch of mathematics; modelling himself on Euclid's foundation of geometry, he formulated a number of axioms on statics from which, with the aid of certain implicit suppositions concerning the theory of the centre of gravity, he logically derived the fundamental rule for the equilibrium of a lever. With him, statics came to be an autonomous science.

In the Middle Ages only the former of these two methods was applied. In the school which is named after Jordanus Nemorarius it developed from its original form, which may be characterized as a germ of the principle of virtual velocities, into that of virtual displacements. This was applied not only in the theory of the lever, but also for the derivation of the law of the inclined plane. In the 16th century this current was continued by the Italian scholars Tartaglia and Cardano ¹⁾.

In the forties of that century the Archimedean approach became known through the publication of his works. The Italian mathematicians Commandino, Maurolyco, Guido Ubaldo del Monte, Benedetti, and Luca Valerio followed this method. However, they mainly made use of it for the determination of centres of gravity, thus enriching the science of Statics with new results, without finding new possibilities for its further development.

It is the abiding merit of Stevin that he did find these new possibilities. He not only studied and supplemented the work of Archimedes, but he also continued it.

In accepting the Archimedean method he radically rejects that of Aristotle and Jordanus. He thinks it perfectly absurd to derive a condition of equilibrium from the consideration of a situation which cannot possibly present itself as long as the state of equilibrium continues, viz. the simultaneous displacements of the balancing bodies. This severe and rather unjust criticism, however, does not prevent him from appealing sometimes to rules proper to the theory condemned by him.

In accordance with the prevalent custom of his day, Stevin as a rule quotes other authors only in order to dispute their opinions, and carefully conceals any sources from which he has drawn. This makes it extremely difficult to appreciate the degree of his originality. However, as long as no other works besides the Archimedean treatise *On the Equilibrium of Planes* have come to light in which his particular treatment of the subject is anticipated, we may credit him

¹⁾ For more detailed information on this period of the history of Mechanics the reader may consult: P. Duhem, *Les origines de la statique*, 2 vol., Paris, 1905-06.

with the merit of having been the first to take over and pass on the torch lit by the great Syracusan.

§ 2. SUMMARY OF THE WORK

We now proceed to give a summary of the contents of the *Art of Weighing* in present-day terminology.

In Part I of Book I the premisses of the *Art of Weighing* are set forth in 14 Definitions and 5 Postulates. In the same way as Archimedes had done, Stevin presupposes a theory of the centre of gravity without specifying its logical foundations and its results. His definition of centre of gravity (Def. 4) is substantially identical with that given by Pappus in his *Collectio Mathematica*²⁾: a point such that if the solid is conceived to be suspended from it, the solid remains at rest in any position given to it. It is assumed that any body has such a point, and only one such point. In a Note to Postulate 5, Stevin is seen to be aware of the fact that this supposition is valid only if the verticals through the different points of the body are considered to be parallel.

In Definition II the fundamental concept of *evenstaltwichtigheid* is introduced. Bodies balancing one another at unequal arms of a lever are not really *evenwichtig* (of equal weight), but they only appear to be so. Stevin therefore carefully distinguishes between *evenwicht* or *evenwichtigheid* (equality of weight) and *evenstaltwichtigheid*. Since the first term has become current for denoting the second concept, whereas the term proposed by Stevin never penetrated into scientific terminology, it is rather difficult to translate the latter and its derivatives. Neither Snellius nor Girard succeeded in finding a Latin or French equivalent of one word; bodies called *evenstaltwichtig* by Stevin are designated by the former as *ex situ equilibria*³⁾, by the latter as *équilibres selon leur disposition*⁴⁾. Starting from Stevin's explanation that bodies balancing one another have a *ghelaet* (appearance) *van evenwichticheyt* (Def. II, Explanation), we translate *evenstaltwichtig* by "of equal apparent weight" and *evenstaltwichtigheid* by "apparent equality of weight" or "equality of apparent weight". It will be seen that Stevin not only speaks of bodies of equal apparent weight, but also attributes to a body a certain *staltwicht* depending on the circumstances (Prop. 19). This term, which is nowhere defined explicitly, was translated by Snellius by *sacoma*⁵⁾ (from Greek Σήκωμα = weight) and by Girard by *puissance* or *pouvoir*⁶⁾. In accordance with the above we render it by "apparent weight", though this is not a current term in modern physics. It would, however, be impossible to find a modern equivalent, since the concept which Stevin denoted by the word "staltwicht" had not yet taken a definite form in his mind. It may denote the moment of a force with regard to a point, but also the component of a force along a line, while in other cases the meaning is not quite clear. It should be remembered that in the 16th century the science of mechanics was only just coming into being,

²⁾ *Pappi Collectionis Mathematicae quae supersunt*, ed. Hultsch, 3 vols, Berlin, 1875-78. VIII 5; III 1030.

³⁾ Xlb. *De Staticis Elementis* 8.

⁴⁾ XIII 435 b.

⁵⁾ Xlb. *De Staticis Elementis* 34.

⁶⁾ XIII 448a. The current term in the Middle Ages is *gravitas secundum situm*. Tartaglia has *gravita secondo el luoco over sito*. *Quesiti et Inventioni diverse*. VIII Def. 13. Venetiis, 1546.

and that it was to take a couple of centuries before its fundamental conceptions could be fixed with a reasonable degree of exactitude.

Part II of Book I contains 27 propositions, which may be divided into two groups.

The first group (Prop. 1—18) refers to the theory of the lever and its applications, the second (Prop. 19—27) to the theorem of the inclined plane and its consequences.

Group I. The first of this group is to be found in Prop. 1, which contains a mathematical demonstration of the state of equilibrium of a straight lever. Since the method is closely akin to that of Archimedes (*Equilibrium of Planes*, Prop. 6), we first summarize the latter in a modern form. It is to be proved that, if two weights are suspended from a horizontal lever at distances from the fulcrum which are inversely proportional to the weights, the lever will be in equilibrium. Translating Archimedes' argument into modern symbols, we may say that he supposes the weights G_1 and G_2 to be suspended from a lever with fulcrum O at the points A_1 and A_2 respectively. Putting $OA_1 = l_1$, and $OA_2 = l_2$, he supposes

$$G_1 : G_2 = l_2 : l_1.$$

He now puts: $G_1 = n_1 \cdot G$

$$G_2 = n_2 \cdot G \quad (n_1 \text{ and } n_2 \text{ are integers; } G \text{ is a common measure of } G_1 \text{ and } G_2)$$

and accordingly

$$l_1 = n_2 \cdot l$$

$$l_2 = n_1 \cdot l.$$

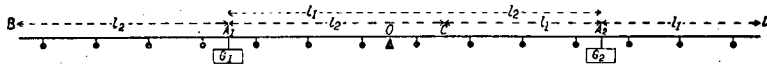
He now makes $A_1B = A_1C = l_2$

$$A_2D = A_2C,$$

which implies

$$A_2C = A_2D = l_1.$$

He now replaces the weight G_1 by $2n_1$ weights $G/2$ hanging in the middle points of the $2n_1$ segments l into which BC can be divided, and likewise G_2 by $2n_2$ weights $G/2$ in the middle points of the $2n_2$ parts of CD . Since $OB = OD$, the distribution of weights is now symmetrical with respect to O , and the lever is therefore in equilibrium (the validity of this inference has been granted by Postulate I).



The case that G_1 and G_2 , and consequently l_1 and l_2 are incommensurable requires a separate demonstration, which is given by *reductio ad absurdum*.

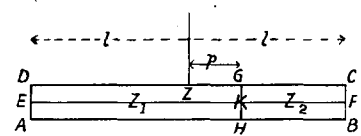
It is to be noted that Archimedes also makes use of the converse theorem: if the lever is in equilibrium when the weights G_1 and G_2 are hanging from it at distances l_1 and l_2 from the fulcrum, then $G_1 : G_2 = l_2 : l_1$, but he does not prove this.

Stevin now modifies the Archimedean demonstration in two respects:

- 1) Starting with a symmetrical distribution of the weight along the lever, he proves the converse theorem.
- 2) By replacing the discrete magnitudes $G/2$ of Archimedes by continuous ones he eliminates the necessity of distinguishing between commensurable and incommensurable weights.

His demonstration may be translated into modern symbols as follows:

Let the rectangular parallelepiped $ABCD$ be suspended from a point in the vertical through its centre of gravity Z . $AB = 2l$. Cut the body by a plane GH at right angles to the axis EF and meeting this axis in K . $ZK = p$. The centres of gravity of the parts $AHGD$ and $BHGC$ are Z_1 and Z_2 respectively. Now suppose the weights G_1 and G_2 of these parts to be concentrated in their centres of gravity. Since



$$ZZ_1 = \frac{l-p}{2} \quad ZZ_2 = \frac{l+p}{2}$$

and

$$G_1 : G_2 = (l+p) : (l-p)$$

it follows that

$$G_1 : G_2 = ZZ_2 : ZZ_1$$

so the weights are inversely proportional to the arms.

The converse theorem (which is the original one of Archimedes) is enunciated, but not proved.

Obviously Stevin's demonstration is open to the same objection as was raised by E. Mach ⁷⁾ against the proof given by Archimedes: the argument is based on the assumption that an existing state of equilibrium of the lever will not be disturbed, if a weight hanging at a given point is so distributed along the lever that the centre of gravity of this distribution remains in the original position or, conversely, if a body attached to a lever is replaced by its weight acting at its centre of gravity. This, however, is by no means evident. In the simplest case, in which a weight G at a distance l from the fulcrum is replaced by two weights $G/2$ at distances $l \pm a$, the assumption amounts to the functional equation.

$$f(G, l) = f(G/2, l-a) + f(G/2, l+a)$$

where $f(G, l)$ denotes the influence exerted on the lever by a weight G at the distance l from O , while moreover it is assumed that the influences of two separate weights can be combined by addition. Now it is clear that if the form of $f(G, l)$ were, for example, $G.l^2$ instead of $G.l$, the equality would not hold. The problem, however, consists in determining the form of the function $f(G, l)$, and the assumption is therefore unwarranted.

It has been urged ⁸⁾ against Mach's argument that at all events Archimedes did make this assumption explicitly in Postulate 6 of *Equilibrium of Planes*, which states that any body suspended from the lever may be replaced by any other having the same weight and hanging at the same place. It is then contended that this body may also consist of a number of bodies, the common centre of gravity of which is in the vertical through the original point of suspension. However, it is doubtful in the first place whether this is the real meaning of the text, and secondly whether it is permissible to make so far-reaching a statement in the form of a postulate. In any case this justification of Archimedes' procedure — if it be one — does not apply to that of Stevin, who has no such postulate.

After having satisfied himself as to the truth of the first proposition, Stevin

⁷⁾ E. Mach, *Die Mechanik in ihrer Entwicklung historisch-kritisch dargestellt*. Leipzig, 1912, p. 14.

⁸⁾ W. Stein, *Der Begriff des Schwerpunktes bei Archimedes*. Quellen und Studien zur Geschichte der Mathematik, der Astronomie und der Naturwissenschaften, Studien B I (1931) 221.

finds no difficulty in solving a number of simple statical problems: given two weights balancing one another at a lever, to determine the fulcrum (Prop. 2); given one of said weights and the fulcrum, to find the other (Prop. 3); given two weights and one of the arms, to find the other arm (Prop. 4); given a prism, to determine a weight in a given ratio to the weight of the prism by means of statics (Prop. 5).

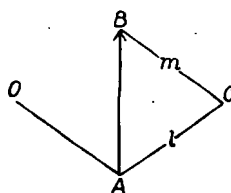
The propositions 6—8 deal with the difference between stable, indifferent, and unstable equilibrium (without, however, making use of these terms). By way of introduction to the following problems, Prop. 9 states that if two weights are hanging in equilibrium from a lever at right angles to their lines of action, this lever may be replaced by another inclined to these lines, all the fulcrums remaining in the original vertical lines. After this it has become possible to solve various problems: to determine whether the equilibrium of a prism from which two weights are hanging so as to balance one another is stable, indifferent or unstable (Prop. 10); to determine the common centre of gravity of a prism and certain weights attached to it (Prop. 11); to find the weight which should be attached at a given point in a given prism loaded with known weights in order to keep the prism in a given position (Prop. 12).

Hitherto the prism has been subjected to forces directed vertically downwards only. Proposition 13 now introduces upward vertical forces, and shows how to replace them by equivalent downward forces. Proposition 14 shows that a body with one point fixed in the axis may be kept in equilibrium by an upward force acting at another given point on the axis. The magnitude of this force is determined (Prop. 15) and shown to be independent of the position of the body (Prop. 16). Proposition 17 shows how the weight of the prism is distributed between two points of support, both in the axis; the same problem is solved in Prop. 18 for the case that the two points are arbitrarily chosen.

Group II. Proposition 19 contains the famous demonstration of the so-called law of the inclined plane by means of the "*clootcrans*" (wreath of spheres). Since the argument is perfectly clear, it does not seem necessary to reproduce it here in a modern form; critical remarks about its validity and about the corollaries will be given in the notes.

It is remarkable that in the *Weeghconst* the inclined plane is not considered at all as a mechanical instrument; this will only be done in the *Weeghdaet*. In

the *Weeghconst* it is used as a lemma for a theory of the equilibrium of a body with one fixed point; the transition is brought about by considering the point in which the body rests on the inclined plane as fixed, and omitting the plane. The main contents of the following propositions may be summarized as follows: given a rigid body, one point O of which has been fixed, the body is to be held in equilibrium by a force acting along a given line l in the vertical plane through both the fixed point and the centre of gravity of the body; to determine the magnitude of this force. Let the vertical force at A which keeps the body in equilibrium be represented by AB ; this force has been determined in Prop. 14. If now the line m is drawn through B parallel to OA to meet l in C , then AC will represent the required force: The truth of this is evident to us, the forces AB and AC having equal statical moments about O .



This fundamental theorem is proved in Prop. 20 for upward, and in Prop. 21 for downward forces. In Prop. 23 it is shown that the value of AC is the same for the two positions of AC which make equal angles with OA . In Prop. 24 the minimum value of AC is found to be perpendicular to OA .

In the remaining propositions, no point of the body is supposed to be fixed; it is now suspended from two lines. It is proved that if these lines are non-parallel, they will meet in the vertical through the centre of gravity (Prop. 25); further that either both lines must be vertical or neither of them, and finally that in the latter case they incline one to the right and the other to the left of the vertical (Prop. 26). By considering as fixed either of the points of the body at which the lines are attached, it is proved that the fundamental proposition holds. Finally, in Prop. 28 the prisms hitherto considered are replaced by bodies of arbitrary form.

Book II deals with the determination of centres of gravity a) in plane figures (Prop. 1—13); b) in solids (Prop. 14—24). In an introductory note to the first group it is observed that such terms as weight, centre of gravity, etc. with reference to plane geometrical figures are to be understood metaphorically; a similar note to group b), referring to geometrical solids, is, however, lacking.

Prop. 1 shows, with reference to some examples, that if a plane figure has a geometrical centre, this point is at the same time its centre of gravity. It is then proved that the centre of gravity of a triangle is in a median (Prop. 2), from which follows its determination as the point of intersection of two medians (Prop. 3) and the ratio of the segments into which it divides a median (Prop. 4). Prop. 5 amounts to no more than a simple corollary to the preceding theorem. It is then shown how to determine the centre of gravity of a plane polygon (Prop. 6), of a trapezium (Prop. 7, 8), and of the remainder of a plane figure after removal of a given part (Prop. 9). The propositions 10—12 deal with the centre of gravity of a parabolic segment: this is proved to be in the diameter (Prop. 10), and to divide it in a ratio which is the same for any parabola (Prop. 11), this ratio (3 : 2) being determined in Prop. 12. Finally, in Prop. 13 the centre of gravity of a portion of a parabolic segment, cut off by a line parallel to the base, is determined.

In group b), Prop. 14 repeats Prop. 1 for solids. The centre of gravity of a prism is determined in Prop. 15. The propositions 16—18 (centre of gravity of a pyramid) correspond to Prop. 2—4 for the triangle, Prop. 19 to Prop. 9. In Prop. 20 the centre of gravity of a truncated pyramid is found, in Prop. 21 that of any solid. Prop. 22—23 deal with the centre of gravity of a segment of a paraboloid of revolution, Prop. 24 with that of a truncated segment.

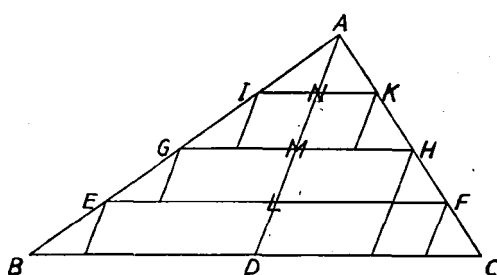
Stevin's method of dealing with centres of gravity is substantially identical with that introduced by Archimedes in his work *On the Equilibrium of Planes*⁹⁾ and subsequently used by Federigo Commandino in his *De centro gravitatis solidorum*¹⁰⁾. It amounts to approximating a plane or solid figure, the centre of gravity of which has to be found, by means of a series of inscribed polygonal (or polyhedral) figures with known centres of gravity, and then determining the limiting position of the latter when the number of the sides (or faces respectively) of the inscribed figure is indefinitely increased. Whereas,

⁹⁾ *Archimedes Opera Omnia*, ed. J. L. Heiberg. Vol. II, 124–213. Leipzig, 1913.

¹⁰⁾ Federici Commandini *Liber de centro gravitatis solidorum*. Bononiae, 1555.

however, Archimedes and Commandino felt obliged to prove on each fresh occasion the correctness of the result obtained by the *reductio ad absurdum*, which is characteristic of the ancient method of treating infinite processes, Stevin makes use of the general consideration that two quantities, the difference between which can be proved to be less than any assigned magnitude, are equal to one another.

The working of the method may be illustrated by the following rendering of Prop. 2, in which it is proved that the centre of gravity of a triangle is in a median.



Let ABC be a triangle, D the middle point of BC . The median AD is divided into n equal segments (in the drawing $n = 4$). Through the points of division L, M, N are drawn lines parallel to BC , intersecting AB in E, G, I respectively and AC in F, H, K respectively. Through these points on AB and AC are drawn lines parallel to AD which, in a manner suf-

ficiently clear from the drawing produce a figure Π_n consisting of $(n - 1)$ parallelograms. It is easily seen that the difference between the area of Π_n and

that of the triangle ABC (Δ) is $\frac{\Delta}{n}$ and can therefore, through the choice of n , be made less than any assigned area.

By applying Prop. 1 it is seen that the centre of gravity of Π_n is in AD , so that, if Π_n be suspended from A , the line AD will be vertical, and the two parts into which the median AD divides Π_n will balance one another, or, in Stevin's terminology, will have the same "staltwicht" (apparent weight). It is now contended that the "staltwichten" of ADB and ADC differ less from one another than any given quantity, from which it is inferred that they are equal to one another. The final conclusion that the centre of gravity of ΔABC is in AD is then easily reached.

The decisive point of the demonstration obviously consists in the contention relating to the "staltwichten" of the parts ADB and ADC ; this is based on the preceding observation that the areas of Π_n and ΔABC can be made to differ less than any given quantity. It is, however, to be doubted whether this is a sufficient reason. It is indeed obvious that the areas (and therefore the weights) of these two triangles can, through the choice of n , be made to differ less than any assigned quantity from the areas (and weights respectively) of the two parts into which Π_n is divided by AD , but the transition from this statement to that on "staltwichten" seems unwarranted, since it has not been shown anywhere how the "staltwicht" of a part of the figure relatively to AD is to be determined. At the back of Stevin's mind there seems to be a consideration, which may be expressed in modern terms by saying that the "staltwicht" of a part of the figure relatively to AD is a continuous function of the variables on which it depends (viz. area and distance of the centre of gravity from AD).

The method used by Stevin in Prop. 2 and repeated without any substantial

modification in the propositions 10, 15, 16, 18, 22 forms an important contribution to the historical development of the treatment of infinite processes. As far as we know, he was the first to emancipate himself from the obligation to give each time again a proof by *reductio ad absurdum*, which the rigorism of the great Greek predecessors still imposed on mathematicians, and thus to pave the way for the newer and simpler methods which the Calculus was to provide. It is, however, characteristic of the powerful influence exerted by the Greek tradition even on those who strove to grow independent of it that Stevin cannot yet bring himself to formulate his innovation in the form of a general proposition to be applied in each individual case presenting itself, but repeats it *in extenso* each time again, just as Archimedes had done with the *reductio ad absurdum*. Just as in important passages of Book I, he clothes his reasoning in the classical form of a syllogism, the mood of which is *Baroko*; so, for example, in Prop. 2 of Book II which has been rendered above, he argues as follows:

- A. Beside any different "staltwichten" there may be placed a gravity less than their difference.
- O. Beside the present "staltwichten" ADC and ADB there cannot be placed any gravity less than their difference.
- O. Therefore the present "staltwichten" ADC and ADB do not differ.

The fact that he uses this form of exposition is undoubtedly due to his desire to stress the importance of his innovation. He certainly had a right to do so: 16th century mathematics had profited immensely by the Greek source of knowledge, but it could not develop beyond the ancient boundaries unless it succeeded in emancipating itself from the burdensome Greek style of demonstration, even if this movement were to result — which indeed it did — in a temporary decline of mathematical rigour.

Stevin deserves to get credit for his clear insight into this necessity, and for the resolution with which he took the new road. It is yet another manifestation of his strong desire to make mathematics a practical tool for the investigation of nature, fit to be handled by all clear-minded people.

The immediate result was an enormous simplification of the treatment of centres of gravity. This becomes clear at once when his work is compared with that of Commandino, his only predecessor in this field¹¹⁾ besides Archimedes. The modern reader, impatient at Stevin's prolixity, need only compare his Prop. 23 on the centre of gravity of a segment of a paraboloid with the corresponding Prop. 29 in Commandino's *Liber de centro gravitatis solidorum* to see what remarkable progress Stevin had made.

§ 3. DISCOURSE ON THE WORTH OF THE DUTCH LANGUAGE¹²⁾

In § 4 of the biographical introduction to this edition we mentioned Stevin's considerable influence on the development of the Dutch language. That which

¹¹⁾ Commandino expressly states in the preface to his work that he is the first to write on centres of gravity of solids. He cannot indeed believe that no one should have dealt with the subject before him, seeing that Archimedes in his work *On Floating Bodies* considers the position of the centre of gravity of a paraboloid a thing of common knowledge. He has not, however, succeeded in tracing any treatise about it.

¹²⁾ We owe the following discussion of Stevin's philological ideas to Prof. Dr C. G. N. de Vooy, Former Professor of Dutch Language and Literature at the University of Utrecht.

now follows is the most important of the passages in which he expounds his linguistic ideas, the *Uytspraeck van de Weerdicheyt der Duytsche*¹³⁾ *Tael* (Discourse on the Worth of the Dutch Language)¹⁴⁾.

Sympathetic consideration and great esteem of the vernacular is an international feature of the 16th century. Italy took the lead, followed by France, which in turn stimulated the movement in the Netherlands and in Germany. In order to understand Simon Stevin's curious linguistic views it is necessary to have regard to this historical background. On a first view it appears strange that the same Renaissance artists and humanists who were such fervent admirers of classical Latin should also have advocated the elaboration and the use of the vernacular. That this inconsistency is only apparent has been shown very clearly by F. Brunot in his *Histoire de la langue française* II, 1 : *L'émancipation du français*. The reversion to Ciceronian purity and the close imitation of the classical style rendered Latin useless as a living language. Mediaeval Latin, with its greater simplicity of structure and its capacity of adapting itself to every requirement, could no longer find favour with the Renaissance scholars. The consequences of this were inevitable: "On cherchait l'élégance; on perdit la commodité". It began to be realized that the only suitable medium for the dissemination of knowledge and art in wide circles was the vernacular, which, however, had to be made as effective as possible for the purpose in view¹⁵⁾. Attention was therefore paid to an adequate systematization of spelling, to syntax, to purification from foreign elements, and to extension of the vocabulary by new word formations.

The new appreciation of the vernacular sometimes resulted in overestimation. Thus the Antwerp scholar, Johannes Goropius Becanus, in his *Origines Antwerpianae* (1569) believed he could prove Dutch to be the oldest language of the world; nay, he even held that this language had been spoken by the inhabitants of Paradise and their immediate descendants. The name *Adam* was none other but the Dutch word *adem* (breath), for had not God breathed into his nostrils at the Creation? *Noach* (Noah) was the man who "acht op de noot" (minds the distress), *Babel* had a very apt meaning, for *babelen* "est tam confuse et inarticulate loqui, ut non intelligatur"¹⁶⁾. The very name of *Duyts* furnishes evidence in support of the theory, for it means *Douts*, i.e. "the oldest". Philology

¹³⁾ As has been remarked in Note 16 to the biographical introduction, *Duytsch* has to be translated by Dutch, not by German. Sometimes Stevin distinguishes the language of the *Overlanders* (Germans) as *Hoogduytsch* from the *Neerduytsch* spoken by the *Neerlanders*. He considers the former to be a variety of the latter, which is spoken in its purest form in the province of North Holland (cf. p. 46 below).

¹⁴⁾ He had dealt with this subject before in the *Dialectikelicke Tsamespraeck* at the end of the work *Dialectike* (III), and he returns to it once more in the introduction to the *Waterwicht* (cf. p. 385 below), the *Stercktenbouwing* (IX, p. 87), and the *Spiegel der Singconst* (XV or XVI, p. 56-57). A second, somewhat modified version of the *Uytspraeck* is given in the *Wiskonstighe Gbedachtenissen* (XI; i, 21, Bepaling 6), where it is supplemented with an elaborate exposition of Stevin's theory of the *Wijentijt* (Age of the Sages) (cf. General Introduction, § 4). This passage is to be inserted in our Volume III.

¹⁵⁾ Brunot's argument has been given more fully by the writer in *De Nieuwe Taalgids* 1917. This article was reprinted in *Verzamelde Taalkundige Opstellen* I, p. 255. Cf. also the introduction to K.W. de Groot's article on *Het purisme van Simon Stevin* (Simon Stevin's Purism) in *De Nieuwe Taalgids* 1919.

¹⁶⁾ Vide Dr. K. Kooiman: *Twespraack van de Nederduitsche Letterkunst* (1913) pp. 77 et seq.

of such kind seems rather naïve to us, moderns, but it is remarkable that among the writer's contemporaries men such as Coornhert and Spieghel took it seriously, and even had implicit confidence in the "irrefutable arguments".

Simon Stevin did not go as far as that: he does not go back to the language of Paradise, nor does he look for arguments in the Old Testament, but in his own way he assumes a certain evolution of linguistic history in order to account for the great antiquity of Dutch. Language as an invention of the human mind — which is to Stevin a miracle — presupposes an advanced stage of civilization. Consequently, in remote times, which he calls the "Wijsentijt" (Age of the Sages), the "Duytsen", i.e. the Germanic tribes in general and our ancestors, the Dutch, in particular, must have been a very powerful race with a highly developed culture. Through all sorts of causes this race must have fallen into a condition of barbarism, which lasted to the days of Julius Caesar. After that, another age of progress dawned, and the Germans grew more and more to be the masters of Europe. Stevin finds evidence for this in the fact that the Gauls — i.e. the French —, who conquered Southern Europe, must originally have spoken Dutch or at least held this language in great esteem. The Spaniards, too, have either been Dutch or have modelled their language on that of the Dutch.

Besides its venerable antiquity, Stevin also points out the inherent excellency of Dutch. His principal aim is to show that Dutch is more suitable for scientific purposes than any other language. This claim is based on four arguments:

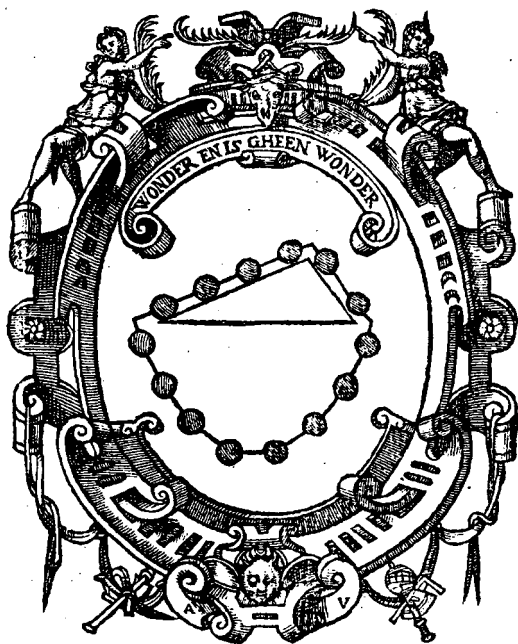
- 1) Since the end of language consists in expressing thoughts by words, the more it is capable of denoting single things by monosyllables, the greater will its value be. By means of a statistical investigation Stevin proves Dutch to be superior in this respect to both Latin and Greek.
- 2) A second criterion for judging the merits of a language is the facility with which compound words are formed. In this respect, too, Dutch is found to excel.
- 3) The *Art of Weighing* contains examples of concise formulation of mechanical theorems not equalled in any other language.
- 4) The Dutch language possesses in a superlative degree the faculty which Stevin calls *beweegblichkeit*, i.e. the power to move, the emotional appeal of which it is capable. In a most remarkable passage (cf. p. 87 below) this quality is illustrated by the strong influence exercised by religious orators on the people in the Low Countries.

A curious consequence of Stevin's views is that on account of its preference for monosyllables he considers the language of North Holland to be of older origin than his own native Western Flemish, whereas in reality it is the longer nouns ending in -e which are the earlier forms.

From a statement in some editions of *Materiae Politicae* (XIV) ¹⁷⁾ it appears that Stevin had also intended to write some treatises on rhetoric and poetry, in which he was to have laid down his views about moot points of grammar, in particular spelling. Since these treatises, if written at all, at any rate have not come down to us, we cannot make any statement about his views on these subjects. We only know of his aspirations after purification of the language and the creation of a practical, well-considered terminology, so that the vernacular might in due time be fully appreciated for its services to the nation and to science.

¹⁷⁾ Dijksterhuis p. 60.

D E
BEGHINSELEN
DER WEEGHCONST
BESCHREVEN DVER
SIMON STEVIN
van Brugghe.



TOT LEYDEN,
Inde Druckerij van Christoffel Plantijn,
By François van Raphelinghen.
c1o. 1o. LXXXVI.

OM DE WEEGHCONST,

WATERWICHT, ENDE

*LOCHTWICHT.

Liber hic
hodieq. lu-
cifuga, pau-
corumq. ad-
huc testium.



LARA Rhodos Clario, clarum celebrare

Colossium

Desine, miraclo tanto nil tantula moles

Dignum habet, annorum labor ut fuerit
duodenium.

* Quintuplum tantum haud steterat, cum
viscera terra

LVI, & quod
excurrit, an-
nos steterat,
cum terre-
motu con-
cidit.

Spiritus intus agens vastos disuerberat artus.

Corruit illa, iacet ignobile littore corpus,

Pesque, caputque, aliaque incerto nomine partes.

Stulte quid inspectas, admiratusque iacentem

Quisnam erexerit, & quam ratione modoque

Quæris? at heus! quam mirandum, multoque tremendum

Hoc magis est, totam leuis ut disflauerit aura

Molem? adeo totam leuis ut disflauerit aura?

Sed neque mirandum fuit hoc, stulte ve tremendum.

Si potis es rerum penetrantes discere causas,

Et maiora videbis, & hæc mirabere nullus.

„Pondera ponderibus nitantur maxima paruis,

„Nutibus ut minimis firmissima queque trahantur.

Hoc Natura parens, Naturaque anterior Mens

Omnibus in rebus statuit, seruatque statutum.

Ecce onus hoc Matris diuinum, & quicquid parit illa,

Cæruleus * Pater innixum sibi baiulat: illum

Non impro-
banda Tha-
letis opinio,
à qua faciūt
& sacræ lit-
teræ ipsa que
adeo mini-
me fallax
experientia.

Pondere iam tanto grauidum in se sustinet aër.
 Spiritus hic lenisque leuisque his corporibus par,
 Spirituque leui faciunt hæc corpora. Tantum
 Hic Natura potest. Natura Ars emula, tantum.

Het Al-
 machuch
 multo ma-
 iorum viriū
 quā Ar-
 chimedis
 Trispastus,
 multo & tri-
 bile magis.

Fare age * Pantocrator cæco sit carcere clausum,
 Incipiantque foris ades ac templa moueri,
 Incipiant siluæ, montes migrare, videnti
 Quæ tibi mens animi, quæ sit constantia? rectè
 Cum Natura agitur, tecum & Natura agit, hæc si
 Credideris licitis Natura legibus isse.

* qui tamen
 immobilis.
 † Inferos, sci-
 licet centrū
 quod etiam
 immobile.
 * Ita diuinus
 Plato, in co-
 trarium iuit
 Aristoteles.

Legibus hæc quod eant licitis, etiam hoc cape. Tellus
 Tellurem hanc præter sit, si licet, altera: firmo
 Hæc mihi me sistas talo, ne vixero, si non
 Hunc ego cum Tellure * Polum, atque † Acheronta mouebo.

Hoc Trutinaria nos docet. hæc eadem monet omnia
 Undique ponderibus consistere, pondere cassum
 Esse * nihil, non hanc nostram, non ætheream auram.

Aliis arcus
 pluuius re-
 ipsa colora-
 tur, Aristoteli
 in speciem, quod
 & magis est.

Vt maris, & terræ, numerique potentis arena,
 Sint, fuerint aliàs, mensores, non tamen auræ
 Dimensi spatium, aut vim ponderis appenderunt.
 Te certè, Alhazene, loci quem nubila tranent;
 Qua regione color * appareat Iridis, utrum
 Hanc supra Notus, & Zephyrus, Boreasve, vel Eurus,
 Sitne Cometaryum certus locus, utque sit, eius
 Mensorem video, veneror. potiora mouentem
 Credo secuturus facili ratione fuisses.

In de Loch-
 wicht.

Non bene permensum spatium tibi. iustius illud
 Iusta pensauit * trutina Steuimus, ille
 Ipse tui studiose senex studiosus amator.

L. M. huic
 gloriolæ ces-

Ergo per hæc * princeps graditur loca nullius ante

Trita

*Trita solo, venit, ecce, videt, penitusque penetrat
 Nature anfractus, ut quæ miranda videntur
 Illorum trepida solusamur religione.
 „ Nil admirari res maxima, Lector, at illa
 „ Hæc erat, auctrices miri cognoscere causas.*

senit auctor,
 si quis dica-
 tur hanc viā
 munisse,
 strauisse, co-
 gitasse prior.
 Ick spreek
 vāde Licht-
 wicht.



OM DE SELFDE.



ΠΡΟΜΕΝΟΙΣ ἔργοις πλῆλιν χρῆια πο-
 σάμεν.

Τέτο μάλισθ' ὁρῶμεν τὴν μῦθον διάνοιαν.

Ἐπ' αὖτε, οἱ ποτ' ἔτι μεγαλὸν τ' ἀνδρὸς μέγα τ' ἔργον

Μεῖζον αὖτ' ἢ διὸν Στόλνιος ἔργον ἔδει;

Καὶ γὰρ χαλδαῖοι σοφίαν λάχον, ἢ δ' αὖ ἐρεῶσι,

Ἢ ποτὲ ῥωμαῖοι. Μῦθον τὸ δ' ἔλλατ' ἔφες,

Πυθαγόραντι, πλάτωνά τ', ἀριστοτέλει περ οἶδ' ἔστιν

Ἀρεῶν ἀρίστηλον τ' φυσικῆς σοφίας.

Τοὶ δὲ λέγουσι τί γὰρ, τί δ' ἔσθ' ὁρῶμεν ὑπερδυν,

Πόντος τ' ἥλιος τ', ἡμεῖς τ' ἄλλοι χάος.

Οἶδ' ὅτι δ' ἀκλεῖδ' ὡς Μαθηματικῆς φίλον ἦτορ,

Ἀρεῶν ἀρίστηλον μυστικῆς σοφίας,

Οἷος περ πάντα μέγιστον. καὶ ὅτι δὲ ψάμμος ἀεὶ μὲν

Ἀλλοθεν, ἀλλὰ μάλιστα τ' ἐνὶ ἡμῶν ἔστιν ἱρὰ.

αΑ 3

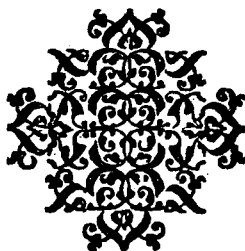
Archime.
 demdico.

Ἐπ'

Εἰπ' ἄρα, εἰ ποτ' ἔπι μεγάλα τ' ἀνδρὸς μέγα τ' ἔργον
Μεῖζον ἂν ἢ οἶον Στάλνιος ἔργον ἔθῃ;

„Ἐπεὶ ἀριστέει μὲν ὕδωρ, κτεάνων δ' ὅλα χεύσας
„ Λαμπρότατός τε, κρατεῖ δ' ἄλλος ἑσπέρῳ,
Εὐκλείδης γὰρ μέγας, μηδὲς δὲ τὸ μεῖζον ἑπαίνοισι,
Ἦρα γ' ἂν Εὐκλείδης Στάλνιος εἶχε νόον;

IAN DE GROOT.



Simon Steuin wenscht

RVDOLF DEN II^{en}

ROOMSCH KEYSER

VEEL GHELVCX.

DAT Ghetal Grootheyt ende Ghewicht, in yder wesentlicke saeck * onscheydelicke an-
cleuinghen sijn, vol diepe ende nutte eyghen-
schappen, en betuyghen niet alleen verschey-
den gheleerden, maer tis duer d'euaring in allen an elcken
bekent. Tis oock openbaer, dat d'eerste twee tot grooten
voordeele der menschen, ter form van beschreuen consten
gherocht sijn, namelick * Telconst ende Meetconst, maer
niet also t'Ghewicht, om dat sijn oirsproncklicke eyghen-
schappen den voorighen verborghen bleuen. Wel is waer
dat inde Rechrwichten duer euaring bemerckt is, twee
cuestaltwichtighe met haer ermen * euerednich te wesen.
Doch sy hebben ghemeent * soodanighe eueredenheyt te
schuylen onder de ronden beschreuen op t'vastpunt duer
d'uytersten der ermen; Uyt het welck, na den ghemeen
aert der dwaling, gheen kennis der oirsaken en volghde.
Wat de Scheefwichten belangt, daer en is niet met allen af
gheweten. Inder voughen dat dese * stof gheen form van
Const als d'ander ghecrighen en conde. Maer doen t'ghe-
ual anders lucte, ende dat sulck langverborghen hem duer
sijn uyterste beghinselen openbaerde, sy is eintlick daer toe
ghecomen, in sulcker ghedaente als die uyve Keyserlicke
M. hier toegheeyghent wort. Maer anghesien byde
voordachtighe niet sonder oirsaeck ende bestaende reden
anghe-

*Inseparabilia
accidentia.*

*Arithmetica
& Geome-
tria.*

*Proportiona-
les.*

*Als Avillo-
teles in Me-
chanica met
sijn nannol-
ghers.*

Materia.

SIMON STEVIN
WISHES MUCH HAPPINESS TO
RUDOLPH II,
HOLY ROMAN EMPEROR

That number, magnitude, and weight are in all essential things inseparable attributes, full of profound and useful properties, is attested not only by several scholars, but it is also known to all by experience in all things. It is also known that the first two, to the great profit of man, have reached the status of recorded arts, viz. arithmetic and geometry; but not so weight, because its fundamental properties have remained hidden from our predecessors. It is true that with regard to vertical weights it has been observed by experience that two gravities of equal apparent weight are proportional to their arms¹⁾. But they thought that this proportionality was due to the circles described about the fixed point by the extremities of the arms²⁾. From which, as is usual with errors, there followed no knowledge of the causes. As to the oblique weights, these were not known at all, with the result that this subject matter could not be shaped into an art like the others. But when the situation changed, and this long-hidden matter was revealed through its fundamental elements, it at last reached this status, in the form in which it is here being dedicated to Your Imperial Majesty. But since, by the thoughtful, nothing is started without any cause and reason, the question

¹⁾ Read: inversely proportional.

²⁾ Cf. the Introduction to the *Art of Weighing*; p. 37.

angheuanghen en wort, soo mocht hier de vraegh van
 reinde mijns doens sijn, te weten of ick na de ghebruyck
 van velen, uwe K. M. tot beschermer mijns wercx ver-
 souck? Verre van daer, so doch de bescherming ende re-
 gering des Rijcx, niet alleen tot sulcx, maer tottet uytlefen
 der voorredens an haer eyghentlick gheschreuen, selden
 eenighe tijdt toelaet: Te meer dat ick van meyning was
 (wie can sijn vermoeden weerstaen?) soo wel Form als
 Stof gheen verdedighing te behouuen. Ten is oock niet
 om met een groote Const, in een grooter spraek eerst uyt-
 ghegaen, den grootsten van Europa te vereeren, hoe lijk-
 formich sulcx nochtans de reden soude mueghen wesen.
 Waerom dan? Op dat de Weeghconstens *daden strec-
 kende tot merckelicke verbetering der Ghemeensaeck, int
 werck ghebrocht worden, van sulcx als daer ick duer be-
 sonder brieuen van v Oetroy af versouck. Waerom sal
 yemant mueghen segghen, dit niet bestelt duer leegher
 (naer de ghebruyck) totten Hoochsten vrie toeganck heb-
 bende? Yghelick, om dattet ronghehoort is, soude vreesen
 niet alleen met een lacherlick *voorstel te verschijnen, maer
 selfs oock belacht te worden: Nu op dat der spotters
 schamp tot ghetuych haerder onwetenheyt strecke, wy
 hebbent duer t'ghene willen versoucken, dat voor den
 verstandighen alsulcx ende meerder veruaet. daerajinan
 wyder en breeder soude connen segghen; Maer want ons
 einde tot Saken streckt, niet tot Woorden, sullen dese ver-
 latende en die verwachtende, uwe K. M. in alle ootmoe-
 dighe eerbieding veel ghelucx wenschen. Uyt Leyden
 in Oogstmaent des 1586^o laers.

Effecta.

Propositione.

might here arise what my object in this is, to wit whether, after the custom of many, I request Your Imperial Majesty to be the protector of my work? Far be it from me, seeing that the protection and government of the Empire seldom leaves any time, not only for this, but also for reading through the prefaces written to Your Imperial Majesty. The more so because I was of opinion (who can resist his own supposition?) that neither the form nor the matter needs any defence. Nor is it my object to honour the greatest man of Europe with a great art, first published in a greater language, however much in accordance with reason this might be. Why then? In order that the effects of the Art of Weighing tending to a considerable improvement of the commonwealth may be realized, to wit those for which I request a patent from you in special letters. Why, someone might say, do not you (according to the custom) set about this by means of inferiors, who have free access to the Greatest? Because it is unheard-of, everyone would be afraid not only to be coming with a ridiculous proposition, but also to be scoffed at. Now in order that the taunts of the scoffers may bear witness to their ignorance, we have attempted to do it by means of that which for the intelligent comprises all this and more. About which we might speak more amply and fully, but because our end is things, not words, we will, leaving the latter and waiting for the former, wish Your Imperial Majesty in all humility and respect much happiness. From Leyden, in Harvest Month of the year 1586.

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SIMON STEVINS
 V Y T S P R A E C K *Elogium.*
 V A N D E W E E R D I C H E Y T
 D E R D V Y T S C H E T A E L.

Is wel waer datter inde Natuer niet wonderlick en is, nochtan tot onderscheyt der dinghen die wy duer de oirsaken verstaen, vande ghene welker redenen ons onbekent sijn, soo gheuen wy dese met recht de naem van wonder, niet dat sijt eyghentlick sijn, maer om dat ter hem voor ons alsoo ghelaet. Twelck soo wesende, wy sullen ons in desen an sien billichlick mueghen verwonderen, duer wat middel de Natuer mocht wercken, doen sy ons voorouders sich haer spraeck dede maken; ouermidts ons van soo constighen werck, der oirsaken ghenouchsaem wetenschap ghebreeft. Maer want een beclaghelicke verblintheit, als duer *T S C H I C S E L veroirdent, t'verstant van *Fatum.* ven alsoo verduystert ofte betoouert heeft, dat sy elicht vande Sonne bouen dat der Sterren, ick meen de weerdicheit deses Taelen bouen al d'ander, niet en connen bemercken, tot groot achterdeel des Duytschen gheslachts; Ghemerct daerbeneuen, dat wy voorgheuen hebben inde selue te beschrijven de W E E G H C O N S T, wiens diepsinnighe *ghe- *Qualitas.* daenten duer slechter spraken ten eersten niet wel bedietlick en sijn, soo sullen wy naer ons vermueghen daer wat af segghen, versouckende of t'uytsterste des S C H I C S E L S bestemden uijts noch niet en naect. ende eerst van haer oudtheit als volght:

Tis te weten dat de Duytschen in die seer oude tijden vande welcke ter weerelt gheen opentlicke schriften ghebleuen en sijn, gheweest hebben een treffelick seermachtich Gheslacht. t'welck duer sulcke verderuende oirsaken als meer ander machughe volcken weeruaren sijn, als oirloghe en dierghelijcke, voorderende uytroeying der wetten, breking van goe oirdens, verwoesting der steden, &c. tot manier van wiltheit gherocht is, doch niet soo volcommen, of den ouden aert der grootmoedicheit, rechtueerdicheit, ende ghetrachtheit, daer Tacitus oock af be- *Li. de Morib. Germanor.* tucht, en bleef al tijt in hemlien ghewortelt. Dese haer woestheit heeft gheduert tot ontrent de tijden van Iulius Caesar, welcke daer naer tot beteren staet begon te keeren, soo dat sy einlick weder ghecomen sijn ter regering ouer t'eertrijcxdeel Europa, als kenlick is. Maer want yemandt van haer voornomde eerste macht twyffelen mocht, ouermits wy

b B

daer

SIMON STEVIN'S
DISCOURSE
ON THE WORTH OF THE DUTCH LANGUAGE.

It is true indeed that in Nature there is nothing mysterious; nevertheless, in order to differentiate between the things we understand through their causes and those whose causes are unknown to us, we rightly call the latter miraculous, not because they are really so, but because they appear so to us. This being so, we may justly wonder in this respect by what means Nature may have operated when she caused our ancestors to frame their language, seeing that we lack sufficient knowledge of the causes of this ingenious creation. But since a deplorable blindness, as if ordained by Fate, has so clouded and bewitched the minds of many people that they cannot perceive the superiority of the light of the sun to that of the stars — I mean the superiority of this language to all others — to the great detriment of the Dutch race; considering further that we intend to describe in this language the Art of Weighing, the profound nature of which cannot well be expounded at once in inferior languages, we will say something about this to the best of our ability, studying to discover whether the time ordained by Fate is not yet drawing to an end, and first we will discuss its antiquity, as follows.

The reader should know that in those very ancient times, of which no public records have been preserved in the world, the Dutch were an excellent, very powerful race, which fell into barbarism through such corruptive causes as other powerful nations have also experienced, such as wars and the like, increasing abolition of laws, disturbance of order, destruction of the cities, etc.; however, not so completely but the ancient character of magnanimity, justice, and loyalty, to which Tacitus also bears witness, always remained rooted in them. This their barbaric state lasted approximately until the days of Julius Caesar, after which their condition began to improve again, so that at last they have regained dominion over the part of the world called Europe, as is apparent. But because someone might doubt their former power, since we have no public records about it,

S. STEVINS

daer af, soo weynich als van veel ander volcken diens tijts, gheen open-
licke schriften en hebben, soo sullen wy die aldus beuestighen.

Tis openbaer dat de Gallen die by ons Walen ghenoecht worden, en-
de int ghemeeen nu Françoysen heeten, ouer oude tijden een machtich
volck gheweest sijn, welcke Griecken, Spaigne, Italie, strengelick be-
crijghden ende ouerwonnen: vande welcke noch Gallogracia ofte Gala-
tia in Griecken, ende Celtiberia in Spaignie, de naem behouden heb-
ben: In Italie stichten sy Milaen, Coma, Brescia, Verona, Bergamo, Tren-
te, Vicentia. De selue Françoysen hebben ofte voormael Duytsch ghe-
sproken, ofte het Duytsch in grooter eere ghehad, ende voor hun wit
ghehouden ghelijck sy nu r'Latijn doen. Teerste wort aldus bethoont:
Tis yder spraeck ghemeen datse inden eenen oirt des landts wat anders
uytgheproken wort als op den anderen. By voorbeelt, daer de Neer-
landers segghen *Dat, Wat, Vat*, d'Ouerlanders segghen *Das, Was, Vas*:
Voor der Parisienen *Chanter, Charbon, Chaleur*, de Picarden ghebruy-
cken *Canter, Carbon, Caleur*; Daer de Castilianen segghen *Hazer, Hierro,*
Harina, in Portugael seymen *Fazer, Ferro, Farina*, &c. Nu alfulck ver-
schil van het Duytsch dat de Françoysen voormael sprake, tottet Duytsch
van dese landen, was, dat sy voor ons *W* int ghemeene ghebruycken *Gu*.
Als daer wy segghen *ick Winde*, sy seyden ende segghen noch *ie Guinde*.
Ende voor ons *Windas* (t'welck een ghecoppelt woort is van Windt en
as, als oftmen wilde segghen een as die windt) sy ghebruycken *Guindas*,
sommighe *Guindal*, vande welcke oock commen hun *Guindement, Guin-*
dant, Guindeur, &c. Wederom voor *Wincket, Wimpel, Want, Wesp, Weet,*
Wincken, Melcwey, Wildemalue, sy segghen *Guichet, Guimple, Guant, Guespe,*
Guedde, Guigner, Megue, Guimalue. Voor *Waren*, ofte *Bewaren*, *Guarder*,
daer af commen la *Guarde, Gardeur, Gardebras, Gardemenger, Gar-*
derobbe, &c. oock *Guarnir, Guarison*, dat me bewaren ende bewaring be-
teeckent, want de * Ghenefers achten dat sy duer drancken, cruyden,
saluen &c. alleenlick t'ghebreck bewaren voor ongheual, ende dat sy
gheensins en heelen, maer dat de Natuer altijt haer seluen gheneest: Van
t'voornomde commen oock *Guarnir, Guarnison, Guarniture*, &c. welcke
oock *Bewaren* ende bewaring bedien: Sghelijcx *Warande*, daer sy *Guaren-*
ne voor nemen. Wederom *Gue* (achterlatende *ch*, die sy soo als wy niet
noemen en connen) dat is by ons *wech*, te weten den wech daer t'water
van een riuier ouer loopt, waer af hun *Gaeer* ende meer ander commen.
Wederom *Guerdonner*, als oft sy wilden segghen *Weerdonner* dat is *Weer-*
gheuen oft verghelden, daer af gheseyt wort *Guerdon, Guerdonnement, Guer-*
donneur, &c. Voorts *Mot de Guet*, dat is *woort vande Wet*, ouermits sulck
woort inde steden vande wet comt, ende duer haer ghegheuen wort: Oft
andersins mach *Guet* van *Wachs* commen, achterlatende *ch* die sy so niet
en ghebruycken als voor gheseyt is, ende *e* voor *a* ghenomen, t'welck by

ons

Medici.

no more than about many other nations of that time, we will prove it as follows.

It is generally known that the Gauls, who are named Walloons among us and are now usually called French, in ancient times were a powerful nation, which waged fierce wars against Greece, Spain, and Italy, and conquered them; from which Gallograecia or Galatia in Greece and Celtiberia in Spain still have their names. In Italy they founded Milan, Como, Brescia, Verona, Bergamo, Trento, Vicenza. These same Frenchmen formerly either spoke Dutch or greatly esteemed the Dutch language, and considered it their example, as they now do with Latin. The first is proved as follows: It is a common feature of all languages that they are pronounced somewhat differently in different parts of the country. For example, where the Dutch say *Dat, Wat, Vat*, the Germans say *Das, Was, Vas*; for the Parisians' *Chanter, Charbon, Chaleur*, the Picards use *Canter, Carbon, Caleur*; where the Castilians say *Hazer, Hierro, Harina*, in Portugal they say *Fazer, Ferro, Farina*, etc. Now a similar difference between the Dutch language formerly spoken by the French and the Dutch of these regions was that they generally used *Gu* for our *W*. Thus, where we say *Ik winde*, they said and still say *Ie Guinde*. And for our *Windas* (which is a compound of *Windt* and *as*, as who should say an *as* that *windt*) they use *Guindas*, some of them *Guindal*, from which are also derived their *Guindement, Guindant, Guindeur*, etc. Again, for *Wincket, Wimpel, Want, Wesp, Weet, Wincken, Melcwey, Wildemalve* they say *Guichet, Guimple, Guant, Guespe, Guedde, Guigner, Megue, Guimalve*. For *Waren*, or *Bewaren*, they use *Guarder*, from which are derived *Garde, Gardeur, Gardebras, Guardemenger, Guarderobbe*, etc., and also *Guarir, Guarison*, which also means to preserve and preservation, for the physicians deem that by means of potions, herbs, ointments, etc. they merely "preserve" the disease from becoming fatal, and that they do not cure men at all, but that Nature always cures itself. From the aforesaid are also derived *Guarnir, Guarnison, Guarniture*, etc., which also mean preserve and preservation; similarly *Warande*, for which they take *Guarenne*. Again *Gue* (omitting *ch*, which they cannot pronounce as we do), that is with us *Wech*, to wit the course taken by the water of a river, from which their *Gueer* and others are derived. Again there is *Guerdonner*, as if they would say *Weerdonner*, that is *Weergheven* or requite, from which are derived *Guerdon, Guerdonnement, Guerdoneur*, etc. Further *Mot de Guet*, that is *Woort vande Wet*, because this word takes the place of the law and is given by it. Or otherwise *Guet* may be derived from *Wacht*, omitting *ch*, which they do not use in this way, as has been said before, and taking *e* for *a*, which is common with us, for

V Y T S P R A E C K.

ons ghemeeen is, want men seght so veel Bert, Swert, als Bart, Swart: Tis ooc kennelick dat sommige *Wecht* voor *Wacht* ghebruycken: Van *Guet* commen *Gueter*, *Guette*, *Guetteur*, &c. Wederom voor *Tet Weere*; *Ala Guerre*, daer af gheseyt wort *Guerroyer*, *Guerroyeur*, &c. Voor *Op de Wyse*; *Ala Guise*, daer af ghemaect wordt *Guisarme*, *Deguiser*, *Deguisement*, ende soo met meer anderen die wy om cortheyt achterlaten. Vyt dese ghemeeene reghel dan van *W* tot *Gu* (bouen de groote menichte van d'ander ghemeeene woorden die sy ghewillick uyt het Duytsch hebben, welcke wy om cortheyt verswyghen, te meer dattet boueschreuen an t'voornemen voldoet) schijnt ghenouch te mueghen besloten worden, de Françoysen voormael Duytsch ghesproken te hebben, dat is Duytschen gheweest te sijne, ende veruolghens dat de Duytschen eertijts een bekent machtich volck waren.

Doch so v dat niet en gheuiel, maer dat sy die woorden voormael uyt het Duytsch vergaert hebben, ghelijck sy nu sulcx uyttet Latijn doen (want een van twee is nootfaelick) t'selue valter uyt te besluyten. Want dat soo ghenomen, tis gheschiet naer hemlien verwoestheyt, daer in, ofte daer vooren: Niet daer naer, want de Duytsche tael by haer sedert in gheen acht gheweest en heeft, maer de Latijnsche, daer sy de hunne naer verandert hebben: Oock niet daer in, want dat een machtich volck t'welck Spaaigne, Grieken, Italie, conde beuechten ende verwinnen, haer spraeck soudent gheformit hebben naer de wildens tael, ten sluyt niet; Dat sulcken Gheslacht vande wilden soude leeren an t'windae een naem gheuen, tis te belachelick, soo sy doch seluer eer dan wilden, windassen ghebruycken. Tis dan nootfaelick dat sy dese woorden na der Duytschen ghemaect hebben voor haer verwoesting, te weten doen sy grooten machtich waren, ende dat yder Gheslacht d'ooghe op hun had.

Hier toe helpt noch dit, dat hemliedertael wyder strecke als ander, t'welck sy in haer woestheyt daer toe niet ghebrocht en hebben, want dat wilden die niet en handelen, noch verre en reysen, haer tael wyder soudent doen verbreyden als ander machtighe volcken, die groote landen en Koninckrijcken onder haer ghebiedt hadden, het strijt teghen den ghemeeenen loop des weerelts, t'waer ongheschiedt sulcx toe te laten. Dese wyde verbreyding dan der tael is gheschiet voor de verwoesting: Waer uyt oock te bemercken is in wat macht sy doen moesten wesen, anghefien t'verstroeyde eenalich ouerblijfsel na soo grooten menichte van iaren, hem wyder strecke dan die teghenwoordelick in groote macht waren: T'sijn voorwaer oirsaken die metgaders d'ander redenen, ons dwinghen te gheloouen, dat de Spaengnaerden voormael oock, of Duytschen gheweest sijn, of dat sy hun tael naer het Duytsch gherecht hebben. want sy, ghelijck de Françoysen, oock segghen *Guante*, *Guardar*, waer af com-

we say both *Bert*, *Swert* and *Bart*, *Swart*. It is also known that some people use *Wecht* instead of *Wacht*. From *Guet* are derived *Guetter*, *Guelle*, *Guetteur*, etc. Again, for *Ter Weere* they use *A la Guerre*, from which are derived *Guerroyer*, *Guerroyeur*, etc. For *Op de Wyse*, they use *A la Guise*, from which are derived *Guisarme*, *Deguiser*, *Deguisement*, and in the same way with several others, which we omit for brevity's sake. From this general rule therefore of *W* into *Gu* (in addition to the great number of other common words which they have undoubtedly borrowed from Dutch and which we omit for brevity's sake, the more so because the above is sufficient for this purpose) it seems we may safely conclude that the French formerly spoke Dutch, that is that they were Dutch, and consequently that in former days the Dutch were a well-known and powerful nation.

But if this should not satisfy you, and you should think that they formerly borrowed these words from Dutch as they now do from Latin (for one of the two is necessary), the same conclusion can be drawn from it. For on this assumption it must have happened after, during or before their period of barbarism. Not after this period, for the Dutch language has not been held in esteem by them since that time, but rather the Latin tongue, on which they have modified their own. Not during this period either, for it is not plausible that a powerful nation, which could fight Spain, Greece, and Italy, and conquer them, should have modelled their speech on the language of the barbarians; it is all too ridiculous to assume that such a race should have learned from the barbarians to give a name to the windlass, since they themselves surely used windlasses before the barbarians did. Therefore it cannot be but that they have modelled these words on those of the Dutch before their period of barbarism, that is when they were great and powerful, and the eyes of all nations were upon them.

This is supported by the fact that their language was more widely distributed than any other, which they have not achieved in their period of barbarism, for it is contrary to the common course of the world that barbarians, who do not trade or make long voyages, should succeed in propagating their language more widely than other powerful nations, which ruled over large countries and kingdoms; it would be absurd to admit such a thing. Therefore this wide propagation of the language took place before the period of barbarism; from which it may also be gathered how powerful they must have been at that time, since the dispersed monolingual remnant is scattered further afield after such a long time than the nations which are powerful in our days. These are truly causes which, combined with the other reasons, compel us to believe that formerly even the Spaniards either were Dutch or modelled their language on Dutch, for — just like the

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men *Guarda, Guardador, Guardoso*, oock *Guarida*, ende *Guarnecer, Guarnicion, &c.* Wederom *Guerra*, daer sy af segghen *Guerrear, Guerreador, Guerrero, &c.* Wederom *Guinar*. Voort *Guisá*, daer *Guisar* af comt, &c. Ia dat etlicke Indianen welcke men segt veel duytsche woorden te ghebruycken; oock verscheyden ander contreyen in Asie wiens spraeck Hieronymus betuycht tsiender tijt bycans de selue gheweest te sijne met die van Trier; ende meer ander volcken wiens talen met Duytsch ghemengt sijn, sulcx uyt het Duytsch hebben, al van die oude tijden af, dat sy in haer eerste groote macht waren.

Effectum.

Tvoornomde wort noch stercker, opentlicker, ende nootsakelicker beuesticht, duer haer talens constich maecsel, voorwaer gheen werck van slechte wilden, maer te verwonderen hoe gheleerde tammen, sulcx hebben connen ter * daet brengen. daer af wy breeder spreken, ende haer weerdicheyt bouen al d'ander, met merckelicke redenen beuestighen moeten, aldus: Teinde der spraken is, onder anderen, te verclaren 'inhoude des ghedachts, ende ghelijck dat cort is, also begheert die verclaring oock cortheyt, de selue can bequamelicx gheschien, duer ynckel faken met ynckel gheluyden te beteecken; Oock soodanighe, datse oueral de T'saemvoughing bequamelick lijd; Datse de Consten grontlick leeren; Ende den Hoorders heftelick beweghen tot des sprekers voornemen. Nu of dese alle vier, byden Duytschen beter ghetrossen sijn dan by eenighe ander, dat sullen wy oirdentlick verclaren, eerst bethoonnende, ende dat metter daet, op datment ghelooue, der Duytschen 742 eenfilbighe woorden inden eersten persoon; daerder de Latinen alleenlick 5 hebben; De Griecken gheen eyghentlicke, maer langhe vercort tot 45. Daernaer sullen wy segghen vande namen, bynamen, &c. welcke wy alle metter haest vergaert hebben yder uyt sijn Woortbouck als volgt.

DUYTSCH

French — they also say *Guante*, *Guardar*, from which are derived *Guarda*, *Guardador*, *Guardoso*, also *Guarida* and *Guarnecer*, *Guarnicion*, etc. Again *Guerra*, from which they derive *Guerrear*, *Guerreador*, *Guerrero*, etc. Again *Guinar*. Further *Guisa*, from which is derived *Guisar*, etc. Nay, we are even bound to believe that several tribes in India, which are said to use many Dutch words, also many other regions in Asia, whose speech is stated by St. Jerome ¹⁾ to have been almost identical with that of Treves in his day, and other nations whose languages are interspersed with Dutch, have borrowed this from the Dutch, already from those ancient times when they were at the height of their power.

The above is proved even more strongly, clearly, and inevitably by the ingenious structure of their language, which is certainly not the work of simple barbarians; it is even to be wondered at how learned civilized beings have succeeded in effecting this, a subject which we must discuss more fully, while confirming with clear arguments its superiority to all the other languages, as follows. The object of language is, among other things, to expound the tenor of our thought, and just as the latter is short, the exposition also calls for shortness; this can best be achieved by denoting single things by single sounds ²⁾; also in such a way that in every respect they properly admit of composition; that they thoroughly teach the arts; and that they violently move the hearers to act after the speaker's intention. Now we will set forth systematically that all these four points have been hit off better by the Dutch than by any other people, first showing — such with facts, so that the reader may believe it — the 742 Dutch monosyllabic words in the first person, where the Latins have only 5 and the Greeks have no monosyllables proper, but only 45 long words that have been contracted. After that we will deal with the nouns, adjectives, etc., all of which we have hurriedly collected from the respective dictionaries, as follows:

¹⁾ As is well known, St. Jerome (347-c.420) passed a part of his youth at Treves. The passage from St. Jerome that Stevin has in mind seems to be: Hieronymus, *Comment. in epist. ad Galatas lib. II*; prol. cap. 3 (Migne, *Patrologia Latina* Vol. XXVI. col. 382 C): *Unum est quod inferimus . . . Galatas excepto sermone Graeco, quo omnis Oriens loquitur, propriam linguam eandem pene habere quam Treviros, nec referre, si aliqua exinde corruerint, cum et Afri Phoenicum linguam nonnullam ex parte mutaverint, et ipsa Latinitas et regionibus quotidie mutetur et tempore* (communicated by Dr J. W. Ph. Borleffs, The Hague).

²⁾ The term „single sound” is naturally used here in a different sense from that of modern phonetics. It here means: what is pronounced in a single effort of speech.

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V Y T S P R A E C.
D V Y T S C H E E E N S I L B I G H E V V O O R D E N.

A cht. l'estime. Estimatio.	C aerd. le cardé. Carmino.	Inedo cuprum aqua forti.
Acs. l'apaste. Inciso.	Caert. le ioué aux cartes. Ludo chartis.	Eych. le demande. Peto
B ack. le cuis. Pisto.	Caets. le ioué la paume. Ludo pila	Fael. le faille. Fallor
Baect. Pono pharum.	Cap. le hache. Concido	Fluyt. le ioué à la flûte. Canso stula.
Baen. le prepare le chemin. Præparatio viam.	Cler. l'orne. Orno	Fryt. le fresse. Frigo
Ban. le barne. Proscribo.	Clær. le fai clair. Clarifico	Frons. le front. Rugo
Baer. l'enfante. Pario.	Colf. le croche. Lugo claua	G aen. le voy. Eo
Bas. l'abbaye. Latio.	Cop. le cariste. Scarifico	Gaep. le baye. Oscito
Baet. le prouffite. Commodus sum.	Cort. le couste. Consto	Gect. le moque. Lasciuio
Bel. le tire la clochette. Tintinno.	Coot. le ioué aux os. Talis ludo.	Gheef. le donne. Do
Ben. le suis. Sum.	Cook. le cuisine. Coquo	Gheel. le fay laune. Ruto
Berit. le creue. Creps.	Crab. le racle. Rado	Gheui. le basille. Oscito
Bey. l'attens. Expecto.	Cræy. le crie. Cornicor	Gheld. le vau. Valeo
Bid. le prie. Precor.	Cræch. le croque. Crepito	Ghier. le fonde. Fundo
Biecht. le confesse. Confiteor.	Croon. le couronne. Coronio	Ghisp. le fouette. Flagello
Bied. l'offre. Presento.	Cruys. le crucife. Crucifigo	Ghis. le souspeçonne. Suspicio
Bies. le bere. Mugio.	Cuyt. le faictes. Vicio dolia.	Glat. le polie. Polio.
Bijt. le mors. Mordeo.	D ag. le patrouille. Palpo	Glic. le giste. Labor.
Bind. le lie. Ligo.	Dacl. le descens. Descendo.	Gloey. le deuen rouge. Candefco.
Black. le flamboy. Flammo.	Danch. le remercie. Habeo gratiam	Gom. le goume. Lino gummi.
Blac. le couleure de bleu. Colore	Dans. le danse. Tripudio	Gord. le cède. Cingo.
Blaculo pingo.	Dau. le fai rose. Roro	Grac. l'engraue. Sculpo.
Black. le couvre de lames. Bracteo	Deck. le couure. Tego	Greys. le refongne. Obduco fronté
Black. le paffi. Palleo.	Delf. l'enfou. Fodio	Grip. le gripe. Capio.
Bleer. le beelle. Balo.	Denck. le pense. Cogito	Grim. le rugit. Rugio.
Bleyck. le blanchi du linge. Candefacio.	Derf. le poste. Audco	Groey. le verdoie. Verno.
Bliff. le demeure. Manco.	Derf. le bats en grange. Trituro	Grou. l'ai en horreur. Abomines
Blinck. le labeure. Resplendo.	Derf. l'ay besoing. Egco	Groen. le pain verd. Virido.
Block. le labeure assiduellement. Assidu laboro.	Deys. le recule. Recedo	Gruet. le salue. Saluto.
Bloc. le saigne. Sanguino.	Dicht. le compose en rime. Cõpono	Gun. le fauorise. Fauco.
Bloey. le fleuronne. Floreo.	Dick. l'espèss. Denso	L aack. le hache. Concido.
Bloot. le mets nud. Nudo.	Dien. le fers. Seruio	Haest. l'haite. Festino.
Bloos. le vermeillon. Rubeo.	Dier. le fay profond. Profundum facio.	Haack. l'accroche. Iunco.
Blufch. l'estain. Extinguo.	Dijck. le fay vne dique. facio aggeré	Hacl. le porte. Adfero.
Bluts. le froisse. Collido.	Ding. le barguine. Licoor	Hang. le pend. Pendo.
Boerd. le bourde. Nugor.	Ding. le plaide. Litigo	Harp. l'harpe. Lyræ pulso.
Boet. le remedie. Medeor.	Doem. le damne. Damno	Haet. le hay. Odio habeo.
Boey. le mets des pieges aux pieds Compedio.	Doe. le fai. Facio	Heb. l'ai. Habeo.
Boock. le bats. Cudo.	Dool. l'erre. Erro	Hecht. le attache. Figo.
Bol. le boule. Voluo globum.	Doo. le tue. Occido.	Heel. le guarri. Sano.
Boord. le borde. Fimbrio.	Doog. le vau. Valco	Heet. le caufe. Calefacto.
Boor. le forte. Perforo.	Doop. le baptize. Baptizo.	Heet. le nomme. Nomino.
Borg. le pleige. Fidei iubeo.	Dor. le deuen aride. Arecco	Heet. le commande. Iubeo.
Bot. le rebouche. Hebetor.	Dorif. l'ay foit. Sitto	Hef. le leue. Leuo.
Bot. le boutonne. Gemmo.	Dou. le preff. Premio	Hel. le panche. Acclino.
Bou. l'edific. Aedifico.	Draey. le tourne. Torno	Heel. le cele. Celo.
Bra. le rosti. Asto.	Draep. le porte. Porto	Help. l'aide. Iuuo.
Brack. le vome. Euoio.	Dracl. le tarde. Tardo.	Herd. le durcis. Duro.
Brand. le brule. Ardeo.	Dracl. le trotte. Succusso.	Hey. le hie. Fiftuco.
Bras. le bauffre. Epulor.	Dreich. le menace. Minor	Hygh. l'abaine. Anhele.
Brec. le fay large. Dilato.	Drijf. le chaffe. Agito.	Hinck. le cloche. Claudico.
Breack. le romps. Rumpo.	Dring. le poule. Penetro turbam	Hiz. l'incite. Instigo.
Brey. le entre lache. Reticulo.	Drooch. le seiche. Sicco	Hoed. le garde. Custodio.
Brich. le rugis. Rugio.	Droom. le fong. Somnio	Hoef. le touse. Tuffio.
Bring. le apporte. Adporto.	Droop. le arroie quelque chose de greffe. Conspergo pinguedine.	Hol. le creufe. Cauo. (Cumulo).
Brock. le coupe des merces aux de pain. In frusta frango.	Druck. l'imprime. Imprimo	Hoop. le comble en monceaux.
Brod. le radoub. Relardo.	Dub. le doute. Dubito	Hoor. l'oy. Audio.
Broc. le couue. Incubo ouis.	Ducht. l'ai doute. Vercoor	Hoop. l'espere. Spero.
Broey. le fourbouille. Subfructuacio	Duer. le dure. Duro	Houd. le tiens. Teneo.
Brou. le brasse. Coquo cereuissiam	Duld. le souffre. Patior	Hou. le coupe. Seco.
Brul. le murle. Mugio.	Dun. le tenue. Extenuo	Hou. le marie. Nubo.
Bruyck. l'ufe. Fruo	Dvael. l'erre. Erro	Hoy. le fene du foin. Sicco forni sole
Buck. le plie dos. Curuo	Dwing. le contrains. Cogco	Huc. le croupe. Sido.
Burt. le troque. Commuto	E er. le honnore. Honoro	Huer. le loue. Conduco.
Buyg. le pie. Flecto	Eg. l'herse. Occo	Hul. le coiffe. Orno caput.
Buyck. le fappe. Pulso	Eind. le fine. Finio	Hurs. l'hoche. Quatio.
	Eet. le menge. Edo	Huyt. l'hurle. Viulo.
	Eet. le mors en cuire de l'eau forte	I aack. le chaffe. Venor.
		Laack. le glappe. Gannio.
		Inck. le moque. Icor.
		Kaack. le cabasse. Suppilo.
		Kan. le fay. Scio.

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Kau. Je marche. Mando.
Keer. Je tourne. Verto.
Keer. Je baille. Scopo.
Kem. Je peigne. Pecto.
Kem. Je cognoy. Cognosco.
Kern. Je lamente. Lamentor.
Kern. Je bats le beure. Butyrū pulso
Keur. Je visite. Genfo.
Kick. Je gronde. Musfo.
Kies. Je lis. Eligo.
Kijck. Je voy. Intruor.
Kuff. Je tence. Litigo.
Kip. Je clos poulins. Pullulo.
Klack. Je creusse avec vn son esclatant. Cum fragore rimas ago.
Klad. Je corte. Penicillo vestem à luto detergo.
Klaeg. Je plains. Queror.
Klap. Je baille. Fabulor.
Kler. Je veste. Vestio.
Klem. Je pince. Premo.
Klers. Je frappe du fouet le faisan sonner. Scutica ferio.
Klesf. L'attache. Visco.
Klein. L'appetisse. Minuo.
Kliet. Je fends. Fendo.
Klim. Je monte. Scando.
Klinck. Je sonne. Sono.
Kluen. Je frappe. Pulso.
Kloot. Je boue. Voluo globum.
Kloos. Je hurte. Pulso.
Kloos. Je ouë à la boule par trauers d'vn anneau. Ludo globo per annulum.
Kluicht. Je plaifante. Facetas narro
Knaeg. Je orange. Rodo.
Knau. Je marche. Mando.
Kne. Je peffris. Deplo.
Kners. Je grince les dens. Denti-
bus strideo.
Knich. Je phoce la teste. Nuo.
Knich. Je agnouille. Geniculator.
Knip. Je chiquenaude. Talitratrain-
Knop. Je noue. Necto. (digo).
Knor. Je gronde. Grunio.
Kocl. Je fait tied. Tepido.
Kom. Je viens. Venio.
Koep. Je chaspte. Emo.
Kout. Je deusse. Fabulor.
Kraets. Je gratte. Scabo.
Krau. Je gratte. Scabo.
Kranck. Je debile. Debilito.
Kriel. Je remue comme entre les
fourmis. Mobilito per turbas.
Krijg. Je acquiers. Acquiror.
Krijt. Je pleure. Eiulo.
Krimp. Je retrecis. Arcto.
Krooch. Je geins. Queror.
Kroock. Je fronce. Rugo.
Krol. Je crepille. Crispo.
Kroin. Je courbe. Curuo.
Krop. Je pemple le gouion. Ingla-
uiem facio anium.
Kruy. Je poulse. Vipello.
Kruy. Je pspice. Aromatibus condio
Kruyp. Je rampe. Repo.
Kuch. Je touffe. Tuffito.
Kus. Je baife. Ofculor.
Kuyfch. Je nettoye. Nitido.
Lach. Je ris. Rideo.
La. Je charge. Onero.
Laeck. Je diminue. Diminuo.
Laeck. Je desprise. Contemno.
Lang. Je alonge. Prolongo.
Lang. Je aueins. Porrigo.
Lap. Je rapitte. Interpole.

Laet. Je laisse. Linquo.
Laef. Je rafraichis. Foveo.
Laeu. Je fait tiede. Tepido.
Leck. Je lache. Lambo.
Leg. Je abbaisse. Humille.
Leem. Je tendus d'argille. Luto.
Leem. Je preste. Mutuo.
Leer. Je peigne. Docco.
Leg. Je mets. Pono.
Leck. Je coule. Stillo.
Leen. Je pappuis. Cubito.
Lees. Je lis. Lego.
Lefch. Je peins. Extinguo.
Let. Je empeche. Impedio.
Let. Je confidere. Confidero.
Leef. Je vis. Viuo.
Ley. Je meine. Duco.
Licht. Jeclairis. Luceo.
Licht. Je leue. Leuo.
Lieg. Je ments. Mentior.
Lig. Je couche. Iacco.
Lijd. Je endure. Fero.
Lijp. Je begaye. Balbutio.
Lock. Je palliche. Allico.
Loer. Je lorne. Obseruo.
Lol. Je grongonne. Eiulo ad
instar felis.
Lol. Je me chauffe comme les viel-
les qui vent d'vn pot plein de feu
le mettant sous elles.
Lonck. Je ocillade. Oculo.
Loon. Je baille de loyer. Remunero
Loop. Je cours. Curro
Los. Je delaische. Lazo.
Loot. Je iette le fort. Sortior.
Loef. Je loue. Laudo.
Lub. Je chaste. Castro.
Lul. Je finge selon. Sonum imitor.
Luyt. Je lorne. Insidiantibus
oculis intueor.
Luys. Je espulche des pouils. Pedi-
culos lego.
Lic. Je passe. Transeo.
Mach. Je puis. Possum.
Macy. Je fauche du foin. De-
feco prata.
Maek. Je fay. Facio.
Macl. Je peins. Pingo.
Macl. Je moule. Molo.
Maen. Je stipule. Stipulor.
Maen. Je coniure. Adiuro.
Melck. Je traite le lait. Mulgeo.
Meng. Je melle. Misceo.
Men als peerden oft vvaghen. Je
meyne. Duco.
Merck. Je marque. Noto.
Meit. Je engraisse. Sagino.
Meet. Je mesure. Metior.
Mets. Je massonne. Extruomuros.
Mein. Je cuide. Opinor.
Mick. Je ay l'oeil à quelque chose.
Collimo.
Myc. Je contregarde. Cauco.
Min. Je aime. Amo.
Moet. Il me faut. Debeo.
Moey. Je moleste. Molesto.
Moord. Je meurtris. Trucido.
Mor. Je murmure. Murmuro.
Muf. Je sens le relant. Sitū redoleo
Munt. Je mannoye. Cudo nummos.
Muyt. Je rechigne. Contraho vultū
Muyt. Je mutine. Seditionem facio
Naey. Je coufe. Suo.
Naack. Je approche. Propinquo
Nau. Je fay estroit. Angusto.
Neem. Je prena. Accipio.

Nest. Je niche. Nidifep.
Nyg. Je incline. Inclino.
Nies. Je sternue. Sternuo.
Nieu. Je fay nouveau. Noue.
Nijp. Je pince. Vellico.
Noem. Je nomme. Nomino.
Noo. Je huite. Inuito.
Noop. Je aguillonne. Stimulo.
Oogst. Je mouffonne. Medem
facio.
Oos. Je vuide l'eau. Exhaustio.
Pacl. Je borne. Termino.
Paet. Je mets pair à pair. Bi-
nos pono.
Paert. Je partis. Partior.
Pacy. Je payse. Peco.
Pand. Je mets en gage. Pignero.
Pap. Je colle. Glutino.
Pas. Je le fais accorder. Apto.
Peck. Je poiffe. Pico.
Pecl. Je pece. Decortico.
Pers. Je perist. Premo.
Pick. Je beque. Rostro.
Piip. Je pipe. Pipo.
Pis. Je pisse. Meio.
Plaeg. Je vex. Vexo.
Plasch. Je pateline en l'eau. In
aqua palpo.
Pleck. Je macule. Macule.
Pleeg. Je foulois. Soleo.
Pleyt. Je plaide. Litigo.
Ploeg. Je are. Aro.
Plomp. Je rebouche. Hebeo.
Plomp. Je plonge en l'eau. Mergo
Ploy. Je pic. Plico.
Pluck. Je cueille. Carpo.
Pluys. Je pluche. Polio.
Poch. Je vante. Iacho.
Pomp. Je vuide lofset. Sentinam
expurgo.
Pnog. Je tafche. Nitor.
Por. Je incite. Incito.
Pot. Je amasse en pot. In ollulis
coacervo. (nibus).
Poy. Je boy. Indulgeo potatio-
Præm. Je oppresse. Opprimo.
Prang. Je oppresse. Opprimo.
Praet. Je baille. Fabulor.
Prick. Je aguillonne. Stimulo.
Pretu. Je derobbe à nemet. Surripio.
Pris. Je prise. Laudo.
Print. Je imprime. Imprimo.
Proef. Je pousse. Probo.
Proef. Je goute. Gusto.
Pronck. Je tiens grauité comme
vne espouffe.
Put. Je pisse. Haurio aquam.
Quel. Je fache. Molesto.
Quets. Je bleffe. Lædo.
Quitt. Je degaste. Dissipo.
Quijl. Je baur. Stillo pituitā ex ore
Ra. Je aduine. Dimino.
Raack. Je touche. Tango.
Raep. Je amasse. Colligo.
Raes. Je rage. Lasciuio.
Reck. Je tens. Tendo.
Ren. Je cours. Curfio.
Reul. Je retroque. Commuto.
Rey. Je danse. Duco choras.
Reyck. Je tends. Porrigo.
Reys. Je chemine. Proficifcor.
Richt. Je rig. Erigo.
Rieck. Je sens. Sentio.
Riem. Je rame. Remigo.
Rie. Je cheuache. Equito.
Rijm. Je rhyme. Versifico.

ENSYLLABISCHE VOORDEN.

Ryp. le meurt. Præcoque.	Suer. le trompe. Impomo.	Glomere. (concito).
Rys. le meleur. Subleuo.	Seyl. le vogué. Veliſco.	Spoor. l'esperonne. Calcaribus
Rips. le route. Ruſto.	Sie. le boullis. Feruco.	Spot. le moque. Derideo.
Reck. l'estrique la quenouille.	Sie. le voy. Video.	Spou. le craiche. Spuo.
Pensumstruo.	Sift. le crible. Cribro.	Spreck. le parle. Loquor.
Rœru. le mevente. Iafco.	Sijp. le degoute. Mano.	Sprey. l'estens. Tendo.
Rœp. leerie. Clamo.	Sinck. l'enfonse. Mergo.	Spring. le saute. Salto.
Rœr. le remue. Mouco.	Sing. le chante. Canto.	Spruyt. le germe. Germino.
Rœst. le enrouille. Rubiginé traho	Sit. l'assis. Sedito.	Spuel. le reinse. Bluo.
Rœy. le rame. Remigo.	Slab. le baue. Squalco.	Spuyt. le iecte l'eau par vn esclif-
Rol. le rouille. Voluo.	Sla. le frappe. Verbero.	soire. Ejecto aquam syringe.
Ronck. le ronfle. Sterto.	Slacht. le ressemble de condition.	Stack. le paiffele. Palo.
Rond. l'arondis. Rotundo.	Affimilor.	Sta. le me tiens sur les pieds. Sto
Rœof. le rautis. Spolio.	Slæp. le dors. Dormio.	Stack. le cesse. Desito.
Rot. le pourris. Putreo.	Slap. le lasche. Laxo.	Stamp. le pile. Tundo.
Ruck. l'arrache. Auellio.	Slact. le travail comme esclave.	Stap. l'aiambe. Pleno gradu incedo.
Ruet. l'engraisse. Sebo.	Laboro.	Steck. le pique. Pungo.
Rua. le coagule. Coagulo.	Slem. le faibonne chere. Comeſſor.	Steel. le decrobbe. Furor.
Run. le flotte. Fluo.	Sleur. le traine. Traho.	Stek. le poſe. Pono.
Ruſt. le repose. Quiesco.	Sleyp. le traine. Traho.	Stelp. l'estanche le sang. Sisto
Ruym. l'amplisse. Amplifico.	Slicht. le fais vni. Plano.	fanguinem.
Ruyſch. le bruy. Strepo.	Slijp. l'aguise. Acuo.	Stem. le baille ma voix. Suffra-
Sæy. le seme. Semino.	Slijt. l'vſt. Tero.	gium fero.
Sæp. le ſe. Serro.	Slis. l'appaise. Paco.	Stern. le ahenne. Gemo.
Sælf. l'oung. Vugo.	Slick. l'aualle. Gurgito.	Sterf. le meurt. Morior.
Scha. l'endommage. Nocceo.	Slof. le trouffe. Replico.	Streu. l'appuye. Fulcio.
Schaf. le traiche. Traſto.	Slorp. l'hume. Sorbeo longotratu.	Steyl. le dresse contremont. Erigo.
Schack. le prens vne fille par for-	Sluym. le fommeile. Dormito.	Sticht. le fonde. Fundo.
ce. Rapio virginem.	Sluyt. le voy en cachette. Clau-	Stijg. le monte. Scando.
Schaem. l'honnis. Erubesco.	culum ingredior.	Stijf. le roidis. Rigeo.
Schamp. le glisse. Labor.	Sluyt. le ferme. Claudio.	Stil. l'appaise. Paco.
Schantis. le fortifie rempars. Mu-	Smacht. le suffoque. Suffoco.	Stinck. le sens mal. Oboto.
nio vallis.	Smack. le rue de roideur. Projicio	Stoor. le trouble. Turbo.
Schat. l'estime. Aſtumo.	Smaeck. le gouffe. Guſto.	Stoot. le hurte. Trudo.
Schæf. le rabotte. Dolo.	Smal. l'attenué. Extenuo.	Strop. l'estoupe. Obthuro.
Scheer. le tends. Tondeo.	Sme. le forge. Cudo.	Stort. l'espans. Effundo.
Scheld. le tenſe. Obiurgo.	Smeeck. le amadoué. Adulor.	Stoof. l'estuue. Fouco.
Schel. le ſcorche. Decortico.	Smeer. l'oins. Linio.	Stou. l'incite de faire ou d'aller.
Schenck. le verse. Infundo.	Smelt. le fonde. Fundo.	Propello.
Schend. le gaste. Corrumpto.	Smet. le macule. Maculo.	Strael. le rayonne. Radio.
Schep. le cret. Cret.	Smets. le bauffe. Comeſſor.	Straf. le punis. Punio.
Scheru. l'escrime. Digladior.	Smoeck. le fume. Fumo.	Streck. le tens. Tendo.
Scherp. l'aguise. Acuo.	Smoor. l'estouffe. Suffocor.	Streel. le peine. Peſto.
Scherf. le bace. Concido.	Smijt. le bats. Verbero.	Strick. le noue. Nodo.
Schuer. le deſchire. Lacero.	Snap. le baille. Garrio.	Stroop. l'escorche. D-glubo.
Scheyd. le ſepare. Separo.	Snau. le parle rudement. Duri-	Stroey. l'espars. Spargo.
Schick. l'ordonne. Ordino.	rer loquor.	Strijck. le frotte. Linio.
Schiet. le tire. Sagitto.	Snic. le coupe. Scindo.	Stric. le combat. Pugno.
Schijn. le luis. Luceo.	Snoep. le friande en cachette. Clam-	Stier. le conduict. Duco.
Schil. le differe. Difſero.	cupedias edo.	Stuyt. l'encline. Inclino.
Schimp. le brocarde. Iocor.	Snoer. l'enſle. Filo traicio.	Stuyt. le vante. Iafco.
Schock. le secoue. Succuſſo.	Snoey. l'esbranche. Frondo.	Stuyt. le bonds. Refulto.
Schors. le trouffe. Succingo.	Snoeck. le ſanglotte. Sterto.	Stuyt. le pouldroye. Puluero.
Schau. le contemple. Contemplor	Snuyt. le mouche. Mungo.	Sucht. le ouſpire. Suſpiro.
Schrab. l'efgrattigne. Vnguibus	Soeck. le cherche. Quzro.	Suf. le radotte. Deliro.
icabo.	Soen. le reconcille. Reconcilio.	Suyg. le tette. Sugo.
Screu. le ecrieſclatant. Exclamo.	Soog. l'aleſte. Laſco.	Suym. le chome. Moror.
Screy. le pleure. Lachrymo.	Sorg. l'ay ſoing. Curo.	Suyt. l'hume. Sorbeo.
Schrick. le faillis. Diſſilio.	sout. le fale. Salio.	Svack. l'afolbit. Inſiruo.
Scroef. le vire. Torqueo cochleam	Spa. l'houe. Fodio.	Svvaer. l'appellant. Grauo.
Seroem. l'effrye. Horreo.	Span. le tends. Tendo.	Svalp. le ſore. Aſuo.
Schrie. l'aiambe. Faciogradum.	Spær. l'espargne. Parco.	Svvert. le noircis. Nigro.
Schrijff. l'ecrrips. Scribo.	Speck. le larde. Lardo.	Svvert. le ſue. Sudo.
Schud. le secoue. Quaiſſo.	Speel. le ioue. Ludo.	Svvelg. l'aualle. Glutio.
Schup. l'houe. Palcaleuo.	Spen. le ſeure. Ablactio.	Svvel. l'enſle. Tumco.
Schur. l'escure. Tergo. (occludo	Spect. le tends. Tendo.	Svven. le nage. Nato.
Schut. le contregarde. Affamentis	Spect. l'embroche. Figo veru.	Svver. le iure. Iuro.
Schu. l'euite. Euito.	Speur. le trache. Indago.	Svveru. l'eschene. Examinio.
Schuil. le m'embuſche. Lateo.	Spic. le cheuille. Clauis lignis figo	Svicht. le ceſſe. Ceſſo.
Schuyf. le coute. Trudo.	Spic. l'espie. Inſidior.	Svrig. le tais. Tacco.
Seep. le ſauonne. Saponellnio.	Spin. le file. Nco.	Tael. l'enquiers. Inquiro.
Seg. le di. Dico.	Spits. le ſar poiure. Acuo.	Tap. le tire. Prouo.
Send. l'enuoie. Mitto.	Spit. le ſue. Fodio.	Tal. l'entraſte. Acerbo.
Seng. le brulle quelque peu. Suburo	Splijt. le ſeuds. Findo.	Taſt. le taſte. Palpo.
Set. le mets. Pono.	Spoey. le me haſte. Accelero.	Tees. l'espliche. Carpo.
Stel. le mets. Pono.	Spoel. le deuide au fil pour tistre.	Tel. le nombre. Numero.

Tem. Iedempte. Donoe.
 Tems. Ietamide. Cribro.
 Tcer. Ietendre. Coguo cibum.
 Tier. Ietempere. Tumultuo.
 Toef. Ietaters. Expefo.
 Toel. Ie donne gabelle. Tributū da.
 Ton. Ie poutonne. Infundū in dolia.
 Toog. Ie monstre. Ofendo.
 Toom. Ie bride. Freno.
 Toon. Ie sonne. Tono.
 Top. Ie ioue de la coupie. Trocho
 Iudo.
 Tau. Ie ranne. Coria perficio.
 Tracht. Ie delibere. Delibero.
 Traen. Ie larmoye. Lachrymo.
 Tract. Ie tire. Traho.
 Terd. Ie marche. Gradio.
 Tref. Ie touche. Tango.
 Trooft. Ie console. Confolator.
 Trau. Ie pouse. Ducuo vocem.
 Treur. Ie contristie. Mereo.
 Tuyg. Ie lefmoinne. Testor.
 Tuyn. Ie penuironne de hayes. Sepio.
 Tuyfch. Ie ioue à dets. Alealudo.
 Tuvist. Ie s'urie. Alterco.
 Tuvyn. Ie tors du sil. Torqueo silū
 Val. Ie tombe. Cado.
 Valsch. Ie saulce. Falso.
 Vang. Ie prans. Capio.
 Vaer. Ie voy par chariot ou par
 traire. Meco.
 Vast. Ie ieuze. Ieiuno.
 Vaet. Poutonne. Infundū modios.
 Vaet. Ie pempoinne. Comprehendo.
 Vecht. Ie combats. Pugno.
 Vel. Ie abbats. Profero.
 Verg. Ie metcs au deuant. Propono.
 Vest. Ie confirme. Confirmo.
 Veyl. Ie metcs en vente. Venale
 propono.
 Veins. Ie dissimule. Dissimulo.
 Vier. Ie faifseudeioye. Celebro
 Vulcania.

Do. ie donne. ick geeef. No. ie naige. ick (v)em. Sum. iesuis. ick ben.
Flo. iesouffle. ick blaes. Sto. iesuis debout. ick stae.

Bδῶ	Bδiō of Bδῆμι	Kεῶ	Kεiō ende Kīrō	Σχῶ	Σχιō
Bλδ	Bλῆμι	Kτῶ	Kτiā ende Ktiō	Σῶ	Σiā ende Σiō
Bῶ	Biō	Kῶ	Kiō ende Kīō	Τλῶ	Τλiō
Γiῶ	Γiōiō	Λῶ	Λiō	Τμῶ	Τμiō
Γεῶ	Γeōiō	Μiῶ	Μiāiō of Mīiō	Τεῶ	Τeōiō
Γῶ	Γiō ende Γiō	Nῶ	Niō of Niō	Τῶ	Tiō
Δμῶ	Δmāiō of Δmīō	Ξῶ	Xiō	Φiῶ	Φiāiō Φθiā Φθiō
Δεῶ	Δeāiō	Πλῶ	Πlāiō of Πliōiō	Φλῶ	Φlāiō ende Φliō
Δῶ	Δiō ende Δiδōμι	Πνῶ	Πniō of Πniōiō	Φρῶ	Φrīō
Ζῶ	Ziā ende Ziō	Πτῶ	Πtāiō of Πtῆμι	Φῶ	Φiō Φῆμι
Θλδ	Θlāiō	Πῶ	Πiō Πῆμι	Χεῶ	Χrīōiō χrīōiō
Θνῶ	Θiōiō of Θnētiō	Ρῶ	Ρiō Ρiōiō cñ Eriō	Χῶ	Χiō χiōiō χrēiō
Θῶ	Θiō	Σκλῶ	Klīō	Ψῶ	Ψiō ende Ψiō
Κλῶ	Klāiō, κlīō, κlīōiō	Σμῶ	Σmiō of Σmīō	Ω	Ωiō
Κεῶ	Kēiō of κεῆμι	Σπῶ	Spāiō	Ω	Ωiō

Dander

D' A N D E R D V Y T S C H E Y N C K E L

GHELVDEN, ALS DER NAMEN, BYNAMEN, VOORSET-

tinghen, &c. *fin in ghetale tot 1418 de Latijnsche (tot de ysaem-
vougung onbequaem) alleenlick 158 de Grijsche 220 Als volght.*

D V Y T S C H E E E N S I L B I G H E N A M E N , B Y N A M E N , & c .

A cht. Huist. Octo.	Bie. Mouche à miel. Apes.	Broeck. Marec. Palus.
Acl. Anguille. Anguilla.	Bier. Bierre. Cercuisa.	Broeck. Brayette. Subligaculum.
Acm. Caque. Cadus.	Bies. Ionc. Iuncus.	Broer. Frere. Prater.
An. Aupres. Apud.	Biest. Caille. Colostre.	Broot. Pain. Panis.
Aep. Sing. Sima.	Bladt. Focuille. Folium.	Broosch. Fragile. Fragilis.
Aer. Elye. Spica.	Blas. Soufflement. Flarus.	Brug. Pont. Pons.
Aert. Complexion. Complexio.	Blaes. Vessie. Vesica.	Bruyck Vlage. Vfus.
Aes. Apaill. Elica.	Blaeu. Bleu. Cæculus.	Bruydt. Eipouse. Sponsa.
Aes. Hache. Afcia.	Black. Focuille ou lame de quelque metal. Lamina.	Bruyn. Brun. Beticus color.
Af. Ius. De.	Bleek. Palle. Pallidus.	Bry. Boullie de farine de panis. Puls
Al. Tout. Totus.	Blein. Empouille. Pustula.	Buel. Bourreau. Carnifex.
Alf. Fee. Fatifer.	Blic. Joyeux. Hilaris.	Buer. Vofin. Vicinus.
Als. Quand. Cum.	Blindr. Aueugle. Cæcus.	Buyt. Butin. Præda.
Am. Nourrice. Nutrix.	Block. Tronc. Truncus.	Bult. Boffe. Gibbus.
Ampt. Office. Officium.	Blect. Sang. Sanguis.	Burn. Fontaine. Fons.
Angit. Anxiété. Anxietas.	Bloem. Fleur. Flos.	Bus. Canon. Tormentum.
Arm. Bras. Brachium.	Blont. Blont. Flauus.	Bur. Boite. Pyxis.
As. Effieu. Axis.	Bloo. Timide. Timidus.	Buyck. Ventre. Venter.
B ack. Auge. Linther.	Blout. Nud. Nudus.	Buil. Gibeciere. Mariupium.
Badt. Bain. Balneum.	Bock. Bouc. Hircus.	Buil. Boffe. Tuber.
Baeck. Machoire. Maxilla.	Bo. Menagier. Nuncius.	Buis. Canal. Canalis.
Baeck. Pharus.	Boeck. Liure. Liber.	By. Pres. Propt.
Bael. Bale. Sarcina.	Boef. Ribaud. Nebulo.	C acl. Chauue. Caluus.
Baen. Parterre. Sphaeristerium.	Boel. Amoureuse. Amica.	Caen. Caniffure. Canus.
Baer. Bierre. Erettrum.	Boer. Villageois. Rusticus.	Caerd. Chardon. Virga Pastoris.
Baer. Onde. Vnda.	Boerd. Bourde. Nugæ.	Caets. Chaffe. Metæ.
Baerf. Barbe. Barba.	Boett. Penitence. Penitentia.	Caf. Paille. Acus.
Baerfch. Perche. Perca.	Boey. Piege. Pedica.	Calck. Chau. Calx.
Baer. Hôte. Herus.	Boog. Arc. Arcus.	Cant. Bord. Extremitas.
Baer Gaing. Commodum.	Bolck. Molue. Molus.	Cap. Cappe. Cuculla.
Baeg. Bague. Monile.	Bol. Boule. Globus.	Car. Chariot. Carrus.
Bal. Effieu. Pila.	Bom. Bedon. Tympanum.	Caes. Fourmage. Caseus.
Balch. Panche. Bestiarum venter.	Bont. Fourrure. Pelles.	Car. Caffé. Capsa.
Balck. Poutre. Trabs.	Boom. Arbre. Arbor.	Car. Chat. Felis.
Bald. Incontinent. Breui.	Boon. Febue. Faba.	Cau. Chucas. Monedula.
Ban. Excommunication. Excommu- nicatio.	Boord. Bord. Margo.	Cijs. Cens. Censur.
Banck. Banc. Scammum.	Boos. Mauuais. Malus.	Cier. Chere. Vultus letus.
Bandt. Lien. Vinculum.	Bout. Bateau. Scapha.	Claer. Clair. Clarus.
Bang. Anglois. Anxius.	Borg. Bourg. Castrum.	Clerck. Clerc. Clericus.
Bar. Present. Præfens.	Boor. Vibrequin. Terebrum.	Cloof. Fente. Fiffura.
Barg. Porceau chaftré. Maialis.	Bors. Bourfe. Bursa.	Cluit. Farce. Facetie.
Bas. Abbey. Latratus.	Borst. Poitrine. Pectus.	Cock. Cuiffier. Coquus.
Bait. Canepin. Scheda.	Bofch. Bois. Sylva.	Colf. Massue. Claua.
Bast. Har. Laqueus.	Bot. Petoncle. Passer.	Com. Escuille. Scutella.
Bat. Micux. Melior.	Bot. Stolidé. Stolidus.	Comst. Venue. Aduentus.
Bay. Bayette. Badiuscolor.	Bot. Boutonde fleur. Gemma.	Cond. Notoire. Notus.
Beck. Bec. Roftrum.	Bou. Hardi. Audax.	Coord. Corde. Chorda.
Bed. List. Leçus.	Bout. Bougeon. Sagitta capitata.	Cop. Chef. Caput.
Be. Petition. Petitio.	Brack. Sentant la marine. Marinum.	Cop. Coupe. Calix.
Beelt. Image. Imago.	Brack. Bracque. Canis legax.	Coff. Corbeille. Corbis.
Beemt. Pral. Pratum.	Bracy. Legras de la jambe. Sura.	Corst. Crouste. Crassa.
Been. Os. Os.	Braem. Ronce. Rubus.	Cort. Court. Curtus.
Been. Iambe. Crus.	Brand. Vn grand feu brulant vne maison ou semblable. Incendium.	Coft. Coust. Sumptus.
Beer. Verrat. Verrus.	Brau. Sourcil. Supercilium.	Cot. Taniere. Caus.
Beer. Ours. Vrfus.	Breet. Large. Latus.	Coot. Offelet. Talus.
Beest. Beste. Bestia.	Breuck. Amande. Mulcta pecuniaris.	Cous. Chausse. Caliga.
Bel. Clochette. Timinabulum.	Brief. Lettre. Littere.	Coy. Estable à brebis. Ouille.
Ben. Banne. Sporta.	Brijn. Saumure. Muria.	Crab. Escruffte. Cammarus.
Berd. Ais. Affer.	Brij. Lunette. Specillum.	Craem. Boutique. Officina.
Berg. Mont. Mons.	Brim. Genest. Genista.	Craen. Robinet. Epistomium.
Best. Micux. Melior.	Brock. Vn petit morceau du pain taillé ou rompu. Frustum.	Craen. Grue. Grus.
Bey. Tous deux. Ambo.		Cray. Corneille. Cornix.
Biecht. Confession. Confessio.		Crap. Garance. Erythrodanum.
		Croon. Couronne. Corona.

cC

Cruca

Croyn. Sommet. de la tette. Ver-
tex capitis.
Croys. Croix. Crux.
Cuy p. Cauce. Cupa.
Cuy t. Oeufs d'un poisson. Ouspiscia
Dach. Jour. Dies.
Dack. Toist. Tectum.
Daet. Effect. Effectus.
Dac.r. La. Ibi.
Dal Val. Vallis.
Dam. Terrain. Agellus.
Damp. vapeur. vapor.
Dan. Donc. Tunc.
Danck. Gre. Gratia.
Dans. Danse. Saltatio.
Darm. Boiau. Intestinum.
Das. Daim. Dama.
Dat. Ce. Hoc.
Dau. Roter. Ros.
De. Le. Illa.
Deech. Parte. Massa.
Deel. Part. Pars.
Deern. Servante. Ancilla.
Den. Le.
Des. Du.
Dees. Celui. Hic.
Deuch. vertu. virtus.
Dicht. Solide. Solidus.
Dicht. Fin. Richmus.
Dick. Espec. Densius.
Dic. Cuisse. Femur.
Die. Le. Ille.
Dief. Laron. Latro.
Di-p. Profond. Profundus.
Dier. Animal.
Dier. Cher. Carus.
Dies. Attecondition. Subcondi-
tione.
Dije. Dique. Agger.
Dinc. Chose. Res.
Disch. Table. Mens.
Dir. Ceci. Hoc.
Doch. Aumoins. Saltem.
Doer. Alors. Tunc.
Doec. Toile. Tela.
Doel. Bur. Scopus.
Door. Par. Per.
Door. Huis. Fores.
Doost. Duuet. Plumulae molliores.
Doort. Mort. Mors.
Doof. Sourd. Surdus.
Doop. Baptême. Baptismus.
Door. Fol. Scultus.
Doos. Boitte. Capsa.
Dop. Escaille entier d'un oiseau quand
le dedens est oité. Ouum exinanit.
Dorp. village. Pagus.
Dorit. Soif. Sitis.
Doy. Degel. Regelatio.
Dra. Incontinent. Strepitus.
Drac. Dragon. Draco.
Dract. Fil. Filum.
Draf. Bran. Furfur.
Draf. Troc. Succussatio equi.
Dranc. Beuvrage. Potus.
Drec. Bone. Lutum.
Dreec. Vne longue rangee d'arbres
plantees. Series arborum.
Droef. Triste. Tristis.
Droes. vne emasure venant à la gor-
ge derriere les aureilles ou es cines.
Panus.
Droom. Le fil de la treme du tisse-
rant. Lignum.
Bronc. Yvre. Ebrius.
Drooch. Sec. Siccus.

Droom. Songe. Somnium.
Druc. Tristesse. Tristitia.
Drop. Goute. Gutta.
Druyf. Gzappe. Vna.
Dric. Trois. Tres.
Dul. Enragé. Furiosus.
Dun. Tenue. Tenuis.
Duyt. Mille. Mille.
Duym. Poulce. Pollex.
Duyt. Dune. Agger arenosus.
Duyt. Colomb. Columba.
Dvvaet. Touaille. Mantile.
Dvvas. Sot. Stultus.
Dvvan. Contraincte. Vis.
Dvvec. Mol. Mollis.
Dvveers. Detrauers. Extransuerso.
Dvverch. Nain. Nanus.
Dy. Toi. Tibi.
Dijn. Tien. Tuus.
Echt. Mariage. Matrimonium.
Eec. Vinaigre. Acetum.
Eedr. Serment. Iuramentum.
Een. Vn. Vnus.
Eer. Auant. Prius.
Eer. Honneur. Honor.
Eerd. Terre. Terra.
Eerst. Premier. Primus.
Eg. Here. Occa.
Eist. Onze. Undecima.
Eist. Aloft. Alofta.
El. Aulne. Vlna.
Eind. Fin. Finis.
Eng. Estroict. Angustus.
Erech. Maling. Malignus.
Erp. Pauvre. Pauper.
Erust. A bon escient. Serid.
Erf. Heritage. Hereditum.
Esch. Fresne. Fraxinus.
Ey. Oeuf. Ouum.
Eyc. Chêne. Quercus.
Eyfch. Pétition. Petitio.
Fael. Faute. Error.
Faem. Renom. Fama.
Fest. Feste. Festum.
Fiel. Felon. Crudelis.
Feil. Fante. Error.
Fieft. Gucu. Mendicus.
Fier. Fier. Ferus.
Fijn. Fin. Exilis.
Fiesch. Flacon. Lagena.
Flux. Subit. Subito.
Fluyt. Fleume. Phlegma.
Fluyt. Fluite. Fistula.
Foc. Trioquet. Artemon.
Foey. Phi.
Form. Forme. Forma.
Fracy. Ioli. Lepidus.
Fret. Foret. viciosa.
Frisch. vigoureux. viuulus.
Fruit. Fruit. Fructus.
Gacr. Pur. Purus.
Gacr. Totalement. Ompino.
Gay. Gai. Pstacus.
Gal. Fiel. Fel.
Galgh. Gibbet. Patibulum.
Galm. Rérentissemr de la voix. Echo.
Ganc. Allure. Incessus.
Ganc. Allée. Ambulacrum.
Gans. Oie. Anser.
Gants. Entier. Integer.
Garst. Ranci. Rancidus.
Gast. Hoste. Hospes.
Gat. Trou. Foramen.
Gaef. Don. Donum.
Gau. Prompt. Promptus.
Gec. Badin. Sannio.

Geel. Jaune. Rufus.
Gheen. Nul. Nullus.
Gheest. Esprit. Spiritus.
Gheft. Argent. Pecunia.
Gheft. Porceau chaire. Malafis.
Gheft. Lot. Quatuor hominaz.
Ghent. Iar. Anser mas.
Geur. Fonte. Fusura.
Gheft. Cheure. Capra.
Ghi. vous. Tu.
Ghier. vautour. vultur.
Ghist. Don. Donum.
Ghild. Homme liberal. Prodigus.
Ghins. verila. Illuc.
Ghist. Lie. Fex.
Ghants. Resplendeur. Splendor.
Glas. verre. vitrum.
Glat. Poli. Politus.
Goudt. Dieu. Deus.
Goedt. Bon. Bonus.
Gom. Gomme. Gummi.
Gort. Orge seiche. Hordeum aridum.
Goot. Ruiffeau. Aqueductus.
Goudt. Or. Aurum.
Gracht. Fosse. Fossa.
Graef. Comte. Comes.
Graen. Grain. Granum.
Gract. Areste. Arista.
Graf. Sepulchre. Sepulchrum.
Gram. Courroucé. Iratus.
Gras. Herbe. Gramen.
Gracu. Gris. Glaucus.
Grents. Frontiere. Ora.
Grep. Poigne. Manipulus.
Griel. Gripaille.
Gris. Gris. Canus.
Groef. Fosse. Fovea.
Gros. Gros. Grossus.
Gron. Fond. Fundum.
Groot. Grand. Grandis.
Grou. Horreur. Horror.
Gruen. verd. viridis.
Gruet. Saluation. Saluatio.
Gruis. Moillon. Rudus.
Gunth. Faveur. Favor.
Guir. Gucu. Mendicus.
Hacr. Croc. Hama.
Hacr. Haie. Sept.
Haen. Coc. Gallus.
Haer. Poil. Pilus.
Haest. Haie. Properatio.
Haer. Haine. Odium.
Half. Demi. Dimidius.
Hals. Col. Collum.
Ham. Iambon. Pernis.
Handt. Main. Manus.
Harp. Harpe. Lyra.
Hars. Resine. Resina.
Haes. Lieure. Lepus.
Hecht. Manche. Capulus.
Hec. Claie. vacerra.
Heel. Tout. Totus.
Heerd. Fouier. Focus.
Heer. Seigneur. Herus.
Heedh. Enroué. Rancus.
Heet. Chaud. Calidus.
Hel. Clair. Clarus.
Held. Homme Noble. Herus.
Hel. Enfer. Infernus.
Helm. Heaume. Galea.
Hem. Lui. Illi.
Hemd. Chemise. Indulum.
Hengh. Etallon. Caballus.
Hert. Dur. Durus.
Herst. Automne. Autumnus.
Herst. Etchine. Spina porci.

ENSLIGHE VOORDEN

Hert. Cœur. Cor.	Kemp. Chanure. Cannabia.	Kruym. Mle. Mles.
Hesp. Iambon. Versus.	Kerck. Eglise. Templum.	Kruydt. Herbe. Herba.
Her. Ce. Id.	Kerf. Crenne. Crena.	Kud. Troupeau de bestes. Grex.
Heur. Sieme. Sua.	Kern. Pepin. Semen.	Kund. Notoire. Notus.
Hensch. Courrois. Ciuilis.	Kers. Guine. Cerafa.	Munst. Art. Ars.
Heus. Anse. Ansa.	Kers. Creston. Narflutium.	Mus. en baifer. Basium.
Hex. Sorciere. Venefica.	Kert. Cren. Crena.	Muyt. Spelonque. Spelunca.
Hey. Lande. Campus Aerialis.	Keur. Chois. Optio.	Muytch. Chaste. Caltus.
Hey. Hie. Fiftuca.	Keurs. Coriet. Cycias.	Muyt. Le mol derrière la lampa.
Heyl. Salus. Salus.	Key. Caillon. Cautus.	Sura.
Heur. Armée. Exercitus.	Kiel. Carine. Carina.	Muyt. Petite biere.
Hic. Hoquet. Singultus.	Kim. Cipeau d'Yntonneau. Oravasis.	Lach. Ris. Risus.
Hiel. Talon. Talus.	Kindr. Enfant. Puer.	Lary. Layette. Capsa.
Hier. Ici. Hic.	Kim. Menton. Mentum.	Lact. Tard. Sero.
Hind. Biche. Cerna.	Kist. Coffre. Cista.	Laf. Fade. Flaccidus.
Hin. Pouille. Gallina.	Kit. Boisson. Brochus.	Laegh. Rang. Series.
Hirs. Mil. Milium.	Klacht. Querelle. Querels.	Lam. Affoible. Paralyticus.
Hoe. Comment. Quomodo.	Klack. Creuasse. Crepichio.	Lam. Agneau. Agnus.
Hoeck. Coing. Angulus.	Klad. Crote. Maculaluti.	Lamp. Lampe. Lampas.
Hoer. Pouille. Gallina.	Klamp. Membrane d'en huis. Mem-	Lanc. Flanc. Femen.
Hoer. Paillarde. Meretrix.	brum aferis.	Lanc. Long. Longus.
Hoest. Toux. Tuffis.	Klanck. Tintement. Tinnigmentum.	Landt. Terre. Terra.
Hoet. Chapeau. Pileus.	Klap. Babil. Loquacitas.	Lap. Piece de drap. Segmentum.
Hoef. Maître. Villa.	Klau. Patte. Vnguis.	Lait. Charge. Moles.
Hof. Iardin. Hortus.	Klect. Vêtement. Vestis.	Lar. Larc. Adula.
Hoir. Heritier. Heres.	Klef. Attachant comme glu. Tenax.	Lacu. Tiede. Tepidus.
Mol. Caue. Causus.	Kley. Argille. Argilla.	Leals. Lecschip.
Mondr. Chien. Canis.	Klein. Petit. Parvus.	Leer. Cuir. Corium.
Hoorth. Hault. Altus.	Klier. Apotreme. Tonfilla.	Leech. Oïff. Otiofus.
Hoest. Tête. Caput.	Klinck. Clichet. Pessulus.	Ledt. Membre. Membrum.
Hoop. Monceau. Aceruna.	Klis. Grateron. Aparine.	Leech. Bas. Humilis.
Hoop. Bipoir. Spes.	Klock. Cloche. Campana.	Lecc. Lay. Laicus.
Hop. Moubelon. Lupulus.	Kleek. Hardi. Audax.	Leer. Eschelle. Scala.
Hop. Hupe. Vpupa.	Kloet. Rabie. Rutabulum.	Leedt. Desplaisir. Luctus.
Hord. Caye. Crates.	Klomp. Fillet. Massa.	Leem. Argille. Argilla.
Hoos. Chauffe. Caliga.	Kloot. Boule. Sphera.	Leen. Fief. Prædium beneficiarium.
Hout. Bois. Lignum.	Klop. Coup. Idus.	Leop. Chasseux. Lippus.
Hou. Compde Caille. Iaus.	Kloof. Creuasse. Rima.	Leop. Caureux. Astutus.
Hoy. Fein. Foenum.	Kluobt. Farce. Facies.	Leop. Doctrine. Doctrina.
Hulp. Aide. Auxilium.	Kluys. Hermitage. Sacellum.	Leot. Forme de corduanier. Forma.
Hult. Boue. Aquifolia.	Knap. Struteur. Seruus.	Leou. Lion. Leo.
Hupfch. Elegant. Elegans.	Knecht. Carion. Seruus.	Lia. Le mollet du bout de l'oreille.
Hut. Loge. Mapale.	Knick. Hocement de la tache. Natus.	Carilago.
Huyt. Lutte en la gorge. An-	Knif. Genouil. Genu.	Leen. Appuy. Podium.
gina.	Knip. Chiquenaude. Talitrum.	Lest. Dernier. victimus.
Huyck. Huque. Cucullus.	Knol. Naveau. Napus.	Lets. Laiffe. Loxum.
Huyt. Cheueche. Vlula.	Knoop. Neud. Nodus.	Leur. Ranauderie. Resnullius valoris.
Huyt. Maison. Domus.	Knop. Bouton. Bulla.	Ley. Ardoise. Ardosis.
Hy. Il. Ille.	Ker. Vache. Vacca.	Licht. Lumiere. Lux.
Yas. Qui. Ita.	Kock. Gateau. Libum.	Licht. Legier. Leuis.
Iacht. Chasse. Venatus.	Koel. Tiede. Tepidus.	Licht. Poulmon. Pulmo.
Iacr. An. Annus.	Koen. Hardi. Audax.	Liet. Chanfon. Cantio.
Ick. Je. Ego.	Koets. Couche. Cubile.	Lief. Cher. Charus.
Iet. Quelque chose. Aliquid.	Kool. Charbon. Carbo.	Lief. Ami. Amicus.
Iuecht. Jeunesse. Iuventus.	Kool. Chou. Brassica.	Lier. Lire. Lyra.
Ys. Glace. Glacies.	Koop. Achapt. Emptio.	Lijc. Funerailles. Exequiz.
In. En. In.	Korck. Liege. Suber.	Lijf. Corps. Corpus.
Inf. Encre. Atramentum.	Koudt. Froid. Frigus.	Lijm. Colle. Colla.
Iock. Joue. Jugum.	Kout. Deuis. Fabula.	Lijn. Lin. Linum.
Iock. Raillerie. Iocus.	Kracht. Force. Virtus.	Lijst. Bordure. Limbus.
Iouck. Jeune. Iuuenis.	Krack. Son estillant. Crepitus.	Lind. Tillet. Tilia.
Is. Est. Est.	Kraegh. Gouion. Iugulus.	Lint. Ruben. vitta.
Kack. Machoire. Maxilla.	Kramp. Crampe. Spasmus.	Lip. Leure. Labium.
Kacy. Cai. Acha.	Kram. Crampon. Fibula.	Lis. Rante. Carex.
Kack. Pilon. Numelle versatiles.	Kranck. Debile. Debilis.	Lift. Finesse. Astutia.
Kalf. Veu. Vitulus.	Krans. Chapeau de fleurs. Sertum.	Loen. Lourdaut. Idiota.
Kam. Peigne. Pecten.	Kreest. Escrueille. Cancor.	Lof. Los. Laus.
Kan. Pot. Amphora.	Krib. Creche. Praefepium.	Lont. Meiche.
Kans. Chanse. Causus alex.	Krick. Cerise. Cerasus.	Looc. Des aulx. Allium.
Kant, als brood. Chanteau. Frustum.	Krijck. Guerre. Bellum.	Loof. Foeuille. Frons.
Keel. Garderobe. Supparum.	Krijch. Crois. Creta.	Loogh. Lexius. Lixiuum.
Keer. Tour. Circuitus.	Krijt. Braiment. Elusatio.	Leon. Salaire. Salarium.
Keers. Chandelle. Candela.	Krocs. Goblet. Scyphus.	Loop. Cours. Curfus.
Kelck. Calice. Clix.	Krom. Tortu. Tortus.	Loos. Subtil. Subtilis.
Keel. Gorge. Guttur.	Krop. Cropion. Ingulum.	Loos. Poulmon. Pulmo.
Kpes. Fourmage. Causus.	Kruyck. Cruche. Vna.	Loot. Plomb. Plumbum.

S. STEVINS

Loof. Armier d'une maison. Vm.
braculum.
Loos. Mot de guet. Tesseræ.
Los. Delle. Laxus.
Lofch. Loufche. Strabo.
Lot. Sort. Sors.
Lucht. Air. Aer.
Lul. Refonance d'une chanfon.
Lust. Volupté. voluptas.
Luy. Pareilleux. Ignavus.
Luis. Pouil. Pediculus.
Luyt. Luc. Testudo.
Macht. Puissance. Potestas.
Macch. Affin. Affinis.
Maccht. vierge. virgo.
Mael. Malle. Mantica.
Mael. Foie.
Maen. Lune. Luna.
Maent. Mois. Mensis.
Maer. Mais. Sed.
Maer. Bruict. Rumor.
Maer. Mesure. Mensura.
Maer. Compagnon. Socius.
Maegh. Estomach. Stomachus.
Mal. Fol. Stultus.
Mals. Tendre. Tener.
Mam. Mammelle. Mamma.
Man. Homme. vir.
Manck. Boiteux. Claudus.
Mat. Matz. Malus.
Mat. Las. Defectus.
Me. Auec. Cum.
Mec. Garance. Rubra.
Meel. Farine. Farina.
Meeps. Fragile. Fragilis.
Meer. Mer. Mare.
Meers. Hunc. Carcassum.
Meer. Plus. Plus.
Meersch. Mare. Palus.
Mees. Maufrange. Paris.
Meest. Tout le plus. Plurimus.
Meeu. Oiseau marin. Aquila marina.
Melck. Laïc. Lac.
Men. Nourrice. Nutrix.
Men. On.
Mensch. Homme. Homo.
Merch. Moelle. Medulla.
Merck. Marque. Signum.
Merc. Marché. Forum.
Mes. Couiteau. Culter.
Met. Auec. Cum.
Mey. May. Frons festa.
Mier. Fourmi. Formica.
Mili. Lieue. Milliare.
Mijn. Mon. Meus.
Mijne. Mine. Fodina.
Mijt. Mite. Mira.
Milt. Liberal. Liberalis.
Min. Moins. Minus.
Mis. Faute. Defectus.
Mest. Fiens. Fimus.
Mist. Bruyne. Bruma.
Moe. Las. Lassus.
Moer. Mere. Mater.
Moes. Porcé. Holus.
Moet. Courage. Animus.
Moey. Tante. Matertera.
Mol. Taulpe. Talpa.
Mondt. Bouche. Os.
Moort. Meurtre. Internecio.
Mos. Mouffe. Muscus.
Mout. Mouit. Mustum.
Mot. Teigne. Tinea.
Mout. Grain apparcillé pour brasser
de la biere. Polenta.
Mau. Manche. Manica.

Moy. Orné. Ornatus.
Muer. Mur. Murus.
Muf. Relant. Situs.
Mug. Moucheron. Culex.
Munt. Monnoie. Moneta.
Murs. Boner. Pileus.
Muil. Mulet. Mulus.
Muyt. Mufkau. Rostrum.
Muys. Sourris. Sorex.
Muyt. Muc. Cauea.
Na. Apres. Post.
Nacht. Nuit. Nox.
Naecht. Nud. Nudus.
Naem. Nom. Nomen.
Naen. Nain. Nanus.
Nacht. Tout le plus prochain. Pro-
ximus.
Nact. Cousture. Sutura.
Nap. Platcrux. Catinus.
Nues. Nez. Nasus.
Nat. Mouillé. Madidus.
Nau. Étroit. Strictus.
Neck. Chainon. Cervix.
Neer. Bas. Inferus.
Neef. Neveu. Nepos.
Neen. Non. Non.
Neep. Pinfure. Compressio.
Neft. Nid. Nidus.
Net. Net. Nidus.
Net. Retz. Rete.
Neet. Lende. Lens.
Nicht. Nieppe. Neptis.
Nier. Rein. Ren.
Niet. Rien. Nihil.
Nieu. Nouveau. Novus.
Nijdt. Enuie. Invidia.
Nech. Encore. Adhuc.
Noen. Midi. Meridies.
Noo. A regret. Inuitus.
Noort. North. Septentrion.
Noot. Nécessité. Necessitas.
Nop. Floe. Florens.
Nuet. Noix. Nux.
Noit. Jamais. Nunquam.
Nu. Maintenant. Nunc.
Nut. Vile. Vtilis.
Och. Ah. Hei.
Oft. Ou. Vel.
Olm. Orme. Vlnus.
Om. Pour. Ob.
Ons. Nostre. Noster.
Oock. Aussi. Etiam.
Oog. Oeil. Oculus.
Oogft. Moisson. Messis.
Oom. Oncle. Patruus.
Oor. Aureille. Auris.
Oort. Lieu. Locus.
Oost. Orient. Oriens.
Op. Dessus. Super.
Os. Beuf. Bos.
Oudt. Viel. Vetus.
Oyt. Onques. Vnquam.
Pacht. Ferme. vestigal.
Pack. Fardeau. Sarcina.
Pael. Pau. Palus.
Paer. Paire. Par.
Paert. Part. Pars.
Palm. Paulme. Palma.
Pand. Hypothèque. Pignus.
Pande. Pand. Lacinia.
Pan. Pacille. Sartago.
Pap. Papin. Pappa.
Pas. En point. Commodum.
Par. Sentier. Semita.
Pacu. Paon. Pauo.

Pais. Paix. Pax.
Peck. Poix. Pix.
Peert. Cheval. Equus.
Peersch. Pers. Coruleus.
Pels. Peau. Pellis.
Pen. Plume. Calamus.
Pens. Trippe. Intestina.
Perck. Parc. Septum.
Peer. Poire. Pirum.
Pers. Presse. Torcular.
Peer. Corde d'arc. Chorda arcus.
Pijp. Tuyau. Tubus.
Pier. Vers de terre. Lumbricus.
Pijck. Pique. Hasta.
Pie. Manteau à marinier. Nautica.
penula.
Pijl. Flesche. Sagitta.
Pijn. Doleur. Dolor.
Pin. Baston pointu. Verruculum.
Pips. Pepie. Piruita.
Plack. Ferule. Ferula.
Plaet. Planché. Syrtex.
Plaets. Place. Locus.
Plaegh. Vexation. Vexatio.
Plauck. Planché. Planca.
Plas. Marée. Lacuna.
Plat. Plat. Planus.
Pleck. Tasche. Macula.
Pleit. Bateau large & plat. Sclata.
Plicht. Office. Officium.
Ploech. Charrue. Aratrum.
Plomp. Rebouché. Hebes.
Ploy. Plis. Plica.
Pluym. Plume. Pluma.
Pock. Verolle. Lues veneres.
Peer. Pouldre. Pulvis.
Poel. Lac. Lacuna.
Pol. Concubinaire. Concubinus.
Pols. Pouls. Pulsus.
Pomp. Offe. Sentina.
Pondt. Liure. Pondo.
Poort. Porte. Porta.
Poos. Petit espace de temps. Mo-
mentum.
Poort. Patte. Palmipedis.
Pop. Poupée. Pupa.
Post. Posteau. Postis.
Post. Laposte. Cursor.
Poort. Sion. Tala.
Pot. Pot. Olla.
Pracht. Magnificence. Magnificencia.
Prat. Pier. Arrogans.
Prick. Lamproie. Mufula.
Priem. Poinçon. Pugionculus.
Prijs. Pris. Laus.
Proef. Preuve. Proba.
Pruym. Prume. Prunum.
Pruts. Superbe. Superbus.
Prye. Charogne. Cadaver.
Punt. Point. Punctum.
Put. Puis. Puteus.
Puyft. Empoule. Pustula.
Quact. Mauvais. Malus.
Quael. Langueur. Languor.
Quart. Gallant. Scitus homo.
Quijt. Quête.
Radt. Roue. Rota.
Raedt. Conseil. Consilium.
Raem. Chassis. Fulcrum fenestra.
quadratum.
Raep. Naueau. Rapum.
Ram. Belier. Arics.
Ramp. Malheur. Infelicitas.
Ranck. Branche. Ramus.
Ranck. Finesse. Astutia.
Ranck. Gresse. Gracilis.

Randt

EENSILBICHE VOORDEN.

Randr. Bord. Ora.
Rafch. Soudain. Cito.
Rafp. Rape. Scalprum.
Rat. Rat. Glis.
Raeft. Corbeau. Corvus.
Razu. Cru. Crudus.
Recht. Droit. Rectus.
Ree. Biche. Cerva.
Reep. Cercle. Circulus.
Reit. Rete. Residuum.
Rueck. Odeur. Odor.
Ruc. Chienmaffe. Canis mas.
Rues. Geant. Gigas.
Key. Danfe. Choreas.
Reyn. Pur. Purus.
Reys. Fois.
Reys. voyage. Profectio.
Reb. Coite. Coita.
Riet. Canne. Arundo.
Riem. Ceinture. Cingulum.
Riem. Rame. Remus.
Rijck. Riche. Dives.
Rijm. Gelée. Pruna.
Rijm. Rhythme. Rhythmus.
Rijp. Meur. Maturus.
Rijs. Riz. Oriza.
Rijs. Branche. Ramus.
Rinck. Anneau. Annulus.
Rindt. Beuf. Bos.
Ring. vifte. velox.
Rin. Cage. Cages.
Rinck. Auccnement fur. Subacidus
Roch. Rave. Raia.
Rock. Saie. Toga.
Rock. Ogenouille. Colus.
Roe. verge. virga.
Roedt. Suide de cheminée. Fuligo.
Roef. Poupe. Puppis.
Roem. vanterie. Iactantia.
Roep. Cri. Clamor.
Roer. Gouvernail. Gubernaculum.
Roest. Enrouilleure. Rubigo.
Roogh. Oeuf de poisson. Ovipiscium
Rog. Seigle. Siligo.
Rol. Roule. Phalanga.
Rondt. Rond. Rotundus.
Rook. Fumée. Fumus.
Root. Rouge. Ruber.
Roof. Butin. Præda.
Room. Crème. Cremor.
Roos. Rose. Rosa.
Ros. Roux. Rufus.
Ros. Ceual. Equus.
Nor. Pourri. Putris.
Rot. Bende de gens. Classis.
Rou. Rude. Rudis.
Ruet. Suif. Seumum.
Rug. Dos. Dorsum.
Rups. Chenille. Bruchus.
Ruit. Repos. Quies.
Ruim. Ample. Amplus.
Ruyt. Hongre. Cantherius.
Ruyt. Rue. Ruta.
Ruyt. Lozange. Tesseræ.
Ric. Rang. Series.
Sacht. Mol. Mollis.
Sac. Sac. Saccus.
Sacl. Selle. Ephippium.
Sacl. Salle. Atrium.
Saen. Toit. Statum.
Saen. Crème. Cremor.
Sart. Semence. Semen.
Sary. Siette.
Sarg. Sie. Serra.
Sacc. Cause. Causa.
Salf. Onguent. Unguentum.

Salm. Saulmon. Salmo.
Sandr. Arene. Arena.
Sap. Suc. Succus.
Sack. Tombe. Cippus.
Sadt. Saul. Satar.
Saus. Sauffe. Condimentum.
Saut. Sel. Sal.
Schacht. Flefche. Sagitta.
Scha. Dommage. Damnum.
Schacu. Ombre. Umbra.
Schack. Eschetz. Alueus.
Schael. Tasse. Patra.
Schacp. Brebis. Ovis.
Schærd. Test. Ruma.
Schaers. A peine. Vix.
Schæts. Eschaffe. Gralla.
Schalck. Caut. Cautus.
Schamp. Brocard. Scommæ.
Schand. Dishonneur. Ignominia.
Schants. Rempart. Vallum.
Schaer. Grande multitude de gens.
Caterua.
Schat. Thresor. Thesaurus.
Schæf. Rabor. Dolabra.
Schec. Gaine. Vagina.
Schæf. Bihay. Obliquus.
Schæf. Louche. Lufcus.
Schæf. Creue de la tefte. Separatio coma.
Schæf. Couuercle. Operculum.
Schæf. Force. Forfex.
Schel. Sonnerre. Tympanabulum.
Schel. Escorce. Cortex.
Schelm. Mefchant. Nequam.
Schelp. Coquille. Calix.
Schenck. Don. Donum.
Scheen. Creue de la jambe. Tibia.
Scherf. Test. Testa.
Scherp. Agu. Acutus.
Scheur. Fente. Fidura.
Scheut. Scion. Surculus.
Schicht. Dard. Isculum.
Schier. Tantoit. Mox.
Schijn. Lueur. Splendor.
Schijf. Tableau. Mensa.
Schil. Difference. Differentia.
Schilt. Escu. Scutum.
Schimp. Brocard. Iocus.
Schip. Nauiere. Nautis.
Schoe. Soulier. Calceus.
School. Ecole. Schola.
Schof. Sole. Solea.
Schoof. Gerbe. Fascis spicarum.
Schoon. Beau. Pulcher.
Schoot. Giron. Gremium.
Schors. Escorce. Cortex.
Schout. Preteur. Prætor.
Schrab. Egratigneure. Laceratio vnguium.
Schraegh. Trefteau. Fulcrum menfariū.
Schram. Berlafe. Vibex.
Schre. Adiambée. Passus.
Schreer. Cri. Clamor.
Schreer. Traict. Tractus lineæ.
Schrift. Escripture. Scriptura.
Schoef. Escroue. Cochlea.
Schub. Escaille de poisson. Squamma
Schud. Vautneant. Scurra.
Schult. Debre. Debitum.
Schup. Pelle. Pala.
Schuer. Grange. Granarium.
Schurft. Rougneux. Scabiosus.
Schu. Saunage. Agrestis.
Schuym. Escume. Spuma.
Schuyt. Naffelle. Nauticula.
Se. Coûtume. Mos.

See. Mer. Mare.
Seel. Grosse corde. Funis.
Seem. Sameau. Cortium hazadinum.
Seep. Saumon. Sapo.
Seer. Vlcere. Vlcus.
Seer. Fort. Valde.
Self. Mefme. Ipsum.
Sex. Six. Sex.
Seyl. Voile. Velum.
Seys. Faux. Falx.
Sich. Soy. Se.
Sieck. Malade. Aegrotus.
Siel. Ame. Anima.
Sift. Crible. Cribrum.
Sijn. Son. Suus.
Sim. Singe. Simia.
Sin. Sens. Sensus.
Sint. Depuis. Postilla.
Slab. Bauette. Fascia pituitaria.
Slach. Coup. Ictus.
Slaep. Tempes de la tefte. Tempora.
Slaep. Somne. Somnus.
Slang. Couleuvre. Coluber.
Slap. Lafche. Laxus.
Slæf. Esclau. Seruus emptitius.
Slecht. Simple. Simplex.
Sleck. Limacon. Limax.
Sle. Traincau. Traha.
Slec. Prunc. Acatium.
Sier. Torchon. Peniculamentum.
Sleyp. Longue queue de veitement.
Slijck. Boue. Lutum. (Syrma.
Slijm. Limon. Limus.
Slijm. Abhay. Obliquus.
Slijnx. Gauche. Sinister.
Slip. Pand. Peniculamentum.
Slott. Serrure. Sera.
Sluys. Echuse. Cataracta.
Smart. Calumnie. Calumnia.
Smaeck. Gouff. Gustus.
Smal. Estroict. Arctus.
Smeer. Graisse. Abdomen.
Smert. Doleur. Dolor.
Smer. Macule. Macula.
Smis. Forge. Fabrica ferraria.
Smit. Marechal. Faber ferrarius.
Smout. Gresse. Pinguedo.
Snap. Babil. Garrulitas.
Snaer. Corde de luc. Fides.
Snau. Mor dit avec despit. Iracunda locutio.
Sne. Coupure. Scissura.
Snee. Neige. Nix.
Snel. Vifte. Celer.
Snip. Beccaffe. Gallinago.
Snick. Souffir. Suspirium.
Snoeck. Brochet. Lupus.
Snoer. Cordon. Chorda.
Snuf. Rume. Rheuma.
Snoo. Mefchant. Vilis.
Snoc. Morue. Pituita.
Soo. Ain't. Sic.
Soch. Lait. Succus.
Sock. Chauffon. Soccus.
Soch. Traye. Porca.
Soet. Doux. Dulcis.
Soel. Semelle. Solea.
Soon. Filz. Filius.
Son. Soleil. Sol.
Soom. Bord. Limbus.
Soe. Feste. Fatigium.
Soe. Iue. Ius.
Sorg. Suing. Cura.
Sot. Fol. Stultus.
Spa. Houe. Ligo.
Spay. Tard. Tardus.

c C 3

Spate

S. S T A V I N S

Spaen. Efclet. Affula.
Span. Extension de la paulme. Spithama.
Specht. Picmar. Picus.
Speck. Lard. Lardum.
Spel. Iocu. Lufus.
Speer. Lance. Lancea.
Spear. Tracé du pas qui demeure apres avoir marché. Vestigium.
Spie. Cheuille. Impages.
Spie. Espieur. Infidiator.
Spier. La chair blanche qui est à la poitrine d'un oiseau. Pulpa.
Spies. Pique. Hastia.
Spil. Fufeu. Fufus.
Spind. Puche. Penarium.
Spin. Araigne. Aranea.
Spint. Picotin. Corbula.
Spit. Broche. Veru.
Spits. Haulrain.
Spoot. Haste. Properatio.
Spool. Nauette. Glomus textorius.
Spood. Chailit. Sponda.
Spor. Epéron. Calcar.
Sport. Echillon. Climacter.
Spot. Moquerie. Irrisio.
Spreuck. Langage. Lingua.
Spreuck. Diction. Seruentia.
Spreu. Estourneau. Sturnus.
Spriet. Jaulet. Venabulum.
Sproet. Lentille. Lencge.
Spronck. Saulx. Saltus.
Sprot. Harangade. Membras.
Sprau. Pécie des poules. Pituita in gallinis.
Spruyt. Icton d'arbre. Germen.
Spou. Excluf. Cataracta.
Spout. Escalioire. Syrinx.
Spijs. Viande. Cibus.
Spit. Despit. Contumelia.
Stadt. Ville. Vrbr.
Stack. Pali. Palus.
Stael. Acier. Chalybs.
Stact. Estat. Status.
Stacy. Loisir. Otium.
Staf. Baston. Baculus.
Stal. Estable. Stabulum.
Stam. Lignage. Genus.
Stanck. Puntour. Fortor.
Standt. Cuue. Cupa.
Stang. Perche. Pertica.
Stand. Estat. Status.
Stap. Pas. Passus.
Stack. Baston. Baculus.
Steads. Affuel. Affidue.
Steech. Obstiné. Obstinatus.
Steeck. Coup. Idus.
Steel. Tige. Caulis.
Steer. Pierre. Lapis.
Steert. Queue. Cauda.
Stel. (als sel hier) Estale. Vetus.
Stelt. Eschasse. Gralla.
Stem. Voix. Vox.
Sterck. Fort. Fortis.
Steur. Ehourgeon. Turfis.
Steyl. Contremont. Sursum.
Stuer. Tanteau. Taurus.
Steif. Roide. Rigidus.
Stijl. Poteau. Postis.
Stil. Quoy. Quietus.
Stock. Baston. Baculus.
Stoel. Selle. Sedes.
Stof. Poudre. Puluis.
Ston. Muet. Mutus.
Stompt. Rebouché. Obtusus.
Stoop. Lot. Gelta.

Stoot. Poulement. Concussus.
Storm. Tempete. Tempestas.
Stoof. Estue. Hypocaustum.
Stout. Hardi. Audax.
Strack. Incontinent. Quamprimum.
Strael. Ray. Radius.
Stract. Rue. Platen.
Straf. Rigoureux. Durus.
Strang. Riuage de la mer. Littus.
Streeck. Traict. Tractus.
Streng. Aspre. Seuerus.
Streep. Traict. Stria.
Strick. Lacs. Laqueus.
String. Ridelle. Restis.
Stronck. Tronchet. Truncus.
Stroo. Estrain. Stramen.
Stroom. Cours de l'eau. Fluxus aquae.
Strop. Har. Vinculum.
Struyck. Planfon. Frutex.
Struys. Auftruche. Strutiocamelus.
Struyt. Crepices. Laganum.
Strijt. Bataille. Prolium.
Stuck. Piece. Frustum.
Stuer. Seuer. Seuerus.
Suer. Aigre. Acer.
Sulck. Tel. Talis.
Sus. Ains. Sic.
Sus. Tout quoy. Silentium.
Suydt. Midi. Meridies.
Suyt. Piler. Columna.
Svack. Debile. Debilis.
Svaer. Peint. Grauis.
Svart. Noir. Niger.
Sveem. Becasson. Rufficula minor.
Sverp. Fouet. Flagrum.
Sveert. Espee. Ensis.
Sveer. Vicer. Vicus.
Sveet. Sueur. Sudor.
Sverm. Iecion de mouches. Examen apum.
Svyn. Porceau. Porcus.
Sy. Elle. Illa.
Tack. Rameau. Ramus.
Tack. Certain oeuvre par jour. Pensum.
Tael. Langue. Lingua.
Taert. Tarte. Scriblita.
Taey. Coriace. Lentus.
Tal. Nembre. Numerus.
Tam. Dompce. Mansuetus.
Tang. Tenaille. Forceps.
Tant. Dent. Dens.
Tap. Broche d'un tonneau. Embolum vasis.
Tas (als hoytas) Fenil. Fenile.
Teen. Ofier. vimen.
Teen. Orteil. Digitus pedis.
Teer. Tendre. Tener.
Tel. Haqueneé. Gradarius equus.
Temst. Tamis. Cribrum.
Teuch. Traict. Haustus.
Teyl. vn plat creux de terre. Gataba siglina.
Thien. Dix. Decem.
Tob. Cuue. Cupa.
Toch. Certes. veré.
Tocht (als tocht des heys) le mar-cher de l'armée. Agmen.
Toe (als tot daer toe) A. Ad.
Tol. Gabelle. vectigal.
Teng. Langue. Lingua.
Ton. Tonneau. Dolium.
Toon. Reine d'nebride. Habena.
Toon. Monstre. Demonstratio.
Toon. Son. Tonus.
Top. Teupie. Trochus.

Torn. Ire. Ira.
Torsch. Grappe. Racemus.
Torts. Torche. Fax.
Tot. Iuques. Vique.
Tau. Corde. Funis.
Traech. Lente. Lentus.
Traen. Larme. Lachryma.
Trap. Degré. Gradus.
Treck. Traict. Tractus.
Treet. Trespie. Tripes.
Troch. Auge. Linter.
Tromp. Trompe.
Tronck. Tronck. Truncus.
Troost. Solas. Solatium.
Tros. Bagage qu'on porte à la guerre. Impedimenta exercitus.
Trau. Fidele. Fidelis.
Trijp. Tripe.
Tucht. Modesteté. Modestia.
Turf. Tourbe. Cespes.
Tuych. Harde. Arma.
Tuygh. Tefmoing. Testis.
Tuyt. Jardin. Hortus.
Twaelf. Douze. Duodecim.
Twee. Deux. Duo.
Twyf. Discord. Discordia.
Twyyn. Filzors. Filum retortum.
Tych. Couit. Culitra.
Tijt. Temps. Tempus.
Vaem. Toif. Hexappus.
Vaer. Pere. Pater.
Vaek. Sommeil. Sopor.
Vael. Baillet. Helius.
Vaen. Banier. Vexillum.
Vaer. Péril. Periculum.
Vaert. Fosse nauigable. Fossa.
Varrt. Allure. Profectio.
Valck. Faucon. Falco.
Val. Cheute. Casus.
Val. Trebuchet. Decipulum.
Valck. Faulx. Falsus.
Van. De. A.
Vast. Ferme. Firmus.
Vat. Vaisseau. Vas.
Vee. Bestial. Pecus.
Veel. Beaucoup. Multus.
Veer. Passage. Tractus.
Veers. Vers. Versus.
Veldt. Champ. Campus.
Vel. Peau. Pellis.
Veint. Garçon. Infans.
Versch. Frez. Recens.
Vest. Muraille d'une ville. Moenia.
Vet. Graisse. Pinguedo.
Veul. Poulain. Pullus equinus.
Veych. Qui est prochain de sa mort.
Veyl. Exposé en vente. Venalis.
Vier. Quatre. Quatuor.
Vier. Feu. Ignis.
Vies. Facheux. Morosus.
Viif. Cinc. Quinque.
Viig. Figue. Ficus.
Vijl. Lime. Lima.
Vijs. Vis. Cochlea.
Vilt. Feultre. Cento.
Vinck. Beffigue. Frigilla.
Vin. Lopin de chair. Offa.
Vin. Aile de poisson. Pinna.
Visch. Poisson. Piscis.
Vlack. Plain. Planus.
Vizey. Plan. Scriblita.
Vlaegh. Ondée de pluie. Nimbus.
Vlam. Flambe. Flamma.
Vlas. Lin. Linum.
Vleek. Village. Pagus.
Vleech. Chair. Caro.

Vlieg

EENSILBIGHEN VOORDEN.

Vlieg. Mouche. Musca.	vyt. Heure. Hora.	vverp. Iest. Iactus.
Vlien. Lancette à chirurgien. Scalpèr.	vyt. Hors. Ex.	vveip. Guefp. Volsps.
Vlier. Surreau. Sambucus.	vyl. Chatuan. Bubo.	vvelt. Occident. Occidens.
Vlies. Toifon. Vellus.	v. vous. Tibi.	vver. Loy. Lex.
Vlier. Rive. Ripa.	vurd. Prince. Princeps.	vvey. Megue. Serum.
Vlijt. Diligence. Diligentia.	vuyt. Ord. Sordidus.	vvicht. Enfant. Puer.
Vloek. Floe. Floccus.	vuyt. Poing. Pugnus.	vvicht. Pois. Pondus.
Vloek. Maudifion. Imprecatio.	Wacht. Garde. Custodia.	vvie. Qui. Quis.
Vloet. Flot. Fluctus.	Wvaeck. veille. vigilia.	vvieck. Tente. Pannus.
Vloer. Aire. Ara.	vvaen. Presumption. Præsumptio.	vviegh. Berceau. Cunæ.
Vlooi. Pulce. Pulex.	vvaer. Ou. vbi.	vviecl. voile de Nonnain. velum.
Vlucht. Fuite. Fuga.	vvaer. Marchandise. Merx.	vviecl. Roue. Rota.
Vocht. Humide. Humidus.	vvaer. vray. verus.	vviecl. Sauvage. Siluester.
Voet. Pied. Pes.	vvaegh. Balance. Libra.	vviecl. volonte. voluntas.
Voor. Deuant. Ante.	vval. Kempars. vallum.	vvieck. Cild'œil. Nictus oculi.
Volck. Peuple. Populus.	vvaend. Paroy. Paries.	vvint. vent. ventus.
Vol. Plain. Plenus.	vvang. Ioue. Mala.	vvint. Gain. Quæstus.
Vonck. Etincelle. Scintilla.	vvan. van. vannus.	vvip. Balcule. Tollenen.
Vondt. Invention. Inuentio.	vvant. Car. Nam.	vvieck. Trochen. Penicillus.
Voocht. Tuteur. Tutor.	vvant. Gand. Manica.	vvie. viorne. D'oïser. vimen.
Voort. Avant. Vitra.	vvas. Cere. Cera.	vvie. Certain. Cere.
Voos. Corrompu. Infidus.	vvat. Quoi. Quid.	vvie. Blanc. Albus.
Vorck. Fourche. Furca.	vveb. Filpour tistre. Textura.	vvoest. Desert. Defertus.
Vorsch. Grenouille. Rana.	vvech. Chemin. Iter.	vvolck. Nuée. Nubes.
Vorst. Gellée. Gelu.	vveer. Belier. Aries.	vvolck. Loup. Lupus.
Vort. Pourri. Putridus.	vveer. Temps. Tempus.	vvol. Laine. Lana.
Vos. Regnard. Vulpes.	vveer. Dercheff. Iterum.	vvoend. Plais. Plaga.
Vau. Pli. Plicatura.	vvee. Malheur. vz.	vvoort. Mor. verbum.
Vracht. Voiture. Vectura.	vverch. Paroy. Paries.	vvoort. Saucisse. Botulus intestinaris.
Vraegh. Demande. Interrogatio.	vveyck. Mol. Mollis.	vvooudt. Forck. Silua.
Vranck. Franck. Liber.	vveet. Guedde. Glafam.	vvaeck. vengeance. vindicta.
Vreck. Chiche. Parcus.	vveeld. Delice. Delitiz.	vvrat. verrue. verruca.
Vre. Paix. Pax.	vveer. Toutes armes de défense.	vvreer. Cruel. Crudelis.
Vrees. Crainte. Timor.	Arma.	vvroneck. Torfement. Torfo.
Vreemt. Etrange. Extraneus.	vveret. Hôte. Hospes.	vvelp. Laine Chien. Catulus.
Vreucht. Joye. Gaudium.	vvees. Orphelin. Pupillus.	vvulps. Folaire. Lascivus.
Vriendt. Ami. Amicus.	vveeck. Sepmaine. Septimana.	vvurm. ver. vermis.
vroeck. Tempre. Mans.	vvelck. Quel. Quis.	vvy. Nous. Nos.
vroet. Sage. Prudens.	vvel. Bien. Bene.	vvydt. Large. Amplus.
vroet. Eclair. Parcus.	vvenck. Souhait. Optio.	vvyf. Femme. Mulier.
vro. Dehait. Hilaris.	vverck. Etoupe. Stupa.	vvyt. Temps vacant. Spatium.
vroom. Preux. Probus.	vverck. Oeuure. Opus.	vvyn. vin. vinum.
vrau. Femme. Femina.	vverf. Cay. Acta.	vvys. Sage. Sapiens.
vrecht. Fruit. Fructus.	vverm. Chaud. Calidus.	Zier. Ciron. Chiron.
vrij. libre. Liber.		

LATYNSCHE EENSILBIGHEN NAMEN.

S Y N A M M E N, &c.

A Ab Abs. De. van.	Dux. Duc. Leydtsman.	Hinc. D'ici. Hieraf.
Ac. Et. Ende.	E. De. vyt.	Huc. Ici. Hervvact.
Ad. A. Tot.	En. voici. Siehier.	Hiems. Yuer. vvinter.
Aes. Cuire. Coper.	Er. Et. Ende.	Jam. Ja. Nu.
Ah. Ach. Ach.	Ex. De. vyt.	Id. Cela. Dat.
An. Aduerbium interrogantis.	F. De. lic. Ghift.	In. En. In.
Ans. Art. Conft.	Falk. Faulx. Sichel.	Is. Ea. Id.
Arx. Chasteau. Borch.	Fas. Lichte. Toeghelaten.	Ius. Ius. Sop.
As. Liure. Pont.	Fax. Fallot. Torts.	Ius. Droit. Recht.
Ast. Mais. Maer.	Fel. Fiel. Gal.	Lac. Laidt. Melc.
At. Mais. Maer.	Flos. Fleur. Bloem.	Lanx. Baffin. Schael.
Au. Interiectio confternati animi.	Fons. Fontaine. Born.	Lar. Fouyr. Heerdt.
Aut. On. Oit.	Frons. Fucille. Blat.	Lau. Los. Lof.
B is. Deux fois. Tyveemael.	Frons. Front. Stirn.	Lex. Loy. vvet.
Bos. Bouf. Os.	Fur. Larron. Dief.	Lie. Noife. Tvift.
C alx. Chaulx. Kalck.	Git. Genus feminis.	Lux. Lumiere. Licht.
Cir. Deça. Op dees fide.	Glands. Gland. Eckel.	M e.
Clam. En cachette. Heymelic.	Glos. Seur de mon mari. Mijns	Mel. Miel. Honich.
Cor. Coeur. Hert.	mans fuster.	Mens. Sens. Sin.
Cos. Queue. vversteen.	Grex. Troupeau de bestes. Kud.	Merx. Marchandise. vvaer.
Cras. Demain. Morgen.	Grus. Gruet. Cracen.	Mons. Montaigne. Berch.
Crus. lambe. Been.	H a. A. A.	Mors. Mort. Doot.
Crux. Croix. Cruys.	Hac. Parci. Lancx hier.	Mox. Tantost. Terftont.
Cum. Auec. Met.	Heu. Helas. Eylas.	Mus. Souris. Muys.
Cur. Porquoy. vaerom.	Heus. He. Hau.	N æ. Certainement. vvaerlic.
De. De. van.	Hic Hæc Hoc Hunc Hanc Hi Hæ Hos	Nam. Car. vvan.
Dos. Doff. Houvvellike ghift.	Has Hii.	Ne. Non. Niet.

Nil. Rien. Niet.
 Nix. Neige. Sneeu.
 Non. Non. Neen.
 Nos. Nous. vvy.
 Nox. Nuict. Nacht.
 Num. Aduerb.
 Nunc. Maintenant. Nu.
 Nux. Noix. Nuet.
 O. O. O.
 Ob. Pour. Om.
 Oh. Interiect.
 Os. Bouche. Mondt.
 Os. Os. Been.
 Par. Paire. Paer.
 Pax. Pais. Paey.
 Per. Par. Doot.
 Pes. Pied. voet.
 Phy. Interiect.
 Pix. Poix. Pec.
 Plebs. Peuple. Ghemeinte.
 Plus. Plus. Meer.
 Pons. Pons. Brug.
 Post. Depuis. Nac.
 Pra. Deuant. Voor.
 Pro. pour. voor.
 Prob. Interiect.
 Puls. Papin. Pap.

S. STEVINS.
 Pus. Boue. Etter.
 Quis Qui Quæ Quod Quid.
 Quin. Quene. Dat niet.
 Quot. Combien. Hoeveel.
 Quum. Quand. Als.
 Res. Chose. Dinc.
 Ros. Rosée. Dau.
 Rus. Les champs. Velt.
 Sal. Sel. Saut.
 Sat. Asses. Ghenouch.
 Scobs. Scieur. Sargmeel.
 Scrobs. Fosse. Gracht.
 Se. Accus.
 Sed. Mais. Maer.
 Seps. Haye. Tuyn.
 Seps. Serpens.
 Seu. Ou. Oft.
 Sex. Six. Ses.
 Si. Si. Ist dat.
 Sic. Ainsi. Soo.
 Sin. Mais si. Maer ist dat.
 Sol. Soleil. Son.
 Sons. Coupable. Misdadich.
 Sors. Fortune. Fortune.
 Spes. Esperance. Hoop.
 Spitch. Rate. Milt.

Stips. Denier. Pennine.
 Stirps. Racine. Struy.
 Sub. Soubs. Onder.
 Sus. Porc. Soch.
 Tam. Tant. Soo seer.
 Tax. Son de fouet. Clets.
 Ter. Troisfois. Driemaal.
 Thus. Encens. Vvicrooc.
 Tot. Autant. Soo veel.
 Trabs. Poulxre. Balc.
 Tres. Trois. Drie.
 Trux. Cruel. vvreect.
 Tu. Toy. Ghy.
 Tunc. Adonc. Dan.
 Vx. Vvec.
 Vas. vaisseau. Vat.
 ve. Ou. Oft.
 vel. ou. Oft.
 ver. Printemps. Lenten.
 vir. Homme. Man.
 vis. Forcé. Stercke.
 vix. Agrand paine. Naulick.
 vos. vous. Ghylien.
 vox. voix. Stem.
 vrbs. ville. Stadt.
 vt. Afin. Op dat.

GRIECSCH EENSILBIGHE NAMEN BY NAMEN, &c.

A^ε. mensis September.
 A^α. Capra.
 A^α. Sal, mare.
 A^α. Potentia.
 A^γ. Si.
 A^ε. i^{us}. Viquequod.
 A^o. Autem.
 A^ψ. Aduerbium & coniuuct.
 B^α. Panis.
 B^η. Tuffis.
 B^α. Mollis.
 B^α. Musca.
 B^α. Assidue.
 B^σ. Bos.
 B^ρ. Lactuca.
 B^δ. Profunditas.
 B^δ. Genus piscis.
 B^α. Tergus bubulum.
 Γ^α. Nam.
 Γ^α. Terra.
 Γ^α. Herbæ genus.
 Γ^α. Noctua.
 Γ^ν. Genu.
 Γ^α. siue γ^α. pro γ^α. Igitur.
 Γ^α. Anus.
 Γ^α. Sordes vnguium.
 Γ^α. Vultur.
 Δ^α. vel Δ^ι. Autem.
 Δ^α. Conuiuium.
 Δ^α. Fax.
 Δ^α. Oportet.

Δ^ι. Corpus.
 Δ^α. Sanè.
 Δ^ι. Diu.
 Δ^η. Vermis ligni corrodens.
 Δ^ι. Bis.
 Δ^μ. Seruus.
 Δ^ρ. Caprea.
 Δ^ρ. Manipulus.
 Δ^ρ. Virtus.
 Δ^ρ. Quercus.
 Δ^ω pro δ^ω. Domus.
 Δ^ω. Dos.
 E^ι siue E^ν. Si.
 E^ι. Procella.
 E^ι. Vnus.
 E^α. siue E^ε. Ex.
 E^ν siue E^γ, E^ι E^ς. In.
 E^ε. Sex.
 E^ν. Bene.
 Z^α. Mare.
 Z^ε. Genus vestis.
 Z^ι siue Δ^ι. Z^α. Z^η. Iupiter.
 Z^α. Viuus.
 Z^α. Animal.
 Θ^ν. Cumulus.
 Θ^ι. Debitor.
 Θ^ρ. Fera.
 Θ^ι. Mercenarius.
 Θ^ι. Littus.
 Θ^ι. Nomen piscis.
 Θ^ρ. Nomen auis.

Θ^ι. Capillus.
 Θ^ρ. Vermiculus.
 Θ^ρ. Parcus.
 Θ^ω. September apud Ægy-
 ptios.
 Θ^ω. Genus lupi.
 Θ^ω. Adulator.
 I^ν siue I^ς. Genus mensuræ.
 I^ς. Vermis.
 I^ς. Nerus.
 I^ψ. Vermis.
 K^α. Nam.
 K^α quamuis pro K^α e^α.
 K^α. Caput.
 K^η. Genus auis.
 K^η. Cor.
 K^ι. Vermis.
 K^α. Clauis.
 K^α. Ramus.
 K^α. Fur.
 K^ν. Culex.
 K^ν. Cæcus.
 K^α. Plantæ genus.
 K^α. Vbi.
 K^ρ. Caro.
 K^ρ. caput.
 K^ρ pro K^ρ. H. Hordeum.
 K^ρ. Pecten.
 K^ρ. Possessio.
 K^ρ. Viucra.
 Δ^α. Aduerbium cum calcibus
 Δ^α.

ENSTELLINGEN VOORDEN.

Aās. Lapis.	Παλξ. Stimuli genus.	Στεφς. id est, Στεφδς Passer
Als. Leo.	Παλξ. Gressus.	Σδ. Tu. [ausis,
Als. Pannus lineus.	Παλξ. Navigatio.	Σς. Sus.
Ανγξ. Inanis singultus.	Πνξ. Suffocatio.	Σφν. Cuneus quo ligna scin-
Ανγξ. Feræ genus.	Ποτ. Quodammodo.	duntur.
Αδξ. Lux.	Πδ. Vbi, Partim, Alibi.	Σφξξ. Vespæ.
Αδψ. Chlamys.	Πς. Pes.	Σφνξξ. Sphinx animal.
Μα. Aduerbium iurandi.	Πξδν pro πξδν Dudum.	Σς. Saluus.
Μα. Quidem.	Πξν. Prius.	Ταξ. Autem.
Μαψ. Frustrâ.	Πξδ. Antè.	Τις. Quis, Aliquis.
Μοις pro Μην mensis.	Πξδξ. Animal seruo simile.	Τξς. Tres.
Μην. Tamen, quidem.	Πξς. Per, Ad.	Τξς. Ter.
Μη. aduerb. Ne.	Πξδν. Eminentia montis.	Τξξξ. Fax.
Μην. Mensis & aduerbium ta-	Πξδξ. Ros.	Τξδξ. Gurgulio.
men.	Πτνξξ. Genus ausis.	Τδ. Tu.
Μηδ. Mina.	Πτνξ. Plicatura.	Τδς. Sic.
Μης. Lana tenerrima cum	Πτδξ. Timidus.	Υς. Sus, & piscis nomen.
quæ nascuntur agni.	Πδξ. Aduerb. pugnis.	Φαψ. Ausis genus.
Μυς. Mus.	Πξξ. Ignis.	Φν. Hæu.
Μω. An.	Πξς. Quò.	Φνξ pro Φνξ Fera.
Μαψ. Cui hebesacies oculorū	Πα. vbi pro πξδν.	Φδξξ. Pedunculus, etiam me-
Ναι. Certè.	Πα. Quomodo.	dium clauis.
Ναξ. Nauis.	Ρα. Genus radicis.	Φδξς. Genus placenta.
Νς. Mens.	Ραξ. Acinus vuz.	Φλδξ. Flamma.
Νν. Nunc.	Ρν. Ouis.	Φξν. Præcordia.
Νδξ. Nox.	Ρν. Naris.	Φξξξ. Maris vel fluctuum fre-
Ναψ. Lusciosus.	Ρν. Nasus.	mitus.
Ξδν pro ον Cum.	Ρνψ. Vimen flexile.	Φδξ Aduerbialiter cum fuga.
Ο. Hic.	Ρς. Fluxus.	Φδξ. Fur.
Οι pro ον Vbi.	Ραξ. Rupes.	Φαξ. Genus apium.
Ος. Qui.	Ραψ. Virgultum.	Φδς. Inustio ab igne facta in
Ον. Non.	Σα. Incolumbia.	cruribus.
Ον. Ergo.	Σαξξ. Caro.	Φδς. Vir.
Ον. Auris.	Σαξ. Sol.	Φς. Lux.
Οψ. Vox.	Σδς. Vermis.	Χαξ. Manus.
Πα. Qua, Quò, Vbi.	Σης. Tinea.	Χεξ. Oportet.
Πα. Idem.	Σηψ. Serpens.	Χλς. Herba.
Παξ. Puer.	Σκξ. Merda.	Χνς. Lanugo.
Πας. Omne.	Σκδψ. Ausis loquax.	Χς. Agger.
Πδξ. Genus calcæamenti.	Σς. Tuus.	Χς. Congius.
Παξ pro Παξ. A, Ab, Ex.	Σπλν. Splen.	Χρς, & χρς. Corpus.
Παο pro Παοον. Paula.	Σταξ. Farina.	Ψξ. Mica.
Πη. Qui.	Στιξ. Turma continens viros	Ωλξ. Sulcus.
Πλξξ. Tabula.	trecentos.	Ως. Vt.
Πλν. Præter.	Στενξξ. Ausis.	Ωψ. Facies.

S. STEVINS

ANGAENDE ymandt totte voorschreuen Latijnsche ende Griecsche ynckel gheluyden, die metter haest vergaert sijn, noch eenighe dergelijcke mocht vinden, aldaer niet beschreuen, sulcx soudemen int Duytsch oock connen doen, ende onghelijck in al veel meerder menichte, want wy de schandelicke om noemen, ende ander die ons buyten *Den schat der Duytscher talen* (welcke t'Woortbouck was daerwyse uyt vergaerden) ons wel inden sin quamen, moerwillens uytghelaten hebben, ons daer in vernoughende, dat duer de voorgaende opentlick blijct, d'oude Duytschen met voorset, d'uyterste volmaectheyt in desen, meer dan eenighe ander, naghetracht, ende ghetrossen te hebben.

Grammatica elementis.

Geometria.

Merckt noch dat sy t'selue oock ghedaen hebben in des * Letterconsts beghinselen, dat is inde boucstaffen ofte letteren, die sy al met eenfilbighe gheluyden noemen, t'welck voorwaet d'uyterste volcommenheyt naerder is, dan de contrari; want ghelijct inde * Meetconst ongheschied waer, t'punt, beghin der grootheyt, meerder te stellen dan grootheyt, alsoo ist oock inde Letterconst ombetaemlick, t'beghin van meer gheluyden te sijn, dan t'ghene van verscheyden beghinselen ghemaect wort. Als by voorbeelt int spellen van *Dal*, datmen opt Griecsche seght *Delta*, *Alpha*, *Lambda*, *Dal*; ofte opt Hebreusch *Daleth*, *Aleph*, *Lamed*, *Dal*; alwaer yder beghin ongheschiedelick van meer gheluyden is, dan t'ghene vande drie beghinselen ghemaect wort. Daerom segghen wy veel natuerlicker ende aerdigher, *De*, *A*, *El*, *Dal*; want t'ghene inde Consten beghin is, moet daerin het alder eenvoudichste sijn, t'welck hier, soor de Duytschen ghetrossen hebben, ynckel gheluyt is. Daerom deden de Latinen wel, doe sy leerden lesen en schrijuen, dat sy in desen d'ander lieten varen, en de Duytschen volghden. Wat de onhegronde meyning van hemlien belangt, die soudent duruen segghen de Duytschen sulcx eer vande Latinen te hebbē, die en spreken niet duer beweeghnis der reden, maer ghedreuen van eensinnighe moerwillicheyt, soo doch de Latinen na sulcke cortheyt niet ghetracht en hebben, maer ter contrari, t'gheen by ons cort en goet was, dat hebben sy naer huerlieder ghebruyck gheern verlangt: als *Angst*, *Caes*, *Beeft*, *Put*, *Muer*, *Recht*, *Cael*, *Graen*, *Heer*, &c. daer sy voor segghen *Anxietas*, *Casus*, *Bestia*, *Puerus*, *Murus*, *Rectus*, *Caluus*, *Granum*, *Hermus*. Tis dan vande Duytschen dat de letteren de volmaecte namen hebben.

Wat de Fransche, Italiaensche, Spaensche, ende meer talens eenfilbighe gheluyden belangt, welcke hier yemandt begheeren mocht, wy en hebben die niet ghestelt, om dattet Griecx ende Latijn in volcomenheyt d'ander te bouen gaende, tottet voornemen voldoen; want als wy bewesen hebben, het Duytsch volmaecter dan dese twee te sijne, soo volghet uyt noch stercker reden, dattet veel volmaecter is dan eenighe van dien. Wel is waer dat de Fransche eenfilbighe gheluyden, de Latijnsche

In case the reader should find, in addition to the above-mentioned, hurriedly collected Latin and Greek single sounds, some more of this kind which are not included in the list, this might also be done in Dutch, and even in much greater number, for we have intentionally omitted those that were shameful to mention and others that occurred to us outside *Den schat der Duytscher talen*¹⁾ (which was the dictionary from which we collected them), being satisfied that it is evident from the preceding lists that the ancient Dutch, more than any others, purposely strove after and achieved the highest perfection in this matter.

It is further to be noted that they have also done this in the elements of grammar, i.e. in the letters, all of which they denote by monosyllables, which is certainly nearer to the highest perfection than the contrary; for just as in geometry it would be absurd to consider the point, the element of magnitude, greater than magnitude itself, in the same way it is also improper in grammar for the element to consist of more syllables than that which is made of several elements. Thus for example in spelling the word *Dal*, which is said in Greek: Delta, Alpha, Lambda, *Dal*; or in Hebrew Daleth, Aleph, Lamed, *Dal*, in which each element improperly consists of more syllables than that which is made up of the three elements. Therefore we say, much more naturally and peculiarly: De, A, El, *Dal*, for the elements in the arts should be simplest of all, which in this case, as the Dutch have achieved it, are single sounds. Therefore the Latins did well, when they learned to read and write, to abandon the other method in this and imitate the Dutch. As to the unfounded opinion of those who should dare to say that the Dutch have rather borrowed this from the Latins, such people are not moved to say so by reason, but by obstinate wilfulness, for the Latins did not aim at such brevity: on the contrary, they liked to lengthen in accordance with their custom that which was short and good with us: for example *Angst*, *Caes*, *Beest*, *Put*, *Muer*, *Recht*, *Cael*, *Graen*, *Heer*, etc., for which they say *Anxietas*, *Caesus*, *Bestia*, *Puteus*, *Murus*, *Rectus*, *Calvus*, *Granum*, *Herus*. It is therefore from the Dutch that the letters have received their most perfect names.

As regards the monosyllables in the French, Italian, Spanish, and other languages which the reader might desire to be mentioned here, we have not mentioned them because Greek and Latin, being superior to the others, suffice for the purpose; for if we have proved Dutch to be more perfect than these two, it follows *a fortiori* that it is much more perfect than any of the former. It is true indeed that the French monosyllables are greater in number than the Latin ones, since the

¹⁾ This is a work by Ian van den Werve, a lawyer at Antwerp, who advocated purification of the Dutch language.

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tijnsche in menichte te bouen gaen, ouermids de Françoysen dickmael snoeyen ende vercorten, t'ghene sy vande Latijnen ontleenen, als voor *Facio, Seruio, Venio, Rideo, Sentio*, &c. te segghen *Je Fay, Sers, Vien, Ri, Sens*; welcken aert der vercorting sy noch schijnen behouden te hebben van weghen dat sy, als vooren gheseyt is, eens Duytsch spraken; maer wat iller af? sy en lijden gheen binding, sy sijn ter Tlaemvoughing onbequaem, ende veruolghens van cleinder weerde.

TEN tweeden soo volghet van der woorden voornomde T'saemvoughingh gheseyt te worden, welcke niet t'onrecht voor een der voornaemste ende nutste eyghenschappen die in talen begheert worden, gheacht is; wiens voordering ende nootlicheyte den ghenen die hun inde Consten oeffnen, niet onbekent en is, ouermids der dinghen namen daer daer oock haer corte * bepalinghen sijn. Hier in wort byden gheleerden het Griecx gheluckigher gheacht als d'ander, dat is, als de ghene die by haer verleken wierden, onder welcke het Duytsch gheen plaats en had; anders ten waer gheen oordeel van gheleerden, maer van verkeerden gheweest, want ghelijck gheen menschen die wel by haer sinnen sijn drie grooter ghetal en achten dan Duytst, maer veel cleender; alsoo oock de Griecsche T'saemvoughing niet bouen de Duytsche, maer verre daer onder, want in die sijn hier en daer sommighe woorden diese lijden, maer in dese oueral, ende dat met een ander besonder cortheit, gheschiedtheyt, ende eyghentlicker beteekening haers grondts, welcke nootsakelick volghen uyt de voorgaende groote menichte der yackel gheluyden, daerenbouen ter bequame T'saemvoughing wonderlick ghetrossen. Ymant mocht nu van desen eenighe voorbeelden begheeren; maer wanttet licht ende te slicht waer, uyt de oneindelicke een groote menichte te vergaren (als inde T'saemspraec der * Be-
wysconst begonnen is) soo gheuen wy hem seluer eenighe voor te stellen die hem ter copping onbequaemst duncken. Ick neem dat hy daertoe verkieft (om haer aldermerckelicste versheydenheyt, ende gheduerighen strijt) Water en Vier: voorwaer soot den noot erghens voorderde dese te vergaren, als by ghelijcknis, ymandt willende segghen, *Tot d'incorst des Kueninx Waren vieren ghemaect die van selfs int Water onstaken*, hy sal die noemen (ghelijck wy anders segghen Turfvieren, Eyckevieren) *Watervieren*: ende daer toe en behouft hy gheen gheleerde te sijne, noch hem lang te bedencken, maer de leecken worden, duer de wonderlicke eyghenschap des tael, van selfs daertoe ghedronghen. Ten is den hoorenden oock gheen nieu noch vreemt woort, hoewel hy dat van te vooren noyt ghehoort en had, reden dat sulcke niet alleen duer de ghewoonte verstaen en worden, maer uyt den ghemeenen aert der gheluyden, welcke d'oude Duytschen soo constelick

Definitiones.

Dialectica.

French often curtail and shorten that which they borrow from the Latins, saying for example, for *Facio*, *Servio*, *Venio*, *Rideo*, *Sentio*, etc.: *Je Fay*, *Sers*, *Vien*, *Ri*, *Sens*; which shortening tendency they seem to have retained because, as has been said above, they once spoke Dutch; but what of that? They do not admit of combination, they are unfit for composition, and consequently have less value.

The second point to be discussed is the aforesaid composition, which is not unjustly deemed one of the principal and most useful properties required in languages; the advantage and need of which is not unknown to those who exercise themselves in the arts, since the names of things are thus also short definitions thereof. In this respect Greek is considered by the scholars to be more felicitous than the other languages, that is the languages compared with it, among which Dutch did not figure; otherwise it would not have been a judgment of scholars, but of fools, for just as no man in his senses will deem three to be a greater number than one thousand, but much less, thus Greek composition is not superior to Dutch, but far inferior, for in the former there are occasionally a few words admitting of it, but in the latter it is always possible, and such with a special brevity, suitability, and proper denotation of their fundamental meaning which are the necessary consequences of the above-mentioned multitude of single sounds, which are also wonderfully suited for composition. The reader might now require some examples of this, but because it would be easy and too simple to collect a great many from the infinite multitude (as we started to do in the dialogue in the *Dialektike* ¹⁾), we suggest that he himself should propose some, which seem to him least suited for composition. I assume that he chooses for this (because of their highly obvious difference and continuous conflict): *Water* and *Vier* ²⁾). Indeed, if circumstances should require these to be combined, for example if anyone should wish to say: *Tot d'incomst des Kuenincx waren vieren ghemaect die van selfs int water ontstaken* ³⁾), he would call them *Watervieren* ⁴⁾) (just as we also say *Turfviere* ⁵⁾), *Eyckevieren* ⁶⁾). And for this he need not be a scholar, nor need he take thought about it long, but the unlearned are automatically induced to do this owing to the wonderful character of the language. Nor is it a new or strange word for the hearer, even though he had never heard it before, because such words are understood not only through usage, but owing to the common character of the sounds, which the ancient Dutch have found so ingeniously for the purpose that I, and all those who know no more of the origin,

¹⁾ Work III, *Dialectikelicke Tsamespraek*. This „dialectical dialogue” in part already develops the ideas of the *Uytspraek*.

²⁾ i.e. water and fire.

³⁾ For the King's entry fires had been arranged, which kindled out of themselves in the water.

⁴⁾ water fires.

⁵⁾ peat fires.

⁶⁾ literally: oak fires.

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Grammatica.
Subiectum.
Adiunctum.

daertoe gheuonden hebben, dat ick, met al de ghene die van d'oirsaeck niet meer en weten, ons alsvoren noch met recht mueghen verwonderen duer wat middelen dat mach gheschiet sijn. Merckt bouen al dit noch een besonder, ende weerdighe eyghenschap, by hemlien constelick inde T'saemvoughing veroirdent, ia sulcke, dat gheen Griecx, noch Latijns * Letteraer, soodanighe uyt die talen perssen en sal, al wrong hy tot sweetens toe: Te weten datter laetste der ghecoppelde altijd * Grondt is, ende t'voorgaende * Ancleuing; Als wanneermen seght, *Putwater*, so is water grondt, ende put ancleuing, want onsen sin dan voornamelick tot water streckt, om t'welck t'onderscheyden van stroomwater, reghe-water, &c. men vougtter Put voor: Maer als wy dit verkeerren, segghende *Waterput*, dan is den sin (hoewel het de selue woorden sijn) al een ander, want Put is dan grondt, ende Water anclening, ouermids de voornamelicke meyning dan is van een Put, om welcke t'onderscheyden van een Mespit, Calckput, &c. men stelter water voor. Alfoo oock is *Glasveinster*, een veinster van glas, maer *Veinsterglas*, is glas niet daermen uyt drinckt, maer plat daermen veinsters af maeckt. Wederom *Oli-nuet*, is een nuet des gheslachts daermen olie uyt perst, maer *Nuetolie*, is olie van nueten. Sghelijcx *Iachthondt*, is een hondt daermen mede iaecht, maer *Hondiacht*, een iacht niet met voghelen, dan met honden, &c. Wat den ghenen belangt die noch van meyning mueghen sijn, het Griecx in desen gheuolle voor het Duytsch te gaen, wy achten dat sulcx gheschiet duer dat hem het Duytsch onbekent is, of datter verstant der oirdeling ghebreekt, of dat hy hartneekich sy, of eenich der ghelijcke beletsel heb, niet weerdich een woort daer af meer te roeren.

Sublata materia tollitur effectus,
4. v. 2. strij-
reden der Be-
wysonit.

TEN derden moeten wy segghen vande bequaemheyt deses tael tot de leering der Consten, waer af wy (bouen dien sulcx nootlaecklick volght uyt het voorgaende toeghelaten) de volghende Weeghconst, sulck sy is, tot voorbeelt stellen; welcke ghy, ghemerct de groote rijcheyt onses tael, uyt welcke alles veel beter behoort ghedaen te sijne, daertoe mischien niet weerdich en sult achten, te meer datter duysenden by ons sijn, die se veel beter, ende met beuallicker woorden beschrijven souden: Maer niet teghenstaende al dit, so ist doch sooghedaen, dat gheen van al d'ander Gheslachten der volcken wie hy sy, t'selue, soo veel des spracx grondelicke beteeckening, ende uytbeelding der Saeck belangt (ick en wil niet segghen, soude connen verbeteren, daer gheen vreesse voor te hebben en is, want hemlien ghebreekt Stof * welcke gheweert soo wort oock gheweert de daet) soude connen soo nauolghen. want waer wildy spraken halen daermen duer segghen sal, Euestaltwichtich, Rechthefwicht, Scheefdaellini, en dierghelijcke daer de Weeghconst vol af is? sy en sijnder niet, de Natuer heeft daer toe aldereyghentlicx het Duytsch veroirdent.

TEN

may justly wonder, as said before, by what means this may have happened. In addition to all this, a special and valuable property should also be noted, which was ingeniously disposed by them in composition, a property such that no Greek or Latin grammarian can squeeze it from those languages, though he should wring them until he sweat: to wit, that the last member of the compound is always the head word and the preceding one the attribute. For example, when we say *Putwater* ¹⁾, *water* is the head word and *put* ²⁾ the attribute, for then we chiefly mean *water*, to distinguish which from *stroomwater* ³⁾, *regbewater* ⁴⁾, etc. we prefix *put* to it. But if we invert the order, saying *Waterput* ⁵⁾, the meaning is quite different (though the words are the same), for then *put* is the head word and *water* the attribute, since the principal meaning of it is then that of a *put*, to distinguish which from a *Mesput* ⁶⁾, *Calckput* ⁷⁾, etc. we prefix *water* to it. In the same way, *Glasveinster* ⁸⁾ is a *veinster* of *glas*, but *Veinster-glas* ⁹⁾ is *glas* not such as we drink from, but plates from which *veinsters* are made. Again, *Olinuet* ¹⁰⁾ is a *nuet*, of the genus from which *olie* is pressed, but *Nuetolie* ¹¹⁾ is *olie* from *nueten*. Similarly, a *lachthondt* ¹²⁾ is a *hondt* with which one *iaecht*, but *Hondiacht* ¹³⁾ is a *iaecht* not with birds, but with *honden*, etc. As for him who should still be of opinion that Greek is superior to Dutch in this respect, we consider that this is because he ignores Dutch or because he is not competent to judge or is obstinate or has some similar defect not worth wasting any more words about.

Thirdly we have to discuss the suitability of this language for the teaching of the arts, of which (apart from the fact that it follows of necessity from the above assumptions) we are taking the following Art of Weighing, such as it stands, as an example; which, considering the great wealth of our language, in view of which everything should have been done much better, you may not deem worthy, the more so as there are thousands among us who would describe it much better and in more pleasing words. But in spite of all this, it is a fact that none of the other nations, whichever it be, could imitate it as far as the fundamental meaning of the language and the representation of the thing are concerned (I would not say: could improve upon it, which need not be feared, for they lack the material, in the absence of which the effect is also absent); for where would you find any languages in which one can say *Evestaltwichtich*, *Rechthefwicht*, *Scheefdaellini* and the like, in which the Art of Weighing abounds? They do not exist, Nature has specially designed Dutch for it.

¹⁾ well-water.

²⁾ well.

³⁾ river water.

⁴⁾ rain water.

⁵⁾ literally: water well.

⁶⁾ dung pit.

⁷⁾ lime-pit.

⁸⁾ glass window.

⁹⁾ window glass.

¹⁰⁾ literally: oil-nut.

¹¹⁾ nut-oil.

¹²⁾ literally: hunting-hound.

¹³⁾ literally: hound-hunting.

V Y T S P R A E C K.

TEN laetsten moeten wy, na t'voornemen, deses tael's beweeghlic-
 heyt bethoonen, waer toe ons onder anderen tot voorbeeld dienen
 can, Hendrick Glareaen, in sijn Latijnsche * uytſpraeck te Friburg op *Oratione.*
 Suetonius ghedaen, alwaer hy heftelick ontfleken op der Keyfers boof-
 heden, ende gheen Latijnsche noch Griecsche woorden (hoevel hy in
 die ſpraken ſeer eruaeren was) bequaem ghenouch vindende, om den
 hoorders haer afgrijſlichkeiten tot een walghe te maken, heeft dat onder-
 tuſſchen door duytsche beſtelt, als daer hy ſegt: *Quid enim de Tiberio di-*
cam? vlceroso in omnem invidiam animo, quo nihil umquam fucatus toto ter-
rarum orbe, nihil nocentius, nihil turpius vixit. De eo sanè, quod vix Latine
dixeris, nostra lingua ornatissime dici poterit: Ein abgſeimpter, eerloſer,
 znichtigher boeſwicht. *Si licet Graecammiſcere Latinis, ſepe etiam apud*
non intelligentes Græcæ; cur non liceat inferere Celtica ac Germanica non mi-
nus vetuſta lingua verba, apud intelligentes? Sed pudet plura de eo Diuo: di-
xiſſem libentius, von dem leidighen Tüfel. Producatur Caligula Imperator,
merdoſus ille puſio, Das ſchantlich phyſickguckly, pudenda Germanici Ce-
sarij progenies, &c. Ende corts daer na ſprekende van Nero, Galba, Otto,
 Vitellius: *Cui enim monſtro potius comparabuntur belluones illi, bibones, come-*
dones, lurcones, abdomines, ventres, braſſer, ſchlemmer, pfuſer, ſchlucker?
 Næemt noch merckelicker voorbeeld, ande prekinghen ofte verſchey-
 den leeringhen der gheloouen, die inde Duytsche landen gheſchien. waer
 vindtmen ander contreyen daer de ghemeenten alſoo ghetrocken wor-
 den, den eenen tot dit, den anderen tot dat, ende elck tot t'ghene hy
 hoort? wat is d'oirſaec? de beweeghlicheyt der Duytsche woorden, al veel
 heftelicker des menſchen ſin ende ghemoet tot des Redenaers voorne-
 men dringhende, als eenighe ander; want ſoo hy de tong wel t'ſijnen be-
 uele heeft, ende dat hem maer int hooft quaem een beſſem de bruyt te
 ſijne, hy ſal de ghemeente beweghen ter bruyloft te comen; la noch al
 ſlimmer dinghen doen beſtaen; ſtreckende niet alleen tot ellende van
 wyſen kinderen, tot verlies van liſſen en goet, maer oock tot ghemeene
 verderfnis des landts, als metter daet, dat beclaghelick is, te veel blijct;
 Ende dit al door die heftighe beweeghlicheyt deses tael's: Daerom waert
 wel te wenſchen, dat gheen ander begaefde der Duytsche tong, ſulck
 ampt ten deele en viele, dan diens einde tot de ghemeene welvaert
 ſtreckt; want ſoodanigher menſchen Duytsche woorden, vaten inde hoor-
 ders herten als cliſſen an wolle, ſy ſijn als den breydel des peerts, als
 t'roer eens ſchips, duer t'welck de ghemeente gheuoert wort daert den
 ſtierman belieft. Angaende yemandt ſulcx der Duytschen lichtveerdic-
 heyt ſoude willen toefchrijven, ſeker t'waer teghen d'oude oircon-
 den van Cefar, Tacitus, ende verſe ander des huydighen daechs, welcke, re-
 kenende int ghemeene Gheſlacht teghen Gheſlacht, hun voor t'ſtant-
 vaſtichſte ende gheſtadichſte achten: Daerom ſoo wy gheſeyt hebben,

Lastly we have to prove, as we intended, the emotional appeal ¹⁾ of this language, as an example of which may serve, among other things, the case of Hendrick Glarean ²⁾, in his Latin oration made at Friburg on Suetonius, where, being greatly incensed about the vices of the emperors, and not finding any Latin or Greek words (though he was greatly versed in those languages) suitable enough to inspire his hearers with horror of their hideous deeds, he uses German words for it now and then, as where he says: *Quid enim de Tiberio dicam? ulceroso in omnem invidiam animo, quo nihil umquam fucatus toto terrarum orbe, nihil nocentius, nihil turpius vixit. De eo sanè quod vix Latinè dixeris, nostra lingua ornatisime dici poterit: Ein abgfeimpter, eerloser, znichtigher boesswicht. Si licet Graeca immiscere Latinis, saepe etiam apud non intelligentes Graeca, cur non liceat inserere Celtica ac Germanicae non minus vetustae linguae verba, apud intelligenteis? Sed pudet plura de eo Divo: dixissem libentius, von dem leidighen Tüfel. Producat Caligula Imperator, merdosus ille pusio. Das schantlich physickguckly, pudenda Germanici Caesaris progenies, etc.* And a little further on, speaking of Nero, Galba, Otto, Vitellius: *Cui enim monstro potius comparabuntur belluones illi, bibones, comedones, lurcones, abdomines, ventres, brasser, schlemer, pfuser, schlucker?*

Take an even more obvious example, viz. the preachings or different teachings of creeds which take place in the Dutch countries. Where do we find any other regions where the congregations are so much fascinated, one by this, the other by that, and each by that which he hears? What is the cause? The emotional appeal of the Dutch words, which cause men's minds and hearts to be persuaded by the orator's intentions much more vehemently than any other, for if he has his tongue well in his command and should get it into his head that a broom was to be the bride, he will induce the congregation to come to the wedding. Nay, he will provoke even worse things, tending to cause the misery of wife and children, the loss of life and property, but also the general ruin of the country, as is only too evident, a thing to be deplored. And all this is due to the vehement emotional appeal of this language. Therefore it were to be wished that such a function were to fall to no persons expert in the use of the Dutch language but those who have the welfare of all in view; for the Dutch words of such men cling to the hearts of the hearers like burs to wool, they are like a horse's bridle, like a ship's rudder, by means of which the congregation is led as it pleases the steersman. If the reader should be inclined to attribute this to the frivolity of the Dutch, this would be contrary to the ancient records of Caesar, Tacitus, and many others of the present day, who, comparing in general one race with another, consider the Dutch to be the most steadfast and constant. Therefore, as we have

¹⁾ Stevin's term is „beweeeghlicheyd” (mobility), but this means the power to move.

²⁾ Henricus Glareanus, a Swiss scholar (1488-1563), author of the well-known work *Dodekachordon* (1547), was professor at Freiburg im Breisgau from 1529 to 1560.

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tis duer de heftighe beweeghlicheyd der Duytsche woorden, nootfelick volghende uyt haren voorſchreuen conſtighen grondt.

Argumenta. **M**AER wat hebben doch d'uytheemſche verachters der Duytsche ſpraek; die ſchampweerdighe ſchampers, die oordeelders als blinden vande verwe, voor * ſtrijtedens by te brenghen? Ia, ſegghen ſy, als wy al veel iaren die tael gheleert hebben, ſoo ſpreken wy noch ſoo erbarmelick, dat de Duytſchen lachen moeten wanneer ſyt hooren, maer d'onſe hebben ſy terſont gheleert, hoe can d'hare dan goetd ſijn? O aermeongheuallighe ghedierten! O iammerlicke wyſheyt! Om dat een witte muer beter om ſchilderen is dan Paris oordeel, is ſy daerom oock conſtigher? Om dat den volmaecten omtreck eens naecten menſchen lichaems, onder de formen de aldermoeylicſte is die den ſchilder ontmoet, is ſy daerom de verachtſte? Om dat een * Singconſtich ſtick met vier of vyf ſtemmen, vol ſchoonder * vluchten, bequamer * vallen, lieſlicker * teghepuncten, den leerenden luytſlaghers moeylicker valt, als dankens, ende ghemeene ſtraetliſkens, iſt daerom oock het verworpenſte? Iaer voor verworpen plompaerts, die haer groſtheyt bedecken fouden coſten ſy ſwyghen: Alſoo oock, om dat de Duytsche ſpraek, welke de diepe verborghentheden der natuer grondelick uytbeelden can, laſtighe om leeren is als d'ander dieſe verſwyghen moeten, is ſy daerom de ſlichtſte? Ia ſy voor ſlichter dan ſlichte ſlichthoofden, die niet en weten waerin goetheyt of ſoetheyt van talen gheleghe is. Voorwaer fouden woorden an woorden hanghen om eenwoordighe redenen te maecken, ghelijck letteren an letteren woorden baren, ſy moeten als de letteren conſtighe gheluyden hebben, niet naer t'gheual van hier en daer t'ſamen gheſchrapt, als de hare, maer ſulcke als ons voorouders ghetroffen hebben, ende dat duer middelen, daer alle verſtanden (ſoomen uyt het ſijne van eens anders oordeelen mocht) voorruſten moeten; Reden is deſe, dat de ſpraken niet duer eenen, maer duer velen van verſcheyden gheuoelen ghemaect worden, d'een ſus, d'ander ſoo, deſe beter, die ergher willende; maer de voorighe Duytſchen hebben ghedaen, als of ſy altemael de ſaken eruaren daer de talen toe dienen, mer een ſelfde gheneghentheyt aldus eendrachtelick ghedocht hadden: *Angheſien wy duer t'behulp van tong, lippen, tanden, verhemelt, keel, bycans oneindelicke verſcheyden eenſilbighe gheluyden connen uyten, ſoo iſt billich dat wy yder ynnel ſaek een eenſilbighe gheluyt toeyghenen (Want min is onnueghelick, meer is onnut) ende van ſulcker aert, dat ſy de Tſaemvoughing bequamelick lijdén, op dat wy daer duer niet alleen de ghemeene dinghen, maer oock de wonderlicke die de Natuer daghelicx baert, beuallick ende verſtaenlick uytbeelden muezghen.* Wat der woorden langhe ſilben belangt, welke int Griecx, Latijn, ende meer ander talen, ſonder grondt ghenomen

*Musicalis.
Fuga.
Cadentia.
Contrapuncta*

said, the cause is the vehement emotional appeal of the Dutch words, which is an inevitable consequence of their above-mentioned ingenious character.

But what arguments have the foreign despisers of the Dutch language to adduce, those scornors deserving scorn, who judge as blind men judge of colours? Why, they say, when we have studied this language for many years, we still speak so lamentably that the Dutch cannot help laughing when they hear it, but they very quickly learn our language; so how can theirs be a good one? Oh, thou miserable, despicable vermin! Oh, lamentable wisdom! Because a white wall is easier to paint than the judgment of Paris, is it also more artistic for that? Because the perfect contour of a naked human body is the most difficult of all forms encountered by the painter, is it the most despicable for that? Because a piece of music in four or five parts, full of beautiful fugues¹⁾, apt cadences, pleasant counterpoints is more difficult for people learning to play the lute than dances and common street songs, is it the most abject for that? Yes, so it is for abject churls, who would conceal their coarseness if they could be silent. In the same way also, because the Dutch language, which is capable of thoroughly interpreting the profound secrets of Nature, is more difficult to learn than the other languages, which have to keep silent about them, is it the simplest because of that? Yes, so it is for simpler than simple simpletons, who do not know in what consists the excellence or sweetness of languages. In truth, if words are to be combined with other words into compounds forming one word, just as letters combined with letters produce words, like the letters they should consist of artful sounds, not scraped together at random, as in the languages of other nations, but such as our ancestors have created them, and this by means which pass all understanding (if one may judge of others by one's own). The reason is that languages are not made by one man, but by many with different ideas, one like this, another like that, one having better intentions, the other worse; but the ancient Dutch did as if, having all together learned for what purpose languages are meant, they had thought of one accord, and one and the same mind: *Since by means of the tongue, lips, teeth, palate, and throat we can utter an almost infinite variety of monosyllabic sounds, it is fit that we should assign to every single thing a monosyllabic sound (for less is impossible, and more is useless), and of such a nature that they are fit for composition, so that we may pleasingly and intelligibly represent by them not only ordinary things, but also the strange things which Nature creates daily.* As to the long syllables of the words which, having been groundlessly accepted in Greek and Latin, and other languages besides, have reduced Latin to

¹⁾ In the sixteenth century a „fugue” was what we now describe as a canon.

V Y T S P R A E C K.

men sijnde, t'Latijn daertoe ghebrocht hebben, dattet in rwyffel is of de ghesproken woorden der ouden nu ter deghe soude connen verstaen worden, daer schijnen sy aldus gheseyt te hebben: *Nadien de Natuer als duer ghemeen insturing allen menschen inghebeelt heeft, dat de ghesproken woorden een manier van ghesanck eysschen als * hoochbyclanck * leeghbyclanc en dierghelijcke onder welke des woorts langhe silb van meesten anstien is de reden wil dat wy des spraecx soo besonder * ancleuing niet na t'gheual, maer na yet behoirlicx ende sekens veroirdenen: Wat sal dat wesen? dit, datse voor ghemeen regel alijst comme op des * doende woorts ende. dieder wyt spruyten, voornaemste silb, als in Hōoren, Verhōorende, Ghehōort, Behōorende, Hōorende, dat de langhe silb. alijst op Hōor valle, die aldaer de weerdichste is, want men inden eersten persoon seght ick Hōor, d'ander silben als en, ver, ende, ghe, bo, en sijn maer by ghesette, daermen alle woorden me verandert. Maer inde ghecoppele, daer false alijst op d'eerste vallen, als Sautvat, Hāumes, Vytgaen, jnsien, en dierghelijcke. Soo hebben sijt afgheclaert, ende dit noch als sommige meenen, in haer wiltheyt, waeruytmen aldus strijden mocht: Hebben sy sulcx ghedaen in haer wiltheyt, wat connen sy in haer remdheit! Voorwaer wy souden dese eere wel draghen, maer ghemerct de Natuer niet teghen Natuer en doet, de reden wil dat wy ons met een minder vernoughen, te weten, dattet voormael een seer wys, gheleert, ende ouertreflick Gheslacht is gheweest, als vooren bethoont is, daertoe ghecommen sijnde met langher tijdt, duer veel ervaeringhen. Ende soo wy van dese voorghanghers weerdighe navolghers willen gheacht sijn, en sullen niet duer een beestelick ghetuych van ondanchaertheyt, so groote gauen ons naghelaten, duer onwetenheyt vermaen, noch, den lasteraers diese niet en kennen, sordelick ghelooouen, noch verlatende den spieghel der talen, ons dickmael behaghen in haer leelick schrapfel van schuym der vuyllicheyte, maer sullen ter contrari die clouclick beschermen, niet met ydel woorden als d'hare sijn, noch na t'onuerstandt van hemlien die de goetheyt der Saken in haer talen beschreuen, onbescheydelick de goetheyt der talen meenen te wesen; maer ghelijck rgout duer t'vier beproeft wort, alsoo salmen haer weerdicheyt duer de * daer bethoonen: Welcke sal die sijn? dese, neemt voor * grondt rghene in al d'ander spraken tot noch toe der Naturen diepe verholentheden sijn, welke sy niet ter deghe bedien en connen, als dat sy v (onder duyfentich anderen daer het Duytsch vol af is) dit na segghen: *Ghelijck rechtheflini tot scheefhef- lini, alsoo rechthefwicht tot scheefhefwicht*; ende dier soo doen connen, be- loofte vrielick een koeck; Ia dat sy t'auent op sullen blijuen (voor kin- deren dienen doch kinder prijsen) ende ick verseker v dat ghyder sonder schade sult afcommen, want het is blijckelijck ghenouch wat sy hier in vermueghen, te weten voor dese woorden langhe redenen te stellen, die ronderfcheyt der * palen, ende de form der * eueredenheyt oueral seer*

Accentus a-
cutus.
Accentus
grauis.
Adiunctum.
Verbi actiui.

Strijdereden
van het on-
gheloofticker
tot het ghe-
loofticker
door de 130
strijtereden des
4^e voorstels
der Bewyssoft.

Effectum.
Subiecto.

20, v. 1. B. der
Begh. vande
Waegbconst.

Kinderen in
kennis der
weerdicheyt
vande Duyt-
sche taal.

Terminorum
Proportionis.

such a state that it is doubtful whether the speech of the Ancients would now be completely understood, the Dutch seem to have said as follows: *Since by universal inspiration Nature has impressed on all people the idea that the spoken words require a kind of intonation, such as strong and weak accent and the like, under the influence of which the long syllable of a word has the greatest importance, reason demands that we design this peculiar affix of language not by chance, but properly and methodically. What shall this be? That as a general rule the accent shall always fall on the chief syllable of the active verb and its derivatives, for example that in Hōoren, Verhōorende, Ghehōort, Behōorende, Hōorende the long syllable shall always be Hōor, which is the most important in these words, for we say in the first person Ick Hōor*¹⁾; *the other syllables, such as en, ver, ende, ghe, be are only affixes, with which all the words are altered. But in compounds the accent shall always fall on the first syllable, for example Sautvat, Häumes, Uytgaen, Insien*²⁾, *and the like. Thus they achieved it, and such, as some suppose, in their barbarian condition, from which it might be argued: If they have done this in their barbarian condition, of what are they capable in their civilized state! In truth, we should the like. Thus they achieved it, and such, as some suppose, in their barbarian condition, from which it might be argued: If they have done this in their barbarian condition, of what are they capable in their civilized state! In truth, we should accept this compliment, but seeing that Nature does not act contrary to Nature, reason demands that we shall be content with a lesser one, viz. that formerly there was a very wise, learned, and excellent race, as has been proved before, which reached this condition after a long time and through much experience. And if we wish to be considered worthy followers of these predecessors, we must not disdain with beastly ingratitude such great gifts bequeathed to us, through ignorance, nor foolishly believe the slanderers, who do not know them, nor, abandoning the mirror of languages, frequently revel in their ugly dregs; but on the contrary should valiantly protect it, not with idle words as are those of the others, nor according to the unwisdom of those who have so little humility as to deem the excellence of things described in their language to be the excellence of the language. But just as gold is tested by fire, so shall its superiority be proved by practice. What shall this be? Take for subject those things which have hitherto been Nature's profound secrets in all the other languages, which they cannot completely denote; for example let them imitate this (among thousands of other things in which Dutch abounds): Ghelijck rechtheflini tot scheefheflini, alsoo rechthefwicht tot scheefhefwicht*³⁾. You may safely promise a cake to those who succeed in this; nay, that they may stay up late (children should after all be given children's rewards), and I assure you that you will get off cheaply, for it is sufficiently evident of what they are capable in this respect, viz. that they can only replace these words by long circumlocutions, which greatly obscure the distinction between the terms and the form of the proportion throughout. But if

¹⁾ I hear, of which the other words are derivatives.

²⁾ salt-tub, chopping knife, to go out, to see.

³⁾ Prop. XX of Book I of the *Art of Weighing*.

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verduyfteren. Maer soo sy vraghen wat sulcke woorden te bedien hebben, men mach antwoorden, dattet de opening is van t'ghene tot nocht toe den voorighen sterflicken seerbegheerde verborghentheden gheweest sijn, streckende tot groot voordeel van t'menschelick gheslacht, want hoewel yder lichaem in sijn eyghen plaets licht noch swaer en is, nochtans t'ghewicht des lochts is duer sulcx nu volcomelick ghelijck van ander stoffen, metgaders ettelicke sijn noytbekende * ancleuinghen, openbaer gheworden, soo de * daet van dies ende meer anderen cortelick betuyghen sal. Laetter maer cloelick anuallen, want hebbender Reuchlinus, Valla, Erasmus, Barbarus, Picus, Politianus, &c. me duer gheroicht, die maer Latijn en beschermden, Sghelijcx de Françoysen, wiens * strijtredden ende talens * stof ons kennelick ghenouch sijn, wat sullen wy die het (O weerdighen * grondt!) DUYTSCH voorstaen? Seker niet alleen de spraek ophelpen, noch ons seluen voorderen, maer oock ander volcken, welcke aldan niet alleen huer wooninghen ende lichamen met der Duytschen constighe wercken versieren sullen, maer oock haren gheeft met wetenschap, want de Consten welcke ander met haer eyghen woorden niet uyten en connen, die sal den ghemeenen man alhier duer de beghintelen grondelick naeghen verstaen, ende door sijn ingheboren gheneghentheyt tot de selue, die tot yder volcx baet al andersins connen voorderen dant den anderen mæghelick is.

Dit is t'ghene wy vande weerdicheyt der Duytsche tael. voorghenomen hadden te verclaren; Inde selue sullen wy de WEGHCONST, die de wonderlicste der vrie is, eerst tot Constens form laten comen, als spraek die der Natueren eyghenschappen grondelicx beteeckenencan, ende als bequaemste wit, daer al d'ander die willen, tot yder ghe-meentens grootste nut, na micken, ende haer * bepalinghen, daer inde Consten veel an ghelegghen is, na rechten mæghen. Oock by aldien der Duytschen vliet daerin alsoo vermeerderde, ghelijct de reden wel eyscht, t'selue soude ons voornemen verstercken om met ander angheuanghen voort te varen: Doch soo de contrarie gheschiede, ick can my vernoughen in een eerlick voornemen mijn goede wille te verclaren, welcke in haer beroup tot yders dienst ghecyghent is.

CORT-

they ask what these words signify, it may be replied that it is the revelation of those things which had hitherto been secrets greatly coveted by earlier mortals, things which greatly benefit mankind, for though a body is neither light nor heavy in its own place, through this it has now become manifest that the air has weight just as well as other substances, while also several of its unknown attributes have thus become known, as the practical discussion of this and other matters will shortly prove. Let them attack valiantly, for what have Reuchlinus, Valla, Erasmus, Barbarus, Picus, Politianus, etc.¹⁾ achieved, who merely protected Latin, and likewise the French, whose arguments and linguistic material we know well enough? What then shall *we* achieve, who propagate *Dutch* (O worthy subject!)? Certainly not only bring the language on a higher level or advance ourselves, but also other nations, which will then adorn not only their houses and bodies with the artistic products of the Dutch, but also their minds with knowledge, for the arts which other nations cannot express in their own words, will here be thoroughly understood from the elements by the common man, and through his inborn disposition thereto he will be able to advance it, to the profit of all nations, in quite a different way from what is possible to the others.

This is what we had proposed to declare with regard to the worth of the Dutch language. In this language we will first cause the Art of Weighing, which is the most miraculous of the free arts, to attain to the status of an art, being a language which is capable of describing Nature's properties most thoroughly and the most suitable object at which all the others who may wish to be aiming, to the great profit of every community, and on which they may model their definitions, which are of great importance in the arts. Also, if the zeal of the Dutch in this art should increase, as reason demands, this would strengthen our resolve to continue with other planned studies. But if this does not happen, I can be content with an honest resolve to declare my good will, which is at everyone's service, if an appeal is made to it.

¹⁾ Stevin here enumerates some famous humanists: Johann Reuchlin (1455-1522), Lorenzo Valla (c. 1406-1457), Desiderius Erasmus (1469?-1536), Ermolaus Barbarus (1454-1493), Giovanni Pico della Mirandola (1463-1494), Angelo Poliziano (1454-1494).

C O R T B E G R Y P.

Argumenta.

DE Beghinselen der Weeghconst, welcke van swaerheyt sijn duer t'ghedacht van natuerlicke stof gheweert, sullen in twee boucken begrepen worden. Des eersten boucx eerste deel sal van 14 * bepalinghen wesen: T'ander van 28 * voorstellen, vande ghedaenten der ghe-*Definitionib. Propositionibus.* wichten, die tweederhande sijn, als Rechtwichten, ende Scheefwichten. Der Rechtwichten sijn twee * afcomsten, te weten Rechtdaelwichten, *Species.* ende Rechthefwichten, beschreuen inde achtien eerste voorstellen. Der Scheefwichten sijn oock twee afcomsten, als Scheefdaelwichten, ende Scheefhefwichten, verclaert inde rest der voorstellen.

HET tweede bouck der Beghinselen sal vande vinding der * swaer-*Centrorum granitatum. Planorum.* heyts middelpunten sijn, wiens eerste deel vande * Platten is; T'ander vande lichamen. Twelck wy tot meerder claarheyt int corte ende tafelwys aldus vervaten :



THE ARGUMENT

The Elements of the *Art of Weighing* ¹⁾, which deal with gravity, dissociated in thought from physical matter, are to be contained in two books. The first part of the first book is to consist of 14 definitions, the second part of 28 propositions about the properties of weights, which are of two kinds, viz. vertical weights and oblique weights ²⁾. There are two kinds of vertical weights, viz. vertical lowering weights and vertical lifting weights; these are described in the first eighteen propositions. There are also two kinds of oblique weights, viz. oblique lowering weights and oblique lifting weights; these are explained in the remaining propositions.

The second book of the Elements is to deal with the finding of the centres of gravity, to wit: in the first part those of plane figures, and in the second, those of solids. For the sake of greater clearness we summarize this shortly in a scheme as follows:

The Elements of the Art of Weighing comprise two books, the	1st of the properties of weights, of which	the 1st part contains 14 definitions	vertical weights	vertical lowering weights	described in the first 18 propositions.
		the 2nd part contains 28 propositions concerning		vertical lifting weights	
	2nd of the finding of the centres of gravity of	plane figures	oblique weights	oblique lifting weights	explained in the remaining propositions,
		solids		oblique lowering weights	

¹⁾ *Weeghconst* is Stevin's translation of the Latin term *Ars ponderaria*.

²⁾ i.e. weights by means of which vertical and oblique forces respectively are exerted.

HET EERSTE BOVCK¹

VANDE BEGHINSELEN

DER WEEGCONST,

Beschreuen door Simon Steuin.

TEERSTE DEEL

vande Bepalinghen.

I. BEPALING.

Definitie.

WEEGCONST is die, welcke leert de Redenen, Euereidenheden, ende ghedaenten vande ghewichten ofte swaerheden der lichamen.

VERCLARING.



HELIICK de * Meetconst anset der formen grootheden niet hare swaerheden, houdende die alleenelick voor euen ofte oneuen, diens grootheden euen ofte oneuen sijn; Alsoo anset ter contrarie de Weegconst haer swaerheden, niet haer grootheden, houdende die voor euen ende oneuen, diens ghewichten euen ofte oneuen sijn: Ende ghelijck diens voornamelicke wercking bestaet int onderfoucken der * Redenen, Euereidenheden, ende Ghedaenten haerder grootheden, Also defens int onderfoucken der Redenen Euereidenheden, ende ghedaenten haerder swaerheden ofte ghewichten, welcker bescriuing t'voornemen is deses handels.

Geometria

** Rationum,
Proportionū
& qualita-
tum.*

II. BEPALING.

Swærheydt eens lichaems, is de macht sijnder daling in ghestelde plaets.

VERCLARING.

DE swærheydt ofte lichticheydt die wy ghemeenelick segghen een lichaem te hebben, en is niet sijn eyghen welentlicke ghedaente, maer veroirsaect uyt sijn ghemeenschap met een ander (wiens breeder verclaring wy elders gheschiedt hebben) want veel Stoffen die swaer sijn inde locht, worden licht beuonden int water, ende de lichte inde locht, sijn el-

Materia.

A ders

THE FIRST BOOK

OF THE ELEMENTS
OF THE ART OF WEIGHING,
Described by Simon Stevin

THE FIRST PART OF THE DEFINITIONS

DEFINITION I.

The art of weighing is the art which teaches the ratios, proportions, and properties of the weights or gravities of solids.

EXPLANATION.

Whereas geometry relates to the magnitudes of figures, not their gravities, holding only those to be equal or unequal whose magnitudes are equal or unequal, the art of weighing on the contrary relates to their weights, not their magnitudes, holding those to be equal or unequal whose weights are equal or unequal. And just as the chief task of the former consists in examining the ratios, proportions and properties of their magnitudes, so the task of the latter consists in examining the ratios, proportions, and properties of their gravities or weights, the description of which is the object of this treatise.

DEFINITION II.

The gravity of a solid is the power of its descent in a given place.

EXPLANATION.

The gravity or levity which we commonly state a solid to have is not its own essential property, but is caused by reference to something else (the more detailed explanation of which is given elsewhere), for many substances which are heavy in the air are found to be light in water, and those which are light in the air are

ders swaer; daerom als wy segghen een haudtte wegghen hondert pondt; wy verstaen daer by de macht sijnder daling in ghestelde plaets, dat is in dien * Grondt daert in gheweghen was.

* *Subiecto.*

DOOR tverkerde deser bepaling is te verstaen, dat lichticheyt eens lichaems de macht is sijnder rijfing, maer in ghestelde plaets, want eyghentlick is alle lichaem swaer.

III. BEPALING.

BEKENDE swaerheyt is diemen door bekent ghewicht uytet.

VERCLARING.

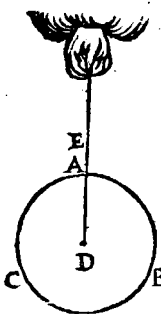
ALS wannermen seght een lichaem ofte swaerheydt te wegghen ses pont, ofte acht marck, oft drie oncen, &c. Om datse door sulcke bekende ghewichten gheuytet wort, wy noemense Bekende swaerheydt.

IIII. BEPALING.

SWAERHEYDTS middelpunt is, an twelck het lichaem door ons ghedacht hanghende, alle ghestalt houdt diemen hem gheeft.

VERCLARING.

LAET ABC een cloot sijn, diens stof ouer al eue swaer is, welcke wy met haer middelpunt D door ons ghedacht nemen te hanghen ende lini ED; Ende is kennelick dat dien cloot ghekeert wordende, sal houden alle ghestalt diemen haer gheeft, want soomen B keerde daer A is, B sal daer bliuen, ende voort yder deel op sijn plaets, want soo dat niet en gheschiede, de stof soude an deen sijde swaerder sijn als an d'ander, twelck teghen tghestelde waer. D dan naer luyt deser bepaling is Swaerheydts middelpunt des cloots ABC; Ende alsoo salmen verstaen dat binnen alle lichamen soo wel ongeschietter form ende van stof oncenuaerdigher swaerheydt als gheschietter ende eenvaerdigher, is eenich sulcken punt, waer an tlichaem also hanghende, alle ghestalt houdt diemen hem gheeft, welck punt ghenoeemt wort sijn Swaerheydts middelpunt. Ende op datter door eenighe sijne eyghenschappen kennelicker sy, sullender noch dit toe segghen: Het swaerheydts middelpunt der oirdentlicke lichamen als Pilaren, Clooten, * Lancworpighe Clooten, der Vijf gheschiette lichamen, &c. ouer al eue wighigher



* *Sphaeroidium.*

heavy elsewhere. If we therefore state a piece of wood to weigh a hundred pounds, we understand by this the power of its descent in a given place, that is in the medium in which it has been weighed.

By the converse of this definition is to be understood that the levity of a solid is the power of its ascent, but in a given place, for properly speaking any solid is heavy.

DEFINITION III.

A known gravity is one expressed by a known weight.

EXPLANATION.

As when a solid or gravity is said to weigh six pounds, or eight marcks¹⁾, or three ounces, etc. Because this gravity is expressed by such known weights, we call it a known gravity.

DEFINITION IV.

The centre of gravity is the point such that if the solid is conceived to be suspended from it, it remains at rest in any position given to it.

EXPLANATION.

Let ABC be a sphere whose material is everywhere equally heavy, which is conceived to be suspended with its centre D from the line ED . It is evident that if this sphere is turned, it will remain at rest in any position given to it, for if B were turned to the position of A , still B would remain there, and every other part would also remain in its place, for if this did not happen, the material would be heavier on one side than on the other, which would be contrary to the supposition. According to this definition therefore D is the centre of gravity of the sphere ABC . And in the same way it shall be understood that within all solids, both those of an irregular form and made of a material of non-uniform weight and those of a regular form and made of a material of uniform weight, there is one and only one point such that the solid, if suspended from it, remains at rest in any position given to it, which point is called its centre of gravity. And in order that it may be better known through some of its properties, we shall add the following remarks. The centre of gravity of the ordinary solids such as prisms, spheres, spheroids, of the five regular solids, etc. — the material being everywhere equally

¹⁾ The *marck* is a unit of weight of German origin. For particulars the reader is referred to K. M. C. Zevenboom and Dr D. A. Wittop Koning, *Nederlandse gewichten. Stelsels, ijkeuzen, vormen, makers en merken*. Leiden, 1953, p. 15 et seq.

wichtigher Stof sijnde, is tselue der Form ofte grootheydt, datmen anders Meetconstich middelpunt noemt. Maer die niet ouer al euewichtigher Stof en sijn, en hebben dese twee punten niet nootfaeckelick tot een selfde plaets. Wat de *naelden, ende ongheschiecte lichamen belangt, sy en hebben gheen formens ofte grootheydts middelpunt, maer alleen des swaerheydts. Het ghebuert oock in veel lichamen als Rynghen, Haecken, Beckens, ende dier ghelijcke, dat haer swaerheydts middelpunt niet en valt inde stof des lichaems, maer binnen tlichaem uyt de stof. Pyramides.

DAER wort inde bepaling gheseydt *Duer ons ghedacht* reden datmen int bepalen moet nemen, tghene den aert van tbepaelde best verclaert, twelck Pappus (daer hy int 8^e bouck het swaerheydts middelpunt bepaelt) door tghedacht oock bequamelick ghedaen heeft. Men soudet oock mueghen aldus bepalen: *Swaerheydts middelpunt eens lichaems, is door twelck alle plat, tlichaem deelt in euestaltwichtighe deelen.* Wat Euestaltwichtigheyt is sal door de 11^e Bepaling verclaert worden.

V. BEPALING.

SWAERHEYTS middellini eens lichaems, is de oneindelicke hanghende door sijn swaerheydts middelpunt.

VERCLARING.

ALS inde form der 4^e bepaling, de oneindelicke hanghende lini door tswaerheydts middelpunt D, daer an de swaerheyt door ons ghedacht hangt, ghelijck D E ouer beyden sijden oneindelick voortghe-trocken, noemen wy de Swaerheydts middellini des lichaems A B C.

VI. BEPALING.

SWAERHEYTS middelplat eens lichaems, is alle plat hem deelende door sijn swaerheydts middelpunt.

VERCLARING.

ALS eenich plat sijnende den Cloot der 4^e bepaling door sijn middelpunt D, wort des selfden Swaerheyts middelplat gheseyt, ende alsoo met allen anderen. Sijn eyghenschap is tlichaem alins te deelen in twee euestaltwichtighe stucken.

VII. BEPALING.

ALLE rechte lini begrepen tusschen twee swaerheyts middellinien, noemen wy dier swaerheden Balck.

heavy — is identical with that of the figure or magnitude, which is otherwise called the geometrical centre. But those solids whose material is not everywhere equally heavy do not necessarily have these two points in the same place. As to the pyramids and irregular solids, they do not have a centre of figure or magnitude, but only a centre of gravity. It may also occur with many solids, such as rings, hooks, basins and the like, that their centre of gravity does not fall within the material of the solid, but inside the solid and outside the material.

The definition contains the word "conceived", because in formulating a definition we should make use of that which is best adapted to explain what is being defined, a method which has also been aptly applied by Pappus by the use of the term "conceived" (where in the 8th book ¹⁾ he defines the centre of gravity). The definition might also be worded as follows ²⁾: *The centre of gravity of a solid is the point through which any plane divides the solid into parts of equal apparent weight.* The meaning of the expression "equal apparent weight" will be explained in Definition XI.

DEFINITION V.

Centre line of gravity of a solid is the infinite vertical through its centre of gravity ³⁾.

EXPLANATION.

Thus, in the figure of Definition IV, the infinite vertical through the centre of gravity D , from which the gravity is conceived to be suspended, as DE , produced indefinitely on either side, is called the centre line of gravity of the solid ABC .

DEFINITION VI.

Centre plane of gravity of a solid is any plane dividing it through its centre of gravity.

EXPLANATION.

Thus, any plane cutting the sphere of Definition IV through its centre D is called a centre plane of gravity of the said sphere, and the same applies to all the others. Its property is to divide the solid always into two parts of equal apparent weight.

DEFINITION VII.

Any straight line contained between two centre lines of gravity, we call the beam of these gravities.

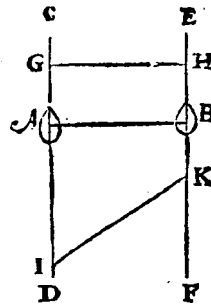
¹⁾ *Pappi Alexandrini Collectionis Mathematicae quae supersunt* ed. F. Hultsch. 3 vols, Berlin, 1875-78. VIII 5. III. 1030.

²⁾ This definition is also given by Pappus l.c. On the meaning of the term „equal apparent weight”, see p. 38.

³⁾ This definition has been somewhat modified in XI; iv, 1 with a view to the applications to be made in the *Byvouch*. There any line through the centre of gravity is called centre line of gravity, whereas the vertical line through the centre of gravity is called vertical centre line of gravity. In the subsequent applications the term „centre line of gravity” is usually taken to mean the vertical through the point of suspension. See the note on Prop. 6.

VERCLARING.

LAET A ende B twee lichamen wesen, ende haer swaerheyds middellinien C D ende E F, tusschen de welke ghetrocken sijn, eenighe linien soot valt als G H, A B, I K, yder van dien, ende alle ander alsoo begrepen tusschen twee swaerheyds middellinien, noemen wy den Balck der swaerheden A B.

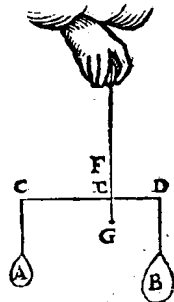


VIII. BEPALING.

WESENDE den Balck ghedeelt met de swaerheyts middellini daer de twee swaerheden eucstaltwichtich an sijn, wy noemē de deelen Ermen.

VERCLARING.

LAET A B, twee lichamen wesen, diens balck sy C D, welke ghedeelt is in E, met de swaerheyds middellini F G, daer de twee swaerheden eucstaltwichtich an hanghen; de twee deelen des balcx als E C ende E D worden Ermen ghe-noemt.



IX. BEPALING.

ENDE die swaerheyds middellini der twee swaerheden, heeten wy Handthaeft.

VERCLARING.

ALS E F, der 8^e bepaling wort Handthaeft ghe-noemt.

X. BEPALING.

ENDE des Handthaefts punt inden balck, Vastpunt.

VERCLARING.

ALS E, der 8^e bepaling wort Vastpunt gheseyt.

XI. BEPALING.

ENDE die twee swaerheden noemen wy Evcstaltwichtighe.

VERCLA-

EXPLANATION.

Let A and B be two solids, and their centre lines of gravity CD and EF , between which there shall be drawn at random a number of lines, as GH , AB , IK ; we call each of these, and any others contained in the same way between two centre lines of gravity, the beam of the gravities A and B .

DEFINITION VIII.

The beam being divided by the centre line of gravity at which the two gravities are of equal apparent weight, we call the parts arms ¹⁾.

EXPLANATION.

Let A and B be two solids, and their beam CD , which is divided in E by the centre line of gravity FG at which the two gravities are of equal apparent weight. The two parts of the beam, as EC and ED , are called arms.

DEFINITION IX.

And the centre line of gravity of the two gravities is called the handle.

EXPLANATION.

Thus, EF of Definition VIII is called the handle.

DEFINITION X.

And the point of the handle on the beam is called the fixed point.

EXPLANATION.

Thus, E of Definition VIII is called the fixed point.

DEFINITION XI.

And the two gravities are said to be of equal apparent weight.

¹⁾ Evidently this definition is based on the assumption that the centre of gravity of the system of the two bodies is somewhere between their centre lines of gravity, and on the application of the second definition of centre of gravity given in the Explanation of the 4th Definition.

VERCLARING.

ALS A ende B, inde form der 8^e bepaling, tſy haer eyghenwichten euen ofte oneuen ſijn, wy noemen die Eueſtaltwichtighe, ouermidts ſy naer de gheſtalt euewichtich ſijn, want A doet anden balck door tghetſelde ſoo grooten ghewelt als B, ende B als A.

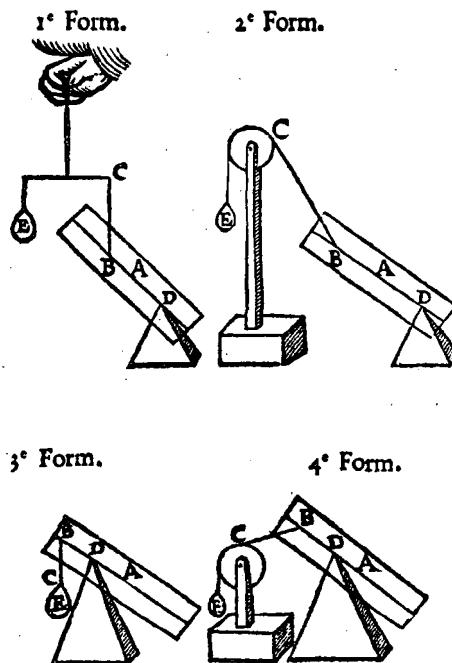
Deſe Eueſtaltwichtigheyt dient nootſaekelick verſtaen, ende onderſcheyden vande Eueneyghenwichtigheyt, anghesien dit al wat anders is als dat, want om by voorbeeld daer af te ſpreken, tghewicht ande cortſte ſijde des onſels hanghende, is ſomtjts thienmael ſwaerder als tander, nochtan hebben ſy een ghelaet van euewichticheyt, maer ten is niet eyghen, dan alleenlick na de gheſtalt.

XII. BEPALING.

HEFWICHT is t'ghene oirſaek is van eens ſwaerheyds verheffing-, ende Daelwicht van eens ſwaerheyds daling.

VERCLARING.

LAET den pilaer A, een ſwaerheyt weſen, diens lini daer ſy alſoo by ghehouden wort ſy B C, en tpunt daer ſy op ruſt D, ende E, ſy t'ghewicht dat lichaem A in die gheſtalt houdt. Wy noemen E der eerſte ende tweede Form Hefwicht, overmidts tſelve wicht, het lichaem A verheft, oft in die verheven gheſtalt houdt. Maer E der derde ende vierde Form, Daelwicht, om dattet het lichaem an ſijn gehechte ſijde B doet dalen, ofte in die ghedaelde geſtalt houdt.



EXPLANATION.

Thus, A and B in the figure of Definition VIII, no matter whether their proper weights are equal or unequal, are said to be of equal apparent weight, since in appearance they have the same weight, for by the supposition A exerts the same force on the beam as B , and B as A .

It is essential that this term equality of apparent weight be understood, and be distinguished from equality of proper weights, since the latter is quite a different thing from the former. In fact, to give an example, the weight hanging at the shorter side of a steelyard is sometimes ten times heavier than the other weight, and yet they seem to have the same weight, but this is not actually so, but only in appearance ¹⁾.

DEFINITION XII.

Lifting weight is that which causes the ascent of a gravity, and lowering weight is that which causes the descent of a gravity.

EXPLANATION.

Let the prism A be a gravity, and let the line by which it is thus held be BC , and the point on which it rests D , and let E be the weight keeping the solid A in that position. We call E in the first and the second figure lifting weight, since this weight lifts the solid A or keeps it in such a lifted position. But E in the third and the fourth figure we call lowering weight, because it lowers the solid on the side B where it is attached, or keeps it in such a lowered position.

¹⁾ See the remarks in the *Introduction*, p. 38.

XIII. BEPALING.

ENDE de rechte lini vande verheven swaerheyt naer thefwicht, noemen wy Heflini, maer vande ghedaelde swaerheyt naer het daelwicht, Daellini, en alfulcke linien in t'gemeen, Trecklini.

VERCLARING.

ALS de rechte lini C B der 12^e bepaling noemen wy inde 1^e ende 2^e form Heflini, maer inde 3^e ende 4^e Daellini. Ende fulcke linien (die bouen de voorgaende ons oock euewidich vanden fichtender connen ontmoeten) in t'ghemeen Trecklini.

XIIII. BEPALING.

Horizon.

ENDE als de Heflini ofte Daellini rechthouckich is op den * Sichteinder, soo noemen wy die Rechthefflini, Rechtdaellini, ende hare ghewichten Rechthefwicht, Rechtdaelwicht: Maer op den Sichteinder scheefhouckich wesende, alsdan Scheefhefflini, Scheefdaellini, ende hare ghewichten Scheefhefwicht, Scheefdaelwicht.

VERCLARING.

ALS de Heflini en Daellini C B der 1^e ende 3^e form vande 12^e bepaling, om dat sy door t'ghestelde rechthouckich sijn op den fichteinder, wy noemen die Rechthefflini, en dese Rechtdaellini, ende haer ghewichten E Rechthefwicht, Rechtdaelwicht: Maer wesende de Heflini ofte Daellini C B, scheefhouckich op den fichteinder, als inde 2^e ende 4^e form, dan heeten wy die Scheefhefflini, ende dese Scheefdaellini, ende haer ghewichten E Scheefhwicht, Scheefdaelwicht.

I^e MERCK.

Astrologia.

WAER Sichteinder by ons een woort soo ghemeen ende bekend als byden Grieken Horizon, welck de Latinen oock ghebruycken, ende daer vooren altemet Finitor, ofte terminator visus, wy en souden daer af hier niet segghen, ouermits sijn eyghen plaets inde * Sterconst is; Maer want den ongheualighen slaep des Spieghels der talen sulcx niet toeghelaten en heeft, oock dat dit woort hier naer dickmael sal ghenoemt worden, sullen dat verclaren, doch niet als wesentlike bepaling deses boucx, om de redenen als vooren, Aldus: Sichteinder is des

DEFINITION XIII.

And the straight line from the lifted gravity to the lifting weight we call lifting line, and the one from the lowered gravity to the lowering weight we call lowering line, and all such lines in general we call drawing lines ¹⁾).

EXPLANATION.

Thus, the straight line *CB* of Definition XII is called lifting line in the 1st and the 2nd figure, and lowering line in the 3rd and the 4th figure. And such lines (which besides the above may also be parallel to the horizon) are called in general drawing lines.

DEFINITION XIV.

And when the lifting or the lowering line is at right angles to the horizon, we call it vertical lifting line and vertical lowering line, and the respective weights: vertical lifting weight and vertical lowering weight. But when it is oblique to the horizon, we call it oblique lifting line and oblique lowering line, and the respective weights oblique lifting weight and oblique lowering weight.

EXPLANATION.

As the lifting and the lowering line *CB* of the 1st and the 3rd figure of Definition XII; because by the supposition they are at right angles to the horizon, we call the former vertical lifting line and the latter vertical lowering line, and their weights *E*: vertical lifting weight and vertical lowering weight. But when the lifting line or lowering line *CB* is oblique to the horizon, as in the 2nd and the 4th figure, we call the former oblique lifting line and the latter oblique lowering line, and their weights *E* oblique lifting weight and oblique lowering weight.

NOTE I ²⁾).

If "*sichteinder*" were with us a word as common and familiar as "horizon" with the Greeks, which the Latins also use, and formerly sometimes "*finitor*" or "*terminator visus*", we should not mention it here, because its proper place is in astronomy. But because the regrettable sleep of the mirror of languages ³⁾ has not permitted this, and also because this word will frequently be used hereinafter, we shall explain it, but not as an essential definition of this book, for the reasons mentioned before, thus: Horizon is the world's greatest circle, which

¹⁾ This definition has been somewhat changed in XI; iv, 1 in accordance with the change in Definition 5.

²⁾ This Note has been omitted in XI; iv, 1.

³⁾ This expression can only be understood in connection with Stevin's theory about the superiority of the Dutch language, as developed in the *Uytſpraeck van de Weerdicheyt der Duytſe Taal*. Dutch is the mirror of languages, i.e. all languages have to take example by it; but unfortunately this mirror has long slept, i.e. it has been neglected.

des weerelts grootste ronds, dat haer sienlick deel scheydt van het onfienlick: Dat is, onder veel ronden die inde Sterconst bepaelt worden, soo isser een het aldermerckelicste, scheydende ooghschynlick den oppersten baluen weereltsloot vanden ondersten, ende in ons ansien den hemel met sin omtreck naeckende, welck volcommentslicxt schijnt vande hooschte plaets eender contreyen, ofte op een water daer hem nergens landt en vertoocht; Ende ouermits ons ghesicht langs der eerden ofte langs het water niet voorder strecken en can dan tot diens ronds voornoemden omtreck, ende daer in eindet, soo wort dat ronds gheenoemt den Sichteinder, dat is den Einder van 't ghesicht. Ende alle platten die op 't eertrick vanden Sichteinder * euewydich sin (welcke by ons ghemeenelick gheseyt worden op waterpas te ligghen) worden * lijck spreuckelick oock sichteinders gheenoemt. Ick seg lijck spreuckelick want eyghenlick ofte * wisconstelick en isser gheen ander, dan dat door des weerelts middelpunt strect.

II. MERCK.

De form vanden Weegconstighen * Pilaer, is de selue der * Meetconst, maer Columna. Wy nemen hier sin stof eenuuerdigheer swaerheyt te wesen, ende sin grondt ende deusel viercanten. Wat de ghemeene constwoorden belangt int Latijn aldus ghebruyck.

Materia	Stof
Forma	Form
Effectus	Daet
Subiectum	Grondt
Alimentum	Ancleuing
Genus	Gheslacht
Species	Afcomst
Definitio	Bepaling
Propositio	Voorstel
Problema	Eysch
Theorema	Vertooch
Ratio	Reden
Proportio	Everedenheyt
Aequales	Even
Similes	Ghelijcke
Exemplum	Voorbeeld
Centrum grauitatis	Swaecheys middelpunt
Axis	As
Diameter	Middellini
Circumferentia	Omtreck
Parallela	Euewydeghe
Homologa latera	Lijckstandighe sijden
Superficies	Vlack

Daer voor
tullen wy
soodanige
Duytsche
stellen

separates its visible from its invisible part. That is, among many circles which are defined in astronomy there is one which is most notable, apparently separating the upper half of the celestial sphere from the lower half and touching in our view the heaven with its periphery, a fact which becomes most completely apparent from the highest place of a region or on an expanse of water where no land presents itself to our view. And since our sight cannot extend along the earth or the water beyond the aforesaid periphery of that circle, and ends therein, that circle is called the "sichteinder", that is the "einder van t' ghesicht" (terminator of sight). And all those planes which on the earth are parallel to the horizon (which with us are usually said to be level) are also called horizons metaphorically. I say metaphorically, for in the proper or mathematical sense there is no other horizon but that passing through the centre of the world.

NOTE II.

The form of the prism as considered in the art of weighing is the same as in geometry, but we here take its material to be everywhere equally heavy, and its base and top to be squares. As to the common technical words, used as follows in Latin:

materia	we shall use the following Dutch words for them	stof	— material
forma		form	— form, figure
effectus		daet	— practice
subiectum		grondt	— medium
adiunctum		ancleuing	— attribute
genus		gheslacht	— genus
species		afcomst	— species
definitio		bepaling	— definition
propositio		voorstel	— proposition
problema		eysch	— problem
theorema		vertooch	— theorem
ratio		reden	— ratio
proportio		everedenheyt	— proportion
aequales		even	— equal
similes		ghelijcke	— similar
exemplum		voorbeelt	— example
centrum gravitatis		swaerheys	— centre of
		middelpunt	gravity
axis		as	— axis
diameter		middellini	— diameter
circumferentia		omtreck	— circumference
parallelae		euwydeghe	— parallel lines
homologa latera		lijkstandighe	— homologous
		sijden	sides
superficies		vlack	— surface

Planum	Plat
Columna	Pilaer
Arithmetica	Telconst
Geometria	Meetconst
Ars Mathematica	Wisconst
Mathematicus	Wisconstnaer
Mathematice	Wisconstlick.

WELCKE Latijnsche met eenighe ander dier by mueghen vallen Wy tot meerder claarheyt, somwylen inden cant sullen scriven neven haer duytsche. Dese drie letteren v. b. E. altemet inde cant gheselt beteecken om cortheyt, voorstel, bouck, Euclides, als 2 v. 6. b. E. dat is te segghen het 2^e voorstel des 6^{en} boucx van Euclides.

B E G H E E R T E N.

ANGHESIEN sommige saken als beghinselen door ghemeene wetenschap bekendt sijn, ende gheen bewijs en behouven; Ander bedestelicker den berispsers tot stof souden dienen, om te straffen t'ghene gheen straf en verdient. Wy sullen naer * Wisconstnaers gebruyck, eer wy tot de voorstellen comen, begheeren dat ons alsulcke toegelaten worden.

I. BEGHEERTE.

WY begheeren datmen toelate euen ghewichten an euen crimen oock cuestaltwichtich te sijne.

II. BEGHEERTE.

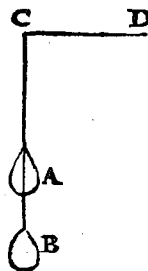
ENDE ande* wisconstighe lini alle ghewicht te connen hanghen ofte daer op te connen rusten, sonder dat sy breke ofte buyghe.

III. BEGHEERTE.

ENDE de swaerheydt hoogher ofte leegher hangende, alijt van een selfde gewicht te blijven.

VERCLARING.

ALS de swaerheydt A neerghetrocken sijnde tot B, aldaer euen soo swaer te wesen, ofte sulcken machtan C D te doen, als sy ter plaats van A dede.



IIII. BE-

<i>planum</i>		plat	— plane (figure)
<i>columna</i>		pilaer	— prism
<i>arithmetica</i>		telconst	— arithmetic
<i>geometria</i>		meetconst	— geometry
<i>ars mathematica</i>		wisconst	— mathematics
<i>mathematicus</i>		wisconstnaer	— mathematician
<i>mathematice</i>		wisconstlick	— mathematically

We shall sometimes give these Latin words, with some more that may occur, in the margin by the side of the Dutch words, for the sake of greater clearness. The three letters v.b.E., sometimes found in the margin, signify for brevity's sake "voorstel" (proposition), "bouck" (book), "Euclides"; for example: 2v.6b.E. means the 2nd proposition of the 6th Book of Euclid.

POSTULATES.

Since some matters of an elementary nature are common knowledge, and need not be proved, while other matters of a more veiled character might give the critics cause to criticize that which does not deserve criticism, we shall, after the custom of mathematicians, before arriving at the propositions, postulate that the following things be granted.

POSTULATE I.

We postulate that it be granted that equal weights at equal arms are also of equal apparent weight.

POSTULATE II.

And that at the mathematical line any weight can hang or rest without its breaking or bending.

POSTULATE III.

And that the gravity always keeps the same weight, no matter whether it hangs higher or lower.

EXPLANATION.

And that the gravity *A*, being pulled down to *B*, has the same weight in that place or exerts on *CD* the same force as it did in the place *A* ¹⁾.

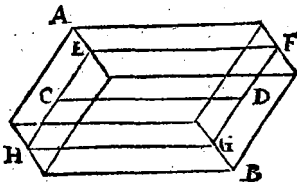
¹⁾ It is only in the Explanation that the meaning of the Postulate becomes manifest: it is not only the weight of the body which is unchanged when it is elevated or lowered, but also the influence it exerts on the lever.

III. BEGHEERTE.

ENDE datmen by des pilaers beschreuen plat
r'welck hem door de langde des as deelt, verstaen
sal den voorghestelden pilaer.

VERCLARING.

Als wifende A B een pilaer diens
as C D, ende de felue doorfneen met
eenich plat als E F G H, datmen
door t'bescreuen plat E F G H, al de
rest achterghelaten, verstaen sal den
ghegheuen pilaer.



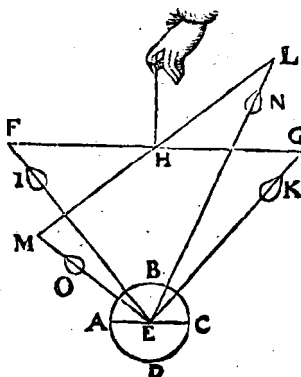
V. BECHERTE.

ENDE alle hanghende linien voor * ewewy- *Parallel.*
dighe ghehouden te worden.

VERCLARING.

D^e reden is dese; Laet A B C D den eertsfloot sijn, wiens middel-
punt E, ende * sichteinder A C, ende F G een balck, euewydich vanden *Horizon.*
sichteinder A C, diens balcx even ermen H F, H G, ende euen swaer-
heden daer an I, K, alwaer het blyct, dat de hanghende linien F I, ende
G K, gheen euewydigheen sijn, maer
onder naerder malcander dan bo-
uen; Laet daer naer den balc F G ghe-
keert worden op t'vastpunt H, alsoof
dat G comme daer nu L is, ende F
daer M, ende K sal commen daer nu
N, ende I daer nu O is, ende den
houck L M E is naerder den recht-
houck dan M L E, waer duer O (als
in het volghende 22^e voorstel blij-
ken sal) naer de ghestalt swaerder is
dan N. Vyt defen volght oock dat
onder alle lichamelicke vormen die
inde natuer bestaen, so en isser gheen
ander, * wisconfelick sprekende,
dan den cloot, an wiens swaerheyds

*Mathem.
ca.*



**Mathemati-
cs.**

middelpunt het lichaam door ons ghedacht hanghende, alle gheftalt houdt diemen hem gheeft; Ofte door twelck alle plat, lichaam deelt

B in eve-

POSTULATE IV.

And that a plane through the axis ¹⁾ of the prism shall stand for the given prism.

EXPLANATION.

Let AB be a prism, its axis CD , and let the prism be cut by any plane, as $EFGH$. That the plane $EFGH$, all the rest omitted, shall stand for the given prism.

POSTULATE V.

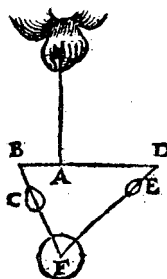
And that all verticals be held to be parallel lines.

EXPLANATION.

The reason is the following. Let $ABCD$ be the terrestrial sphere, the centre of which is E , the horizon AC , and FG a beam parallel to the horizon AC , the equal arms of this beam HF , HG , and let there be hanging therefrom equal gravities I , K ; from which it may be seen that the verticals FI and GK are not parallel lines, but are nearer to one another at the bottom than at the top. Let then the beam FG be turned about the fixed point H so that G comes where L is now, and F where M , and K will come where N , and L where O is now, and the angle LME is nearer to a right angle than MLE , owing to which (as will become apparent in the 22nd proposition hereinafter) O is heavier in appearance than N . From this it also follows that among all the corporeal forms existing in Nature there is, mathematically speaking, none but the sphere which remains at rest in any position given to it, when conceived to be suspended from its centre of gravity, or which is divided by any plane through the centre of gravity into parts

¹⁾ The axis of a prism or a cylinder is the straight line joining the centres of gravity of the parallel faces. The axis of a pyramid or a cone is the straight line joining the vertex with the centre of gravity of the base.

in eueftaltwichtighe deelen, maer om de oneindelicke verfcheyden gheftalten, fullender oneindelicke verfcheyden fwaerheys middelpunten in fijn. Oock en foude (teghen t'volgende 1^e voorftel) de fwaerfte fwaerheyt niet fulcken reden hebben tot de lichtfte, als den langften erm tot den cortften, maer d'eene foude naer de gheftalt fwaerder fijn, om dat haer houck plomper ende den rechthouck naerder is dan des anders houck. Maer om t'selue by voorbeeld te verclaren, laet AB den cortften erm fijn, diens ghewicht C, ende AD den langften erm, diens ghewicht E in fulcken reden fy tot t'ghewicht C, als AB tot AD, ende F fy t'sweerefts middelpunt; Alwaer blijft dat den houck FBA plomper ende den rechthouck naerder is, dan den houck ADF, waer uyt volght (door tvootnoemde 22^e voorftel) dat C naer de gheftalt fwaerder fal fijn dan E.



Alle defe ongheualen spruyten daer uyt, dat FE met GE in d'eerste form, ofte BF met DF der tweede form, gheen ewewydighe linien en fijn: Maer ouermits dat verfchil in alle t'ghene de menfchen wegghen, onbemerckelick is, want den balck foude al veel milen lanck moeten fijn eer hem dat foude connen openbaren, foo begheeren wy datse voor ewewydighe ghehouden worden. Wel is waer dat wy die anfiende voor t'ghene fy fijn, volcommelick fouden connen wercken na haerlieder ghedaente, maer want dat moeyelicker foude wesen, ende tot de faeck, dat is de **W E E G D A E T** nochtans niet voordrlicker, fo ist beter ghelaten.

H E T

of equal apparent weight, but because of the infinite variety of forms there will be an infinite number of different centres of gravity in them. Also (contrary to the 1st proposition hereinafter) the heavier gravity would not have to the lighter the same ratio as the longer arm to the shorter, but the one would be heavier in appearance than the other, because its angle is more obtuse and nearer to a right angle than the angle of the other. But in order to explain this by an example, let AB be the shorter arm, its weight C , and AD the longer arm, whose weight E shall have to the weight C the same ratio as AB to AD , and let F be the centre of the earth; then it appears that the angle FBA is more obtuse and nearer to a right angle than the angle ADF , from which it follows (by the aforesaid 22nd proposition) that C will be heavier in appearance than E .

All these difficulties result from the fact that FE and GE in the first figure, or BF and DF in the second figure, are not parallel lines. But since this difference is imperceptible in all things weighed by us — for the beam would have to be many miles long before it would become perceptible — we postulate that they be held to be parallel lines. It is true that, taking them for what they are, we should be able to operate exactly according to their nature, but because this would be more difficult, and yet would be of no advantage for the practice of weighing, it is better not to do this.

HET ANDER DEEL VANDE VOORSTELLEN.

I. VERTOOGH.

I. VOORSTEL.

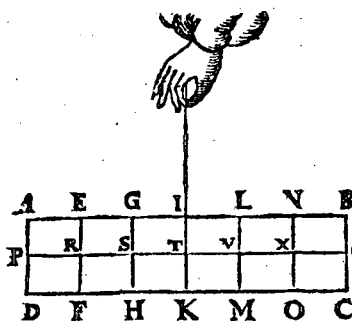
*Theorema.
Propositio.*

W E S E N D E twee eueftaltwichtighe fwaerheden, de fwaerfte heeft fulcken reden tot de lichtfte, als den langften erm tot den cortften.

1^e VOORBEELT.

T G H E G H E V E N. Laet A B C D een pilaer fijn weghen- *Datum.*
de 6 lb. welcke ghedeelt fy in 6 euen deelen, door * plat- *Plana parab-*
ten euewydich van fijn grondt A D, als E F, G H, I K, *ola.*
L M, N O, fniende den as P Q in R, S, T, V, X: Laet
ons nu nemen L M D A voor de fwaerfte fwaerheydr,
wiens fwaerheys middelpunt is S, ende L M C B voor

de lichtfte fwaerheydt, wiens
fwaerheys middelpunt is X,
en SX is dier deelen balck door
de 7^e bepaling, en T is t'fwaer-
heys middelpunt des heelen
pilaers, ende T I d'hanthaef,
waer an L M D A ende L M
C B eueftaltfwichlich hangen,
ende T X is den langften erm,
ende T S den cortften door de
8^e bepaling. T B E G H E E R D E.
wy moeten bewyfen dat ghe-
lijck de fwaerfte fwaerheydt



Quaestio.

L M D A, tot de lichtfte L M C B, also den langften erm T X, tot den
cortften T S. T B E W I I S. De fwaerfte fwaerheydt L M D A weeght *Demonstra-*
4 lb, ende de lichtfte L M C B 2 lb, ende den langften erm T X *io.*
heeft fulcken reden tot de cortfte T S, ghelijck 2 tot 1 door t'ghegheven:
Maer ghelijck 4 tot 2, alsoo 2 tot 1, ghelijck dan de fwaerfte fwaerheydt
L M D A, tot de lichtfte L M C B, also den langften erm T X, tot den
cortften T S.

M A E R op datmen niet en dencke dit daer also by gheualle ghe- *Mathemati-*
fciendt te sijne, wy fullender * Wifconstich bewys af doen aldus: *cam demon-*
strationem.

11^e VOORBEELT.

T G H E G H E V E N. Laet A B C D wederom een pilaer fijn, ghedeelt
B 2 met een

THE SECOND PART OF THE PROPOSITIONS

THEOREM I.

Given two gravities of equal apparent weight, the heavier has to the lighter the same ratio as the longer arm to the shorter.

PROPOSITION I.

EXAMPLE I.

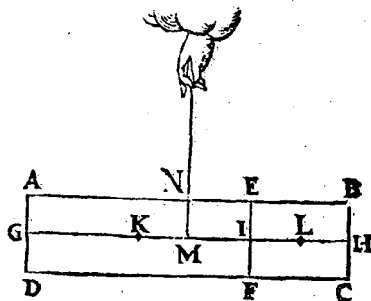
SUPPOSITION. Let $ABCD$ be a prism weighing 6 lbs, which shall be divided into 6 equal parts by planes parallel to its base AD , as EF, GH, IK, LM, NO , meeting the axis PQ in R, S, T, V, X . Let us now take $LMDA$ for the heavier gravity, whose centre of gravity is S , and $LMCB$ for the lighter gravity, whose centre of gravity is X ; then SX is the beam of these parts by the 7th definition, and T is the centre of gravity of the whole prism, and TI the handle at which $LMDA$ and $LMCB$ are hanging in equality of apparent weight, and TX is the longer arm and TS the shorter arm by the 8th definition. **WHAT IS REQUIRED TO PROVE.** We have to prove that as the heavier gravity $LMDA$ is to the lighter $LMCB$, so is the longer arm TX to the shorter TS . **PROOF.** The heavier gravity $LMDA$ weighs 4 lbs and the lighter $LMCB$ 2 lbs, and the longer arm TX by the supposition has to the shorter TS the ratio of 2 to 1. But as 4 is to 2, so is 2 to 1; therefore, as the heavier gravity $LMDA$ is to the lighter $LMCB$, so is the longer arm TX to the shorter TS .

But in order that it may not be thought that this happened only accidentally, we shall give a mathematical proof of it, as follows.

EXAMPLE II.

SUPPOSITION. Let $ABCD$ again be a prism, divided by a plane parallel to

met een plat euewydich van A D, als E F, sniende den as G H, waert sy in I, ende het swaerheys middelpunt van het deel E F D A sy K, int middel van G I, ende van het deel E F C B, sy L int middel van I H, en des heels A B C D sy M int middel van G H, ende M N sal der deelen E F D A ende E F C B handthaeft sijn, daer an sy euestaltwichtich hanghen.



L Ghestalt.

T B E G H E E R D E. Wymoc-
ten bewyfen dat ghelijck het
lichaem ofte de swaerheydt
(twelck hier een selfde is om

Proportionē.

* haer eueredenheydt, want
ghelijck lichaem E F D A, tot lichaem E F C B, alsoo diens swaer-
heyt tot defens, ouermits den pilaer door tghestelde oueral eenuaerdi-
gher swaerheyt is) van E F D A, tot E F C B, also den langsten erm M L,
tot den cortsten M K. **T B E W Y S.** 1^e L I D T. M H is euen an M G
door tghheueu, laet tot elck doen K M, soo sal dan K H euen sijn an
M G met K M; daer naer van d'ene ghetrocken G K, ende van d'ander
K I (welcke G K ende K I euen sijn door tghheueu) soo sal K M met
K M euen bliuen an I H; Ende haer helften als K M ende I L sullen oock
euen sijn. 1¹ L I D T. Laet tot elck (te weten K M ende I L) doen
M I, Ende M L sal euen sijn an I K. 1¹¹ L I D T. Ghelijck G I tot haer
helft K I, also I H tot haer helft I L, ende door * oueranderde euereden-
heydt ghelijck G I tot I H, also K I tot I L, maer K I is euen an M L door
het 2^e lidt, ende I L an M K door het 1^e lidt, daerom ghelijck G I tot
I H, alsoo M L tot M K; Maer ghelijck G I tot I H, also het lichaem ofte
de swaerheyt E F D A, tot E F C B. Ghelijck dan de swaerste swaerheyt
E F D A, tot de lichtste E F C B, also den langsten erm M L, tot den
cortsten M K.

*Alternam
proportionem*

NV mocht yemant segghen, ghy hebt dat voorstel wel bewesen in
deelen die tsamen een heel pilaer maken eenuaerdigher swaerheyt,
maer wie weet of dat also plaets sal houden in allen anderen verscheyden
deelen van ongheschieder form, ende oneueswaerder stof, daerom sul-
len wy de ghemeenheydt des voorstels aldus bethoonen: Laet ons ach-
ten dat den balck K L der 1^e ghestalt hier bouen, in haer plaets bliue,
ende dat het stick E F D A neerghetrocken wordt, ende dat het blyue
hanghende met een lini uyt sijn swaerheydts middelpunt an tpunt K,
ende dat in ghelijcx oock neerghetrocken sy het ander stick E F C B,
ende dat het blyue hanghende by sijn swaerheydts middelpunt an
tpunt L,

AD , as EF , meeting the axis GH in I , and let the centre of gravity of the part $EFDA$ be K in the middle of GI , and that of the part $EFCB$, L in the middle of IH , and that of the whole $ABCD$, M in the middle of GH , and MN shall be the handle of the parts $EFDA$ and $EFCB$, at which they are hanging in equality of apparent weight. WHAT IS REQUIRED TO PROVE. We have to prove that as the solid or the gravity (which is the same thing in this case on account of their proportionality, for as the solid $EFDA$ is to the solid $EFCB$, so is the gravity of the former to that of the latter, since by the supposition the prism is everywhere equally heavy) $EFDA$ to $EFCB$, so is the longer arm ML to the shorter MK . PROOF, FIRST SECTION. MH is equal to MG by the supposition; add to each KM , then KH will be equal to MG plus KM . If then GK is subtracted from the one and KI from the other (which GK and KI are equal by the supposition), KM plus KM will remain equal to IH , and their halves, as KM and IL , will also be equal. SECOND SECTION. Add to each (viz. KM and IL) MI ; then ML will be equal to IK . THIRD SECTION. As GI is to its half KI , so is IH to its half IL , and by taking the terms alternately: as GI is to IH , so is KI to IL . But KI is equal to ML by the second section, and IL to MK by the first section, therefore, as GI is to IH , so is ML to MK ; but as GI is to IH , so is the solid or gravity $EFDA$ to $EFCB$. Consequently, as the heavier gravity $EFDA$ is to the lighter $EFCB$, so is the longer arm ML to the shorter MK ¹⁾.

Now someone might say: You have proved this proposition indeed of parts which together constitute a complete prism made of material which is everywhere equally heavy, but who knows whether the proposition will also hold with regard to all other different parts of irregular form and made of material which is not everywhere equally heavy. Therefore we shall prove the general validity of the proposition as follows. Let us assume that the beam KL of the 1st figure above remain in its place, and that the part $EFDA$ be pulled down, and remain hanging at the point K by a line from its centre of gravity, and that likewise the other part $EFCB$ be also pulled down, and remain hanging at the point L in its centre

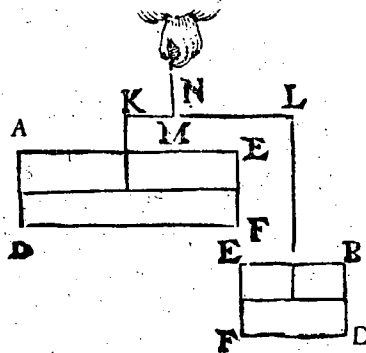
¹⁾ For a criticism of this argument, see the *Introduction*, p. 40.

rpunt L, ende dat E F C B niet en ghenake an E F D A, ende haer ghestalt sy dan soo dees form uytwyft. Nu doen het lichaem in d'eerste ghestalt hinck ande hanthaef M N, alldoen was E F D A euestaltwichtich met E F C B; Maer tghewicht E F D A in dees tweede ghestalt neerghetrocken synde, en brengt an K L gheen meerder noch minder swaerheyt dan in d'eerste ghestalt door de 3^e begheerte. Sghelijcx en brengt tghewicht

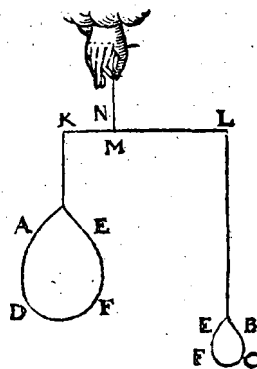
E F C B der tweede ghestalt, an L K gheen meerder swaerheyt dan in d'eerste ghestalt, daerom de ghewichten der tweede ghestalt sijn an K L de selfde die sy in d'eerste waren, daerom oock de balck K L blijft noch inde selue eerste ghestalt, waer duer E F D A noch euestaltwichtich blijft met E F C B. De sticken dan des pilacs bliuen soo wel euestaltwichtich verscheyden, als doen sy an malcanderen waren, ende de ermen oock inde selue reden.

Dit so synde, laet ons de lichamen E F D A ende E F C B der tweede ghestalt ander formen gheuen, die alsoo duwende (neemt dat de stof sy van was, cleye, ofte yet foodanich twelck sulcx lijde) dat E F D A der tweede ghestalt, sy E F D A deser derde ghestalt, ende dat E F C B der tweede ghestalt, sy E F C B deser derde ghestalt; Ende is openbaer dat K L noch in haer selue ghestalt sal bliuen, ende de ermen M L, M K, inde selue reden, ende veruolgens E F D A noch euestaltwichtich met E F C B, want dees verandering der form (al de stof bliuende) en verouirsaect gheen verandering des ghewichts.

LAET ons ten laetsten weeren E F D A der derde ghestalt ende hanghen in diens plaets een lichaem van loot des selfden ghewichts, ende inde plaets van E F C B een hanten lichaem des seluen ghewichts,



2. Ghestalt.



3. Ghestalt.

of gravity, and that *EFCB* do not touch *EFDA*; then their position will be such as this figure shows. Now when the solid in the first figure hung at the handle *MN*, *EFDA* was of equal apparent weight to *EFCB*. But when the weight *EFDA* is pulled down in this second figure, it does not cause any more or less gravity to hang at *KL* than in the first figure by the 3rd postulate. Likewise the weight *EFCB* of the second figure does not cause any more gravity to hang at *LK* than in the first figure. Therefore the weights of the second figure at *KL* are the same as in the first figure, and therefore also the beam *KL* remains in the position of the first figure, owing to which *EFDA* remains of equal apparent weight to *EFCB*. Thus the parts of the prism, when separated, remain of equal apparent weight just as when they were joined, and the arm also have the same ratio.

This being so, let us give other forms to the solids *EFDA* and *EFCB* of the second figure, moulding them in such a way (assuming the material to be wax, clay or something of the kind, which shall admit of it) that *EFDA* of the second figure shall become *EFDA* of this third figure, and that *EFCB* of the second figure shall become *EFCB* of this third figure. Then it is manifest that *KL* will remain in the same position, and the arms *ML*, *MK* will have the same ratio, and consequently *EFDA* will still be of equal apparent weight to *EFCB*, for this change of the form (all the material remaining) does not cause any change in the weight.

Let us finally take away *EFDA* of the third figure, and hang in its place a solid of lead of the same weight, and in the place of *EFCB* a wooden solid of the

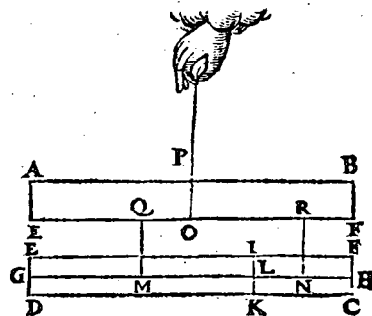
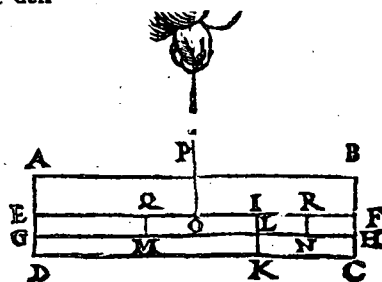
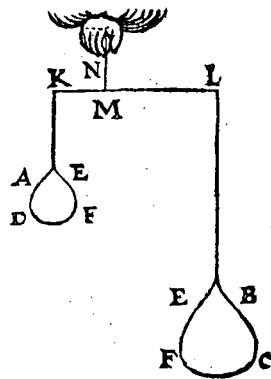
4. *Ghestalt.* wichts, wiens vierde ghestalt alsdan sy als hier neuens. Ende is kennelick dat KL noch inde selue ghestalt sal blyuen, ende veruolghens EFD A noch eueftaltwichtich met EFCB, ende de ermen noch inde selue reden.

III. VOORBEELT.

MEN can tvoorgaende oock be-
thoonen, blyuende de twee swaerhe-
den hanghende an eenen lichamelick-
ken balck, in defer voughen: Laet
den pilaer ABCD ghesneen sijn in
twee deelen, met een plat door den
as EF, ende den as des on-
dersten deels EC sy GH,
ende EC sy doorsneen met
een plat IK euewydich van-
den grondt ED, sniende den
as GH in L, ende het swaer-
heydts middelpunt van het
deel IKDE sy M int mid-
del van GL, ende van het
deel IKCF sy N int mid-
del van LH, ende des heels

ABCD sy O in middel van EF, ende OP sy swaerheydts middelli-
ni des heels ABCD, ende MQ van IKDE, ende NR van IKCF.
Dit soo sijnde tis kennelick dat des heels pilaers rechter sijde, euewich-
tich is teghen haer sincker.

LAET ons nu het onderste
deel EFC D neertrecken, also
dat het blyue hanghende ande
linien ML ende NR, als hier
neuens. Ende is openbaer dat
den lichamelickē balc ABFE
noch in haer eerste ghestalt sal
blyuen. Laet ons nu achten
dat het deel IKDE, ghesneen
sy van IKCF, ende dat elck
deel vallē mach daert wil, maer
sy hanghen an haer swaerheys



middel-

same weight, the situation then being as shown in the annexed fourth figure. It is then evident that KL will again remain in the same situation, and consequently $EFDA$ will still be of equal apparent weight to $EFGB$, and the arms will still have the same ratio.

EXAMPLE III.

The above may also be shown when the two gravities remain hanging from a physical beam, in the following way: Let the prism $ABCD$ be cut into two parts by a plane through the axis EF , and let the axis of the lower part EC be GH , and let EC be cut by a plane IK parallel to the base ED , meeting the axis GH in L , and let the centre of gravity of the part $IKDE$ be M in the middle of GL , and that of the part $IKCF$, N in the middle of LH , and that of the whole $ABCD$, O in the middle of EF , and let OP be the centre line of gravity of the whole $ABCD$, and MQ of $IKDE$, and NR of $IKCF$. This being so, it is evident that the right side and the left side of the whole prism are of equal weight ¹⁾).

Now let us pull down the lower part $EFCD$, in such a way that it shall remain hanging from the lines ML and NR , as shown in the annexed figure. Then it is manifest that the physical beam $ABFE$ will still remain in the situation of the first figure. Now let us suppose that the part $IKDE$ be cut from $IKCF$, and that either part is free to fall at will; but they are hanging at their centres of gravity

¹⁾ Read: of equal apparent weight.

middelpunten M, N, sy houden dan haer eerste ghegheuen ghestalt door de 4^e bepaling, daerom A B F E blijft oock noch in sijn eerste ghedaente. Maer I K D E, sulcken reden te hebben tot I K C F, als den erm O R, tot den erm O Q, is vooren beproeft; Inder voughen dat tghene eerst betoocht was anden weegconstighen balck (dat is een lini) sulcx hebben wy hier verclaert an een lichamelicken. **T B E S L V Y T.** *We- Conclusio.* fende dan twee euestaltwichtighe swaerheden, de swaerste heeft sulcken reden tot de lichtste (van wat stof ofte form oock de lichamen sijn) als den langsten erm tot den cortsten, twelck wy bewysen moesten.

V E R V O L G.

V Y T het verkeerde des voorgaenden voorstels volcht, dat hebbende de swaerste swaerheydt sulcken reden tot de lichtste, als den langsten erm tot den cortsten, dat die twee swaerheden euestaltwichtig sijn.

I. EYSCH.

II. VOORSTEL.

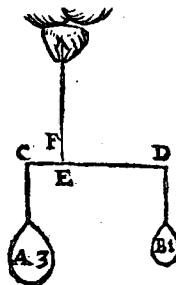
Problema.

WESSENDE ghegheuen bekende swaerheden, haer handthaef te vinden.

I^e VOORBEELT.

T G H E G H E V E N. Laet d'een swaerheyt A sijn weghende 3 lb, hanghende an C, d'ander B van 1 lb hanghende an D; ende C D si balck.

T B E G H E E R D E Wy moeten haer hanthaef vinden. **T W E R C K.** Men sal C D also deelen, dat haer meeste stick naest de swaerheyds middellini van de minste swaerheydt, sulcken reden hebbe tot het minste stick, ghelijck de meeste swaerheyt tot de minste, twelck sy in E, te weten dat E D sulcken reden hebbe tot E C, als 3 lb van A, tot 1 lb van B. Ick seg dat de hanghende door E, als E F, d'hanthaef is.



II VOORBEELT.

T G H E G H E V E N. Laet d'een swaerheydt sijn den pilaer A B C D weghende 6 lb, ghedeelt als den pilaer int beghin des eersten voorstels; Ende an Q hanghe een ghewicht Y van 12 lb. **T B E G H E E R D E.** Wy moeten d'handthaef vinden. **T W E R C K.** De swaerheyds middellini des pilaers is I T, en van tghewicht Y is B Q, ende T Q is balck, de selue

M , N , and they therefore remain in their first given situation by the 4th definition. Hence $ABFE$ also still remains in its first situation. But it has been proved before that $IKDE$ has to $IKCF$ the same ratio as the arm OR to the arm OQ . Therefore, what was first proved with regard to a beam as considered in the art of weighing (that is a line), we have here explained with regard to a physical beam. CONCLUSION. Given therefore two gravities of equal apparent weight, the heavier one has to the lighter (no matter of what material the solids consist and what form they have) the same ratio as the longer arm to the shorter, which we had to prove.

COROLLARY.

From the converse of the preceding proposition it follows that if the heavier gravity has to the lighter the same ratio as the longer arm to the shorter, the two gravities are of equal apparent weight¹⁾.

PROBLEM²⁾ I.

Given two known gravities, to find their handle.

PROPOSITION II.

EXAMPLE I.

SUPPOSITION. Let the one gravity be A , weighing 3 lbs, hanging at C , the other B of 1 lb, hanging at D , and let CD be the beam. WHAT IS REQUIRED TO FIND. We have to find their handle. CONSTRUCTION. CD shall be divided in such a way that the longer segment of it, adjacent to the centre line of gravity of the lighter gravity, shall have to the shorter segment the same ratio as the heavier gravity to the lighter, and let the point of division be at E , to wit so that ED shall have to EC the same ratio as A (3 lbs) to B (1 lb). I say that the vertical through E , as EF , is the handle.

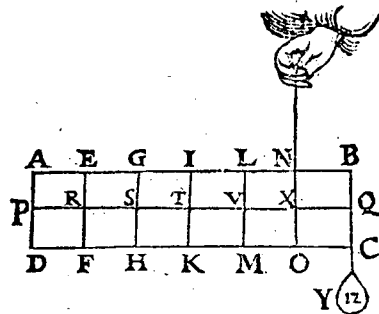
EXAMPLE II.

SUPPOSITION. Let the one gravity be the prism $ABCD$, weighing 6 lbs, divided like the prism at the beginning of the first proposition; and let there be hanging at Q a weight Y of 12 lbs. WHAT IS REQUIRED TO FIND. We have to find the handle. CONSTRUCTION. The centre line of gravity of the prism is IT , and that of the weight Y is BQ , and TQ is the beam. The latter shall

¹⁾ As has been remarked in the *Introduction*, p. 39-40, this converse proposition ought to have been proved as well.

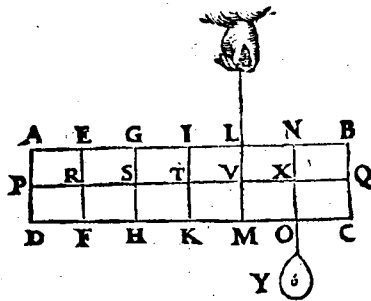
²⁾ The Dutch term *Eysch* has been changed into *Werkstick* in XI; iv, i and may accordingly be translated by *Problem*.

de selue salmen in twee deelen, alsoo dat de stikken de reden hebben als 12 lb van Y, tot 6 lb vanden pilaer, welverstaende tcoortste stick naer de swaerheyds middellini vande swaerste swaerheydt Y, twelck vallen sal in X, inder voughen dat N X de begheerde handhaef is.



III. VOORBEELT.

TGHEGHEVEN. Laet A B C D wederom den pilaer sijn, ghedeelt als vooren, hanghende nu Y 6 lb an X. **TBEGHEERDE.** Wy moeten d'handhaef vinden. **TWERCK.** De swaerheyds middellini des pilaers is I T, ende van Y is N X, ende T X is balck: de selue salmen in twee deelen, alsoo dat de stikken de reden hebben als 6 lb van Y, tot 6 lb des pilaers, twelck vallen sal in V, inder voughen dat V L de begheerde handhaef sijn sal.



TVOORNOEMDE WERCK

OP EEN ANDER MANIER.

DE swaerheyds middellini van M L B C Y, is N X, ende van M L A D is S G, ende S X is balck, de selue salmen in twee deelen, alsoo dat de stikken de reden hebben als 8 lb van M L B C Y, tot 4 lb van M L A D: welverstaende tcoortste stick naer de swaerheys middellini van tswaerste deel, twelck vallen sal in V, inder voughen dat V L wederom de begheerde handhaef sijn sal als vooren.

IIII. VOORBEELT.

TGHEGHEVEN. Laet A B C D wederom den pilaer sijn, ghedeelt als vooren, hanghende Y 6 lb an X, ende Z 24 lb an R. **TBEGHEERDE.** Wy moeten d'handhaef vinden. **TWERCK.** De swaerheys

be divided in two in such a way that the segments have the ratio of 12 lbs (Y) to 6 lbs (the prism), to wit: the shorter segment adjacent to the centre line of gravity of the heavier gravity Y ; let the point of division be at X , so that NX is the required handle.

EXAMPLE III.

SUPPOSITION. Let $ABCD$ again be the prism, divided as above, Y (6 lbs) now hanging at X . **WHAT IS REQUIRED TO FIND.** We have to find the handle. **CONSTRUCTION.** The centre line of gravity of the prism is IT , and that of Y is NX , and TX is the beam. The latter shall be divided in two in such a way that the segments have the ratio of 6 lbs (Y) to 6 lbs (the prism); let the point of division be at V , so that VL will be the required handle.

THE ABOVE CONSTRUCTION

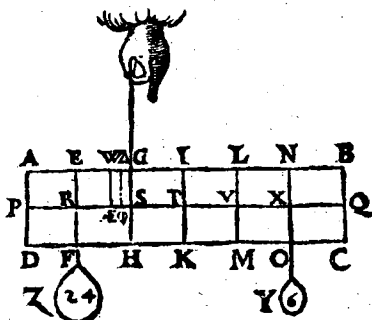
in a different manner.

The centre line of gravity of $MLBCY$ is NX , and that of $MLAD$ is SG , and SX is the beam. The latter shall be divided in two in such a way that the segments have the ratio of 8 lbs ($MLBCY$) to 4 lbs ($MLAD$), to wit: the shorter segment adjacent to the centre line of gravity of the heavier part; let the point of division be at V , so that VL will again be the required handle, as before.

EXAMPLE IV.

SUPPOSITION. Let $ABCD$ again be the prism, divided as before, Y (6 lbs) hanging at X , and Z (24 lbs) at R . **WHAT IS REQUIRED TO FIND.** We have to find the handle. **CONSTRUCTION.** The centre line of gravity of $ABCDY$

swaerheyds middellini van ABCDY, is LV door het 3^e voorbeeld, ende van Z is RE, daerom is RV balck: de selue salmen in twee deelen, alsoo dat de stucken de reden hebben als 12 lb van ABCDY, tot 24 lb van Z: wel verstaende toortste stic naer de swaerheys middellini van tswaerste deel, twelck vallen sal in S, inder voughen dat SG de begheerde handthaeftijn sal.



TVOORNOEMDE WERCK OP
EEN ANDER MANIER.

DE swaerheyds middellini van ABCDZ is EW door het 3^e voorbeeld, alsoo dat S doet $\frac{2}{3}$ van SR, ende de swaerheyds middellini van Y is XN, ende EX is balck: de selue salmen in twee deelen, alsoo dat de sticken de reden hebben als 30 lb van ABCDZ, tot 6 lb van Y: wel verstaende toortste stic naer de swaerheyds middellini van tswaerste deel, twelck vallen sal in S, inder voughen dat SG wederom de begheerde handthaeft is als vooren.

TVOORNOEMDE WERCK
OP EEN ANDER MANIER.

DE swaerheyds middellini van YZ, is (door het eerste voorbeeld) $\Phi\Delta$, alsoo dat S doet $\frac{1}{2}$ van SR, ende de swaerheyds middellini vande pilaer is TI, ende T Φ is balck: de selue salmen in twee deelen, alsoo dat de sticken de reden hebben als 30 lb van Y met Z, tot 6 lb vande pilaer, te weten toortste stic naer de swaerheyds middellini van tswaerste deel, twelck vallen sal in S, inder voughen dat SG wederom de begheerde handthaeft is als vooren.

V^e VOORBEELT.

TGHEGHEVEN. Laet ABCD wederom den pilaer sijn ghedeelt als vooren, hanghende Y 6 lb an X, ende Z 24 lb an R, en AE 12 lb an Q. TBEGHERDE. Wy moeten d'handthaeft vinden. TWERCK. De swaerheyds middellini van ABCDYZ is SG door het 4^e voorbeeld, ende van AE is QB, ende SQ is balck: de selue salmen in twee deelen, alsoo dat de sticken de reden hebben als 36 lb vanden pilaer met Y
C ende

is LV , by the 3rd example, and that of Z is RE ; therefore RV is the beam. The latter shall be divided in two in such a way that the segments have the ratio of 12 lbs ($ABCDY$) to 24 lbs (Z), to wit: the shorter segment adjacent to the centre line of gravity of the heavier part; let the point of division be at S , so that SG will be the required handle.

THE ABOVE CONSTRUCTION

in a different manner.

The centre line of gravity of $ABCDZ$ is AEW , by the 3rd example, in such a way that SAE makes $\frac{2}{5}$ ¹⁾ of SR , and the centre line of gravity of Y is XN , and AEX is the beam. The latter shall be divided in two in such a way that the segments have the ratio of 30 lbs ($ABCDZ$) to 6 lbs (Y), to wit: the shorter segment adjacent to the centre line of gravity of the heavier part; let the point of division be at S , so that SG will again be the required handle, as above.

THE ABOVE CONSTRUCTION

in a different manner.

The centre line of gravity of YZ is (by the first example) $\Phi\Delta$, in such a way that $S\Phi$ makes $\frac{1}{5}$ of SR , and the centre line of gravity of the prism is TI , and $T\Phi$ is the beam. The latter shall be divided in two in such a way that the segments have the ratio of 30 lbs (Y with Z) to 6 lbs (the prism), to wit: the shorter segment adjacent to the centre line of gravity of the heavier part; let the point of division be at S , so that SG will again be the required handle, as above.

EXAMPLE V.

SUPPOSITION. Let $ABCD$ again be the prism, divided as above, Y (6 lbs) hanging at X , and Z (24 lbs) at R , and AE (12 lbs) at Q . WHAT IS REQUIRED TO FIND. We have to find the handle. CONSTRUCTION. The centre line of gravity of $ABCDYZ$ is SG , by the 4th example, and that of AE is QB , and SQ is the beam. The latter shall be divided in two in such a way that the segments have the ratio of 36 lbs (the prism with Y and Z) to 12 lbs (AE), to

¹⁾ Read $\frac{3}{5}$.

ende Z, tot 12 lb van A, te weten toortste stick naer de swaerheyds middellini van tswaerste deel, welck vallen sal in T, inder voughen dat T I de begheerde handthaeft sal sijn.

Ende soomen noch hinghe an P 24 lb, d'handthaeft soude S G sijn, ende so voorts met allen anderen swaerheden die men anden pilaer soude mueghen hanghen. T B E W Y S.

De swaerste swaerheydt A int eerste voorbeeld, heeft fulcken reden tot de lichtste B, als den langsten erm E D, tot den cortsten E C, daerom E F door de 9^e bepaling is d'handthaeft. Sghelijcx sal oock tbewijs sijn van al dander voorbeelden, welck wy om de cortheydt achterlaten.

T B E S L V Y T. Wesende dan ghegheuen bekende swaerheden, wy hebben haer handthaeft gheuonden naer den eysch.

M E R C K T.

SOOMEN tghewicht T des 2^{en} voorbeelts verswaerde van 1 lb, ende daermen an V hinghe 1 lb, inder voughen dat haer gheslalt dan waer als hier onder, T is kennelick uyt het voorgaende dat X N noch handthaeft blijft, ende alles an haer euestalrwichich hangt. Tselue sal X N oock bliuen, soomen Z 1 lb hangt an T, ende dat T doe 14 lb, ofte Z 1 lb an S, ende dat T doe 15 lb, ofte Z 1 lb an R, ende dat T doe 16 lb, ofte Z 1 lb an P, ende dat T doe 17 lb, ende soo oirdentlick voort by aldien den pilaer langher waer; te weten, verswaerde T alijt van 1 lb, voor elike langde als X V, daermen Z voordere an verschuift. Waer uyt de * Ghedaenten des Onfils bekent sijn, als inde Weegdaes breeder daer af sal ghehandelt worden.

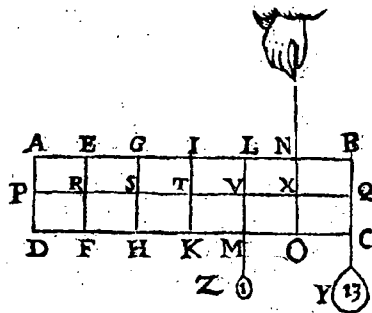
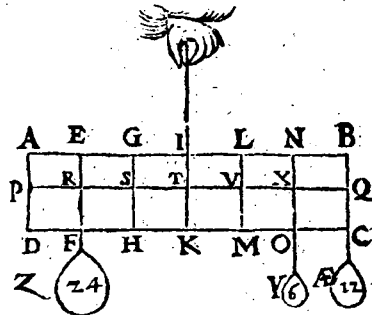
Qualitates.

I. EYSCHE.

III. VOORSTEL.

W E S E N D E ghegheuen twee euestalrwichighe swaerheden, d'een bekent dander onbekent, ende d'handthaeft: Die onbekende bekent te maken.

I. VOOR-



wit: the shorter segment adjacent to the centre line of gravity of the heavier part; let the point of division be at T , so that TI will again be the required handle.

And if in addition 24 lbs were hung at P , the handle would be SG , and so on with any other gravities that might be hung from the prism. PROOF. The heavier gravity A in the first example has to the lighter one B the same ratio as the longer arm ED to the shorter EC ; therefore EF is the handle by the 9th definition. A similar proof can also be given of all the other examples, which we omit for brevity's sake. CONCLUSION. Given therefore two known gravities, we have found their handle as required.

NOTE.

If the weight Y of the 2nd example were made heavier by 1 lb, and 1 lb were hung at V , in such a way that the situation would be as below, it is evident from what precedes that XN still remains the handle, and that the whole hangs from it in apparent equality of weight. The handle will also remain XN , if Z (1 lb) is hung at T and Y is made 14 lbs, or Z (1 lb) is hung at S and Y is made 15 lbs, or Z (1 lb) is hung at R and Y is made 16 lbs, or Z (1 lb) is hung at P and Y is made 17 lbs, and so regularly on if the prism were longer, viz. always making Y heavier by 1 lb for every segment of the beam equal to XV along which Z is displaced; by which the properties of the steelyard are known, which will be dealt with more in detail in the Practice of Weighing ¹).

PROBLEM II.

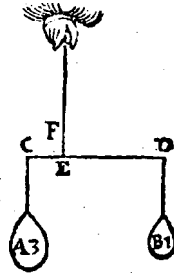
PROPOSITION III.

Given two gravities of equal apparent weight, one of them known and the other unknown, and the handle: to make known the unknown.

¹) See *The Practice of Weighing*, Prop. 5

I. VOORBEELT.

TGHEGHEVEN. Laet A ende B twee eueftaltwichtighe swaerheden sijn, welcker A hanghende an C weeght 3 lb, maer B hanghende an D is onbekent, ende E F sy d'handhaef. TBEGHEERDE. Wy moeten tghewicht van B bekenet maken. TWERCK. Men sal ondersoecken wat reden den erm E D heeft, tot den erm E C, wort beuonden, neem ick, als van 3 tot 1, daerom seg ick, E D 3, gheeft E C 1, wat A 3 lb? comt voor B 1 lb.



II. VOORBEELT.

TGHEGHEVEN. Laet inde form des 2^m voorbeelts van het 2^e voorstel den pilaer A B C D voor d'een swaerheyt weghen 6 lb, ende dander onbekende swaerheyt sy tghewicht daer an hanghende Y, ende d'handhaef sy X N. TBEGHEERDE. Wy moeten tghewicht van Y bekenet maken. TWERCK. Anghesien T I swaerheyds middellini is des pilaers, ende Q B van Y, so sal T Q balck sijn, diens cortsten erm X Q, ende langsten X T, Daerom salmen ondersoucken wat reden den erm X Q, heeft tot X T, wort beuonden neem ick, als van 1 tot 2. Ich seg dan, X Q 1, gheeft X T 2, wat den pilaer 6 lb? comt voor Y 12 lb. Der ghelijcke voorbeelden mochten wy hier stellen op dander formen der voorbeelden des 2^m voorstels, ten waer die door de voorgaende kennelick ghenouch sijn. TBEWYS. Laet B int eerste voorbeelt, soot mueghelick waer, swaerder sijn dan 1 lb, de swaerste swaerheydt dan en sal niet sulcken reden hebben tot de lichtste, als den langsten erm tot den cortsten; twelck teghen het 1^e voorstel is; B dan en is niet swaerder dan 1 lb. Sghelijcx salmen oock bethoonen dat sy niet lichter en is, sy weeght dan effen 1 lb, twelck wy bewysen moesten. TBESLVYT. Wesende dan ghegeuen twee eueftaltwichtighe swaerheden, d'een bekenet dander onbekent, ende d'handhaef: Wy hebben die onbekende bekenet ghemacét, naer den eysch.

III. EYSCH.

IIII. VOORSTEL.

WESENDE ghegeuen twee bekende eueftaltwichtighe swaerheden met de langde van d'eenen erm: de langde des anderen erms te vinden.

TGHEGHEVEN. Laet A ende B twee eueftaltwichtighe swaerheden sijn, welcker A hanghende an C weeght 3 lb, ende B hanghende an D 1 lb, ende de langde des erms D E sy 6 voeten. TBEGHEERDE.

C 2 Wy

EXAMPLE I.

SUPPOSITION. Let A and B be two gravities of equal apparent weight, of which A , hanging at C , weighs 3 lbs, but B , hanging at D , is unknown, and let EF be the handle. **WHAT IS REQUIRED TO MAKE KNOWN.** We have to make known the weight of B . **CONSTRUCTION.** It shall be ascertained what ratio the arm ED has to the arm EC . I assume this is found to be 3 to 1. Therefore I say: ED 3 gives EC 1, what A 3 lbs? B becomes 1 lb¹⁾.

EXAMPLE II.

SUPPOSITION. In the figure of the 2nd example of the 2nd proposition let the prism $ABCD$ be the one gravity, weighing 6 lbs, and let the other — unknown — gravity be the weight Y hanging therefrom, and let the handle be XN . **WHAT IS REQUIRED TO MAKE KNOWN.** We have to make known the weight of Y . **CONSTRUCTION.** Since TI is the centre line of gravity of the prism, and QB that of Y , TQ will be the beam, the shorter arm of which will be XQ and the longer XT . It shall therefore be ascertained what ratio the arm XQ has to XT . I assume this is found to be 1 to 2. I therefore say: XQ 1 gives XT 2, what the prism 6 lbs? Y becomes 12 lbs. We might give similar examples with regard to the other figures of the examples of the 2nd proposition, if these were not sufficiently evident from what precedes.

PROOF. Let B in the first example, if this were possible, be heavier than 1 lb; the heavier gravity will not then have to the lighter the same ratio as the longer arm to the shorter, which is contrary to the first proposition. B therefore is not heavier than 1 lb. In the same way it can also be shown that it is not lighter. It therefore weighs precisely 1 lb, which we had to prove. **CONCLUSION.** Given therefore two gravities of equal apparent weight, one of them known and the other unknown, and the handle, we have made known the unknown, as required.

PROBLEM III.

PROPOSITION IV.

Given two known gravities of equal apparent weight, and the length of one arm: to find the length of the other arm.

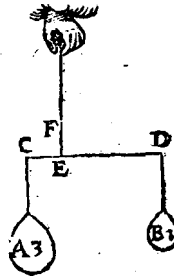
SUPPOSITION. Let A and B be two gravities of equal apparent weight, of which A , hanging at C , weighs 3 lbs, and B , hanging at D , 1 lb, and let the length of the arm DE be 6 feet. **WHAT IS REQUIRED TO FIND.** We have to

¹⁾ The meaning of this elliptical way of formulating the rule of three will be clear: If $ED = 3$, $EC = 1$. Therefore, if $A = 3$ lbs, $B = 1$ lb.

Wy moeten de langde des anderen erms vinden. **TWERCK.** Men sal segghen A 3 lb, gheeft B 1 lb, wat D E 6 voeten? comt voor E C 2 voeten. Ende der ghelijcke voorbeelden mochten wy stellen op de formen der voorbeelden des 1^e voorstels, ten waer die duer voorgaende kennelick ghenouch sijn.

TBEWYS. Laet E C, soot mueghelick waer, langher sijn dan 2 voeten; den langsten arm sal dan minder reden hebben tot den corsten, dan de swaerste swaerheyt tot de lichtste, twelck teghen het eerste voorstel is, E C dan en is niet langher dan 2 voeten; Sghelijcx salmense oock bewysen niet corter te sijn, sy is dan effen van twee voeten, twelck wy bewysen moesten.

TBESLVT. Wesende dan ghegheuen twee euefstalwichtighe swaerheden met de langde van d'eenen arm, wy hebben de langde des anderen erms gheuonden, naer den eysch.



IIII. EYSCH.

V. VOORSTEL.

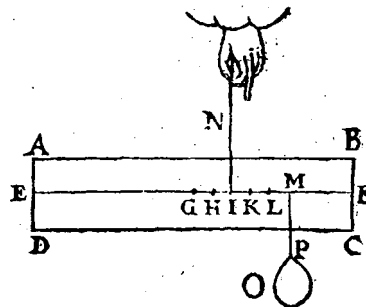
WESENDE ghegheuen een pilaer: te vinden een ghewicht in ghestelde reden tot des pilaers ghewicht.

Centrum.

TGHEGHEVEN. Laet ABCD een pilaer wesen, diens as E F, ende haer * middelpunt G, ende de ghestelde reden sy van 2 tot 3.

*Geometrica
& Arithmetica
propositiones.*

TBEGHEERDE. Wy moeten een ghewicht vinden in sulcken reden tot den pilaer, als van 2 tot 3, dat is euen an sijn $\frac{2}{3}$. **MERCKT.** Ghelijck de * Meetconstighe ende Telconstighe voorstellen verscheyden werckinghen hebben, alsoo oock de Weegconst, want men soude vanden pilaer een stuck connen snien in sulcken reden tot den heelen pilaer, als van 2 tot 3, Oft andersins om den pilaer heel te laten, men mocht hem teghen ander stof wegghen, daer af nemende de $\frac{2}{3}$, maer wy willent Weegconstlicker doen in deser voughen. **TWERCK.** Men sal van t'middelpunt G af, naer F, teekenen eenighe vijf punten (te weten 5 voor de somme der ghegheuen



find the length of the other arm. **CONSTRUCTION.** It can be said: if A 3 lbs gives B 1 lb, what DE 6 feet? EC becomes 2 feet. We might give similar examples with regard to the figures of the examples of the 2nd proposition, if these were not sufficiently evident from what precedes.

PROOF. Let EC , if this were possible, be longer than 2 feet; the longer arm will then have to the shorter a ratio less than the heavier gravity to the lighter, which is contrary to the first proposition. Therefore EC is not longer than 2 feet. In the same way it can also be proved not to be shorter. It is therefore precisely 2 feet long, which we had to prove. **CONCLUSION.** Given therefore two gravities of equal apparent weight, and the length of one arm, we have found the length of the other arm, as required.

PROBLEM IV.

PROPOSITION V.

Given a prism: to find a weight which shall have to the weight of the prism a given ratio.

SUPPOSITION. Let $ABCD$ be a prism, its axis EF and its centre G , and let the given ratio be that of 2 to 3. **WHAT IS REQUIRED TO FIND.** We have to find a weight having to the prism the ratio of 2 to 3, i.e. being equal to $\frac{2}{3}$ of the latter.

NOTE.

Just as geometrical and arithmetical propositions have different operations, so also the Art of Weighing, for one might cut from the prism a piece having to the whole prism the ratio of 2 to 3. Or in another way, to keep the prism intact, it might be balanced against some other material, after which $\frac{2}{3}$ of the latter would be taken; but we will do it more in accordance with the Art of Weighing, as follows. **CONSTRUCTION.** There shall be marked from the centre G , towards F , five points (to wit 5, for the sum of the given terms 2 and 3), as H, I, K, L ,

ghegheuen palen 2. 3.) als H, I, K, L, M, van malcanderen euewyte; Ende van het tweede punt I (van het tweede om dat 2 het ander der ghegeuen ghetalen is) salmen den pilaer ophangen byde swaerheys, middellini I N; Daer naer salmen an twijfde punt M een ghewicht hanghen als O, euen so swaer dat alles in euestaltwichticheyt sy, welck soo wesen, ick seg dat tghewicht van O, in sulcken reden is tot tghewicht des pilaers, als 2 tot 3, ofte dat O euen is ande $\frac{2}{3}$ des pilaers.

T B E W Y S. G is * swaerheyds middelpunt des pilaers A B C D, *Centrum grauitatis.* ende M P swaerheys middellini van O, daerom ghelijck den erm I G tot den erm I M, alsoo O tot den pilaer door het 1^e voorstel, maer I G heeft sulcken reden tot I M, als 2 tot 3, daerom O heeft sulcken reden tot den pilaer, als 2 tot 3, welck wy bewysen moesten. **T B E S L V Y T.** Wefende dan ghegheuen een pilaer, wy hebben gheuonden een ghewicht in ghestelde reden tot des pilaers ghewicht, naer den eylich. **M E R C K T.** Wy souden oock mueghen voorbeelden stellen met Redenen van * onmetelicke palen, maer sulcx is openbaer ghenouch door *Incommensurabilium terminorum.* tvoorgaende, mergaders tghene wy vande onmetelicke grootheden elders ghescreuen hebben.

II VERTOOC.

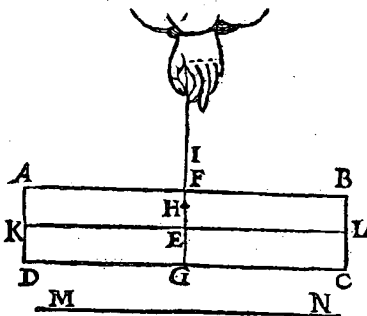
VI VOORSTEL.

W E S E N D E een hanghende pilaer ghesneen door sijn swaerheyds middelpunt, met een plat euewydich vanden gront, en wesen de tvaftpunt in dat plat boue het swaerheys middelpunt: Den as des pilaers blijft euewydich vanden * sichteinder. *Horizon.*

T G H E G H E V E N. Laet A B C D een pilaer sijn, ghesneen door sijn swaerheyds middelpunt met een plat F G, euewydich vanden grondt A D, ende laet H vaftpunt inde swaerheyds middellini I G wesen, bouen het swaerheys middelpunt E, ende K L sy as, ende M N sichteinder.

T B E G H E E R D E. Wy moeten bewysen dat den as K L euewydich blijft vanden sichteinder. M N.

T B E W Y S. Laet K L foot mueghelijck waer, oneuewydich sijn vanden sichteinder M N, als in



M , equidistant from one another; and from the second point I (from the second because 2 is the second of the given numbers) the prism shall be hung by the centre line of gravity IN . After this, at the fifth point M a weight O shall be hung, just heavy enough for the whole to be of equal apparent weight. This being so, I say that the weight of O has to the weight of the prism the ratio of 2 to 3, or that O is equal to $\frac{2}{3}$ of the prism. PROOF. G is the centre of gravity of the prism $ABCD$, and MP is the centre line of gravity of O ; therefore, as the arm IG is to the arm IM , so is O to the prism, by the 1st proposition. But IG has to IM the ratio of 2 to 3; therefore O has to the prism the ratio of 2 to 3, which we had to prove.

CONCLUSION. Given therefore a prism, we have found a weight in a given ratio to the weight of the prism, as required.

NOTE.

We might also give examples with ratios of incommensurable terms, but this is sufficiently manifest from what precedes and from what we have said elsewhere about incommensurable magnitudes ¹⁾.

THEOREM II.

PROPOSITION VI.

Given a hanging prism, cut through its centre of gravity by a plane parallel to the base, and the fixed point being in that plane above the centre of gravity, the axis of the prism remains parallel to the horizon.

SUPPOSITION. Let $ABCD$ be a prism, cut through its centre of gravity by a plane FG , parallel to the base AD , and let H be the fixed point in the centre line of gravity IG , above the centre of gravity E ; and let KL be the axis, and MN the horizon. WHAT IS REQUIRED TO PROVE. We have to prove that the axis KL remains parallel to the horizon MN .

PROOF. Let KL , if this were possible, be non-parallel to the horizon MN , as in

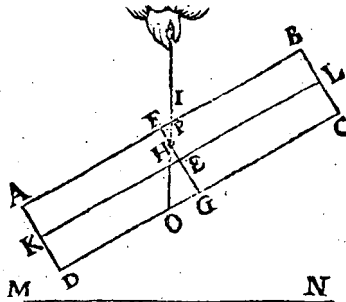
¹⁾ This remark refers to Stevin's ideas on irrational numbers, which are discussed in V (*Thèses mathématiques*). In his opinion there is no reason to call certain numbers absurd or irrational; irrational ratios should be called numbers on a par with rational ratios.

dees tweede form, ende laet I H voortghetrocken worden tot 'in O, sijnende A B in P, ende laet het stuck des pilaers P O C B alsoo euewichtich blijen hanghen teghen P O D A, maer dat is grooter ende swaerder dan dit (want F G D A, is euen an F G C B, ende minder is den driehouck F H I ghesneen van F G C B, dan de driehouck O H G ghesneen van F G D A, daerom, &c.) het swaerder dan sal euewichtich sijn an een lichter twelck ongheschielt is, K L dan blijft euewydich vanden sichteinder M N, als in d'eerste form.

Tis oock te anmercken als voor ghemeenen Weegconstighen Reghel, dat

Alle swaerhejts middelpunt eens hanghenden lichaems is in sijn swaerheydis middellini.

Maer (swaerheyds middelpunt hier bouen E en is inde tweede form niet in sijn swaerheyds middellini I O, tis dan een onmoeghelicke ghestalt. **T B E S L V Y T.** Wesende dan een pilaer ghesneen, &c.



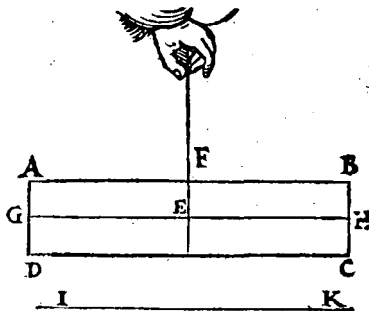
III VERTOOGH.

VII VOORSTEL.

W E S E N D E tvaftpunt het swaerheyds middelpunt des hanghenden pilaers, hy houdt alle ghestalt diemen hem gheeft.

T G H E G H E V E N. Laet A B C D een pilaer wesen, diens swaerheyds middelpunt E vast sy, daer by hanghende ande lini E F, ende den as G H sy euewydich vanden sichteinder I K.

T B E G H E E R D E. Wy moeten bewysen dat den pilaer A B C D alle ghestalt houdt diemen hem gheeft.



T B E W Y S. Laet ons den ghegheuen pilaer (tpunt E vast blijuende) een ander ghestalt gheuen dan d'eerste, als in dees tweede form, ende

this second figure, and let IH be produced to O , meeting AB in P , and let the part $POCB$ of the prism remain hanging in equilibrium ¹⁾ against $PODA$; now the former is greater and heavier than the latter (for $FGDA$ is equal to $FGCB$, and the triangle FHI cut from $FGCB$ is less than the triangle OHG cut from $FGDA$; therefore, etc.). The heavier part will therefore be of equal weight ²⁾ with the lighter part, which is absurd ³⁾. KL therefore remains parallel to the horizon MN , as in the first figure.

It is also to be considered a general rule in the Art of Weighing that: *The centre of gravity of a hanging solid is always in its centre line of gravity* ⁴⁾. But the centre of gravity E above is not in its centre line of gravity IO in the second figure; this is therefore an impossible situation. CONCLUSION. Given therefore a prism, cut etc.

THEOREM III.

PROPOSITION VII.

The fixed point being the centre of gravity of the hanging prism, the latter remains at rest in any position given to it.

SUPPOSITION. Let $ABCD$ be a prism, the centre of gravity E of which shall be the fixed point by which the prism is hanging from the line EF , and let the axis GH be parallel to the horizon IK . WHAT IS REQUIRED TO PROVE. We have to prove that the prism $ABCD$ remains at rest in any position given to it. PROOF. Let us give the given prism (the point E remaining fixed) a different position from the first, as in this second figure, and let FE be produced to L ,

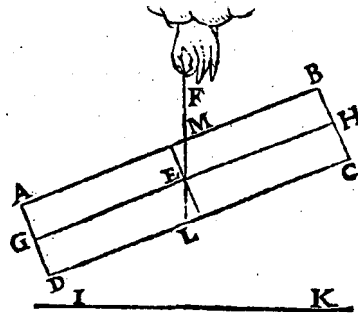
¹⁾ Read: in apparent equality of weight.

²⁾ Read: equal apparent weight.

³⁾ As Girard (XIII 441a) rightly remarks, this conclusion is not justified. It does not matter at all that $OCBP$ and $DOPA$ are not of equal weight; it has to be proved that they are not of equal apparent weight.

⁴⁾ According to Definition 5, centre line of gravity is the vertical through the centre of gravity. Evidently the word is taken here in the sense of vertical through the point of suspension. Here, as elsewhere, Stevin seems to make use of a certain theory of gravity, which, however, is explained nowhere.

ende laet FE voortghetrocken worden tot in L, sniende AB in M, ende en laet den pilaer soot mueghelick waer niet in die ghestalt bliuen, dan het stick MLDA, ofte MLCB neervallen; Maer dees twee deelen sijn ghelijck euegroot, ende daerom oock euefwaer, het eene dan van euewichtighe sal swaerder sijn dan t'ander, welc ongeschickt is: Den pilaer dan blijft in die ghestalt, en sgelijcx in allen anderen diemen hem soude mueghen gheuen. **T B E S L V Y T.** Wefende dan vastpunt het swaerheydts middelpunt des pilaers, hy houdt alle ghestalt diemen hem gheeft, welck wy bewysen moesten.

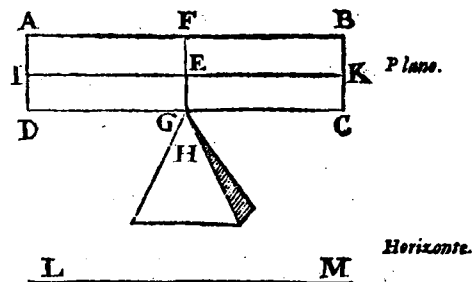


IIII VERTOOC.

VIII VOORSTEL.

W E S E N D E den pilaer ghesneen door sijn swaerheyts middelpunt, met een plat euewydich vanden gront, ende wefende vastpunt in dat plat beneden het swaerheydts middelpunt: Den pilaer (natuerlick verstaen) keert om tot dat sijn swaerheydts middelpunt is in sijn swaerheydts middellini.

T G H E G H E V E N. Laet ABCD een pilaer wesen, ghesneen door sijn swaerheyts middelpunt E, met een * plat FG euewydich vanden gront AD, ende laet G vastpunt sijn, beneden swaerheydts middelpunt E, met welck punt G den pilaer ligt ofte rust op tpunt des pins H, ende IK sy as, euewydich vanden * sichteinder LM.



T B E G H E E R D E. Wy moeten bewysen dat den pilaer omkeeren sal, tot dat sijn swaerheydts middelpunt is in sijn swaerheyts middellini. **maer**

meeting AB in M . And let the prism, if this were possible, not remain in that position, but let the part $MLDA$ or $MLCB$ fall down. But these two parts are equal in size, and consequently also equally heavy; one of two parts of equal weight will therefore be heavier than the other, which is absurd ¹⁾. The prism therefore remains in that position, and likewise in any other position that might be given to it. CONCLUSION. The fixed point therefore being the centre of gravity of the prism, the latter remains at rest in any position given to it, which we had to prove.

THEOREM IV.

PROPOSITION VIII.

Given the prism, cut through its centre of gravity by a plane parallel to the base, and the fixed point being in that plane below the centre of gravity, the prism (physically speaking) turns upside down until its centre of gravity is in its centre line of gravity ²⁾.

SUPPOSITION. Let $ABCD$ be a prism, cut through its centre of gravity E by a plane FG parallel to the base AD , and let G be the fixed point, below the centre of gravity E , with which point G the prism lies or rests on the point of the peg H , and let IK be the axis, parallel to the horizon LM . WHAT IS REQUIRED TO PROVE. We have to prove that the prism will turn upside down until its centre of gravity is in its centre line of gravity, such physically speaking, for, conceived

¹⁾ Of course this is not absurd at all: two bodies of equal weight need not be of equal apparent weight.

²⁾ Here again, centre line of gravity is not taken in the sense of Definition 5; it again means vertical through the point of suspension.

Mathemati-
co.

maer dit natuerlick verstaen, want * Wiskonstelick ghenomen soo can sy daer op rusten,

T B E W Y S.

- A. Al dat ligt moet gronds hebben daert op rust,
E. Dees pilaer en heeft gheen grondt daer by op rust,
B. Dees pilaer dan en can soo niet ligghen.

Syllogismi.

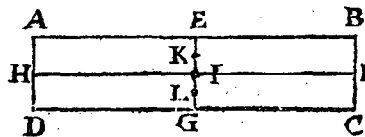
D E S * Bewysfredens tweede voorstel is daer uyt openbaer, dat het punt gheen grootheyt en is, ende veruolghens gheen grondt: wel is waer dat wy dickmael nemen door tghestelde een lichaem alsoo te rusten, maer metter daer en connen wy dat niet te weeg brenghen. Inder voughen dat hoewel den as I K ewwydich ghestelt is vanden sichteinder L M, soo sal nochtans den pilaer (tpunt G vast blyuende) omkeeren ouer die sijde daer hy eerst beghint. Maer dat hy so lang keeren sal tot dat sijn swaerheyds middelpunt inde swaerheyds middellini si, is door het 6^e voorstel openbaer. T B E S L Y T, Wefende dan den pilaer ghesneen, &c.

I^e M E R C K.

Y E M A N T mocht hier noch de verclaring begheeren des verschils tusschen hanghen en ligghen, waer op d'antwoort is dat wy een lichaem voor hanghende houden, als sijn swaerheys middelpunt is onder, ofsint tghenaecsel daers op rust; Maer is swaerheys middelpunt daer bouen sinde, aldaer houden wy't voor ligghen, staen, oft sitten; Ligghen, als de langste sude des lichaems haer streck langs den sichteinder: Staen, als sy daer op rechthouckich is; daerom ist oock dat wy den teerlinck (ouermits sijn siden al euen lanck sijn) soo eyghentlick segghen te staen als te ligghen, ende te ligghen als te staen. Sitten is wat tusschen ligghen en staen.

II^e M E R C K.

Soo yemant thinhondt der voorgaende drie voorstellen door eenighe ervaering wilde sien, hy mocht nemen een reghel van hout ofte ander stof eenvaerdigher dickte ende swaerheyt, als A B C D, treckenende de punten E, F, G, H, inde middelen der linien A B, B C, C D, D A, treckende E G, ende H F, malcander sniende in I, maeckende daer naer een seer cleen gaetken an I, ende daer bouen een gaetken als K, ende onder I een gaetken als L. Ende stekende een naelde door igatken K, die vrielick daerindragen mach, d'ervaring sal bethoonen dat H F altydt ewwydich sal bliuen vanden sichteinder. Maer de naelde in I stekende, de reghel sal daerop alle ghestalts houden diemen haer gheeft. Ende de naelde in L ghesteken, alles sal omkeeren ouer



die

mathematically ¹⁾, it can rest thereon.

PROOF.

A ²⁾. *Everything that lies must have a base on which it rests;*

E . *This prism has no base on which it rests;*

E . *Therefore this prism cannot lie in this way.*

The second proposition of the syllogism is apparent from the fact that a point is no magnitude, and consequently no base; it is true that by the supposition we often assume a body to rest in this way, but in actual fact we cannot bring it about. Therefore, though the axis *IK* be put parallel to the horizon *LM*, the prism (the point *G* remaining fixed) will nevertheless turn upside down on the side where it begins to turn. But it is manifest by the 6th proposition that it will turn until its centre of gravity is in the centre line of gravity. CONCLUSION. Given therefore the prism, cut, etc.

NOTE I.

If anyone should here desire the explanation of the difference between hanging and lying, the answer is that we hold a solid to be hanging when its centre of gravity is below or in the support on which it rests; but if the centre of gravity is above the latter, we hold the solid to be lying, standing or sitting: lying, if the longest side of the solid is parallel to the horizon; standing, if the said side is at right angles thereto. This is also the reason why we may just as well say of a cube (because all its sides are of equal length) that it stands as that it lies, and that it lies as that it stands. Sitting is something intermediate between lying and standing.

NOTE II.

*If anyone should wish to see the contents of the preceding three propositions by some experience, he might take a ruler of wood or some other material which is everywhere equally thick and heavy, for example *ABCD*, marking the points *E, F, G, H* in the centres of the lines *AB, BC, CD, DA*, joining *EG* and *HF*, which meet in *I*, making thereafter a very small hole at *I*, and above it a hole *K* and below *I* a hole *L*. And if he puts through the hole *K* a needle which can freely pivot therein, experience will show that *HF* will always remain parallel to the horizon. But if he puts the needle in *I*, the ruler will remain at rest in any position given to it. And if the needle is put in *L*, the whole prism will turn*

¹⁾ Stevin here introduces a distinction between a real or physical and a mathematical body. His proposition relates to the first one, which cannot in practice remain at rest in the position shown in the figure. The distinction is expressed by the terms *natuerlick verstaen* (physically speaking) and *wisconstelick gbenomen* (conceived mathematically). Evidently the consideration of physical possibility introduces quite a new element into the system, which disturbs its mathematical coherence.

²⁾ Stevin here makes use of the symbolism of ancient formal logic, in which *A* denotes a universal affirmative proposition (All *X*'s are *Y*'s) and *E* a universal negative proposition (No *X*'s are *Y*'s). The mood of the syllogism is *Camestres*. Stevin makes use of the syllogistic formulation whenever he wants to stress the importance or the originality of his reasoning. Other examples are to be found in Prop. 24 of Book I and in Props 2, 10, 15, 16, 18, 22 of Book II of *The Art of Weighing*. There is not the slightest ground for Vailati's contention (*Il principio dei lavori virtuali da Aristotele a Erone d' Alessandria*, Atti R. Acc. d. Sc. di Torino 82 (1896-97), 949) that the syllogistic formulation was meant ironically.

die syde daert eerst beghint, tot dat I is in haer swaerheys middellini, waer af d'oirsaek inde voornoemde 6^e, 7^e, 8^e, voorstellen * Wisconslick blijkt. *Mathemati-
cè.*

V. VERTOOGH.

IX. VOORSTEL.

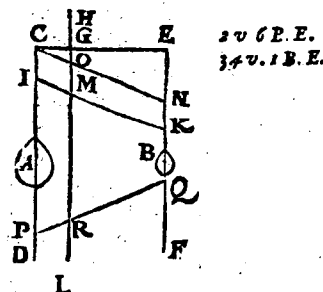
D'HANTHAEF oneindelick voortghetrocken, deelt alle balcken twee swaerheden in haer ermen.

TGHEGHEVEN. Laet AB twee swaerheden sijn ende haer middellinen CD, EF, ende haer balck CE, ende d'anthaef GH, inder voughen dat CG is tot GE, als de swaerheydt B tot A, Laet IK noch een balck wesen, oneuwydich van CE, ende laet GH oneindelick voortghetrocken worden naer L, snijende den balck IK in M.

TBEGHEERDE. Wy moeten bewyfen dat IM ende MK, oock de ermen sijn der swaerheden AB; dat is ghelijck B tot A, alsoo MI tot MK.

TBEREYTSSEL. Laet ghetroken worden CN, euwydich van IK, snijende HL in O. TBEWYS. Ghelijck CG tot GE, alsoo CO tot ON, Maer CO is euen an IM, ende ON an MK, daerom ghelijck CG tot GE, alsoo IM tot MK, maer ghelijck B tot A, alsoo CG tot GE, door tghegheuen, daerom ghelijck B tot A, alsoo MI tot MK, tselfde sal alsoo bewesen worden van allen balcken tusschen CD ende EF, als PQ, doorsneen in R, ende allen anderen diemen soude mueghen trecken.

TBESLUYT. D'anthaef dan oneindelick voortghetrocken, deelt alle balcken twee swaerheden in haer ermen, twelck wy bewyfen moesten.



I. V E R V O L G H.

HIER uyt blijkt datmen om te vinden de swaerheyds middellini twee swaerheden, niet nootfaeckelick en moet nemen een * euwydige vanden * sichteinder, maer alsulcke als men wil, ende als best te *Parallela
Horizonte.* pas comt.

II V E R V O L G H.

ANGHESIEN alle swaerheyds middelpunt inde swaerheys middellini is, soo volght dat alle rechte lini begrepen tusschen twee swaerheyds middelpunten, oock dier swaerheden balck is, ende het onderscheydt der ermen diens balcx, oock het swaerheyds middelpunt te wesen der twee swaerheden.

D 5 EYsch.

upside down on the side where it begins to turn, until I is in its centre line of gravity, the cause of which appears mathematically from the aforesaid 6th, 7th, and 8th propositions.

THEOREM V.

PROPOSITION IX.

The handle, produced indefinitely, divides any beam of two gravities into its arms.

SUPPOSITION. Let A and B be two gravities, and their centre lines of gravity CD , EF , and their beam CE , and the handle GH , so that CG is to GE as the gravity B is to the gravity A . Let IK be another beam, not parallel to CE , and let GH be produced indefinitely to L , meeting the beam IK in M . WHAT IS REQUIRED TO PROVE. We have to prove that IM and MK are also the arms of the gravities A and B ; i.e. as B is to A , so is MI to MK . PRELIMINARY. Let CN be drawn parallel to IK , meeting HL in O . PROOF. As CG is to GE , so is CO to ON . But CO is equal to IM , and ON to MK , therefore as CG is to GE , so is IM to MK . But as B is to A , so is CG to GE by the supposition. Therefore, as B is to A , so is MI to MK . The same can also be proved of any beam between CD and EF , as PQ , cut in R , and any others that might be drawn. CONCLUSION. The handle therefore, produced indefinitely, divides any beam of two gravities into its arms, which we had to prove.

COROLLARY I.

From this it appears that in order to find the centre line of gravity of two gravities one need not take a line parallel to the horizon, but may take any line one likes and which suits best.

COROLLARY II.

Since the centre of gravity is always in the centre line of gravity, it follows that any straight line contained between two centres of gravity is also the beam of those gravities, and the dividing point of the arms of that beam is also the centre of gravity of the two gravities.

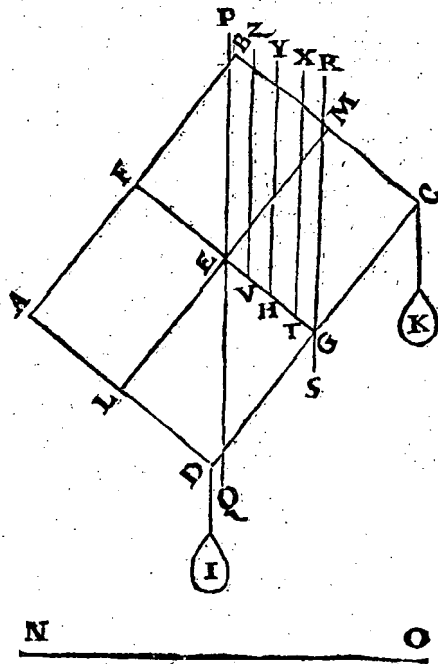
Horizonte.

W E S E N D E ghegheuen een vastpunt des bekenden pilaers, ende bekende euestaltwichtighe swaerheden an hem hangende: Te vinden of den as euewydich sal blijuen vanden * sichteinder, oft alle ghestalt houden diemen hem gheeft, ofte omkeeren tot dat sijn swaerheyds middelpunt is in sijn swaerheys middellini.

T G H E G H E V E N. Laet ABCD een pilaer sijn weghende 4 lb, ende ghesneen door sijn swaerheyds middelpunt E, met een plat FG euewydich vanden grondt A D, ende laet H vastpunt wesen beneden t'middelpunt E int middel van E G; Ende anden pilaer twee ghewichten hanghen als I, K, elck weghende 4 lb, welcker middellinien vastpunten sijn D, C, ende laet L M den as, ende NO sichteinder wesen.

T B E G H E R D E. Wy moeten vinden of den as L M euewydich sal connen blijuen vanden sichteinder N O; ofte alle ghestalt houden diemen haer gheeft; Ofte ommekeeré tot dat haer swaerheyds middelpunt E is inde swaerheys middellini door H, welke verscheydenheden vallen connen naer de reden der swaerheyt des pilaers, tot de ghewichten dieder anhangen.

T W E R C K. Men sal trecken door E de swaerheys middellini P O des pilaers, daer naer door G de



PROBLEM V.

PROPOSITION X.

Given a fixed point of the known prism, and known gravities of equal apparent weight hanging therefrom: to find out whether the axis will remain parallel to the horizon, or will remain at rest in any position given to it, or will turn upside down until its centre of gravity is in its centre line of gravity.

SUPPOSITION. Let $ABCD$ be a prism weighing 4 lbs and cut through its centre of gravity E by a plane FG parallel to the base AD , and let H be the fixed point below the centre E , in the middle of EG . And let there hang from the prism two weights, as I, K , each weighing 4 lbs, whose centre lines are fixed points D, C , and let LM be the axis, and NO the horizon. WHAT IS REQUIRED TO FIND OUT. We have to find out whether it will be possible for the axis LM to remain parallel to the horizon NO , or whether it will remain at rest in any position given to it, or will turn upside down until its centre of gravity E is in the centre line of gravity through H , which different possibilities may occur according to the ratio of the gravity of the prism to the weights hanging therefrom. CONSTRUCTION. Through E there shall be drawn the centre line of gravity PQ of the prism, and

Gde swaerheys middellini R S der ghewichten I, K, ende E G sal balck
sijn, daer naer salmen sien door het 2^e voorstel waer vastpunt der hant-
haef valt: want commet onder H, soo keert L M tot sy euewydich blijft
vanden sichteinder N O; Maer commet in H, sy houdt alle ghestalt
die men haer gheeft; Commet bouen H, alles keert om. Maer
den pilaer weeght 4 lb, ende I, K, elck 4 lb samen 8 lb door tge-
gheuen, daerom ghedeelt E G in T, alsoo dat E T, sulcken reden heb
tot T G, als 8 tot 4: Ick seg dat L M keeren sal (ouermits T onder H
comt) tot sy euewydich is vanden sichteinder. Laet nu den pilaer we-
ghen 4 lb, ende I en K elck 2 lb, samen 4 lb, daerom ghedeelt E G in H
(welcke H middel van E G is door tgegheuen) alsoo dat E H sulcken
reden heb tot H G, als 4 tot 4: ick seg dat L M (ouermits het in H viel)
alle ghestalt sal houden diemen haer gheeft. Laet nu den pilaer we-
ghen 4 lb, ende I, K, elck 1 lb, samen 2 lb, daerom ghedeelt E G in V,
also dat E V sulcken reden hebbe tot V G, als 2 tot 4. Ick seg dat den
pilaer met al de rest omkeeren sal (ouermits V bouen H comt) tot dat
H is in haer swaerheyts middellini. **T B E W Y S.** Ten eersten I en K
elck 4 lb weghende, dat dan L M keert tot sy euewydich is vanden sicht-
einder, blijft aldus: De hanghende door T ghelijck T X, is swaerheys
middellini des heels, daerom die latende, ende hanghende tgeheel an-
de * hanghende door H, als H Y (welcke H ons ghegheuen vastpunt is)
so sal de sijde naer B C K, swaerder sijn dan naer A D I, daerom oock
sal de sijde B C K neerdalen, tot dat H inde swaerheyts middellini is
des heels, ende dan sal L M euewydich sijn vanden sichteinder N O.

*Perpendicu-
larem.*

Ten tweeden I, K, elck 2 lb weghende, dat dan L M alle ghestalt
houdt, wordt aldus bethoont: Laet ons achten dat I ende K opghe-
schorft sijn, alsoo dat D tswaerheyts middelpunt sy van I, ende C van
K, ende door de 3^e begheerte sy en sullen anden pilaer gheen oirsaek
van verandering der swaerheydt wesen; Twelck soo sijnde, H is tswaer-
heys middelpunt van soodanighen lichaem vergaert uyt den pilaer ende
de twee ghewichten I K, ende door de 4^e bepaling tsal daer op alle ghe-
stalt houden diemen hem gheeft, tselfde sal also bewesen worden in alle
ghestalten daermen L M in soude connen stellen.

Ten laetsten I, K, elck 1 lb weghende, dat dan alles omkeert, wort
aldus bethoont: De hanghende door V ghelijck V Z, is swaerheys mid-
dellini des heels, daerom die latende, ende hanghende tgeheel an de
hanghende H Y door H ghegheuen vastpunt, so sal de sijde naer A D I,
swaerder sijn dan naer B C K, daerom oock sal de sijde A D I neerda-
len, tot dat H inde swaerheys middellini is des heels, ende ofmen schoon
L M (alles op vastpunt H draeyende) euewydich stelde vanden sicht-
einder N O, sy en can so niet blyuen door het 8 voorstel, maer alles sal
omkeeren, twelck wy bewysen moesten.

then through G the centre line of gravity RS of the weights I, K ; EG will then be the beam. After this, it shall be ascertained by the 2nd proposition where the fixed point of the handle will fall. For if it comes below H , LM will turn until it remains parallel to the horizon NO . But if it comes at H , it will remain at rest in any position given to it. If it comes above H , the whole prism turns upside down. Now the prism weighs 4 lbs, and I, K each 4 lbs, together 8 lbs, by the supposition. Then, EG being divided in T so that ET has to TG the ratio of 8 to 4, I say that LM will turn (since T comes below H) until it is parallel to the horizon. Now let the prism weigh 4 lbs, and I and K each 2 lbs, together 4 lbs. Then, EG being divided in H (which H is the middle of EG , by the supposition) so that EH shall have to HG the ratio of 4 to 4, I say that LM (since it fell in H) will remain at rest in any position given to it. Now let the prism weigh 4 lbs, in I, K each 1 lb. together 2 lbs. Then, EG being divided in V so that EV shall have to VG the ratio of 2 to 4, I say that the prism with all the rest will turn upside down (since V comes above H) until H is in its centre line of gravity. PROOF. Firstly, the fact that if I and K each weigh 4 lbs, LM will turn until it is parallel to the horizon is proved as follows: The vertical through T , as TX , is the centre line of gravity of the whole; therefore, omitting this one and hanging the whole from the vertical through H , as HY (which H is the given fixed point), the part adjacent to BCK will be heavier than that adjacent to ADI . Therefore the part BCK will descend until H is in the centre line of gravity of the whole, and then LM will be parallel to the horizon NO .

Secondly, the fact that if I, K each weigh 2 lbs, LM will remain at rest in any position given to it is shown as follows: Let us suppose I and K to have been pulled up in such a way that D is the centre of gravity of I , and C of K ; then, by the 3rd postulate, they will not be the cause of any change of the gravity at the prism. This being so, H is the centre of gravity of a solid made up of the prism and the two weights I, K , and by the 4th definition it will remain at rest in any position given to it; the same can likewise be proved of any position in which LM could be put.

Lastly, the fact that if I, K each weigh 1 lb, the whole turns upside down is shown as follows: the vertical through V , as VZ , is the centre line of gravity of the whole; therefore, omitting this one and hanging the whole from the vertical HY through H , the given fixed point, the side adjacent to ADI will be heavier than that adjacent to BCK , and therefore also the side ADI will descend until H is in the centre line of gravity of the whole. And though one should put LM (the whole turning about the fixed point H) parallel to the horizon NO , it cannot remain at rest in that position, by the 8th proposition, but the whole will turn upside down, which we had to prove.

T B E S L V Y T. Wefende dan ghegheuen een vastpunt des bekenden pilaers, &c.

Uyt het voorgaende is ghenouch blijckelick den ghemeenen voortganck in allen anderen, als van pilaren welcker vastpunt is buyten de lini als F G, ende der ghewichten vastpunten op ander plaetsen dan D C; Maer ouermits wy hier voornamelick trachten de oirfaccken vande gedaenten des waegs grondelick te openbaren (daer af inde Weeghdaet breeder sal gheseyt worden) so en gheuen wy van sulcke ongheschiede ghestaltheden gheen besonder voorbeelden.

6. EYSCHE.

I I. VOORSTEL.

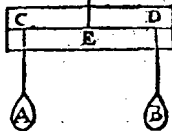
W E S E N D E ghegheuen een bekende pilaer, ende bekende swaerheden daer an hanghende: Te vinden het vastpunt daer op hy alle ghestalt houdt diemen hem gheeft.

I^e M E R C K.

So o twee euewichten als A, B, vastpunten C, D, waren in des pilaers als, euewyt van middelpunt E, als in dees form, is kennelick door het tweede deel des bewys van het 1^o voorstel, dat E begheerde punt soude sin, maer wy sullen voorbeelt van ongheschieder ghestalt gheuen.

II^e M E R C K.

T is openbaer dat wesende de twee vastpunten der ghewichten als C D, ende vastpunt des handthaefs als E, alle drie in een rechte lini als hier bouen, ende an C D een ghewichten ghehanghen, soo groot of te cleen als valt: E sal altyt vastpunt bliuen, daer sy alle ghestalt op houden diemen haer gheeft. Maer soo die drie punten als C E D in een rechte lini wesende C ende D niet euewyt en waren van E, ende datmen *Proportional.* an haer ghewichten hinghe* euewedich met de ermen, dat E noch altyt vastpunt sal bliuen daer sy alle ghestalt op houden diemen haer gheeft.



T G H E G H E V E N. Laet A B C D een pilaer sijn, weghende 10. lb, diens swaerheyds middelpunt E, ende laet de ghewichten daer an hanghende wesen F 1 lb, diens vastpunt G, ende H 4 lb, wiens vastpunt I. **T B E G H E E R D E.** Wy moeten het vastpunt vinden daerop sy alle ghestalt houden diemen haer gheeft. **T W E R C K.** Men sal trecken

CONCLUSION. Given therefore a fixed point of the known prism, etc.

From the preceding the common procedure in all other cases is sufficiently clear, for example with prisms whose fixed points are outside the line FG , while the fixed points of the weights are in places other than D, C . But since we are here mainly attempting to disclose the causes of the properties of the balance in principle (of which we will speak more fully in the Practice of Weighing¹⁾), we do not give any specific examples of such irregular forms.

PROBLEM VI.

PROPOSITION XI.

Given a known prism, and two known gravities hanging therefrom: to find the fixed point on which it remains at rest in any position given to it.

NOTE I.

If the fixed points C, D of two equal gravities A, B were in the axis of the prism, equidistant from the centre E , as in the annexed figure, it is evident from the second section of the proof of the 10th proposition that E would be the required point, but we will give the example with regard to an irregular form.

NOTE II.

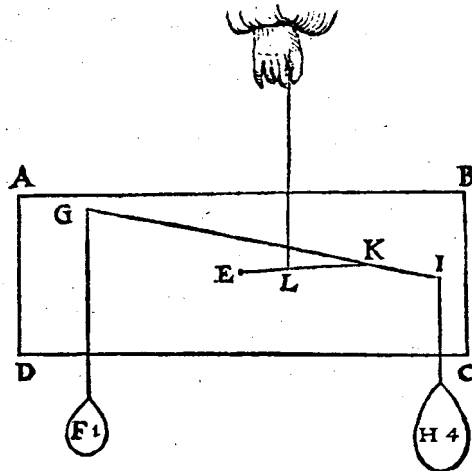
It is manifest that if the two fixed points of the weights are C, D , and the fixed point of the handle is E , all three being on a straight line, as above, equal weights hanging at C, D , as great or small as the case may be, E will always remain the fixed point on which they remain at rest in any position given to them. But if, these three points, as C, E, D , being on a straight line, C and D were not equidistant from E , and weights were hung at them proportional to the arms, E will still remain the fixed point on which they remain at rest in any position given to them.

SUPPOSITION. Let $ABCD$ be a prism weighing 10 lbs, its centre of gravity E ; and let the weights hanging therefrom be F (1 lb), and its fixed point G , and H (4 lbs), and its fixed point I . WHAT IS REQUIRED TO FIND. We have to find the fixed point on which they remain at rest in any position given to them. CONSTRUCTION. There shall be drawn GI , the beam of the weights

¹⁾ See *The Practice of Weighing*, Prop. 2.

trecken G. I balck
der gewichten F H,
daer naer salmen
vinden haer ermen
door het 2^e voor-
stel, dat is ghelijck
F 1 lb, tot H 4 lb,
also den erm K I, tot
K G, daer naer sal-
men trecken E K
balck des pilaers ter
eender, ende der
ghewichten F H ter
ander sijden, de sel-
ue E K ghedeelt in
L, also dat den erm
E L sulcken reden
hebbe tot L K, als

5 lb van F H, tot 10 lb des pilaers, L sal tbegeerde punt sijn op welck
sy alle ghestalt sullen houden diemen haer gheeft, waer af tbewys open-
baer is door het 7^e voorstel.



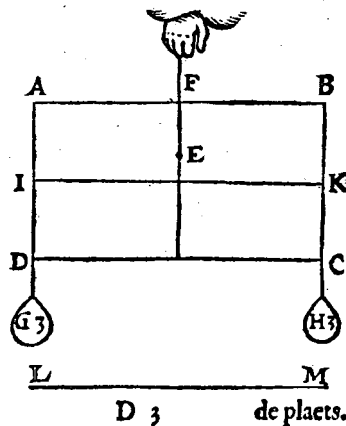
VII. EYSCH.

XII. VOORSTEL.

W E S E N D E ghegheuen een bekende pilaer,
met sijn vastpunt ende bekende ghewichten daer
an hanghende die den as euewydich houden van-
den * sichteinder: Te vinden een ghewicht han-
ghende ter begheerder plaets des pilaers, dat
den as in ghegheuen ghestalt houde. *Horizonte.*

I. VOORBEELT.

T G H E G H E V E N. Laet A B C D
een pilaer sijn weghende 6 lb,
diens vastpunt E, ende handthaeft
E F, ende twee ghewichten G, H,
elck 3 lb weghende, welckervast-
punten C, D; en I K, sy as, eue-
wydich vanden sichteinder L M,
ende D sy tpunt voor de begheer-



D 3 de plaets.

F , H , and then their arms shall be found by the 2nd proposition; i.e. as F (1 lb) is to H (4 lbs), so is the arm KI to the arm KG . After this, there shall be drawn EK , the beam of the prism on the one hand and of the weights F , H on the other hand. This beam EK being divided in L in such a way that the arm EL shall have to the arm LK the same ratio as F , H (5 lbs) to the prism (10 lbs), L will be the required point on which they will remain at rest in any position given to them. The proof of this will be manifest from the 7th proposition.

PROBLEM VII.

PROPOSITION XII.

Given a known prism, with its fixed point, and known gravities hanging therefrom, which keep the axis parallel to the horizon: to find a weight hanging in a required place of the prism which shall keep the axis in the given position.

EXAMPLE I.

SUPPOSITION. Let $ABCD$ be a prism weighing 6 lbs, its fixed point E and its handle EF , and let there be two weights G , H , each weighing 3 lbs, whose fixed points are C , D . And let IK be the axis, parallel to the horizon LM , and let D be the point indicating the required place. Then the axis IK (the whole turning

about E) is lifted, as in the second figure. **WHAT IS REQUIRED TO FIND.** We have to find a weight at D which shall keep the axis IK in that position. **CONSTRUCTION.** By the 11th proposition, there shall be found the fixed point on which the axis shall remain in any position given to it. Let this be N . Then there shall be drawn DN and the vertical EO , meeting ND in O . After this, it shall be ascertained what ratio NO has to OD . I take this to be that of 1 to 2. I therefore hang at D a weight P of 6 lbs, to wit in the ratio of 1 to 2 to the prism with the two weights G, I , being together 12 lbs. I say that P , weighing 6 lbs, is the required weight. **PROOF.** The heavier weight (12 lbs) of the arm ON has to the lighter weight (6 lbs) of the arm OD the same ratio as the longer arm OD to the shorter arm ON . Therefore the whole hangs, by the first proposition, from the handle EF in apparent equality of weight. And consequently the axis IK remains in its given position.

EXAMPLE II.

Let $ABCD$ be a prism weighing 6 lbs, its fixed point E and its handle EF , and let G be a weight of 2 lbs and its fixed point H , and I a weight of 1 lb and its fixed point K . And let the axis LM be parallel to the horizon NO , and let P be a point in the prism for the required place. Then the axis LM (the whole turning

TWERCK.

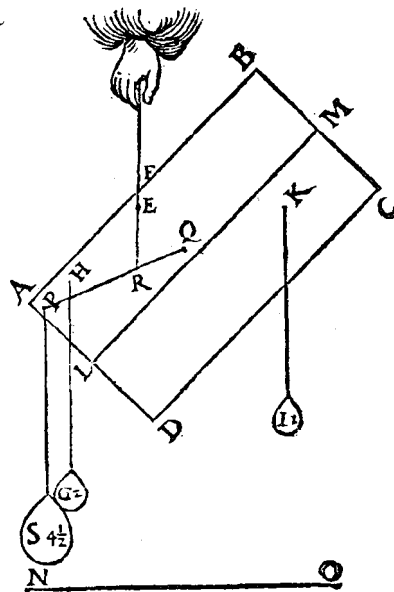
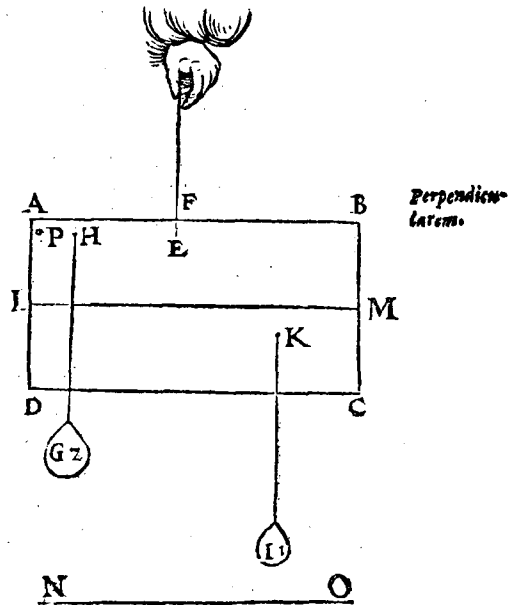
Men sal vinden door het 11^e voorstel vastpunt daerop ghegheuen alle ghestalt houdt diemen hem gheeft, welck Q sy, daer naer salmen trecken PQ , ende de * hanghende ER , sniende PQ in R : siende daer naer wat reden RQ heeft tot RP , ick neem als van 1 tot 2, so hang ick an P een ghewicht S van $4\frac{1}{2}$ lb, te weten in sulcken reden tot den pilaer met de twee ghewichten G, I , al tsamen 9 lb, als van 1 tot 2; ick seg $S 4\frac{1}{2}$ lb te wesen het begheerde ghewicht.

TBEWYS.

Tswaertste ghewicht 9 lb des erms RQ , heeft sulcken reden tot het lichtste ghewicht $4\frac{1}{2}$ lb des erms RP , ghelijck den langsten arm RP , tot den cortsten RQ , daerom hanghet al cunstelwichtich ande handthaeft EF door het 1^e voorstel, en veruolghens den as LM blijft in haer ghegheuen ghestalt, welck wy bewysen moesten.

TBESLVT.

Wesende dan ghegheuen een bekenden pilaer met sijn vastpunt, &c.

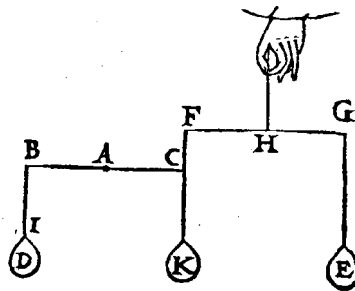


about E) is lifted, as in the second figure. **WHAT IS REQUIRED TO FIND.** We have to find a weight at P which shall keep the axis LM in that position. **CONSTRUCTION.** There shall be found, by the 11th proposition, the fixed point on which the given prism shall remain at rest in any position given to it. Let this point be Q . Then there shall be drawn PQ and the vertical ER meeting PQ in R . After this, it shall be ascertained what ratio RQ has to RP . I take this to be that of 1 to 2. I therefore hang at P a weight S of $4\frac{1}{2}$ lbs, to wit in the ratio of 1 to 2 to the prism with the two weights G, I , weighing together 9 lbs. I say that S , weighing $4\frac{1}{2}$ lbs, is the required weight. **PROOF.** The heavier weight (9 lbs) of the arm RQ has to the lighter weight ($4\frac{1}{2}$ lbs) of the arm RP the same ratio as the longer arm RP to the shorter arm RQ . Therefore the whole hangs, by the 1st proposition, from the handle EF in apparent equality of weight, and consequently the axis LM remains in its given position, which we had to prove. **CONCLUSION.** Given therefore a known prism, with its fixed point, etc.

EEN daelwicht ende een hefwicht an hem euen, doen met euen houcken an euen ermen euen ghewelden.

I^e. VOORBEELT met rechtwichten.

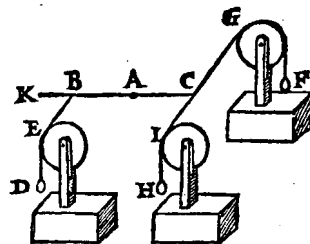
TGHEGHEVEN. Laet A des balcx BC vastpunt, ende AB met AC twee euen ermen sijn, ende an B hanghe het rechtreckwicht D, ende an C sy het rechthefwicht E, euewichich an D, ende sijn balck sy FG, diens vastpunt H, ende euen ermen HF, HG, ende den houck ABI, sy euen anden houck ACF. TBEGHEERDE. Wy moeten bewyfen dat het rechtdaelwicht D, ende trechthefwicht E, ande euen ermen AB, AC, euen ghewelden doen. TBEREYTSSEL. Laet an C een ghewicht K hanghen, euen an D. TBELWYS. Laet ons weeren E, en is blyckelijck dat de macht van D is de ermen AB, AC, in die ghegheuen ghestalt te houden, want D is euen an K, ende AB an AC. Laet nu D weeren, ende E wederom anhanghen, ende de macht van E is oock de ermen AB, AC, in die ghegheuen ghestalt te houden, want K is euen an E, ende HF an HG, daerom E ende D doen an, euen ermen AB, AC, euen ghewelden.



II^e VOORBEELT met scheefwichten.

TGHEGHEVEN. Laet A des handthaefs vastpunt, ende AB met AC twee euen ermen sijn, ende an B hanghe t'scheefdaelwicht D, diens scheefdaellini BE, ende an C sy t'scheefhefwicht F, euen an D, en sijn scheefheffini sy CG, ende den houck ABE, sy euen anden houck ACG.

TBEGHEERDE. Wy moeten bewyfen dat het scheefdaelwicht D, ende t'scheefhefwicht F, ande euen ermen AB, AC, euen ghewelden doen. TBEREYTSSEL. Laet an C een scheefdaelwicht H



hanghen

THEOREM VI.

PROPOSITION XIII.

A lowering weight and a lifting weight equal to it, acting at equal angles on equal arms, exert equal forces.

EXAMPLE I, with vertical weights.

Let A be the fixed point of the beam BC , and AB and AC two equal arms. And let there hang at B the vertical lowering weight D and let there be at C the vertical lifting weight E of equal weight to D ; and let the beam of the latter be FG , its fixed point H , and the equal arms HF , HG , and let the angle ABI be equal to the angle ACF . WHAT IS REQUIRED TO PROVE. We have to prove that the vertical lowering weight D and the vertical lifting weight E exert equal forces on the equal arms AB and AC . PRELIMINARY. Let there hang at C a weight K , equal to D . PROOF. Let us take away E , then it is apparent that the power of D is to keep the arms AB , AC in that given position, for D is equal to K , and AB to AC . Now let us take away D and attach E again, then the power of E is also to keep the arms AB , AC in that given position, for K is equal to D , and HF to HG . Therefore E and D exert equal forces on equal arms AB , AC .

EXAMPLE II, with oblique weights.

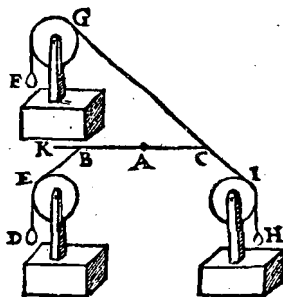
SUPPOSITION. Let A be the fixed point of the handle, and AB and AC two equal arms. And let there hang at B the oblique lowering weight D , whose oblique lowering line shall be BE , and let there be at C the oblique lifting weight F , equal to D , whose oblique lifting line shall be CG , And let the angle ABE be equal to the angle ACG . WHAT IS REQUIRED TO PROVE. We have to prove that the oblique lowering weight D and the oblique lifting weight F exert equal forces on the equal arms AB , AC . PRELIMINARY. Let there hang at C

hanghen euen an D, diens scheefhefline CI,* euewydich sy van B E, *Parallela.* ende C B sy wat voortghetrocken tot in K. **TBEWYS.** Laet ons weeren F, ende is kennelick dat de macht van D teghen H, is de ermen A B, A C, in die ghegheuen ghestalt te houden, want D is euen an H, ende den erm A B, an A C, ende den houck A C I, anden houck K B E. Laet nu D weeren, ende F wederom anhanghen, ende de macht van F is oock de ermen A B, A C, in die ghegheuen ghestalt te houden, ouermids H euen is an F.

III VOORBEELT.

TGHEGHEVEN. Laet A des handthaefts vastpunt, en A B met A C twee euen ermen sijn, ende an B hanghe het scheefdaelwicht D, diens scheefdaellini B E, ende an C sy het scheefhefwicht F, euen an D, diens scheefhefline sy C G, ende den houck K C G, sy euen anden houck K B E. **TBEGHEERDE.** Wy moeten bewyfen dat het scheefdaelwicht D, ende het scheefhefwicht F, ande euen ermen A B, A C, euen ghewelden doen. **TBEREYTSSEL.**

Laet an C een scheefdaelwicht H hanghen euen an D, diens scheefdaellini C I, also dat den houck A C I, euen sy anden houck A B E. **TBEWYS.** Laet ons weeren F, ende is kennelick dat de macht van D is de ermen A B, A C, in die ghegheuen ghestalt te houden, want D is euen an H, ende den erm A B an A C, ende den houck A C I, anden houck A B E. Laet nu D weeren, ende F wederom anhanghen, ende de macht van F is oock de ermen A B, A C, in die ghegheuen ghestalt te houden, ouermits H euen is an F.



TBESLVT. Een daelwicht dan ende een hefwicht an hem euen, doen, met euen houcken an euen ermen euen ghewelden, twelck wy bewyfen moesten.

VIII. EYSCH.

XIII. VOORSTEL.

WESENDE ghegheuen een pilaer, ende twee punten inden as, t'een vast tander int langste deel verroerlick: Te vinden een rechthefwicht an tverroerlick, dat den pilaer in sijn ghegheuen standt houde.

TGHEGHEVEN. Laet A B C D een pilaer sijn, weghende 6 lb,
E ende

an oblique lowering weight H , equal to D , whose oblique lifting line CI shall be parallel to BE , and let CB be somewhat produced to K . PROOF. Let us take away F ; then it is evident that the power of D against H is to keep the arms AB , AC in that given position, for D is equal to H , and the arm AB to AC , and the angle ACI to the angle KBE . Let us now take away D and attach F again; then the power of F is also to keep the arms AB , AC in that given position, because H is equal to F .

EXAMPLE III.

SUPPOSITION. Let A be the fixed point of the handle, and AB and AC two equal arms. And let there hang at B the oblique lowering weight D , whose oblique lowering line shall be BE , and let there be at C the oblique lifting weight F , equal to D , whose oblique lifting line shall be CG . And let the angle KCG be equal to the angle KBE . WHAT IS REQUIRED TO PROVE. We have to prove that the oblique lowering weight D and the oblique lifting weight F exert equal forces on the equal arms AB , AC . PRELIMINARY. Let there hang at C an oblique lowering weight H , equal to D , whose oblique lowering line shall be CI , in such a way that the angle ACI shall be equal to the angle ABE . PROOF. Let us take away F ; then it is evident that the power of D is to keep the arms AB , AC in that given position, for D is equal to H , and the arm AB to AC , and the angle ACI to the angle ABE . Let us now take away D and attach F again; then the power of F is also to keep the arms AB , AC in that given position, because H is equal to F . CONCLUSION. A lowering weight therefore and a lifting weight equal to it, acting at equal angles on equal arms, exert equal forces, which we had to prove.

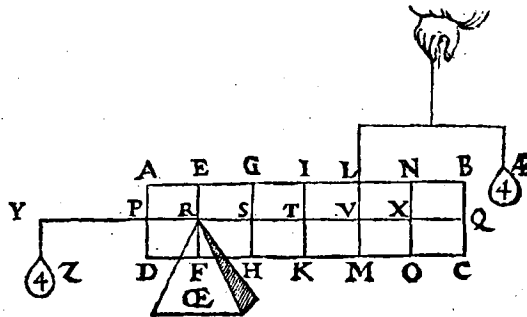
PROBLEM VIII.

PROPOSITION XIV.

Given a prism, and two points in the axis, one being fixed and the other in the longer part being movable: to find a vertical lifting weight at the movable point which shall keep the prism in its given position.

ende die ghedeelt als int beghin des 1^e voorstels, ende vastpunt sy R, ende roerlick V, int langste deel des as R Q, want int cortste R P ist onmueghelick dat eenich rechthefwicht den as in haer ghegheuen stant houde. **T B E G H E E R D E.** Wy moeten een rechthefwicht an V vinden, dat den pilaer in die standt houde. **T W E R C K.** Men sal de lini

Q R voorttrecken tot in Y, also dat R Y euen sy an R V: Daer naer salmen vinden tghewicht Z an Y, euestaltwichtich met de pilaer, tselue (gedenckede dat R vastpunt is) sal van 4 lb wesen door het 3^e voorstel; Ick seg daerom dat het begheerde rechthefwicht twelck Æ sy, van 4 lb sal wesen.



T B E W Y S. Ouermidts den erm R V des rechthefwichts Æ, euen is anden erm R Y des ghewichts Z, ende Æ euen an Z, soo is de ghewelt Æ euen an de ghewelt van Z door het 13^e voorstel. Maer de ghewelt van Z is (Æ gheweert sijnde) den pilaer in die standt te houden, de ghewelt dan van Æ (Z gheweert sijnde) is oock den pilaer in die standt te houden, twelck wy bewysen moesten. **T B E S L V Y T.** Wefende dan ghegheuen een pilaer, ende twee punten inden as, t'een vast, t'ander int langste deel verroerlick: Wy hebben gheuonden een rechthefwicht an tverroerlick, dat den pilaer in sijn ghegheuen standt houdt naer den cysch.

M E R C K T.

M E N soude oock mueghen segghen metten cortsten V R 3, gheeft R T 2, wat den pilaer 6 lb? comt voor Æ 4 lb als vooren, waer af de reden int volghende 15 voorstel bliken sal.

1^e V E R V O L G H.

A N G H E S I E N den heelen pilaer door tghestelde 6 lb weeght, waer af Æ de 4 lb verheft, so volgt nootfaeckelick datter opt punt R, dat is op ttop des keghels OE, 2 lb rusten.

O P T E

SUPPOSITION. Let $ABCD$ be a prism weighing 6 lbs, and let it be divided as at the beginning of the 1st proposition; and let the fixed point be R and the movable point V in the longer part of the axis RQ , for it is impossible for any vertical lifting weight in the shorter part RP to keep the axis in its given position. WHAT IS REQUIRED TO FIND. We have to find a vertical lifting weight at V which shall keep the prism in that position. CONSTRUCTION. The line QR shall be produced to Y , in such a way that RY shall be equal to RV . Then there shall be found the weight Z at Y , of equal apparent weight to the prism. Bearing in mind that R is the fixed point, this weight will be of 4 lbs, by the 3rd proposition. I therefore say that the required vertical lifting weight, which shall be $\mathcal{A}E$, will be of 4 lbs. PROOF. Since the arm RV of the vertical lifting weight $\mathcal{A}E$ is equal to the arm RY of the weight Z , and $\mathcal{A}E$ is equal to Z , the force of $\mathcal{A}E$ is, by the 13th proposition, equal to the force of Z . But ($\mathcal{A}E$ being taken away) the power of Z is to keep the prism in that position; therefore (Z being taken away), the power of $\mathcal{A}E$ is also to keep the prism in that position, which we had to prove. CONCLUSION. Given therefore a prism, and two points in the axis, one being fixed and the other in the longer part being movable, we have found a vertical lifting weight at the movable point which shall keep the prism in its given position, as required.

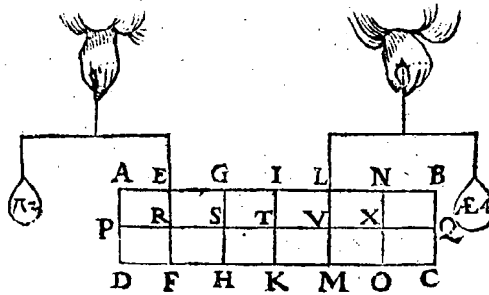
NOTE.

One might also say: with the shorter arm VR 3 gives RT 2, what the prism 6 lbs? $\mathcal{A}E$ becomes 4 lbs, as above, the cause of which will become apparent in the 15th proposition hereinafter.

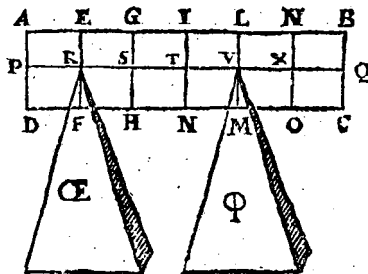
COROLLARY I.

Since by the supposition the whole prism weighs 6 lbs, of which $\mathcal{A}E$ lifts the 4 lbs, it necessary follows than on the point R , that is the top of the cone $\mathcal{A}E$, there rest 2 lbs.

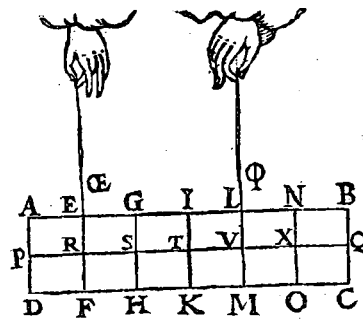
OFTE soomen an R een rechthefwicht II vougde, inde plaets des keghels OE, als hier neuen, dat II sal weghen 2 lb.



OFTE soomen an V een keghel * vougde, inde plaets des rechthefwichts E, als hier neuen, dat op den keghel OE rusten sal 2 lb, ende op den keghel * 4 lb.



OFTE soomen den pilaer ophinghe an twee * euewidighe linien OE R, ende * V, als hier neuen, dat ande lini OE R hanghen sal 2 lb, ende ande lini * V 4 lb.



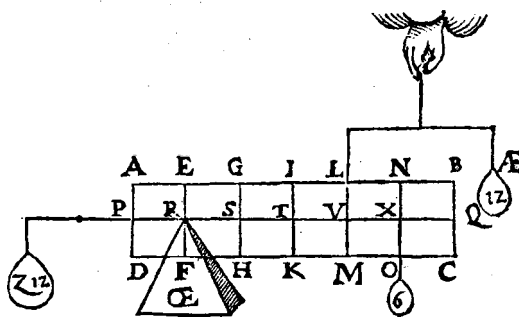
Paralleles.

Or if a vertical lifting weight Π were attached at R instead of the cone $\mathcal{C}E$, as shown in the annexed figure, that Π will weight 2 lbs.

Or if a cone Φ were applied at V instead of the vertical lifting weight $\mathcal{A}E$, as shown in the annexed figure, that there will rest 2 lbs on the cone $\mathcal{C}E$ and 4 lbs on the cone Φ .

Or if the prism were hung by two parallel lines $\mathcal{C}E R$ and ΦV , as shown in the annexed figure, that there will hang 2 lbs by the line $\mathcal{C}E R$ and 4 lbs by the line ΦV .

SO anden pilaer (tpunt R vast sijnde als vooren) eenich gewicht ofte gewichten hingen, trechthefwicht sal oock beken worden. Laet by voorbeeld an X hanghen 6 lb, so sal Z moete weghen 12 lb door het 3^e voorstel, ende vervolgens Æ 12 lb.



VII VERTOCH.

XV VOORSTEL.

WESENDE twee punten inden as des pilaers, t'een vast t'ander verroerlick: Trechthefwicht an t'verroerlic met den pilaer euestaltwichtich, heeft sulcken reden tot den pilaer als het afftick tusschen het swaerheys middelpunt des pilaers, ende het vastpunt, tot het afftick tusschen t'vastpunt ende t'verroerlick punt.

VERCLARING.

*Mathemati-
ed.* **L**AET ons nemen de formen des 14 voorstels, al waer blijft dat ghelijck Æ 4 lb, tot tghewicht des pilaers 6 lb, alsoo TR tot R V. Maer om d'oirsaek hier af * Wisconstelick te verclaren, soo is te weten dat ghelijck t'ghewicht Z, tottet ghewicht des pilaers, alsoo R T tot R Y door het 1^e voorstel; Maer Æ is euen an Z, ende R V is euen an R Y door t'ghegeuen, ghelijck dan Æ tot den pilaer, alsoo TR tot R V.

TBESLUYT. Wesende dan twee punten inden as des pilaers t'een vast t'ander verroerlick, &c.

VIII VERTOCH.

XVI VOORSTEL.

Wesende twee punten inden as des pilaers t'een vast t'ander verroerlick: Trechthefwicht an t'verroerlick dat den pilaer in een ghestalt houdt, sal hem in alle ghestalten houden.

TGHE-

COROLLARY II.

If there hung one or more weights from the prism (the point R being fixed, as above), the vertical lifting weight will also become known. For example, let there hang 6 lbs at X , then Z , by the 3rd proposition, will have to weigh 12 lbs, and consequently $\mathcal{A}E$ 12 lbs.

THEOREM VII.

PROPOSITION XV.

If there are two points in the axis of the prism, one being fixed and the other movable, the vertical lifting weight at the movable point having equal apparent weight to the prism has to the prism the same ratio as the part of the axis between the centre of gravity of the prism and the fixed point to the part of the axis between the fixed point and the movable point.

EXPLANATION.

Let us take the figures of the 14th proposition, where it is apparent that as $\mathcal{A}E$ (4 lbs) is to the weight of the prism (6 lbs), so is TR to RV . But in order to explain the cause of this mathematically, it has to be known that as the weight Z is to the weight of the prism, so, by the 1st proposition, is RT to RY . Now $\mathcal{A}E$ is equal to Z , and RV is equal to RY by the supposition; therefore, as $\mathcal{A}E$ is to the prism, so is TR to RV . CONCLUSION. If therefore there are two points in the axis of the prism, one being fixed and the other movable, etc.

THEOREM VIII.

PROPOSITION XVI.

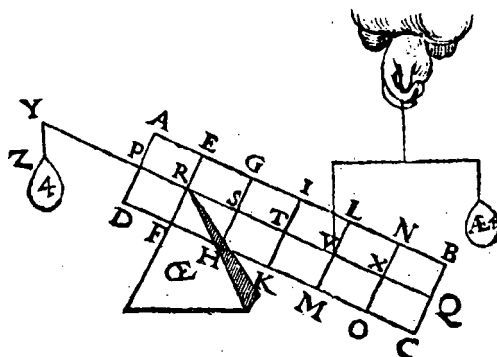
If there are two points in the axis of the prism, one being fixed and the other movable, the vertical lifting weight at the movable point which keeps the prism in one position will keep it in any position.

T G H E G H E V E N. Laet ons den pilaer met sijn ghewichten des 14^e voorstels wat verkeeren op vastpunt R, ende dat $\mathcal{A}$ 4 lb noch sy rechthefwicht, alsoo dat dan alles van gestalt sy als hier neuen.

T B E G H E E R D E. Wy moett bewy-
sen dattet rechthefwicht $\mathcal{A}$ den pilaer oock in die ghegheuen ghestalt houdt.

T B E W Y S.
Laet ons weeren $\mathcal{A}$ ende anhanghen Z 4 lb, ende

door het 10^e voorstel den pilaer sal in die ghestalt bliuen: Maer $\mathcal{A}$ doet by V soo grooten ghewelt anden pilaer als Z by Y door het 13^e voorstel, daerom gheweert Z, ende $\mathcal{A}$ anghehanghen, soo sal $\mathcal{A}$ den pilaer oock in die ghestalt houden. **T B E S L V Y T.** Wesende dan twee punten in den as des pilaers t'een vast tander verroerlick, trechthefwicht an tverroerlick, dat den pilaer in een ghestalt houdt, sal hem in alle ghestalten houden, twelck wy bewysen moesten.



IX VERTOCH.

XVII VOORSTEL.

R V S T E N D E een pilaer op twee punten inden as. Ghelijck het astick tusschen t'swaerheys middelpunt ende t'slinckerpunt, tottet astick tusschen t'swaerheys middelpunt ende trechterpunt, alsoo tghewicht des pilaers rustende op trechterpunt, tottet ghewicht rustende op t'slinckerpunt.

T G H E G H E V E N. Laet A B C D een pilaer sijn weghende 6 lb, ghedeelt als int 1^e voorstel, rustende met de twee punten R, V, op de punten van O E, $\mathcal{A}$. **T B E G H E E R D E.** Wy moeten bewysen dat ghelijck het astick T R, tottet astick T V, also tghewicht rustende metter punt V op tpunt van $\mathcal{A}$, tottet tghewicht rustende metter tpunt R op tpunt van O E. **T B E W Y S.** T R is dobbel an T V door tghesteelde,

F 3 ende

SUPPOSITION. Let us turn the prism with its weights, of the 14th proposition, somewhat about the fixed point R , and let $\mathcal{AE}$ (4 lbs) still be the vertical lifting weight, in such a way that the situation shall be as shown in the annexed figure. WHAT IS REQUIRED TO PROVE. We have to prove that the vertical lifting weight $\mathcal{AE}$ also keeps the prism in that given position. PROOF. Let us take away $\mathcal{AE}$ and attach Z (4 lbs); then, by the 10th proposition, the prism will remain in that position. Now, by the 13th proposition, $\mathcal{AE}$ exerts on the prism at V the same force as Z at Y ; therefore, Z being taken away and $\mathcal{AE}$ attached again, $\mathcal{AE}$ will also keep the prism in that position.

CONCLUSION. If therefore there are two points in the axis of the prism, one being fixed and the other movable, the vertical lifting weight at the movable point which keeps the prism in one position will keep it in any position, which we had to prove.

THEOREM IX.

PROPOSITION XVII.

If a prism rests on two points in the axis: as the part of the axis between the centre of gravity and the lefthand point is to the part of the axis between the centre of gravity and the righthand point, so is the weight of the prism resting on the righthand point to the weight resting on the lefthand point.

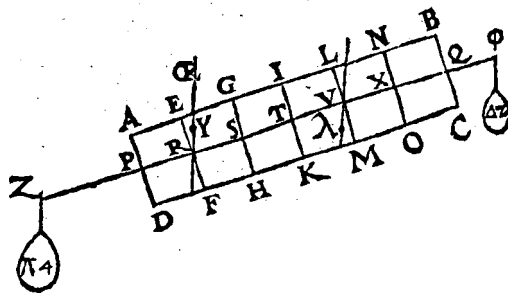
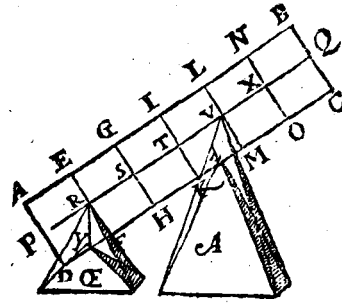
SUPPOSITION. Let $ABCD$ be a prism weighing 6 lbs, divided as in the 1st proposition, resting with the two points R, V on the points of $\mathcal{CE}, \mathcal{AE}$. WHAT IS REQUIRED TO PROVE. We have to prove that as the part of the axis TR is to the part of the axis TV , so is the weight resting with the point V on the point of $\mathcal{AE}$ to the weight resting with the point R on the point of $\mathcal{CE}$. PROOF. TR is double of TV by the supposition; then on the point of $\mathcal{AE}$ there rest 4 lbs and

ende opt tpunt van Δ rust 4 lb, ende van CE 2 lb door 1^o veruolg des 14^{en} voorstels, maer 4 lb is tot 2 lb oock dubbel, ghelijck dan TR tot TV , alsoo tghewicht rustende op tpunt van Δ , tot tghewicht rustende op tpunt van CE .

Maer om tghemeen noot-saeckelick veruolgh in allen te bewysen, laet ons voorttrecken VR tot in Z , also dat RZ euen

sy an RV ; aen-siende daer naer R voor vastpunt, so sal an Z moeten hanghen Π 4 lb, om de pilaer in die ghestalt te houden door het 3^o voorstel. Maer tghene an V den pilaer in die ghestalt houdt als Δ , doet daer an alsulcken gheweldt als Π , door

het 1^o voorstel; An Δ dan rust een ghewicht euen an Π . Laet ons insghelick voorttrecken, RV tot in Φ , also dat $\text{V}\Phi$ euen sy an VR , aen-siende daer naer V voor vastpunt, soo sal an Φ moeten hanghen Δ 2 lb, om den pilaer in die ghestalt te houden door het 3^o voorstel, maer tghene an R den pilaer in die ghestalt houdt als OE , doet daeran alsulcke gewelt als Δ door het 1^o voorstel, An OE dan rust een ghewicht euen an Δ . Nu anghesien Π euestalwichtich is teghen den pilaer op tghemeen vastpunt R , so heeft den erm TR , sulcken reden tot den erm RZ , als Π tot den pilaer door 1^o voorstel. Insghelijcx nemende V voor vastpunt, soo heeft den erm TV sulcken reden tot den erm $\text{V}\Phi$, als Δ tot den pilaer, maer RZ is alijt euen an $\text{V}\Phi$: Wy hebben hier dan twee * eueredenheden elck van vier * palen, welcker tweede palen an malcanderen euen sijn, ende welcker laeste palen an malcanderen oock euen sijn. Maer alle twee eueredenheden elck van vier palen, welcker tweede palen an malcanderen



*Proportiones.
Terminus.*

on that of CE 2 lbs, by the 1st corollary of the 14th proposition. But 4 lbs is also double of 2 lbs; therefore, as TR is to TV , so is the weight resting on the point of AE to the weight resting on the point of CE .

But in order to prove the general necessary consequence in all cases, let us produce VR to Z , in such a way that RZ shall be equal to RV . If we then consider R as the fixed point, there will have to hang at Z a weight Π of 4 lbs in order to keep the prism in that position, by the 3rd proposition. Now that which, acting at V keeps the prism in that position, as AE , exerts on it the same force as Π by the 13th proposition. At AE therefore rests a weight equal to Π . Let us likewise produce RV to Φ , in such a way that $V\Phi$ shall be equal to VR . If we then consider V as the fixed point, there will have to hang from Φ a weight Δ of 2 lbs in order to keep the prism in that position, by the 3rd proposition. But that which, acting at R , keeps the prism in that position, as CE , exerts on it the same force as Δ , by the 13th proposition. At CE therefore rests a weight equal to Δ . Since Π is of equal apparent weight to the prism on the common fixed point R , the arm TR has to the arm RZ the same ratio as Π to the prism, by the 1st proposition. Likewise, taking V for the fixed point, the arm TV has to the arm $V\Phi$ the same ratio as Δ to the prism. But RZ is always equal to $V\Phi$. We therefore have two proportions, each of four terms, the second terms of which are equal to one an-

cander euen sijn, ende welcker laetste palen an malcander oock euen sijn, die hebben dander palen oock euerednich, daerom ghelijck T R tot T V, alsoo Π tot Δ ; maer Π is euen an tghewicht des pilaers rustende met tpunt V op tpunt van ΔE , en tghewicht Δ is euen an tghewicht des pilaers rustende met tpunt R op tpunt van ΔE , daerom ghelijck T R tot T V, also tghewicht rustende mettpunt V op tpunt van ΔE , tottet ghewicht rustende mettpunt R op tpunt van ΔE . T B E S L V Y T. Rustende dan een pilaer op twee punten inden as, &c.

VERVOLGH.

So o de twee punten daer den pilaer op rust, waren inde * hanghende linien door R en V, de selue ghewichten die hier vooren op elck rustende punt waren, soudender nu oock op sijn. Laet by voorbeeld door de punten R, V, hanghende linien ghetrocken worden, ende punten inde selue ghestelt als Y λ , Ghenomen nu dat Y ende λ de punten sijn daer den pilaer op rust, tis kennelick dat op Y rusten sal 2 lb, ende op λ 4 lb, waer uyt alfulcken vertooch openbaer is.

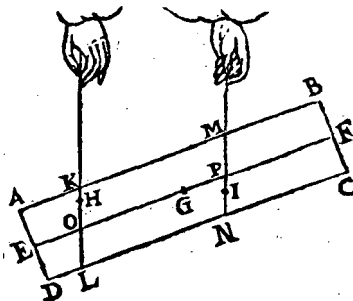
Perpendicularibus.

IC. VERTOCH.

XVIII. VOORSTEL.

R V S T E N D E een pilaer op eenighe twee punten, ghelijck het astick tusschen tswaerheys middelpunt ende de hanghende door t'sinckerpunt, tottet astick tusschen t'swaerheyds middelpunt ende de hanghende door trechterpunt, also tghewicht des pilaers rustende op trechterpunt, tottet ghewicht rustende op t'sinckerpunt.

T G H E G H E V E N. Laet A B C D een pilaer wesen, diens as E F, ende swaerheyds middelpunt G, ende de twee puntē daer d'een pilaer op rust H I, waer duer ghetrocken sijn de hanghende linien K L, M N, liniende den as in O, P; Ick seg dat ghelijck G O tot G P, alsoo de swaerheydt rustende op tpunt I, tot de swaerheydt rustende op H, waer af tbewys openbaer is door tvervolgh des voorgaenden 17^{en} voorstels, nochtans



other and the last terms of which are also equal to one another. But if of any two proportions, each of four terms, the second terms are equal to one another and the last terms are also equal to one another, then they have the other terms also proportional. Therefore, as TR is to TV , so is Π to Δ . But Π is equal to the weight of the prism resting with the point V on the point of $\mathcal{AE}$, and the weight Δ is equal to the weight of the prism resting with the point R on the point of $\mathcal{CE}$. Therefore, as TR is to TV , so is the weight resting with the point V on the point of $\mathcal{AE}$ to the weight resting with the point R on the point of $\mathcal{CE}$. CONCLUSION. If therefore a prism rests on two points in the axis, etc.

COROLLARY.

If the two points on which the prism rests were in the verticals through R and V , the same weights which were above on each point of support would then also be thereon. Let there, for example, be drawn verticals through the points R , V , and let there be marked points in them, as Y , Λ . Now considering Y and Λ to be the points on which the prism rests, it is evident that there will rest 2 lbs on Y and 4 lbs on Λ , from which the theorem is manifest.

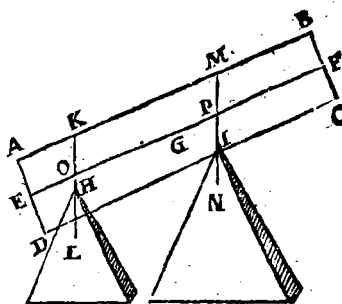
THEOREM X.

PROPOSITION XVIII.

If a prism rests on any two points: as the part of the axis between the centre of gravity and the vertical through the lefthand point is to the part of the axis between the centre of gravity and the vertical through the righthand point, so is the weight of the prism resting on the righthand point to the weight resting on the lefthand point.

SUPPOSITION. Let $ABCD$ be a prism, its axis EF , the centre of gravity G , and the two points on which the prism rests H , I , through which are drawn the verticals KL , MN , meeting the axis in O , P . I say that as GO is to GP , so is the gravity resting on the point I to the gravity resting on H , from which the proof is manifest by the corollary of the 17th proposition hereinbefore. Nevertheless, in

nochtans om alhier wat breeder vande nootsakelicheyt te segghen, so laet ons achten al of H ter plaats van O waer, twelck soo ghenomen tghewicht alsdan op H rustende, heeft sulcken reden tottet ghewicht op P rustende, ghelijck G P, tot G O, duer het 17^e voorstel; Laet ons voort nemen dattet tpunt H vast blijuende, den pilaer in haer ghegheuen ghestalt neerghe-



trocken worde, soo verre als van H tot O, ende duer de 3^e begheerte, de swaerheydt an H rustende blijft de selue. Sghelijcx salmen bethoonen de swaerheydt dieder op P rust, oock te rusten op I, daerom ghelijck G O tot G P, alsoe de swaerheydt rustende op I, tot de swaerheydt rustende op H. **T B E S L V Y T.** Rustende dan een pilaer op eenighe twee punten, &c.

VERVOLGH.

T B L I I C T uyt het voorgaende dat soomen begheerde te weten de reden van tghewicht rustende op I, tottet tghewicht rustende op H, datmen trecken soude de hanghende linien K L, M N, sniende den as E F in O, P, ende de reden van G O tot G P soude de begheerde sijn waer uyt oock openbaer is, dat des pilaers swaerheydt bekend wesende, soo is oock tghewicht bekend rustende op yder punt als H ende L.

T O T H I E R T O E S I I N D E G H E D A E N T E N D E R R E C H T - W I C H T E N V E R C L A E R T : I N T *volgende sullen de eyghenschappen der scheefwichten bescreuen worden, wiens ghemeene gronds dit volgende vertooch begrijpt.*

XI. VERTOOC.

XIX. VOORSTEL.

Planum.

Horizontem.

W E S E N D E een driehouc wiens * plat recht-
houckich op den * sichteinder is, met sijn gronde
daer af euewidich, ende op elck der ander sijden
een rollende clood met malcanderen euewichtich:
Ghelijck

order to speak here somewhat more fully about the necessity, let us suppose H to be in the place of O . On this assumption, the weight then resting on H has to the weight resting on P the same ratio as GP to GO , by the 17th proposition. Let us further suppose that, the point H remaining fixed, the prism be pulled down in its given position as far as H is from O ; then by the 3rd postulate the gravity resting on H remains the same. In the same way the gravity resting on P will be shown to rest also on I ; therefore, as GO is to GP , so is the gravity resting on I to the gravity resting on H . CONCLUSION. If therefore a prism rests on two points, etc.

COROLLARY.

It appears from the above that if it should be required to know the ratio of the weight resting on I to the weight resting on H , the verticals KL , MN , meeting the axis EF in O , P , should be drawn; then the ratio of GO to GP would be the one required, from which it is also manifest that, the gravity of the prism being known, the weight resting on each of the points, as H , I , is also known.

UP TO THIS POINT

THE PROPERTIES OF VERTICAL WEIGHTS HAVE BEEN EXPLAINED;

in the following pages the properties of oblique weights will
be described, the common principle of which
is contained in the following theorem.

THEOREM XI.

PROPOSITION XIX.

Given a triangle, whose plane is at right angles to the horizon, with its base parallel thereto, while on each of the other sides there shall be a rolling sphere, of equal weight to one another: as the right side of the triangle is to the left

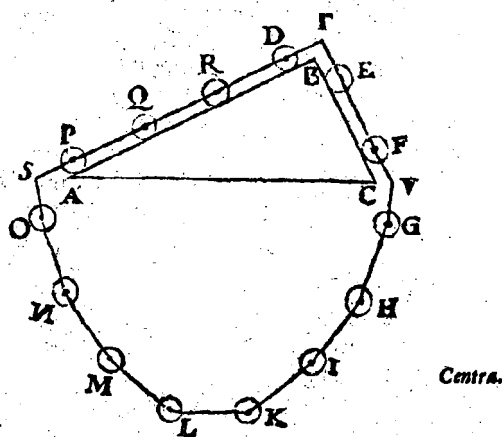
Ghelijck des driehoucx rechter sijde tot de slincker, also t'staltwicht des cloots op de slincker sijde, tottet staltwicht des cloots op de rechter sijde.

T G H E G H E Y E N. Laet ABC een driehouck wesen diens plat sy rechthouckich op den sichteinder, ende den grondt AC euewydich vanden sichteinder, ende op de sijde AB ; die dobbel sy an BC , ligge een cloot D , ende op de sijde BC een cloot E , euewichtich ende euegroot met den cloot D . **T B E G H E E R D E.** Wy moeten bewysen dat ghelijck de sijde AB 2, tot BC 1, alsoo t'staltwicht des cloots E , tottet staltwicht des cloots D .

T B E R E Y T S E L. Laet ons maecten rondtom den driehouck ABC eenen crans van veertien clooten, euegroot, euewichtich, ende euewichtich van malcanderen, als $E, F, G, H, I, K, L, M, N, O, P, Q, R, D$, al ghesnoert an een lini, streckende door haer * middelpunten, also dat sy op die middelpunten drayen mueghen; Datter

oock twee clooten passen op de sijde BC , ende vier op BA , dat is ghelijck lini tot lini, also clooten tot clooten; laet oock an S, T, V , drie vastpunten staen, ouer welcke de lini ofte t'snoer der clooten slieren mach, also dat de twee deelen des snoers die bouen den driehouck staen, * euewydich sijn vande sijden AB, BC ; Inder voughen dat als men den crans an d'een ofte d'ander sijde neertrect, soo rollen de clooten op de linien AB, BC . **T B E W Y S.** Soo t'staltwicht der vier clooten D, R, Q, P , niet euen en waer met het staltwicht der twee clooten E, F , t'een of t'ander sal swaerder sijn, latet wesen (soot mueghelick wacen) der vier D, R, Q, P ; Maer de vier clooten O, N, M, L , sijn euewichtich met de vier clooten G, H, I, K , de sijde dan der acht clooten D, R, Q, P, O, N, M, L , is swaerder na de ghestalt dan de sijde der ses clooten E, F, G, H, I, K ; maer want het swaerste altydt het lichtste ouerweeght, de acht clooten sullen neerwaert rollen, ende d'ander ses rijfen: Latet soo wesen, ende D

F sy ghe-



Parallela.

side, so is the apparent weight ¹⁾ of the sphere on the left side to the apparent weight of the sphere on the right side.

SUPPOSITION. Let ABC be a triangle, whose plane shall be at right angles to the horizon, and the base AC parallel to the horizon, and on the side AB , which shall be double of the side BC , let there lie a sphere D , and on the side BC a sphere E , of equal weight and equal size to the sphere D . **WHAT IS REQUIRED TO PROVE.** We have to prove that as the side AB (2) is to BC (1), so is the apparent weight of the sphere E to the apparent weight of the sphere D . **PRELIMINARY.** Let us make about the triangle ABC a wreath of fourteen spheres, of equal size and equal weight, and equidistant from one another, as $E, F, G, H, I, K, L, M, N, O, P, Q, R, D$, all of them strung on a line passing through their centres, in such a way that they can revolve about those centres; let there also fit two spheres on the side BC and four on BA , i.e. as line to line, so spheres to spheres. Let there also be three fixed points at S, T, V , over which the line or the string of the spheres can slide, in such a way that the two parts of the string above the triangle shall be parallel to the sides AB, BC , so that if the wreath is pulled down on one side or the other, the spheres roll on the lines AB, BC . **PROOF.** If the apparent weight of the four spheres D, R, Q, P were not equal to the apparent weight of the two spheres E, F , either one or the other will be the heavier. Let us suppose (if this were possible) this to be the one of the four spheres D, R, Q, P . But the four spheres O, N, M, L are of equal weight to the four spheres G, H, I, K . The side therefore of the eight spheres D, R, Q, P, O, N, M, L is heavier in appearance than the side of the six spheres E, F, G, H, I, K ²⁾. But because that which is heavier always preponderates over that which is lighter, the eight spheres will roll downwards and the other six will rise. Let

¹⁾ Here for the first time the term *staltwicht* is used, which has not been defined anywhere. It means the component of the weight which in the given situation is the only active one, i.e. the component along the inclined plane.

²⁾ Of course this contention, though true in this special case, is not valid generally: if to two systems of unequal apparent weight, equal weights whose apparent weights are not equal to one another are added on both sides, the systems may become of equal apparent weight.

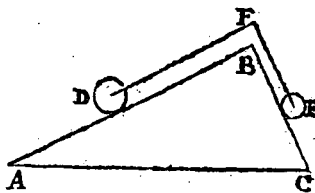
sy gheuallen daer nu O is, ende E, F, G, H, sullen sijn daer nu P, Q, R, D, ende I, K, daer nu E, F, sijn. Maer dit soo wesen, den crans der clooten sal alfulcken ghestalt hebben als sy te vooren dede, ende sullen om de selue redenen de acht clooten ter sliicker sijde wederom staltwichtighe sijn dan de ses clooten ter rechter, waer duer de acht clooten wederom neer sullen rollen, ende d'ander ses rijzen, welcke vallig ter eender, ende rijzing ter ander, om dat de reden altijd de selue is, altijd ghedueren sal, ende de clooten sullen uyt haer seluen een eeuwich roersel maken, t'welck valsich is. Het deel dan des crans D, R, Q, P, O, N, M, L, is euefaltwichtig met het deel E, F, G, H, I, K: Maer van sulcke euewichtighe ghetrocken euewichtighe, de resten sijn euewichtig, laet ons dan van dat deel trecken de vier clooten O, N, M, L, ende van dit de vier clooten G, H, I, K, (welcke euen sijn ande voornoemde O, N, M, L,) de resten D, R, Q, P, ende E, F, sullen euefaltwichtig sijn, Maer wesen de dese twee euefaltwichtig met die vier, E sal tweemaal staltwaerder sijn als D. Ghelijck dan de lini B A 2, tot de lini B C 1, also t'faltwicht des cloots E, tottet staltwicht des cloots D.

T'beslyt. Wefende dan een driehouck wiens plat, &c.

I. V E R V O L G H.

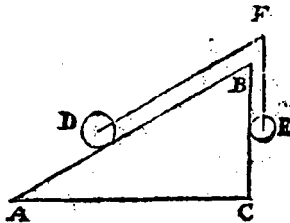
Centro.

LAET ABC een driehouck sijn als vooren, wiens sijde A B dobbel sy an B C, ende laet op A B ligghen een cloot D, ende op de sijde B C een cloot E euewichtig anden helft van D, ende an F sy een vastpunt daer ouer de lini D F E. (te weten uyt het *middelpunt des cloots D ouer F tot int middelpunt des cloots E) slieren mach, alsoo dat D F euewijdich blijue van A B, ende F E van B C. Dit also sijnde, anghesien de vier clooten P, Q, R, D, hier vooren, euefaltwichtig waren met de twee clooten E, F, so sal desen cloot D, euefaltwichtig sijn teghen den cloot E: want ghelijck die P, Q, R, D, tot E, F, also dese D tot E: Daerom ghelijck de lini A B, tot B C, alsoo den cloot D tot den cloot E.



II V E R V O L G H.

LAET ons nu d'een sijde des driehouckx als B C (ande welcke A B dobbel is) rechthouckich stellen op A C als hier neuën; Ende den cloot D die dobbel is an E, sal noch met E euefaltwichtig sijn, want ghelijck A B tot B C, also den cloot D tot den cloot E.



III VER-

this be so, and let D have fallen where O is now, then E, F, G, H will be where P, Q, R, D are now, and I, K where E, F are now. But this being so, the wreath of spheres will have the same appearance as before, and on this account the eight spheres on the left side will again have greater apparent weight than the six spheres on the right side, in consequence of which the eight spheres will again roll down and the other six will rise. This descent on the one and ascent on the other side will continue for ever, because the cause is always the same, and the spheres will automatically perform a perpetual motion, which is absurd ¹⁾. The part of the wreath D, R, Q, P, O, N, M, L therefore is of equal apparent weight to the part E, F, G, H, I, K . But if from such equal weights there are subtracted equal weights, the remainders will have equal weight ²⁾. Let us therefore subtract from the former part the four spheres O, N, M, L , and from the latter part the four spheres G, H, I, K (which are equal to the aforesaid O, N, M, L); then the remainders D, R, Q, P and E, F will be of equal apparent weight. But the two latter being of equal apparent weight to the four former, E will have twice the apparent weight of D . As therefore the line BA (2) is to the line BC (1), so is the apparent weight of the sphere E to the apparent weight of the sphere D . CONCLUSION. Given therefore a triangle, whose plane, etc.

COROLLARY I.

Let ABC be a triangle, as before, whose side AB shall be double of BC , and on AB let there lie a sphere D and on the side BC a sphere E being of equal weight to half of D . And in F let there be a fixed point, over which the line DFE (to wit, from the centre of the sphere D via F to the centre of the sphere E) can slide, in such a way that DF shall remain parallel to AB , and FE to BC . This being so, since the four spheres P, Q, R, D in the preceding case were of equal apparent weight to the two spheres E, F , this sphere D will be of equal apparent weight to the sphere E ; for as the former P, Q, R, D are to E, F , so is the latter D to E . Therefore, as the line AB is to BC , so is the sphere D to the sphere E ³⁾.

COROLLARY II.

Now let us put one side of the triangle, as BC (AB being double of it) at right angles to AC , as in the annexed figure. Then the sphere D , which is double of E , will still be of equal apparent weight to E , for as AB is to BC , so is the sphere D to the sphere E .

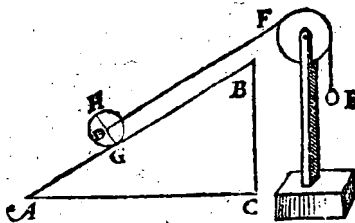
¹⁾ The conviction that a perpetual motion is impossible in physical reality is not a sufficient ground for qualifying it as absurd in the ideal sphere of rational mechanics, where friction and resistance of the air are absent.

²⁾ Obviously, Stevin should here have said: but if from equal apparent weights there are subtracted equal weights, the remainders will have equal apparent weight. This again is not generally true, but it does hold in the case here considered.

³⁾ Here, the so-called law of the inclined plane is finally reached.

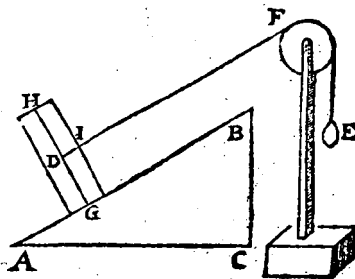
III. VERVOLGH.

LAET ons nu inde plaats van r^{e} punt F, stellen een caterol als hier neuen, also dat de scheefheslini van D naer F ewewydich blijue van A B, ende inde plaats vanden cloot E sy eenich wicht van form foot valt, maer ewewichtich anden cloot E: r^{e} selue is noch ewestaltwichtich met D, Daerom ghelijck A B tot B C, alsoo noch den cloot D tottet ghewicht E.



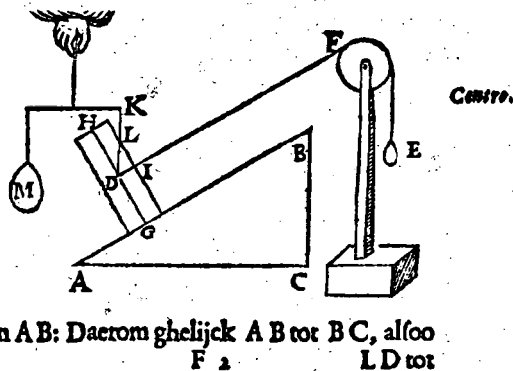
IIII. VERVOLGH.

ANGHESIEN den cloot des 3^o veruolgs naect de lini A B, in r^{e} punt G, als vastpunt, soosalden as G H rechthouckich sijn op r^{e} v. 3. DE. A B; Daerom laet ons weren den cloot, ende stellen in die plaats den pilaer D ewewichtich met den cloot, alsoo dat den as G H (diës vastpunt G) rechthouckich sy op A B, en de scheefheslini tusschen D F noch ewewydich van A B, ende sniende de sijde des pilaers in I, Als hier neuens. Ende is openbaer dat ghelijck A B tot B C, (dat is dobbel als vooren) also den pilaer D tottet rghewicht E.



V. VERVOLGH.

LAET ons trekken de hanghende lini uyt het * middelpunt des pilaers D als D K, sniende de sijde des pilaers in L, r^{e} welck soo sijnde, den driehouck L D I is ghelijck an den driehouck A B C, want de houcken A C B ende L I D sijn recht, ende L D is ewewydich van B C ende D I van A B: Daerom ghelijck A B tot B C, alsoo



F 2

LD tot

COROLLARY III.

Let us now put in the place of the point F a pulley, as shown in the annexed figure, in such a way that the oblique lifting line from D to F shall remain parallel to AB . And in the place of the sphere E let there be some arbitrary weight, but which is of equal weight to the sphere E . This weight will still be of equal apparent weight to D . Therefore, as AB is to BC , so is the sphere D to the weight E .

COROLLARY IV.

Since the sphere of the 3rd corollary touches the line AB in the point G as fixed point, the axis GH will be at right angles to AB . Therefore let us take away the sphere, and put in its place the prism D , of equal weight to the sphere, in such a way that the axis GH (its fixed point being G) shall be at right angles to AB , and the oblique lifting line between D , F still parallel to AB and meeting the side of the prism in I , as shown in the annexed figure. Then it is manifest that as AB is to BC (i.e. double of it, as above), so is the prism D to the weight E .

COROLLARY V.

Let us draw the vertical from the centre of the prism D , as DK , meeting the side of the prism in L . This being so, the triangle LDI is similar to the triangle ABC , for the angles ACB and LID are right angles, and LD is parallel to BC , and DI to AB . Therefore, as AB is to BC , so is LD to DI . But as AB is to BC ,

so is the prism to the weight E , by the 4th corollary; therefore, as LD is to DI , so is the prism to E . Let us now attach at the line KD the vertical lifting weight M of equal apparent weight to the prism. This weight M will be of equal weight to the prism, by the 14th proposition. Therefore, as LD is to DI , so is M to E ¹⁾.

COROLLARY VI.

Let us draw BN ²⁾, meeting AC produced in N ; in the same way DO , meeting LI produced, that is the side of the prism, in O , and in such a way that the angle IDO shall be equal to the angle CBN . Let us also attach at DO the oblique lifting weight P , which shall keep the prism (the weights M and E being taken away) in that position. Now since DL of the triangle DLI is homologous to BA of the triangle BAC , and DI to BC , it may thus be concluded: As BA is to BC , so is the apparent weight of BA ³⁾ to the apparent weight of BC (by the 2nd corollary). And also, as DL is to DI , so is the apparent weight of DL ⁴⁾ to the apparent weight of DI , that is M to E . But the homologous lines of these similar triangles ABN , LDO are BA to DL , and BN to DO . Therefore we say, as above: as BA is to BN , so is the apparent weight of BA to the apparent weight of BN (by the 1st corollary); and also, as DL is to DO , so is the apparent weight of DL to the apparent weight of DO , that is M to P ⁵⁾. But if the line BN were drawn from B on the other side of BC , the line DO would also fall from D on the other side of DI , i.e. whereas DO now falls below DI , it would then fall above it, and the foregoing proof would also be true of such a situation, to wit that we should still say: as BA is to BN , so is the apparent weight of BA to the apparent weight of BN , and as DL is to DO , so is the apparent weight of DL to the apparent weight of DO , that is M to P . Therefore this proportion is true not only in the examples where the lifting line DI is at right angles to the axis, but also with any angle.

The above may also be understood of a sphere lying on a line AB , as shown in the annexed figure, in which case we say, as before, as LD is to DO , so is M to P (to wit: if CL is drawn at right angles to AB , i.e. parallel to the axis

¹⁾ As it is irrelevant that the body on the inclined plane is a prism, the following theorem may be considered to have been proved. If the vector DL is opposite to the weight of a body lying on the inclined plane AB , and LI is perpendicular to AB , DI represents the force along the plane which keeps the body in equilibrium. We may also say: DL is the resultant of the forces DI and IL (normal reaction). This is the triangle of forces for the special case that the two components are at right angles to one another.

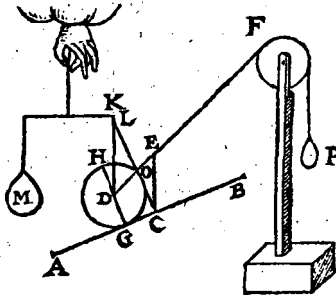
²⁾ Stevin does not specify the direction of BN ; probably it is intended to be at right angles to AB in accordance with the fact that the line of action of P is horizontal. However, this special position is irrelevant.

³⁾ Evidently Stevin here takes the word *staltwicht* in quite a different sense from that used above, viz. proper weight of the prism on AB . Likewise apparent weight of BC now means the proper weight of a body on BC balancing the prism; Girard (XIII 449a) therefore has the correct expression: le poids sur BA , le poids sur BC .

⁴⁾ Apparent weight of DL here means the force acting along DL , i.e. the weight of M . Likewise: apparent weight of $DI = E$. Girard (XIII 449a) translates: le poids appartenant à DL , à DI .

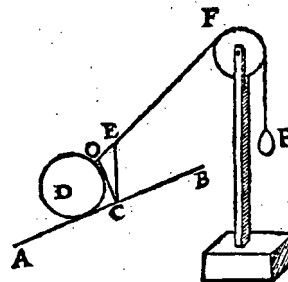
⁵⁾ This argumentation gives the proper weight of a body P balancing the prism on AB , if it were connected with it by a string, which is first parallel to AB and then, after having passed over a pulley, parallel to BN . It is not at all clear how Stevin is able to make any inference about the force P considered by him, the line of action of which is arbitrary. It is true that his result is correct, the force $DO (= P)$ and the normal reaction OL having DL as resultant, but the object of the proposition was to prove the triangle of forces for the case of forces not mutually perpendicular, and not to apply it.

rechthouklich ghetrocken is op AB, dat is ewewydich met den as GH des cloots D) maer 'ghewicht M is euen an den clood D, daerom segghen wy ghelijck LD tot DO, also 'ghewicht des cloots, tot P. Maer want LD ende DO binnen 't lichaem des cloots metter daet niet bequaemelick en connen beschreuen worden, so laet ons trecken de hanghende CE, ende sullen dan hebben buyten 't lichaem een driehouck CEO, ghelijck anden driehouck LDO, welker *lijckstandighe sijden sijn LD met CE, ende DO met EO, daerom ghelijck LD tot DO, alsoo CE tot EO, ende veruolghens ghelijck CE tot EO, alsoo 'ghewicht des cloots, tot P.

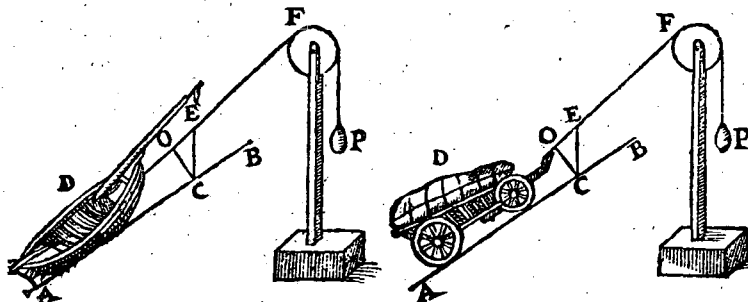


Homologa

LAET onsnut tot meerder clareheydt dit alleen stellen sonder d'ander linien als hier neuen, alwaer wy segghen ghelijck CE tot EO, also 'ghewicht des cloots D tot P.



ENDE dit niet alleen van clooten maer van ander lichamen sliende, ofte rollende, op puntē ofte linien als hier onder (daerwy eyghentlicker



af handelen sullen inde Weegdaet) alwaer wy noch segghen ghelijck CE to EO, also 'ghewicht des lichaems D totter ghewicht P.

F 3

WAER

GH of the sphere D). But the weight M is equal to the sphere D , therefore we say: as LD is to DO , so is the weight of the sphere to P . But because LD and DO cannot easily be drawn in practice inside the body of the sphere, let us draw the vertical CE ; we shall then have outside the said body a triangle CEO similar to the triangle LDO , the homologous sides of which triangles are LD to CE , and DO to EO . Therefore, as LD is to DO , so is CE to EO ; and consequently: as CE is to EO , so is the weight of the sphere to P .

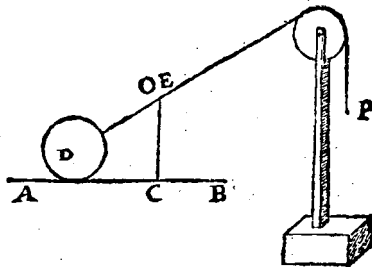
Now for the sake of greater clearness let us put this separately, without the other lines, as shown in the annexed figure, where we say: as CE is to EO , so is the weight of the sphere D to P .

And this is true not only of spheres, but also of other solids sliding or rolling on points or lines, as shown below (a subject with which we will deal more properly in the *Practice of Weighing* ¹⁾), where we still say: as CE is to EO , so is the weight of the solid D to the weight P .

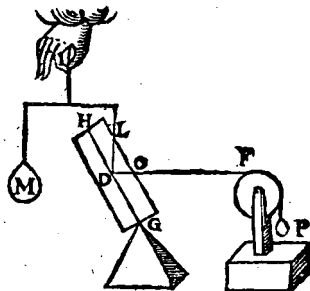
¹⁾ See *The Practice of Weighing*, Prop. 9, Examples 3 and 4.

*Horizonte.**Mathemati-
cè.*

WAER uyt oock blijktt, dat wesende de lini A B euewydich vanden *sichteinder als hier neuens, dat C E ende C O dan in een selfde lini sullen vallen, waer duer tusschen E en O gheen langde en sal sijn, ende veruolghens C E en sal tot E O gheen reden hebben, daermen by verstaen sal dat een swaerheydt inde plaets van P hoe cleen sy mocht wesen, en sal niet euefaltwichtich connen sijn teghen t'lichtaem D, maer salt (*wisconstelick verstaende) voorttrecken hoe swaer het sy: Waer uyt volght, dat alle swaerheden voortghetrocken langs den sichteinder, als schepen int water, waghens langs t'platte landt, &c. en behouuen gheen vlieghesterckens macht tot haer verroersel, meer dan de omstaende verhindernissen en veroirsaecken, als Water, Locht, Naecfel der assen, teghen de buffen, naecfel der rayers teghen de straet, ende dierghelijcke.

*Proportionē.*

MAER anghesien den driehouck A B N int 6^e veruolg, tot dese *eueredenheydt niet en gheeft noch en neemt, laet ons hem weeren, ansiende G voor vastpunt des pilaers rustede op een pin als hier neuens, ende sullen noch segghen ghelijck L D tot D O, also M tot P.



VII. VERVOLGH.

*Proportionē.**Perpendicu-
laris.*

MAER op dat nu blijcke dese *eueredenheydt niet alleen also te bestaen inde pilaren alwaer de rechtheflini als D L, comt uyt t'middelpunt des pilaers, ende diens vastpunt is des assens uysterste, als hier vooren G int 6^e vervolg; Soo laet A B C een driehouck sijn, wiens sijde A B dobbel is an B C, ende B C sy *hanghende op A C: Ende laet D E een pilaer sijn diens as F G rechthouckich op A B. ende sniende A B in t'punt H, ende I sy eenich ander punt inden seluen as;
Lact

From the above it also appears that, the line AB being parallel to the horizon, as in the annexed figure, CE and CO will fall on the same line, so that there will be no distance between E and O ; consequently CE will not have any ratio to EO , by which it is to be understood that a gravity taking the place of P , however small it may be, cannot be of equal apparent weight to the solid D , but will pull it along (mathematically speaking), however heavy it may be. From this it follows that all gravities pulled along parallel to the horizon, such as ships in the water, wagons along the level land, etc., to be moved do not require the force of a fly beyond that which is caused by the surrounding obstacles, viz. water, air, contact of the axles with the bearings, contact of the wheels with the road, and the like.

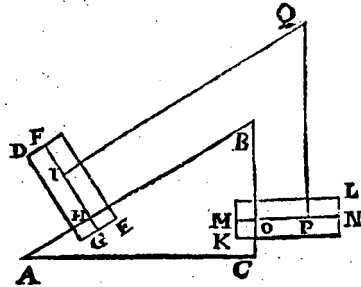
But since the triangle ABN in the 6th corollary is irrelevant to this proportion, let us take it away ¹⁾, considering G as the fixed point of the prism resting on a peg, as in the annexed figure; then we shall still say: as LD is to DO , so is M to P .

COROLLARY VII.

But in order that it may be evident that this proportion is true not only of prisms where the vertical lifting line, as DL , starts from the centre of the prism, while the fixed point of the latter is the extremity of the axis, as G above, in the 6th corollary: let ABC be a triangle, whose side AB shall be double of BC , and let BC be vertical to AC . And let DE be a prism whose axis FG shall be at right angles to AB and meet AB in the point H , and let I be some other point in this

¹⁾ The inclined plane is now dismissed; instead of resting on it in G , the body is supposed to have a fixed point at G .

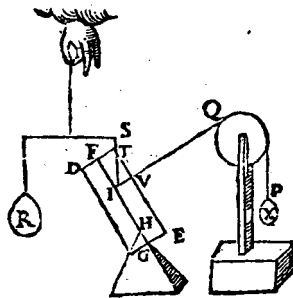
Laet oock KL een ander pilaer sijn, euen ende ghelijck anden pilaer DE, wiens as MN, ende O een punt des as naeckende BC, ende van ghelijcke gestalt in sijn pilaer, als H inden pilaer DE; Laet oock P een ander punt sijn van sulcker ghestalt inden pilaer KL, als I inden pilaer DE; Ende laet Q een vastpunt sijn daer ouer de lini



IQP slieren mach, alsoo dat de lini IQ ^{*euewydich} sy van AB, ende *Parabola.* QP ^{euewydich} van BC. Ende om de redenen die int 19^e voorstel vande 14 clooten verclaert sijn (twelck wy hier deur soodanighe veel slierende pilaren oock souden connen bewysen, maer want sulcx uyt r'voorgaende kennelick is, wy slaent ouer) het statwicht des pilaers KL, sal dobbel sijn an t'staltwicht des pilaers DE.

VIII. VERVOLGH.

LAET ons nu an I des 7^e veruolgs voughen trechthefwicht R euestatwichtich met den pilaer, diens rechtheflini sy IS, sniende de sijde des pilaers in T, ende IQ (nie de sijde des pilaers in V, ende laet an de lini PQ hanghen een ghewicht X, inde plaats vanden pilaer KL, twelck euen sy anden helft van t'staltwicht des selfden pilaers KL. Laet ons oock weeren den driehouck ABC, ende den pilaer DE doen rusten op t'punt H als hier neuen. Ende om de redenen als vooren, ghelijck TI tot IV, alsoo R tot X. Ende dit niet alleen als IV rechthouckich is op dē as FG, maer cromhouckich soot valt, waeramen besondet bethoogh soude mueghen doen, maer is openbaer ghenouch door het 6^e veruolgh.



IX. VERVOLGH.

Wy hebben int 8^e veruolgh dese ^{*eueredenheyt} verclaert, alwaer *Proportion8.* t'roerende punt I, hoogher was dan t'vastpunt H, ende alwaer de scheefhefline IV helde naer de sijde des vastpunts H; Wy moeten nu betooghen

same axis. Let LK be another prism, equal and similar to the prism DE , and its axis MN , and O a point of the axis touching BC and similarly placed in its prism to H in the prism DE . Let P also be another point, similarly placed in the prism KL to I in the prism DE . And let Q be a fixed point, over which the line IQP can slide, in such a way that the line IQ shall be parallel to AB , and QP parallel to BC . Then, for the reasons explained in the 19th proposition with regard to the 14 spheres (which we might also prove here by means of an equal number of sliding prisms, but we omit this because it is evident from what precedes), the apparent weight of the prism KL will be double of the apparent weight of the prism DE .

COROLLARY VIII.

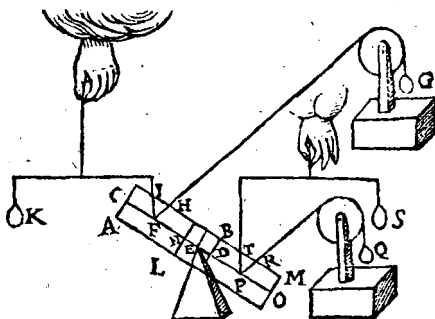
Let us now attach at I of the 7th corollary the vertical lifting weight R having equal apparent weight to the prism, whose vertical lifting line shall be IS , meeting the side of the prism in T , and let IQ meet the side of the prism in V . And let there hang from the line PQ a weight X , instead of the prism KL , which shall be equal to half the apparent weight of this same prism KL . Let us also take away the triangle ABC , and cause the prism DE to rest on the point H , as shown in the annexed figure. Then, for the same reasons as above, as TI is to IV , so is R to X . And this is true not only when IV is at right angles to the axis FG , but also when it is at any oblique angle thereto, which might be proved specifically; but it is sufficiently manifest from the 6th corollary.

COROLLARY IX.

We have explained this proportion in the 8th corollary, where the movable point I was higher than the fixed point H , and where the oblique lifting line IV

berhooghen de selue cueredenheyt oock so te bestaen in d'ander ghestalten, ende eerst alwaer troerende punt leegher sy dan t'vastpunt, ende alwaer de scheefheflini afwyct vande sijde des vastpunts in deser voughen:

Laet A B een pilaer sijn, diens as C D, ende vastpunt E, ende t'verroerlick punt F, ende t'scheefheflini F H, ende F I sy rechtheffini, diens rechtheffwicht K. Laet L M oock een pilaer sijn, enen ende ghelijck an den pilaer A B, wiens as sy N O, ende vastpunt E, ende verroerlick punt P, also dat E N enen sy an E D, ende E F an E P, ende t'scheefheflini Q sy enen an G, ende sijn scheefheflini sy P R, *euewichtich van F H, ende rechtheffwicht S sy enen an K, ende sijn rechtheffini sy P T. Dit soo sijnde laet ons vergaren de twee pilaren A B ende L M, ansiende A M voor een heel pilaer, wiens *swaerheys middelpunt ende vastpunt sal E sijn door t'ghestelde. Laet ons nu weeren de ghewichten K, G, S, Q, ende den pilaer A M sal op E alle ghestalt houden diemen hem gheeft door het 7^e voorstel, hy sal dan soo bliuen, ende den pilaer A B sal alsoo euewichtich bliuen teghen den pilaer L M. Laet ons nu de ghewichten Q G weder andoen, hanghende euewichtighe van ghelijcke ghestalt, an euewichtighe, ende door het 1³^e voorstel, Q sal anden pilaer A M enen sulcken macht doen als G: Ende veruolghens Q doet sulcken macht an huer pilaer L M, als G an huer pilaer A B; maer de macht van G is A B in die ghestalt te houden door het 6^e vervolg, de macht dan van Q is oock L M in die ghestalt te houden. Inghelijcx soo is oock de macht van K, den pilaer A B in die ghestalt te houden, daerom oock is de macht van S den pilaer L M in die ghestalt te houden; Nu ghelijck I F tot F H, also K tot G door het 8^e veruolg, Maer T P. is enen an I F, ende P R an F H, ende S an K, ende Q an G, ghelijck dan T P tot P R, alsoo S tot Q. Dese cueredenheydt dan, als wy gheseyt hebben, is so wel inde voorbeelden alwaer t'roerende punt P leegher is dan t'vastpunt E, ende alwaer de scheefheflini P R afwyct vande sijde des vastpunts E, als daert hoogher is, ende daer de scheefheflini helde naer t'vastpunt.



Parallela.

*Centrum
gravitatis.*

verged towards the side of the fixed point H . We now have to prove that this proportion is also true in other situations, in the first place when the movable point is lower than the fixed point, and when the oblique lifting line verges away from the side of the fixed point, as follows:

Let AB be a prism, its axis CD , and the fixed point E and the movable point F . And let the oblique lifting weight keeping it in that position be G , its oblique lifting line FH ; and let FI be the vertical lifting line, and its vertical lifting weight K . Let LM also be another prism, equal and similar to the prism AB , whose axis shall be NO , the fixed point E and the movable point P , in such a way that EN shall be equal to ED , and EF to EP , and that the oblique lifting weight Q shall be equal to G , and its oblique lifting line shall be PR , parallel to FH , while the vertical lifting weight S shall be equal to K , and its vertical lifting line shall be PT . This being so, let us combine the two prisms AB and LM , considering AM as a complete prism, whose centre of gravity and fixed point shall be E , by the supposition. Let us now take away the weights K , G , S , Q ; then the prism AM will remain at rest on E in any position given to it, by the 7th proposition. It will therefore remain thus, and the prism AB shall in this way remain of equal weight ¹⁾ to the prism LM . Let us now attach the weights Q , G again, hanging equal weights of the same appearance at equal weights ²⁾; then by the 13th proposition ³⁾ Q will exert the same force on the prism AM as G . And consequently Q exerts on its prism LM the same force as G on its prism AB . But the power of G is to keep AB in that position, by the 6th corollary; the power of Q therefore is also to keep LM in that position. In the same way the power of K is also to keep the prism AB in that position, and therefore also the power of S is to keep the prism LM in that position. Now as IF is to FH , so is K to G , by the 8th corollary. But TP is equal to IF , and PR to FH , and S to K , and Q to G ; therefore, as TP is to PR , so is S to Q . As we have said, therefore, this proportion is true in the examples where the movable point P is lower than the fixed point E and where the oblique lifting line PR verges away from the side of the fixed point E , as well as in the examples where the said movable point is higher and where the oblique lifting line verged towards the fixed point.

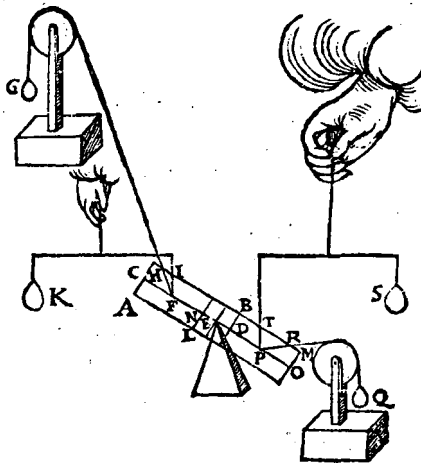
¹⁾ It is not relevant that AB and LM are of equal weight, but that they balance one another; they are of equal apparent weight.

²⁾ The meaning of the words „of the same appearance” can be no other than that the lines of action of the forces G and Q make equal angles with CO , and that the points of application are equidistant from E . However, there is no postulate or proposition relating to this case.

³⁾ It is not clear that this proposition can be applied here. Accordingly the subsequent argumentation is far from being convincing.

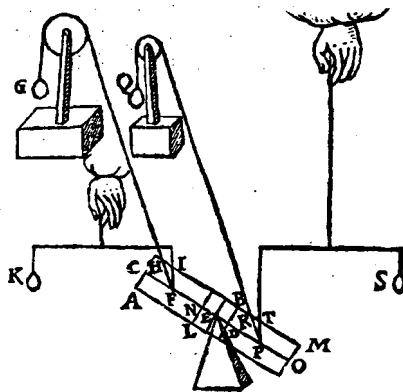
X. VERVOLGH.

LAET ons stellen een form ghelijck an die des 9 veruolghs, alleen daer in verschillende dat dese FH wyct ouer d'ander sijde van FI, ende dat den houck HFC, euen sy anden houck RPO, waer duer G anden pilaer AM euen soo grooten ghewelt doet als Q, ende om de redenen des 9 veruolghs (die wy om cortheyt ouerslaen) G doet euen sulcken ghewelt anden pilaer AB, als Q anden pilaer LM; Nu ghelijck TP tot PR, alsoo S tot Q door het 9^e veruolgh, maer IF is euen an TP, ende FH an PR, ende K an S, ende G an Q, daerom ghelijck IF tot FH, also K tot G.



XI VERVOLGH.

LAET ons stellen een form ghelijck an die des 10 veruolghs, alleen daer in verschillende dat dese PR wyct ouer d'ander sijde van PT, ende dat PR euewydich sy met FH, waer duer Q anden pilaer AM, euen soo grooten ghewelt doet als G, ende om de redenen des 9^e veruolghs, Q doet euen sulcken gheweldt anden pilaer LM, als G anden pilaer AB; Nu ghelijck IF tot FH, also K tot G door het 6 veruolgh: Maer TP is euen an IF, ende PR an FH, ende S an K, ende Q an G, daerom ghelijck TP tot PR, also S tot Q. Ende inder seluer voughen salmen vanden anderen ghestalten door haer contrarien altyt dese eueredenheyt bewyfen.



COROLLARY X.

Let us take the same figure as that of the 9th corollary, with the only difference that in this case FH verges towards the other side of FI , and let the angle HFC be equal to the angle RPO , in consequence of which G exerts on the prism AM the same force as Q ; then, for the reasons mentioned in the 9th corollary (which we omit for brevity's sake), G exerts on the prism AB the same force as Q on the prism LM . Now as TP is to PR , so is S to Q , by the 9th corollary. But IF is equal to TP , and FH to PR , and K to S , and G to Q ; therefore, as IF is to FH , so is K to G .

COROLLARY XI.

Let us take the same figure as that of the 10th corollary, with the only difference that in this case PR verges towards the other side of PT , and let PR be parallel to FH , in consequence of which Q exerts on the prism AM the same force as G ; then, for the reasons mentioned in the 9th corollary, Q exerts on the prism LM the same force as G on the prism AB . Now, as IF is to FH , so is K to G , by the 6th corollary. But TP is equal to IF , and PR to FH , and S to K , and Q to G ; therefore, as TP is to PR , so is S to Q . And in the same way this proportion can always be proved to be true of the other positions, through their contraries ¹⁾.

¹⁾ The meaning of this phrase seems to be: by taking the line of action of one of the oblique lifting weights on the other side of the vertical.

Propertio.
Horizonte.

Parallela.

MAER dat dese * cueredenheydt oock bestaer inde ghestalt daer den as euewydich is vanden * sichteinder, wort aldus bethoont:

Laet A B een pilaer sijn, diens as C D * euewydich sy vanden sichteinder, ende t'vastpunt daer in E, ende t'roerlick punt F, ende G t'scheefhefwicht dat den pilaer in die ghestalt houdt, wiens scheefheslini F H, ende I trechthefwicht dat den pilaer oock in die ghestalt houdt, wiens rechtheslini F K; T welck soo sijnde, Laet K F tot F H een ander reden hebben (soot muelghelick waer) dan I tot G. By voorbeelt K F sy tot F H, als 1 tot 2, maer I tot G, als 3 tot 7. Dit so ghenomen, laet ons den pilaer der eerste form neerdruwen, ofte der tweeder form oplichten, tot dat K F sulcken reden hebbe tot F H, als 3 tot 7, ende aldan sal G oock euefaltwichtich sijn teghen den pilaer door de voorgaende vervolghen; Inder voughen dat den pilaer hoogher ende leegher verheuen, sal teghen G euefaltwichtich bliuen, t welck openbaer onmuelghelick is, als oock * wisconstlick sal blijcken door t'volghende 22 voorstel. K F dan en heeft tot F H gheen ander reden dan I tot G.

Mathemati-
ca.

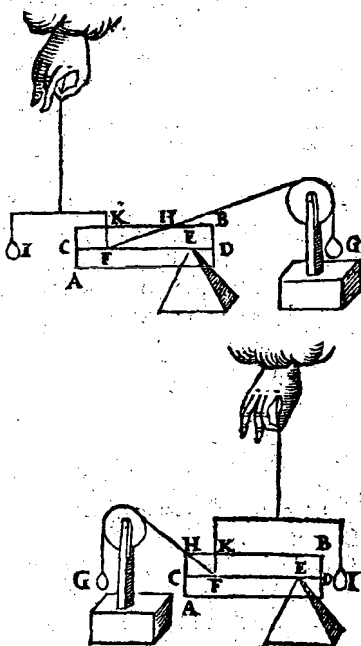
U Y T, dese voorgaende beschriuen wy een vertooch soodanich.

XII. VERTOCH.

XX. VOORSTEL.

WESENDE inden as des pilaers een vastpunt, ende een roerlick, daer an hy door een rechthefwicht ende scheefhefwicht in seker standt ghehouden wort: Ghelijck rechtheslini tot scheefheslini, also rechthefwicht tot scheefhefwicht.

T'GHEGHEVEN. Laet A B een pilaer sijn diens as C D, ende t'vastpunt



COROLLARY XII.

Now it is proved as follows that this proportion is also true of the position where the axis is parallel to the horizon. Let AB be a prism, whose axis CD shall be parallel to the horizon, and let the fixed point therein be E , and the movable point F , and G the oblique lifting weight keeping the prism in that position, whose oblique lifting line shall be FH , and I the vertical lifting weight also keeping the prism in that position, whose vertical lifting line shall be FK . This being so, let KF have to FH a different ratio (if this were possible) from I to G , for example let KF be to FH as 1 to 2, but I to G as 3 to 7. On this assumption let us push down the prism of the first figure or lift that of the second figure until KF have to FH the ratio of 3 to 7; then G will also be of equal apparent weight to the prism, by the preceding corollaries. Therefore, the prism, lifted or lowered, will remain of equal apparent weight to G , which is manifestly impossible, as will also be proved mathematically by the 22nd proposition hereinafter. The ratio between KF and FH therefore is not different from that between I and G .

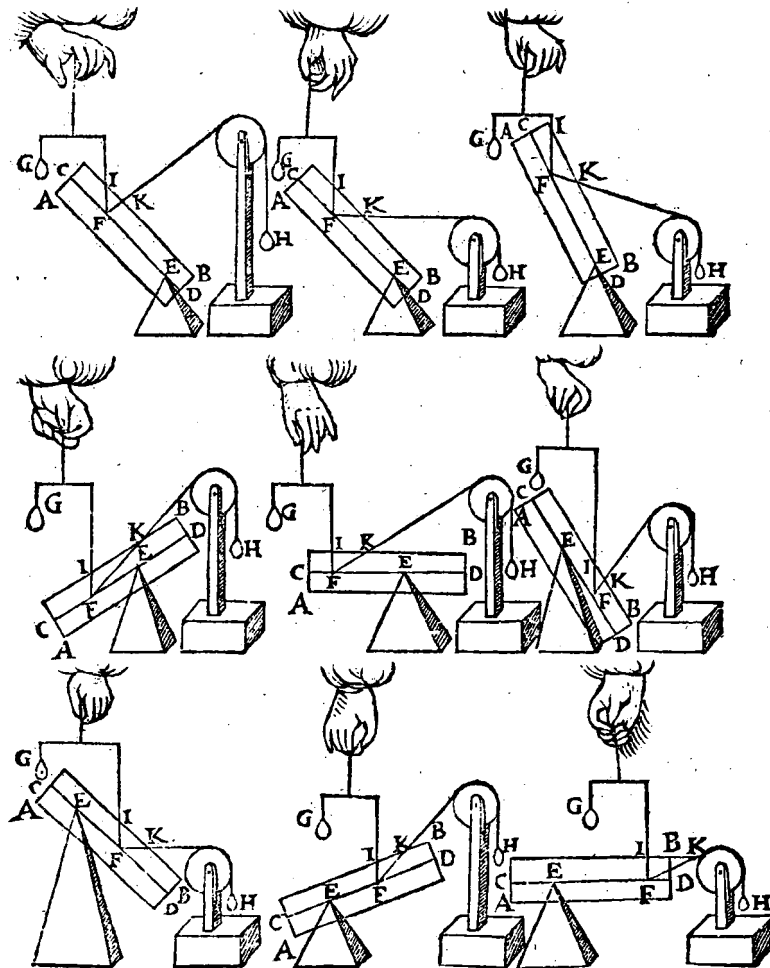
From the preceding we derive the following theorem.

THEOREM XII.

PROPOSITION XX.

If there is in the axis of the prism a fixed point and a movable point, at which it is kept in a certain position by a vertical lifting weight and an oblique lifting weight: as the vertical lifting line is to the oblique lifting line, so is the vertical lifting weight to the oblique lifting weight.

punt E, ende roerlick punt F, daeran den pilaer door trechthefwicht G in die ghestalt ghehouden wort, daer an oock den pilaer door t'scheefhefwicht H (welverstaende G gheweert sijnde) in die ghestalt ghehouden wort, ende de rechtheflini snie de sijde des pilaers in I, maer de scheefhefline de selue sijde in K: Ick seg dat ghelijck de rechtheflini I F, tot de scheefhefline F K, alsoo trechthefwicht G, tot het scheefhefwicht H. waer af t'bewys uyt de voorgaende openbaer is.



G 2

TRECHT

SUPPOSITION. Let AB be a prism, its axis CD , the fixed point E , and the movable point F , at which the prism is kept in that position by the vertical lifting weight G , at which the prism is also kept in that position by the oblique lifting weight H (to wit: G being taken away). And let the vertical lifting line meet the side of the prism in I , and let the oblique lifting line meet this same side in K . I say that as the vertical lifting line IF is to the oblique lifting line FK , so is the vertical lifting weight G to the oblique lifting weight H , the proof of which is

BESLUYT. Wefende dan inden as des pilaers een vastpunt, &c.

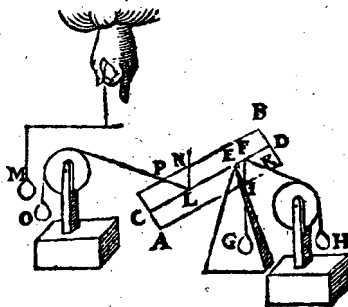
MERCKT. Soo eenighe der linien als IF, FK, de sijde des pilaers niet en sneen, men sal die sijde voorder trecken tot dat sy ghesneen wort, als inde voorgaende laetste form.

XIII. VERTOOGH.

xxi. VOORSTEL.

W E S E N D E inden as des pilaers een vastpunt, ende een roerlick, daer an hy door een rechtdaelwicht en scheefdaelwicht in seker standt gehouden wort: Ghelijck rechtdaellini tot scheefdaellini, also rechtdaelwicht tot scheefdaelwicht.

TGH EGHEVEN. Laet A B een pilaer sijn, diens as C D, ende vast-
punt E, ende roerlick punt F, daer an den pilaer door trechtdaelwicht G
in die gheftalt ghehouden wort, daer an oock den pilaer door t'scheef-
daelwicht H (welverstaende G gheweert sijnde) in die gheftalt ghehou-
den wordt, ende de rechtdaellini sijn de sijde des pilaers in I, maer de
scheefdaellini sijn de selue sijde in K. **T**BE GHEERDE. Wy moe-
ten bewyfen dat ghelijck de rechtdaellini I F tot de scheefdaellini F K,
alsoo t'rechtdaelwicht G tot het scheefdaelwicht H. **T**BEREYSEL.
Laet ons teeken en t'punt L, alsoo dat E L euen sy an E F, ende vou-
ghen an t'punt L t'rechthefwicht M, dat den pilaer in die gheftalt can
houden, diens rechthefsi-
ni L N: In ghelijcx t'scheefhef-
wicht O, dat den pilaer oock
in die gheftalt can houden,
wiens scheefhesini L P eue-
wydich sy met F K.



TBEWYS. Ghelijck N L tot L P, alsof M tot O, duer het 20^e voorstel maer de macht van G is anden pilaer euen met de macht van M, en de macht van H met die van O duer het 13^e voorstel, ende IF is euen an LN, ende FK, an L P; Daerom ghelijck de rechtdaellini IF tot de scheefdaellini F K, alsof rechedaelwicht G totter scheefdaelwicht H, Sghelijcx sal oock rbewys sijn van alle d'ander ghestalten als inde formen hier na volghende.

TRESLVYT

manifest from what precedes. **CONCLUSION.** If therefore there is in the axis of the prism a fixed point, etc.

NOTE.

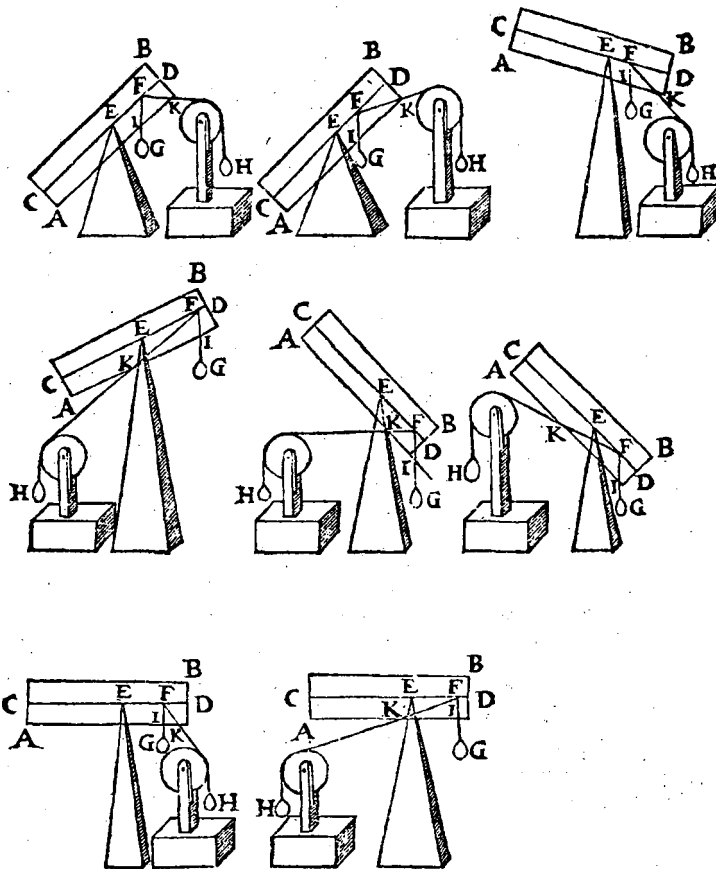
If any of the lines, as IF , FK , should not meet the side of the prism, this side shall be produced until such lines do meet it, as in the last of the preceding figures.

THEOREM XIII.

PROPOSITION XXI.

If there is in the axis of the prism a fixed point and a movable point, at which it is kept in a certain position by a vertical lowering weight and an oblique lowering weight: as the vertical lowering line is to the oblique lowering line, so is the vertical lowering weight to the oblique lowering weight.

SUPPOSITION. Let AB be a prism, its axis CD , and the fixed point E and the movable point F , at which the prism is kept in that position by the vertical lowering weight G , at which the prism is also kept in that position by the oblique lowering weight H (to wit: G being taken away). And let the vertical lowering line meet the side of the prism in I , and let the oblique lowering line meet this same side in K . **WHAT IS REQUIRED TO PROVE.** We have to prove that as the vertical lowering line IF is to the oblique lowering line FK , so is the vertical lowering weight G to the oblique lowering weight H . **PRELIMINARY.** Let us mark the point L in such a way that EL shall be equal to EF , and let us attach at the point L the vertical lifting weight M , which can keep the prism in that position, whose vertical lifting line shall be LN . In the same way the oblique lifting weight O , which can also keep the prism in that position, whose oblique lifting line LP shall be parallel to FK . **PROOF.** As NL is to LP , so is M to O , by the 20th proposition, but the power of G on the prism is equal to the power of M , and the power of H to that of O , by the 13th proposition; and IF is equal to LN , and FK to LP . Therefore, as the vertical lowering line IF is to the oblique lowering line FK , so is the vertical lowering weight G to the oblique lowering weight H . A similar proof can be given with regard to all the other positions, as in the following figures.



T'besluyt. Wesende dan inden as des pilaers een vastpunt ende een roerlick, &c.

IX. EYSCH.

XXII. VOORSTEL.

W E S E N D E ghegheuen een bekenden pilaer, met een vastpunt inden as, ende een roerlick punt, an t'welck eenich onbekent treckwicht den pilaer in ghegheuen ghestalt houdt: Dat treckwicht bekent te maken.

G 3

T G H E

CONCLUSION. If therefore there is in the axis of the prism a fixed point and a movable point, etc.

PROBLEM IX.

PROPOSITION XXII.

Given a known prism, with a fixed point in the axis and a movable point at which an unknown drawing weight keeps the prism in the given position: to make known the said drawing weight.

T'GHEGHEVEN. Laet $A B C D$ een pilacr zijn weghende 6 lb, ende ghedeelt als int 1^o voorstel, ende t'vastpunt sy X , ende het roerende punt S , an t'welck gheuoecht sy een onbekent scheefhewicht Y , met den pilacr euefstaltwichtich, ende sijn scheefhesfijn (nie de sijde des pilacers $A B$ in OE . **T'BEGHEERDE.** Wy moeten dat onbekende scheefhewicht Y bekent maken.

TWERCK. Men sal sien wat rechthefwicht an S den pilae in die ghes-
talt soude houden, wort beuonden
door 14^e voorstel, van 4 lb, daer naer
salme onderfoucken wat reden eeni-

*Perpendicu-
laris.*

ghe * hangende lini als Z Æ, heeft tot
Z OE, ick neme als van z tot i, daer
uyt seg ick z gheeft i, wat t'rechthef-
wicht van 4 lb? comt voor Y z lb,
t'welck ick seg sijn waer ghewont
te sijne. TBEREYSEL. Laet ons
trecken de hanghende door S welcke
sy A S. TBEWYS. Ghelijck A S
tot S OE, also t'rechthefwicht tottet
scheefthefwicht Y door het zo^e voor-
stel, maer den driehouck OE Z B, is
ghelijck anden driehouck OE S A,
welcker * lijckstandighe linien sijn
OE Z mer OE S, ende Z Æ met S A:

Homologs

Daerom ghelijck A S tot S OE, alsoo Æ Z tot Z OE, ende vervolgens ghelijck Æ Z 1, tot Z OE 1, alsoo rechtthefwicht 4 lb tot Y, daerom Y weghende 2 lb is bekennt ghemaect, 'twelck wy bewysen moesten. Ende ghelijcx sal den voortganck sijn in allen anderen voorbeelden.

T B E S L V Y T. Wefende dan ghegheuen een bekenden pilaer met een vastpunt inden as, &c.

I^c M E R C K.

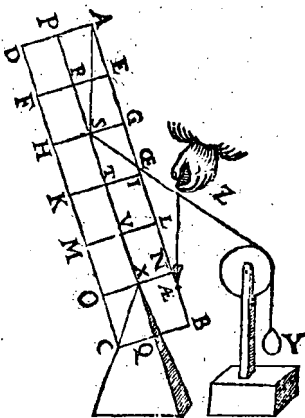
Wy fouden inde wercking hebben mueghen segghen, AS 2, gheeft S CE 1, wat rechtefwich 4 IB? comt voor T 2 IB, maer op dat sy lijckformigher soude fin an t'ghene inde daet gheschiet (want men can binnen int lichaem qualick de linien AS, S OE trecken) wy hebben de hanghende lini Z AE int voorbeeldt uytwendich ghenomen.

II^c · M E R C K.

Inversam &
alternā pro-
portionem.
Terminorum

*TIJ openbaer door de * verkeerde ende overanderde Eueredenbeyds, hoe dat elck van d'ander onbekende* palen als Rechthefwicht, Rechthefini, Scheefhefini, Pilaer, door drie bekende palen alijds bekent sullen worden, welcker beschrijving Wy om de cortheyt achterlaten.*

XIII. VER-



SUPPOSITION. Let $ABCD$ be a prism weighing 6 lbs and divided as in the 1st proposition, and let the fixed point be X and the movable point S , at which let there be attached an unknown oblique lifting weight Y , of equal apparent weight to the prism, and let its oblique lifting line meet the side of the prism AB in $\mathcal{C}\mathcal{E}$. WHAT IS REQUIRED TO MAKE KNOWN. We have to make known that unknown oblique lifting weight Y . CONSTRUCTION. It shall be ascertained what vertical lifting weight at S would keep the prism in that position. This is found, by the 14th proposition, to be 4 lbs. Then it shall be ascertained what ratio a vertical line, as $Z\mathcal{A}\mathcal{E}$, has to $Z\mathcal{C}\mathcal{E}$. I take this to be 2 to 1, from which I conclude: 2 gives 1; what the vertical lifting weight of 4 lbs? Y becomes 2 lbs, which I say is its true weight. PRELIMINARY. Let us draw the vertical through S , which shall be AS . PROOF. As AS is to $S\mathcal{C}\mathcal{E}$, so is the vertical lifting weight to the oblique lifting weight Y , by the 20th proposition. But the triangle $\mathcal{C}\mathcal{E}ZB$ is similar to the triangle $\mathcal{C}\mathcal{E}SA$, whose homologous lines are $\mathcal{C}\mathcal{E}Z$ to $\mathcal{C}\mathcal{E}S$ and $Z\mathcal{A}\mathcal{E}$ to SA . Therefore, as AS is to $S\mathcal{C}\mathcal{E}$, so is $\mathcal{A}\mathcal{E}Z$ to $Z\mathcal{C}\mathcal{E}$, and consequently as $\mathcal{A}\mathcal{E}Z$ (2) is to $Z\mathcal{C}\mathcal{E}$ (1), so is the vertical lifting weight (4 lbs) to Y . Therefore Y weighing 2 lbs has been made known, which we had to prove. And the procedure will be the same in all the other examples. CONCLUSION. Given therefore a known prism with a fixed point in the axis, etc.

NOTE I.

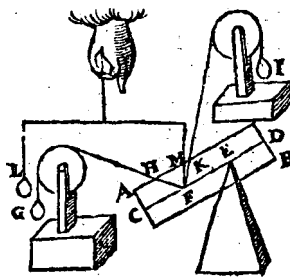
In the construction we might have said: AS 2 gives $S\mathcal{C}\mathcal{E}$ 1; what the vertical lifting weight of 4 lbs? Y becomes 2 lbs. But in order that it might be more in agreement with what happens in practice (for it is hardly possible to draw the lines AS , $S\mathcal{C}\mathcal{E}$ inside the solid), we have taken the vertical $Z\mathcal{A}\mathcal{E}$ in the example outside the solid.

NOTE II.

It is manifest by taking the terms inversely and alternately that each of the other unknown terms, as the vertical lifting weight, the vertical lifting line, the oblique lifting line, and the prism, can always be made known from three known terms, the discussion of which we omit for brevity's sake.

EVEN ghewichten der trecklinien van een selfde punt des as, ende op verscheyden sijden met den as euen houcken makende, doen anden pilaer euen ghewelden.

T G H E. Laet A B een pilaer sijn diens as C D, ende vastpunt daer in E, en t'roerlick punt F, an t'welck een scheefhefwicht G sy, dat den pilaer in die ghestalt houde, ende diens scheefhefline F H. Laet oock an t'selue punt F gheuocht wesen een scheefhefwicht I, ouer d'ander sijde, ende met G euenwichtich, ende diens scheefhefline F K, den houck K F D euen make anden houck H F C.



T B E G H E E R D E. Wy moeten bewysen dat I anden pilaer euen sulcken ghewelt doet als G, te weten dat I (G gheweert sijnde) den pilaer oock in die ghestalt sal houden.

T B E R E Y T S E L. Laet an t'punt F gheuocht worden t'rechthefwicht L dat den pilaer oock in die ghestalt can houden, ende sijn rechtheffline sy F M.

T B E W Y S. Want de linien F H, F K, sijn tusschen de * euewydighe *Parallelas*. H K, C D, ende dat den houck H F C, euen is (door t'ghegheuen) an den houck K F D, so sijn F H ende F K euen. waer uyt volght dat ghelijck M F tot F H, alsoo M F tot F K, Maer ghelijck M F tot F H, alsoo L tot G, daerom oock ghelijck M F tot F K, also L tot G; maer I is euen an G door t'ghefelde, ghelijck dan M F tot F K, alsoo L tot I. T'welck so sijnde, I houdt den pilaer in die ghestalt door her 20 voorstel. Sghelijcx sal oock t'bewijs sijn in alle anderen voorbeelden.

T B E S L V Y T. Euen ghewichten dan der trecklinien van een selfde punt des as, ende op verscheyden sijden met den as euen houcken makende, doen anden pilaer euen ghewelden, t'welck wy bewysen moesten.

A L S des ghewichts trecklini rechthouckich op den as is; Soo doetet anden pilaer ghegeuener ghestalt de grootste ghewelt.

THEOREM XIV.

PROPOSITION XXIII.

Equal weights of the drawing lines of the same point in the axis and making equal angles with the axis on different sides exert equal forces on the prism.

SUPPOSITION. Let AB be a prism, its axis CD , and the fixed point therein E and the movable point F , at which let there be an oblique lifting weight G which shall keep the prism in that position, and let its oblique lifting line be FH . Let there also be attached at this same point F an oblique lifting weight I on the other side and having equal weight to G , and let its oblique lifting line FK make the angle KFD equal to the angle HFC . WHAT IS REQUIRED TO PROVE. We have to prove that I exerts on the prism the same force as G , to wit that I (G being taken away) will also keep the prism in that position. PRELIMINARY. Let there be attached at the point F the vertical lifting weight L , which can also keep the prism in that position, and let its vertical lifting line be FM . PROOF. Because the lines FH , FK are contained between the parallel lines HK , CD , and the angle HFC is equal (by the supposition) to the angle KFD , FH and FK are equal. From this it follows that as MF is to FH , so is MF to FK . But as MF is to FH , so is L to G ; therefore also as MF is to FK , so is L to G . But I is equal to G , by the supposition; therefore as MF is to FK , so is L to I . This being so, I keeps the prism in that position by the 20th proposition. A similar proof can also be given in all the other examples. CONCLUSION. Equal weights therefore of the drawing lines of the same point in the axis and making equal angles with the axis on different sides exert equal forces on the prism, which we had to prove.

THEOREM XV.

PROPOSITION XXIV.

If the drawing line of the weight is at right angles to the axis, this weight exerts the greatest force on the prism in the given position.

SUPPOSITION. Let AB be a prism, its axis CD , and the fixed point E and the

T'GHEGHEVEN. Laet A B een pilaer sijn diens as C D, ende vastpunt E, ende toetlick punt F, waer an gheuocht is t'scheefhefwicht G, dat den pilaer in die ghestalt houdt, ende also dat sijn scheefhefline H F rechthouckich op den as C D is; Laet oock an F gheuocht worden t'scheefhefwicht I, euen an G, ende sijn scheefhefline sy K F.

T'BEGHEERDE. Wy moeten bewyfen dat G meerder ghewelt doet anden pilaer, dan I, oock gheen meerder ghewelt daer andoen en can.

T'BEREYTSSEL. Laet ons an F voughen t'rechthefwicht L dat den pilaer in die ghestalt houden can, diens rechtheffline F M. T'BEWYS.

A. *Alle befwicht dat minder reden heeft tot L, dan sijn heffline tot F M, is te licht om den pilaer in die ghestalt te houden, duer het 20^e voorstel:*

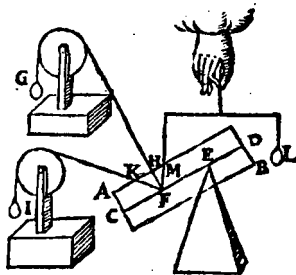
I. *I is befwicht dat minder reden heeft tot L, dan sijn heffline K F tot F M:*

I. *T'befwicht I dan is te licht om den pilaer in die ghestalt te houden.*

Syllogismi.

DES * bewysredens tweede voorstel wort aldus bethoont, t'ghewicht G (t'welck den pilaer in die ghestalt houdt) heeft sulcken reden tot L, als H F tot F M, maer I is euen an G, ende K F is meerder dan F H, daerom I heeft minder reden tot L, dan K F tot F M, waer duer soo wy bouen gheseyt hebbē, t'gewicht I is te licht om den pilaer in die ghestalt te houden; maer G cander hem in houden, G dan doet anden pilaer meerder ghewelt dan I. Maer dat G daer an gheen meerder doen en can, is daer uyt openbaer, dat van F op de sijde des pilaers gheen corter lini en can ghetrocken worden dan F H, anghesien sy daer op rechthouckich is.

T'BSLVT. Als dan des ghewichts treckline rechthouckich op den as is, soo doedert an den pilaer ghegheuer ghestalt de grootste ghewelt, t'welck wy bewyfen moesten.



VERVOLGH.

HET blijft dat hoe de houcken der trecklinien vande ghewichten, op den as den rechthouck naerder sijn, hoe de ghewichten meerder ghewelt doen; Ende ter contrarie hoe sy vanden rechthouck meer verschillen, hoe de ghewichten minder ghewelt doen.

movable point F , at which there is attached the oblique weight G , which keeps the prism in that position, in such a way that its oblique lifting line HF is at right angles to the axis CD . Let there also be attached at F the oblique lifting weight I , equal to G , and let its oblique lifting line be KF . WHAT IS REQUIRED TO PROVE. We have to prove that G exerts on the prism a greater force than does I , and cannot exert on it any greater force. PRELIMINARY. Let us attach at F the vertical lifting weight L , which can keep the prism in that position, and let its vertical lifting line be FM .

PROOF.

- A ¹⁾. Any lifting weight which has to L a ratio less than its lifting line to FM is too light to keep the prism in that position, by the 20th proposition;
 I . I is a lifting weight which has to L a ratio less than its lifting line KF to FM ;
 I . Therefore the lifting weight I is too light to keep the prism in that position.

The second proposition of the syllogism is shown as follows. The weight G (which keeps the prism in that position) has to L the same ratio as HF to FM . But I is equal to G , and KF is greater than FH ; therefore I has to L a ratio less than KF to FM , in consequence of which, as we have said above, the weight I is too light to keep the prism in that position. But G can keep it in that position, therefore G exerts on the prism a greater force than does I . But that G cannot exert on it any greater force is manifest from the fact that from F to the side of the prism no line shorter than FH can be drawn, since it is at right angles thereto. CONCLUSION. If therefore the drawing line of the weight is at right angles to the axis, this weight exerts the greatest force on the prism in the given position, which we had to prove.

COROLLARY.

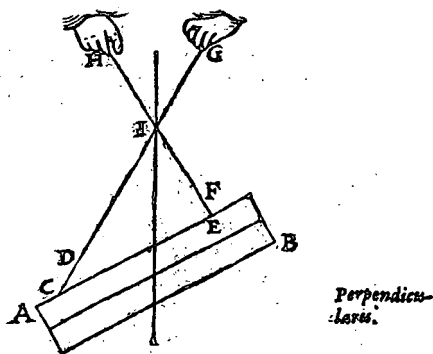
It appears that according as the angles of the drawing lines of the weights with the axis are nearer to a right angle, the weights exert greater forces. And conversely: the more they differ from a right angle, the less force the weights exert.

¹⁾ Cf. note 2) on page 143. I denotes a particular affirmative proposition. The mood of the syllogism is *Darii*.

T W E E oneuwydighe linien daer een pilaer an hangt beyde oneindelick voortghetrocken, snien malcanderen inde swaerheydts middellini des pilaers.

I^e VOORBEELT.

T G H E G H E V E N. Laet AB een pilaer sijn hanghende ande twee oneuwydighe linien CD, EF, welcke voortghetrocken sijn tot G, H, sniende malcanderen in I. **T B E G H E E R D E.** Wy moeten bewysen dattet punt I inde swaerheyts middellini is des pilaers AB. **T B E W Y S.** Den houck FEC, ofte IEC, ofte HEC, is al een selfden houck, also oock is DCE, ofte ICE, ofte GCE, daerom wat punten wy indelinien HE, ende CG voor uystersten nemen, den pilaer houdt daer an sijn ghegeuen standt. Laet ons nemen I, ghemeen uysterste punt van d'een ende d'ander lini, den pilaer dan houdt daer an sijn ghegeuen standt. Maer hanghede den pilaer an t'punt I, so is de * hanghende door I des pilaers swaerheydts middellini inde welcke I is.



II^e VOORBEELT.

T G H E G H E V E N. Laet AB een pilaer sijn hanghende ande oneuwydighe linien CD, EF, welcke voortghetrocken sijn tot G, H, sniende malcanderen in I. **T B E G H E E R D E.** Wy moeten bewijsen dattet punt I, inde swaerheydts middellini is des pilaers AB. **T B E W Y S.** Laet ons DG ende FH ansien voor stylen ofte stue linien daer den pilaer op rust, welcke door de 1^e begheerte niet en breken noch en buyghen; der seluer ghewelt is euen ande ghewelt der linien CD, EF, want ghelijck dese den pilaer in sijn ghegeuen standt houden alsoo oock die. Ende wat punten wy inde linien DG, FH voor uystersten nemen, den pilaer
 H houdt

THEOREM XVI.

PROPOSITION XXV.

Two non-parallel lines, from which a prism is hanging, both of them produced indefinitely, meet in the centre line of gravity of the prism.

EXAMPLE I.

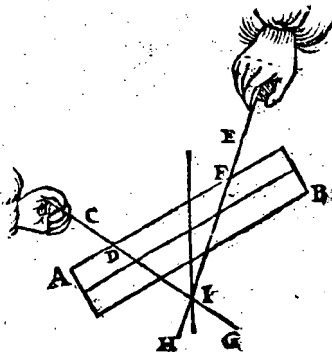
SUPPOSITION. Let AB be a prism hanging from two non-parallel lines CD , EF , which are produced to G , H , meeting in I . WHAT IS REQUIRED TO PROVE. We have to prove that the point I is in the centre line of gravity of the prism AB . PROOF. The angle FEC , or IEC , or HEC is always the same angle; similarly DCE , or ICE , or GCE . Therefore, whatever points in the lines HE and CG we take as their extremities, the prism will keep its given position thereon. Let us take I , the common extremity of both lines; the prism then keeps its given position thereon. But if the prism is hanging on the point I , the vertical through I is the centre line of gravity of the prism, in which lies I .

EXAMPLE II.

SUPPOSITION. Let AB be a prism hanging from the non-parallel lines CD , EF , which are produced to G , H , meeting in I . WHAT IS REQUIRED TO PROVE. We have to prove that the point I is in the centre line of gravity of the prism AB . PROOF. Let us consider DG and FH to be laths or rigid lines on which the prism rests, which by the 2nd postulate do not break or bend; the forces exerted by them are equal to the forces exerted by the lines CD and EF , for just as the latter keep the prism in its given position, so do the former. And whatever points in the lines DG , FH we take as their extremities, the prism keeps its given po-

Mathemati-
ca.

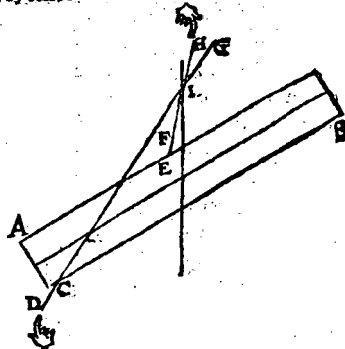
houdt daer op sijn ghegheuen standt. Laet ons nemen I, ghemeeu uyterste punt van d'een en d'ander lini; den pilaer dan houdt daer op (*Wisconftlick verstaende) sijn ghegheuen standt, maer rustende den pilaer op t'punt I, soo is de hanghende door I des pilaers swaerheyts middellini, inde welke I is.



III. VOORBEELT.

T'GHEGHEVEN. Laet AB een pilaer sijn welcke in die standt ghehouden wort door de scheefdaellini CD, ende scheefheffini EF, de selue sijn voortghetrocken tot G, H, sniende malcanderen in I.

T'BEGHEERDE. Wy moeten bewysen dat I inde swaerheyts middellini is des pilaers AB. **T'BEWYS.** Laet ons GC ansien voor styl, ofte stijue lini ende nemen dat de macht die an D int neertrecken was, nu neerstekende sy in yder punt tusschen C en G daermen haer stelt, ende den pilaer AB, sal alsoo op allen punten diemen tusschen C, G en E, H voor uytersten neemt, sijn ghegheuen standt houden. Laet ons nemen I ghemeeu uyterste van d'een en d'ander lini, den pilaer dan houdt daer an sijn ghegheuen standt; maer hanghende den pilaer an t'punt I, de hanghende door I is des pilaers swaerheyts middellini, inde welke I is.



III. VOORBEELT.

T'GHEGHEVEN. Laet AB een pilaer sijn, welcke in die standt ghehouden wort door de scheefdaellini CD, ende de scheefheffini EF, de selue sijn voortghetrocken tot G, H, sniende malcanderen in I.

T'BEGHEERDE. Wy moeten bewysen dat I inde swaerheyts middellini.

sition thereon. Let us take I , the common extremity of both lines; the prism then (mathematically speaking) keeps its given position thereon, but if the prism rests on the point I , the vertical through I is the centre line of gravity of the prism, in which lies I .

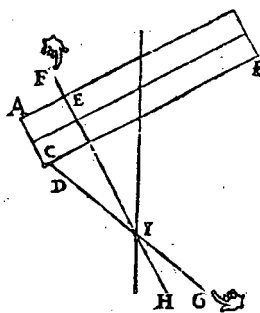
EXAMPLE III.

SUPPOSITION. Let AB be a prism which is kept in that position by the oblique lowering line CD and the oblique lifting line EF , which are produced to G , H , meeting in I . WHAT IS REQUIRED TO PROVE. We have to prove that I is in the centre line of gravity of the prism AB . PROOF. Let us consider GC to be a lath or rigid line, and let us suppose that the force which was drawing downwardly in D be now pushing downwardly in any point between C and G where it is put; then the prism AB will keep its position in any point taken as extremity between C , G , and E , H . Let us take I , the common extremity of both lines; the prism then keeps its given position thereon. But if the prism hangs at the point I , the vertical through I is the centre line of gravity of the prism, in which lies I .

EXAMPLE IV.

SUPPOSITION. Let AB be a prism which is kept in that position by the oblique lowering line CD and the oblique lifting line EF , which are produced to G , H , meeting in I . WHAT IS REQUIRED TO PROVE. We have to prove that I is in

dellini is des pilaers A B. T'BEWYS. Laet ons H E ansien voor stijl, ofte stue lini, ende nemen dat de macht die an E int opheffen was, nu opstekende sy in yder punt tusschen E en H, daermen haer stelt, ende den pilaer A B sal alto op allen punten diemen tusschen C G ende E H voor uystersten neemt, sijn ghegheuen standt houden. Laet ons nu nemen I ghemeen uysterste punt van d'een en d'ander lini, den pilaer dan houdt daer op sijn ghegheuen standt, maer rustende den pilaer op punt I, soo is de hanghende door I des pilaers swaerheyds middellini, inde welcke I is.



T'BESELYT. Twee oneuwydighe linien dan, daer een pilaer an hangt beyde oneyndelick voortghetrocken, snien malcanderen inde swaerheyds middellini des pilaers, welck wy bewyfen moesten.

XVII. VERTOOC.

XXVI. VOORSTEL.

So o d'een der twee linien daer een pilaer an hangt rechthouckich op den sichteinder is, d'ander salder oock rechthouckich op sijn: Ende sooder d'een scheefhouckich op is, d'ander salder oock scheefhouckich op wesen: Ende soo dese naer die neigt, die sal naer dese neighen: Maer so dese van die wyckt, die sal oock van dese wycken.

Horizontem.

T'GHEGHEVEN. Laet A B een pilaer sijn hanghende an twee linien, d'een C D rechthouckich op den sichteinder, d'ander E F (soot mueghelick waer) scheefhouckich, ende G H sy des pilaers swaerheyds middellini. T'BEGHEERDE. Wy moeten bewyfen 'tinhoudt des voorstels. T'BEREYTSSEL. Laet C D ende E F voortghetrocken worden, sniende malcander in I. T'BEWYS. Soo den pilaer in die ghestalt blijft hanghende ande linien C D, E F, sy sal op alle vastpunten in die voortghetrocken linien de selue ghestalt houden, ouermidts de houcken I C E, ende I E C, niet en veranderen: Daerom ghenomen I

H 2

ghemeen

the centre line of gravity of the prism AB . PROOF. Let us consider HE to be a lath or rigid line, and let us suppose that the force which was lifting upwardly in E be now pushing upwardly in any point between E and H where it is put. Then the prism AB will keep its given position in any point taken as extremity between C , G , and E , H . Let us now take I , the common extremity of both lines; the prism then keeps its given position thereon. But if the prism rests on the point I , the vertical through I is the centre of gravity of the prism, in which lies I . CONCLUSION. Two non-parallel lines therefore, from which a prism is hanging, both of them produced indefinitely, meet in the centre line of gravity of the prism, which we had to prove.

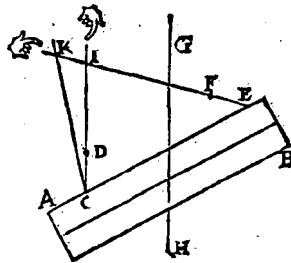
THEOREM XVII.

PROPOSITION XXVI.

If one of two lines from which a prism is hanging is at right angles to the horizon, the other will also be at right angles thereto; and if one is at oblique angles thereto, the other will also be at oblique angles thereto; and if the latter verges towards the former, the former will verge towards the latter, but if the latter verges away from the former, the former will also verge away from the latter.

SUPPOSITION. Let AB be a prism hanging from two lines, one CD , at right angles to the horizon, the other, EF (if this were possible), at oblique angles thereto, and let GH be the centre line of gravity of the prism. WHAT IS REQUIRED TO PROVE. We have to prove the contents of the proposition. PRELIMINARY. Let CD and EF be produced until they meet in I . PROOF. If the prism remains hanging in that position from the lines CD , EF , it will keep the same position on any fixed point in those lines produced, since the angles ICE and IEC do not change. Therefore, I being taken as the common fixed point of

ghemeen vastpunt dier twee linien, den pilaer sal daer an in sijn ghelgheuen standt bliuen hanghende, ende I C sal swaerheyds middellini sijn: maer dat is onmueghelick, wantter G H haer euewydeghe is. T'selue sal oock alsoo bethoont worden als de lini E F ouer dander sijde neigt. Wefende dan I C rechthouckich op den sichteinder, d'ander lini als E F en cander niet scheefhouckich op sijn; nootfaecklick dan rechthouckich:



Ende veruolghens sooder E F scheefhouckich op is, dander moeder oock scheefhouckich op sijn.

VOORDER, anghesien E F neigt naer de sijde van A, soo sal de lini die den pilaer in die ghestalt houdt moeten neighen naer E F. Want laetse (soot mueghelick waer) daer van wycken, als C K, sijnende de voortghetrocken E I in K, inder voughen dat de hanghende lini door K, sal om de redenen als bouen swaerheyds middellini wesen des pilaers, t'welck noch ongheschieter is dan doen wy die seyden door I te vallen: D'ander lini dan die den pilaer in de ghestalt can houden, en wyckt van E F niet, sy en is met haer oock gheen euewydighe als bouen bethoont is, ende ter sijden uyt te wijcken is openbaer onmueghelick, sy neigt dan nootfaecklick naer E F. Ende soo E F ouer d'ander sijde neigde, men sal insghelijcx bethoonen dat d'ander lini van haer wycken sal.

T'BESELYT. Sood'eene dan der twee linien, &c.

XVIII. VERTOOC.

XXVII. VOORSTEL.

HANGHENDE een pilaer euestaltwichtich teghen twee scheefhefwichten: Ghelijck scheefheflini tot rechthefflini, alsoo elck scheefhefwicht tot sijn rechtheffwicht.

T'GHEGHEVEN. Laet A B een pilaer sijn wiens as C D, ende twee punten daer in E, F, welcker scheefhefwichten die hem in die standt houden sijn G, H, ende rechtheffwichten I, K, ende scheefhefflinien E L, F M, ende rechthefflinien E N, F O. T'BEGHEERDE. Wy moeten bewysen dat ghelijck L E tot E N, alsoo G tot I, ende ghelijck M F tot F O, alsoo H tot K. T'BEWYS. Laet ons F ansien voor vastpunt, ende E voor troerlick, daerom (door het 20^e voorstel) ghelijck L E tot E N,

those two lines, the prism will remain hanging thereon in its given position, and IC will be the centre line of gravity; but this is impossible, because it is parallel to GH . The same can also be shown in this way if the line EF verges towards the other side. Therefore, IC being at right angles to the horizon, the other line, as EF , cannot be at oblique angles thereto, so that necessarily it is at right angles thereto. And consequently, if EF is at oblique angles to the horizon, the other must also be at oblique angles thereto.

Further, since EF verges towards the side of A , the line keeping the prism in that position will have to verge towards EF . For let the said line (if this were possible) verge away therefrom, as CK , meeting EI produced in K ; then the vertical through K will, for the reasons mentioned above, be the centre line of gravity of the prism, which is even more absurd than saying that it passes through I . The other line, therefore, which can keep the prism in that position, does not verge away from EF , nor is it parallel thereto, as proved above, and it is evidently impossible for it to verge sidelong; therefore it necessarily verges towards EF . And if EF verged towards the other side, it can be shown similarly that the other line will verge away therefrom. CONCLUSION. If therefore one of two lines, etc.

THEOREM XVIII.

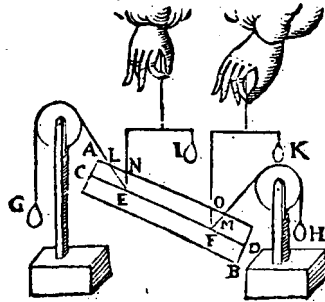
PROPOSITION XXVII.

If a prism is hanging in equality of apparent weight with two oblique lifting weights: as the oblique lifting line is to the vertical lifting line, so is also each oblique lifting weight to its vertical lifting weight.

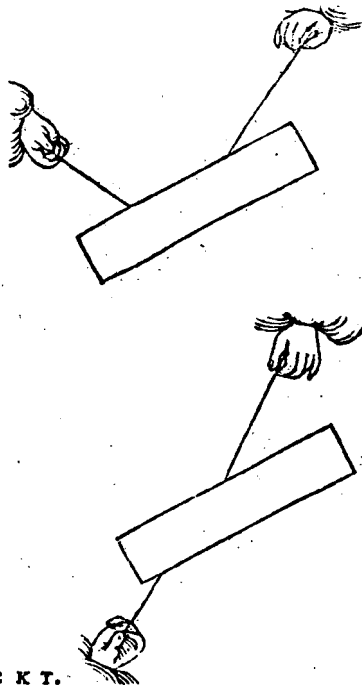
SUPPOSITION. Let AB be a prism, its axis CD , and two points therein E , F , whose oblique lifting weights keeping the prism in that position are G , H , and the vertical lifting weights I and K , the oblique lifting lines being EL , FM , and the vertical lifting lines EN , FO . WHAT IS REQUIRED TO PROVE. We have

tot E N, alsoo G tot I. Laet ons ten tweeden E an sien voor vastpunt, ende F voor roerlick; Daerom (door t'voornoomde 20^e voorstel) ghelijck M F tot F O, alsoo H tot K.

T B E S L V Y T. Hanghende dan een pilaer cuestaltwichtich reghen twee scheefhefwichten: Ghelijc scheefhef lini tot recht hef lini, alsoo elck scheefhefwicht tot sijn rechthefwicht, twelck wy bewysen moesten.



VERVOLGH.
HANGHENDE een bekende pilaer an twee oneuwydighe linien als hier neuen; Tblyckt dat bekend sal worden hoe veel ghewichts an yder lini hangt, ofte hoe veel ghewelts yder lini doet.



M E R C K T.

W y hebben tot veel voorbeelden der voorstellen deses boucx; ghenomen den pilaer, als bequaemste form tot de verclaring des voornemens oock vastpunt ende roerlickpunt ghestelt inden as. Wy sullen nu door dit laetste voorstel, beschoonen de reghelen van dien ghemeen te wesen ouer alle formen der lichamen bodanich sy sijn met vastpunt ende roerlickpunt daert valt.

to prove that as LE is to EN , so is G to I , and as MF is to FO , so is H to K . PROOF. Let us consider F to be the fixed point and E the movable point; therefore (by the 20th proposition), as LE is to EN , so is G to I . Let us secondly consider E to be the fixed point, and F the movable point. Therefore (by the aforesaid 20th proposition), as MF is to FO , so is H to K . CONCLUSION. If therefore a prism is hanging in equality of apparent weight with two oblique lifting weights: as the oblique lifting line is to the vertical lifting line, so is each oblique lifting weight to its vertical lifting weight, which we had to prove.

COROLLARY.

If a known prism is hanging from two non-parallel lines, as shown in the annexed figure, it appears that it will become known what weight is hanging from either line, or what force either line exerts.

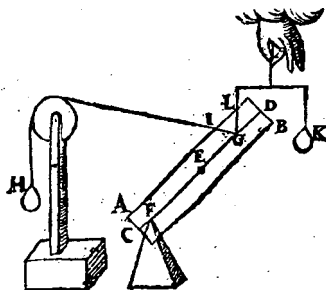
NOTE.

For many examples of the propositions of this book we have taken the prism, as being the most suitable form for explaining the meaning, and we have also put the fixed and the movable point in the axis. By the last proposition we shall now show that the respective rules are true generally of solids of whatever form, with arbitrary fixed and movable points.

Proportio-
nes.

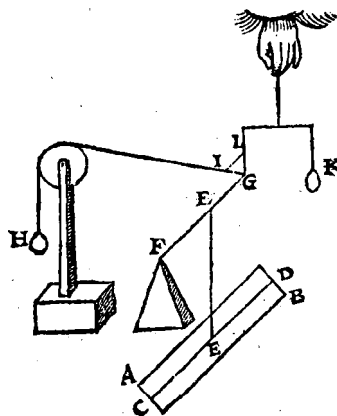
ALLE de *eueredenheden, welke hier vooren beschreuen sijn vanden pilaer tot de ghewichten an hem hanghende, ende dier ghewichten linien: De selue te wesen van yder lichaem tot de ghewichten an hem alsoo hanghende, ende dier ghewichten linien.

TGHEGHEVEN. Laet ons t'voorbeelt nemen der eueredenheydt des 20^e voorstels aldus: Het sy een pilaer A B, diens as C D, ende swaerheys middelpunt E, ende vastpunt daer in F, ende roerlick punt G, an t'welc gheuocht sy een scheefhefwicht H, dat den pilaer in die ghestalt houde, diens scheefhef lini G I. Daer naer trecht hefwicht K, dat den pilaer oock in die ghestalt houde, diens rechthe lini G L, alwaer wy segghen, ghelijck I G tot G L, also H tot K. T'BEGHEERDE. Wy moeten bewysen dat dese eueredenheydt niet alleenlick also en bestaet in t'lichaem A B een pilaer sijnde, maer van sulcke form alst valt.



Centrum
gravitatis.

T'BEWYS. Laet ons den pilaer A B (blijuende de linien F G ende I L op haer plaetsen) neertrecken, alsoo dat hy bliue hanghende an sijn *swaerheys middelpunt E, wiens ghestalt dan sy als hier neuens. Ende door de 3^e begheerte den pilaer en veroirsaect op de punten F, G, gheen ander swaerheydt dan d'eerste; ende alles blijft noch euestaltwichtich, ende ghelijck I G tot G L, alsoo noch H tot K,



LAET

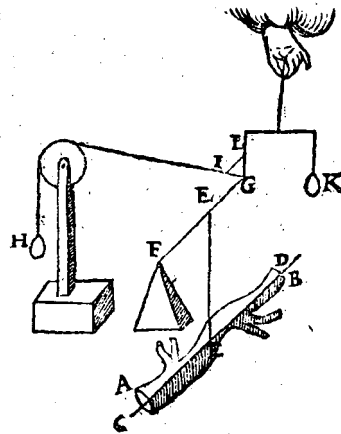
THEOREM XIX.

PROPOSITION XXVIII.

Any proportions discussed above of the prism in relation to the weights hanging therefrom, and the lines of such weights, are true of any solid in relation to the weights thus hanging therefrom, and the lines of such weights.

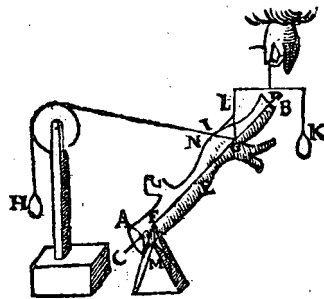
SUPPOSITION. Let us take as example the proportion of the 20th proposition, as follows. Let there be a prism AB , its axis CD , and the centre of gravity E , and the fixed point therein F and the movable point G , at which let there be attached an oblique lifting weight H , which shall keep the prism in that position, its oblique lifting line being GI . Then let there be the vertical lifting weight K , which shall also keep the prism in that position, its vertical lifting line being GL . We then say: as IG is to GL , so is H to K . **WHAT IS REQUIRED TO PROVE.** We have to prove that this proportion is true not only when the solid AB is a prism, but of any form whatever. **PROOF.** Let us pull down the prism AB (the lines FG and IL remaining in their places) in such a way that it shall remain hanging at its centre of gravity E , the situation then being as shown in the annexed figure. Then by the 3rd postulate the prism does not cause any other gravity on the points F , G than the first, and the whole still remains in equality of apparent weight; and as IG is to GL , so is H to K .

LAET nu de form des pilaers (al de stoff bliuende) verandert worden in eenighe ander ongheschickte form, als AB hier neuens, diens swaerheyds middelpunt E sy, ende een rechte lini daer deur CD (welcke vinding des swaerheys middelpunts ende rechter linien inde Weeghdaet verclaert sal worden, *werckelick, niet Wisconstelick) ende alles blijft noch eueftaltwichtich, ende ghelijck IG tot GL, alsoe noch H tot K.



*Mechanica
non Mathematica.*

LAET nu lichaem AB opghetrocken worden, tot dat FG is inde lini CD, wiens ghestalt dan sy als hier neuens, ende alles blijft noch eueftaltwichtich: want het lichaem AB hooger ofte leegher hanghende, blijft van een selfde ghewicht door de 3^e begheerte, ende veruolghens ghelijck IG tot GL, alsoe noch H tot K.



De eueredenheydt dan des 20^e voorstels en is niet alleenelick alsoo mer den pilaer, maer met yder lichaem: Ende der ghelijcke salmen oock alsoo beshoonen van al t'gheue hier vooren in alle d'ander voorstellen vanden pilaer gheseyt is.

T B E S L V Y T. Alle de eueredenheden dan, welcke hier vooren bescreuen sijn vanden pilaer tot de ghewichten an hem hanghende, ende dier ghewichten linien; de selue sijn van yder lichaem tot de ghewichten an hem alsoo hanghende, ende dier ghewichten linien, t'welck wy bewylen moesten.

VERVOLGH.

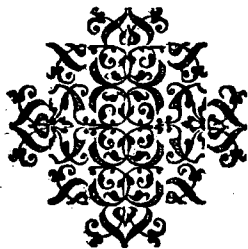
Now let the form of the prism (all the material remaining) be changed into some other — irregular — form, as AB in the annexed figure, whose centre of gravity shall be E , and let there be drawn a straight line through it, as CD (which determination of the centre of gravity and of the straight lines will be explained in the Practice of Weighing¹), not mathematically, but mechanically); then the whole still remains in equality of apparent weight, and as IG is to GL , so is H to K .

Now let the solid AB be pulled up until FG is in the line CD , the position of which shall then be as shown in the annexed figure. The whole still remains in equality of apparent weight, for the solid AB , no matter whether it hangs higher or lower, keeps the same weight, by the 3rd postulate, and consequently as IG is to GL , so is H to K . The proportion therefore of the 20th proposition is true not only of the prism, but of any solid. And the same can also be shown of all that has been said before of the prism in all the other propositions. CONCLUSION. Any proportions, therefore, discussed above of the prism in relation to the weights hanging therefrom, and the lines of such weights, are true of any solid in relation to the weights thus hanging therefrom, and the lines of such weights, which we had to prove.

¹) See *The Practice of Weighing*, Prop. 1.

*Perpendicu-
lars.* T i s oock openbaer dat de ghegheuen punten als F, G, nietnoot-
faeckelick en moeten inde lini CD lijn, maer daert valt. by voorbeeld
I N tot inde rechte CD, *welck ick neem te vallen in G, ghelijcx
ghetrocken door M de * hanghende tot inde lini C D, welcke ick neem
te vallen in F, de voornoemde eueredenheydt, te weten ghelijck I G
tot G L, alsoo H tot K, blijft noch staende.

EINDE DES EERSTEN BOVCK.



COROLLARY.

It is also manifest that the given points, as F , G , need not be in the line CD , but may be in any place, for example at the extremities of the solid, M , N . For the line IN being produced to meet the straight line CD , which I take to be in G , and likewise the vertical through M being drawn to meet the line CD , which I take to be in F , the aforesaid proportion, viz. as IG is to GL , so is H to K , is still true.

END OF THE FIRST BOOK.

HET TWEEDE BOVCK
V A N D E * B E G H I N S E L E N Elementa.
D E R W E E G H C O N S T , D W E L C K I S
V A N D E V I N D I N G D E R S W A E R -
H E Y D T S M I D D E L P V N T E N ,
Beschreuen door Simon Stevin.

WY hebben in t'eerste bouck tot het beschriuen der wichtige ghedaenten, ghenomen een pilaer (voldoende aldaer het voornemen) diens swaerheys middelpunt door ghemeene wetenschap bekend is, maer in veel ander lichamen en ghebueret niet also; wel is waer dattet door een corte ghemeene reghel in allen werckelick te vinden is, so door t'eerste voorstel der * Weeghdaet blijcken sal, maer met de * Wif- Praxis. Mathema-constighe vinding ist anders ghestelt; Daer af heeft eerst gheschreuen sica. Archimedes in platten, ende naer hem Frederic Commandin in lichamen: Wy sullen tottet een en t'ander (ouermits het een* afcoemst van Species. beghinselfen is, byde voorgaende wel dienende, ende tottet volghende, so wel W A T E R W I C H T , als W E E G H D A E T , seer noodich) het onse voughen, ende alles naer onse oirden verspreyden, daer af beschrijuen- de der Beghinselfen tweede bouck.

Wat de * bepalinghen belangt vande Meetconstighe formen, die by- Definitiones. gheualle hier yemandt begheeren mocht, wy nemen die * door t'ghestel- Per hypothe-de voor bekend uyt de * Meetconst; Alleenelick dit daer af segghende, sin. dat wy t'woort *Parabola*, ofte *Rectanguli conis sectio*, beteecken met Geometria. *Brantsne*: Ende *Conoidale Rectangulum*, met *Brander*; Reden, dat dier for- men * daet voornamelick bestaet int ontfteken ofte branden. Effectus.

E E R S T V A N D E V I N D I N G D E R
S W A E R H E Y T S M I D D E L P V N T E N
V A N D E * P L A T T E N . Planis.

BY aldien de platten eenich ghewicht hadden, ende datmen toeliete die te wesen inde reden haerder grootheden, wy souden eyghentlick mueghen spreken van haer Swaerheydt, Swaerheys middelpunt, Swaerheys middellini, &c. Maer
I angheffen

THE SECOND BOOK

OF THE ELEMENTS

OF THE ART OF WEIGHING, WHICH DEALS WITH THE FINDING OF THE CENTRES OF GRAVITY, Described by Simon Stevin

In the first book we have, for the description of the qualities of weights, taken a prism (which in that case was satisfactory for our purposes), whose centre of gravity is known by common knowledge, but in many other solids it does not so happen; it is true indeed that it can be found constructionally in all forms by means of a short common rule, as will become apparent from the first proposition of the Practice of Weighing, but with the mathematical finding it is a different matter. The first to write about this was Archimedes ¹⁾, viz. about plane figures, and after him Frederick Commandinus ²⁾, about solids. We will add our own observations to both (since the subject forms a kind of elements, which have been useful for what precedes and will be highly necessary for what follows: Hydrostatics as well as the Practice of Weighing), and arrange the subject matter according to our own method, thus describing the second book of the Elements.

As to the definitions of the geometrical figures which anyone might require in this place, we assume these to be known by hypothesis from geometry, merely stating that we denote the word *Parabola*, or *Rectanguli conic section* ³⁾, by "*Brantsne*" ⁴⁾, and *Conoidale Rectangulum* ⁵⁾ by "*Brander*" ⁶⁾, because the effect of these figures chiefly consists in igniting or burning.

FIRST ABOUT THE FINDING OF THE CENTRES OF GRAVITY OF PLANE FIGURES

If plane figures had any weight, and these were admitted to be proportional to their magnitudes, we might properly speak of their gravity, centre of gravity, centre line of gravity, etc., but since a plane figure has no weight, properly speak-

¹⁾ Archimedes, *De Planorum Equilibriis Libri II. Opera Omnia*, ed. J. L. Heiberg, Vol. II. Leipzig, 1913, 122-213.

²⁾ Federici Commandini Urbinatis *Liber de centro gravitatis solidorum*. Bononiae, 1555.

³⁾ This is the current term for parabola in older Greek geometry; the curve was generated by cutting a cone of revolution the vertical angle of which is a right angle by a plane perpendicular to a generating line.

⁴⁾ This is one of Stevin's neologisms, for which there is no English equivalent. The meaning of "*brantsne*" might be rendered by "*burning section*".

⁵⁾ *Conoidale rectangulum* (ῥοθωγωνιον κωνοειδές) is the Archimedean term for a paraboloid of revolution.

⁶⁾ Another neologism. The English equivalent is "*burner*".

Metaphori-
sch.

anghesien in 't plat gheen ghewicht en is, soo en isser eyghentlick sprekende gheen swaerheydt, swaerheyds middelpunt, noch swaerheys middellini in; Daerom moetmen dit alles* lijk spreucklick verstaen, ende nemen als door 't ghestelde, dat der platten ghewichten inde reden haerder grootheden sijn, want T'valliche wort toeghelaten, op datmen t'waerachtighe daer duer leere.

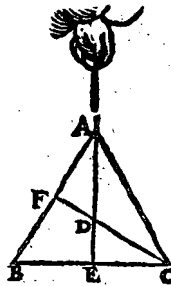
I. VERTOOGH.

I. VOORSTEL.

YDER plats middelpunt der form, is oock sijn swaerheys middelpunt.

I^e VOORBEELT.

TGHEGHEVEN. Laet $A B C$ een euezijdich driehouck wesen, diens formens middelpunt sy D . T'BEGHEERDE. Wy moeten bewyfen dat D oock het swaerheys middelpunt is des driehouck $A B C$. T'BEREYTSSEL. Laet ghetrocken worden van A tot int middel van $B C$, de lini $A E$, ghelijcx van C tot int middel van $A B$, de lini $C F$. T'BEWYS. Wefende de driehouck $A B C$ opghehanghen byde lini $A E$, het deel $A E C$ sal euewichtich hanghen teghen $A E B$, want sy sijn euen groot, ghelijck, ende van ghelijcker ghestalt; $A E$ dan is swaerheys middellini des driehouck $A B C$, Ende om de selue reden sal $F C$ oock des driehouck swaerheys middellini sijn, maer dese snien malcanderen in des formens middelpunt D , ende elck dier linien heeft in haer het swaerheys middelpunt, tis dan D .

II^e VOORBEELT.

TGHEGHEVEN. Laet $A B C D$ een euewydich vierhouck sijn, diens formens middelpunt E . T'BEGHEERDE. Wy moeten bewyfen dat E oock het swaerheys middelpunt is. T'BEREYTSSEL. Laet ghetrocken worden $F G$, tusschen de middelpunten van $A D$ ende $B C$, insghelijcx $H I$, tusschen de middelpunten van $A B$ ende $D C$.

T'BEWYS. Wefende den vierhouck opghehanghen byde lini $H I$, Het deel $H I D A$ sal euewichtich hanghen teghen $H I C B$, want sy sijn euegroot ghelijck ende van ghelijcker ghestalt; $H I$ dan is swaerheys

ing there is no gravity, centre of gravity, nor centre line of gravity therein¹⁾. Therefore all this has to be understood metaphorically, and it has to be assumed by hypothesis that the weights of plane figures are proportional to their magnitudes, for:

THE FALSE IS ADMITTED IN ORDER THAT THE TRUE MAY BE LEARNED THEREFROM.

THEOREM I.

The geometrical centre of any plane figure is also its centre of gravity.

PROPOSITION I.

EXAMPLE I.

SUPPOSITION. Let ABC be an equilateral triangle, whose geometrical centre shall be D . WHAT IS REQUIRED TO PROVE. We have to prove that D is also the centre of gravity of the triangle ABC . PRELIMINARY. Let there be drawn from A to the middle point of BC the line AE ; likewise from C to the middle point of AB the line CF . PROOF. The triangle ABC being suspended by the line AE , the part AEC will balance²⁾ AEB , for they are equally large, similar, and of the same form. Therefore AE is centre line of gravity³⁾ of the triangle ABC , and for the same reason FC will also be centre line of gravity of the triangle. But these lines intersect in the geometrical centre D , and each of these lines contains the centre of gravity; therefore the latter is D .

EXAMPLE II.

SUPPOSITION. Let $ABCD$ be a parallelogram, whose geometrical centre shall be E . WHAT IS REQUIRED TO PROVE. We have to prove that E is also the centre of gravity. PRELIMINARY. Let FG be drawn, joining the middle points of AD and BC , and likewise HI ⁴⁾, joining the middle points of AB and DC . PROOF. The quadrilateral being suspended by the line HI , the part $HIDA$ will balance $HICB$, for they are equally large, similar, and of the same form. HI is

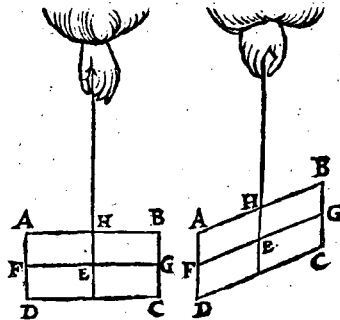
¹⁾ It is highly remarkable that this observation is made with regard to plane figures only, and not for solids as well, as if the latter did have weight, etc.

²⁾ It is to be noted that here as elsewhere Stevin uses the term „*evenwichtig*“ (of equal weight) instead of „*evestaltwichtig*“ (of equal apparent weight), as might have been expected.

³⁾ As we remarked in our notes to Definition 5 and Proposition 6 of Book I, the term „centre line of gravity“ is usually taken to mean the vertical through the point of suspension of the figure at rest. It is then assumed „by a common rule of Statics“ that the centre of gravity is in the so defined centre line of gravity. This remark applies to the whole of Book II.

⁴⁾ The letter I denoting the middle point of DC is lacking in the drawings.

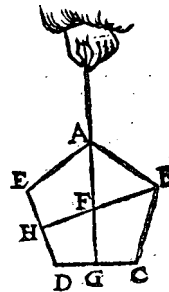
heyls middellini des vierhoucx $ABCD$, Ende om de selue reden sal FG oock des vierhoucx swaerheyls middellini sijn, maer dese doorsnien malcanderen in E , ende elck dier linien heeft in haer het swaerheyls middelpunt, tis dan E .



III. VOORBEELT.

T'GHEGHEVEN. Laet $ABCD$ een gheschickt ofte inschriuelick vijfhouck wesen, diens formens middelpunt F sy. **T'BEGHEERDE.** Wy moeten bewyfen dat F oock het swaerheyls middelpunt is.

T'BEREYTSSEL. Laet ghetrocken worden van A tot int middel van DC , de lini AG ; (ghelijcx van B tot int middel van ED , de lini BH). **T'BEWYS.** Wefende den vijfhouck opghehanghen byde lini AG , het deel $AGDE$ sal euewichtich hanghen teghen het deel $AGCB$, want sy sijn euegroot, ghelijck, ende van ghelijcker ghestalt: AG dan is swaerheyls middellini des vijfhoucx, ende om de selue reden sal BH oock des selfden vijfhoucx swaerheyls middellini wesen; maer dese doorsnien malcanderen in des formens middelpunt F , ende elck dier linien heeft in haer het swaerheyls middelpunt, tis dan F . Sghelijcx sal oock t'bewys sijn in allen anderen hebbende een formens middelpunt als Sefhoucken, Ronden, Scheefronden, &c.



T'BEELVYT. Yder plats middelpunt der form dan, is oock sijn swaerheyls middelpunt, t'welck wy bewyfen moesten.

II. VERTOOC.

II. VOORSTEL.

YDER driehoucx swaerheydts middelpunt, is inde lini ghetrocken vanden houck tot int middel der sijde.

T'GHEGHEVEN. Laet ABC een driehouck sijn van form foot

then centre line of gravity of the quadrilateral $ABCD$, and for the same reason FG will also be centre line of gravity of the quadrilateral. But these lines intersect in E , and each of these lines contains the centre of gravity. Therefore the latter is E .

EXAMPLE III.

SUPPOSITION. Let $ABCDE$ be a regular or inscribed pentagon, whose geometrical centre shall be F . **WHAT IS REQUIRED TO PROVE.** We have to prove that F is also the centre of gravity. **PRELIMINARY.** Let there be drawn from A to the middle point of DC the line AG ; likewise from B to the middle point of ED the line BH . **PROOF.** The pentagon being suspended by the line AG , the part $AGDE$ will balance the part $AGCB$, for they are equally large, similar, and of the same form. AG is therefore centre line of gravity of the pentagon, and for the same reason BH will also be centre line of gravity of the same pentagon. But these lines intersect in the geometrical centre F , and each of these lines contains the centre of gravity; therefore the latter is F . The same proof holds for all other figures having a geometrical centre, such as hexagons, circles, ellipses, etc. **CONCLUSION.** The geometrical centre of any plane figure therefore is also its centre of gravity, which we had to prove.

THEOREM II.

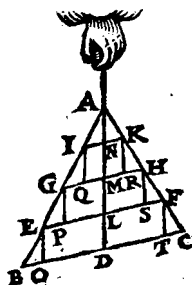
PROPOSITION II.

The centre of gravity of any triangle is in the line drawn from the angle to the middle point of the side.

SUPPOSITION. Let ABC be a triangle of any form, in which from the angle

valt, waer in vanden houck A tot in D middel vande sijde B C, ghetrocken is de lini A D. **T B E G H E E R D E.** Wy moeten bewyfen dat des driehoucx swaerheys middelpunt inde lini A D is. **T B E R E Y T S E L.** Laet ons trekken E F, G H, I K, euewydighe van B C, sniende A D in L, M, N, daer naer E O, G P, I Q, K R, H S, F T, euewydighe met A D.

T B E W Y S. Ouermits E F euewydighe is van B C, ende E O, F T met L D, soo sal E F T O, euewydich vierhouck sijn, wiens E L euen is met L F, oock met O D ende D T, waer deur het swaerheys middelpunt des vierhoucx E F T O in D L is, door het 1^o voorstel deses boucx. Ende om de selue reden sal het swaerheys middelpunt des euewydichs vierhoucx G H S P wesen in L M, ende van I K R Q in M N, ende vervolghens het swaerheys middelpunt der form I K R H S F T O E P G Q ghemaect vande voornoemde drie vierhoucken, sal wesen inde lini N D, ofte A D. Nu



ghelijck hier in beschreuen sijn drie vierhoucken, also can mender oneindelicke sulcke vierhoucken in beschrijuen, ende des binneschreuens formens swaerheys middelpunt, sal altijt sijn (om de redenen als vooren) inde lini A D. Maer hoe datter sulcke vierhoucken meer sijn, hoe dat den driehouck A B C min verschilt vande binneschreuen form der vierhoucken; want trekkende linien euewydich van B C door de middelen van A N, N M, M L, L D, t'verschil des laetsten ghestalts, sal effen den helft sijn van t'verschil des voorgaenden ghestalts. Wy connen dan door dat oneindelic naerderen sulck een form binnen den driehouck stellen, dattet verschil tusschen haer ende den driehouck, minder sal wesen dan eenich ghegheuen plat hoe cleen het sy: Waer uyt volghet, dat stellende A D als swaerheyds middellini, so sal t'staltwicht des deels A D C, min verschillen van t'staltwicht des deels A D B, dan eenich plat datmen soude connen gheuen hoe cleen het sy, waer uyt ick aldus strie.

A. *Neuen alle verschillende statfwaerbeden, can een swaerheydt ghestelt worden minder dan haer verschil;*

O. *Neuen dese statfwaerbeden A D C ende A D B, en can gheen swaerheydt ghestelt worden minder dan haer verschil;*

O. *Dese statfwaerbeden dan A D C ende A D B en verschillen niet.*

Daerom A D is swaerheys middellini, ende vervolghens t'swaerheys middelpunt des driehoucx A B C is in haer. **T B E S L V Y T.** Yder driehoucx swaerheyds middelpunt dan is inde lini ghetrocken vanden houck tot int middel der sijde, t'welck wy bewyfen moesten.

A to D , the middle point of the side BC , there is drawn the line AD . WHAT is REQUIRED TO PROVE. We have to prove that the centre of gravity of the triangle is in the line AD . PRELIMINARY. Let us draw EF , GH , IK parallel to BC , intersecting AD in L , M , N ; after that EO , GP , IQ , KR , HS , FT , parallel to AD . PROOF Since EF is parallel to BC , and EO , FT to LD , $EFTO$ will be a parallelogram, in which EL is equal to LF , also to OD and DT , in consequence of which the centre of gravity of the quadrilateral $EFTO$ is in DL , by the 1st proposition of this book. And for the same reason the centre of gravity of the parallelogram $GHSP$ will be in LM , and of $IKRQ$ in MN ; and consequently the centre of gravity of the figure $IKRHSFTOEPGQ$, composed of the aforesaid three quadrilaterals, will be in the line ND or AD . Now as here three quadrilaterals have been inscribed in the triangle, so an infinite number of such quadrilaterals can be inscribed therein, and the centre of gravity of the inscribed figure will always be (for the reasons mentioned above) in the line AD . But the more such quadrilaterals there are, the less the triangle ABC will differ from the inscribed figure of the quadrilaterals. For if we draw lines parallel to BC through the middle point of AN , NM , ML , LD , the difference of the last figure will be exactly half of the difference of the preceding figure¹⁾. We can therefore, by infinite approximation, place within the triangle a figure such that the difference between the latter and the triangle shall be less than any given plane figure²⁾, however small. From which it follows that, taking AD to be centre line of gravity³⁾, the apparent weight of the part ADC will differ less from the apparent weight of the part ADB than any plane figure that might be given, however small, from which I argue as follows⁴⁾:

- A. Beside any different apparent gravities there may be placed a gravity less than their difference;
- O. Beside the present apparent gravities ADC and ADB there cannot be placed any gravity less than their difference;
- O. Therefore the present apparent gravities ADC and ADB do not differ.

Therefore AD is centre line of gravity, and consequently the centre of gravity of the triangle ABC is in it. CONCLUSION. The centre of gravity of any triangle therefore is in the line drawn from the angle to the middle point of the side, which we had to prove.

¹⁾ It is obviously assumed that the side AB is divided into n equal segments (in the drawing $n = 4$). The difference between the area Δ of the triangle ABC and that of the figure Π_n consisting of $(n - 1)$ parallelograms is $\frac{\Delta}{n}$.

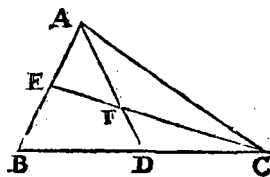
²⁾ Euclid X 1; porism.

³⁾ As in Prop. 6 of Book I, the term „centre line of gravity” cannot here be meant in the sense attributed to it by Defin. 5 of Book I (vertical through the centre of gravity). It is to be proved that the centre of gravity is in the line AD , and it would be begging the question to suppose AD to be centre line of gravity. Stevin’s meaning may be rendered as follows: Suppose AD to be held in the vertical of A . It is then proved that the „staltwichten” of the triangles ADB and ADC relatively to AD are equal to one another. If now AD is released, it will remain in the vertical. Hence the conclusion: AD is the centre line of gravity (in the sense of vertical through the point of suspension of the solid in rest), and hence (by the common rule of statics, quoted in Prop. 6 of Book I) the centre of gravity is in AD .

⁴⁾ See note 2 to p. 143.

W E S E N D E ghegheuen een driehouck: Sijn swaerheys middelpunt te vinden.

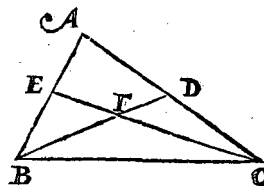
T G H E G H E V E N. Laet ABC een driehouck wesen. T B E G H E E R D E. Wy moeten sijn swaerheyts middelpunt vinden. T W E R C K. Men sal van A tot int middel van BC , trecken de lini AD , sghelijcx van C tot int middel van AB , de lini CE , sniende AD in F : Ick seg dat F r begheerde swaerheyts middelpunt is. T B E W Y S. T swaerheys middelpunt des driehoucx ABC , is inde lini AD , ende oock in CE , duer het 2^e voorstel, tis dan F , twelck wy bewyfen moesten. T B E S L V Y T. Wefende dan ghegheuen een driehouck: Wy hebben sijn swaerheyts middelpunt gheuonden naer den eysch.



H E T swaerheys middelpunt eens driehoucx deelt de lini vanden houck tot int middel der sijde alsoo, dattet stick naer den houck, dobbel is an rander.

T G H E G H E V E N. Laet ABC een driehouck sijn, ende vanden houck B een lini ghetrocken worden tot D int middel van AC , sghelijcx van C een lini tot E int middel van AB , sniende BD in F voor swaerheys middelpunt des driehoucx ABC . T B E G H E E R D E. Wy moeten bewyfen dat CF dobbel is an FE . T B E W Y S. Ghetrocken de reden EB 1 tot BA 2, vande reden CD 1 tot DA 1 (dat is Reden $\frac{1}{2}$ van Reden $\frac{1}{1}$) * daer rest de reden van CF tot FE , maer treckende Reden $\frac{1}{2}$ van Reden $\frac{1}{1}$ daer blijft Reden $\frac{2}{1}$. CF dan is tot FE , als van 2 tot 1.

T B E S L V Y T. Het swaerheys middelpunt dan eens driehoucx deelt de lini vanden houck tot int middel der sijde alsoo, dattet stick naer den houck dobbel is an rander, twelck wy bewyfen moesten.



Door t'over-
keerde des 12
cap. 1 lib.
Almag.
Ptolem.

PROBLEM I.

Given a triangle: to find its centre of gravity.

PROPOSITION III.

SUPPOSITION. Let ABC be a triangle. WHAT IS REQUIRED TO FIND. We have to find its centre of gravity. CONSTRUCTION. There shall be drawn from A to the middle point of BC the line AD , likewise from C to the middle point of AB the line CE , intersecting AD in F . I say that F is the required centre of gravity. PROOF. The centre of gravity of the triangle ABC is in the line AD , and also in CE , by the 2nd proposition. It is therefore F , which we had to prove. CONCLUSION. Given therefore a triangle, we have found its centre of gravity, as required.

THEOREM III.

PROPOSITION IV.

The centre of gravity of a triangle divides the line from the angle to the middle point of the side in such a way that the segment adjacent to the angle is double of the other.

SUPPOSITION. Let ABC be a triangle, and let there be drawn from the angle B a line to D in the middle of AC , likewise from C a line to E in the middle of AB , intersecting BD in F , the centre of gravity of the triangle ABC . WHAT IS REQUIRED TO PROVE. We have to prove that CF is double of FE . PROOF¹⁾. The ratio of EB (1) to BA (2) being subtracted from the ratio of CD (1) to DA (1) (i.e. ratio $\frac{1}{2}$ from ratio $\frac{1}{1}$), there remains the ratio of CF to FE , but the ratio $\frac{1}{2}$ being subtracted from the ratio $\frac{1}{1}$, there remains the ratio $\frac{2}{1}$. Therefore CF is to FE as 2 to 1. CONCLUSION. The centre of gravity of a triangle therefore divides the line from the angle to the middle point of the side in such a way that the segment adjacent to the angle is double of the other, which we had to prove.

¹⁾ In the margin Stevin quotes the converse of Ptol my, *Almagest* I 12. In the modern edition of the work by Heiberg (Claudii Ptolemaei *Opera quae exstant omnia*, Vol. I, *Syntaxis Mathematica* Pars I, Leipzig, 1898) this is I 13, which begins with Menelaus's Theorem. Applying this theorem to $\triangle ACE$ with transversal DFB , we obtain:

$$\frac{DA \cdot FC \cdot BE}{DC \cdot FE \cdot BA} \text{ hence } \frac{CF}{FE} = \frac{\frac{DC}{DA}}{\frac{BE}{BA}} = \frac{1}{2} = 2.$$

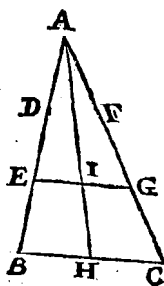
According to the ancient terminology, which was still current in Stevin's time, the division of the ratio $DC : DA$ by the ratio $BE : BA$ was called subtraction. In Greek mathematics the theorem was enunciated in the form

$\frac{DC}{DA} = \frac{CF \cdot BE}{FE \cdot BA}$. This explains perhaps why Stevin, when calculating $\frac{CF}{FE}$, refers to the converse of the theorem, the term „converse” not being taken in its ordinary sense.

W E S E N D E twee zijden eens driehouck elck ghedeelt in drie euen deelen: De lini tusschen de twee punten der deeling naest de derde sijde, streckt door des driehouck swaerheys middelpunt.

T'GHEGHEVEN. Laet ABC een driehouck wesen, van t'welck yder sijde AB ende BC ghedeelt sy in drie euen deelen, met de punten D, E, F, G , ende tusschen de punten E, G , naest de derde sijde BC , sy ghetrocken de lini EG . T'BEGHEERDE. Wy moeten bewyfen dat E, G duer des driehouck ABC swaerheys middelpunt streckt

T'BEKEVTSSEL. Laet ons trekken van A tot int middel van BC , de lini AH , sniende EG in I . T'BEWYS. Ouermits AE sulcken reden heeft tot EB , als AG tot GC , soo is EG euewydighe met BC , ende veruolghens EI is euewydighe met BH , daerom ghelijck AE tot EB , alsoo AI tot IH , maer AE is dobbel tot EB door t'ghegheuen, daerom AI is dobbel tot IH , maer wefende AI dobbel tot IH , soo is I t'swaerheys middelpunt des driehouck ABC door het 4^e voorstel, daerom EG streckt door des ghegheuen driehouck swaerheys middelpunt. T'BESLVT. Wefende dan twee syden eens driehouck elck ghedeelt in drie euen deelen, de lini tusschen de twee punten der deeling naest de derde syde, streckt door des driehouck swaerheys middelpunt, t'welck wy bewyfen moesten.



Planum re-
ctilineum.

W E S E N D E ghegheuen een *rechtlinich plat: Sijn swaerheys middelpunt te vinden.

I^e VOORBEELT.

T'GHEGHEVEN. Laet $ABCD$ een ongheschiët vierhouck wesen. T'BEGHEERDE. Wy moeten sijn swaerheys middelpunt vinden.

T'WERCK. Men sal den vierhouck deelen in twee driehoucken met de lini AC , ende vinden het swaerheys middelpunt van elck driehouck, duer het 3^e voorstel, dat van ACB sy E , ende van ACD sy F ,
ende

THEOREM IV.

PROPOSITION V.

Two sides of a triangle each being divided into three equal segments, the line joining the two points of division adjacent to the third side passes through the centre of gravity of the triangle.

SUPPOSITION. Let ABC be a triangle, of which each of the sides AB and AC be divided into three equal segments by the points D, E, F, G , and between the points E, G , adjacent to the third side BC , let there be drawn the line EG . WHAT IS REQUIRED TO PROVE. We have to prove that EG passes through the centre of gravity of the triangle ABC . PRELIMINARY. Let us draw from A to the middle point of BC the line AH , intersecting EG in I . PROOF. Since AE has to EB the same ratio as AG to GC , EG is parallel to BC , and consequently EI is parallel to BH ; therefore, as AE is to EB , so is AI to IH . But AE is double of EB by the supposition, therefore AI is double of IH . But AI being double of IH , I is the centre of gravity of the triangle ABC by the 4th proposition; therefore EG passes through the centre of gravity of the given triangle. CONCLUSION. Two sides of a triangle therefore each being divided into three equal segments, the line joining the two points of division adjacent to the third side passes through the centre of gravity of the triangle, which we had to prove.

PROBLEM II.

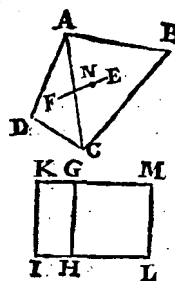
PROPOSITION VI.

Given a rectilineal plane figure: to find its centre of gravity.

EXAMPLE I.

SUPPOSITION. Let $ABCD$ be an irregular quadrilateral. WHAT IS REQUIRED TO FIND. We have to find its centre of gravity. CONSTRUCTION. The quadrilateral shall be divided into two triangles by the line AC , and the centre of gravity of each triangle shall be found by the 3rd proposition. That of ACB shall be

ende de lini EF sal balck wesen. Daer naer salmen maken twee euewydige vierhoucken van een selfde hoogte, als GHK, euen anden driehouck ACD, ende GHLM, euen anden driehouck ACB, daer naer deelende den balck FE in N, alsoo dat den erm NE, sulcken reden hebbe tot den erm NF, als HI tot HL; Ick seg dat N begheerde swaerheys middelpunt is.



Door het 45.
v. 1. B. E.

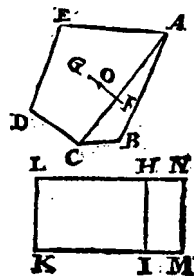
Door het 10.
v. 6. B. E.

II^e VOORBEELT.

TGHEGHEVEN. Laet ABCDE een ongheschickt vijfhouck sijn.

TBEGHEERDE. Wy moeten sijn swaerheys middelpunt vinden.

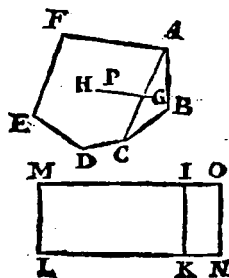
TWERCK. Men sal trecken AC, ende vinden tswaerheys middelpunt des driehouck ACB door het 3^e voorstel, t welck F sy, en vande vierhouck ACDE duer t voorgaende 1^e voorbeelt, t welck G sy, ende de lini FG sal balck wesen, daer naer salmen maken twee euewydighe vierhoucken van een selfde hoogte, als HIKL euen anden vierhouck ACDE, ende HIMN euen anden driehouck ACB, deelende den balck GF in O, alsoo dat den erm OF, sulcken reden hebbe tot den erm OG, als IK tot IM; Ick seg dat O begheerde swaerheys middelpunt is.



III^e VOORBEELT.

TGHEGHEVEN. Laet ABCDEF een ongheschickt seshouck sijn. TBEGHEERDE. Wy moeten sijn swaerheys middelpunt vinden.

TWERCK. Men sal trecken AC, ende vinden tswaerheys middelpunt des driehouck ACB duer het 3^e voorstel, t welck G sy, ende vanden vijfhouck ACDEF, duer het voorgaende 2^e voorbeelt, t welck H sy, ende de lini GH sal balck wesen. Daer naer salmen maken twee euewydighe vierhoucken van een selfde hoogte, als IKLM, euen anden vijfhouck ACDEF, ende IKNO



euen

E , and that of ACD shall be F ; then the line EF will be beam. After this, there shall be constructed two parallelograms of the same height, as $GHIK$ equal to the triangle ACD , and $GHLM$, equal to the triangle ACB , upon which the beam FE shall be divided at N in such a way that the arm NE shall have to the arm NF the same ratio as HI to HL . I say that N is the required centre of gravity.

EXAMPLE II.

SUPPOSITION. Let $ABCDE$ be an irregular pentagon. WHAT IS REQUIRED TO FIND. We have to find its centre of gravity. CONSTRUCTION. The line AC shall be drawn, and the centre of gravity of the triangle ACB shall be found by the 3rd proposition, which shall be F , and that of the quadrilateral $ACDE$ by the preceding 1st example, which shall be G . Then the line FG will be beam. After this, there shall be constructed two parallelograms of the same height, as $HIKL$, equal to the quadrilateral $ACDE$, and $HIMN$, equal to the triangle ACB , upon which the beam GF shall be divided at O in such a way that the arm OF shall have to the arm OG the same ratio as IK to IM . I say that O is the required centre of gravity.

EXAMPLE III.

SUPPOSITION. Let $ABCDEF$ be an irregular hexagon. WHAT IS REQUIRED TO FIND. We have to find its centre of gravity. CONSTRUCTION. The line AC shall be drawn, and the centre of gravity of the triangle ACB shall be found by the 3rd proposition, which shall be G , and that of the pentagon $ACDEF$ by the preceding 2nd example, which shall be H . Then the line GH will be beam. After this, there shall be constructed two parallelograms of the same height, as $IKLM$, equal to the pentagon $ACDEF$, and $IKNO$, equal to the triangle

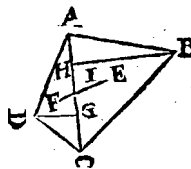
euen anden driehouck ACB , deelende den balck HG in P , alsoo dat den erm PG , sulcken reden hebbe tot den erm PH , als de lini KM tot KN ; Ick seg dat P t'begheerde swaerheyds middelpunt is. Welcke maniere van wercking in allen anderen veelsijdeghe platten ghelijck sal sijn ande voorgaende.

M E R C K T.

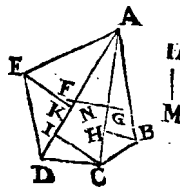
Wy hebben hier bouen voorbeelden beschreuen alwaer t'ghegheuen plat verkeert wort in euenhooghe ende euewydighe vierhoucken, wy connen t'selfde oock doen sonder foodanighe verkeerking, daer af wy verscheden voorbeelden beschrijven sullen als volght.

IIII^e VOORBEELT.

T'GHEGHEVEN. Laet $ABCD$ een ongheschickt vierhouck wesen. T'BEGHEERDE. Wy moeten sijn swaerheyds middelpunt vinden. T'WERCK. Men sal den vierhouck deelen in twee driehoucken, met de lini AC , ende vinden t'swaerheyds middelpunt van elcken driehouck door het 3^e voorstel, dat van ACB sy E , ende vanden driehouck ACD sy F , de lini dan EF is balck. Daer naer salmen trecken DG ende BH , beyde rechthouckich op AC , deylende den balck FE en I , alsoo dat den erm IE , sulcken reden hebbe tot den erm IF , als DG tot BH ; Ick seg dat I t'begheerde swaerheys middelpunt is.

V^e VOORBEELT.

T'GHEGHEVEN. Laet $ABCDE$ een ongheschickt vijfhouck sijn. T'BEGHEERDE. Wy moeten sijn swaerheys middelpunt vinden. T'WERCK. Men sal den vijfhouck deelen in drie driehoucken, met eenighe linien als AD , AC , vindende daer naer het swaerheys middelpunt des vierhouck $ACDE$ duer het 4^e voorbeelt, t'welck F sy, ende des driehouck ACB duer het 3^e voorstel, t'welck G sy, ende de lini FG , is balck. Daer naer ghetrocken BH rechthouckich op AC ; Ende CI met EK rechthouckich op AD , men sal der drie linien AD , AC , HB , vinden de vierde^{*} euerednighe, welke sy LM , deelende^{*} den balck FG in N , alsoo dat den erm NG sulcken reden hebbe tot den erm NF , ghelijck CI met EK , tot LM ; Ick seg dat N het begheerde swaerheyds middelpunt is.



*Proportiona-
lis.
Door het 12.
v. 6. B. E.*

VI VOOR-

ACB , upon which the beam HG shall be divided at P in such a way that the arm PG shall have to the arm PH the same ratio as the line KM to KN . I say that P is the required centre of gravity. This manner of construction will be identical with the preceding in all other polyilateral plane figures.

NOTE.

In the above we have described examples where the given plane figure is transformed into parallelograms of the same height; we can also do the same without such transformation, of which we will describe several examples, as follows.

EXAMPLE IV.

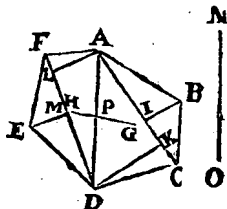
SUPPOSITION. Let $ABCD$ be an irregular quadrilateral. **WHAT IS REQUIRED TO FIND.** We have to find its centre of gravity. **CONSTRUCTION.** The quadrilateral shall be divided into two triangles by the line AC , and the centre of gravity of each triangle shall be found by the 3rd proposition. That of ACB shall be E , and that of the triangle ACD shall be F ; the line EF then is beam. After this, there shall be drawn DG and BH , both at right angles to AC , upon which the beam FE shall be divided at I , in such a way that the arm IE shall have to the arm IF the same ratio as DG to BH . I say that I is the required centre of gravity.

EXAMPLE V.

SUPPOSITION. Let $ABCDE$ be an irregular pentagon. **WHAT IS REQUIRED TO FIND.** We have to find its centre of gravity. **CONSTRUCTION.** The pentagon shall be divided into three triangles by some lines, as AD , AC , after which the centre of gravity of the quadrilateral $ACDE$ shall be found by the 4th example, which shall be F , and that of the triangle ACB by the 3rd proposition, which shall be G ; and the line FG is beam. After this, BH being drawn at right angles to AC , and CI and EK at right angles to AD , the fourth proportional to the three lines AD , AC , HB shall be found, which shall be LM , and the beam FG shall be divided at N in such a way that the arm NG shall have to the arm NF the same ratio as CI with EK to LM . I say that N is the required centre of gravity.

VI. VOORBEELT.

T'GHEGHEVEN. Laet $AB C D E F$ een ongheschickt seshouck sijn. **T'BEGHEERDE.** Wy moeten sijn swaerheyds middelpunt vinden. **T'WERCK.** Men sal den seshouck deelen in vier driehoucken, met eenighe linien als $A C, A D, F D$, vindende daer naer het swaerheys middelpunt des vierhouck $A D C B$ door het 4^e voorbeelt, twelck G sy, ende des vierhouck $A D E F$, twelck H sy, ende de lini $H G$ is balck. Daernaer ghetrocken $B I$ ende $D K$ rechthouckich op $A C$, inghelijcx $A L$ ende $E M$ beyde rechthouckich op $F D$, men sal der drie linien welcke eerste $F D$, de tweede $A C$, de derde $B I$ met $K D$, vinden de vierde euerednighe, welcke $N O$ sy, deelende den balck $H G$ in P , also dat den erm $P G$, sulcken reden hebbe tot den erm $P H$, ghelijck $A I$ met $E M$, tot $N O$; Ick leg dat P het begheerde swaerheys middelpunt is. En soo salmen voort mueghen varen met ander veelhouckeghe platten.



T'BEWYS. Ghelijck int eerste voorbeelt $H I$ tot $H L$, also den erm $N E$ tot den erm $N F$, maer ghelijck $H I$ tot $H L$, also den vierhouck $G H I K$, tot den vierhouck $G H L M$, ghelijck dan $G H I K$ tot $G H L M$, also $N E$ tot $N F$, maer $G H I K$ is euen an den driehouck $A C D$, ende $G H L M$ an den driehouck $A C B$ door t'werck, ghelijck dan den driehouck $A C D$ tot $A C B$, also den erm $N E$ tot $N F$. Het punt dan N is (door het 1^{ste} voorstel des 1^{sten} boucx) des vierhouck swaerheys middelpunt. Sghelijcx sal oock bewys sijn des 2^{den} ende 3^{den} voorbeelts.

T'vierde voorbeelt is openbaer als wy bewesen hebben dat ghelijck $D G$, tot $H B$, also den driehouck $A C D$, tot $A C B$ in deser voughen: Nemende $A C$ voor hoochde, ende $D G$ ende $H B$ voor gronden, soo heeft den rechthouck begrepen onder $A C$ ende $D G$, sulcken reden tot den rechthouck onder $A C$ ende $H B$, ghelijck $D G$ tot $H B$; Maer ghelijck dien rechthouck tot desen, alsoo de driehouck $A C D$ tot $A C B$, want elck driehouck is sijn rechthoucx helft, ghelijck dan $D G$ tot $H B$, also den driehouck $A C D$ tot $A C B$.

Des 5^{den} voorbeelts bewys sal oock claer sijn als wy bewesen hebben, dat ghelijck $E K$ met $I C$ tot $L M$, also den vierhouck $A C D E$ tot den driehouck $A C B$, aldus: Anghesien $L M$ vierde euerednighe is der drie $A D, A C, H B$, de rechthouck begrepen onder $A D$ ende $L M$, sal euen sijn an den rechthouck begrepen onder $A C$ ende $H B$, Laet ons nu $E K, I C, L M$, ansien voor gronden, wiens ghemeene hoochde $A D$; Maer ghelijck die gronden

K tot mal-

EXAMPLE VI.

SUPPOSITION. Let $ABCDEF$ be an irregular hexagon. **WHAT IS REQUIRED TO FIND.** We have to find its centre of gravity. **CONSTRUCTION.** The hexagon shall be divided into four triangles by some lines, as AC , AD , FD , after which the centre of gravity of the quadrilateral $ADCB$ shall be found by the 4th example, which shall be G , and that of the quadrilateral $ADEF$, which shall be H . And the line HG is beam. After this, BI and DK being drawn at right angles to AC , likewise AL and EM both at right angles to FD , the fourth proportional shall be found to the three lines, of which the first is FD , the second AC , the third BI with KD ; this shall be NO , upon which the beam HG shall be divided at P in such a way that the arm PG shall have to the arm PH the same ratio as AL with EM to NO . I say that P is the required centre of gravity. In the same way one may proceed with other polygonal plane figures. **PROOF.** As in the first example HI is to HL , so is the arm EN to the arm NF . But as HI is to HL , so is the quadrilateral $GHIK$ to the quadrilateral $GHLM$; therefore as $GHIK$ is to $GHLM$, so is NE to NF . But $GHIK$ is equal to the triangle ACD , and $GHLM$ to the triangle ACB , by the construction; therefore, as the triangle ACD is to ACB , so is the arm NE to NF . The point N therefore (by the 1st proposition of the 1st book) is the centre of gravity of the quadrilateral. The same proof holds for the 2nd and the 3rd example.

The fourth example is manifest when we have proved that as DG is to HB , so is the triangle ACD to ACB , as follows. Taking AC for the height, and DG and HB for bases, the rectangle contained by AC and DG has to be rectangle contained by AC and HB the same ratio as DG to HB . But as the former rectangle is to the latter, so is the triangle ACD to ACB , for each triangle is half of its rectangle. Therefore, as DG is to HB , so is the triangle ACD to ACB .

The proof of the 5th example will also be manifest when we have proved that as EK with IC is to LM , so is the quadrilateral $ACDE$ to the triangle ACB , as follows. Since LM is fourth proportional to the three lines AD , AC , HB , the rectangle contained by AD and LM will be equal to the rectangle contained by AC and HB : Let us now consider EK , IC , LM as bases, whose common height shall be AD . But as these bases are to one another, so are the rectangles contained

1. v. 6. B. E. tot malcanderen, alsoo de rechthoucken begrepen onder haer ende hare ghemeene hoochde, daerom oock ghelijck de twee gronden E K, I C, tot den grondt L M, alsoo dier gronden rechthoucken tot deses grondts rechthouck; maer die twee rechthoucken sijn elck het dobbel haers driehoucx; Ghelijck dan E K met I C tot L M, also het dobbel vanden vierhouck A C D E tot den rechthouck begrepen onder A D ende L M: Maer desen is euen an den rechthouck begrepen onder A C ende H B als vooren betoocht is, ende de selue rechthouck begrepen onder A C ende H B is het dobbel des driehoucx A C B, daerom ghelijck E K met I C tot L M, alsoo het dobbel des vierhoucx A C D E tot het dobbel des driehoucx A C B, ende veruolghens ghelijck E K met I C tot L M, alsoo den vierhouck A C D E tot den driehouck A C B, waer uyt de reste openbaer is. Tbewys van het 6^e voorbeelt is duer dit oock kennelick ghenouch. T B E S L V Y T. Wefende dan ghegheuen een rechthouckich plat: Wy hebben sijn swaerheyds middelpunt gheuonden naer den eysch.

M E R C K T.

Commentarius in quadraturam parabolae.

MY is onder het drucken ter hands ghecomen, Fredric Commandins * verclaring ouer de viercanting der Brantsne van Archimedes, alwaer hy onder het 6^e voorstel de manier beschrijft, om t swaerheys middelpunt te vinden van yder rechtlinich plas, ende dat op een ander wyse als de twee voorgaende. So ymant tottet ouersien der selue begheerich waer, salse daer vinden.

V. VERTOOGH.

VII. VOORSTEL.

Parallelis.

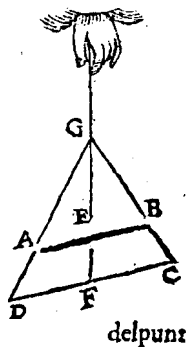
HET swaerheys middelpunt des vierhoucx met twee * euewydighe sijden, is inde lini tusschen dier sijden middelpunten.

T G H E G H E V E N. Laet A B C D een vierhouck sijn, diens twee euewydighe sijden A B ende D C, ende de lini uyt E middel van A B, tot F middel van D C, sy E F.

T B E G H E E R D E. Wy moeten bewysen dat t swaerheys middelpunt des vierhoucx A B C D inde lini E F is. T B E R E Y T S E L. Laet de drie linien D A, F E, C B, voortghetrocken worden, welcke om de * eueredenheit der linien A E, E B, D F, F C, vergaren sullen in een selfde punt t welck G sy.

Proportioni.

T B E W Y S. Laet ons den driehouck G D C ophanghen byde lini G F, ende het deel G F C sal euestalwichtich sijn, teghen G F D door het 2^e voorstel, waer deur oock t swaerheys mid-



by them and their common height; therefore also, as the two bases EK , IC are to the base LM , so are the rectangles on the former bases to the rectangle on the latter base. But those two rectangles are each double of their triangle. Therefore, as EK with IC is to LM , so is the double of the quadrilateral $ACDE$ to the rectangle contained by AD and LM . But the latter is equal to the rectangle contained by AC and HB , as has been argued above, and this same rectangle contained by AC and HB is double of the triangle ACB . Therefore, as EK with IC is to LM , so is the double of the quadrilateral $ACDE$ to the double of the triangle ACB ; and consequently, as EK with IC is to LM , so is the quadrilateral $ACDE$ to the triangle ACB , from which the rest is manifest. The proof of the 6th example is also evident enough from this. **CONCLUSION.** Given therefore a rectilineal plane figure, we have found its centre of gravity, as required.

NOTE.

While this book was being printed, there came into my hands Frederick Commandinus' explanation of the quadrature of the parabola by Archimedes¹⁾, where in the 6th proposition he describes the method for finding the centre of gravity of any rectilineal plane figure, such in a manner different from the preceding two. If anyone should be desirous to see this, he may find it there.

THEOREM V.

PROPOSITION VII.

The centre of gravity of the quadrilateral with two parallel sides is in the line joining the middle points of those sides.

SUPPOSITION. Let $ABCD$ be a quadrilateral, whose two parallel sides shall be AB and DC , while the line from E , the middle point of AB , to F , the middle point of DC , shall be EF . **WHAT IS REQUIRED TO PROVE.** We have to prove that the centre of gravity of the quadrilateral $ABCD$ is in the line EF . **PRELIMINARY.** Let the three lines DA , FE , CB be produced, which on account of the proportionality of the lines AE , EB , DF , FC will meet in one and the same point, which shall be G . **Proof.** Let us hang the triangle GDC by the line GF then the part GFC will be of equal apparent weight to GFD by the 2nd propo-

¹⁾ *Archimedis Opera Nonnulla a Federico Commandino Urbinate nuper in Latinum conversa, et commentariis illustrata.* Venetiis 1558. *Commentarii*, p. 22 v.

delpunt des driehouck GDC inde lini GF is. Maer den driehouck GEB , is oock euefaltwichtich teghen den driehouck GEA , daerom van euefaltwichtighe ghetrocken euefaltwichtighe, de resten als de vierhoucken EFA , EFB , fullen noch euefaltwichtich bliuen, ende haer swaerheys middelpunt noch inde lini GF , maer niet uyt de form in EG ; Nootfacecklick dan in EF . **T B E S L V Y T.** Het swaerheyds middelpunt dan des vierhouck met twee euewydighe sijden, is inde lini tusschen dier sijden middelpunten, t welck wy bewyfen moesten.

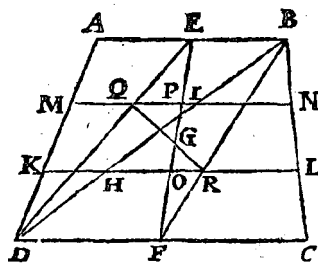
VI. VERTOOGH.

VIII. VOORSTEL.

H E T swaerheys middelpunt des vierhouck met twee euewydighe sijden, deelt de lini tusschen dier euewydighens middelpunten also, dat het stick naer de minste sijde, tot het ander, sulcken reden heeft, als tweemaal de meeste sijde met eenmaal de minste, tot tweemaal de minste met eenmaal de meeste.

T G H E G H E V E N. Laet $ABCD$ een vierhouck wesen met twee euewydighe sijden AB, DC , ende de lini tusschen haer middelpunten sy EF , ende t swaerheyds middelpunt sy G . **T B E G H E E R D E.** Wy moeten bewyfen dat ghelijck tweemaal DC met eenmaal AB , tot tweemaal AB met eenmaal DC , also GE tot GF . **T B E R E Y T S E L.** Laet ghetrocken worden DB , ende ghedeelt in drie euen deelen met de punten H, I , ende door de selue ghetrocken worden KL , ende MN , euewydich van DC , sniende EF in O en P . Daer naer de lini DE , sniende MI in Q , Ende BF sniende KL in R , Ende ten laetsten RQ .

T B E W Y S. Anghesien het swaerheyds middelpunt des driehouck BDC , is in BF , duer het 2^e voorstel, ende oock in HL duer het 3^e voorstel, soo is R , sijn swaerheys middelpunt, en om de selue reden is Q swaerheys middelpunt des driehouck ABD , ende QR is dier driehoucken balck, inden welcken haer beyder, dat is des vierhouck $ABCD$, swaerheys



K 2 middelp.

sition, in consequence of which the centre of gravity of the triangle GDC is also in the line GF . But the triangle GEB is also of equal apparent weight to the triangle GEA . Therefore, equal apparent weights being subtracted from equal apparent weights, the remainders, viz. the quadrilaterals $EFDA$, $EFCB$, will still remain of equal apparent weight¹⁾, and their centres of gravity will still be in the line GF , but not outside the figure in EG ; therefore it is necessarily in EF . CONCLUSION. The centre of gravity therefore of the quadrilateral with two parallel sides is in the line joining the middle points of those sides, which we had to prove.

THEOREM VI.

PROPOSITION VIII.

The centre of gravity of the quadrilateral with two parallel sides divides the line joining the middle point of those parallel sides in such a way that the segment adjacent to the shorter side has to the other the same ratio as twice the longer side plus once the shorter to twice the shorter plus once the longer.

SUPPOSITION. Let $ABCD$ be a quadrilateral with two parallel sides AB , DC , and the line joining their middle points shall be EF , and the centre of gravity shall be G . WHAT IS REQUIRED TO PROVE. We have to prove that as twice DC plus once AB is to twice AB plus once DC , so is GE to GF . PRELIMINARY. Let DB be drawn, and let this be divided into three equal segments by the points H , I , and let there be drawn through these points KL and MN , parallel to DC , intersecting EF in O and P . After this, let the line DE be drawn intersecting MI in Q , and BF intersecting KL in R , and finally RQ . PROOF Since the centre of gravity of the triangle BDC is in BF , by the 2nd proposition, and also in HL , by the 5th proposition, R is its centre of gravity. And for the same reason Q is the centre of gravity of the triangle ABD , and QR is the beam of these triangles, in which lies the centre of gravity of the two, that is of the quadrilateral

¹⁾ As has been remarked in notes 2 to p. 177 and p. 179, this inference is not generally valid.

middelpunt is, t'selue is oock in E F duer het 7^e voorstel, daerom G is t'swaerheys middelpunt des vierhoucx A B C D. Maer want de twee driehoucken C D B ende A B D sijn tusschen twee euewydighe A B ende D C, so sijn sy inde reden van haer gronden, dat is ghelijck den driehouck C D B tot A B D, alsoo D C tot A B: Maer ghelijck den driehouck C D B tot A D B, also den erm G Q tot G R duer het 1^e voorstel des 1^{en} boucx, ghelijck dan D C tot A B, alsoo G Q tot G R; maer ghelijck G Q tot G R, alsoo P G tot G O (want sy tusschen de euewydeghe M N, K L sijn) ghelijck dan D C tot A B, alsoo G P tot P O, daerom oock ghelijck tweemael D C met eenmael A B, tot tweemael A B met eenmael D C, also tweemael G P met eenmael G O, tot tweemael G O met eenmael G P. Maer G E is euen an tweemael G P met eenmael G O, ende G F is euen an tweemael G O met eenmael G P, daerom ghelijck tweemael D C met eenmael A B, tot tweemael A B met eenmael D C, alsoo G E tot G F. T'beslyt. Het swaerheys middelpunt dan des vierhoucx met twee, &c.

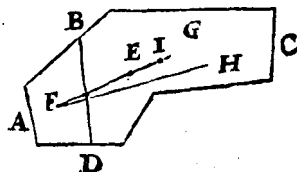
III. EYSCH.

IX. VOORSTEL.

W E S E N D E ghegheuen t'swaerheys middelpunt eens plats ende sijns deels, wiens reden an t'ander deel kennelick is: Het swaerheys middelpunt van t'ander deel te vinden.

I. VOORBEELT.

T'GHEGHEVEN. Laet A B C D een rechtlinich plat wesen, diens swaerheys middelpunt E, ende B D A deel des plats, wiens swaerheys middelpunt F. T'BEGHEERDE. Wy moeten t'swaerheys middelpunt vinden des ander deels B D C. T'WERCK. Men sal trecken F E tot in G, alsoo dat F E sulcken reden hebbe tot E G, als t'stick B D C tottet stick B D A: Ick seg dat G t'begheerde swaerheys middelpunt is des ander deels B D C. T'BEWYS. Anghesien t'swaerheys middelpunt van B D A is F, ende des heels A B C D is E, soo moet t'swaerheys middelpunt des ander deels B D C sijn in de rechte F E oneindelick voortghetrocken. Want soot muelghelick waer, latet daer buyten wesen als H, ende laet ons trecken F H, het swaerheys middelpunt dan des heels



Hypothesin.

sal in F H sijn, maer dat is teghen * t'ghestelde, wantet E is; Ten is dan niet buyten F E oneindelick voortghetrocken maer daer in. Latet nu wesen (foot

ABCD. This centre of gravity is also in *EF* by the 7th proposition, therefore *G* is the centre of gravity of the quadrilateral *ABCD*. But because the two triangles *CDB* and *ABD* are contained between two parallel lines *AB* and *DC*, they are to one another in the ratio of their bases, that is: as the triangle *CDB* is to *ABD*, so is *DC* to *AB*. But as the triangle *CDB* is to *ADB*, so is the arm *GQ* to *GR*, by the 1st proposition of the 1st book; therefore as *DC* is to *AB*, so is *GQ* to *GR*. But as *GQ* is to *GR*, so is *PG* to *GO* (for they are contained between the parallel lines *MN*, *KL*); therefore, as *DC* is to *AB*, so is *GP* to *PO*, and therefore also as twice *DC* plus once *AB* is to twice *AB* plus once *DC*, so is twice *GP* plus once *GO* to twice *GO* plus once *GP*. But *GE* is equal to twice *GP* plus once *GO*, and *GF* is equal to twice *GO* plus once *GP*. Therefore as twice *DC* plus once *AB* is to twice *AB* plus once *DC*, so is *GE* to *GF*. CONCLUSION. The centre of gravity therefore of the quadrilateral with two etc.

PROBLEM III.

PROPOSITION IX.

Given the centre of gravity of a plane figure, and of one part of it, whose ratio to the other part is known: to find the centre of gravity of the other part.

EXAMPLE I.

SUPPOSITION. Let *ABCD* be a rectilineal plane figure, whose centre of gravity shall be *E*, while *BDA* shall be a part of the figure, whose centre of gravity shall be *F*. WHAT IS REQUIRED TO FIND. We have to find the centre of gravity of the other part *BDC*. CONSTRUCTION. The line *FE* shall be drawn up to *G*, in such a way that *FE* shall have to *EG* the same ratio as the part *BDC* to the part *BDA*. I say that *G* is the required centre of gravity of the other part *BDC*. PROOF. Since the centre of gravity of *BDA* is *F*, and that of the whole *ABCD* is *E*, the centre of gravity of the other part *BDC* must be in the straight line *FE* produced indefinitely. For, if it were possible, let it be outside said line, as *H*, and let us draw *FH*. The centre of gravity of the whole will then be in *FH*, but this is contrary to the supposition, because it is *E*. It is not therefore outside *FE* produced indefinitely, but in it. Now let it be (if this were possible) between the points *E* and *G*, as *I*. But then the longer arm *EF* will have to the

(foot mueghelick waer) tusschen de punten E G als I; Maer den langsten erm EF sal dan meerder reden hebben tot den cortsten EI, dan de swaerste swaerheyt B D C tot de lichtste B D A, twelck teghen het 1^e voorstel des 1^{en} boucx waer. Ten is dan tusschen E G niet: Sghelijcx salmen oock bethoonen dattet bouen G niet en is. Tis dan nootfaecklick G, t'welck wy bewyfen moesten.

II VOORBEELT.

TGHEGHEVEN. Laet A B C D een rondt wesen diens *half-*Semidiam-* middellini E A, ende swaerheys middelpunt E sy, ende trondt A F G H, *ter*. deel des rondts A B C D, ende sijn swaerheys middelpunt I, ende *mid-*Diameter.* dellini A G. T'BE GHEERDE. Wy moeten het swaerheys middelpunt vinden des ander deels, dat is der maen A B C D H G F.

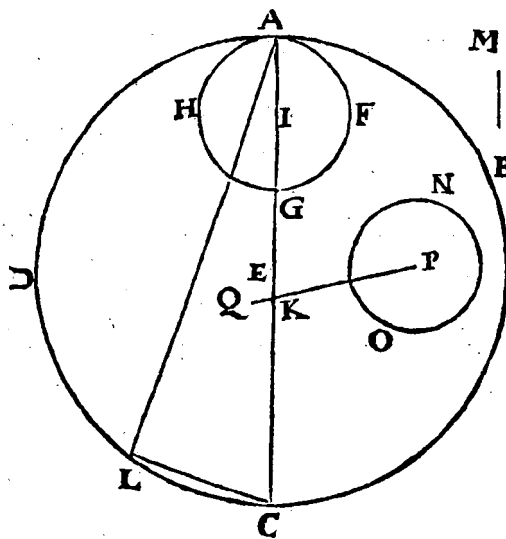
TWERCK. Men sal I E voorttrecken tot in K, also dat I E sulcken reden hebbe tot E K, als de maen A B C D H G F tot het rondt A F G H, ende K sal rbe-

gheerde swaerheys middelpunt wesen, Daer af r'bewys ghelijck sal sijn an voorgaende. Maer om de reden dier maen tot dat rondt te vinden, men sal trecké C L enen met A G, daernaer A L, vindende de derde everednighe welcker eerste A L, de tweede L C, ende de derde sy M, Ende A L tot M,

sal de reden sijn der maen tot het rondt A F G H. Want ouermits A L C rechthouck is (reden dat sy int half rondt staet) het ront diens middellini A L, sal enen sijn ande maen, ende A L tot M is de *ghedobbelde reden van A L tot L C, dat is van A L tot A G, daerom &c. *31.v.3. B.E. Duplicata ratio.*

Sghelijcx soudemen voortvaren dat int rondt A B C D meer ronden ghebraken; by voorbeelt het rondt N O, wiens middelpunt P. Want

K ; P K



11.2.6. B.E.

shorter EI a greater ratio than the heavier gravity BDC to the lighter BDA , which would be contrary to the 1st proposition of the 1st book. It is not therefore between E and G . In the same way it can also be shown not to be above G . It is therefore necessarily G , which we had to prove.

EXAMPLE II.

SUPPOSITION. Let $ABCD$ be a circle, whose semi-diameter shall be EA , and its centre of gravity E , while the circle $AFGH$ shall be a part of the circle $ABCD$, and its centre of gravity I , and its diameter AG . **WHAT IS REQUIRED TO FIND.** We have to find the centre of gravity of the other part, that is of the lune $ABCDHGF$. **CONSTRUCTION.** IE shall be produced to K , in such a way that IE shall have to EK the same ratio as the lune $ABCDHGF$ to the circle $AFGH$; then K will be the required centre of gravity, the proof of which will be identical with the preceding one. But in order to find the ratio of the said lune to the said circle, CL shall be drawn equal to AG ; after that AL , upon which the third ¹⁾ proportional shall be found, the first of which shall be AL , the second LC , and the third shall be M . Then AL to M will be the ratio of the lune to the circle $AFGH$. For since ALC is right-angled (because it is contained in a semicircle), the circle having AL as diameter will be equal to the lune, and AL to M is the duplicated ratio ²⁾ of AL to LC , that is of AL to AG ; therefore, etc.

In the same way one would proceed if more circles were missing from the circle $ABCD$, for example the circle NO , whose centre is P . For PK being produced to Q , in such a way that PK should have to KQ the same ratio as the

¹⁾ M is the third proportional to AL and LC , i.e. $AL : LC = LC : M$. Now $\frac{\text{circle } (AC)}{AC^2} = \frac{\text{circle } (AG)}{LC^2} = \frac{\text{lune}}{AL^2}$.

But since $AL : M = AL^2 : LC^2$, $\frac{\text{lune}}{\text{circle } (AG)} = \frac{AL}{M}$.

²⁾ In the ancient terminology to which we referred in note ¹⁾ to p. 233, doubling of a ratio means squaring.

PK voortghetrocken tot in Q, alsoo dat PK sulcken reden hadde tot K Q, als het restende tot het rondt N O, so soude Q t'begheerde swaerheys middelpunt sijn. Ende alsoo met allen anderen formen welcker deelen reden kennelick is. **T'beslyt.** Welende dan ghegheuen de swaerheys middelpunten eens plats ende sijns deels, wienus reden an t'ander deel kennelick is: wy hebben het swaerheys middelpunt gheuonden des ander deels naer den eysch.

VII. VERTOOGH.

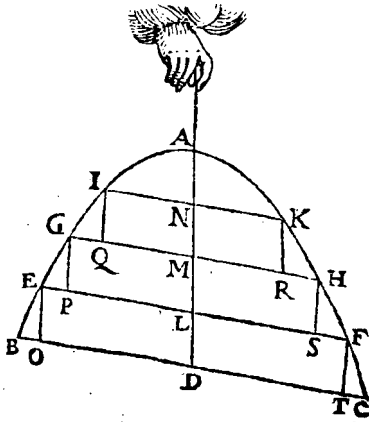
X. VOORSTEL.

Parabola.

YDER * brandtsnees swaerheys middelpunt is in haer middellini.

T'ghegheuen. Laet ABCD een brandtsne sijns diens middellini AD. **T'begheerde.** Wy moeten bewysen dat t'swaerheys middelpunt inde lini AD is. **T'bereyt sel.** Laet ons trekken de linien EF, GH, IK, ewewydighe van BC, ende liniende AD in L, M, N, daer naer EO, GP, IQ, KR, HS, FT, ewewydighe van AD.

T'bewys. Ouermidts EF ewewydighe is van BC, ende EO, FT, van LD, so sal EFTO ewewydich vierhouck sijn, wienus EL euen is met LF, oock met OD ende DT, waer duer t'swaerheys middelpunt van EFTO, in DL is duer het 1^e voorstel, Ende om de selue reden sal t'swaerheys middelpunt des ewewydich vierhouck GHSP in LM wesen, ende van IKRQ in MN, ende veruolghens t'swaerheys middelpunt der form IKRHSFTO EP G Q, ghemaectt vande voornoemde drie vierhoucken sal inde lini ND oft AD sijn. Maer hoe datter sulcke vierhoucken meet gheschreuen worden, hoe dattet verschil des brandtsnees ABC, ende der binnenschreuen form van die vierhoucken vergaert, minder is, wy connen dan door dat on-eindelick naerden sulck een form binnen de brandtsne stellen, dattet verschil tusschen haer en de brandtsne, minder sy dan eenich ghegheuen plat hoe cleen het sy, waer uyt volght, dat stellende AD als swaerheys middellini, so sal t'staltwicht des deels ADC, min verschillen van t'staltwicht des deel ADB, dan eenich plat datmen



remainder to the circle NO , Q would be the required centre of gravity. And the same holds for all other figures the ratio of whose parts is known. **CONCLUSION.** Given therefore the centres of gravity of a plane figure and of a part thereof, whose ratio to the other part is known: we have found the centre of gravity of the other part, as required.

THEOREM VII.

PROPOSITION X.

The centre of gravity of any parabola is in its diameter.

SUPPOSITION. Let $ABCD$ be a parabola, whose diameter shall be AD . **WHAT IS REQUIRED TO PROVE.** We have to prove that the centre of gravity is in the line AD . **PRELIMINARY.** Let us draw the lines EF , GH , IK , parallel to BC and intersecting AD in L , M , N , and then EO , GP , HQ , KR , HS , FT , parallel to AD . **PROOF** Since EF is parallel to BC , and EO , FT to LD , $EFTO$ will be a parallelogram, in which EL is equal to LF , and also to OD and DT , in consequence of which the centre of gravity of $EFTO$ is in DL , by the 1st proposition. And for the same reason the centre of gravity of the parallelogram $GHSP$ will be in LM , and that of $IKRQ$ in MN ; and consequently the centre of gravity of the figure $IKRHSFTOEPPGQ$, composed of the aforesaid three quadrilaterals, will be in the line AD or AD . But the more of such quadrilaterals there are inscribed, the less will be the difference between the parabola ABC and the inscribed figure of those quadrilaterals. We can therefore, by infinite approximation, place such a figure within the parabola that the difference between said figure and the parabola shall be less than any given plane figure; however small, from which it follows that, AD being taken as centre line of gravity¹⁾, the apparent weight of the part ADC will differ less from the apparent weight of the

¹⁾ See Note 3 to p. 227.

datmen soude connen gheuen, hoe cleen het sy, waer nytick aldus strije:

- A. Neuen alle verschillende statfwaerbeden, can een swaerheyt gheslelt worden minder dan haer verschil;
- O. Neuen dese statfwaerbeden ADC ende ADB , en can gheen swaerheyt gheslelt worden minder dan haer verschil;
- O. Dese statfwaerbeden dan ADC ende ADB en verschillen niet.

Daerom AD is swaerheys middellini, ende veruolghens het swaerheys middelpunt des branders ABC is in haer. **T B E S L V Y T.** Yder brandtsnees swaerheys middelpunt dan, is in haer middellini, t welck wy bewyfen moesten.

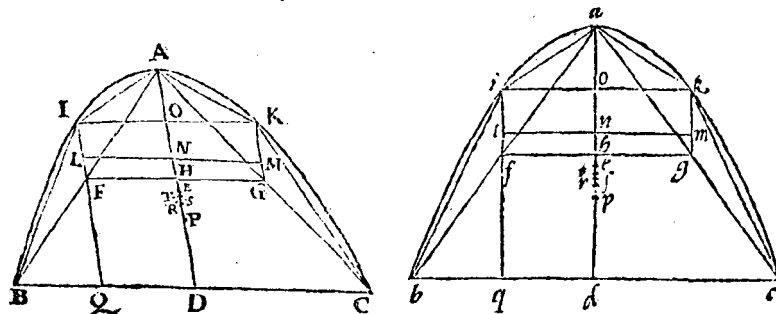
VIII. VERTOOC.

XI. VOORSTEL.

ALLER brantfneens middellinien worden van het swaerheys middelpunt *Proportiona-
liker.* eueredelick ghedeelt.

T G H E G H E V E N. Laet $ABCD$ ende $abcd$ twee onghelijke brantfneen sijn, diens middellinien AD , ende ad , ende swaerheys middelpunten E , ende e . **T B E G H E E R D E.** Wy moeten bewyfen dat ghelijck AE tot ED , alsoo ae tot ed .

T B E R E Y T S E L. Laet ons trecken de linien AB , AC , die deelende in haer middelen F , G , ende trecken FG sniende AD in H , daer naer FI ende GK ewewydghe van AD , ende daer naer IA , IB , KA , KC , ende laet ons stellen L in IF , alsoo dat IL dobbel sy an LF , (ghelijcx M ; alsoo dat KM dobbel sy an MG , ende laet ons trecken LM , sniende AD in N , ende IK , sniende AD in O , ende laet ons stellen P , alsoo dat AP dobbel sy an PD , ende laet ons IF voorttrecken tot



Q inden grondt BC . Nu anghesien AP dobbel is an PD , so is P swaerheys middelpunt des driehoucx ABC , ende omme de selue reden L , M , swaerheys

part ADB than any plane figure that might be given, however small, from which I argue as follows ¹⁾:

- A. *Beside any different apparent gravities there can be placed a gravity less than their difference;*
- O. *Beside the present apparent gravities ADC and ADB there cannot be placed any gravity less than their difference;*
- O. *Therefore the present apparent gravities ADC and ADB do not differ.*

Therefore AD is centre line of gravity, and consequently the centre of gravity of the parabola ABC is in this line. CONCLUSION. The centre of gravity therefore of any parabola is in its diameter, which we had to prove.

THEOREM VIII.

PROPOSITION XI.

The diameters of all parabolas are proportionally divided by the centre of gravity.

SUPPOSITION. Let $ABCD$ and $abcd$ be two dissimilar parabolas, whose diameters shall be AD and ad , and the centres of gravity E and e . WHAT IS REQUIRED TO PROVE. We have to prove that as AE is to ED , so is ae to ed . PRELIMINARY. Let us draw the line AB , AC , dividing them in their middle points F , G , and let us draw FG , intersecting AD in H ; after this, FI and GK parallel to AD , and then IA , IB , KA , KC . And let us take L in IF in such a way that IL shall be double of LF , and in the same way M so that KM shall be double of MG , and let us draw LM intersecting AD in N , and IK intersecting AD in O . And let us take P in such a way that AP shall be double of PD , and let us produce IF to Q in the base BC . Now since AP is double of PD , P is the centre of gravity of the triangle ABC , and for the same reason L , M are the centres of

¹⁾ See note 2 to p. 143.

20. v. 1. B.
Appol.

Proportional.
loc.

swaerheys middelpunten der twee driehoucken A B I, ende A C K, en veruolghens, want sy euen sijn, soo is N haer beyde swaerheys middelpunt. N P dan is balck, de selue ghedeelt in R, alsoo dat den erm N R sy tot R P, als den driehouck A B C tot de twee driehoucken A B I, A C K, dat is, als 4 tot 1 (want alle brantsne is tot den driehouck als A B C ghelijck 4 tot 3, duer het 24 voorstel der viercanting des brantsnees van Archim. daerom, &c.) Laet ons nu dergelijcke linien ende punten oock beschrijuen inde brantsne *abc*. T'BEWYS. Ghelijck A D tot A O, alsoo het viercant van D B tottet viercant van O I; Maer D Q is euen an O I, ende D Q is den helft van D B (want F is t'middel van A B, ende F Q is euewydich van A D) daerom het viercant van D B, is viervoudich an t'viercant van D Q, ofte van O I, ende veruolghens A D is viervoudich tot A O, daerom A O is $\frac{1}{4}$ van A D, ende O H oock $\frac{1}{4}$ (want A H is den helft van A D, ouermits F G ghetrocken is uyt de middelen van A B, A C) daerom doet N H $\frac{1}{2}$ van A D, daer toe ghedaen H D $\frac{1}{2}$, comt voor N D $\frac{2}{3}$, daer af ghetrocken P D $\frac{1}{3}$, rest voor P N $\frac{1}{3}$: Maer N R is viervoudich tot R P, daerom R P doet $\frac{1}{4}$, daer toe P D $\frac{1}{3}$, doet voor R D $\frac{7}{12}$, daerom R A de reste der lini, doet $\frac{5}{12}$, Ghelijck dan 37 tot 23, alsoo A R tot R D, ende met de selue reden is bethoont dat *ar* tot *rd*, oock is als 37 tot 23. Dese twee rechteghe formen dan ghelijckelick beschreuen in verscheyden brandtsneen, hebben het swaerheys middelpunt in haer middellinien, alsoo dat de deelen onder malcanderen euerednich sijn. Ende so wy inde brandtsnekens B I, I A, A K, K C, driehoucken beschreuen, soo ghedaen is inde brantsneen A B I, A C K, vindende daer naer t'swaerheys middelpunt des heels binnescreuen rechtlinich plats, t'welck ick neem dat hier S soude wesen, ende daer *s*, wy souden inder selue voughen als vooren bethoonen, dat ghelijck A S tot S R, alsoo *a* tot *s*. Maer wy connen duer sulck oneindelick inschriuen der rechtlinighe formen oneindelick naerderen nae E, ende *e*, ende ghelijckideghe platten sullen alrijt der middelliniens A D twee sticken euerednich ghedeelt hebben duer haer swaerheys middelpunt, ende veruolghens de heele brantsneen A B C, *abc*, sullen die deelen euerednich hebben. Want laet (soot mueghelick waer) T t'swaerheys middelpunt sijn des brantsneen A B C, ende *e* van *abc*, ende laet ons teekenen *t*, dat ghelijck E T tot T S, alsoo *e* tot *t*. Nu als men duer t'inschrijuen veelsidegher formen in *abc*, sal ghecomen sijn tot *t*, men sal met ghelijcke veelsidegher formen in A B C, ghecomen sijn tot T, daerom T sal t'swaerheys middelpunt sijn der binneschreuen form, ende oock des heelen brantsneen A B C, t'welck ongheschickt is. T'BESELYT. Aller brantsneen middellinien dan, worden van het swaerheys middelpunt eueredelick ghedeelt, t'welck wy bewysen moesten.

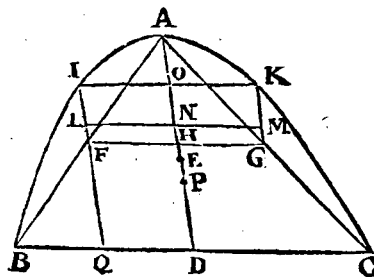
gravity of the triangles ABI and ACK ; and consequently, because they are equal, N is the centre of gravity of both. NP therefore is beam, which shall be divided at R in such a way that the arm NR shall have to RP the same ratio as the triangle ABC to the two triangles ABI , ACK , that is 4 to 1 (for any parabola has to the triangle, as ABC , the ratio of 4 to 3, by the 24th proposition of the quadrature of the parabola of Archimedes¹⁾, therefore, etc.). Now let us also mark similar lines and points in the parabola abc . PROOF. As AD is to AO , so is the square of DB to the square of OI . But DQ is equal to OI , and DQ is equal to one-half DB (for F is the middle point of AB , and FQ is parallel to AD), therefore the square of DB is equal to four times the square of DQ or OI , and consequently AD is equal to four times AO ; therefore AO is $\frac{1}{4} AD$, and OH is also $\frac{1}{4}$ (for AH is equal to one-half AD , since FG has been drawn from the middle points of AB , AC). Therefore NH makes $\frac{1}{12} AD$. If to this is added HD ($\frac{1}{2}$), ND becomes $\frac{7}{12}$. If from this is subtracted PD ($\frac{1}{3}$), there remains for PN $\frac{1}{4}$. But NR is equal to four times RP , therefore RP makes $\frac{1}{20}$. If to this is added PD ($\frac{1}{3}$), RD becomes $\frac{23}{60}$. Therefore RA , the remainder of the lines, makes $\frac{37}{60}$. Therefore as 37 is to 23, so is AR to RD , and in the same way it is shown that ar is also to rd as 37 to 23. These two rectilinear figures therefore, similarly inscribed in different parabolas, have the centre of gravity in their diameters, so that the segments are proportional to one another. And if we inscribed triangles in the small parabolas BI , IA , AK , KC , as has been done in the parabolas ABI , ACK , finding thereafter the centre of gravity of the whole inscribed rectilinear plane figure, which I assume would be S in the first figure and s in the second, we should show in the same way as above that as AS is to SR , so is as to sr . But we can, by such infinite inscription of rectilinear figures, approximate infinitely to E and e , and equilateral plane figures will always have the two segments of the diameters AD divided proportionally at their centre of gravity. And consequently the complete parabolas ABC , abc will have these segments proportional. For (if this were possible) let T be the centre of gravity of the parabola ABC , and e of abc , and let us mark t so that as ET is to TS , so et to ts . Now, when by inscribing polyilateral figures in abc , the point t has been reached²⁾, with similar polyilateral figures in ABC the point T will have been reached. Therefore T will be the centre of gravity of the inscribed figure, and also of the complete parabola ABC , which is absurd. CONCLUSION The diameters therefore of all parabolas are proportionally divided by the centre of gravity, which we had to prove.

¹⁾ The ratio of the triangle ABC to the sum of the triangles ABI and ACK can indeed be derived from the Archimedean proposition on the ratio of a parabolic segment to its inscribed triangle, but it is not logical to do this, the said proposition being demonstrated with the aid of this ratio.

²⁾ The assumption that any point of AD can be obtained as centre of gravity of an inscribed figure Π_n is, of course, unwarranted.

W E S E N D E ghegheuen een * brantsne : Huer *Parabola.*
swaerheysts middelpunt te vinden.

T'GHEGHEVEN. Laet ABC een brandtsne sijn, diens middellini AD . T'BEGHEERDE. Wy moeten haer swaerheysts middelpunt vinden. T'WERCK. Men sal de middellini AD , deelen in E , alsoo dat AE tot ED de reden hebbe van 3 tot 2 : Ick seg dat E r'begheerde swaerheysts middelpunt is. T'BEREYTSSEL. Laet ghetrocken worden de rechte linien AB , ende AC , ende de selue ghedeelt in haer middelen F , G , ende ghetrocken worden FG sniende AD in H , daer naer FI ende GK euwydighe van AD , ende laet ghestelt worden t'punt L in IF , inder voughen dat IL sy tot LF , als AE tot ED : Laet oock ghestelt worden t'punt M in KG , alsoo dat MG euen sy an LF , ende laet ghetrocken worden LM sniende AD in N , ende IK sniende AD in O , ende laet IF voortghetrocken worden tot Q , in den grondt BC , ende laet ghestelt worden t'punt P , alsoo dat AP dobbel sy an PD , ende P sal swaerheysts middelpunt sijn des driehouck ABC . ende want L , M , als swaerheysts middelpunten ghestelt sijn der brantsnekens ABI , ende ACK , soo sal N swaerheysts middelpunt sijn dier twee brandtsnekens, daerom ghedeelt den balck PN , also dat d'een erm sulcken reden hebbe tot d'ander, als den driehouck ABC tot die twee brantsnekens, wy sullen r'begheerde hebben; maer de heele brantsne heeft sulcken reden tot den driehouck ABC als 4 tot 3 (duer het 24 voorstel vande viercanting der brantsne van Archimed.) daerom den driehouck ABC heeft sulcken reden tot de twee brantsnekens, als 3 tot 1; Ghedeelt dan PN alsoo dat het opperste stick, drievoudich sy tot het onderste, wy sullen r'swaerheysts middelpunt des heels hebben. Ist dan dat wy bethoonen t'selue, te vallen in E (welcke E duer t'werck soo staet dat AE is tot ED inde reden van 3 tot 2) so is E het ware swaerheysts middelpunt.



T'BEWYS. AO en OH soo wy verclaert hebben int 11^e voorstel, sijn elck $\frac{1}{4}$ van AD , Maer ghelijck 3 tot 2, alsoo AE tot ED , ende IL tot LF , ende ON tot NH , daerom ghedeelt OH $\frac{1}{4}$, in sulcken reden
L als 3

PROBLEM IV.

Given a parabola: to find its centre of gravity.

PROPOSITION XII.

SUPPOSITION. Let ABC be a parabola, whose diameter shall be AD . **WHAT IS REQUIRED TO FIND.** We have to find its centre of gravity. **CONSTRUCTION.** The diameter AD shall be divided at E , in such a way that AE shall have to ED the ratio of 3 to 2. I say that E is the required centre of gravity. **PRELIMINARY.** Let the straight lines AB and AC be drawn, and let these be divided in their middle points F , G , and let FG be drawn, intersecting AD in H ; after that, FI and GK parallel to AD . And let the point L be taken in IF , in such a way that IL shall be to LF as AE is to ED . Let also the point M be taken in KG , in such a way that MG shall be equal to LF , and let LM be drawn, intersecting AD in N , and IK intersecting AD in O . And let IF be produced to Q , in the base BC , and let the point P be taken, in such a way that AP shall be double of PD . Then P will be centre of gravity of the triangle ABC , and because L , M have been taken as centres of gravity of the small parabolas ABI and ACK , N will be the centre of gravity of those two parabolas. Therefore, the beam PN being divided in such a way that one arm shall have to the other the same ratio as the triangle ABC to these two parabolas, we shall have the required centre of gravity. But the whole parabola has to the triangle ABC the ratio of 4 to 3 (by the 24th proposition of the quadrature of the parabola of Archimedes); therefore the triangle ABC has to the two parabolas the ratio of 3 to 1. PN therefore being so divided that the upper segment shall be three times the lower segment, we shall have the centre of gravity of the whole. If we then show this to be at E (which E , by the construction, is so disposed that AE has to ED the ratio of 3 to 2), E is the true centre of gravity. **PROOF.** As we have explained in the 11th proposition, AO and OH are each $\frac{1}{4} AD$. But as 3 is to 2, so is AE to ED , and IL to LF , and ON to NH . Therefore, OH ($\frac{1}{4}$) being divided in the ratio of 3 to 2, the segment NH will make $\frac{1}{10} AD$. If to this is added $\frac{1}{2}$ for HD , ND becomes $\frac{3}{5}$. If from this is subtracted PD ($\frac{1}{3}$), there remains for NP $\frac{4}{15}$. This is divided, by the preliminary, at E in such a way that NE is to EP as 3 to 1. Therefore EP makes $\frac{1}{15}$. If to this is added PD ($\frac{1}{3}$), ED becomes $\frac{2}{5} AD$. But ED being $\frac{2}{5}$, EA will make $\frac{3}{5}$. Therefore AE has to ED the ratio of 3 to 2, and consequently E is the centre of gravity of the parabola ABC , which

als 3 tot 2, so sal t'stick NH doen $\frac{1}{10}$ van AD, daer toeghedaen $\frac{1}{2}$ voor HD, doet voor ND $\frac{3}{10}$, daer af ghetrocken PD $\frac{1}{3}$ rest voor NP $\frac{4}{15}$, de selue is duer t'bereytsel ghedeelt in E, alsoo dat NE is tot EP, als 3 tot 1, daerom EP doet $\frac{1}{15}$, daer toe ghedaen PD $\frac{1}{3}$, comt voor ED $\frac{2}{3}$ van AD: Maer wesende ED $\frac{2}{3}$, so sal EA doen $\frac{2}{3}$, daerom AE heeft sulcken reden tot ED, als 3 tot 2, ende veruolghens E is t'swaerheys middelpunt des brantsnees ABC, t'welck wy bewysen moesten.

T B E S L V Y T. Wesende dan ghegheuen een brantsne: Wy hebben huer swaerheys middelpunt gheuonden naer den eysch.

M E R C K T.

HET schijnt dat Archimedes ter kennis deses voorstels ghecommen is, duer een deser twee manieren: D'eerste dat hy lichamelicke brantsneen makende, tot het formen sinder brandtspieghels, ofte om andersins hem daer in te oefnen, beuandt duer de daet, dit deel tot dat te wesen, als 3 tot 2, souckende daer naer de sekerheyt van dien in deser voughen: Anghesten BAI ende BAC beyde brandtsneen sijn, soo worden haer middellinien IF ende AD* eueredelick ghe-deelt van haer swaerheys middelpunten (soo int 11^e voorstel bewesen is) daerom moet IL tot LF sijn, als AE tot ED, maer ON is euen an IL, ende NH an LF, daerom moet ON sulcken reden hebben tot NH, als AE tot ED. Maer als N swaerheys middelpunt waer der twee brantsneken, ende P des driehouck ABC, so moet (ouermits desen driehouck drievoudich is tot die twee brantsneken) den erm NE drievoudich sijn anden erm EP, waer uyt sulcken voorstel rijst: Te vinden twee punten als N, E, alsoo dat de lini ON sulcken reden hebbe tot NH, als AE tot ED. Stellende daer naer AE te doen $\frac{3}{4}$ van AD, ende ED de $\frac{2}{4}$ ende versouckende alsoo watter uyt volghen soude, heeft beuonden naer de maniere als bouen, sulcx waerachtelick te ouercommen metter begheerde. Ofte soo hy dit aldus niet ghesocht en heeft al tastende, duer de voornoemde reden van 3 tot 2, maer duer lauter cracht der const, soo schijnet dat hy hem t'voornoemde in ghetalen voorghestelt heeft in deser voughen: Het sijn twee ghetalen OH $\frac{1}{4}$ ende HP $\frac{1}{6}$; deelt elck alsoo, dat het minste van OH, met het meeste van HP, drievoudich sy an t'minste van HP, ende dat t'meeste van OH sulcken reden hebbe tot sijn minste, als t'meeste van HP $+$ $\frac{1}{2}$ tot t'minste van HP $+$ $\frac{1}{3}$.

V. EYSCH.

XIII. VOORSTEL.

W E S E N D E ghegheuen een ghecorte brantsne: Huer swaerheys middelpunt te vinden.

T G H E G H E V E N. Laet ABCD een ghecorte brantsne sijn (welverstaende dat AB euewydighe sy met DC) wiens middellini EF.

T B E G H E E R D E. Wy moeten haer swaerheys middelpunt vinden.

T W E R C K

we had to prove ¹⁾. CONCLUSION. Given therefore a parabola: we have found its centre of gravity, as required.

NOTE.

It seems that Archimedes discovered this proposition by one of the following two methods. The first that, making corporeal parabolas for the formation of his burning mirrors, or in order to practise the matter in some other way, he found by experience that one segment had to the other the ratio of 3 to 2, after which he sought to verify this as follows: Since BAI and BAC are both parabolas, their diameters IF and AD are proportionally divided by their centres of gravity (as has been proved in the 11th proposition); therefore IL must be to LF as AE to ED, but ON is equal to IL, and NH to LF, therefore ON must have to NH the same ratio as AE to ED. But if N were the centre of gravity of the two parabolas, and P that of the triangle ABC, then (since the latter triangle is three times the two parabolas), the arm NE must be three times the arm EP, from which arises the following proposition: To find two points, as N, E, so that the line ON shall have to NH the same ratio as AE to ED. Then taking AE to make $\frac{3}{5}$ AD, and ED $\frac{2}{5}$, and trying to find what would follow from this, he found in the above manner that this is in perfect agreement with what was required to prove. But if he has not sought it thus by experience, by starting from the aforesaid ratio of 3 to 2, but purely theoretically, it is probable that he argued in numbers as follows: There are two numbers OH ($\frac{1}{4}$) and HP ($\frac{1}{6}$); divide each of these in such a way that the lesser segment of OH plus the greater segment of HP shall be three times the lesser segment of HP, and that the greater segment of OH shall have to its lesser segment the same ratio as the greater segment of HP + $\frac{1}{2}$ to the lesser segment of HP + $\frac{1}{3}$ 2).

¹⁾ It may seem that the demonstration contains a *circulus in probando*, since it is assumed that the points K and M, which are found by dividing IF and KG respectively in the ratio 3 : 2, are centres of gravity of the parabolic segments AIB and AKC respectively. By Prop. 11 this assumption is equivalent to the supposition that E is the centre of gravity of the segment ABC. Nevertheless, the reasoning is valid; it should, however, be borne in mind that Stevin does not aim at finding out the position of the centre of gravity of a parabolic segment, but only at verifying the statement that it divides the diameter in the ratio 3 : 2. The derivation of this result might have been given by algebra. Putting the ratio $ED : AD = \lambda$ and $AD = a$, we have: $PD = \frac{1}{3}a$, $ND = \frac{1}{2}a + \lambda \cdot \frac{1}{4}a$, $PN = \frac{1}{6}a + \frac{\lambda}{4}a$, $TE = \frac{1}{4}[\frac{1}{6}a + \frac{\lambda}{4}a]$, $DE = [\frac{3}{8} + \frac{\lambda}{16}]a$.

This gives the equation for λ : $\frac{3}{8} + \frac{\lambda}{16} = \lambda$,

from which: $\lambda = \frac{2}{5}$.

Stevin's procedure amounts to putting in the first member $\lambda = \frac{2}{5}$, and then calculating:

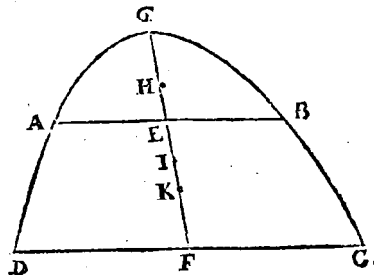
$$\lambda = \frac{3}{8} + \frac{2}{5} \cdot \frac{1}{16} = \frac{2}{5}.$$

²⁾ Putting $AD = 1$, we have $OH = \frac{1}{4}$, $HP = \frac{1}{2} - \frac{1}{3} = \frac{1}{6}$.

Now by Prop. 1 of Book I: $NE = 3 PE$, or $NH + HE = 3 TE$, therefore if $ED = \lambda$, $\lambda \cdot \frac{1}{4} + (1 - \lambda) \frac{1}{6} = 3(\lambda - \frac{1}{3})$, from which: $\lambda = \frac{2}{5}$.

The second relation is not a condition for finding the centre of gravity, but expresses the property of this centre enunciated in Prop. 11, viz.: The diameters of all parabolas are proportionally divided by the centre of gravity.

TWERCK. Men sal de ghecorte brantsne volmaken, daer an stellende t'ghebrekende ABG , daer naer salmen teekenen H , alsoo dat GH sy tot HE , als 3 tot 2: Inghelijcx I , alsoo dat GI sy tot IF , als 3 tot 2, daer naer K , alsoo dat IH sulcken reden hebbe tot IK , ghelijck de ghecorte brandtsne $ABCD$, tot de brantsne ABG ; Ick seg dat K t'begheerde swaerheys middelpunt is. **T'BEWYS.** I is swaerheys middelpunt des heels, ende H des deels, ende ghelijck t'ander deel tot dit, alsoo HI tot IK , daerom K , duer het 9^e voorstel, is t'begheerde swaerheys middelpunt, t'welck wy bewysen moesten. **T'BESLUYT.** Wesende dan ghegheuen een ghecorte brantsne, wy hebben huer swaerheys middelpunt gheuonden naer den eysch.



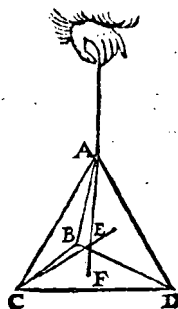
NV VANDE VINDING DER
SWAERHEYTS MIDDELPUNTEN
VANDE LICHAMEN.

IX. VERTOOC. XIII. VOORSTEL.

YDER lichaems formens middelpunt, is oock sijn swaerheys middelpunt.

T'GHEGHEVEN. Laet $ABCD$ een viergrondich wesen, diens formens middelpunt E sy, ende den as van A duer E , tot in F , middelpunt des driehoucx BCD , sy AF . **T'BEGHEERDE.** Wy moeten bewysen dat E oock is sijn swaerheys middelpunt. **T'BEWYS.** Laet ons t'lichaem ophanghen byde lini AF , maer het viergrondich bestaet uyt vier euen ende ghelijcke naelden een selfder ghestalt, wiens ghemeene sop E , daerom AF is des lichaems swaerheys middellini, ende om de selue reden sal de lini CE oock des swaerheys middellini sijn: E dan is oock het swaerheys

L 1



Tetraedron.

middelpunt

PROBLEM V.

Given a truncated parabola: to find its centre of gravity.

PROPOSITION XIII.

SUPPOSITION. Let $ABCD$ be a truncated parabola (to wit, that AB shall be parallel to DC), whose diameter shall be EF . WHAT IS REQUIRED TO FIND. We have to find its centre of gravity. CONSTRUCTION. The truncated parabola shall be completed, by adding the missing part ABG . After that, H shall be marked so that GH shall be to HE as 3 to 2. In the same way I , so that GI shall be to IF as 3 to 2. After that K , so that IH shall have to IK the same ratio as the truncated parabola $ABCD$ to the parabola ABG . I say that K is the required centre of gravity. PROOF. I is the centre of gravity of the whole, and H of the part, and as the other part is to this part, so is HI to IK . Therefore, by the 9th proposition, K is the required centre of gravity, which we had to prove ¹⁾. CONCLUSION. Given therefore a truncated parabola, we have found its centre of gravity, as required.

NOW ABOUT THE FINDING OF THE CENTRES OF
GRAVITY OF SOLID FIGURES ²⁾

THEOREM IX.

PROPOSITION XIV.

The geometrical centre of any solid is also its centre of gravity ³⁾.

SUPPOSITION. Let $ABCD$ be a tetrahedron, whose geometrical centre shall be E , while the axis from A through E to F , the centre of the triangle BCD , shall be AF . WHAT IS REQUIRED TO PROVE. We have to prove that E is also its centre of gravity. PROOF. Let us hang the solid by the line AF . But the tetrahedron consists of four equal and similar pyramids of the same form ⁴⁾, whose common vertex is E ; therefore AF is centre line of gravity of the solid, and for the same reason the line CE will also be centre line of gravity. Therefore E is also the centre of gravity. The same proof also holds for all solids having

¹⁾ This result is rather disappointing: the demonstration contains no more than the principle on which the determination of the centre of gravity of the truncated segment might be based. The determination itself had been given by Archimedes in the highly elaborate propositions 9 and 10 of the second book of the work *On the Equilibrium of Plane Figures*. The result is far from simple. If $GF = a$, $GE = b$, it is as follows:

$$\frac{IF}{EF} = \frac{3b\sqrt{b} + 6b\sqrt{a} + 4a\sqrt{b} + 2a\sqrt{a}}{5b\sqrt{b} + 10b\sqrt{a} + 10a\sqrt{b} + 5a\sqrt{a}}$$

²⁾ Most of the propositions of the second section of Book II being simply three-dimensional analogues of propositions on plane figures in the first section, they give rise to the same remarks; accordingly, we will confine ourselves to stating the correspondence.

³⁾ Cf. Prop. 1.

⁴⁾ The tetrahedron is obviously supposed to be regular.

middelpunt. Sghelijcx sal oock bewys sijn van allen lichamen hebbende middelpunten der form, soo wel vermeerde ende ghecorte gheschicte lichamen, als gheschicte, want soomense ophangt by de middellinien deur eenighen lichamelicken houck, ofte duer het middelpunt haerder gronden ende des formens middelpunt, soo hebben al de naelden (wiens ghemeene sop het formens middelpunt, ende gronden de platten des lichaems sijn) tot allen sijden ghelijcke ghestalt, daerom oock duer ghemeene wetenschap, ende duer de 1^e begheerte des 1^{en} boucx, alles hangen die lini euewichtich, ende veruolghens de sne sulcker twee swaerheys middellinien malcander sniende in des formens middelpunt, is oock het swaerheys middelpunt. **T'beslyt.** Yder lichaems formens middelpunt dan, is oock sijn swaerheys middelpunt.

X. VERTOCH.

XV. VOORSTEL.

YDER pilaers swaerheys middelpunt is int middel vanden as.

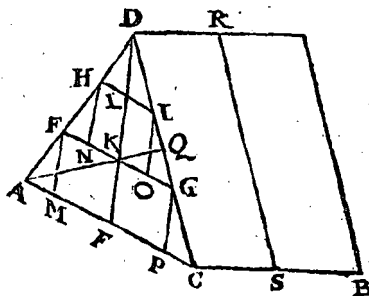
1^e VOORBEELT.

T'gheghe. Laet AB een driehouckich pilaer sijn diens grondt ACD. **T'begheerde.** Wy moeten bewysen dat sijn swaerheys middelpunt int middel vanden as is. **T'bereytse.** Laet ons trekken van D tot E int middel van AC de lini DE: Daer naer FG ende HI euewydighe van AC, sniende DE inde punten K, L, daer naer de linien FM, HN, IO, GP, euewydighe met DE, daer naer van A tot Q int middel der sijde DC, de lini AQ: Laet sghelijcx oock beschreuen worden het deefel, ende laet ons den pilaer doorsnien met een *plat RS euewichtich met den grondt ADC, ende S sy int middel van CB.

Plano.

Homologam.

T'bewys. T'plat ghetrocken duer DE, ende duer haer *lijckstandighe int deefel, deelt den binneschreuen pilaer uyt die twee vierhouckighe pilaren vergaert, in twee euen ende ghelijcke deelen, ende van ghelijcke ghestalt; het doorsnijt dan dier binneschreuen pilaers swaerheys middelpunt. Maer hoe datter sulcke vierhouckighe pilaren meer beschreuen sijn inden ghegheuen driehouckighen, hoe dat dese min verschilt van die; wy connen dan duer dat oneindelick naerderen



geometrical centres, augmented and truncated regular solids as well as regular solids, for if they are hung by the centre lines through some corporeal angle, or through the centre of their bases and the geometrical centre, all the pyramids (whose common vertex is the geometrical centre, while the bases are the faces of the solid) have the same form in every direction. Therefore also, by common knowledge and by the 1st postulate of the 1st book, everything hangs balanced from that line. And consequently the point of intersection of two such centre lines of gravity, intersecting in the geometrical centre, is also the centre of gravity. CONCLUSION. The geometrical centre therefore of any solid is also its centre of gravity.

THEOREM X.

PROPOSITION XV.

The centre of gravity of any prism is in the middle point of the axis ¹⁾.

EXAMPLE I.

SUPPOSITION. Let AB be a triangular prism, whose base shall be ACD . WHAT IS REQUIRED TO PROVE. We have to prove that its centre of gravity is in the middle point of the axis. PRELIMINARY. Let us draw from D to E ²⁾ in the middle point of AC the line DE , after this FG and HI parallel to AC , intersecting DE in the points K, L ; then the lines FM, HN, IO, GP parallel to DE , after this from A to Q , in the middle point of the side DC , the line AQ . Let the cover also be constructed in the same way, and let us intersect the prism by a plane RS parallel to the base ADC , and S shall be in the middle point of CB . PROOF. The plane drawn through DE and through its homologue in the cover divides the inscribed prism, composed of the two quadrangular prisms, into two equal and similar parts having the same form. It therefore passes through the centre of gravity of the said inscribed prism. But the more of such quadrangular prisms there are inscribed in the given triangular prism, the less the latter will differ from the former. We can therefore, by infinite approximation, inscribe

¹⁾ Here, as well as in subsequent propositions, axis means the line joining the centres of gravity of the two parallel faces of a prism, or the line joining the vertex of a pyramid with the centre of gravity of the base.

²⁾ In the drawing this letter has been erroneously replaced by F .

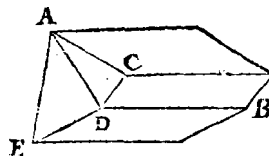
naerderen sulck een form binnen den ghegheuen pilaer beschrijven, dat haer verschil vande binnescreuen minder sal wesen dan eenich ghegheuen lichaem hoe cleen het sy, waer uyt volgt dat het staltwicht des deels D E C B ouer d'een sijde des plats, min verschillen sal van t'staltwicht des deels ouer d'ander sijde des plats, dan eenich lichaem datmen soude connen gheuen hoe cleen het sy, waer uyt ick aldus strie:

- A. Neuen alle verschillende staltswaerbeden can een swaerheyt ghestelt worden minder dan haer verschil;
- O. Neuen dese staltswaerbeden en can gheen swaerheyt ghestelt worden minder dan haer verschil;
- O. Dese staltswaerbeden dan en verschillen niet.

Daerom t'plat duer D E ende haer *lickstandighe int decksel, lijct duer *Homologam.* t'swaerheys middelpunt des ghegheuen pilaers, ofte het swaerheys middelpunt is in dat plat. Ende om de selue reden ist oock int plat duer A Q, ende haer lijckstandighe int decksel. Maer deser twee platten gheemeene sne is de rechte lini tusschen de swaerheys middelpunten des grondts ende deckfels, welcke lini den as is des ghegeuen pilaers, t'swaerheys middelpunt dan is inden as, het is oock int plat duer R S, want t'selue deelt den pilaer in twee euen, ghelijcke, ende lijckstandighe deelen; Maer dat plat doorsnijt den as in sijn middel, het swaerheys middelpunt dan is in des as middel.

II^e VOORBEELT.

T'GHEGHEVEN. Laet A B een vierhouckich pilaer wesen, diens grondt A C D E. T'BEGHEERDE. Wy moeten bewysen dat sijn swaerheys middelpunt int middel vanden as is. T'BERYTSSEL. Laet ons trecken een plat duer A D, ende haer lijckstandighe int decksel, deelende also den ghegheuen pilaer in twee driehouckighe pilaren, welcker yder het swaerheys middelpunt int middel vanden as heeft duer het 1^e voorbeelt, daerom ghetrocken den balck tusschen die twee punten, ende den seluen ghedeelt in sijn ermen, het onderscheydt der ermen sal het swaerheys middelpunt sijn des ghegheuen pilaers, welck punt valt in t'swaerheys middelpunt des plats cewydych vanden grondt den pilaer in twee euen stucken deelende, ende t'selue int middel der lini tusschen de swaerheys middelpunten des grondts ende deckfels, dat is int middel vanden as; T'selue salmen oock alsoo be-
thoonen in yder pilaer. T'BESELYT. Yder pilaers swaerheys middelpunt dan, is int middel vanden as, t'welck wy bewysen moesten.



within the given prism a figure such that its difference from the inscribed figure shall be less than any given solid, however small, from which it follows that the apparent weight of the part *DECB* to one side of the plane will differ less from the apparent weight of the part to the other side of the plane than any solid that might be given ¹⁾, however small, from which I argue as follows ²⁾:

- A. *Beside any different apparent gravities there can be placed a gravity less than their difference;*
- O. *Beside the present apparent gravities there cannot be placed any gravity less than their difference;*
- O. *Therefore the present apparent gravities do not differ.*

Therefore the plane through *DE* and its homologue in the cover passes through the centre of gravity of the given prism, or the centre of gravity is in the said plane. And for the same reason it is also in the plane through *AQ* and its homologue in the cover. But the common intersecting line of these two planes is the straight line joining the centres of gravity of the base and the cover, which line is the axis of the given prism. The centre of gravity therefore is in the axis. It is also in the plane through *RS*, because the latter divides the prism into two equal, similar, and homologous parts. But this plane intersects the axis in its middle point. The centre of gravity therefore is in the middle point of the axis.

EXAMPLE II.

SUPPOSITION. Let *AB* be a quadrangular prism, whose base is *ACDE*. WHAT IS REQUIRED TO PROVE. We have to prove that its centre of gravity is in the middle point of the axis. PRELIMINARY. Let us draw a plane through *AD* and its homologue in the cover, thus dividing the given prism into two triangular prisms, each of which has the centre of gravity in the middle point of the axis by the 1st example. Therefore, the beam being drawn between those two points and being divided into its arms, the point of division of the arms will be the centre of gravity of the given prism, which point falls in the centre of gravity of the plane, parallel to the base, dividing the prism into two equal parts. It is then in the middle point of the line joining the centres of gravity of the base and the cover, that is in the middle point of the axis. This can also be shown of any prism. CONCLUSION. The centre of gravity of any prism therefore is in the middle point of the axis, which we had to prove.

¹⁾ Cf. the note on Prop. 2.

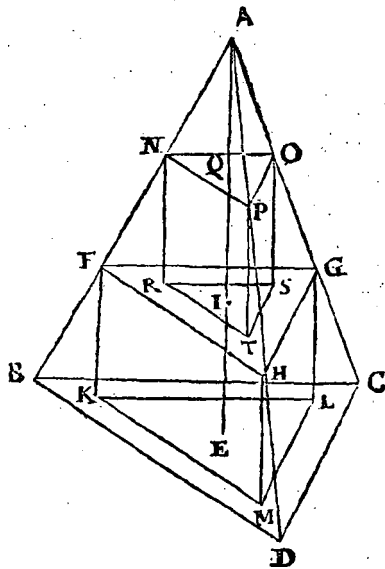
²⁾ See note 2 to p. 143

Pyramidu.

Y DER * naeldens swaerheys middelpunt is inden as.

TGHEGHEVEN. Laet ABCD een naelde sijn, diens grondt den driehouck BCD, wiens swaerheys middelpunt E, ende den as AE.

TBECHERDE. Wy moeten bewysen dat des naeldens swaerheys middelpunt inden as AE is. **TBEREYTSSEL.** Laet ons de naelde snien met een plat FGHeuewydich met BCD, ende sniende den as AE in I: Laet oock ghetrocken worden FK, GL, HM, euewydich vanden as AE, alsoo dat de punten K, L, M, int plat sijn des driehouck BCD, inder voughen dat FG HKLM een pilaer is, wiens grondt IKL euen ende ghelijck is an het decksel FGH, ende ghelijck anden grondt BCD; Daer naer ghelijck de naelde doorsneen was met FGH, laetse noch eenmaal alsoo doorsneen sijn met het plat NOP, sniende den as in Q, ende daer uyt oock alsoo ghemaect den pilaer NOPRST, te weten NR, OS, PT, euewydich vanden as AE, ende de punten R, S, T, int plat FGH. **TBEWYS.** Anghesien de driehoucken NOP, RST, FGH, KLM, alle ghelijck sijn anden driehouck BCD, ende dat haer punten Q, I, E, in haer sulcken stant hebben als E indé driehouck BCD, ende dat E des driehouck BCD swaerheys middelpunt is, soo sijn oock die Q, I, E, haer driehouckens swaerheys middelpuntē, waer duer IE as is des pilaers FG HKLM, in wiēs middel huer swaerheys middelpūt is duer het 14^e voorstel. Sghelijcx is QI oock as des pilaers NOPRST, in wiens middel huer swaerheys middelpūt is, en vervolgens het swaerheys middelpunt des lichaems uyt die twee pilaren vergaert is in QE, daerō oock in AE; Maer hoe datter inde naelde sulcke pilaren meer beschreuen worden, hoe dattet verschil der naelde ende der binne-



THEOREM XI.

PROPOSITION XVI

The centre of gravity of any pyramid is in the axis ¹⁾.

SUPPOSITION. Let $ABCD$ be a pyramid, whose base is the triangle BCD , the centre of gravity of the latter being E , and the axis being AE . **WHAT IS REQUIRED TO PROVE.** We have to prove that the centre of gravity of the pyramid is in the axis AE . **PRELIMINARY.** Let us intersect the pyramid by a plane FGH , parallel to BCD and intersecting the axis AE in I . Let there also be drawn FK , GL , HM parallel to the axis AE , in such a way that the points K , L , M are in the plane of the triangle BCD , so that $FGHKLM$ is a prism, whose base IKL is equal and similar to the cover FGH , and similar to the base BCD . After this, as the pyramid was intersected by FGH , let it be intersected once more in this way by the plane NOP , intersecting the axis in Q , and let there also be constructed from this the prism $NOPRST$, to wit NR , OS , PT parallel to the axis AE , and the points R , S , T in the plane FGH . **PROOF.** Since the triangles NOP , RST , FGH , KLM are all similar to the triangle BCD , and the points Q , I , E therein have the same position as E in the triangle BCD , and E is the centre of gravity of the triangle BCD , those points Q , I , E are also the centres of gravity of their triangles, in consequence of which IE is the axis of the prism $FGHKLM$, in whose middle point is its centre of gravity, by the 14th proposition. In the same way QI is also the axis of the prism $NOPRST$, in whose middle point is its centre of gravity, and consequently the centre of gravity of the solid composed of these two prisms is in QE , and therefore also in AE . But the more of such prisms there are inscribed in the pyramid, the less will be the difference between the pyramid and

¹⁾ Cf. Prop. 2.

binneschreuen form van sulcke pilaren vergaert, minder is, blijuende nochtan het swaerheys middelpunt der binneschreuen form altijd inden as AE; Wy connen dan duer dat oneindelick naerderen sulcken form binnen de naelde stellen, datter verschil tusschen haer ende de naelde, minder sal wesen dan eenich ghegheuen lichaem hoe cleen het sy, waer uyt volght dat stellende A E voor swaerheys middellini, der naelde, soo sal het statwicht van d'een sijde tot d'ander, min verschillen dan eenighe swaerheyt diemen soude connen gheuen, waer uyt ick aldus stree:

A. *Neuen alle verschillende statfwaerheden can een swaerheyt ghestelt worden minder dan haer verschil;*

O. *Neuen dese statfwaerheden en can gheen swaerheyt ghestelt worden minder dan haer verschil;*

O. *Dese statfwaerheden dan en verschillen niet.*

Sghelijcx sal oock t'bewys sijn val naelden wiens gronden sijn Vierhoucken, Veelhoucken, Ronden &c. **T'beslyt.** Yder naeldens swaerheys middelpunt dan is inden as.

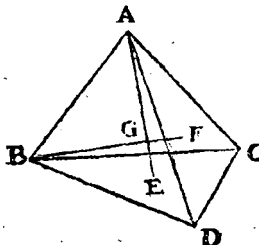
VI. EYSCH.

XVII. VOORSTEL.

W E S E N D E ghegheuen een naelde: Huer swaerheys middelpunt te vinden.

T'ghegheuen. Laet ABCD een naelde wesen, diens grondt sy den driehouck BCD. **T'begheerde.** Wy moeten haer swaerheyt middelpunt vinden. **T'werck.** Men sal de swaerheys middelpunten vinden van eenighe twee driehoucken, als E van BCD, ende F van ADC, treckende de linien A E, B F, welcker sine G, ick seg te wesen het begheerde swaerheys middelpunt.

T'bewys. Des naeldens ABCD swaerheys middelpunt is in A E, ende oock in B F, duer het 16 voorstel, het is dan nootsaecklick G. **T'beslyt.** Wefende dan ghegheuen een naelde: Wy hebben huer swaerheys middelpunt gheuonden naer den eysch.



XII. VERTOCH.

XVIII. VOORSTEL.

H E T swaerheys middelpunt van yder naelde deelt den as alsoo, dat het stiek naer den houck dreyvoudich is an t'ander.

T'ghe-

the inscribed forms of such prisms, the centre of gravity of the inscribed form, however, always remaining in the axis AE . We can therefore, by infinite approximation, place within the pyramid a form such that the difference between the latter and the pyramid shall be less than any given solid, however small, from which it follows that, taking AE as centre line of gravity of the pyramid, the apparent weight of one side will differ less from the other than any gravity that might be given, from which I argue as follows ¹⁾:

- A. *Beside any different apparent gravities there can be placed a gravity less than their difference;*
- O. *Beside the present apparent gravities there cannot be placed any gravity less than their difference;*
- O. *Therefore the present apparent gravities do not differ.*

The same proof also holds for pyramids whose bases are quadrilaterals, polygons, circles, etc. **CONCLUSION.** The centre of gravity of any pyramid therefore is in the axis.

PROBLEM VI.

PROPOSITION XVII.

Given a pyramid: to find its centre of gravity ²⁾.

SUPPOSITION. Let $ABCD$ be a pyramid, whose base shall be the triangle BCD . **WHAT IS REQUIRED TO FIND.** We have to find its centre of gravity. **CONSTRUCTION.** The centres of gravity shall be found of any two triangles, as E of BCD and F of ADC , and the lines AE , BF shall be drawn; whose point of intersection G , I say is the required centre of gravity. **PROOF.** The centre of gravity of the pyramid $ABCD$ is in AE , and also in BF , by the 16th proposition. It is therefore necessarily G . **CONCLUSION.** Given therefore a pyramid: we have found its centre of gravity, as required.

THEOREM XII.

PROPOSITION XVIII.

The centre of gravity of any pyramid divides the axis in such a way that the segment adjacent to the angle is three times the other segment ³⁾.

¹⁾ See note 2 to p. 143.

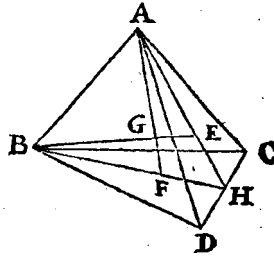
²⁾ Cf. Prop. 3.

³⁾ Cf. Prop. 4.

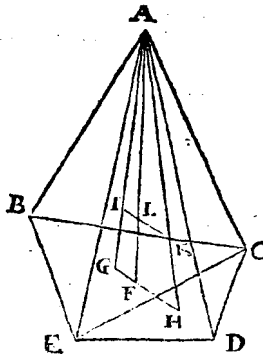
T'GHEGHEVEN. Laet $ABCD$ een driehouckighe naelde wesen, diens sop A , ende grondt BCD , ende den as van B tot int swaerheys middelpunt E des driehoucx ADC , sy BE , ende van A tot int swaerheys middelpunt F des driehoucx BCD sy AF , sniende BE in G , voor t' swaerheys middelpunt der ghegheuen naelde. **T'BEGHEERDE.** Wy moeten bewysen dat BG drievoudich is an GE . **T'BEREYTSSEL.** Laet ons trecken van H middel van CD , de linien HA , HB .

T'BEWYS. Ouermits HA ghetrocken is uyt het middel van DC tot inden houck A , ende dat E t' swaerheys middelpunt is des driehoucx ACD , soo sal AE dobbel sijn an EH duer het 4^e voorstel, ende om de selue reden sal BF dobbel wesen an FH . Dit soo sijnde, ghetrocken de reden EA 2, tot AH 3, vande reden BF 2, tot FH 1

Door t' ver- (dat is Reden $\frac{3}{2}$ van Reden $\frac{2}{1}$) * daer
keerde des 12 rest de reden van BG tot GE : Maer
cap. 1. lib. Al- treckende Reden $\frac{2}{3}$ van Reden $\frac{3}{1}$ daer
mag. Ptol. blijft Reden $\frac{3}{1}$. BG dan is tot GE , als
3 tot 1.



MAER soo des ghegheuen naeldens grondt een vierhouck waer, t'voorstel sal in die oock also bewesen worden: Laet by voorbeeld $ABCDE$ een naelde wesen, wiens grondt een vierhouck $BCDE$, ende as AF sy. Nu dese vierhouckighe naelde ghedeelt in twee driehouckighe, wiens gronden ECB , ende ECD , diens affen AG , ende AH , wiens swaerheys middelpunten I , K , des heelen naeldens swaerheys middelpunt sal inde lini IK wesen, tis oock in AF duer het 16^e voorstel, tis dan L : Maer want AGH een driehouck is, ende IK ewewydich van GH (want IG is t'vierendeel van GA , ende HK t'vierendeel van HA daerom &c.) soo sal AL sulcken reden hebben tot LF , als AI , tot IG , dat is drienoudich. Sghelijcx sal oock t'bewys sijn in alle naelde met veelsidighen grondt.



MAER de naelde een keghel sijnde, te weten dat den grondt waer een rondt ofte lanckrondt, t'selfde sal daerin oock alsoo bewesen worden

SUPPOSITION. Let $ABCD$ be a triangular pyramid, whose vertex shall be A and the base BCD , and the axis from B to the centre of gravity E of the triangle ADC shall be BE , and that from A to the centre of gravity F of the triangle BCD shall be AF , intersecting BE in G , which is the centre of gravity of the given pyramid. **WHAT IS REQUIRED TO PROVE.** We have to prove that BG is three times GE . **PRELIMINARY.** Let us draw from H , the middle point of CD , the lines HA , HB . **PROOF.** Since HA has been drawn from the middle point of DC to the angle A , and E is the centre of gravity of the triangle ACD , AE will be double of EH by the 4th proposition, and for the same reason BF will be double of FH . This being so, if the ratio of EA (2) to AH (3) is subtracted from the ratio of BF (2) to FH (1) (i.e. ratio $\frac{3}{2}$ from ratio $\frac{2}{1}$), there remains the ratio of BG to GE . But if the ratio $\frac{2}{3}$ is subtracted from the ratio $\frac{2}{1}$, there remains the ratio $\frac{3}{1}$. GB therefore is to GE as 3 to 1¹⁾.

But if the base of the given pyramid be a quadrilateral, the proposition will also be proved of this in the following way. Let, for example, $ABCDE$ be a pyramid, whose base shall be a quadrilateral $BCDE$ and the axis AF . Now if this quadrangular pyramid is divided into two triangular ones, whose bases shall be ECB and ECD , the axes AG and AH , and the centres of gravity I , K , the centre of gravity of the whole pyramid will be in the line IK . It is also in AF , by the 16th proposition; it is therefore L . But because AGH is a triangle, and IK is parallel to GH (for IG is the fourth part of GA , and HK the fourth part of HA ; therefore, etc.), AL will have to LF the same ratio as AI to IG , that is 3 to 1. The same proof also holds for any pyramid with a polylateral base.

But if the pyramid be a cone, to wit that the base be a circle or an ellipse, the same proposition will also be proved thereof as follows. For it is obvious from

¹⁾ Cf. the note on Prop. 4 (p. 233). Menelaus's theorem is here applied to the triangle EBH with transversal GFA , giving:

$$\frac{GE \cdot FB \cdot AH}{GB \cdot FH \cdot AE} = 1; \text{ therefore } \frac{GB}{GE} = \frac{2}{1} \cdot \frac{3}{2} = \frac{3}{1}.$$

worden, want het is duer r'voorgaende kennelick, dat alle veelhouckighe naelde in haer beschreuen, r'swaerheys middelpunt alsoo sal hebben, dattet opperste deel drievoudich is teghen het onderste. Maer hoe de naelde daer in beschreuen van meer houcken is, hoe die binneschreuen naeldens grootheyt vande ronde naelde min verschilt, daerom oock connen wy bin het oneindelick naerderen, een binneschreuen setten, min verschillende vande veruutende, dan eenich ghegheuen lichaem hoe cleen het sy; Daerom oock de langde der plaets van diens swaerheys middelpunt tot deses, corter soude moeten wesen dan eenighe langde die mueghelick is ghegheuen te worden, waer uyt ick aldus stric:

- A. Neuen alle twee punten in verscheyden plaetsen staende, connen twee punten ghestelt worden die malcander naerder sijn ;
- O. Neuen dese twee punten en connen gheen twee punten ghestelt worden die malcander naerder sijn ;
- O. Dese twee punten dan en staen in gheen verscheyden plaetsen.

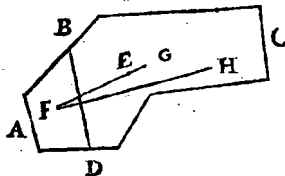
T B E S L V V T. Het swaerheyts middelpunt dan van yder naelde, deelt den as alsoo, dat het stuck naer den houck drieuoudich is an t'ander.

vii Eysch.

xix VOORSTEL.

W E S E N D E ghegheuen t'swaerheys middelpunt eens lichaems ende sijns deels, wiens reden an t'ander deel kennelick is: Het swaerheys middelpunt des ander deels te vinden.

TGHGHEVEN. Laet ABCD een lichaem sijn, diens swaerheys middelpunt E, ende BDA deel des lichaems, wiens swaerheys middelpunt F. TBEGHEERDE. Wy moeten t' swaerheys middelpunt vinden des ander deels BCD. TWERCK. Men sal trecken FE tot in G, alsoo dat FE sulcken reden hebbe tot EG, als t'stick BDC tot t'stick BDA; Ick seg dat G t' begheerde swaerheys middelpunt is, des ander stix BDC; waer af t' bewys ghe-lijck sal sijn an t' bewys des 9^m voorstels. Wy souden oock moghen voorbeelt setten van een heele cloot, wiens ander deel oock een cloot sy, maer sulcx is openbaer ghenouch duer het tweede voorbeelt des boueschreuen 9^m voorstels in ronden. TBESLVT. Westende dan ghegheuen t' swaerheys middelpunt eens lichaems ende sijns deels, wiens reden an t' ander deel



M kennelick

the above that any polygonal pyramid inscribed therein will have the centre of gravity in such a place that the upper part shall be three times the lower part. But the more angles the pyramid described therein has, the less the magnitude of this inscribed pyramid will differ from the cone. We can therefore, by infinite approximation, inscribe a pyramid differing less from the containing cone than any given solid, however small. Therefore also the distance between the place of the centre of gravity of the former and that of the latter would have to be less than any distance that may be given, from which I argue as follows ¹⁾:

- A. *Beside any two points in different places there can be placed two points which are nearer to one another;*
- O. *Beside the present two points there cannot be placed two points which are nearer to one another;*
- O. *Therefore the present two points are not in different places.*

CONCLUSION. The centre of gravity therefore of any pyramid divides the axis in such a way that the segment adjacent to the angle is three times the other segment.

PROBLEM VII.

PROPOSITION XIX.

Given the centre of gravity of a solid and that of a part thereof, the ratio of which to the other part is known: to find the centre of gravity of the other part ²⁾.

SUPPOSITION. Let $ABCD$ be a solid whose centre of gravity shall be E , and BDA a part of the solid, whose centre of gravity shall be F . WHAT IS REQUIRED TO FIND. We have to find the centre of gravity of the other part BCD . CONSTRUCTION. FE shall be drawn up to G , in such a way that FE shall have to EG the same ratio as the part BDC to the part BDA . I say that G is the required centre of gravity of the other part BDC , the proof of which will be similar to the proof of the 9th proposition. We might also take as example a complete sphere, whose other part shall also be a sphere, but this is sufficiently manifest from the second example of the 9th proposition described above with regard to circles. CONCLUSION. Given therefore the centre of gravity of a solid

¹⁾ See note 2 to p. 143.

²⁾ Cf. Prop. 9.

kennelick is, wy hobben t' swaerheys middelpunt des ander deels gheuonden, naer den eysch.

VIII EYSCHE.

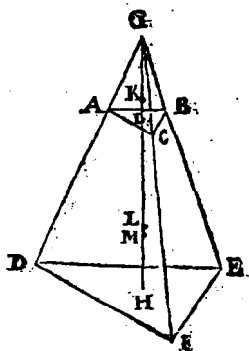
XX VOORSTEL.

*Pyramis
symmetrica.*

W E S E N D E ghegheuen een* ghecorte naelde:
Huer swaerheys middelpunt te vinden.

T'GHEGHEVEN. Laet $ABCDEF$ een ghecorte naelde sijn, diens deckfel ABC , ende grondt DEF . T'BEGHEERDE. Wy moeten huer swaerheys middelpunt vinden. T'WERCK. Men sal de ghecorte naelde volmaken, daer an stellende het ghebrekende $ABCG$, vindende H swaerheys middelpunt des driehouck DEF , treckende den as GH , wiens punt inden driehouck ABC , sy I , daernaer salmen teekenen K , alsoo dat GK drievoudich sy an KI : Inghelijcx L , alsoo dat GL drievoudich sy an LH , teekene M , alsoo dat KL sulcken reden hebbe tot LM , ghelijck de ghecorte naelde $ABCDEF$, tot de naelde $ABCG$, Ick seg dat M t'begheerde swaerheys middelpunt is.

T'BEWYS. L is swaerheys middelpunt des heels, ende K des deels, ende ghelijck t'onderste deel tottes bouenste, alsoo KL tot LM , Daerom M , door het 1^o voorstel des 1^o bouck is t'begheerde swaerheys middelpunt, t'welck wy bewysen moesten. Sghelijcx sal oock den voortganek sijn in allen anderen ghecorte naelden. T'BESLUYT. Wesende dan ghegheuen een ghecortenaelde: Wy hebben huer swaerheys middelpunt gheuonden, naer den eysch.



IX. EYSCHE.

XXI. VOORSTEL.

W E S E N D E ghegheuen een platgrondich lichaem foodanigher form alst valt: Sijn swaerheys middelpunt te vinden.

T'GHEGHEVEN. Laet A een ongheschiekt platgrondich lichaem sijn, dat is omvanghen in platten so veel alst sy. T'BEGHEERDE. Wy moeten sijn swaerheys middelpunt vinden. T'WERCK. Men sal t'lichaem

and that of a part thereof, the ratio of which to the other part is known, we have found the centre of gravity of the other part, as required.

PROBLEM VIII.

PROPOSITION XX.

Given a truncated pyramid: to find its centre of gravity.

SUPPOSITION. Let $ABCDEF$ be a truncated pyramid, whose cover shall be ABC and the base DEF . WHAT IS REQUIRED TO FIND. We have to find its centre of gravity. CONSTRUCTION. The truncated pyramid shall be completed, adding thereto the missing part $ABCG$, finding H , the centre of gravity of the triangle DEF , drawing the axis GH , whose point in the triangle ABC shall be I , and after this, K shall be marked in such a way that GK shall be three times KI . Likewise L in such a way that GL shall be three times LH , marking M in such a way that KL shall have to LM the same ratio as the truncated pyramid $ABCDEF$ to the pyramid $ABCG$. I say that M is the required centre of gravity. PROOF. L is the centre of gravity of the whole, and K that of the part, and as the lower part is to the upper, so is KL to LM . Therefore, by the 1st proposition of the 1st book, M is the required centre of gravity, which we had to prove. The same procedure may also be applied to any other truncated pyramid. CONCLUSION. Given therefore a truncated pyramid, we have found its centre of gravity, as required.

PROBLEM IX.

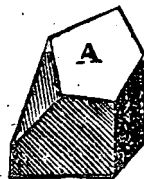
PROPOSITION XXI

Given a polyhedron of any form whatever: to find its centre of gravity ¹⁾.

SUPPOSITION. Let A be an irregular polyhedron, enveloped by any number of faces that may occur. WHAT IS REQUIRED TO FIND. We have to find its centre of gravity. CONSTRUCTION. The solid shall be divided into the smallest

¹⁾ Cf. Prop. 6.

t'lichaem deelen inde*naelden dieder ten weynichsten ende bequamelicxt *Pyramids.*
uyt vallen willen. Ten quaetsten commende men can als duer ghemee-
ne reghel, alle platgrondich lichaem in soo veel naelden deelen alst plat-
ten heeft, stellende eenich punt int lichaem voor haer ghemeene sop.
Dit soo sijnde, men sal yder naeldens swaerheys middelpunt vinden
duer het 17^e voorstel. Daer naer om te vinden
t'ghemeene swaerheyds middelpunt van twee
naelden, men sal tusschen haer swaerheys mid-
delpunten een balck trecken, die deelende in sul-
ken reden als haer twee naelden tot malcanderen
sijn, weluerstaende t'corste deel naer de swaerste
naelde. Ende inder seluer voughen salmen daer-
toe vergaderen de derde naelde, ende alle d'ander,
ende t'punt den balck alsoo ten laetsten deelende,
sal t'begheerde swaerheys middelpunt sijn, waer af t'bewys openbaer is.



T'BESLVT. Wefende dan ghегheuen een platgrondich lichaem
soodanigher form alst valt, Wy hebben sijn swaerheys middelpunt ghe-
uonden, naer den heysch.

XIII. VERTOOGH.

XXII. VOORSTEL.

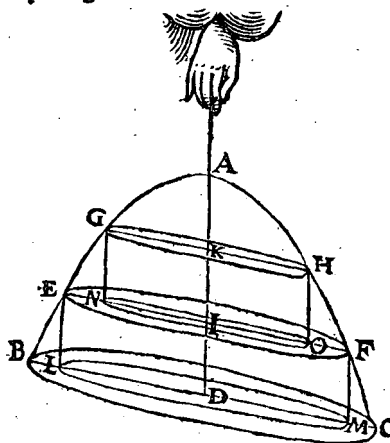
YDER branders swaerheys middelpunt is *Conoidale*
inden as. *reGangulid.*

Het swaerheys middelpunt des rechten branders inden as te wesen
is duer ghemeene wetenschap openbaer, wy sullen dan alleenelick t'voor-
beeldt stellen des gheens diens as opden grondt cromhouckich is.

T'GHEGHEVEN. Laet
A B C een brander wesen
diens grondt B C sy, ende
den as A D daerop crom-
houckich.

T'BEGHEERDE. Wy
moetē bewysen datter swaer-
heys middelpunt in A D is.

T'BEREYTSSEL. Laet
ons den brander snien met
twee platten E F, G H eue-
wydich vanden grondt B C,
welcker ghemeene sneen
met den as A D, sijn I, K;
Ende laet ons trecken de li-
nien E L, F M, G N, H O:



M 2

ende

and most suitable number of pyramids that can be made from it. In the worst case, any polyhedron can by a common rule be divided into as many pyramids as it has faces, taking some point in the solid for their common vertex. This being so, the centre of gravity of each pyramid shall be found by the 17th proposition. After this, in order to find the common centre of gravity of two pyramids, there shall be drawn between their centres of gravity a beam, which shall be divided in the same ratio as its two pyramids have to one another, that is to say: the shorter segment adjacent to the heavier pyramid. And in the same way the third pyramid shall be combined therewith, and all the others, and the final point of division of the beam will be the required centre of gravity, the proof of which is manifest. **CONCLUSION.** Given therefore a polyhedron of any form whatever, we have found its centre of gravity, as required.

THEOREM XIII.**PROPOSITION XXII.**

The centre of gravity of any paraboloid is in its axis ¹⁾).

It is manifest by common knowledge that the centre of gravity of a right paraboloid is in its axis. We shall therefore only give the example of a paraboloid whose axis is at oblique angles to the base.

SUPPOSITION. Let ABC be a paraboloid, whose base shall be BC , and the axis AD at oblique angles thereto. **WHAT IS REQUIRED TO PROVE.** We have to prove that the centre of gravity is in AD . **PRELIMINARY.** Let us intersect the paraboloid by two planes EF , GH , parallel to the base BC , whose points of intersection with the axis AD shall be I , K . And let us draw the lines EL , FM ,

¹⁾ Cf. Prop. 10. The reader may be reminded that paraboloid means a segment of a paraboloid of revolution. If this segment is cut off by a plane perpendicular to the axis of revolution, the segment is called „rechte brander“ (right paraboloid).

Ellipses.

ende LM, NO, GH, sullen* lancronden wesen ghelijck an t'lanckront BC; ende laet EM met GO pilaren sijn uyt de selue beschreuen.

Semidiameter.

T'BEWYS. Want LD* halfmiddellini des lancronds LM euen is an DM, oock an NI, ende IO, soo sal ID as sijn des pilaers EM, inde welcke diens pilaers swaerheys middelpunt is: Ende om de selue reden sal t'swaerheys middelpunt des pilaers GO oock wesen in KI, ende vernolghens t'swaerheys middelpunt des lichaems uyt die twee pilaren vergaert is in KD, daerom oock in AD. Maer hoe datter sulcke pilaren indé brander meer beschreuen worden, hoe dattet verschil des branders ende der binneschreuen form van sulcke pilaren vergaert, minder is. Wy connen dan duer dat oneindelick naerderen sulcken form binnen den brander stellen, dat huer verschil minder sal wesen, dan eenich ghègheuen lichaem hoe cleen het sy; Waer uyt volght dat stellende AD voor swaerheys middellini des branders, soo sal t'staltwicht van d'een sijde tot d'ander, min verschillen dan eenighe swaerheyt diemen soude connen gheuen, waer uyt ick aldus stie:

- A. Neuen alle verschillende statfwaerbeden, can een swaerheyt gheslelt worden minder dan haer verschil;
- O. Neuen dese statfwaerbeden van d'eene en dander side des branders, en can gheen swaerheys gheslelt worden minder dan haer verschil;
- O. Dese statfwaerbeden dan van d'eene ende dander side des branders en verschillen niet.

Daerom AD is haer swaerheys middellini. T'BESELYT. Yders branders swaerheys middelpunt dan, is inden as; t'welck wy bewysen moesten.

X. EYSCH.

XXIII. VOORSTEL.

W E S E N D E ghegheuen een brander: Huer swaerheys middelpunt te vinden.

T'GHEGHEVEN. Laet ABC een brander wesen diens sop A, ende as AD sy. T'BEGHEERDE. Wy moeten sijn swaerheys middelpunt vinden. T'WERCK. Men sal den as AD in E deelen, alsoo dat AE dobbel sy an ED, ende E sal t'begheerde swaerheys middelpunt sijn; T'welck bewesen is duer Frederick Commandin int 29 voorstel, waer af den sin verclaert naer onse manier soodanich is. T'BEWYS. Laet den brander doorsneen worden met een plat FG, eüewydich vanden grondt BC, ende duer t'middel des as H, ende sniende de uystersten des branders in I, K, ende laet BCGF ende IKLM twee pilaren sijn, beschreuen omme den brander, wiens middelpunten N, O, ende IK P Q een pilaer binnen den brander, wiens swaerheys middelpunt oock O sijn sal. Nu want ghelijck AD tot AH, t'welck is als 2 tot 1, alsoo t'ronds

GN , HO ; then LM , NO , GH will be ellipses similar to the ellipse BC . And let EM and GO be prisms described thereon. PROOF. Because LD , the semi-diameter of the ellipse LM , is equal to DM , also NI to IO ¹⁾, ID will be the axis of the prism EM , in which is the centre of gravity of the said prism. And for the same reason the centre of gravity of the prism GO will also be in KI , and consequently the centre of gravity of the solid composed of those two prisms is in KD , and therefore also in AD . But the more of such prisms there are inscribed in the paraboloid, the less will be the difference between the paraboloid and the inscribed figure composed of such prisms. We can therefore, by infinite approximation, place within the paraboloid a form such that its difference with the paraboloid shall be less than any given solid, however small. From which it follows that, taking AD for the centre line of gravity of the paraboloid ²⁾, the apparent weight of one side will differ less from that of the other than any gravity that might be given, from which I argue as follows ³⁾:

- A. *Beside any different apparent gravities there can be placed a gravity less than their difference;*
- O. *Beside the present apparent gravities of one side of the paraboloid and the other there cannot be placed any gravity less than their difference;*
- O. *Therefore the present apparent gravities of one side of the paraboloid and the other do not differ.*

Therefore AD is its centre line of gravity. CONCLUSION. The centre of gravity therefore of any paraboloid is in the axis, which we had to prove.

PROBLEM X.

PROPOSITION XXIII.

Given a paraboloid: to find its centre of gravity ⁴⁾.

SUPPOSITION. Let ABC be a paraboloid, whose vertex shall be A and the axis AD . WHAT IS REQUIRED TO FIND. We have to find its centre of gravity. CONSTRUCTION. The axis AD shall be divided in E in such a way that AE shall be double of ED ; then E will be the required centre of gravity, which has been proved by Frederick Commandinus in the 29th proposition, the purport of which, explained in our own way, is as follows. PROOF. Let the paraboloid be intersected by a plane FG , parallel to the base BC and through the middle point of the axis, H , and intersecting the extremities of the paraboloid in I , K . And let $BCGF$ and $IKLM$ be two prisms circumscribed about the paraboloid, the centres of said prisms being N , O , and let $IKPO$ be a prism within the paraboloid, the centre of gravity of this prism also being O . Now because as AD is to AH ,

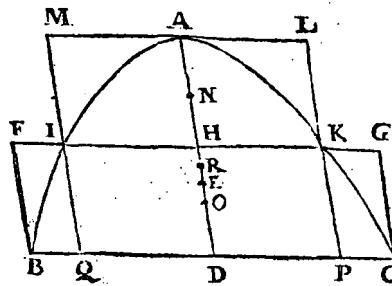
¹⁾ The text has „also to NI and NO “, but this is obviously a mistake.

²⁾ Cf. note 3 to p. 227.

³⁾ See note 2 to p. 143.

⁴⁾ It is to be noted that the reasoning here deviates from that of Props 11 and 12. A lemma corresponding to Prop. 11 is not needed here.

r'rondt B C tottet rondt I K, soo sal den pilaer B G sulcken reden hebben tot den pilaer I L, als 2 tot 1, daerom laet B G weghen 2 lb, ende I L 1 lb: Maer huer swaerheyts middelpunten sijn N, O, de lini dan N O sal balck sijn de selue ghe-deelt in huer ermen, dat is in R, alsoo dat NR dobbel sijn an R O, soo sal R swaer-



20 v. 1 B. v. d.
Apollonius.

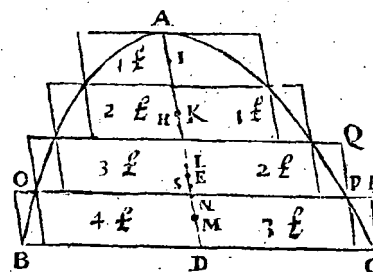
13. v. 12. B. E

heyts middelpunt sijn der twee omfeschreuen pilaren, ende O ist vande binneschreuen, ende R sal soo verre van E vallen, als O van E, te weten elck $\frac{1}{12}$ van A D: Ende sulcx sal in alle anderen der ghelijcke voorbeelden oock alsoogheshien. Maer op datter claerder sy, Wy sullender noch een besonder voorbeelt af beschrijuen aldus:

Laet ons den brander A B C nocheinmael snien duer de middelen van A H, ende H D, daer uyt teeckenende vier omfeschreuen, ende drie binneschreuen pilaren, als hier onder, alwaer A D des branders as is, ende der pilaren middelpunten sijn I, K, L, M; ende A E synoch dobbel an E D als vooren. Nu want ghelijck A D tot A N (t'welck is als 4 tot 3) alsoo het rondt B C tottet rondt O P, soo sal den pilaer-

20 v. 1 B. v. d.
Apollonius.

B F sulcken reden hebben tot den pilaer O Q, als 4 tot 3, ende om de selue oirfaeck sal B F sulcken reden hebben tot de derde diens middelpunt K, als 4 tot 2, ende tot den omfeschreuen pilaer wiens middelpunt I, als 4 tot 1: Daerom laet d'onderste der omfeschreuen pilaren weghen 4 lb, d'ander 3 lb, de volghende 2 lb, de hoochste 1 lb: Laet oock om



de selue reden de leegste der binneschreuen pilaren weghen 3 lb, d'ander 2 lb, de laetste 1 lb. T'welck soo sijnde, ende anghesien alle de swaerheyts middelpunten ende der pilaren swaerheden bekend sijn, soo ist openbaer duer het 2^e voorsteldes 1^{en} boucx, datter swaerheyts middelpunt der vier omfeschreuen pilaren sal vallen in L, also dat L E sal doen $\frac{1}{24}$ van A D, ende der drie binneschreuen pilaren sal vallen in S, alsoo

M 3 dat

viz. 2 to 1, so is the circle BC to the circle IK , the prism BG will have to the prism IL the ratio of 2 to 1. Therefore, let BG weigh 2 lbs and IL 1 lb. Their centres of gravity are N , O ; therefore the line NO will be beam, which being divided into its arms, viz. in R , in such a way that NR shall be double of RO , R will be the centre of gravity of the two circumscribed prisms, and O is the centre of gravity of the inscribed prism; then R will be the same distance from E as O from E , to wit each $\frac{1}{12} AD$ ¹⁾. And this will be the same in all other examples of the kind. But in order to make it clearer, we will describe a special example thereof, as follows:

Let us intersect the paraboloid ABC once again through the middle points of AH and HD , drawing four circumscribed and three inscribed prisms, as shown below, where AD is the axis of the paraboloid, while the centres of the prisms are I , K , L , M , and AE shall still be double of ED , as before. Now because as AD is to AN (viz. 4 to 3), so is the circle BC to the circle OP , the prism BF will have to the prism OQ the ratio of 4 to 3, and for the same reason BF will have to the third prism, whose centre is K , the ratio of 4 to 2, and to the circumscribed prism, whose centre is I , the ratio of 4 to 1. Therefore, let the lowermost of the circumscribed prisms weigh 4 lbs, the second 3 lbs, the next 2 lbs, and the topmost 1 lb. Let also, for the same reason, the lowermost of the inscribed prisms weigh 3 lbs, the second 2 lbs, the last 1 lb. This being so, and all the centres of gravity and the gravities of the prisms being known, it is manifest by the 2nd proposition of the 1st book that the centre of gravity of the four circumscribed prisms will fall in L , in such a way that LE will make $\frac{1}{24} AD$ ²⁾, and that the centre of gravity of the three inscribed prisms will fall in S , in such

¹⁾ This may be seen by applying Prop. 2 of Book I:

If $AD = a$, $DR = DO + \frac{1}{3} ON = \frac{1}{4} a + \frac{1}{6} a = \frac{5}{12} a$.

Therefore $RE = DR - DE = (\frac{5}{12} - \frac{1}{3}) a = \frac{1}{12} a$, whereas $EO = \frac{1}{3} a - \frac{1}{4} a = \frac{1}{12} a$.

²⁾ This has probably been found by repeated application of Prop. 2 of Book I. It may be verified by using the formula for the centre of mass of a compound figure relatively to BC , which gives

$$\text{for the circumscribed figure: } DL = \frac{4 \cdot \frac{1}{8} + 3 \cdot \frac{3}{8} + 2 \cdot \frac{5}{8} + 1 \cdot \frac{7}{8}}{10} a = \frac{3}{8} a.$$

$$\text{for the inscribed figure: } DS = \frac{3 \cdot \frac{1}{8} + 2 \cdot \frac{3}{8} + 1 \cdot \frac{5}{8}}{6} a = \frac{7}{24} a.$$

$$\text{Hence } LE = \frac{3}{8} a - \frac{1}{3} a = \frac{1}{24} a \text{ and } SE = \frac{1}{3} a - \frac{7}{24} a = \frac{1}{24} a.$$

dat S E oock sal doen $\frac{1}{24}$ van A D. Dees twee punten dan L ende S vallen wederom euen verre van E.

Maer soomen om den brander schreue sulcke acht pilaren, ende seuen daer binnen, men sal sulcke punten noch euewydich vinden van E; te weten elck $\frac{1}{48}$ van A D.

Maer soomen om den brander schreue soodanighe seshien pilaren, ende vijftien daer binnen, men sal sulcke punten noch euewydich vinden van E, te weten elck $\frac{1}{96}$ van A D: Inder voughen dat het verschil der volghende inschrijving, alijt den helft is der voorgaende, daer af wy naer t'nootsaecklick veruolg in allen souden trachten, ten waer wy dat lieten om de cortheyt.

Dit soo synde E is t'swaerheys middelpunt des ghegheuen branders: want later (soot mueghelick waer) daer buyten sijn tusschen E L ofte E S, men sal dan duer de oneindelicke omschrijving en binneschrijving veler pilaren, daer toe comen, dattet swaerheys middelpunt des omschreuen forms, leegher sal comen dan des branders: ofte der binneschreuen form, hoogher dan des branders, t'welck ommueghelick is. Ten is dan gheen ander punt dan E, t'welck wy bewyfen moesten.

T' BESLVT. Wefende dan ghegheuen een brander, wy hebben sijn swaerheys middelpunt gheuonden, naer den eysch.

M E R C K T.

ANGHESIEN des driehoucx lini vanden houck tot int middel der sijde, van t'swaerheys middelpunt in sulcken reden ghedeelt wordt, als desen as des branders duer het 4^e voorstel, soo volgt dat inden driehouck der ghelijcke ghedaenten sullen beuonden worden duer omschreuen ende binneschreuen vierhoucken, ghelijck hier vooren gheschiet is met omschreuen ende binneschreuen pilaren.

XI. EYSCH.

XXIIII. VOORSTEL.

W E S E N D E ghegheuen een ghecorten brander: Huer swaerheys middelpunt te vinden.

T'GHEGHEVEN. Laet A B C D een ghecorten brander sijn, diens decsel A B, ende grondt D C, ende as E F.

T'BEGHEERDE. Wy moeten huer swaerheys middelpunt vinden.

T'WERCK. Men sal den ghecorten brander volmaken, daer an stellende t'ghebrekende A B G, Daernaer salmen teekenen H, alsoo dat G H dobbel sy an H E, ghelijcx, alsoo dat G I dobbel sy an I F, daernaer K, alsoo dat I H sulcken reden hebbe tot I K, als den ghecorten brander A B C D, tottet branderken A B G: Ick seg dat K t'be-
gheerde

a way that SE will also make $\frac{1}{24} AD$. These two points L and S therefore are again at the same distance from E .

But if eight such prisms be circumscribed about the paraboloid and seven be inscribed therein, similar points will be found still equidistant from E , to wit each $\frac{1}{48} AD$.

But if sixteen such prisms be circumscribed about the paraboloid and fifteen be inscribed therein, similar points will be found still equidistant from E , to wit each $\frac{1}{96} AD$. In such a way that the distance found in the next inscription is always half that of the preceding, and in this way we might continue but that we have omitted it for brevity's sake.

This being so, E is the centre of gravity of the given paraboloid. For let it be (if this were possible) beyond it, between E and L or E and S ; by infinite circumscription and inscription of many prisms the result will be attained that the centre of gravity of the circumscribed figure will come below that of the paraboloid, or that of the inscribed figure above that of the paraboloid, which is impossible. Therefore it is none other point but E , which we had to prove. CONCLUSION. Given therefore a paraboloid, we have found its centre of gravity, as required.

NOTE.

Since the line in the triangle from the angle to the middle point of the side is divided by the centre of gravity into segments having the same ratio as those of the axis of the paraboloid, by the 4th proposition, it follows that in the triangle by means of circumscribed and inscribed quadrilaterals figures analogous to those found above with circumscribed and inscribed prisms will be found.

PROBLEM XI.

PROPOSITION XXIV.

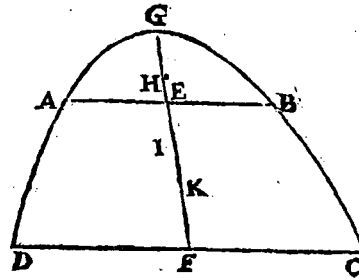
Given a truncated paraboloid: to find its centre of gravity ¹⁾.

SUPPOSITION. Let $ABCD$ be a truncated paraboloid, whose cover shall be AB and the base DC , and the axis EF . WHAT IS REQUIRED TO FIND. We have to find its centre of gravity. CONSTRUCTION. The truncated paraboloid shall be completed, adding thereto the missing part ABG . After this, H shall be marked in such a way that GH shall be double of HE , and likewise I in such a way that IG shall be double of IF ; after this K in such a way that IH shall have to IK the same ratio as the truncated paraboloid $ABCD$ to the small paraboloid

¹⁾ Cf. Prop. 13.

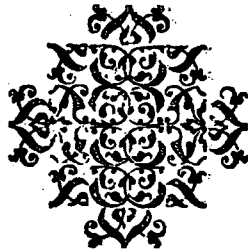
gheerde swaerheys middelpunt is.

T'BEWYS. Is swaerheys middelpunt des heels DCG , ende H des deels ABG , ende ghelijck t'ander deel $ABCD$, tot dit deel ABG , alsoo HI tot IK duer t'werck, daerom K , duer het 19^e voorstel, is t'begheerde swaerheys middelpunt, t'welck wy bewyfen moesten.



T'BESELYT. Wesende dan ghegheuen een ghecorten brander, wy hebben huer swaerheys middelpunt gheuonden naer den eysch.

EINDE DES TWEEDEN BOECK.

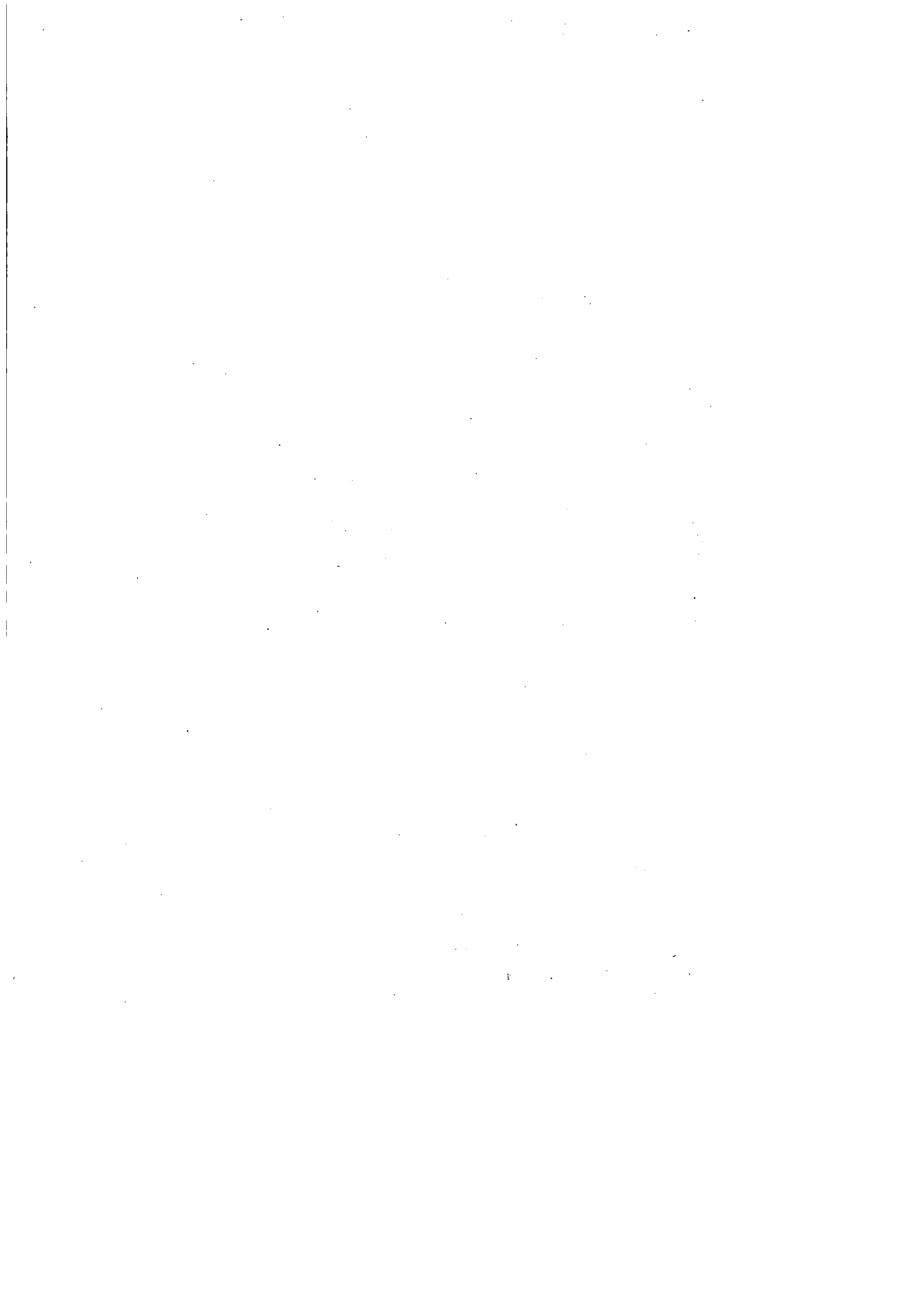


ABG. I say that *K* is the required centre of gravity. PROOF. *I* is centre of gravity of the whole *DCG*, and *H* of the part *ABG*; and as the other part *ABCD* is to the latter part *ABG*, so is *HI* to *IK* by the construction. Therefore, by the 19th proposition, *K* is the required centre of gravity, which we had to prove. CONCLUSION. Given therefore a truncated paraboloid, we have found its centre of gravity, as required.

END OF THE SECOND BOOK.

DE WEEGHDAET

THE PRACTICE
OF WEIGHING



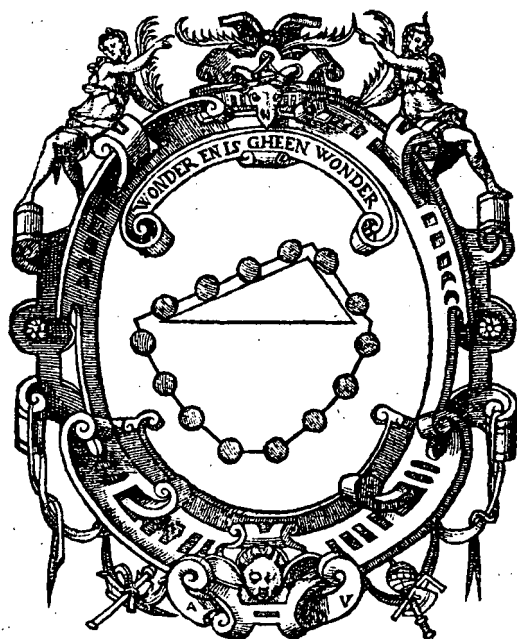
INTRODUCTION

In this work Stevin deals with the practical applications of the Art of Weighing in tools and engines. In the Preface the reader is warned that the theory developed in the preceding work does not permit of treating this subject with anything like completeness: it only leads to conditions of equilibrium, but does not provide information about the additional force required to produce motion and to maintain it.

Proposition I teaches an empirical method of determining the centre of gravity of any body by suspending it in different ways and determining the point in which the verticals through the points of suspension meet. The propositions 2-4 are devoted to the ordinary balance. According to Stevin the ideal form of this instrument is that in which the equilibrium of the non-loaded balance is indifferent. In Prop. 2, instructions for its construction are given with this end in view; for practical reasons, however, a slight deviation in the direction of stable equilibrium is permitted. Prop. 3 deals with the problem what weight has to be put in one of the pans of a balance in stable equilibrium in order to keep the beam in a given position. In Prop. 4 the equilibrium of the beam is unstable, and it is asked what weight the pans must have if the equilibrium of the completed balance is to be indifferent. In Prop. 5 the construction of a steelyard is described, in Prop. 6 that of a so-called oblique balance, i.e. an instrument enabling given forces to be exerted in given directions. Prop. 7 contains the practical applications of the lever; in Prop. 8 various cases in which weights have to be carried are treated by means of the theorems on the composition of parallel or concurrent forces, proved in the Art of Weighing. In Prop. 9 the theory of the lever is applied to the windlass and to other implements for displacing heavy loads, especially ships to be hauled.

The final Prop. 10 gives a very elaborate treatment of an instrument designed by Stevin, to which he gives the name of Almighty, thereby expressing that theoretically the mechanical advantage may be increased indefinitely.

DE
WEEGHDAET Præcis ærth
Ponderatio.
BESCHREVEN DVER
SIMON STEVIN
van Brugghe.



TOT LEYDEN,
Inde Druckerye van Christoffel Plantijn,
By François van Raphelinghen.
c10. 10. LXXXVI.

3

Simon Steuin wenscht
DEN BVRGMEESTERS
ENDE REGIERDERS DER
STADT NVRRENBERG
VEEL GHELVCKX.



HELIJCK onnutte cost vvaer,
een groote stercke grondt te leg-
ghen, die een svvaer ghesticht
draghen can, sonder eintlick
eenich ghebau daerop te vvillen
brenghen; Alsoo is de * spiegheling inde be-^{Theoria.}
ghinselen der consten verloren arbeydt, daer
t'einde totte * daet niet en strect. Ghelijck oock ^{Effectum.}
na de natuerlicke oirden, dien grondt voor
t'opperghebau gaedt, alsoo dese spiegheling
voor huer daet. Dit soo sijnde, ende vooren
beschreuen hebbende de Beghinselen der
Weeghconst, soo ist vouglick de WEEGHDAET ^{Praxis.}
te volghen; Oock my niet onbetamelick, de
selue an ulieder E. Heeren toe te eyghenen.
ende dat om drie besonder redenen: D'eerste,
dat haer * voorstellen niet alleen onghehoort, ^{Propositio-}
maer oock nut sijn. D'ander, dat haer voorde-
ring van nerghens merckelicker te vervvach-

SIMON STEVIN WISHES
THE BURGOMASTERS AND RULERS OF THE
CITY OF NUREMBERG
MUCH HAPPINESS.

Just as it would be useless to lay large and strong foundations which can support a heavy edifice without ultimately wishing to erect any building thereon, thus in the elements of the arts theory is lost labour when the end does not tend to practice. Just as, in the natural order of things, the foundation precedes the building, thus theory precedes practice. This being so, it is appropriate that, having described the Elements of the Art of Weighing hereinbefore, I should follow this up with the Practice of Weighing. Nor is it improper for me to dedicate same to Your Worships, such for three special reasons. The first, that its propositions are not only unheard-of, but also useful. The second, that its furtherance was to be expected from nowhere more clearly than from those with whom

4
Effet. ten en schein, dan vande ghene daer de Con-
sten inde grootste acht sijn, vvaer af niet alleen
en ghetuyghen de schriften veler gheleerden,
maer oock selfs u onderfatens constighe* da-
den, verspreydt in allen houcken des vveerelts.
De laetste ende voornaemste, dat ick my duer
sulcx toecommende voorthelpers hoopte te
bereyden, tot seker daden mijns voornemens;
Met vvelcke eindelicke meining hier eindende,
vvensch V. H. alle voorspoet ende vvel-
varen. Vyt Leyden in Oogstmaendt des
1586. Iaers.

ANDEN

the arts are held in the greatest esteem, to which is borne witness not only by the writings of many scholars, but also by the ingenious works, of your subjects, scattered in all parts of the world. The last and most important, that in this way I hope to gain future collaborators, for carrying out my projects. With which final thought I here conclude, and wish Your Worships all prosperity and health. From Leyden, in Harvest Month of the year 1586.

A N D E N L E S E R. ⁵



VANT in ettelicke * voorstellen der *Propositiones*
 Weeghdaet ghehandelt sal worden vande
 roerselen der lichamen, soo heeft my goet
 ghedocht, eer wy tot de saeck commen, den
 Leser van dies wat te verclaren. Te we-
 ten dat de Weeghconst ons alleenlick leert,
 het roerende ter euelstaetwichticheyt brenghen mettet teroe-
 ren. Angaende t'ghewicht ofte de macht, die t'roerende bo-
 uen dien noch behouft, om het teroeren ter roerlicke daet te
 crygen (welck ghewicht ofte macht, ouerwinnen moet des te-
 roerens beletsel, dat in yder teroeren * onscheydelicke ancleuing *Inseparabile*
 is) de Weeghconst en leert dat ghewicht ofte die macht niet *accidens.*
 * Wisconstlick vinden, d'oirsaek is dattet een gheroerde ende *Mathema-*
 sin beletsel niet * euerednich en is mettet ander gheroerde ende *tica.*
 sin beletsel. Maer op dat den sin van desen duer ghelijcknis *Proportionalis.*
 opentlicker verstaen worde, soo laet by voorbeelt een waghen
 bekender swaerheyt, te trecken sin op een berch ofte hoochde
 bekender steylheyt, Ick seg dat de Weeghdaet leert, soo duer
 het 4^e voorbeelt des 9^m vcorstels blycken sal, hoe groote macht
 met die waghen euelstaetwichtich, ofte euemachtich sal slaen,
 sonder t'ansien roersel met sin belet, a's assen teghen de bussen,
 raeyers teghen de straet, waghen teghen de locht, &c. welke
 macht des beletsels de Weeghconst niet en leert vinden, om dat
 sulcke beletselen ende haer gheroerden in gheen e'veredenheyt en
 bestaen, so wy hier souden connen bethoemen, weerleggende de
 * strijtredens vande ghene die in vallende swaerheden de contra- *Argumenta.*
 rie meinen, ten waer ons voornemen is, in dese Weeghconst al-
 leenlick met de leering voort te varen, ende d'oude dwalinghen

TO THE READER

Because in several propositions of the Practice of Weighing the motions of bodies will be dealt with, I thought it advisable, before coming to the matter, to explain something of it to the reader. To wit, that the Art of Weighing only teaches us to bring the moving body into equality of apparent weight to the body to be moved. As to the additional weight or the force which the moving body requires in order to set in motion the body to be moved (which weight or force has to overcome the impediment of the body to be moved, which is an inseparable attribute of every body to be moved), the Art of Weighing does not teach us to find that weight or force mathematically; the cause of this is that the one moved body and its impediment are not proportional to the other moved body and its impediment. But in order that the meaning of this may be more openly understood through an example, let a wagon of known gravity have to be drawn up a mountain or height of known steepness. I say that the Practice of Weighing teaches, as will become apparent from the 4th example of the 9th proposition, what force will be of equal apparent weight or equal power to that wagon, without considering the motion with its impediment, such as axles against the bearings, wheels against the road, wagon against the air, etc.; the finding of which force of the impediment the Art of Weighing does not teach, because these impediments are not in any proportion to their moved bodies, as we might here prove, refuting the arguments of those who think the contrary to be true of falling gravities, but that it is our intention merely to continue the instruction in this Art of Weighing and to reject elsewhere the old errors concerning the

der wichtighe ghedaenten elders te verworpen. Meret oock dat dese kennis der euestaltwichticheyt tot de saeck ghenouch doet, want ligghende in elcke schael des waeghs eueveel ghewichts, ghelijck wy dan weten (hoewel de waegh oock haer belet des roersels heeft) dat tottet roersel der schalen luttel machts behouft, alsoo in allen anderen.

Dit is van't belet des roersels tot dien einde gheseyt, op dat yemant duer de daet, de roerende macht altemet wat grooter bewindende, dan de gheroerde, niet en dencke sulcx t'ghebreck der const te wesen, maer nootsaecklick, ouermidts, als vooren gheseyt is, t'roerende bouen de euestaltwichticheyt soo veel swaerder ofte machtigher moet sin, dan het teroeren, dattet sulck belet ouerwint. Ten anderen, op dat niemant, die hem in sulcke schijn van eueredenheyt mocht betrouwen, bedroghen en worde, t'welck den ghenen alderlichtelicx ghebuert, die t'valsche voor warachtich houden.

Argumentii.

C O R T B E G R Y P.

D E S E Weeghdaet sal veruaten de werclicke vinding des swaerheys middelplats, swaerheys middellini, en swaerheys middelpunts: Voort de making des alderuolmaecsten Waeghs, met verclaring van elicke huer ghedaenten. Oock den aldervolmaecsten Onsel. Wyder, de ghedaenten der steerten daermen ghewelt me doet: De ghedaenten der ghedreghen ghewichten; Der Windassen; Der ghetrocken ghewichten; Ende des Almachichs.

DE WEEGH-

qualities of weights¹⁾. Note also that this knowledge of equality of apparent weight is sufficient for the purpose, for if the same weight lies in either pan of the balance, as we then know (though the balance also has its impediment to motion) that little force is required to move the pans, thus it is also in all other cases.

The above has been said about the impediment to motion in order that someone, finding in practice the moving force to be perhaps slightly greater than the force moved, may not think this to be a defect of the art, but may understand it to be necessary, since, as has been said above, the moving body, over and above the equality of apparent weight, has to be so much heavier or more powerful than the body to be moved that it overcomes the impediment to motion. Secondly, in order that no one, relying on this apparent proportionality, shall be deceived, which may very easily happen to those who hold the false to be true.

ARGUMENT

This Practice of Weighing is to contain the finding by practice of the centre plane of gravity, centre line of gravity, and centre of gravity. Further the construction of the most perfect balance, with an explanation of several of its properties. Also the most perfect steelyard. Further the properties of the levers by which a force is exerted, the properties of weights that are being carried, of windlasses, of weights that are being hauled, and of the Almighty.

¹⁾ This promise will be fulfilled in the second chapter of the *Appendix to the Art of Weighing*. See the present volume, p. 509.

DE WEEGHDAET ⁷ Praxis artis Ponderationis.

BESCHREVEN DVER SIMON STEVIN.

I. VOORSTEL.

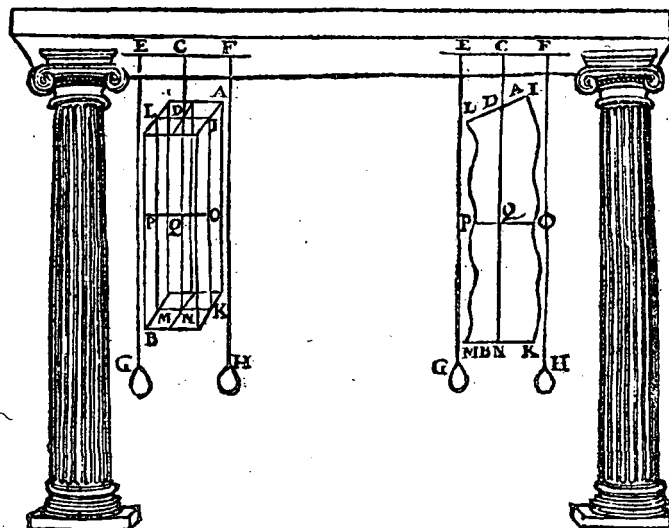
W E S E N D E ghegheuen een lichaem van form foot valt: Sijn swaerheys middelplat, swaerheys middellini, ende swaerheys middelpunt werckelick te vinden.

I^r VOORBEELT.

TGHEGHEVEN. Laet A B een lichaem sijn van form foot valt.

TBEGHEERDE. Wy moeten sijn swaerheys middelplat, swaerheys middellini, ende swaerheys middelpunt werckelick vinden.

TWERCK. Men sal t'lichaem hanghen ande coorde C D, treckende duer t'opperste punt C, de rechte lini E F, hanghende uyt de selue lini twee sijne draen met haren ghewichtkens, als E G, F H, neuens het



lichaem A B, ende t'plat veruaet tusschen de linien G E, F H, t'welck by ghedacht

THE PRACTICE OF WEIGHING

Described by Simon Stevin

PROPOSITION I.

Given a body of any form: to find by practice its centre plane of gravity¹⁾, centre line of gravity, and centre of gravity.

EXAMPLE I

SUPPOSITION. Let AB be a body of any form. **WHAT IS REQUIRED TO FIND.** We have to find by practice its centre plane of gravity, centre line of gravity, and centre of gravity. **CONSTRUCTION.** The body shall be hung from the cord CD , upon which through the highest point C shall be drawn the straight line EF , from which line shall be hung two thin threads with their small weights, as EG , FH , beside the body AB ; then the plane contained between the lines GE , FH , which may be conceived of as passing through the body, is the centre plane

¹⁾ Centre plane of gravity having been defined in Def. 6 of Book I of the *Art of Weighing* as any plane through the centre of gravity of the body, it is clear that it is not a single plane, as the text seems to suggest.

ghedacht duer 't lichaem lij, is des lichaems swaerheys middelplat. Maer om sijn uysterste sijden op 't lichaem te teekenen, men mach de draen EF, GH, bekriten, die ghespannen treckende, ende daer op teekenen de, ghelijck de Saghers haer boomen doen daer sy doorfaeght moeten sijn; ick neme die linien te wesen IK, LM, teekene daer naer insghelijcx de linien LI, ende MK, 't plat LIKM, sal 't begheerde sijn.

Maer om nu de swaerheys middellini te vinden, men sal 't lichaem noch hanghende an C, een weynich draeyen ende teekenen een ander der ghelijcke swaerheys middelplat, sniende 't voorgaende ick neem onder in N, ende bouen in D, ende haer ghemene sne DN sal de begheerde swaerheys middellini sijn: Maer om 't swaerheys middelpunt te vinden, men sal 't lichaem verhanghen inde dwerfde, ick neem by 't punt O, ende vinden aldaer oock des lichaems swaerheys middellini alsvoren, ick neem die te wesen OP, ende daer sy de lini DN snijt, als in Q, is 't begheerde swaerheys middelpunt.

11^e VOORBEELT.

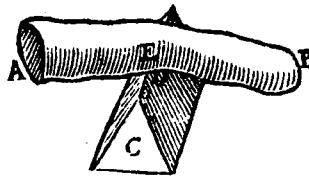
T'GHEGHEVEN. Laet AB een lichaem sijn van form soot valt:

T'BEGHEERDE. Wy moeten sijn swaerheys middelplat, swaerheys middellini, ende swaerheys middelpunt werckelick vinden.

T'WERCK. Men sal 't lichaem AB legghen op eenighen scherpen cant als CD, dat verreckende ter eender ende ander sijde, tot datmen sich

Horizontem.

bemercke de euestaltwichticheyt beyder sijden ghetroffen te hebben, 't welc ick neem te wesen in E, daerom 't plat rechthouckich op den *sichteinder 't lichaem door E sniende, sal 't begheerde swaerheys middelplat sijn. Ende een der ghelijcke plat 't voorgaende plat doorsniende, huet ghemene sne sal swaerheys middellini sijn. Ende soodanighen derde plat snijt die swaerheys middellini in des lichaems swaerheys middelpunt. Welcker bewys uyt de voorgaende openbaer is.



T'BESLIVYT. Wefende dan ghegheuen een lichaem van form soot valt, wy hebben sijn swaerheys middelplat, swaerheys middellini, ende swaerheys middelpunt werckelick gheuonden, naer de begheerte.

II VOORSTEL.

EEN aldervolmaeckste waegh te maken.

T'WERCK. Men sal eerst int middel des balcx AB, wiens tong ter behoorlicker plaets sy, teekenen de lini CD onder 't middel der tong,

of gravity of the body ¹⁾. But in order to draw its boundaries on the body, we can chalk the threads EG , FH , stretching them and thus drawing them on the body, as do the sawyers with the trees where they are to be sawn. I assume these lines to be IK , LM . If thereafter the lines LI and MK are likewise drawn, the plane $LIK M$ will be the required plane.

Now in order to find the centre line of gravity, the body, still hanging from C , shall be turned a little, upon which another, similar centre plane of gravity shall be drawn, intersecting the preceding one, say in N at the bottom and in D at the top; then its line of intersection DN will be the desired centre line of gravity. But in order to find the centre of gravity, the body shall be hung transversely, say at the point O , and the centre line of gravity of the body shall also be found in this position as above. I assume this to be OP , and where it intersects the line DN , as in Q , is the required centre of gravity.

EXAMPLE II.

SUPPOSITION. Let AB be a body of any form. **WHAT IS REQUIRED TO FIND.** We have to find by practice its centre plane of gravity, centre line of gravity, and centre of gravity. **CONSTRUCTION.** The body AB shall be laid on a sharp edge, as CD , and it shall be shifted on either side until we find we have attained the equality of apparent weight of the two sides, which I take to be in E . Therefore the plane at right angles to the horizon intersecting the body in E will be the required centre plane of gravity ²⁾. And a similar plane intersecting the preceding plane, their common line of intersection will be centre line of gravity. And a third such plane intersects this centre line of gravity in the centre of gravity of the body. The proof of which is manifest from the preceding. **CONCLUSION.** Given therefore a body of any form, we have found by practice its centre plane of gravity, centre line of gravity, and centre of gravity, as required.

PROPOSITION II.

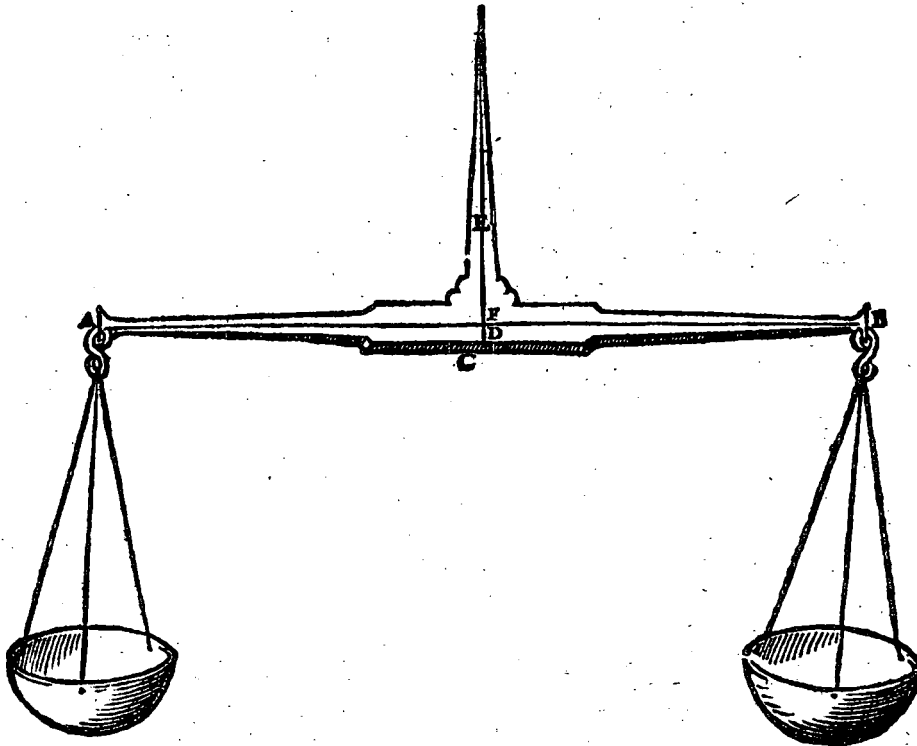
To construct a most perfect balance.

CONSTRUCTION. In the middle of the beam AB , having its tongue at the appropriate place, the line CD shall first be drawn below the middle of the

¹⁾ Read: a centre plane of gravity.

²⁾ Read: any plane at right angles to the horizon intersecting the body in E will be a centre plane of gravity.

tong, rechthouckich op de canten des balcx, ende vylen ofte weeten van d'een ende d'ander sijde soo veel stof, tot dat den balck (ligghende met de lini C D op eenighen scherpen cant) ouer beyden sijden met euen ermen euewichtich beuonden wort. Daer naer salmen trecken D E oock rechthouckich op de canten, ende legghen den balck op eenen scherpen stalen punt, ghenakende inde lini D E, souckende inde selue lini D E des balcx swaerheys middellini, te weten den balck ter eender ende ander sijde vertreckende (welverstaende dat den stalen punt altyt inde lini D E blijue) tot datmen bemerckt de euewicheyde ghetrossen te



sijne, t welck ic neem te wesen in F; Daernaer ghetteckent een derghe-
lijcke punt ouer d'ander sijde, de rechte lini door die twee punten sal de
swaerheys middellini des balcx sijn, beteeckenende t'scherp vanden
dweerlas, soo noem ick t'scherp daer op den balck int huysken rust.

tongue, at right angles to the sides of the beam, and then so much material shall be filed off or taken away from either side until the beam (lying with the line *CD* on some sharp edge) is found to be in equilibrium with equal arms on either side. Thereafter, *DE* shall be drawn, also at right angles to the sides, and the beam shall be laid on a sharp steel point, touching the line *DE*, and the centre line of gravity of the beam shall be sought in this line *DE*, viz. by shifting the beam on either side (it being understood that the steel point shall always remain in the line *DE*), until equilibrium is found to be attained, which I take to be in *F*. Thereafter, a similar point being marked on the other side, the straight line passing through these two points shall be the centre line of gravity of the beam ¹⁾, which determines the sharp edge of the transverse axis ²⁾ (which is the name I give to the pin on which the beam is supported in the fork). Thereafter, if

¹⁾ According to Def. 5 of Book I of the *Art of Weighing* this is only true if the beam is supported vertically in one of the two points mentioned.

²⁾ Obviously Stevin requires the ideal balance to be in a state of indifferent equilibrium.

Daer naer soode schalen an dien balck met haecken moeten hanghen, men sal de plaetsen der ghenaeckfelen des balcx ende dier haecken als an A, B, alsoo stellen, dat sy ende t'scherp vanden dweersas in een rechte lini AFB comen te staen: verstaet wel t'voornomde woort Ghenaeckfelen, want wy spreken vande eyghen wesentlicke ghenaeckfelen der haecken teghen de stof des balcx. Maer soo t'ghene daer mede de schalen anden balck hanghen yet anders waer dan haecken, men sal op haer derghe-lijke naeckfelen letten. Twelck ghedaen sijnde ende thuyken t'sijnder plaets ghevoucht wesende, soodanighē waegh met alle euen ghewichten diemen in haer schalen soude mueghen legghen, sal, so lang den dweersas op haer scerpste rust, alle ghestalt houden diemen haer gheeft, door het 10 voorstel des 1^{en} boucx vande beghinfelen der Weeghconst.

Maer dat alsulcken waegh de aldervolmaecte sy, is openbaer door het 1^e voorbeeld van het 11^e voorstel des voornomden 1^{en} boucx, alwaer be- thoont is, dat wesende E vastpunt, wat ghewicht men an D soude moe- ten hanghen, om den as in ghegheuen ghestalt te houden, maer so t'vast- punt aldaer had gheweest N, te weten het swaerheys middelpunt des ghegheuens, daer en soude gheen ghewicht so cleen connen sijn * Wis- constelick sprekende, dat an D ghehanghen, die sijde niet en soude doen gantselick neerdalen: T'selue is hier oock alsoo te verstaen, te we- ten dat tot d'een ofte d'ander deser euewichtigher deelen een seer cleen ghewicht gheleyt, die sijde sal stracx ten gronde sincken, daer sy van sommeghe ander waghen nau verroeren en soude.

Maer soot den Waeghmakers te moeylick viel die plaets van t'scherp des dweersas, metgaders de ghenaeckfelen der haecken ende des balcx, al- tijt soo puntelick te treffen, sy mueghen t'ghene gheleyt is houden als voor hun wit, dat soo naer commende als sy willen oft connen; Ende so sy van t'volmaecte yet soudē begheeren te verschillen, mueghen ghe- dachtich sijn t'naeckfel der haecken ende des balcx lieuer te stellen een haerken beneden de rechte lini AB, dan daer bouen, want daer bouen ghestelt sijnde, alles keert omme duer het 8^e voorstel des 1^{en} boucx, t'welck onbequaem is om te wegghen; Ia t'ghene t'swaerste waer, soude al- temer t'lichtste schijnē, voornamelick als den as duer de langde des balcx int beghin des weghens niet euewydich en waer vanden * sichteinder, ouermits alles an die sijde keert daert eerst beghint.

Angaende dat de ermen des balcx euelanck moeten wesen, dat is ken- nelick, want soo d'een een honderste deel des erms langher waer als d'ander, dat soude een bedriechlicke waegh sijn, ouermids t'ghene eue- wichtich schene, soude een ten hondert verschillen; ende waer d'een een vyuentwintichste deel langher als dander, t'soude 4 ten hondert schil- len, &c. Want ghelijck den langsten erm tot den corsten, alsoo dit ghe- wicht tot dat, duer het 1^e voorstel des 1^{en} boucx.

MERCT

Mathema-
ticd.

Horizonte.

the pans have to hang on the beam by means of hooks, the places of the points of contact of the beam and the hooks, as A , B , shall be so arranged that they come to fall in a straight line AFB with the sharp edge of the transverse axis. Do not mistake the aforesaid term "points of contact", for we are referring to the proper and real points of contact of the hooks against the material of the beam. But if the device by which the pans hang on the beam does not consist in hooks, similar points of contact shall be noted. Which being done and the fork being arranged in its proper place, such a balance with all the equal weights that might be laid in the pans will, as long as the transverse axis rests on the sharp edge, remain at rest in any position given to it, by the 10th proposition of the 1st book of the elements of the Art of Weighing.

The fact that such a balance is a most perfect one is manifest from the 1st example of the 11th proposition of the aforesaid 1st book, where it has been shown, E being the fixed point, what weight would have to be hung from D to keep the beam in its given position, but if the fixed point there had been N , to wit the centre of gravity of the given body, there would be no weight so small, in the mathematical sense, but would, if hung from D , cause that side to descend altogether. This is also to be so understood in this case, to wit that if a very small weight be added to one or the other of these balanced parts, that side will at once descend, while in some other balances it would scarcely stir.

But if the makers of balances deem it too difficult always to find exactly the place of the sharp edge of the transverse axis, and the points of contact of the hooks and the beam, let them aim at what has been said, approximating it as much as they wish to or are able to. And if they should desire to fall short of perfection a little, let them be mindful of placing the points of contact of the hooks and the beam a hair's breadth below the straight line AB rather than above it ¹⁾, for if they are placed above it, everything will turn upside down by the 8th proposition of the 1st book; which makes weighing impossible. Nay, what is heaviest would sometimes appear to be lightest, especially if the axis through the length of the beam were not parallel to the horizon during the commencement of the weighing, since everything turns upside down on the side where it first begins to turn.

As to the necessity of the arms of the beam being equally long, this is obvious, for if the one were longer than the other by one-hundredth of the arm, the balance would be a deceptive one, since what seemed to be of equal weight would differ by one per cent.; and if the one were longer than the other by one-twenty-fifth, the difference would be 4 per cent., etc. For as the longer arm is to the shorter, so is the latter weight to the former, by the 1st proposition of the 1st book.

¹⁾ As A and B have been introduced to denote the points of contact of the beam and the hooks, this instruction is not clear. It should read: below the horizontal plane through the sharp edge of the transverse axis.

MERCKT oock dat inde langste dunste ende lichtste balcken, 't grootste voordeel is om scherpelick te wegghen. Want wesende twee euefware balcken maer d'een tweemaal langher als d'ander, tis kennelick dat een once, aes oft watter sy, ande langste tweemaal meer ghewelts sal doen dan ande cortste duer t'voornoemde 1^e voorstel.

T' BESLVT. Wy hebben dan een aldervolmaeckste waegh ghemaeckt na t'voornemen.

III. VOORSTEL.

W E S E N D E ghegheuen een waegh diens balck euewydich blijft vanden * sichteinder : T'ghe- *Horizonto.* wicht te vinden dat in d'een schael gheleydt, den balck in begheerde ghestalt houde.

T'GHEBVERT dickmael dat d'een waegh veel stegher gaet als d'ander, sonder datmen weet waer an het liecht, want t'scherp des dweersas is van d'een soo bequaem als van d'ander, ende inde reste en openbaert hem niet ooghenschynelick daermen de reden duer bemercken can: Daerom sullen wy d'oirsaek beschrijven, bethoonende wat ghewicht men in d'een schael van foodanighen waegh sal moeten legghen op dat den balck blijue in begheerde ghestalt aldus: **T'GHEGHEVEN.** Laet de waegh A B C D sulck sijn, dat alles vry hanghende, den balck soude eindlick euewydich vanden sichteinder rusten, ende E sy t'scherp vanden dweersas. **T'BEGHEERDE.** Wy moeten inde schael D eenich ghewicht legghen, sulcx, dat den balck in die ghegheuen ghestalt blijue.

T'WERCK. Men sal t'huysken ende de schalen met haren coorden ende haecken afdoen, vindende des balcx met de tong daeran swaerheys middellini, euewydich metter scherp vanden dweersas E, door het 1^e voorstel deses boucx, t'welck ick neem F te sijne, daer naer salmen trecken een lini tusschen de plaetsen der naefelen des balcx ende der haecken vande schalen, welcke sy G H, wiens middel sy I: Daernaer salmen F I deelen, alsoo dat de stucken inde reden sijn van t'ghewicht des balcx niet de tong, welcke sy 1 lb, tot de schalen met haer coorden ende haecken, welcke ick neem te wegghen oock 1 lb, daerom ghedeelt F I, int middel K, soo sal K t'punt sijn daer an de ghegheuen waegh alle ghestalt soude houden diemen haer gheeft; Daernaer ghetrocken de lini K G, ende de hanghende duer E als E L, sniende K G in M; Ick seg dat een ghewicht in sulcken reden tot 2 lb (te weten 1 lb voor den balck, en 1 lb voor de schalen, t'samen 2 lb) als M K tot M G, t'begheerde sal sijn, t'welck gheleydt inde schael D, de waegh in die standt sal houden. Ghe-

Note also that the longest, thinnest, and lightest beams are those most suitable for accurate weighing. For given two beams of equal weight, but one being twice as long as the other, it is obvious that an ounce, *aes*¹⁾ or whatever it may be will exert on the longer one twice the force it exerts on the shorter one, by the aforesaid 1st proposition.

CONCLUSION. We have therefore made a most perfect balance, as intended.

PROPOSITION III.

Given a balance whose beam remains parallel to the horizon: to find the weight which, when laid in one of the pans, shall keep the beam in a required position.

It often happens that one balance moves much more slowly than another, without the reason for this being known, for the sharp edge of the transverse axis is equally good in both, and in the other parts nothing becomes apparent from which the cause can be learned. Therefore we will describe the cause, showing what weight has to be laid in one of the pans of such a balance in order that the beam shall remain in a required position. SUPPOSITION. Let the balance *ABCD* be such that, everything being freely suspended, the beam would ultimately remain at rest parallel to the horizon, and let *E* be the sharp edge of the transverse axis. WHAT IS REQUIRED TO FIND. We have to lay in the pan *D* some weight such that the beam shall remain at rest in that given position. CONSTRUCTION. The fork and the pans with their cords and hooks shall be taken away, and the centre line of gravity of the beam with the tongue shall be found, parallel to the sharp edge of the transverse axis *E*, by the 1st proposition of this book²⁾, which I assume to be *F*. Thereafter a line shall be drawn between the places of the points of contact of the beam and the hooks of the pans, which shall be *GH*, whose middle point shall be *I*. Thereafter *FI* shall be so divided that the segments have the same ratio as the weight of the beam with the tongue, which shall be 1 lb, to the pans with their cords and hooks, which I take to weigh also 1 lb. Therefore, *FI* being divided in the middle point *K*, *K* will be the point on which the given balance would remain at rest in any position given to it. Thereafter the line *KG* shall be drawn, and the vertical through *E*³⁾ as *EL*, intersecting *KG* in *M*. I say that a weight having to 2 lbs (i.e. 1 lb for the beam and 1 lb for the pans, together 2 lbs) the same ratio as *MK* has to *MG* will be the required weight, which, when laid in the pan *D*, will keep the balance in that position⁴⁾. If then *MK* is assumed to be one-twenty-fifth of *MG*, one-twenty-fifth

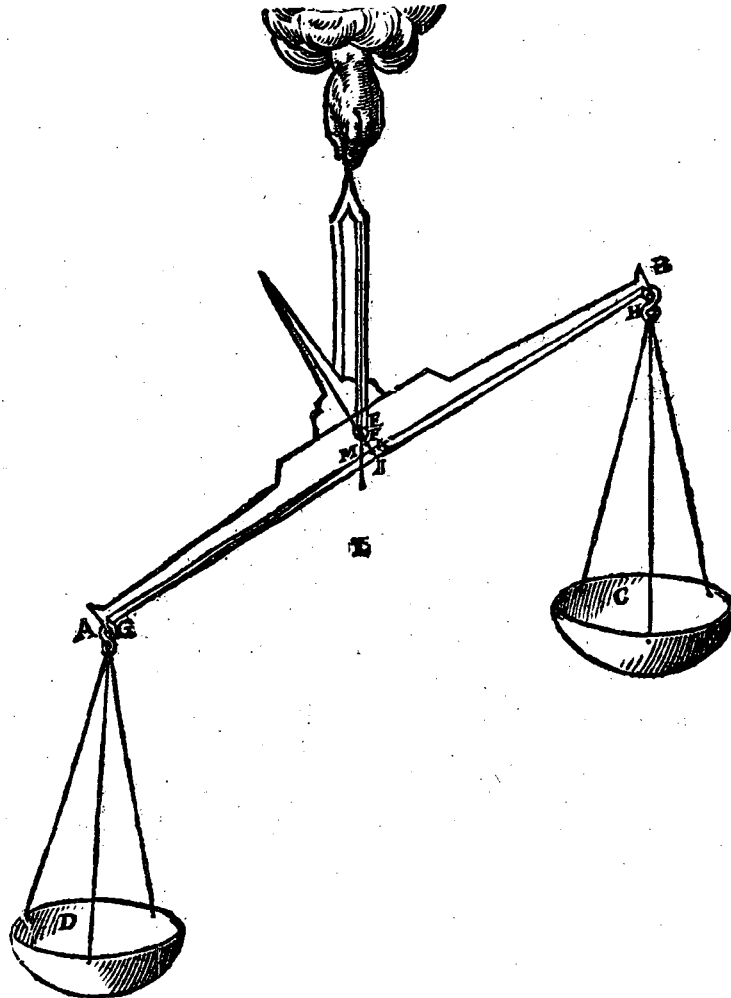
¹⁾ *aes* or *aas* is an old unit of weight, equivalent to 0.048 or 0.046 gramme.

²⁾ According to Prop. I, in the ideal case the centre line of gravity coincides with the sharp edge of the transverse axis.

³⁾ Read *F*.

⁴⁾ Obviously the balance is first given a certain position, as shown in the figure, and then the weight is determined which has to be put in the lower pan to keep the balance in this position. The result obtained is not turned to account by Stevin in order to explain why one balance moves more slowly than the other.

nomen dan dat MK het vijuentwintichste deel waer van MG , so sal het vijuentwintichste deel van $2 lb$ de waegh in die ghestalt houden, waer.



af t'bewys openbaer is duer hier 12 voorstel des 1^{en} boucs, maer wy sul-
lender hier om meerder claetheyt, noch een weynich af segghen.

Talwerks. Angheven K swaerheys middelpunt beteeckent des
ghegheuens

of 2 lbs will keep the balance in that position, the proof of which is manifest from the 12th proposition of the 1st book, but we will say a little more about it here, in order to make it clearer. PROOF. Since K denotes the centre of gravity of the given balance, the vertical through K will be the centre line of gravity of

ghegheuen, so sal de * hanghende duer K, des selfden swaerheys middellini wesen, ende de hanghende duer G, is swaerheys middellini des roegheleyden inde schael D, daerom de lini K G, tusschen die twee swaerheys middellinien, is der seluer weeghconstighen balck; Maer sy is ghedeelt in M, also dat den erm M G, sulcken reden heeft tot den erm M K, als diens swaerheyt tot defens; De hanghende dan duer M, is swaerheys middellini ofte handtaef des heels, ende veruolghens den balck blijft in die ghestalt, t welck wy bewysen moesten.

Perpendien-
laria.

T B E S L V Y T . Wefende dan ghegheuen een waegh, diens balck euewydich blijft vanden sichteinder, wy hebben t ghewicht gheuonden, dat in d'een schael gheleyt, den balck in begheerde ghestalt houdt, na t'voornemen.

IIII^e VOORSTEL.

W E S E N D E ghegheuen een balck, welcke met haer schalen euewydich blijft vanden * sichteinder, maer sonder schalen op t'scherp vanden dweersas niet rusten en can: Te vinden hoe sware schalen men daer an hanghen sal, op dat den balck alle ghestalt houde diemen haer gheeft.

Horizonte.

T G H E B V E R T sommighe balcken, dat sy sonder schalen op t'scherp van haren dweersas niet rusten en connen, maer wel de schalen daer an hanghende, welcker dinghen oirsaken wy duer de daet versoucken moeten. T G H E G H E V E N . Laet A B een balck wesen van ghedaente deses voorstels, wiens dweersassens scherp sy C.

T B E G H E E R D E . Wy moeten an desen balck twee schalen vinden (daerby men verstaen sal schalen met haer coorden en haecken) van sulck ghewicht, dat sy den balck alle ghestalt doen houden diemen haer gheeft. T W E R C K . Men sal vinden des balcx met de tong daer an swaerheys middellini, euewydich van t'scherp des dweersas C duer het 1^o voorstel deses boucx, welcke sy D, bouen C, want in C noch onder C en salse niet vallen, ouermids den balck op C, duer t'ghestelde niet rusten en can, noch min onder C. Daer naer salmen trecken de lini E F tusschen de plaetsen der ghenaeefelen des balcx, ende de haecken der schalen, de selue sal nootfaecklick vallen onder C, want viese daer in, of daer bouen, gheen schalen hoe swaer sy waren, en souden den balck alle ghestalt connen doen houden diemen haer gaue, ofte euewydich doen bliuen vanden sichteinder. Daer naer gheteekent G int middel van E F, men sal trecken de rechte lini D C G, ende gheliick dan C D, tot C G, alsoo moet t ghewicht der begheerde schalen H I sijn, tot

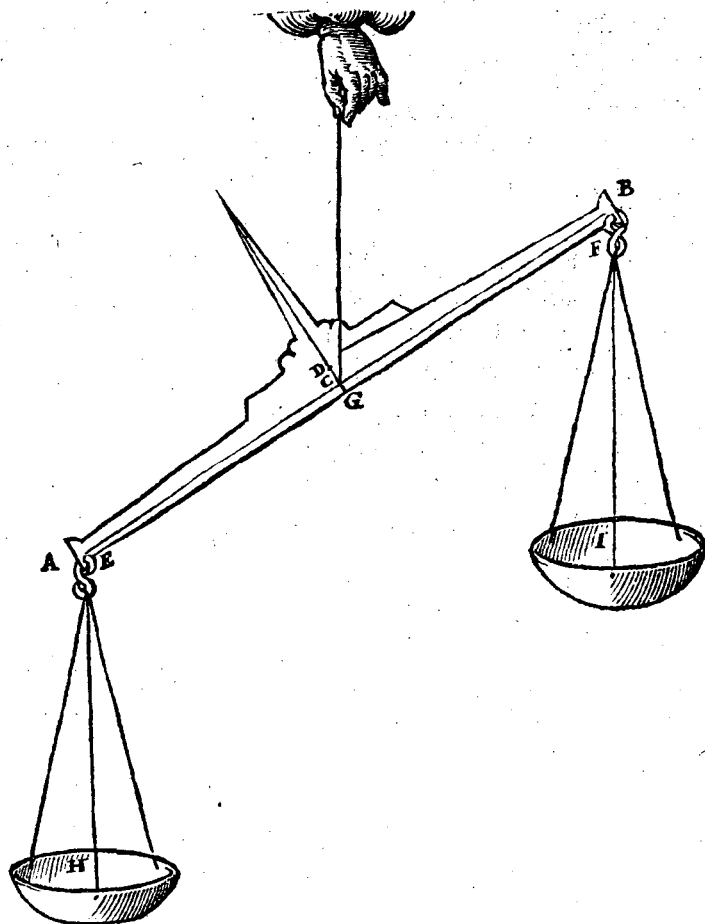
the same, and the vertical through G is centre line of gravity of the weight added in the pan D . Therefore the line KG , between these two centre lines of gravity, is its mathematical beam. But it is divided in M in such a way that the arm MG has to the arm MK the same ratio as the gravity of the former to that of the latter. The vertical through M therefore is centre line of gravity or handle of the whole, and consequently the beam remains at rest in that position which we had to prove. CONCLUSION. Given therefore a balance whose beam remains parallel to the horizon, we have found the weight which, when laid in one of the pans, shall keep the beam in the required position, as intended.

PROPOSITION IV.

Given a beam, which with its pans remains parallel to the horizon, but cannot remain at rest without the pans on the sharp edge of the transverse axis: to find the weight of the pans which have to be hung thereon in order that the beam shall remain at rest in any position given to it.

It happens with some beams that without the pans they cannot remain at rest on the sharp edge of their transverse axis, though they can when the pans hang thereon, the cause of which we have to investigate by practice. SUPPOSITION. Let AB be a beam of the kind referred to in this proposition, the sharp edge of whose transverse axis shall be C . WHAT IS REQUIRED TO FIND. We have to find on this beam two pans (by which are to be understood pans with their cords and hooks) of a weight such that they cause the beam to remain at rest in any position given to it. CONSTRUCTION. The centre line of gravity of the beam with the tongue shall be found, parallel to the sharp edge of the transverse axis C , by the 1st proposition of this book, which shall be D , above C , for it will not fall either in C or below C , since by the supposition the beam cannot remain at rest on C , and even less so below C . Thereafter the line EF shall be drawn between the places of the points of contact of the beam and the hooks of the pans; this will necessarily fall below C , for if it fell in it or above it, no pans, however heavy, could cause the beam to remain at rest in any position given to it or cause it to remain parallel to the horizon. Thereafter, G being marked in the middle point of EF , the straight line DCG shall be drawn, and then, as CD is to CG , so must be the weight of the required pans H and I to the weight

Mathematic. **T**ghewicht des balcx; ick neme dat C D euen sy an C G, tghewicht dan der schalen sal euen moeten wesen an tghewicht des balcx, waer af tbe-
wys * Wilconstlick ghedaen is int 10^e voorstel des 1^{ste} boucx, daer toe wy hier tot meerder clactheyt noch een weynich sullen segghen.



*Perpendicu-
laris.*

TBEWYS. De * hanghende duer D, is swaerheys middellini des balcx ter eender sijden, ende de hanghende door G is swaerheys middellini der schalen ter ander sijde; G D dan is Weegconstighen balck: Maer ghelijck

of the beam. I take CD to be equal to CG . The weight of the pans will then have to be equal to the weight of the beam, the mathematical proof of which has been given in the 10th proposition of the 1st book, to which we will add a little more in order to make it clearer. PROOF. The vertical through D is centre line of gravity of the beam on one side and the vertical through G is centre line of gravity of the pans on the other side. Therefore GD is mathematical beam. But

ghelijck den erm C D tot den erm C G, also dese swaerheyt tot die duer t'ghestelde, het houdt dan op C alle ghestalt diemen hem gheeft, t'welck wy bewyfen moesten. T B E S L V Y T. Wesende dan ghegheuen een balck, welcke met haer schalen euewydich blijft vanden sichteinder, maer sonder schalen op t'scerp vanden dweersas niet rusten en can; wy hebben gheuonden hoe sware schalen men daer an hanghen sal, op dat de balck alle ghestalt houde diemen haer gheeft, naer de begheerte.

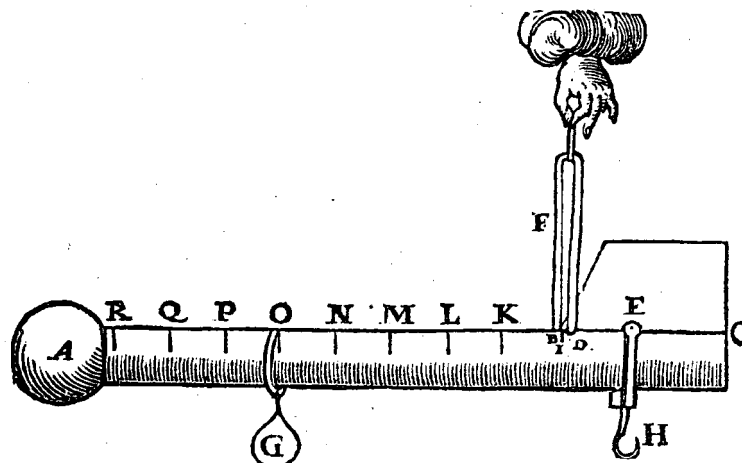
M E R C K T.

Tis openbaer, dat by aldien de schalen yet swaerder waren dan bouen ghefeyt is, ofte dat in haer eenighe euen swaerheden gheleyt wierden, soo en soude den balck dan niet alle ghestalt houden diemen hem gheeft, maer euewydich bliuen vanden sichteinder, daerom en sijn sulcke waghens niet de volmaeckste.

V. V O O R S T E L.

E E N alderuolmaecksten onsel te maken.

T W E R C K. Men sal des lichameliken balcx oppersten cant A B voorttrecken tot in C, ende laten inde lini B C de scherpten commen der twee dweersassen D, E, weluerstaende dat de scherpte van D neerwaert strecke, ende van E opwaert; Daernaer salmen van het dickeinde des balcx naer B C, soo veel afvilen ofte weeren, tot dat alles int huyfken



F ueestaltwichtich hanghe, ende dat bouen dien de scherpte vanden dweersas D (t'huyfken F gheweert sijnde) swaerheys middellini blijue des

as the arm CD is to the arm CG , so is the latter gravity to the former by the supposition. The beam therefore remains at rest in C in any position given to it, which we had to prove. **CONCLUSION.** Given therefore a beam which with its pans remains parallel to the horizon, but cannot remain at rest without the pans on the sharp edge of the transverse axis; we have found the weight of the pans which have to be hung thereon in order that the beam shall remain at rest in any position given to it, as required.

NOTE.

It is manifest that if the pans were somewhat heavier than stated above, or if some equal gravities were laid therein, the beam would not then remain at rest in any position given to it, but would remain parallel to the horizon; therefore such balances are not the most perfect.

PROPOSITION V.

To construct a most perfect steelyard.

CONSTRUCTION. The upper side AB of the corporeal beam shall be extended to C , and the sharp edges of the two transverse axes D, E shall be made to come in the line BC , it being understood that the sharp edge of D shall point downwards and that of E upwards. Thereafter so much material shall be filed off or removed from the thick end of the beam adjacent to BC until everything shall hang in equality of apparent weight in the fork F , and moreover the sharp edge of the transverse axis D (when the fork F has been removed) shall remain centre

des lichamelicken balck A C. Twelck soo sijnde ende den seluen balck int huysken F hanghende, sy sal daer in (soo lang den dweersas D op haer scherpte rust) alle ghestalt houden diemen haer gheeft. Daernaer salmen sien van wat swaerheyt t'schuyswicht G, ende den haeck H sulen sijn, diemen daer an begheert te hanghen; Ick neem G een pondt, ende H een once, dat is t'festhiende deel van G; Daerom salmen teekenen I, alsoo dat de lini tusschen I ende t'scherp des dweersas D, euen sy an t'festiende deel van D E; Daernaer salmen de langde D E (dat is de lini tusschen de scherpten der twee dweersassen) teekenen van I naer A, soo dickmael als sy daerin comen wil, t'welck ick neem te wesen in K, L, M, N, O, P; Q, R, daernaer machmen elcke langde als I K, K L, L M, &c. deelen in soo veel euen deelen alst de plaets toelaet, als in twee, oft in vieren, oft in achten, oft in sestiennen, &c. ende alles sal volmaeckt sijn.

Maer oft dit soo nau passen der dweersassen den onselmaeckers te moeylick viel, sy mueghent (ghelijck int voorgaende 2^e voorstel vande waegh oock gheseyt is) houden als voor hun wir, dat soo naer volghende als sy connen, ende t'scherp des dweersas D lieuer een haerken bouen de lini A C laten comen, dan daer onder.

Wat de ghebruyck belangt, als G an O hangt, ende anden haeck H een swaerheyt met de rest euestaltwichtich, die swaerheyt sal vijf pont weghen, ouermids van I tot O vijf teekenen staen. Maer soo elcke langde als I K, K L, L M, &c. ghedeelt waer in sestiennen, elck deel soude een once bereekenen. By voorbeeld of G hijnghe tusschen P en Q, an het vijftiende deel van P naer Q, de swaerheyt an H soude dan sijn van 6 lb 15 oncen, ende alsoo metten anderen. Nu ouermits desen onsel (ghenomen t'schuyswicht niet neerwaert en siere als d'een sijde leeght daelt) met alle euestaltwichtighe deelen die op beyde sijden hanghen, alle ghestalt houdt diemen huer gheeft soo ist (om de redenen die wy int voorgaende voorstel vanden aldervolmaecksten waegh gheseyt hebben) den aldervolmaecksten onsel. Angaende t'bewys, alles is openbaer door het 2^e voorstel des eersten boucx. T H E S L V V T. Wy hebben dan een alderuolmaecksten onsel ghemaect naer de begheerte.

VI. VOORSTEL.

DE scheefwaeg te maken.

WANT de ghewichten niet altemael rechtneerwaert noch rechtopwaert en roeren, maer sijdeling, ende scheef; ghelijck vooren verscheyden voorbeelden daer af beschreuen sijn, ende hier na beschreuen sullen worden, so behouwen dese een waegh van ander form dan de ghemeene, welke wy tot onderscheyt van d'ander Scheefwaeg noemen: Huer voor-

naemsto

line of gravity of the corporeal beam *AC*. Which being so, and the beam hanging in the fork *F*, it will remain at rest therein (as long as the transverse axis *D* rests on its sharp edge) in any position given to it. Thereafter it shall be ascertained what gravity the sliding weight *G* and the hook *H*, which are required to be hung thereon, must have. I take *G* one pound, and *H* one ounce, i.e. one-sixteenth of *G*. Therefore, *I* shall be marked in such a way that the line between *I* and the sharp edge of the transverse axis *D* shall be equal to one-sixteenth of *DE*. Thereafter the distance *DE* (i.e. the line between the sharp edges of the two transverse axes) shall be plotted from *I* to *A* as often as it will fall therein, which I take to be in *K, L, M, N, O, P, Q, R*. Thereafter each segment, as *IK, KL, LM*, etc., may be divided in as many equal parts as the space permits, as in two, or in four, or in eight, or in sixteen parts, etc.; then everything will be complete.

But if this accurate adjustment of the transverse axes should be too difficult for the makers of steelyards, let them (as stated also in the 2nd proposition hereinbefore with regard to the balance) aim at approximating it as much as possible, causing the sharp edge of the transverse axis *D* to come a hair's breadth above the line *AC* rather than below it.

As to the use of the steelyard, if *G* hangs at *O*, and at the hook *H* hangs a gravity which is of equal apparent weight to the rest, the latter gravity will weigh five pounds, since there are five marks from *I* to *O*. But if each segment, as *IK, KL, LM*, etc., were divided into sixteen parts, each part would denote an ounce. For example, if *G* hung between *P* and *Q*, at the fifteenth part from *P* to *Q*, the gravity at *H* would then be 6 lbs 15 oz., and similarly with others. Now since this steelyard (assuming that the sliding weight shall not slide down when one side descends lowest), with all the parts in equality of apparent weight hanging on either side, remains at rest in any position given to it, it is (for the reasons we gave in the proposition hereinbefore about the perfect balance) the most perfect steelyard. As to the proof, everything is manifest from the 2nd proposition of the first book. **CONCLUSION.** We have therefore constructed a most perfect steelyard, as required.

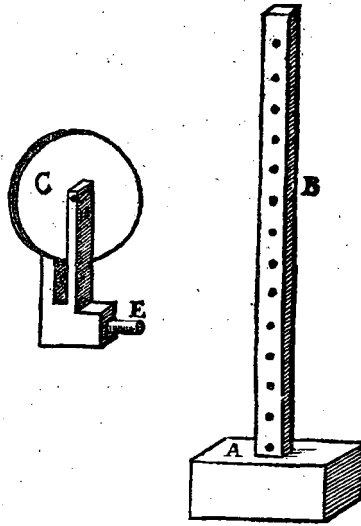
PROPOSITION VI.

To construct the oblique balance.

Because weights do not all move straight downwards or upwards, but also side-long and obliquely, of which many examples have been described hereinbefore and will be described hereinafter, these require a balance of a different form from the usual one, which we call oblique balance in order to distinguish it from

naemste einde is om duer ongeschijnelicke eruaring te sien, onderfoucken, ende verstaen, de waerheydt der voorstellen vande eueredenheydt soodanigher ghewichten int eerste bouck * Wisconstlick beschreuen, op *Mathemati-* datmen hem alsoo te vastelicker betrau in t'ghene men inde Daet tot et t'menschen voordering daer duer uytrecten wil.

T W E R C K. Men sal maken een voet als A, met een reghel daer op tot verscheyden plaetsen duerboort als B, daer naer een caterol als C, met een grouue rondtom inden cant daer een draet in loopen mach, ende in sijn middel sy een as D, rustende met beyde haer einden in een huysken, t'welck met het pinneken E, ghesteken mach worden inde gaetkens der reghel B, soo hooghe ofte leeghe als men wil, ende sal volmaect sijn. Maer t'voornaemste daermen op letten moet (op datmen een scheefwaegh heb die scherpelick weghe) is, dat het caterol ende den as daer in al t'samen moeten ghe-draeyt sijn, ende t'selue caterol ende den as soo dun als men can, ende dat de ronden nergens int huysken en ghenake, latende tusschen de einden des dweersas ende t'plat des caterols, eenighe dickte, wat dicker dan de einden des as. Ick heb voor my daer toe doen drayen een caterol van bosboom, wiens dickte niet meer en was dan als den rugghe van een dun mes, ende des ronds middellini van ontrent vijf duymen, ende den as (al met den anderen ghe-draeyt) van yvoor, vande dickte als een cleermakers naelde, te weten soo dun als den draeybanck lijdten mocht.



VII VOORSTEL.

T O N D E R S O V C K E N de ghedaenten der steerten daermen ghewelt mede doet.

S I E N D E de menschen darmen met langher steerten een merckelicker grooter ghewelt dede dan met de corter, sy hebben veel ghemeene
c reetschappen

the other type. Its main object is to make us see, examine, and understand through visual experience the truth of the propositions on the proportionality of such weights, described mathematically in the first book, in order that we may have all the more confidence in that which we wish to effect therewith in practice for the benefit of mankind.

CONSTRUCTION. A base shall be made, as *A*, with a ruler thereon which is perforated in several places, as *B*; thereafter a pulley, as *C*, with a groove all around the rim, through which may pass a thread, and in its centre there shall be a spindle *D*, supported with both ends in a fork, which can be put with the pin *E* into the holes of the ruler *B*, as high or low as may be desired; then the instrument will be complete. But the main point to be noted (in order to have an oblique balance weighing accurately) is that the pulley and the spindle passing through it should be turned together on the lathe, and that the pulley and the spindle should be as thin as possible, and that the circle shall not touch the fork anywhere, some space being left between the ends of the transverse axis and the surface of the pulley, a little more than the ends of the spindle. For this, I had a pulley turned of boxwood, whose thickness was no greater than the back of a thin knife, the diameter of the circle being about five inches and the axis (turned together with the other parts) being of ivory, of the thickness of a tailor's needle, to wit as thin as the lathe could produce it.

PROPOSITION VII.

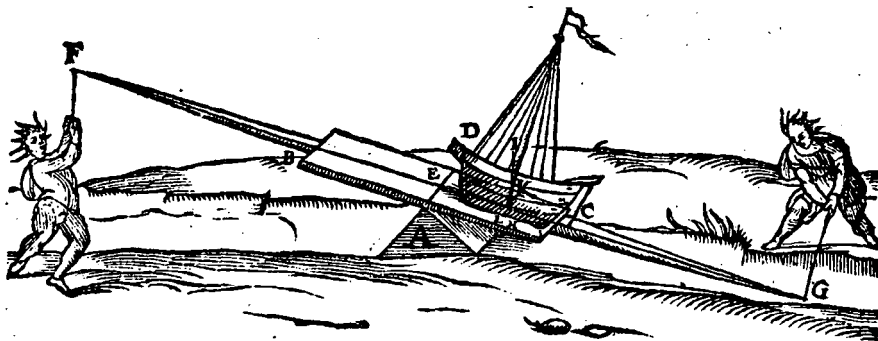
To investigate the forms of the levers by which a force is exerted.

Men having noticed that considerably larger forces could be exerted with longer levers than with shorter ones, they have thereby accomplished the construction

reerschappen tot hueren grooten dienste en voordeele daer duer ter daet ghebrocht: Maer want sulcx alleen gheschiede duer eruarighen, ende niet duer grondelicke kennis der * eueredenheyt in huer bestaende, soo en sijn veel groote nieuwe wercken dickmael niet wel gheluckt, tot groote schade der Makers, ende verachtering des voornemens. Op datmen dan wete eermen beghint, wat de steerten int volmaeckte werck souden connen doen, wy sullen (bouen de * Wisconftighe voorstellen des eersten boucx alsulcx veruatende) eenighe daerlicke voorbeelden daer af beschrijven. Ten eersten, want eenighe personen wel van meyning sijn gheweest, datmen de schepen bequamelicker ende met minder schade ouer een dam soude mueghen brenghen, duer r'behulp van langhe steerten, dan duer een windas, naer de ghemeene ghebruyck, wy sullen r'selue nemen als voorbeelt om te sien wat daer uyt volghen soude in deser voughen:

1^e VOORBEELT.

T'GHEGHEVEN. Laet A een dam wesen, ende B C een plat houten bereytsel daer het schip D weghende 14000 lb op rusten mach (hoe r'ghewichteens schips met al datter in is int water ligghende, bekend can worden, sal int Waterwicht sijn plaets hebben) ende dat E middel van B C passe op r'middel des dams A, ende laet B F den eenen steert sijn, ende C G (euen an B F) den anderen, ende r'schip D gheweert sijnde, so is de sijde E Feuewichtich teghen E G, ende om r'schip ouer den dam te krijghen, men soude trecken an F, ofte heffen an G, ofte an beyde r'samen. Ende laet H I des schips swaerheys middellini wesen, ende F E selsvoudich tot E H: Uyt het welcke men begheert te weten wat macht ofte ghewicht an F of G mer het schip euestaltwichtich sal sijn.



TWERCK. Ouermits F G is als balck eens waeghs, diens vastpunt E, ende schips swaerheys middellini H I, ende dat F E selsvoudich is teghen E H.

of many common tools, to their great service and benefit. But because this was only done by experience, and not by thorough knowledge of the proportionality existing therein, many great new constructions often did not turn out well, to the great detriment of the makers and the delay of the work proposed. Therefore, in order that one may know before one begins what perfectly constructed levers might be capable of, we will (in addition to the mathematical propositions of the first book comprising all this) describe a few practical examples thereof. Firstly, because some people have been of opinion that ships could be hauled better and with less damage across a dam by means of long levers than with a windlass, as is the custom, we will take this as example in order to see what would result from it, in the following way:

EXAMPLE I.

SUPPOSITION. Let A be a dam, and BC a flat wooden device on which the ship D , weighing 24,000 lbs, is adapted to rest (the manner in which the weight of a ship with all that is in it, lying in the water, can be found is to be described in the book on Hydrostatics), and let E , the middle of BC , fit on the middle of the dam A , and let BF be one lever and CG (equal to BF) the other. Then, the ship D being taken away, the side EF balances EG , and in order to haul the ship across the dam one should pull at F or raise at G , or both simultaneously. And let HI be the ship's centre line of gravity, and let FE be six times EH . From which it is required to know what force or weight at F or G will be of equal apparent weight to the ship.

CONSTRUCTION. Since FG resembles the beam of a balance, whose fixed point is E , and the ship's centre line of gravity is HI , and FE is six times EH ,

E H, so sal t'schip sefvoudich sin teghen t'ghewicht dat an F hanghende met hem euestaltwichtich sy, maer t'schip weeght duer t'ghestelde 24000 lb; An F dan soude moeten hanghen 4000 lb: Daerom sooder anhin- ghen 25 menschen elck weghende 160 lb, die soudent teghen t'schip eue- staltwichtich sijn: Maer dit verstaet hem op de stant daert nu in is, want nemende K voor swaerheys middelpunt des schips, ende het deel E G rijfende, soo sal an F min dan 4000 lb behouuen. Om van t'welck met voorbeelt te spreken, Laet ons trecken de lini K L rechthouckich op t'plat E C, inder voughen dat als t'plat E C euewydich sal sijn vanden sichteinder, soo sal K L des schips swaerheys middellini sijn. Ick neem nu dat E F seuevoudich sy teghen E L, daerom t'seuenste deel van 24000 lb als $3428 \frac{4}{5}$ lb, sal t'ghewicht sijn t'welck an F hanghende met de rest aldan in die standt euestaltwichtich sal sijn.

M E R C K T.

W y hebben hier een voorbeelt ghestelt daermen hem in sulcken handel soude naer mueghen rechten, maer tis te ghedencken dat E F sefvoudich ghenomen is teghen E H, t'welck wel eenen seet langhen steert soude moeten wesen ende sterck naer de gheleghentheyt. Ick achte datter in groote schepen (int ansien van beter) gheen goet einde en soude nemen; met cleyne schuytkens mochtet sijn bescheet hebben. Wel is waer, datmen an de einden F G windaffen soude mueghen stel- len, om soo veel volcx daer niet te behouuen, maer wy sullen een beter manier beschrijven int volghende 10^e voorstel, ons hier vernoughende met de rekening van soodanighen voorbeelt verclaert te hebben, wat- men t'sijnen voordeele daer het te pas mocht comen, bequamelicx sal mueghen ghebruycken.

11^e VOORBEELT.

W y hebben in t'eerste voorbeelt verclaert, de ghedaente der steerten die euelanck ende euewichtich sijn, wy sullen nu dit voorbeelt stellen van onen steerten. T G H E G H E V E N. Laet A B C den eenen steert sijn, ende A B D den anderen, rustende met de lini A B op de cant E; Ende de lini D C sniende A B in F, sy den as des heels D A C B weghende 400 lb, ende sijn swaerheys middelpunt sy G, (tis wel waer dattet swaerheys middelplat rechthouckich op den as inde daet ghenouch soude doen, soo wel int volghende 3^e ende 4^e voorbeelt, als in dir, doch om eyghentlic- ker daer af te spreken, wy nemen het swaerheys middelpunt) ende op het deel A B D light een swaerheyt H van 2000 lb, diens swaerheys middellini I K sy, te weten K inden as D C; De vraegh is hoe sterck men an C sal moeten trecken, om H op te lichten.

T W E R C K. Men sal vinden de swaerheys middellini der swaerheyt H, ende des reetschaps D A C B altsamen, aldus: Men sal K G deelen in L alsoo dat G L sulcken reden hebbe tot L K, als 2000 lb tot 400 lb, dat

c 2 is als

the weight of the ship will be six times the weight which, hanging at F , shall be of equal apparent weight thereto. But the ship, by the supposition, weighs 24,000 lbs; therefore 4,000 lbs would have to hang at F . Therefore, if there hung at it 25 men, each weighing 160 lbs, these would be of equal apparent weight to the ship. But this applies to the position in which it is now, for if we take K to be centre of gravity of the ship and the part EG to be rising, less than 4,000 lbs will be required at F . By way of example, let us draw the line KL at right angles to the plane EC in such a way that if the plane EC is parallel to the horizon, KL will be the ship's centre line of gravity. I now take EF to be seven times EL ; therefore the seventh part of 24,000 lbs, i.e. $3.428\frac{4}{7}$ lbs, will be the weight which, hanging at F , will then be of equal apparent weight to the rest in that position.

NOTE.

We have here given an example by which we might be guided in such a case, but it is to be borne in mind that EF has been taken six times EH , which would have to be a very long lever indeed, and strong for the occasion. I think that with big ships (as compared with better means¹⁾) it would not be successful, with small boats it might do. It is true that at the ends F and G windlasses might be placed, in order not to need so many men, but we will describe a better method in the 10th proposition hereinafter, contenting ourselves here with having explained the calculation of this example, which can be turned to account with profit wherever needed.

EXAMPLE II.

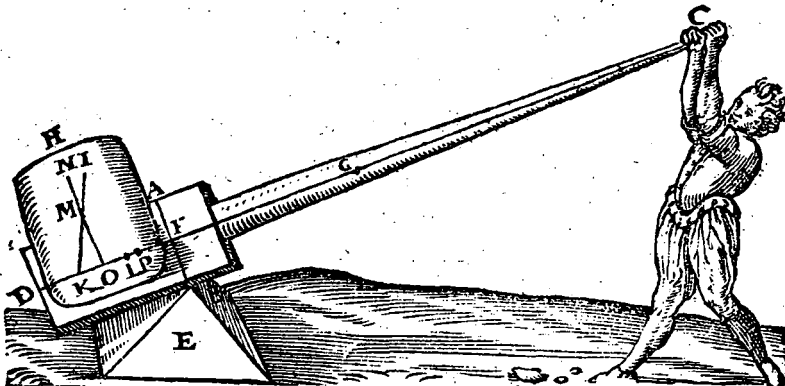
We have explained in the first example the forms of the levers which are equally long and of the same weight; we will now give this example of unequal levers. **SUPPOSITION.** Let ABC be one lever and ABD the other, resting with the line AB on the edge E , and the line DC intersecting AB in F shall be the axis of the whole $DACB$, weighing 400 lbs, and its centre of gravity shall be G (it is true indeed that in practice the centre plane of gravity at right angles to the axis would be sufficient, both in the 3rd and 4th examples hereinafter and in the present, but in order to deal with it more properly, we take the centre of gravity), and on the part ABD lies a gravity H of 2,000 lbs, whose centre line of gravity shall be IK , to wit K in the axis DC . The question is with what force a man will have to pull at C in order to lift H .

CONSTRUCTION. The centre line of gravity shall be found of the gravity H and the device $DACB$ together, thus: KG shall be divided in L in such a way that GL shall have to LK the same ratio as 2,000 lbs to 400 lbs, i.e. as 5 to 1,

¹⁾ This somewhat obscure phrase is omitted by Girard in his French translation (XIII; iv. *Livre de la Statique*, p. 474a).

*Perpendicu-
larn.*

is als 5 tot 1, ende eenighe * hanghende door L sal des heels swaerheys middellini sijn; Ick neem nu dat FC twelfvoudich beuonden sy teghen FL, daerom seg ick FC 12, gheeft FL 1, wat 2400 lb? (te weten de somme des swaerheys ende reetfchaps) comt 200 lb, voort'ghene dat



Horizonte.

an C hanghende met de rest in die gheleghentheyte euestaltwichtich sal sijn, daerom een man weghende 200 lb, ofte so stijf treckende als 200 lb daer an hanghende trecken souden, sal met de reste euestaltwichtich sijn. Maer dit verstaet hem op de ghestalt daert nu in is, want nemende M voor swaerheys middelpunt des ghewichts H, ende het deel A B D ryfende, soo sal an C min dan 200 lb behouuen. Om t'welck opentlicker te verstaen, laet ons trecken de lini N O, duer t'punt M rechthouckich op t'plat A B D, inder voughen dat als t'plat A B D enewydich sal sijn vanden * sichteinder, soo sal N O swaerheys middellini wesen der swaerheyt H, daerom ghedeelt O G in P, alsoo dat P G wederom vijfvoudich sy tot P O, te weten als 2000 lb tot 400 lb, soo sal de hanghende duer P aldan swaerheys middellini wesen des heels; Ick neem nu dat FC vijfthienvoudich sy teghen F P, daerom seg ick FC 15, gheeft F P 1, wat 2400 lb? comt 160 lb, voort'ghene dat an C hanghende met de rest aldan euestaltwichtich sal sijn.

III^e VOORBEELT.

ANGHESTEN de wichtighe ghedaenten der lancien ofte dier ghelijcke, op de schauder ghedraghen, ghelijck ghenouch sijn ande ghedaenten des voorgaende tweede voorbeelts, soo sullen wy daer af dit derde beschrijven. T'GHEGHEVEN. Laet A een man sijn, hebbende op sijn schouder B, een lanci C D, weghende 12 lb, wiens as sy C D, ende haer swaerheys middelpunt sy E, ende van t'punt des naeckfels der lanci

and any vertical through L shall be the centre line of gravity of the whole. I now take that FC be found twelve times FL ; I therefore say: FC 12 gives FL 1, what 2,400 lbs (to wit the sum of the gravity and the device)? comes 200 lbs for that which, hanging at C , will be of equal apparent weight to the rest on this occasion ¹⁾. Therefore a man weighing 200 lbs, or pulling as strongly as would 200 lbs hanging thereat, will be of equal apparent weight to the rest. But this applies to the position in which it is now, for if we take M to be centre of gravity of the weight H and the part ABD to be rising, less than 200 lbs will be required at C . In order to understand this more clearly, let us draw the line NO , through the point M , at right angles to the plane ABD , in such a way that if the plane ABD be parallel to the horizon, NO will be centre line of gravity of the gravity H . Therefore, OG being divided in P so that PG be again five times PO , to wit as 2,000 lbs to 400 lbs, the vertical through P will then be centre line of gravity of the whole. I now take FC to be fifteen times FP . I therefore say: FC 12 gives FP 1; what 2,400 lbs? comes 200 lbs for that which, hanging at C , will then be of equal apparent weight to the rest.

EXAMPLE III.

Since the properties of the weights of lances or the like, carried on the shoulder, are sufficiently similar to the properties of the second example hereinbefore, we will describe the third example with regard thereto. SUPPOSITION. Let A be a man having on his shoulder B a lance CD weighing 12 lbs, whose axis shall be CD and its centre of gravity E ; and from the point of contact of the lance

¹⁾ In this as well as other places we give a literal rendering of Stevin's formulation of the rule of three. The reader will no doubt recognize in his elliptical sentences the proportion: $FC : FL = 2,400 : x$; $12 : 1 = 2,400 : x$; $x = 200$ lbs.

lanci ende sijn schouder, sy ghetrocken de lini B F, rechthouckich op den sichteinder, sniende den as C D in G; Ende sijn handt rechtneerwaert treckende comt an t'punt H inden as, ende G H sy dobbel an G E.

T'BE G H E E R D E. De vraegh is wat ghewelt de handt ande lanci doet.

T'W E R C K. Ouermidts de lini G H dobbel is an G E, soo sal t'ghewicht an E,

dat is der lanci, dobbel sijn an t'ghewicht an H, dat is t'ghene de handt treckt; Maer de lanci weegt 12 lb, de handt dan sal soo stijf trecken als 6 lb soudan an H hanghede.

Maer so den man A waer eē Snaphaen, met een ghesnapt haen I an K hanghende, weghende 3 lb, ende also dat K G drievoudich waer an G H, tis kennelick dat den buyt sijn handt van 9 lb meer verswaren, ende in alles 15 lb trecken soude.

Dit is ghenomen dat de handt recht neerwaert trecke, maer als sy scheef treckt, ghelijck dan rechtdaellini tot scheefdaellini, alsoo rechtdaelwicht tot scheefdaelwicht, duer het 21^e voorstel des 1^{en} boucx der beghinselen, waer wt alles bekent wort duer het 22 voorstel des selfden boucx.

IIII^e VOORBEELT.

Wy hebben tot hier de ghedaente verclaert alwaer twee steerten sijn, ouer elcke sijde des vastpunts een; Wy sullen nu een voorbeelt gheuen vanden steert alleenelick ouer een sijde. T'G H E G H E V E N. Laet A B een steert sijn, vast an t'einde A, de rest verroerlick, weghende 400 lb, diens as A B, ende swaerheys middellini C D, ende de steert A B sy lanck 10 voeten, waerop een ghewicht E light van 1000 lb, diens swaerheys middellini F G. De vraegh is hoe sterck men an B sal moeten heffen om den steert met t'ghewicht E op te lichten.



and the man's shoulder shall be drawn the line BF at right angles to the horizon, intersecting the axis CD in G . And the man's hand, pulling straight downwards, comes in the point H in the axis, and GH shall be double of GE . WHAT IS REQUIRED TO FIND. It is asked what force the hand exerts on the lance. CONSTRUCTION. Since the line GH is double of GE , the weight at E , i.e. that of the lance, will be double of the weight at H , i.e. the force exerted by the hand. But the lance weighs 12 lbs; therefore the hand will pull as strongly as would 6 lbs hanging at H . But if the man A were a poaching soldier, with a poached cock I hanging at K , weighing 3 lbs, in such a way that KG were three times GH , it is evident that the booty would weight his hand by 9 lbs more, and he would pull 15 lbs in all.

It is here assumed that the hand pulls straight downwards, but if it pulls obliquely, then as the vertical lowering line is to the oblique lowering line, so is the vertical lowering weight to the oblique lowering weight, by the 21st proposition of the 1st book of the elements, from which everything becomes manifest by the 22nd proposition of the same book.

EXAMPLE IV.

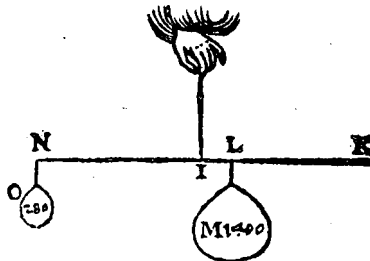
We have so far explained the form where there are two levers, one on each side of the fixed point. We will now give an example of a lever on one side only. SUPPOSITION. Let AB be a lever, fixed at the end A and the rest being movable, weighing 400 lbs, whose axis shall be AB and the centre line of gravity CD . And the lever AB shall be 10 feet long, on which there lies a weight E of 1,000 lbs, whose centre line of gravity shall be FG ¹⁾. It is asked with what force a man will have to lift at B in order to raise the lever with the weight E .

¹⁾ The letters E and F have been transposed in the figure.



TWERCK. Men sal vinden de swaerheys middellini des heels, de-
lende eenighen balck tusschen de middellinien FG en CD , als GD , in
 H , alsoo dat HG sulcken reden hebbe tot HD , als 400 lb des steerts,
tot 1000 lb des ghewichts F , dat is als 1 tot 5 : Ick neem nu dat AH sy 2
voeten, ende seg, AB 10 voeten, gheeft AH 2 voeten, wat 1400 lb voor
t'gheheele ghewicht des steerts ende pacx? comt 280 lb. Men sal dan
an B soo grooten ghewelt moeten doen om met de reste euestaltwichtich
te sijn, als oftmen 280 lb ophielde.

Maer soo den Wegher de voornoemde rekening wilde maken door
naeckter kennis des grondts, hy mach sich selfs Weeghconstighe vormen
beschrijven, ghelijck den * Meter om t'verstercken des ghedachts, hem
Geometria. * Meetconstighe voorstelt, aldus: Ick treck de lini IK , beteeckenende
Geometria. den steert AB van 10 voeten, ende ouermits AH twee voeten was, ende
 H swaerheys middelpunt, ick teecken L , alsoo dat IL beteecken 2 voe-
ten van IK 10 , hanghende M 1400 lb an L , treckende daer naer IN
euen an IK , ende houdende I voor vastpunt, ick sie wat ghewicht an N
sal moeten hanghen, op datter met M euestaltwichtich sy: T'selue is door
het 3 voorstel des 1^{en} boucx openbaer, maer wy sullender tot meerder
claeheynt noch dit af segghen: Ouermits IL is als vijfde deel van IN ,
soo moet an N (door t'voor-
noemde 3 voorstel des 1^{en}
boucx) t'vijsdendeel han-
ghen van M 1400 lb, t'welck
is voor O 280 lb euestalt-
wichtich teghen M ; Maer O
doet so veel an N dalenden-
de, als t'selue ghewicht an K
heffende, door het 13 voor-
stel des 1^{en} boucx der beghin-
felen (want IN is euen an IK)
daerom die an K heft sal
moeten 280 lb heffen om met M euestaltwichtich te sijn, ende vervol-
ghens



CONSTRUCTION. The centre line of gravity of the whole shall be found, which divides any beam between the centre lines FG and CD , as GD , in H in such a way that HG shall have to HD the same ratio as the 400 lbs of the lever to the 1,000 lbs of the weight E , i.e. 2 to 5. I now take AH to be 2 feet, and say: AB 10 feet gives AH 2 feet; what 1,400 lbs for the total weight of the lever and the load? comes 280 lbs. The force one has to exert at B in order to be of equal apparent weight to the rest will therefore be as if 280 lbs were lifted.

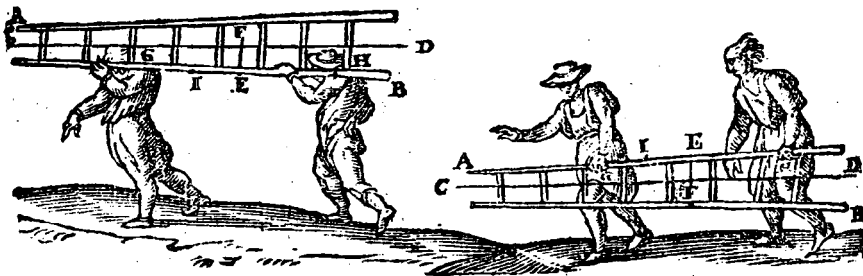
But if the weigher wished to make the above calculation through a nicer knowledge of the subject, let him draw for himself statical figures, just as the geometer, in order to aid his thought, imagines geometrical ones, as follows. I draw the line IK , representing the lever AB of 10 feet, and since AH was two feet and H was centre of gravity, I mark L in such a way that IL shall represent 2 feet of IK 10, M 1,400 lbs hanging at L . If thereafter IN be made equal to IK and I be taken as fixed point, I see what weight will have to hang at N in order that it may be of equal apparent weight to M . This is manifest from the 3rd proposition of the 1st book, but we will say the following about it, for the sake of greater clarity. Since IL is one-fifth of IN , there must hang at N (by the aforesaid 3rd proposition of the 1st book) one-fifth of M (1,400 lbs), which is O (280 lbs), of equal apparent weight to M . But O exerts at N , descending, the same force as the same weight at K , ascending, by the 13th proposition of the 1st book of the elements (for IN is equal to IK). Therefore, a man who lifts at K will have to lift 280 lbs to be of equal apparent weight to M , and consequently

ghens hy moet 280 lb lichten an B, om met de rest eueftaltwichtich te wesen. Der ghelijcke formen mach den Wegher in alle werckelicke voorbeelden sijn seluen altijt voorstellen, welcke hier om cortheyt achterghelaten sijn. **T B E S L U T.** Wy hebben dan ondersocht de ghedaenten der steerten daermen ghewelt mede doet, naer de begheerte.

VIII. VOORSTEL.

T E onderfoucken de ghedaenten der ghedreghen swaerheden.

T G H E G H E V E N. Laet A B een leere wesen, op t'een einde swaerder als op t'ander soo sy ghemeenlick sijn, welcke ghedreghen moet worden van twee mannen, alsoo dat d'een soo veel ghewichts draghe als d'ander, dat is elck den helft, ende haer middellini C D, sal int draghen euewydich vanden * sichteinder bliuen. **T W E R C K.** Men sal de leere op eenighen scherpen cant legghen, die vertreckende voorwaert ende achterwaert, tot datmen bemercke de eueftaltwichticheyt ghetrossen te sijne, t'welck ick neem in E te wesen, ende so sy dicwils moet verdreghen sijn van d'een plaets ten anderen, men mach an E een kerfken stellen; Laet daernaer ghetrocken worden de hanghende E F, sniende C D in F, daernaer salmen teekenen eenighe twee punten euewijt van E F, als G, H, ende die an G draecht sal euen soo veel draghen als die an H: Horizonto.



Maer soomen dien noch soo veel ghewichts wilde doen draghen als desen, men sal desens langde tusschen hem ende E F, dobbel maken an diens. Als H E dobbel sijnde an E I, die an I droughe soude noch soo veel ghewicht draghen als die an H. Ende alsoo salmen de reden des ghewichts vanden eenen tot den anderen, connen stellen naer de begheerte.

T G H E N E

he will have to lift 280 lbs at B in order to be of equal apparent weight to the rest. The weigher can always draw such figures for himself in all the real examples which have here been omitted for brevity's sake. CONCLUSION. We have therefore examined the forms of the levers by which a force is exerted, as required.

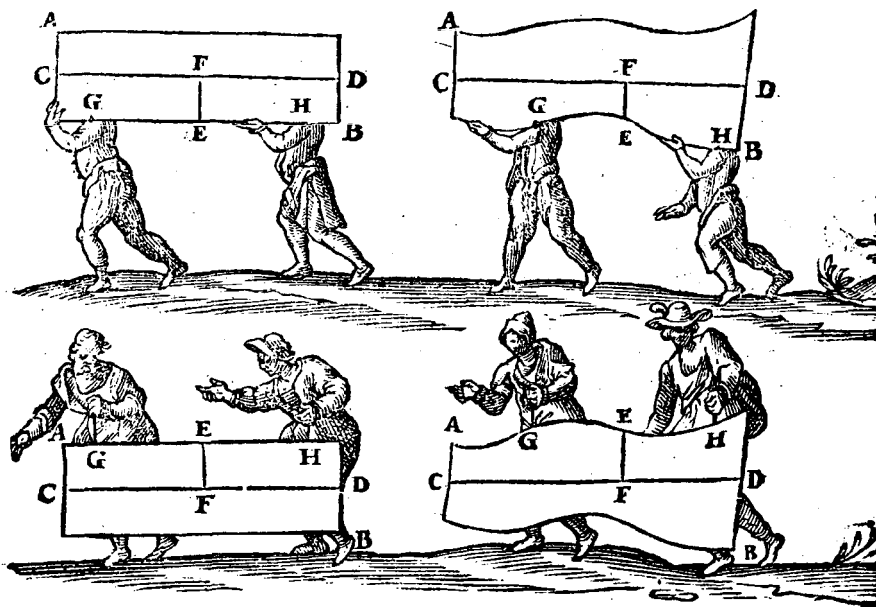
PROPOSITION VIII.

To examine the qualities of gravities that are being carried.

SUPPOSITION. Let AB be a ladder, heavier at one end than at the other end, as they usually are, which is to be carried by two men in such a way that one shall carry as much weight as the other, i.e. each one half; and its centre line CD shall remain parallel to the horizon during the carrying. CONSTRUCTION. The ladder shall be laid on a sharp edge and pulled forwards and backwards until equality of apparent weight is found to be attained, which I take to be in E ; and if the ladder has to be often carried from one place to another, a notch may be made at E . Thereafter let there be drawn the vertical EF , intersecting CD in F , and then two points shall be marked at equal distances from EF , as G and H ; then the man carrying the ladder at G will carry the same weight as the one at H .

But if it be desired to have that one carry twice the weight of this one, the distance between the latter and EF shall be made double of that between the former and EF . If HE were double of EF , the man carrying at I would carry twice the weight of the man at H . And thus the ratio of the weight of the one to that of the other can be fixed according to requirement.

TGHENE bouen gheseyt is vande leere sal hem alsoo verstaen op yder lichaem, als by voorbeelt, de form hier onder, ghedenckende dat der ongheschicter lichamen linien door haer swaerheys middelpunt lijdende als C D, gheuonden worden door het 1^e voorstel deses boucx, oock dat de hangende linien door G en H, eueuerre sijn vande linien E F.

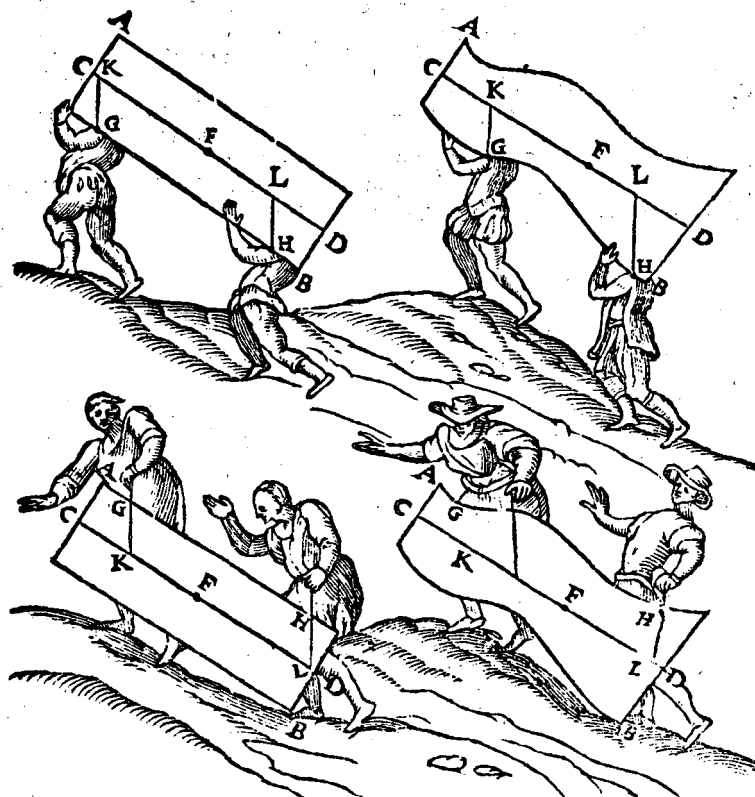


Wy hebben hier voorbeelden ghestelt alwaer de lini C D ghenomen is euewydich vanden lichteinder, maer soo sy daer af onuewydich waer, ende dat de selue mannen eenen berch ofte hoochde opsteghen, de reden vande ghewichten soude veranderen, doch bekent bliuen. Laet tot meerder clærheyt de voornoemde mannen een hoochde opgaen als hier onder, die an G vooren gaende d'ander achter.

Nv ghe-

What has been said above of the ladder will also apply to any body whatever, as for example the figure shown below, bearing in mind that with irregular bodies the lines passing through their centres of gravity, as CD , are found by means of the 1st proposition of the present book, and also that the verticals through G and H are equidistant from the lines EF .

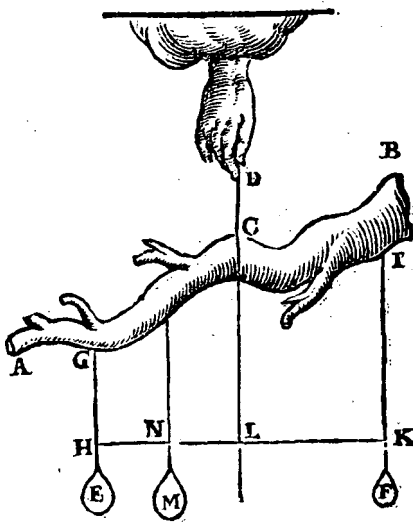
We have here given examples in which the line CD is taken parallel to the horizon, but if it were non-parallel thereto, and the men were ascending a mountain or height, the ratio of the weights would change, but remain known. For the sake of greater clarity, let the aforesaid men ascend a height, as shown below, the man at G going in front and the other behind.



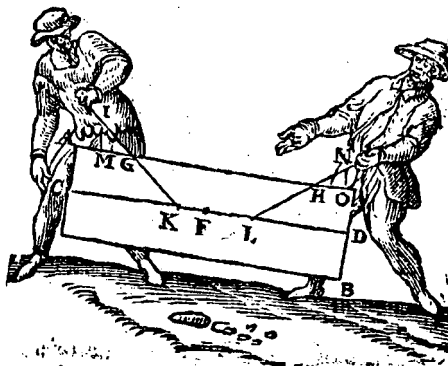
N v ghetrocken * hanghende linien door de punten G, H, siende *Perpendicu-
lares.* C D in K en L, to en sal dan elck niet euen veel draghen als in d'eertie
gheftalt, want F K inde twee opperste formē is meerder dan F L, en inde
onderste formen minder: Ende ghelijck F K tot F L, alsoo t'ghewicht des
draghers an H, tot het ghewicht des draghers an G. Alwaer oock blijkt
dat als de vastpunten G, H, onder de lini C D sijn, soo draecht den
voorsten minst, maer die vastpunten bouen de lini C D wese, soo
draecht den voorsten meest. Tis oock kennelick dat de vastpunten G, H,
inde lini C D sijnde, dat alsdan elck oueral altijt sijn selfde ghewicht sal
draghen, soo wel een berch opstighende, als langs den sichteinder. van
alle welcke de bewysen openbaer sijn door de $14^{\circ} 15^{\circ} 16^{\circ} 17^{\circ} 18^{\circ} 27^{\circ} 28^{\circ}$
voorstellen des 1^{en} boucx. Maer want veler wercklieden gheleghenthey
niet en is die voorstellen te leeren, uoch hemlieden daer in te oefenen,
d ende

The verticals through the points G , H now being drawn, which intersect CD in K and L , each of the men will not then carry as much as in the first figure, for FK in the two figures at the top is greater than FL , and in the figures at the bottom less. And as FK is to FL , so is the weight of the man carrying at H to the weight of the man carrying at G . From which it also appears that if the fixed points G , H are below the line CD , the man in front carries least, but if these fixed points are above the line CD , the man in front carries most. It is also evident that if the fixed points G , H are in the line CD , each of the men will then always carry the same weight everywhere, both when ascending a mountain and when walking parallel to the horizon, the proofs of all of which are manifest from the 14th, 15th, 16th, 17th, 18th, 27th, 28th propositions of the 1st book. But because many workmen have no opportunity to learn these propositions or to

ende nochtans gheerne wat ooghschijnelicx saghen, waer duer sijt ghe-
loofden, die mueghen nemen een rechten gheschickten, ofte crommen
ongheschickten stock,
foot valt, als A B, hem
hanghende tot eenigher
plaets als C, an een
coorde C D. Daer naer
hanghende anden stock
euen ghewichten als E, F,
alsoo dat haer coorden
G H, I K, eueverre sijn
vande lini C D neerwaert
ghetrocken, te weten dat
H L euen sy an L K, den
stock sal haer eerste stant
houden, t'selue sal sy oock
doen soomen E weerde,
ende datmen anhinghe
t'ghewicht M, dobbel an
F, ende also dat L K oock
dobbelt sy an L N, ende
soo met allen anderen,
waer uyt sy de noot saec-
licheyt van t'ghene bo-
uen gheseyt is, lichtelick
ghenoelen sullen.



DE linien daer me-
de de mannen in-
de voorgaende formen
t'lichaem draghen, sijn
rechthouckich op den
sichteinder ghestelt,
maer soo sy daer op
scheefhouckich waren,
als hier neuen, sy sullen
t'samen meerder ghe-
welt moeten doen, dan
de eyghen swaerheydt
des lichaems is. Maer



*Perpendicu-
lares.*

om te weten hoe veel
yegbelick draechr, men sal trecken de* hanghende linien I M, ende N O,
segghende.

exercise themselves therein, and yet would like to see some evidence of it from which they can believe it, let them take any straight, regular or crooked, irregular stick, as AB , suspending it in some place, as C , by a cord CD . Thereafter, equal weights, as E and F , hanging at the stick in such a way that their cords GH , IK are equidistant from the line CD drawn downwards, to wit that HL shall be equal to LK , the stick will keep its first position. It will do the same, if E be taken away and the weight M , double of F , be suspended in such a way that LK shall also be double of LN , and thus with all the others, from which they will easily understand the necessity of what has been said above.

The lines by which the men in the preceding figures carry the body are placed at right angles to the horizon, but if they are at oblique angles thereto, as in the figure opposite, they will have to exert together a greater force than the gravity of the body itself. But in order to know how much each of them is carrying, the verticals IM and NO shall be drawn, and it shall be said: as MI is to IG ,

segghende, ghelijck M I tot I G, alsoo diens rechthefwicht tot 'ghewicht dat den man an G treckt, wederom ghelijck O N tot N H, alsoo diens rechthefwicht tot 'ghewicht dat den man an H treckt, duer het 27^e voorstel des 1^{en} boucx der beghinselen, ende yders macht wort bekend door het 22^e voorstel des seluen boucx.

Wy souden meer verscheyden voorbeelden vande wichtighe ghedaenten der ghedreghen lichamen mueghen beschrijven, maer wy latent eensdeels om de cortheyt, ten anderen dat sy duer 'tvoorgaende kennelick ghenouch schijnen.

IX. VOORSTEL.

TE onderfoucken de ghedaenten der windaffen, ende der ghetrocken swaerheden.

HET treckendwicht ende ghetrockenwicht des windas, sijn *euered-
nich met de * halfmiddellini des as, ende de halfmiddellini des radts,
maer om alles oirdentlicker te beschrijven, wy sullender een * Vertooch
af maken soodanich.

*Proportiona-
les.
Semidiamete-
ter.
Theorema.*

VERTOOCH.

W E S E N D E een windas an diens as een ghewicht hangt, euestaltwichtich segghen 'ghewicht an 'reinde des radts middellini die euewydich is vanden * sichteinder: Ghelijck dan de halfmiddellini des radts, tot de halfmiddellini Horizonti des ronds vanden as, alsoo 'ghewicht anden as, tottes ghewicht an 't radt.

T'GHEGHIVEN. Laet A B C D E F G een windas sijn, diens as E F G, wiens ronds middellini E F, ende middelpunt H sy, ende I een ghewicht anden as hanghende, ende A B C D sy het radt, diens middellini euewydich vanden sichteinder, sy A C, an wiens einde A een ghewicht K hangt, euestaltwichtich teghen I, ende L sy 'tonderste ghenaeccsel des as teghen 'tghene daer sy op rust.

T'HEGHEERDE. Wy moeten bewysen dat ghelijck H A tot H F, alsoo I tot K.

T'BEWYS. Laet ons 't radt A B C D an sien als voor balck eens waeghs, diens hanthaef L B, inder voughen dat de sijde des radts B D A, de ghewichten K, I, gheweert sijnde, euewichtich hangt teghen de sijde B D C. Laet ons nu nemen dattet ghewicht I hanghe an 'tpunt F (want het daer vande selue macht soude sijn, dier 't sijnder plaets is) ende K 't sijnder plaets an A. Dit so wesende, ghelijck den langsten erm H A,

d 2 tot den

so is the vertical lifting weight along MI to the weight pulled by the man at G , and again as ON is to NH , so is the vertical lifting weight along ON to the weight pulled by the man at H , by the 27 proposition of the 1st book of the elements, and the force exerted by each becomes known from the 22nd proposition of the same book.

We might describe several more examples of the qualities of the weights of bodies that are being carried, but we omit it firstly for brevity's sake, and secondly because they seem to be sufficiently clear from the foregoing.

PROPOSITION IX.

To examine the qualities of windlasses and of gravities that are being hauled.

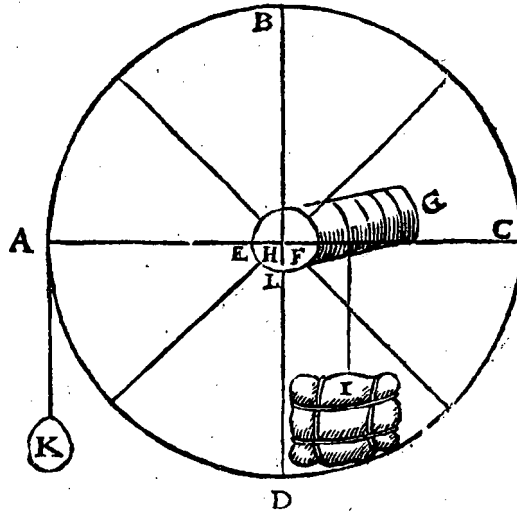
The drawing weight and the drawn weight of the windlass are proportional to the semi-diameter of the axle and the semi-diameter of the wheel, but in order to describe everything more systematically, we will make of it a theorem, as follows.

THEOREM.

Given a windlass on whose axle hangs a weight of equal apparent weight to the weight at the end of the wheel's diameter which is parallel to the horizon: as the semi-diameter of the wheel is to the semi-diameter of the circle of the axle, so is the weight at the axle to the weight at the wheel.

SUPPOSITION. Let $ABCDEFG$ be a windlass, whose axle shall be EFG , the diameter of the latter's circle being EF and its centre H , and I a weight hanging at the axle; and $ABCD$ shall be the wheel, whose diameter parallel to the horizon shall be AC , at whose end A hangs a weight K , of equal apparent weight to I , and L shall be the lowest point of contact of the axle with that on which it rests. **WHAT IS REQUIRED TO PROVE.** We have to prove that as HA is to HF , so is I to K . **PROOF.** Let us consider the wheel $ABCD$ as the beam of a balance, whose handle be LB , in such a way that the side of the wheel BDA , the weights K and I being taken away, balances the side BDC . Let us now take the weight I to hang at the point F (for it would there exert the same force as in its own place), and K in its

tot den cortsten HF, alsoo de swaertste swaerheyt I, tot de lichtste K, duer het 1^e voorstel des 1^{en} boucx der beghinselen. Daerom by aldien H A selsvoudich waer tegen HF, soo sal I selsvoudich wesen teghen K, dat is, weghende I ses hondert pont, K salder hondert weghen, daerom een man treckende an A, so stijf alshodert ponden neerdrucken, die soude teghen I 600 lb euestaltwichtich sijn, en om I te doen rijfen soude (om t'ghenaecsel des as, &c.) wat stijuer moeten trecken dan 100 lb neerdrucken.



II VOORBEELT.

D E ghedaenten der cranen ende der ghelijcke raeyers daer menschen in gaen sijn duer t'voorgaende oock openbaer. Laet tot voorbeelt. A B C D een radt wesen, diens* middellini A C, euewydich sy vanden * sichteinder, ende t'rondt des as sy E F, wiens middelpunt G, ende t'ghewicht anden as sy H, ende I sy een man in tradt euestaltwichtich teghen H, diens swaerheys middellini rechthouckich op A C sy I K. Ende is kennelick dat ghelijck G K tot G F, alsoo t'ghewicht H tot het ghewicht des mans I, ghenomen dan dat G K viervoudich sy teghen G F, so sal t'ghewicht H viervoudich sijn teghen t'ghewicht des mans, daerom soo den man woughe 150 lb, soo sal H weghen 600 lb. Oock en sal den man op die plaets de swaerheyt H niet connen opwinden, ouermids hy aldaer maer euestaltwichtich teghen H en staet; Maer by aldien hy voortgaet naer A, soo sal H rijfen, want de reden vande lini G K tot G F, soude dan grooter wesen dan sy nu is. Maer alser meer menschen int radt gaen dan een, die naest A sijn doen t'meeeste ghewelt, ende

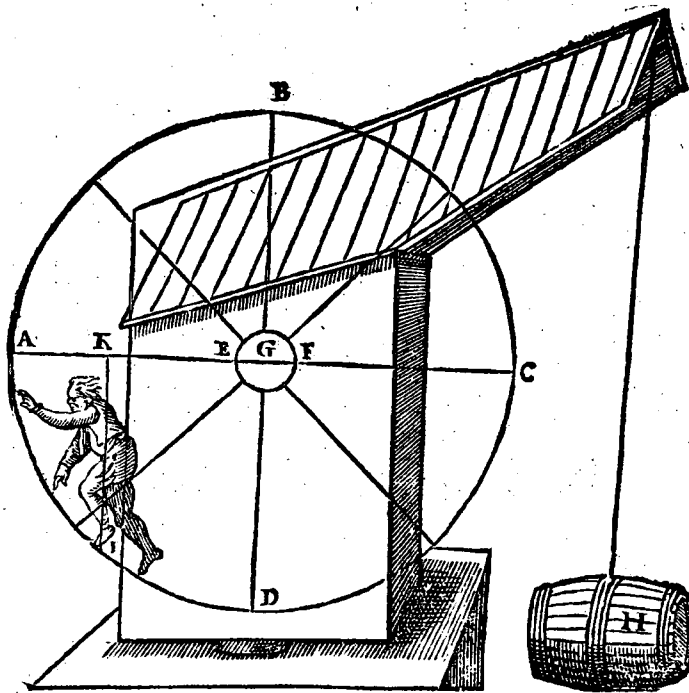
Diameter.

Horizonte.

place at A . This being so, as the longer arm HA is to the shorter arm HF , so is the heavier gravity I to the lighter K , by the 1st proposition of the 1st book of the elements. Therefore if HA be six times HF , I will be six times K , i.e. if I weighs six hundred pounds, K will weigh one hundred pounds. Therefore, a man pulling at H with the same force as one hundred pounds pressing downwards would be of equal apparent weight to I (600 lbs), and in order to raise I (because of contact of the axle, etc.) he would have to pull a little more strongly than 100 lbs pressing downwards.

EXAMPLE II.

The qualities of cranes and similar wheels in which go human beings are also manifest from the foregoing. By way of example, let $ABCD$ be a wheel, whose diameter AC shall be parallel to the horizon, and the circle of the axle shall be EF , whose centre shall be G , and the weight at the axle shall be H , and I shall be a man in the wheel, of equal apparent weight to H , whose centre line of gravity at right angles to AC shall be IK . Then it is evident that as GK is to GF , so is the weight H to the weight of the man I . Taking therefore GK to be four times GF , the weight H will be four times the weight of the man; therefore, if the man weighs 150 lbs, H will weigh 600 lbs. Although in this place the man will not be able to hoist the gravity H , since he is there only of equal apparent weight to H , if he proceed to A , H will rise, for the ratio of the line GK to GF will then be greater than it is now. But if more men than one go in the wheel, those who



ende de reden van haer altsamen ende van yder int besonder tot t'ghe-
wicht H, is openbaer duer het 3^e voorstel des 1^{er} boucx.

III. VOORBEELT.

D I T heeft hem alsoo met de ghewichten die recht op ghetrocken
worden, als packen ende vaten diemen duer cranen uyt schepen windt,
ende dier ghelijcke; Maer de ghewichten die scheef opwaert comen,
als onder anderen de schepen diemen in Neerlandt tot veel plaetsen ouer
dammen windt, tis met de *ueredenheyt van dien war anders ghestelt. *Proportione.*
Laet tot voorbeelt A een dam wesen, ende B een schuyt die daer ouer
ghetrocken moet worden, ende C D het radt, wiens middellini euewy- *Horizonte.*
dich vanden *sichteinder sy C D; ende daer in een man teghen de
schuyt B eueftaltwichtich, wiens swaerheys middellini F E, ende de
coorde sy G H, ende des assens rondt sy I K, ende haer *middelpunt L: *Centrum.*
Laet oock ghetrocken sijn N M, rechthouckich op t'plat des dams, ende
d 3 M inde

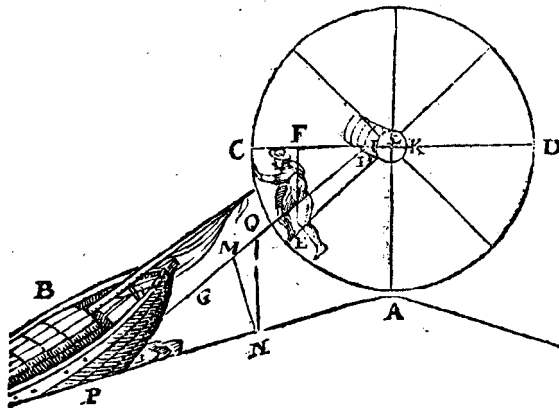
are nearest to A will exert the greatest force, and the ratio of all of them together and of everyone in particular to the weight H is manifest from the 3rd proposition of the 1st book.

EXAMPLE III.

This applies to the weights which are drawn up vertically, such as bales and barrels which are hoisted from ships by means of cranes and the like. But as to the weights which are raised obliquely, such as the ships which in Holland are hauled across dams in many places, the proportion of these is somewhat different. For example, let A be a dam and B a barge which has to be hauled across it, and CD the wheel, whose diameter parallel to the horizon shall be CD , and therein a man being of equal apparent weight to the barge B , whose centre line of gravity shall be FE , and the rope shall be GH , and the circle of the axle shall be IK and its centre L . Let there also be drawn NM , at right angles to the plane of the dam,

*Perpendicu-
laris.*

M inde coorde GH; Daernaer de* hanghende ON; Laet nu LF sefvoudich sijn tot L K, ende NO drievoudich tot OM, ende den man weghen 150 lb. Dit soo sijnde ghelijck LF tot L K, alsoo t'ghewicht dat ande coorde HG rechtneer soude hanghen, tot t'ghewicht des mans van 150 lb, duer t'voorgaende vertooch, maer LF is duer t'ghestelde sefvoudich an L K, t'ghewicht dan dat ande coorde HG rechtneer hinghe, soude sefvoudich sijn an 150 lb, dat is 900 lb; den man dan doet in t'radt soo veel ghewelts ande schuyt B, als ofter met de scheefwacgh 900 lb an hinghen. Twelck so sijnde t'ghewicht der schuyt B, heeft sulcken reden tot die 900 lb, als NO tot OM duer het 20° voorstel des



1^{re} boucx; Maer NO is drievoudich an OM duer t'ghestelde, de schuyt dan weeght driemael 900 lb, dat is 1700 lb, dat is achtienmael den man. Twelck hem soo verstaet wesende de schuyt in die ghestalt, maer als sy hoogher comt, soo sal de coorde GH steylder sijn (ten waer men die ande schuyt versette) ende vervolghens de lini als MO sal wat meerder reden hebben tot ON, dan sy nu doet, waer duer oock het euefstaltwicht teghen de schuyt alsdan meerder soude sijn dan 900 lb; Daerom yemant willende een radt ende as van pas bouwen, niet te groot noch te cleen, mach sijn rekening maken naer de ghestalt daer in een der swaerste schuyten ofte schepen de meeste ghewelt behouft.

Tis oock te ghedencken dat den man E in tradt de meeste ghewelt doet, als de coorde GH euewydich is van t'plat des dams PN, duer het 24^e voorstel des 1^{re} boucx der beghintelen; want dan is HG rechthouckich op den as (op dat ickse soo noem) der schuyt, dat is op de lini duer t'swaer-

with M in the rope GH , and thereafter the vertical ON . Now let LF be six times LK , and NO three times OM , and let the man weigh 150 lbs. This being so, as LF is to LK , so is the weight which would hang down vertically at the rope HG to the weight of the man (150 lbs), by the foregoing theorem. But LF , by the supposition, is six times LK ; therefore, the weight which would hang down vertically at the rope HG would be six times 150 lbs, i.e. 900 lbs. Therefore, the man exerts in the wheel as much force on the barge B as if 900 lbs were hanging at it with the oblique balance. Which being so, the weight of the barge B has to these 900 lbs the same ratio as NO to OM , by the 20th proposition of the 1st book. But NO is three times OM by the supposition; therefore the barge weighs three times 900 lbs, i.e. 2,700 lbs, that is eighteen times the weight of the man. This applies to the case when the barge is in that position, but when it reaches a higher point, the rope GH will be steeper (unless it be displaced at the barge), and consequently the line MO will have a somewhat greater ratio to ON than it has now, as a result of which that which is of equal apparent weight to the barge will then also be more than 900 lbs. Therefore, if anyone wishes to construct a suitable wheel and axle, neither too large nor too small, let him make his calculations according to the position in which one of the heaviest barges or ships requires the greatest force.

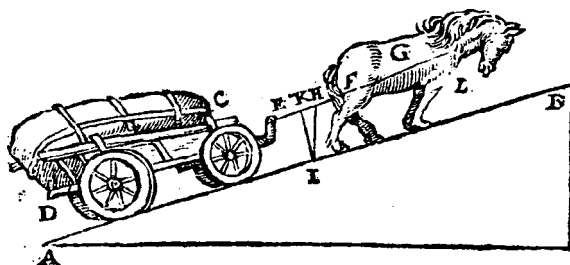
It should also be borne in mind that the man E in the wheel exerts the greatest force when the rope GH is parallel to the plane of the dam PN , by the 24th proposition of the 1st book of the elements. For then HG is at right angles to the axis (if I may so call it) of the barge, i.e. to the line through the centre of

twaerheys middelpunt der schuyt, ende rechthouckich op t'plat P N: Daerom hoe dat G H ende P N de euewydicheyt naerder sijn, hoe lichter werck, ende hoe verder, hoe swaerder.

IIII^e VOORBEELT.

Uyt het voorgaende is oock kennelick, hoe veel ghewichts een peert in een wagen ghespannen, meer treckt een hoochde op styghende, dan opt plat landt. Laet by voorbeeld A B t'plat sijn eens berghs, ende C D een wagen, weghende met datter op is al tlamen 2000 lb, ende E F (inde plaets der strijnghe) sy de coorde, ende G sy t'peert, evenstaltwichtich teghen den wagen. Laet oock ghetrocken sijn de * hanghende H I, ende I K, rechthouckich op t'plat A B, ende laet I H viervoudich sijn tot H K, ende is kennelick duer het 20 voorstel des 1^{en} boucx der beghinselen, dat ghelijck K H tot H I, alsoo t'ghewicht der scheefwaegh sooder een waer (in diens plaets nu t'peert is) tottet ghewicht des waghens, maer K H is t'vierendeel van H I duer t'ghestelde; des scheefwaegs wicht dan soude van 500 lb sijn, te weten t'vierendeel van t'ghewicht des waghens; Daerom t'gareel oft riem oft sulcx alst waer, druckt t'peert soo stijf voor den borst L, als een pack van 500 lb op sijn rugghe duwen soude, ende dat (wel verstaende alst voortgaet) bouen het duytsel dattetlijdt op t'plat landt treckende.

Perpendicu-
laris.



T is oock openbaer duer het 24^e voorstel des 1^{en} boucx, ende duer t'ghene wy hier vooren vande schuyt gheseyt hebben, dat als de strijngen euewydich sijn vande wech daer de wagen ouer vaert, dat de peerden dan de meesten ghewelt ande waghé doen, wel verstaende op eenen harden gantsch effenen wech, maer op eenen oneffenen hobbelighen en sandighen, so voorderet de strijnghe achter wat leegher te doen dan vooren. Twelck den Hollantschen voerliet duer deruaring niet onbekent en is, diens waghens daer naer ghemackt sijnde, doen de strijnghe, langs t'zeestrant varende, ende in derghelijcke eenen harde weghe, achter hoogher

gravity of the barge and at right angles to the plane PN . Therefore, the nearer GH and PN are to being parallel, the lighter will be the work, and the further they are from being so, the heavier will be the work.

EXAMPLE IV.

From the foregoing it is also evident with how much greater weight a horse harnessed to a wagon draws when ascending a height than on level land. For example, let AB be the surface of a mountain and CD a wagon weighing, together with all that is on it, 2,000 lbs; and EF (in the place of the traces) shall be the rope, and G shall be the horse, of equal apparent weight to the wagon. Let there also be drawn the vertical HI , and IK at right angles to the plane AB , and let IH be four times HK ; then it is evident by the 20th proposition of the 1st book of the elements that as KH is to HI , so is the weight of the oblique balance if there were one (whose place is now taken by the horse) to the weight of the wagon. But KH is one-fourth of HI by the supposition; therefore the weight of the oblique balance would be 500 lbs, to wit one-fourth of the weight of the wagon. Therefore the breastharness or strap or whatever it is presses as strongly against the horse's breast L as a bale of 500 lbs would press on its back, such (provided it is proceeding) over and above the pressure it would bear when pulling on level land.

It is also manifest from the 24th proposition of the 1st book and from what we have said above of the ship that if the traces are parallel to the road on which the wagon moves, the horses then exert the greatest force on the wagon, that is to say on a hard, perfectly smooth road, but on a rough, bumpy, and sandy road it is of advantage to attach the traces somewhat lower at the back than in front; which through experience is not unfamiliar to the Dutch carriers, whose wagons, being adapted thereto, have the traces higher at the back when travelling along the

hoogher, dan inde oneuen ende sandighe. Reden is dese, dat wesende de strijnghe euewydich vanden sichteinder, so en sijne niet euewydich met die oneuen verheffelen, t'welck int ouertrecken nootzakelick meerder last anbrengh, dan als de strijnghe achter leegher sijn, ouermidts sy dan de euewydicheyt met die verheffelen naerder sijn. Inde sandighe daer de wagen diep insinckt, daer drucken de raeyers dieper ende moeylicker duer t'sant, wesende de strijnghe euewydich vanden sichteinder, dan als sy achter wat leegher sijn.

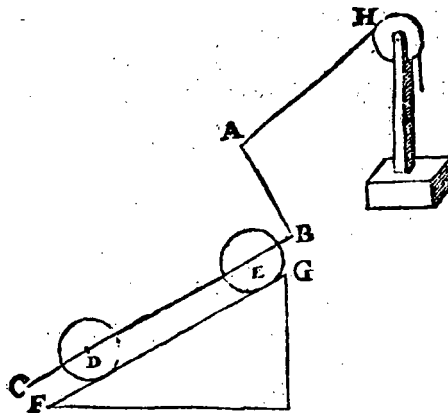
M E R C K T.

YEMANT mocht ons nu twee saken voorwerpen; Ten eersten, waarom wy hier bouen gheseyt hebben: *Ghelijck KH tot HI, alsoo t'ghewicht der (scheefwaegh sooder een waer (in diens plaats nu t'peert is) tottet ghewicht des waghens*, Achtende datmen niet en behoort te segghen *tottet ghewicht des waghens*, maer, *tottet rechthefwicht van t'ghewicht des waghens*.

Ten tweeden waerom wy gheen onderscheyt beichreuen en hebben vande plaats der coorde EF, t'wyselende dat de selue voortghetrocken ende lydende duer t'waerheys middelpunt des waghens, een ander ghewicht voor t'peert mocht veroirsaken, dan als sy daer bouen of daer onder comt. Om op t'welck te verantwoorden, ende * Wisconslick

Mathemati-
ed.

te bewysen dat de boueschreuen eueredenheyt volcommen is: so laet ABC een wagen sijn, al van wisconslinghelinen ghemaeckt, wiens raeyers sijn D, E, ende den wech daer hy op rust sy FG, ende de coorde des toecómen den scheeffhewichts sy AH.



LAET ons nu op desen wagen legghen een pilaer IK als hier onder, alsoo dat HA voortghetrocken come in des pilaers swaerheys middelpunt L, en laet het scheeffhewicht M teghen den pilaer eustaltwichtich sijn; Laet oock an L gheuocht worden t'rechthefwicht N, met

beach and similar smooth and hard roads than on rough and sandy roads. The reason is that if the traces are parallel to the horizon, they are not parallel to those unevennesses, which in being passed across must needs produce a greater load than if the traces are lower at the back, since they are then nearer to being parallel to those unevennesses. On sandy roads, into which the wagon sinks deeply, the wheels force themselves deeper and with more difficulty through the sand if the traces are parallel to the horizon than if they are somewhat lower at the back.

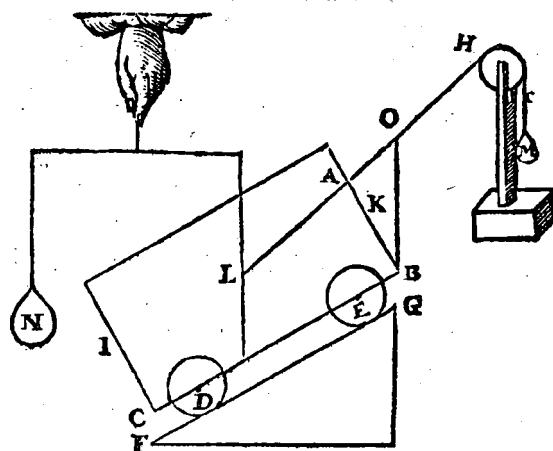
NOTE.

Someone might now raise the following two objections: Firstly, why we have said above: *As KH is to HI, so is the weight of the oblique balance if there were one (whose place is now taken by the horse) to the weight of the wagon; considering that we ought not to say to the weight of the wagon, but to the vertical lifting weight of the weight of the wagon*¹⁾.

Secondly, why we have not made any distinction as to the place of the rope EF, wondering whether this rope, when produced and passing through the centre of gravity of the wagon, would not cause a weight for the horse other than if it comes above or below it. In order to account for this and to prove mathematically that the proportion described above is perfect: let ABC be a wagon, wholly made up of mathematical lines, whose wheels be D and E, and the road on which it rests shall be FG, and the rope of the oblique lifting weight applied shall be AH.

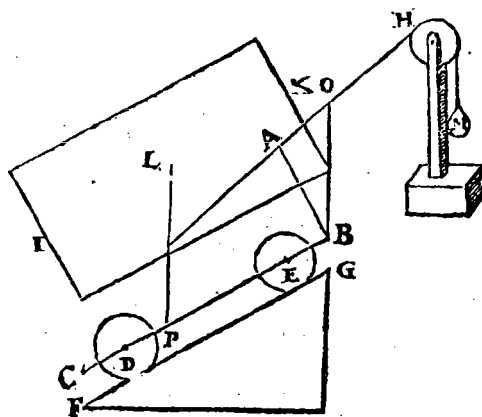
Now let us place on this wagon a prism IK, as shown below, in such a way that HA produced shall come in the centre of gravity L of the prism, and let the oblique lifting weight M be of equal apparent weight to the prism. Let there also be applied to L the vertical lifting weight N, of equal apparent weight to the

¹⁾ This means: to the force directed vertically upwards which keeps the weight of the wagon in equilibrium.



met den pilaer eueftaltwichtich: Laet ons oock trecken de * hanghende *Perpendicu-*
larum. B O, snijende A H in O, t welck fo sijnde, wy segghen duer het 20^e voor-
 stel des 1^{en} boucx, dat ghelijck A O tot O B, alsoo M tot t'rechthefwicht
 N; Maer anghesien N gheuoucht is an t'swaerheys middelpunt L, des
 pilaers I K, soo sal Neuewichtich sijn met den pilaer duer het 14 voorstel
 des 1^{en} boucx; Daerom mueghen wy segghen ghelijck A O tot O B, also
 M tot den pilaer, waer uyt d'eerste voorgeworpen sacck openbaer is
 als A H comt uyt het swaerheys middelpunt L.

MAER om
 nu het tweede
 voorgeworpen
 te bewysen, dat
 is de selue * e-
 ueredenheydt
 dus ooc te be-
 staen al en cõt
 A H niet uyt
 het swaerheys
 middelpunt L,
 so laet ons den
 pilaer I K recht-
 opwaert uyt dē
 waghen trec-
 ken, rustende



Proportional.

op de * hanghende lini L P, als hier neuen: Ende duer de 3 Begheerte, *Perpendicu-*
larum. sy en

prism. Let us also draw the vertical BO , intersecting AH in O , which being so, we say, by the 20th proposition of the 1st book, that as AO is to OB , so is M to the vertical lifting weight N . But since N is applied to the centre of gravity L of the prism IK , N will balance the prism by the 14th proposition of the 1st book. Therefore we can say: as AO is to OB , so is M to the prism, by which the first objection is settled, when AH proceeds from the centre of gravity L .

But in order to settle the second objection, i.e. that the proportion in question also holds even if AH does not proceed from the centre of gravity L , let us pull the prism IK vertically up from the wagon, resting on the vertical LP , as shown opposite. Then according to the 3rd postulate it will not apply to the wagon

sy en brengt op den waghen ABC, gheen meerder swaerheyt dan in d'eerste gheitalt, en vervolghens M en heeft niet meer te trekken dan sy te vooren en dede: Maer HA voortghetrocken comt nu onder t'swaerheys middelpunt L. Commende dan de voortghetrocken HA onder t'swaerheys middelpunt L, soo treckt M t'selfde ghewicht dat sy track doen HA in t'swaerheys middelpunt quam. T'selve salmen oock alsoo bethoonen commende de voortghetrocken lini HA bouen t'swaerheys middelpunt L, dat is treckende den pilaer IK rechtneerwaert onder den waghen. Vyt het welcke t'voornemen openbaer ende bewesen is.

X. VOORSTEL.

T'MAECKSEL ende de eyghenschappen des Almachtichts te verclaren.

PLVTARCHVS ende ander, schrijuen dat Hiero Koninck van Sicilien, dede bouwen een schip van uytmemender grootheyt ende constigher form, om te schencken an Ptolemeus Koninck van Egypten; Twelck, doent volmaeckt was, de burgheren van Syracusa om sijn swaerheyt in zee niet krijghen en konden, maer doen Archimedes daer an ghestelt had sijn reetschap die de Griecken Charistion noemen, Hiero heeft het daer duer selfs alleen metter handt vertrocken. Dese Charistion (naer de form die Jacques Besson daer af heeft laten uytgaen, *Bibliotheca.* gheuonden inde * bouckamer des Kueninx van Vranckrijck) had allen met vijfen, drayende inde canten van ertelicke raeyers: Een werck voorwaer weerdich sijn eeuwighe ghedachtenis, ende soudent hier beschriuen ouermids wy tot sulcke stof ghecommen sijn, ten waer wy in die plaets stelden TALMACHTICH (reetschap die wy om sijne ouergroote macht dien naem gheuen) tot sulcke daet bequamer, te weten Stercker gheduerigher werck; Van minder cost; Duer t'welckmen op corter tijt meer afveerdicht; Ende (ghelijck de Charistion) van oneindelicke cracht, * machtelick welverstaende, niet daetlick. T'maECKSEL daer af is foodanich:

*Potentia non
aem.*

Men sal nemen een boom ofte balck als AB, sterck ende groot naer de cracht dieder duer ghedaen moet sijn: Daer naer salmen maken *Diameter.* een yser stercken als C, ick neem dat sijn * middellini van drie duymen sy, ende dattet ses tanden heb, ende in sijn middel stekende een yseren as CD, an d'einden CD viercantich, ende tusschen beyden ronds, daer naer de sterre E, ick neem met 18 tanden, ende t'stercken F met 6 tanden, daer in stekende een yseren as EF, enen ende ghelijck met den as CD, te weten ande einden viercantich, ende tusschen beyden ronds

ABC any greater gravity than in the first position, and consequently M does not have to draw any greater weight than before. But HA produced now comes below the centre of gravity L . Therefore, HA produced coming below the centre of gravity L , M draws the same weight it did when HA came in the centre of gravity. The same can also be shown if the line HA produced comes above the centre of gravity L , i.e. when the prism IK is pulled vertically down below the wagon. By which the objection is settled.

PROPOSITION X.

To explain the construction and the properties of the Almighty.

Plutarch¹⁾ and others² report that Hiero, King of Sicily, had a ship built, exceptionally large and of ingenious form, in order to present it to Ptolemy, King of Egypt. However, when it was completed, the citizens of Syracuse could not get it into the sea because of its heaviness, but when Archimedes had applied to it his device called Charistion³⁾ by the Greeks, Hiero was able to move the ship by himself, by hand. This Charistion (judging from the drawing which Jacques Besson⁴⁾ made of it, found at the library of the King of France) had shafts with screws, rotating in the sides of several wheels, a device truly worthy to be eternally remembered. We should describe it here, since we have now come to this subject matter, but that we are dealing instead with the Almighty (a device we so call on account of its exceptional power), which is more suited to such work, for the following reasons: sturdier and more durable construction; of lower cost; by which more is done in shorter time; and (like the Charistion) of infinite power, that is to say: potentially, not actually. The construction of this device is as follows:

There shall be taken a tree or beam, as AB , sufficiently strong and big for the force which has to be exerted therewith. Thereafter a small iron gear wheel, as C , shall be made; I assume that its diameter is three inches and that it has six teeth, and in its centre an iron shaft, as CD , of square cross-section at the ends C and D , and round in between. Thereafter the gear wheel E , I take with 18 teeth, and the

¹⁾ Plutarchus, *Vita Marcelli* XIV, 7.

²⁾ An elaborate description of the great ship Surakosia (later Alexandris), which Hiero had had built for King Ptolemy of Egypt, is given by Athenaeus in his *Deipnosophistae*. *Athenaei Naucratis Dipnosophistarum Libri XV*; rec. G. Kaibel (Leipzig 1887) V, 40—44.

³⁾ Not all Greek authors who tell the story of the ship use the name Charistion for the device applied by Archimedes. It is also called a polyspaston (Plutarchus, *Vita Marcelli* XIV, 8) or trispaston (Tzetzes, *Chiliades* II. Hist. 35, 107). The name Charistion is given by Simplicius (*In Aristotelis Physicarum libros Commentaria*; ed. H. Diels, Berlin 1895, p. 1110) to the device Archimedes would have used for moving the earth, if he had been able to find a fixed point outside it. He says that it was a kind of balance, and if the Charistion may be identified with the Charasto mentioned by Gerard of Cremona in a Latin translation of a work by Tabit ibn Qurra (*Liber Charastonis*), this was also the meaning attached to it in the Middle Ages. Tzetzes (*Chiliades* II. Hist. 35, 130) also mentions the Charistion in this connection, without, however, explaining the name. Obviously Stevin and his source, Besson, identify the Charistion with the Baroukos described by Heron (*Mechanicorum Fragmenta*, ed. G. Schmidt, Leipzig 1900, p. 256; *Dioptra*, ed. H. Schöne, Leipzig 1903, p. 306). In this device a windlass is turned by a system of toothed wheels, the last of which is put in motion by means of an endless screw.

⁴⁾ Jacques Besson, *Théâtre des instruments mathématiques et mécaniques*, Lyon. Or: *Theatrum instrumentorum et machinarum*, Lyon, 1582.

den rondt: ende ghelijck EF is, soo salmen oock maken GH, ende IK, dat is G ende K elck met ses tanden, ende H ende I met 18 tanden.

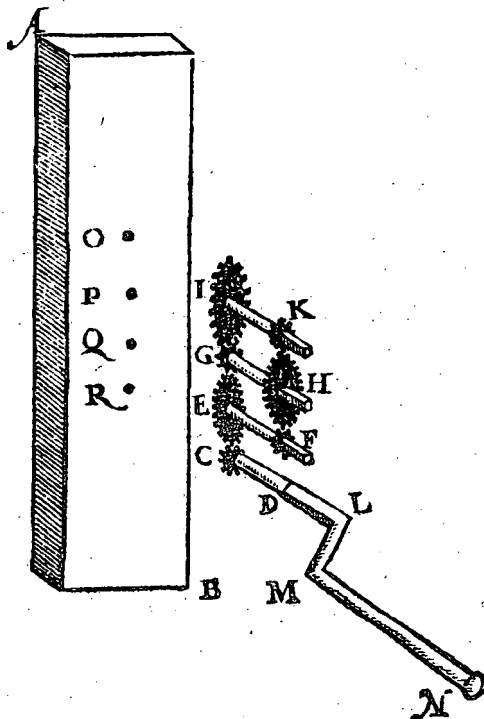
Maer want de bouenste sterren de meeste last sullen lijden, als hier naer blijcken sal, so sullen sy stercker ende grooter sijn dan d'onderste, daer uyt ooc volghen sal dat de assen euewydich van malcanderen wesende, de sterre H sal connen ghenaken an F, ende niet an K, ende de sterre G an I, ende niet an E, t'welck soo wesen moet.

Daer naer salmen maken de kruck I M N, wiens viercantich gat des viercantighen cokers L, passe op alle

de viercantighe einden der assen, als D, F, H, K, ende L M sy lanck een voet, so dickmael sulcke langden in crucken van slijpsteen ende diergheleijcke sijn, ende MN soo lanck als hier naer gheseyt sal worden. Daer naer salmen inden boom A B vier gaten booren, van malcander in sulcker wyde als de vier assen staen, ghelijck de gaten O, P, Q, R, van achter duer commende, daer de vier assen I K, G H, E F, C D in passen muerghen, ende de langde der assen tusschen de sterren, sal euen sijn ande dickte des bocms, ende der assen vierhouckighe einden an K, H, F, D sullen al ontrent de drie ofte vier duymen buyten de sterren steken; Daer naer astreckende de sterre I, men sal den as I K steken, int gat O, ende insghelijcx den as GH in t'gat P, ende EF in Q, ende CD in R, stellende wederom elcke sterre van achter an huer as, also

e 2

dat de



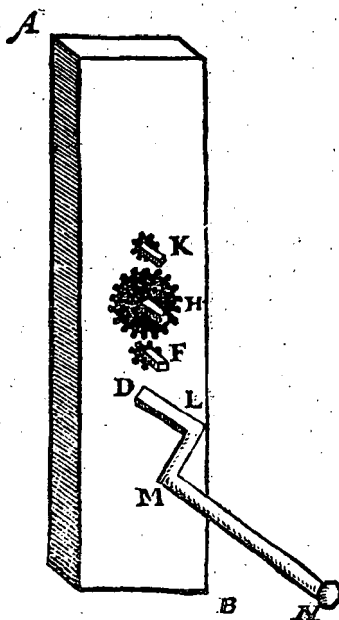
small gear wheel *F* with 6 teeth, in which there is an iron shaft *EF*, equal and similar to the shaft *CD*, to wit of square cross-section at the ends and round in between. And as *EF* is, in the same way *GH* and *IK* shall also be made, that is *G* and *K* each with six teeth, and *H* and *I* with 18 teeth. But because the upper wheels will have to bear the greatest load, as will be found hereinafter, they must be stronger and larger ¹⁾ than the lower, from which it also follows that, the shafts being parallel to each other, the wheel *H* will be able to touch *F* and not *K*, and the wheel *G* will be able to touch *I* and not *E*, which has got to be.

Thereafter the crank *LMN* shall be made, the square hole of whose square tube *L* shall fit all the square ends of the shafts, as *D*, *F*, *H*, *K*, and *LM* shall be one foot long, as is often the length of the cranks of grindstones and the like, and *MN* shall have the length to be specified hereinafter. Thereafter there shall be drilled in the tree *AB* four holes, at the same distances from one another as the four shafts, as the holes *O*, *P*, *Q*, *R*, extending at the back of the tree, in which the four shafts *IK*, *GH*, *EF*, *CD* may fit; and the length of the shafts between the wheels shall be equal to the thickness of the tree, and the square ends of the shafts at *K*, *H*, *F*, *D* shall all extend about three or four inches beyond the wheels. Thereafter, when the wheel *I* has been drawn off, the shaft *IK* shall be put into the hole *O*, and similarly the shaft *GH* into the hole *P*, and *EF* into *Q*, and *CD* into *R*, upon which each wheel shall again be mounted on its shaft at

¹⁾ This entails that the diameters of the wheels should also be enlarged; the drawing however, shows no signs of this.

dat de tanden der sterre F ande voorste sijde, mueghen doen draeyen de sterre H, ende dat de tanden der sterre C ande achterste sijde, mueghen doen drayen de sterre E, ende dat de tanden van G, doen draeyen I, ende haer ghestalt voor volmaeckt Almachtich sal dan sijn als hier neuens.

Nu ghelijck wy t'voorbeelt hier ghegheuen hebben van vier asen, soo machmende meer ofte min stellen: Ende de 18 tanden der groote sterren welcke driefvoudich sijn tot de ses tanden der kleene sterren, die machmen in meerder ofte minder reden stellen, naer gheleghentheydt van t'ghene daermen t'Almachtich toe maeckt.



VAN TGHEBRVYCK ENDE ANDER

ANCLEVING DES ALMACHTICHS.

MAER om de ghebruyck deses Almachtichs te verclaren, wy sul-
len een voorbeelt gheuen daer alle d'ander ghenouch duer sullen
bekent sijn, te weten van schepen daer mede ouer dammen of dijcken
te trecken, want dat den cleynsten dienst niet en schijnt, die dese landen
hier in ghedaen mach worden, voornamelick Hollandt. Laet A B
t'bouenschreuen Almachtich sijn, met de sterren K, H, F, ouer dees sijde
des booms A B, ende de sterren I, G, E, C, ouer d'ander sijde, en L M N
sy de cruck, ende S den as, diens middellini van $1 \frac{1}{2}$ voet sy, commende
duer den boom met een yser sterre an t'einde als T, wiens middellini ick
neem te sijn van 2 voeten (sy moet ten minsten soo veel langher sijn dan
de middellini van t'rondt des as S, dat de sterre I den as S niet en ghe-
raecke) ende te hebben 36 tanden, ende V sy den dam, wiens hoochde
bouen t'onderste des scheps vrielick int water ligghende (dat is int an sien
der * hanghende lini van t'op des dams totter plat cewydich vanden
licht-

Perpendicu-
laris.

the back, in such a way that the teeth of the wheel *F* in front can rotate the wheel *H*, and that the teeth of the wheel *C* at the back can rotate the wheel *E*, and that the teeth of *G* rotate *I*; then the form of the completed Almighty will be as shown opposite.

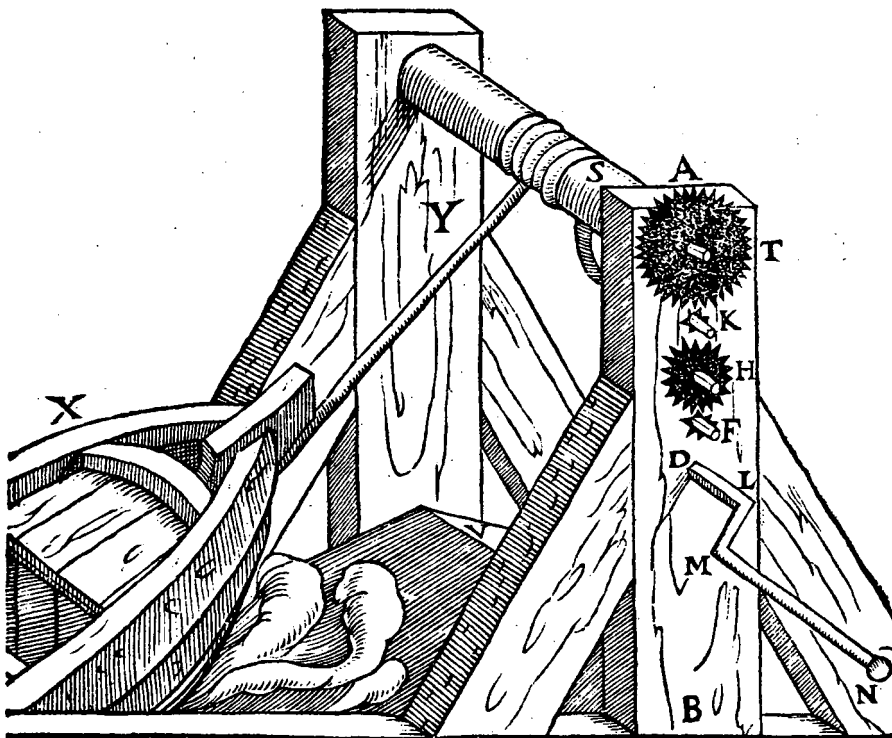
Now just as we have here given an example of four shafts, there may also be more or fewer. And the 18 teeth of the large wheels, which are three times the six teeth of the small wheels, may be taken in a greater or lesser ratio, according to the occasion for which the Almighty is made.

OF THE USE AND OTHER ATTRIBUTES OF THE ALMIGHTY

In order to explain the use of this Almighty we will give an example from which all the others will be sufficiently clear, to wit of hauling ships therewith across dams or dykes, for this does not appear to be the least service it may render to these countries, chiefly Holland. Let *AB* be the Almighty described above, with the wheels *K*, *H*, *F* on this side of the tree *AB* and the wheels *I*, *G*, *E*, *C* on the other side, and let *LMN* be the crank, and *S* the axle, whose diameter shall be $1\frac{1}{2}$ feet, extending through the tree with an iron wheel at the end, as *T*, whose diameter I take to be 2 feet (it should be at least so much longer than the diameter of the circle of the shaft *S* that the wheel *I* cannot touch the axle *S*) and which I take to have 36 teeth¹⁾, and *V* shall be the dam, whose height above the bottom of the ship lying freely in the water (i.e. measured along the vertical from the top of the dam to the plane parallel to the horizon through the bottom of the ship)

¹⁾ Girard (XIII; iv, 482) here finds a difficulty. If the teeth of the wheel *T* are to be equal in size to those of the wheels *C* . . . *K*, *T* cannot have 36 teeth as well as a diameter of 2 feet. The former number calls for a diameter of 1.5 feet, the second for a number of teeth of 48. The solution of this riddle may perhaps be found in the gradual increase of the diameters mentioned in note 1 to page 357. On the strength of this it is possible that both values given by Stevin are correct. Cf. note 1 to page 361.

*lichteinder duer r'onderste des schips) sy vier voeten, ends X sy r'schip. *Horizonte.*
Nu om r'selue ouer te winden, men sal draeyen ande kruck L M N,
daerom sal d'hanthaef M N so lanck sijn, datter al de ghene diemender
toe bruycken wil, ouer beyden sijden bequamelick an staen mueghen.



REDEN DIEDER IS VANDE KEEREN

DES KRUCK TOT DE KEEREN DES AS.

WANT de kruck L M N driemael omdraeyt teghen F eenmael, so
sal sy 9 mael omdraeyen teghen H eenmael, ende 27 mael te-
ghen K eenmael, ende 162 mael teghen T ofte den as S eenmael. Tis
oock kennelick dat de kruck ghestelt an r'einde des as F, sy sal 54 mael
omdraeyen teghen S eenmael, ende ghestelt an K, sal ses mael omdraeyen
teghen S eenmael, ende ghestelt an T, sal so dickmael omdraeyen als S.

e 3

Maer

shall be four feet, and X shall be the ship. Now in order to haul the latter across, one has to turn the crank LMN . Therefore the handle MN must be so long that all the men who are to be employed for it can suitably stand on either side of it.

THE RATIO BETWEEN THE REVOLUTIONS OF THE CRANK AND THE REVOLUTIONS OF THE AXLE

Because the crank LMN revolves three times against F once, it will revolve 9 times against H once, and 27 times against K once, and 162 times against T or the axle S once. It is also evident that the crank, when mounted at the end of the shaft F , will revolve 54 times against S once, and when mounted at K , six times against S once, and when mounted at T , as often as S . But if a man should turn at a shaft higher than D , for example at K , in order that all the lower wheels shall not rapidly revolve too, which would cause unnecessary gravity, he must displace somewhat on its shaft the next in order of the lower ones, which would here be G , until its teeth are free of the teeth of I , and then all the lower wheels will stand still.

RATIO BETWEEN THE FORCE APPLIED BY THE TURNER TO THE CRANK AND THE WEIGHT THAT IS BEING HAULED, AS THE SHIP X

Because LM , one foot long by the supposition, is eight times the semi-diameter of the wheel C , the weight caused by the wheel E on C will be to the gravity or force at MN which is of equal apparent weight to it as 8 to 1, and for the same reason the weight caused by H on F as 24 to 1, and by I on G as 72 to 1, and by T on K as 216 to 1. But the circle of the axle S is potentially equal to the circle of T (we say potentially, for actually the diameter of the circle of S is $1\frac{1}{2}$ feet as against T two feet, by the supposition, but because the teeth of T are six times the teeth of C , its diameter will potentially be six times the diameter of G , being 3 inches, that of T then being 18 inches, i.e. $1\frac{1}{2}$ feet, like the diameter of the axle S); therefore the weight hanging vertically down at the axle S will have to its equal apparent weight or force at MN the ratio of 216 to 1¹⁾. We might

¹⁾ The mechanical advantage of the Almighty may be determined as follows: Put $ML = r$, the radii of the wheels $C, F, G, K = r_1$, the radius of $T = r_2$, that of $S = \rho$; the number of teeth of the wheels $C, F, G, K = n$, that of $E, H, I = n_1$, that of $T = n_2$. In one complete revolution of M, S performs

$$\left(\frac{n}{n_1}\right)^3 \cdot \frac{n}{n_2} \text{ revolutions.}$$

If now the force applied at M is P , and the force exerted by the rope of S, Q , we have

$$2 \pi r \cdot P = \left(\frac{n}{n_1}\right)^3 \cdot \frac{n}{n_2} \cdot 2 \pi \rho \cdot Q;$$

consequently the mechanical advantage.

$$(1) \frac{Q}{P} = \frac{r}{\rho} \cdot \left(\frac{n_1}{n}\right)^3 \cdot \frac{n_2}{n}$$

Taking $r = 1$ ft., $\rho = 3/4$ ft., $n_1 = 18$, $n = 6$, $n_2 = 36$, we find

$$\frac{Q}{P} = \frac{4}{3} \cdot 27 \cdot 6 = 216.$$

This is also Stevin's value; however, he arrives at it in a different way. He first determines the force exerted by the teeth of C against those of E , which is

$$\frac{r}{r_1} \cdot P.$$

Maer als yemant draeyt an een hoogher dan D, by ghelijcknis an K, op dat dan alle de onderste sterren niet snellick mede omdraeyen, t welck onnoodighe swaerheyt soude anbrenghe, soo salmen d'eerstvolghende der onderste, t welck hier G soude wesen, op sijn as wat verschuyuen, tot dat huer tanden buyten de tanden van I sijn, ende dan sullen alle d'onderste stil staen.

REDEN VANDE CRACHT DES

DRAEYERS ANDE CRVC, TOT HET GHETROCKENWICHT als t'schip X.

WANT L M lanck een voet door t'ghefelde, achtvoudich is teghen de halue middellini vande sterre C, so sal t'ghewicht veroirsaeckt uyt de sterre E op C, teghen sijn euefaltwichtighe swaerheyt ofte macht an M N, wesen als van 8 tot 1, ende om de selue reden duer t'veoirsaeckte van H op F, als van 24 tot 1, ende van I op G, als 72 tot 1, ende van T op K, als 216 tot 1: Maer rondt des as S is * machtelick euen an t'rondt T (wy segghen machtelick, want * daetlick, de middellini des rondts van S doet $1\frac{1}{2}$ voet van T twee voeten duer t'ghefelde, maer want de tanden van T selsvoudich sijn an de tanden van C, daerom sal sijn middellini machtelick selsvoudich sijn teghen de middellini van G, doende 3 duymen, die van T dan 18 duymen, dat is $1\frac{1}{2}$ voet, als de middellini des as S) t'ghewicht dan anden as S rechtneerhangende, sal sulcken reden hebben tot sijn euefaltwicht, ofte macht an M N, als van 216 tot 1. Wy souden ooc connen vanden as S neerwaert de rekening maken, ghelijck sy hier van onderen opwaert ghedaen is.

Potentia.
Adu.

Wy connen t'voornomde oock aldus verclaren: Anghesien M N 162 mael omdraeyt, teghen den as S eenmael (als bouen bewesen is) ende dat de middellini des radts beschreuen duer den keer van M N, sulcken reden heeft tot de middellini des rondts vanden as S, als 4 tot 3 (want L M is een voet, ende de * halfmiddellini des rondts vanden as S is $\frac{3}{4}$ voets) soo sal de langde der omtrecken vande 162 ronden beschreuen duer de keeren van M N, sulcken reden hebben tot de langde vanden omtreck des rondts der as S, als 216 tot 1, de selfde reden sullen oock hebben de 216 halfmiddellinien van dat rondt, tot de eenighe halfmiddellini van dit rondt; Daerom oock, duer het 1^e voorstel des 1^{en} boucx, sal t'ghewicht an die, sulcken reden hebben an t'ghewicht ofte de macht an dese, als van 216 tot 1 ghelijck vooren. Waer uyt volght dat wesende an M N een gheduerighe macht soo groot als 25 lb souden neertrecken, t welck ick de macht schat van een man, ende grooter als hy wil (wel is waer dat een man ter noot onghelijck veel grooter macht doen can, maer wy nemen

Semidiamet.
267.

also make our calculations from the axle S downwards, at is has here been done upwards from the bottom.

We can also explain the foregoing as follows: Since MN revolves 162 times against the axle S once (as has been proved above), and the diameter of the circle described by the revolution of MN has to the diameter of the circle of the axle S the ratio of 4 to 3 (for LM is one foot, and the semi-diameter of the circle of the axle S is $\frac{3}{4}$ foot), the length of the circumferences of the 162 circles described by the revolutions of MN will have to the length of the circumference of the circle of the axle S the ratio of 216 to 1; the same will also be the ratio of the 216 semi-diameters of the former circle to the sole semi-diameter of the latter circle. Therefore also, by the 1st proposition of the 1st book, the weight at the former will have to the weight or force at the latter the ratio of 216 to 1, as before. From which it follows that if there is exerted at MN a constant force as great as 25 lbs would pull downwards, which I estimate to be the force that can be exerted by one man — and greater if he cares (it is true that if need be a man can exert a much greater force than this, but we take this by way of example) —,

Consequently the force of F against H is $\frac{r}{r_1} \cdot P \cdot \frac{n_1}{n}$

the force of G against I $\frac{r}{r_1} \cdot P \cdot \left(\frac{n_1}{n}\right)^2$

the force U of K against T $\frac{r}{r_1} \cdot P \cdot \left(\frac{n_1}{n}\right)^3$.

Now putting $n_1 = 18$, $n = 6$, $r = 1$ ft., $r_1 = 1/8$ ft., we find
 $U = 3^3 \cdot 8 \cdot P = 216 P$

This result, however, is given as representing the force exerted by the rope of S . However, continuing this line of reasoning and considering the windlass formed by T and S , we should expect

$$(2) \quad Q = \frac{r_2}{\rho} \cdot \left(\frac{n_1}{n}\right)^3 \cdot \frac{r}{r_1} \cdot P = \frac{4}{3} \cdot 216 \cdot P \text{ or } \frac{Q}{P} = 288.$$

To explain this discrepancy we have to take into account Stevin's remark that T and S are potentially equivalent, which can only mean that the multiplication by $\frac{r_2}{\rho}$ is not necessary. To prove this, he remarks that $r_2 : r_1 = n_2 : n$.

Now (2) takes the form $\frac{Q}{P} = \frac{r}{\rho} \cdot \left(\frac{n_1}{n}\right)^3 \cdot \frac{n_2}{n}$,

which is identical with (1).

But the demonstration naturally is not valid if $r_2 = 1$ ft., $r_1 = 1/8$ ft., $n_2 = 36$, $n = 6$. It is, however, possible to arrive at Stevin's result in a legitimate manner when we suppose, in accordance with note 1 to page 357, that the radii of the wheels increase continually from C to K . Put the radius of $F = \lambda r_1$, that of $G = \lambda^2 r_1$, that of $K = \lambda^3 r_1$. It is then necessary that

$$\frac{n_2}{n} \cdot \lambda^3 r_1 = r_2;$$

consequently $\lambda^3 = \frac{r_2}{r_1} \cdot \frac{n}{n_2}$.

Now the force exerted by the teeth of K against those of T is found to be

$$\left(\frac{r}{r_1}\right) \left(\frac{\lambda}{\lambda}\right)^3 \cdot P,$$

and the mechanical advantage $\frac{Q}{P} = \frac{r_2}{\rho} \cdot \frac{r}{r_1} \left(\frac{\lambda}{\lambda}\right)^3 = \frac{r}{\rho} \cdot \lambda^3 \frac{n_2}{n} \cdot \frac{3^3}{\lambda^3} = \frac{4}{3} \cdot 6 \cdot 3^3 = 216$.

wy nemen dit voorbeeldsche wyse) de selue macht sal euestaltwichtich sijn teghen 3400 lb (dat is 216 mael 25 lb) rechtneerhanghende anden as S: Ghenomen nu dat het schip X sefvoudich sy, teghen dat sijn euestaltwicht an den as S rechtneerhanghende, soo sal t'schip X weghende 32400 (dat is 9 last ghewichts rekenende 3600 lb voor t'last) euestaltwichtich sijn teghen t'ghewicht, ofte die gheduerighe macht van 25 lb an M N.

VANDE MENICHTE DER KEEREN

DES CRVCK OM T'SCHIP OVER DEN DAM TE

Winden: Ende vanden tijds die de aerbeyders bebouuen.

MAER weseude duer t'ghestelde t'schip sefvoudich teghen t'ghewicht anden as S hanghende, so sal de langde van t'sop des dams scheefneerwaert, oock sefvoudich sijn teghen de hoochde diet schip moet verheuen worden (duer het 19^e voorstel des 1^{en} boucx) welcke duer t'ghestelde is 4 voeten, de selue dan ses mael maeckt 24 voeten, voor de voornomde langde, Laet ons nu nemen dat dese 24 voeten ghewonden moeten worden op den as S, om t'swaerheys middelpunt des schips ouer t'middel des damste krijghen; Soo wy dan als vooren, den omtreck van t'rondt des as stellen als drievoudich (die reden is in desen gheualle naer ghenouch) teghen sijn middellini $1\frac{1}{2}$ voet, sy sal $4\frac{1}{2}$ voeten wesen, de selue sijn inde voornomde 24 voeten $5\frac{1}{3}$ mael, den as S dan, sal $5\frac{1}{3}$ keeren moeten omraeyen, maer elcke keer van die behouft 162 keeren van M N als vooren bewesen is, daer sullen dan in als bebouuen 864 keeren van M N.

Wy souden oock mueghen aldus segghen: Elcken keer van M N veruoert 25 lb ses voeten verre, dat is, hanghende een ghewicht an den as S van 3400 lb, elcken keer van M N doet soo veel, als oft het van dien telckemael 25 lb 6 voeten hoogh trocke, ende veruolghens als oft t'sesmael 25 lb, dat is 150 lb des scheeps, trocke 6 voeten verre, daerom ghedeelt 32400 lb duer 150 lb, comt 216. waer duer t'schip met elcke 216 keeren van M N ses voeten voort comen sal, maer t'moet viermael 6 voeten comen, t'moet dan hebben viermael 216 keeren, dat is als vooren 864 keeren. Ofte andersins (anghesien t'schip in als 4 voeten hooch moet comen) men mach aldus segghen, met eenen keer trecktmen 25 lb ses voeten hooch, met hoe veel keeren salmen 32400 lb trecken 4 voeten hooch? comt als vooren met 864 keeren.

Maer wy achten datter een man 1000 can doen op een vierendeel uys, ghenomen dan dat hem alles soo heb als gheseyt is, hy sal t'schip met datter in is t'samen 9 last weghende, alleen ouerwinden op min dan een vierendeel

this force will be of equal apparent weight to 5,400 lbs (i.e. 216 times 25 lbs), hanging down vertically at the axle S . Now assuming that the ship X be six times its equal apparent weight hanging down vertically at the axle S , the ship X weighing 32,400 lbs (that is 9 lasts, taking a last at 3,600 lbs) will be of equal apparent weight to the weight, or the constant force of 25 lbs at M .

OF THE NUMBER OF REVOLUTIONS OF THE CRANK NECESSARY
TO HAUL THE SHIP ACROSS THE DAM, AND OF THE TIME THE
WORKMEN NEED FOR IT

But if, by the supposition, the ship be six times the weight hanging at the axle S , the distance from the top of the dam obliquely downwards will also be six times the height the ship has to be raised (by the 19th proposition of the 1st book), which by the supposition is 4 feet; this, multiplied by six, makes 24 feet for the aforesaid distance. Let us now assume that these 24 feet have to be wound on to the axle S in order to raise the ship's centre of gravity above the centre of the dam. If then, as before ¹⁾, we put the circumference of the circle of the axle at three times (this ratio being near enough in this case) its diameter of $1\frac{1}{2}$ feet, it will be $4\frac{1}{2}$ feet; these are contained $5\frac{1}{3}$ times in the aforesaid 24 feet, therefore the axle S will have to make $5\frac{1}{3}$ revolutions. But each of the revolutions of the axle requires 162 revolutions of MN , as proved above, to that 864 ²⁾ revolutions of MN in all will be required.

We might also say as follows: Each revolution of MN moves 25 lbs six feet further, i.e., a weight of 5,400 lbs hanging at the axle S , each revolution of MN does as much as if it hauled each time 25 lbs thereof 6 feet high, and consequently as if it hauled six times 25 lbs, i.e. 150 lbs of the ship, 6 feet further. Therefore, 32,400 lbs divided by 150 lbs makes 216, so that with every 216 revolutions of MN the ship will be moved six feet further. But it has to be moved four times 6 feet therefore it requires four times 216 revolutions, i.e. 864 revolutions, as stated before. Or otherwise (since the ship has to be raised 4 feet in all) it may be said as follows: with one revolution 25 lbs are hauled six feet up, with how many revolutions will 32,400 lbs be hauled 4 feet up? As before, this works out at 864 revolutions.

Now we consider that a man can perform 1,000 revolutions in a quarter of an hour. Therefore, taking everything to be as said, he will by himself haul the ship with all that is in it, weighing together 9 lasts, across in less than a quarter of an hour. But if there be three men, they may put the crank at F , and then they will haul the ship across in one-third of a quarter of an hour, that is in $\frac{1}{12}$ hour ³⁾. And if there be nine men, they may put the crank at H and will haul it across in $\frac{1}{36}$ hour. It is also possible to provide an Almighty at the other tree Y , as at the tree AB , and place the men on either side.

¹⁾ We do not remember Stevin having used this approximation $\pi = 3$ before.

²⁾ The exact value is $\frac{24}{1.5 \pi} \cdot 162 = \frac{2592}{\pi}$ revolutions.

³⁾ One man, putting the crank at F , would have to exert a threefold force; if he were able to exert this force while at the same time keeping up the same number of revolutions per unit of time, he would be able to perform the work in one third of the time, because the number of revolutions required for a given displacement is now one-third of the original number. However, he alone will not be able to achieve this, but three men working together and each exerting the original force will succeed in doing so.

NOTE.

We have here given an example as if the ship were always of uniform gravity during the hauling by the workmen, though this gravity alters considerably in accordance with the form and shape of the given vessel, for the work will be more difficult at the end than at the beginning, for the reasons given in the 3rd example of the 9th proposition of this book about a similar case. Therefore the foregoing is to be looked upon as an example explaining how to make one's calculations with any form proposed or desired.

As to the wheels placed vertically above each other in the Almighty, they might also be placed side by side, or in pairs, as the occasion demands.

EXPLANATION OF THAT WHICH HAS BEEN
PROMISED HEREINBEFORE

We have promised hereinbefore, at the beginning of this proposition, that the Almighty would be of sturdier construction and of lower cost than the Charistion; by which more is done in shorter time, and of infinite power.

As regards the sturdiness of construction, I consider this to be manifest (though a better one is by no means to be despised), for what better could one wish for, with a view to such work, but a strong tree such as it has grown, whose substance coheres better than any device made up of several pieces. The low cost is also evident.

As regards the shorter time, this follows from the fact that the crank may be put on the shaft of any desired wheel, in accordance with the number of workmen employed and the weight to be hauled, to wit for lighter weights the crank may be put higher and for heavier lower, in such a way that with a suitable effort the weight to be hauled, however heavy, is kept moving, without stopping, which is not possible either with the Charistion or with other windlasses, for in order to haul a small, light ship use is made in windlasses of that which is capable of exerting a much greater force, and this lengthens the time required. But if the weight to be hauled is heavier than can easily be hauled by them, it is necessary to use a great many men or horses, working hard at one time and stopping at another, thus lengthening the time. Nay, in addition they greatly damage the ships, for one of the biggest hauled across the Leyden Dam, weighing thirteen or fourteen lasts requires twenty men going in the wheels,¹⁾ who will often descend all together to the same position of rest, and do severe damage to the ships by the violent shock, which is not possible with the Almighty, since the ship always moves on uniformly and gently.

But in order to speak of its infinite power, let it be known that with the crank above at *D* as great a force may be exerted as with a windlass the diameter of whose wheel should be 324 feet, which is shown as follows. Let there be a wheel whose diameter shall be 324 feet, and its axle shall be *S*, the diameter of whose circle shall be $1\frac{1}{2}$ feet, in consequence of which the semi-diameter of

¹⁾ Prop. IX, Ex. II.

menichte van menschen ofte peerden, welcke met grooten arbeydt altemet voortgaen, altemet stilstaen, ende daer duer den tijt verlanghen; la bouen dien de schepen seer beschadighen, waht een der grootste die ouer den Leydtschen Dam ghetrocken worden van derthien oft veerthien last, behouft twintich menschen die inde raeyers gaen, welcke dickwils naer eenen stillestandt altsamen neerdalen, ende met eenen grooten gheweldighen hurt de schepen seer quetsen, t welck duer t'Almachtich niet gheschien en can, ouermidts t'schip altydt eeuvaerdelick ende sachtkens voortcomt.

Maer om vande oneindelicke cracht te segghen, het is te weten datmen met de kruck hier bouen an D, soo veel vermach als men soude met een windas diens radts middellini van 324 voeten waer, t welck aldus betoocht wort: Laet wesen een radt diens middellini 324 voeten, ende sijn as sy S, wiens rondts middellini sy van $1\frac{1}{2}$ voet, waer duer de halfmiddellini des radts sulcken reden sal hebben tot de halfmiddellini des as, als 216 tot 1, daerom oock t'ghewicht ofte de macht anden as, sal sulcken reden hebben tot sijn euestaltwicht an t'uyterste des radts, als 216 tot 1: duer het 9 voorstel deses boucx, de selue reden isser oock van t'ghewicht an den as S, tot sijn euestaltwicht an M N, daerom so wy gheseyt hebben, wesende de kruck an D, men sal duer haer anden as S soo veel vermueghen, als duer een radt diens middellini lanck waer 324 voeten. Maer de meeste diemen maeckt en schijnen de 30 voeten niet te bereycken, waer uyt opentlick blijktt hoe veel t'Almachtich meer vermach dan de windassen, wel is waer dat eenen gaende int radt eens windas, sijn ghewelt met minder aerbeydt doet, maer gheuerckt de voorgaende omstaende, ten is niet het nutste. Doch soo yemandt sulcken voordeel duer den ganck in tradt begheerde daert noodich viel, hy soude an eenighen as der assen D, F, H, K, T, mueghen steken een schijfsloop, inde plaets vande kruck, stellende tanden an t'uyterste van eenich radt eens windas, die in die schijfsloop draeyen mochten, maer t'voordeel en soude dickmael de onkosten niet weerdich sijn.

Maer soo dees voornomde reetschap niet gheweldich ghenouch beuonden wierde, om daer mede t'voornemen te volbrenghe, daer en is verloren cost, noch onnoodighen aerbeyt ghedaen, want stellende alleenlick onder D noch een as als d'ander, drievoudighende de voorgaende 216, ende daer an de kruck stellende, t soude ande selue euestaltwichtich sijn teghen 648 anden as S. duer welck middel men tot de begheerde ghewelt comen sal.

Maer datmen alsoo maeckte een Almachich met 30 assen, wiens tanden vande grootste sterren thienvoudich waren teghen de tanden vande cleynste sterren, ende het deel des cruck als L M, euen ande halfmiddellini der grootste sterre, ende t'rondt des as als S, euen an t'rondt der

f

cleynste

the wheel shall have to the semi-diameter of the axle the ratio of 216 to 1; therefore also the weight or the force exerted at the axle will have to its equal apparent weight at the rim of the wheel the ratio of 216 to 1, by the 9th proposition of this book. The same is also the ratio of the weight at the axle S to its equal apparent weight at MN . Therefore, as we have said, the crank being at D , the same force can be applied therewith to the axle S as with a wheel whose diameter should be 324 feet. But most wheels that are made do not seem to attain 30 feet, from which it is manifest how much more powerful than windlasses is the Almighty. It is true indeed that a man going in the wheel of a windlass exerts his force with less effort, but considering the circumstances mentioned above this is not the most profitable thing. But if anyone should desire this advantage of going in the wheel where it is necessary, he might put at one of the shafts D, F, H, K, T a wallower¹⁾ instead of the crank, providing teeth at the rim of a wheel of a windlass, which can engage with that wallower, but the advantage frequently would not be worth the expense.

But if the aforesaid device were to be found not sufficiently powerful to perform therewith the proposed work, no cost has been lost and no unnecessary work has been done, for by merely putting below D another such shaft triplicating the foregoing 216, and putting the crank there, 1 would thus be of equal apparent weight to 648 at the axle S , by which means the desired force would be attained.

But if in this way an Almighty with 30 shafts were made, the teeth of whose largest wheels should be ten times the teeth of the smallest wheels, and the part of the crank, as LM , equal to the semi-diameter of the largest wheel, and the circle of the axle, as S , equal to the circle of the smallest wheel (which would not be such a prodigious work), the weight hanging at such an axle would have to its equal apparent weight at the crank the ratio of 1,000,000,000,000,000,000,000,000,000 to 1²⁾. Taking therefore the circumference of the circle of the smallest wheel to be 1 foot, the circumference of the wheel of a windlass (the circumference of the circle of whose axle should also be one foot), in order to exert the same force therewith, would have to be 1,000,000,000,000,000,000,000,000,000 feet³⁾. But the circumference of the greatest circle of the earth (taking the degree at 480 stades, and each stade at 125 geometrical strides, and each geometrical stride at 5 feet) is only 108,000,000

¹⁾ A wallower or lantern wheel is a cylindrical device in which pins are mounted between two circular flanges. The cogs of the wheel of the windlass engage with the pins of the wallower, the (horizontal) axis of which is attached to one of the shafts of the Almighty.

²⁾ Using the notation of note 1 to page 361, we have to put $n_1 = 10n$, $n_2 = n$, $q = r_1$, $r = 10r_1$

and to replace 3 by 29. This gives $\frac{Q}{P} = \frac{10r_1}{r_1} \cdot 10^{29} = 10^{30}$.

³⁾ 10^{30} .

feet ¹). Thus you may see how many hundred thousands of times greater than the greatest circle of the earth the circle of a wheel of a windlass would have to be in order to exert therewith the same force as with this simple Almighty. Let us now place at the crank (put at the lowest shaft of this Almighty) a little child, applying at it a slightly greater force than one pound hanging thereat, this child would wind on to the highest shaft a weight of 100,000,000,000,000,000,000,000,000 lbs ²) (but it should be borne in mind that this highest shaft would not perform a complete revolution on the first day), i.e. a weight heavier than four thousand times the earth with all that is in it, which is proved as follows: The circumference of the greatest circle of the earth is 108,000,000 feet, as stated above; therefore the area of this circle is less than 1,000,000,000,000,000 feet ³), and therefore also the area of the terrestrial globe is less than 4,000,000,000,000,000 feet, and the sixth part of the diameter is shorter than 6,000,000 feet. If the aforesaid 4,000,000,000,000,000 be multiplied thereby, the earth is less than 24,000,000,000,000,000,000 cubic feet ⁴). Let each foot weigh 1,000 lbs (it is not so heavy by far), then the whole earth is lighter than 240,000,000,000,000,000,000,000 lbs ⁵); this is contained more than four thousand times in 1,000,000,000,000,000,000,000,000 lbs ⁶), which we wished to show.

We therefore justly say that this device is potentially of infinite power. Therefore, when Archimedes said that if he were given a fixed place outside the earth where he could put his Charistion ⁷), he would move the earth out of its place, however strange it may sound, yet it is in accordance with reason. For if it were not so, the heavier gravity would not have to the lighter the same ratio as the longer arm to the shorter, which is impossible on account of the 1st proposition of the 1st book ⁸). But to give an example: Assuming that the Charistion or the Almighty were to stand in such a place and the earth weighed — as stated before — 240,000,000,000,000,000,000,000 lbs, and that a man hauled 100

¹) Taking 1 ft. = 0.3 m, this amounts to the value 32,400 km for the earth's circumference.

²) 10^{32} , obviously a mistake for 10^{30} .

³) As a rule Stevin, when dealing with areas and volumes, does not use the correct terms (square feet and cubic feet), but merely writes 'feet'.

⁴) The calculation is equivalent to applying the formula $V = \frac{P^3}{6\pi^2}$ (V = volume of a sphere the greatest circle of which has the circumference P).

For $P = 108 \cdot 10^6$ we find

$$V = \frac{108^3 \cdot 10^{18}}{6\pi^2} < \frac{108^3 \cdot 10^{18}}{6 \cdot 9} = 2 \cdot 108^2 \cdot 10^{18} < 24 \cdot 10^{21}.$$

⁵) $24 \cdot 10^{25}$. It is even lighter than $24 \cdot 10^{24}$ lbs.

⁶) $\frac{10^{30}}{24 \cdot 10^{25}} = \frac{10^5}{24} > 4,000.$

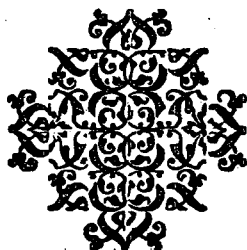
⁷) See note 3 to page 355.

⁸) Here Stevin seems to take the Charistion to be a lever, in accordance with Simplicius.

der kruck 100 lb drie voeten hoog trocke, ende op yder uyr 4000
keeren dede, ende dat gheduerende thien iaren lanck, rekenende t'iaer
op 365^{daghen}, tis openbaer dat hy t'eertrijck op dien tijdt het
^{10 5 12} eens voets, dats bycans ⁷
voets, uyt lijn plaats verrecken soude, t'welck wel is waer een onsenlike
lange is, maer wy moesten de oneindelicke cracht verklaren, die
machtelick hier in bestaet.

Nu hoe alfulcke Almachtighen tot verſcheyden wercken ſullen muen-
ghen ghevoucht worden, als dat een ſchip in hem ſal connen hebben
een ſeer cleen reetſchap van gheringhen coſt, nochtans ſeer crachtich,
hem dienende voor Craen om te laden ende ontladen: Om groote an-
kers op te winden: Voort om perſſen te maken, als lakenperſſen ende
dierghelijcke, gheweldelicker drukkende dan noyt perſſen en druſten:
Om in groote ghebauwen ſware ſteenen op te trecken, ende meer ander
daermen groote ghewelt behouft, van alle deſe en gheuen wy gheen be-
ſonder voorbeelden, ouermits yder uyt het voorſchreuen Almachtich
an ſijn werck een derghelijcke, naer gheleghentheyſt ſal muenghen vou-
ghen, beter dan wy ſegghen connen: Tis voor ons ghenouch ſijn ghe-
daente alhier beſchreuen te hebben.

TEINDE DER WERCHDAIT.



lbs three feet up with each revolution of the crank and performed every hour 4,000 revolutions, such during ten years, taking the year at 365 days, it is manifest that in that time he would move the earth $\frac{10,512}{24,000,000,000,000,000}$ foot, i.e. nearly $\frac{1}{2,400,000,000,000,000}$ foot from its place ¹⁾; which is indeed an invisible distance, but we had to explain the infinite power potentially inherent therein.

Now as to the construction of such Almightyies for various purposes, e.g. that a ship may have on board a very small device of slight cost and yet very powerful, serving as a crane for loading and unloading; for winding up big anchors; further to make presses, such as cloth presses and the like, pressing more powerfully than presses ever did; in order to haul up heavy stones in big buildings, and in many other cases in which great force is required; of all these we are not giving any special examples, since anyone will be able on the basis of the Almighty described above to apply a similar one to his own work, according to the occasion, more effectively than we can tell. For us it suffices that we have here described its form.

THE END OF THE PRACTICE OF WEIGHING.

¹⁾ In 10 years the work done would be $10 \cdot 365 \cdot 24 \cdot 4,000 \cdot 100 \cdot 3 = 10,512 \cdot 10^7$ ft.-pnd. and the displacement of the earth $\frac{10,512 \cdot 10^7}{24 \cdot 10^{25}} = \frac{10,512}{24 \cdot 10^{18}} \approx \frac{10^4}{24 \cdot 10^{18}} = \frac{1}{24 \cdot 10^{14}}$ ft.

DE BEGHINSELEN
DES WATERWICHTS

THE ELEMENTS
OF HYDROSTATICS

INTRODUCTION

Stevin's work *Beghinselen des Waterwichts* (*Elements of Hydrostatics*) which is reproduced hereafter, has the same significance in the history of hydrostatics as his *Beghinselen der Weeghconst* (*Elements of the Art of Weighing*) in the history of statics: it was the first systematic treatise on the subject after the Archimedean work *On Floating Bodies* (3rd century B.C.) and not only contained a new and ingenious treatment of its fundamental theorems, but also extended it beyond the phase of development attained by the Syracusan and never since equalled.

The contents of the work may be summarized as follows:

In the preface Stevin returns to his favourite topic, the excellency of the Dutch language for scientific purposes, which is here illustrated by means of a comparison of the enunciation of two propositions in the *Conica* of Apollonius in Latin and in Dutch respectively.

The definitions 1—5 amount to an introduction of the concept of specific gravity, which is, however, denoted by the term "gravity" alone. An important definition is No 7, in which the concept of "vlackvat" is defined, i.e. the geometrical boundary surface of a physical body from which the material contents are conceived to have been removed; we render this term in English by "surface vessel". The definitions 10 and 11 show the distinction between *ydel* (a vacuum, i.e. containing nothing at all) and *ledich* (empty, i.e. containing only air).

There are 7 postulates, but most of them are never explicitly used. Postulate 3, however, plays a vital part in the demonstration of some fundamental theorems.

The propositions may be classed in five groups:

I. Props 1—9 deal with the behaviour of bodies submerged in a liquid. The principle of Archimedes is demonstrated in a very simple and ingenious way in Prop. 8.

II. In Prop. 10 it is demonstrated that the pressure exerted by a liquid on a horizontal surface immersed therein depends only on the area of this surface and the depth of immersion, and not on the volume of the liquid in the vessel (hydrostatic paradox).

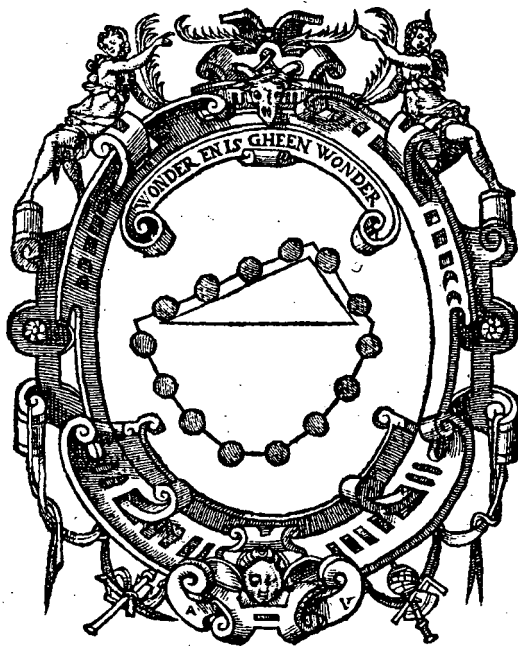
III. Props 11—17 contain theorems and problems concerning the pressure exerted on non-horizontal surfaces.

IV. In Props 18—20 the position of the centre of pressure for non-horizontal surfaces is determined.

V. Props 21 and 22 are simple problems relating to specific gravity.

Just as the *Art of Weighing* is followed by the *Practice of Weighing*, the *Elements of Hydrostatics* was to have been supplemented with a *Practice of Hydrostatics*. Of this work, however, only three propositions are extant, the first of which is a problem about the depth of immersion of a ship, while the second gives experimental illustrations of the hydrostatic paradox, and the third is devoted to the ancient problem why a diver is not crushed by the weight of the incumbent water.

D E
B E G H I N S E L E N ^{Simon.}
D E S W A T E R W I C H T S
B E S C H R E V E N D V E R
S I M O N S T E V I N
v a n B r u g g h e.



T O T L E Y D E N,
I n d e D r u c k e r y e v a n C h r i s t o f f e l P l a n t i j n,
B y F r a n ç o y s v a n R a p h e l i n g h e n.
c l o . l o . l x x x v i .



Simon Steuin Wenscht

DEN STATEN DER
VEREENICHDE
NEERLANDEN VEEL
GHELVEX.



NGHESIEN kennelick ghenouch is, E. Heeren, de gheduerighe oefning die dese landen mettet vvater hebben, meer als ander; vvaer in oock blijckelick is, vvat grooter voordeel hun de oirsaeclicke kennis der vvichtighe ghedaenten des vvaters doen can; ghemerckt daerbeneuen dat onse Weeghconst die duer d'uyterste begheinselen openbaert: Soo sende ick V. H. de beschrijving der seluer, ghelijck sy, vvcl is vvaer, eertijts int vvater bestonden, maer vele van dien (t'vvclck ick te vrielicker seg, omdat my docht t'volghende sulcx ghenouch te beprouuen, ten anderen op dat ick reden gheef, vvat my vervoordert an V. H te schrijven) gheen der sterflicke voor ons bekent.

Aa 2

Wacr

SIMON STEVIN WISHES THE STATES
OF THE UNITED NETHERLANDS
MUCH HAPPINESS

Since it is sufficiently known, Your Worships, what continual practice these countries have with the water, more so than any others; from which it also appears of what greater profit the knowledge about the causes of the statical properties of water may be to them; considering further that our Art of Weighing reveals these through the fundamental elements; I am sending Your Worships the description thereof, as indeed they formerly were present in water, but many of which (which I say the more frankly because I thought the following treatise proves this sufficiently, secondly in order to give the reason inducing me to write to Your Worships) were not known to any mortals before us¹⁾. From which

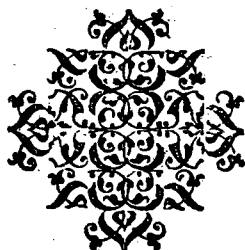
¹⁾ The remark that the newly discovered properties already existed before their discovery may appear somewhat superfluous. It was, however, also made by Archimedes in the preface to his work *De Sphaera et Cyliandro*. Archimedis Opera Omnia, ed. J. L. Heiberg I, p. 4, ll. 9-13. Leipzig 1910.

Effecta.

4

Waer uyt oock * daden sullen volghen, byden
voorighen niet ghesien noch ghevveten. vvelc-
ke, ouermids sy tot grooten voordeele des
Landts strecken, de voordering van V. H. niet
onbilichlick vervvachten. Vaert daerentuf-
schen vvel, in vermeerdering ende alle voor-
spoet. Vyt Leyden in Oogstmaent des
1586^e Iaers.

ANDEN



there will also ensue effects not seen or known among our predecessors; which, since they are to the great benefit of the country, not improperly expect to be furthered by Your Worships. Meanwhile may you thrive in progress and all prosperity. From Leyden, in harvest month of the year 1586.

A N D E N L E S E R.



VAT beweghlicke oirsaeck Archimedes had, om te schrijven rghene by ons in t'Bouck vande dinghen die int water ghedreghen worden, ^{Lib. de its qua vehuntur in aqua.} naghelaten heeft; daer by de natuer heerlick begon te treffen, en weet ick niet, maer wel dit, dat by de myne gheweest is, dat beken ick gheern, in sulcke stof ter form te brengen die wy haer ghegheuen hebben. Belijde oock daerby, dat icker een beter helpende oirsaeck toe ghebadt heb dan Archimedes, namelick de spraeck, welcke D V Y T S C H was, de sune maer Griecx. Want dit moet ghy weten, dat de sprakens goetheyt niet alleen voordelick en is om de Consten bequaemlick daer duer te leeren, maer oock den * Vinders in Inuentorib. haer soucking. Om van t'welck niet reden te spreken, so mer et dat ghelijck inde * Beghinselen der Meetconst, t'punt ghenomen moet worden sonder langde, de lini sonder breedte, t'vlack ^{Elementis Geometriae Superficiis.} sonder dichte, alsoo ist inde Beghinselen des Waterwichts noodich, om * Wisconstlick daer in te handelen, vaten te stellen ^{Mathematicis.} sonder lichamelicke grootheyt, ende sonder ghewicht, sulc noemen wy na hun ghedaenten (want nieuwe Consten brengen nieuwe woorden me.) Vlacvat, ouermits sin stof uyt vlacken bestaet, soo inde volghende t' bepaling gheseyt sal worden. Ende om der ghelijcke redenen moesten wy segghen van Stofswaerheyt, Stoflichtheyt, Eensstofswaer, en diergheijcke, daer t'volghende vut af is, welcke woorden de Griecten soo cort, ende by baren yderman soo verstaenlic, oock so eyghentlick haer gronde beteekenende, noyt en hebben connen segghen, nu niet en connen, noch, dat kennelick ghenouch is, inder eewicheyt niet connen en sullen. Want datter niet in en is en cander niet uytgetrocken.

TO THE READER

What was the cause that moved Archimedes to write that which he left to us in the Book of the things supported in the water, where he began to hit off Nature wonderfully, I do not know. But I do know, and gladly confess, that he was the cause which induced me to cast this matter into the form we have given it. I also avow that I had a better aiding cause therefore than Archimedes, viz. the language, which was Dutch, his only being Greek¹). For you must know that the excellence of language is conducive not only to learning the arts well through it, but also to the search of the inventors. In order to discuss this with good reason, it is to be noted that as in the Elements of Geometry the point is to be taken without length, the line without breadth, the plane without thickness, in the same way it is necessary in the Elements of Hydrostatics, in order to deal therewith geometrically, to assume vessels without any corporeal magnitude and without any weight. These we have called, in accordance with their properties (for new arts call for new words) surface vessels, since their material consists of surfaces, as will be said in the 7th definition hereinafter. And for similar reasons we had to speak of specific gravity, specific levity, being of equal specific gravity, and the like, in which the following abounds; which words the Greeks never were, are not now, and never to all eternity will be able to say so shortly, and so universally intelligibly to everyone of them, and also describing its nature so aptly, as is sufficiently obvious. For what is not in it²), cannot be extracted from it. Its property is to make short, clear, intelligible propositions,

¹) See the *Uytspraak van de Weerdicheyt der Duytsche Taal*. Present volume, p. 58.

²) sc. in a language.

Propositiones trocken worden. Haer eyghenschap is te maken corte clare verstaenlicke * Voorstellen, niet alleen voor den leerlinghen, maer oock self den Vinders, om opentlick t' veruolg van t'een uyt het ander te bemercken. Begheerdi hier af bouen het teghenwoordighe bouck een ander voorbeelt, soo neemt onder sommighe *Conicorum.* Voorstellen des eersten der * Keghelsche boucken van Appollonius het 11^e, t'welck duer Fredericus Commandinus (diens naem ick met eerbieding gheern ghedenck, als van een sterre onder de *Mathematicos.* * Wisconsnaers t' sijnder tijdt, oock duer wiens neersticheyt veel saken die int Griecx verborghen laghen, anden dach ghebrocht sijn) uytet Griecx int Latijn aldus ouergheset is:

Si conus plano per axem secetur, secetur autem & altero plano secante basim coni secundū rectam lineam, quæ ad basim trianguli per axem sit perpendicularis: & sit diameter sectionis vni laterum trianguli per axem æquidistans: recta linea, quæ a sectione coni ducitur æquidistans cōmuni sectioni plani secantis, & basis coni, vsque ad sectionis diametrum, poterit spacium æquale contento lineæ, quæ ex diametro abscissa inter ipsam & verticem sectionis interiicitur, & alia quadam, quæ ad lineam inter coni angulum, & verticem sectionis interiectam, eam proportionem habeat, quam quadratum basis trianguli per axem, ad id quod reliquis duobus trianguli lateribus continetur. dicatur autem huiusmodi sectio parabole.

Daer vooren sullen wy, int eerste dier keghelsche boucken dat wy dencken int Duytsch te laten wygaen, veel corter en claeuder aldus segghen:

T'viercant vande oirdentlicke der brantsne, is euen

not only for the pupils, but also for the inventors themselves, in order to see clearly how one thing follows from another. If you desire some other example in addition to the present book, take among some propositions of the first of the books on Conics by Apollonius the 11th, which has been translated as follows from Greek into Latin by Fredericus Commandinus¹⁾ (whose name I mention with respect as that of a star among the mathematicians of his time, through whose zeal also many matters which lay hidden in Greek have become revealed):

Si conus plano per axem secetur; secetur autem & altero plano secante basim conici secundum rectam lineam, quae ad basim trianguli per axem sit perpendicularis: & sit diameter sectionis uni laterum trianguli per axem aequidistans: recta linea, quae à sectione conici ducitur aequidistans communi sectioni plani secantis, & basis conici, usque ad sectionis diametrum; poterit spatium aequale contento linea, quae ex diametro abscissa inter ipsam & verticem sectionis interiicitur, & alia quadam, quae ad lineam inter conici angulum, & verticem sectionis interiectam, eam proportionem habeat, quam quadratum basis trianguli per axem, ad id quod reliquis duobus trianguli lateribus continetur. dicatur autem huiusmodi sectio parabole.

Instead of this we will say much more shortly and clearly in the first of those books on Conics which we intend to publish in Dutch²⁾:

The square of the ordinate of the parabola is equal to the rectangle compre-

¹⁾ Apollonii Pergaei Conicorum libri quattuor . . . Quae omnia nuper F. Commandino illustravit. Bononiae 1566.

²⁾ This translation never appeared. In this as well as the next example it has escaped Stevin's notice that the cause of the condensation is not the use of Dutch, but the introduction of special mathematical terms, such as *latus rectum*, which the Greek mathematicians were wont to circumscribe.

7
euen anden rechthouck begrepen onder haer middelliniens hoochste deel, ende des brantsnees re-
delicke lini.

*Begheerdy hier by t'volghende 12^e voorstel des boueschre-
uen 1^{en} boucx, soo siet noch langher en duysterder stof, in corte
clare verkeert.*

Si conus plano per axem secetur; secetur autem
& altero plano secante basim coni secundum re-
ctam lineam, quæ ad basim trianguli per axem sit
perpendicularis: & sectionis diameter producta
cum vno latere trianguli per axem, extra verticem
coni conueniat: recta linea, quæ à sectione ducitur
æquidistans communi sectioni plani secantis, &
basis coni vsque ad sectionis diametrum, poterit
spatium adiacens lineæ, ad quam ea, quæ in dire-
ctum constituitur diametro sectionis, subtendi-
turque angulo extra triangulum, eandem propor-
tionem habet, quam quadratum lineæ, quæ dia-
metro æquidistans à vertice sectionis vsque ad ba-
sim trianguli ducitur, ad rectangulum basis parti-
bus, quæ ab ea fiunt, contentum: latitudinem ha-
bens lineam, quæ ex diametro abscinditur, inter
ipsam & verticem sectionis interiectam; excedens-
que figura simili, & similiter posita ei, quæ conti-
netur linea angulo extra triangulum subtenfa, &
ea, iuxta quam possunt quæ ad diametrum appli-
cantur. vocetur autē huiusmodi sectio hyperbole.

Douerfetting daer af is foodanich.

Tviercant vande oirdentlicke der wassendesne,
is euen anden rechthouck begrepen onder haer
middel-

hended by the upper part of its diameter and the latus rectum of the parabola ¹⁾).

If you desire in addition the following 12th proposition of the above-mentioned 1st book, you will see even longer and more abstruse matter made short and clear.

Si conus plano per axem secetur; secetur autem & altero plano secante basim coni secundum rectam lineam, quae ad basim trianguli per axem sit perpendicularis: & sectionis diameter producta cum uno latere trianguli per axem, extra verticem coni conveniat: recta linea, quae a sectione ducitur aequidistans communi sectioni plani secantis, & basis coni usque ad sectionis diametrum, poterit spatium adiacens lineae, ad quam ea, quae in directum constituitur diametro sectionis, subten-diturque angulo extra triangulum, eandem proportionem habet, quam quadratum lineae, quae diametro aequidistans à vertice sectionis usque ad basim, trianguli ducitur, ad rectangulum basis partibus, quae ab ea fiunt, contentum: latitudinem habens lineam, quae ex diametro abscinditur, inter ipsam & verticem sectionis interiectam, excedensque figura simili, & similiter posita ei, quae continetur linea angulo extra triangulum subtensa, & ea, iuxta quam possunt quae ad diametrum applicantur. vocetur autem huiusmodi sectio hyperbole.

The translation of this is as follows::

The square of the ordinate of the hyperbola is equal to the rectangle comprehend-

¹⁾ $y^2 = px$, where p denotes the *latus rectum*.

middelliniens hoochste deel, ende de lini in sulcken reden tot haer redelicke, ghelijck t' hoochste deel met de opstaende, tot de opstaende.

Sulcken helpende oirsaec (als voorghenomen was te verclaren) hebben wy ghehad; Soo ist mettet Duytsch ghestelt, ende diets hem niet en verstaet, bidt hem, beminde leser, dat hijt leere, lieuer dan als een dwaes van het Duytsch dwaeslick te oirdeelen. Angaende v yemandt voortbrengen mocht, dat vele met dese nieuwe costlicheyte der Duytsche tael, daer wy elders breeder af gheseyt hebben, haer spot sullen houden, wanneer sijder af hooren, daer en stoot v niet an, want sulcx is den loop des weereelts, maer denct in v seluen, dat haer ydel woorden, ghetuych van haer verworpen ydelheyt, licht vertreden sullen worden duer v vulle saken, oircondt van v looflicke vulheyt, daerentusschen ghenietende dat sy deruen moeten.

Argumentū.

C O R T B E G R Y P.

Definitiones.

WY sullen ten eersten beschriuen de * bepalinghen van d'eyghen woorden deser Const, metgaders de begheerten. Daer naer de voorstellen, welcker neghen eerste verclaren sullen, ettelicke wichtighe eyghenschappen der lichamen int water. Het 10°, 11°, 12°, 13°, 14°, 15°, voorstel sal sijn vande macht der drucking des waters teghen bodems. Het 16° ende 17° voorstel, vande noodighe langden der sijden des bodems om begheerde drucking des waters daer teghen te hebben. Het 18°, 19°, en 20° voorstel, vande swaerheys middelpunten der gheprangfelen des waters in bodems vergaert. Het 20° voorstel, om duer t'ghewicht des waters sijn grootheyt te vinden. Het 21° ende laetste voorstel, vande

Proportionibus.

* eueredenheden bestaende tusschen der lichamen grootheyt, stoffwaerheyt, ende ghewicht. Achter t'boueschreuen sal noch volghen den Anvang der Waterwichtdaet.

B E G I N.

ed by the upper part of its diameter and the line having to its latus rectum the same ratio as the upper part plus the transverse side to the transverse side ¹⁾).

Such was the aiding cause (as was intended to be explained) that we had. That is what Dutch is like, and when anyone does not understand it, beg him, dear reader, to learn it rather than judge foolishly of Dutch like a fool. If anyone should argue before you that many people will jest about this new cosliness of the Dutch language — about which we have spoken more fully elsewhere ²⁾ — when they hear of it, do not be offended, for such is the way of the world, but think to yourself that their empty words, testifying to their reprehensible emptiness, will easily be trodden down by your full things, the manifestation of your laudable fullness, while you enjoy meanwhile what they must do without.

THE ARGUMENT

We will first describe the definitions of the proper terms of this Art, and also the postulates. Thereafter the propositions, the first nine of which are to explain several statical properties of bodies in water. The 10th, 11th, 12th, 13th, 14th, 15th propositions are to deal with the force of the pressure of the water against surfaces. The 16th and the 17th proposition with the length of the sides of the surface required to have the desired pressure of the water against it. The 18th, the 19th, and the 20th proposition with the centres of gravity of the total pressures of the water on surfaces. The 20th proposition serves to find from the weight of the water its volume. The 21st and last proposition is to deal with the proportions between the volumes, specific gravities, and weights of bodies. The above is to be followed by the Preamble of the Practice of Hydrostatics.

¹⁾ The equation is $y^2 = x \left(\frac{x+q}{q} \right) p$, which follows from $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$, if $p =$
 $\text{latus rectum} = 2 \frac{b^2}{a}$ and $q = \text{transverse side} = 2a$, and the origin is chosen in one of the
 vertices.

²⁾ See the *Uytspraek van de Weerdicheyt der Duytsche Taal*. Present volume p. 58.

D E
B E G H I N S E L E N
D E S W A T E R W I C H T S
B E S C H R E V E N D V E R
S I M O N S T E V I N.

E E R S T D E B E P A L I N G H E N. *Definitiones*

I. B E P A L I N G.

B E K E N D E fwaerheyt noemen wy hier, diens bekende grootheyt duer bekend ghewicht gheuytet wort.

II. B E P A L I N G.

E V E S T O F S W A R E lichamen, diens euegrootheden inde locht euewichtich sijn.

III. B E P A L I N G.

M A E R Stoffwaerfte lichaem, dat der euegrooten t'swaerfte is.

IIII. B E P A L I N G.

E N D E Stoflichtste lichaem, dat dier euegrooten t'lichtste is.

V. B E P A L I N G.

E N D E foo menichmael t'swaerfte der euegrooten fwaerder is dan t'lichtste, so menichmael stoffwaerder segghen wy dat als dit.

VI B E P A L I N G.

S T I I F L I C H A E M is, diens stof niet en vliet, duer t'welck oock water noch locht en dringt.

VII. B E P A L I N G.

V L A C V A T ist gheheel 'Meetconstich vlack *Geometrische superficies.*
eens lichaems, duer t'gedacht daer af scheydelick.

B b

VIII B e-

THE ELEMENTS OF HYDROSTATICS,

Described by Simon Stevin

FIRST THE DEFINITIONS

DEFINITION I.

A known gravity we here call one whose known volume is expressed by a known weight ¹⁾).

DEFINITION II.

Bodies of equal specific gravity we call those bodies, equal volumes of which are equally heavy in air.

DEFINITION III.

But body of greatest specific gravity we call that which is the heaviest of those of equal volume.

DEFINITION IV.

And body of greatest specific levity we call that which is the lightest of those of equal volume.

DEFINITION V.

And as many times as the heaviest of the bodies of equal volume is heavier than the lightest, so many times the latter is called of greater specific gravity than the former ²⁾).

DEFINITION VI.

Solid body is one whose matter does not flow, and through which penetrates neither water nor air.

DEFINITION VII.

Surface vessel is the complete geometrical surface of a body, conceived as separable therefrom.

¹⁾ *Gravity* here means *specific gravity*. The specific gravity of a substance is known when we know the weight of a known volume of the substance. It will be seen that in the following treatise *gravity* may signify both *weight* and *specific gravity*. In all cases the meaning is clear from the context.

²⁾ It may here be remarked that the definitions 1-5 do not yet amount to the definition of specific gravity expressed by the relation

$$S = \frac{G}{V},$$

where G denotes the weight and V the volume of a quantity of a substance with specific gravity S . The low stage of development of symbolic algebra in the 16th century, together with the difficulties inherent in the application of mathematics to physics, prevented this simple formulation.

VIII BEPALING.

B O D E M is alle vlack daer eenich water teghen rust.

IX BEPALING.

*Planum.
Centrum.*

G H E S C H I C K T bodem noemen wy yder * plat, t'welck met alle rechte lini duer sijn * middelpunt, in twee euen deelen ghedeelt wort.

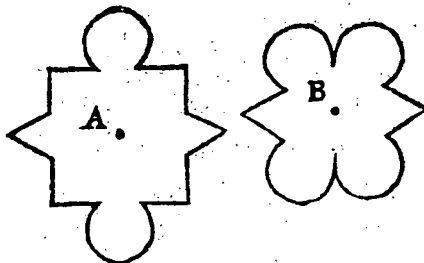
VERCLARING.

Als ronden, scheefronden, euewydighe vierhoucken, ende alle gheschickte veelhoucken in r'rontd beschrijuelick, diens menichte der sijden essential is, en allen anderen van wat form sy souden mueghen wesen, als A, B, ende dierghelijcke, welcke duer haer middelpunt met alle rechte lini in twee euen deelen connen ghedeelt worden, noemen wy Gheschickte bodems, tot

onderscheydt der ghene die met alle rechte lini duer haer middelpunt niet in twee euen deelen ghedeelt en worden, welcke duer t'verkeerde deser bepaling al ongheschickte bodems heeten, als driehoucken, ende veelhoucken met oneuen menichte der sijden, ende dierghelijcke.

D'oirsaek der bepaling deses Gheschickts bodems is (so in t'volghende blijcken sal) dat den pilaer diens grondt een gheschickt bodem is, in twee euen deelen ghedeelt wort, met alle plat duer twee * lijkstandighe punten schoens teghen ouer malcander staende inde omtrecken des grondts ende deckfels.

Homologa.



X BEPALING.

Y D E L noemen wy een plaets daer gheen lichaem in en is.

XI BEPALING.

L E D I C H daer niet dan locht in en is.

B E G H E E R.

DEFINITION VIII.

Bottom is any plane against which rests any water ¹⁾).

DEFINITION IX.

Regular surface we call any plane which is divided into two equal parts by any straight line through its centre.

EXPLANATION.

As circles, ellipses, parallelograms, and all regular polygonal figures that can be inscribed in a circle, the number of whose sides is even, and any others, of whatever form they may be, as A, B, and the like, which can be divided into two equal parts by any straight line through their centre, we call regular surfaces, to distinguish them from those which cannot be divided into two equal parts by any straight line through their centre, which by the inverse of this definition are all called irregular surfaces, as triangles, and polygons with an odd number of sides, and the like. The cause of the definition of this regular surface is (as will become apparent in the following) that the prism whose base is a regular surface is divided into two equal parts by any plane through two homologous points diametrically opposite to each other in the circumferences of the base and of the cover.

DEFINITION X.

Vacuum we call any place in which no body is present.

DEFINITION XI.

Empty we call any place in which there is only air.

¹⁾ The sense in which the Dutch word „bodem” is used by Stevin deviates from general usage in that it denotes not only the bottom of a vessel, but also any surface exposed to the pressure of a liquid.

DES WATERWICHTS.
B E G H E E R T E N.

II

Postulata.

I^e BEGHEERTE.

DER lichamen ghewicht inde locht eyghen gheuoemt te worden, maer in t'water naer de ghestalt.

II BEGHEERTE.

T'VOORGHESTELDE water oueral een-
vaerdigher swaerheyt te sijn.

III BEGHEERTE.

T'GHEWICHT dat een vat ondieper doet sinc-
ken, lichter te wesen, maer dieper, swaerder, ende
euediep, eueswaer te sijn.

IIII BEGHEERTE.

T'VLACKVAT te connen water ende ander
stof houden sonder breken of form te veranderen.

V BEGHEERTE.

T'VLACKVAT vol waters uytghegoten sijn-
de, ledich te bliuen.

VERCLARING.

Ledich te bliuen, dat is niet ydel, want anders t'ghewicht des lochts
sonder ghebreken.

VI BEGHEERTE.

YDER waters oppervlack * plat te wesen, *Esse planum
paralleli cu
Horizonte.*
euewydich vanden sichteinder.

VERCLARING.

T'WELCK int an sien dattet deel des clootvlack ofte weereltvlack is
(weereltvlack noemen wy alle clootvlack diens middelpunt des weerelts
middelpunt is) oock in een droppel erghes op ligghende ofte anhangen-
de, ofte in water daer eenich lichaem me bestreken mocht wesen, so niet
en is, maer in soo cleyne menichvuldicheyt waters als dese, noch in soo
groote als daer t'ghinste in merckelick is, en verkeerde de volgende*voor-*Propositiones*
stellen

B b 2

THE POSTULATES OF HYDROSTATICS

POSTULATE I.

The weights of bodies in air to be called their proper weights, but in water apparent weights.

POSTULATE II.

The water under consideration to be of uniform gravity throughout.

POSTULATE III.

The weight causing a vessel to sink less deep to be lighter, but the weight causing it to sink deeper to be heavier, and that causing it to sink to the same depth, equally heavy.

POSTULATE IV.

The surface vessel to be capable of holding water and other matter without breaking or being transformed.

POSTULATE V.

The surface vessel full of water, the latter being poured out, to be left empty.

EXPLANATION.

To be left empty, that is not a vacuum, for otherwise the weight of the air would be absent.

POSTULATE VI.

Any water's upper surface to be plane, parallel to the horizon.

EXPLANATION.

Since the water's upper surface forms part of the spherical surface or world surface (world surface we call any spherical surface which has its centre in the centre of the world), in reality this is not the case; nor is it with a drop lying on or adhering to something, or with water with which a body has been moistened. The propositions hereinafter, however, relate to quantities of water neither as

stellen niet. Wel is waer dat wy des waters oppervlack soudē mueghen nemen voor deel des weereltvlacks, ende de volghende beschrijving daer na rechten, maer wanttet moeylicker waer, ende tottet einde, dat is de Waterwichdaet, niet voordelicker, soo wortt begheert datmen toelate, yder waters oppervlack plat te wesen, euewydich vanden sichteinder.

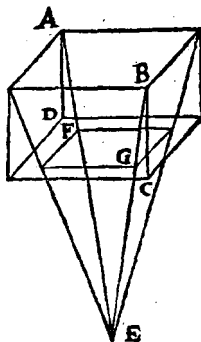
VII BEGHEERTE.

Homologia.

W E S E N D E den grondt ende decksel eens pilaers euewydich vanden sichteinder, en de rechte linien tusschen^a lijkstandighe punten der seluer rechthouckich opden sichteinder: Dat die linien voortghetrocken in t^weerelts middelpunt vergaren; oock sulcke grondt ende decksel deelen van weereltvlakken te sijn.

VERCLARING.

Laet A B C D een pilaer wesen diens decksel A B, ende grondt D C euewydich sijn vanden sichteinder, en B C sy een rechte lini rechthouckich opden sichteinder tusschen twee lijkstandighe punten C, B, maer E sy t^weerelts middelpunt, laet nu ghetrocken worden de linien A E, en B E, naeckende den grondt D C inde punten F, G, tusschen welke beschreuen sy den grondt F G ghelijck met D C. Dit so wesende, t^bblyckt dat de linien B C ende A D voortghetrocken, niet en vergaren in E, want dieder in vergaren sijn A F, ende B G, oock en sijn de platten A B ende D C gheen deelen van weereltvlakken, nochtan begheeren wy toeghelaten te worden, dat B C ende A D voortghetrocken, daer in versamen, ende dat die platten A B, D C deelen van weereltvlakken sijn, reden dat in al t^ghene ons inde Waterwichdaet ontmoet, sulck verschil onbemerckelick is, soot oock is tusschen den pilaer A B C D ende * tⁿaeldensdeel A B G F, schoon ghenomē dat A B ende F G deelen van weereltvlakken waren. Tis wel soo, dat wy inde plaets des pilaers A B C D, soudē mueghen nemen foodanich lichaem A B G F, ende de volghende voorstellen daer naer rechten, maer om sulcke redenen als onder de 6^e begheerte gheseyt sijn, so ist beter ghelaten, want ghelijckt inde * Sterconst slichheyt waer, niet toe te laten t^{re}etriick voor des weerelts middelpunt ghenomen te worden, alsoo dat oock hier.



Pars Pyramiden.

Astrologia.

small as the latter, nor as large as the former. It is true that we might consider the water's upper surface as part of the world surface, and adjust the following description accordingly, but since this would be more difficult and not more conducive to the end in view, viz. the Practice of Hydrostatics, it is postulated that any water's upper surface is plane and parallel to the horizon.

POSTULATE VII.

The base and the cover of a prism being parallel to the horizon, and the straight lines joining homologous points thereof being at right angles to the horizon: that those lines produced meet in the centre of the world; also that such base and cover are parts of world surfaces.

EXPLANATION ¹⁾.

Let $ABCD$ be a prism, whose cover AB and base DC shall be parallel to the horizon, and BC shall be a straight line at right angles to the horizon, joining two homologous points C, B , but E shall be the centre of the world. Let there now be drawn the lines AE and BE , touching the base DC in the points F, G , between which let there be described the base FG , similar to DC . This being so, it is apparent that the lines BC and AD produced do not meet in E , for those meeting therein are AF and BG . Nor are the surfaces AB and DC parts of world surfaces. Nevertheless we postulate that BC and AD produced meet therein, and that those surfaces AB and DC are parts of world surfaces, because in all the cases we shall meet with in the Practice of Hydrostatics this difference is imperceptible, just as it is between the prism $ABCD$ and the part of a pyramid $ABGF$, even if it is assumed that AB and FG are parts of world surfaces. It is true that instead of the prism $ABCD$ we might take such a body $ABGF$, and adjust the following propositions accordingly, but for the reasons mentioned in the 6th postulate it is better not to do so, for just as in astronomy it would be stupid not to grant that the earth be taken for the centre of the world ²⁾, so it is here, too.

¹⁾ Here as well as in the fifth postulate of the *Art of Weighing* Stevin scruples to consider all verticals as parallel. There is, however, one notable difference. In the *Art of Weighing* we were asked to grant that they are parallel; here it is postulated that vertical lines, which are parallel, (viz. the vertical sides of a prism) meet in the centre of the world. As far as we have been able to ascertain, the postulate has not been used on any occasion.

²⁾ This is not to be understood as a rejection of the Copernican system of the world, in which the earth is no longer at the centre of the universe. Indeed, we know from the *Hemelloop* (XI; i, 3) that Stevin was an ardent supporter of this system. The statement only means that the earth is to be considered as a point in regard to the dimensions of the world, and consequently may be called centre, if we take the geocentric view. In a similar way Stevin here assumes the centre of the earth to be at infinity.

NV DE VOORSTELLEN.

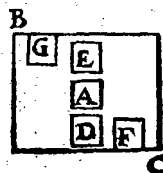
Propositiones

I. VERTOOGH.

I. VOORSTEL.

T'GHESTELDE water houdt alle plaets die-
men hem binnen water gheeft.

T'GHEHEVEN. Laet het water in 'tvlackvat A 'tghestelde water sijn in 'twater B C. T'EGHEERDE. Wy moeten bewyfen dattet water A in die plaets sal bliuen. T'BEWYS. En later water A (soot mueghelick waer) sijn plaets niet houden, maer het sy ghedaelt daer D is; Dit so toeghelaten, 'twater dat daer naer inde plats van A ghecommen is, sal om de selue oirsaek oock ter plaets van D dalen, 'twelck daer naer oock een dergelijcke ander doen sal, inder voughen dat dit water (om dat de reden altydt de selue is) een ewich roersel sal maken, 'twelck ongheschickt is. Sghelijcx sal oock bethoont worden dat A niet rijfen, ofte naer eenighe ander sijden hem begheuen en can. T'blijckt oock dat soomen A stelde binnen 'twater ter plaets van D, E, F, of G, dattet om de voornoemde redenen, op yder van die plaetsen, ende oueral daerment in B C set, bliuen sal.



T'ESLVT. T'ghestelde water dan, houdt alle plaets diemen hem binnen water gheeft, 'twelck wy bewyfen moesten.

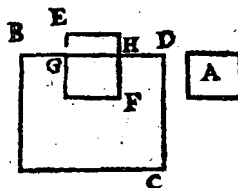
II. VERTOOGH.

II. VOORSTEL.

EEN styflichaem stoflichter dan water, en
sijnckt niet heel daer onder, maer een deel blijfter
nyt stekende.

T'GHEGHEVEN. Laet het styflichaem A, stoflichter sijn dan 'twater B C, diens oppervlack B D. T'EGHEERDE. Wy moeten bewyfen dat A, gheleyt in 'twater B C, niet heel daer onder sincken en sal, maer datter een deel buyten 'twater sal bliuen steken.

T'BEREYTSSEL. Laet E F een vlackvat sijn, wiens deel dat binnen 'twater ende met water ghevult is, sy G F, eue groot ende ghelijck an A, ende sijn oppervlack G H sal in 'tvlack B D sijn, ouermids 'tvlackvat E F licht noch swaer en is.



T'BEWYS. Anghesien A stoflichter is duer 'tghheueu dan 'twater B b ; G F, ende

NOW THE PROPOSITIONS

THEOREM I.

The water under consideration keeps any place given to it in water.

PROPOSITION I.

SUPPOSITION. Let the water in the surface vessel A be the water under consideration in the water BC . **WHAT IS REQUIRED TO PROVE.** We have to prove that the water A will remain in that place. **PROOF.** Let the water A (if it were possible) not remain in that place, but be descended where D is. This being granted, the water which has thereafter reached the place of A will for the same reason also descend to D , which a similar other quantity of water will then also do, in such a way that this water (because the reason is always the same) will perform a perpetual motion, which is absurd. In the same way it can also be shown that A cannot rise or move towards any other side. It also appears that if A be placed within the water in the place of D , E , F or G , it will, for the aforesaid reasons, remain in each of those places, and wherever it is placed in BC .

CONCLUSION. The water under consideration therefore keeps any place given to it in water, which we had to prove.

THEOREM II.

A solid body of greater specific levity than water does not sink completely below the upper surface, but a part continues to stick out of it.

PROPOSITION II.

SUPPOSITION. Let the solid body A be of greater specific levity than the water BC , whose upper surface shall be BD . **WHAT IS REQUIRED TO PROVE.** We have to prove that A , when laid in the water BC , will not sink completely below the upper surface, but that a part will continue to stick out of the water. **PRELIMINARY.** Let EF be a surface vessel, whose part which is in the water and filled with water shall be GF , equal and similar to A , and its upper surface GH shall be in the surface BD , since the surface vessel EF is neither light nor heavy. **PROOF.** Since by the supposition A is of greater specific levity than the water

G F, ende dat G F euegroot is an A, soo is G F swaerder dan A. Laet ons nu t'water G F dat in t'vlackvat E F is, uytghieten, ende legghen daerin t'lichaem A, t'welck die plaets effen vullen sal, ouermits A duer t'bereyt-sel ghelijck en euegroot is an G F; Maer als vooren gheseyt is t'lichaem A is lichter dan t'uytgegoten water; T'vlackvat dan E F en sal van A soo diep niet sincken alst van t'water G F dede, duer de 3^e begheerte; Maer soo veel t'vlackvat E F ondieper sinckt, soo veel moet t'lichaem A nootfakelick buyten t'water steken. **T B E S L V Y T.** Een stijflichaem dan stoflichter als water, en sinckt niet heel daer onder, maer een deel blijft uytstekende; t'welck wy bewyzen moesten.

III. VERTOOC.

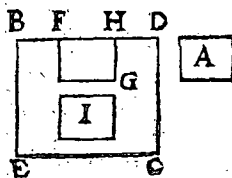
III. VOORSTEL.

E E N stijflichaem stofswaerder dan water sinckt tot den grondt.

T'GHEGHEVEN. Laet A een stijflichaem wesen stofswaerder dan t'water B C, diens oppervlack B D, ende grondt E C sijn.

T'BEGHEERDE. Wy moeten bewyzen dat A gheleyt in t'water B C, sincken sal tot den grondt E C. **T'BEREYTSSEL.** Laet F G een vlackvat sijn met water ghevult, euegroot ende ghelijck an A, wiens oppervlack F H in t'vlack B D sy.

T'BEWYS. Anghesien A stofswaerder is duer ghestelde dan t'water F G, ende dat F G euegroot is an A, soo is A swaerder dan F G. Laet ons nu t'water F G dat in t'vlackvat F G is, uytghieten, ende legghen daerin t'lichaem A, t'welck die plaets effen vullen sal, ouermits A duer t'ghestelde ghelijck en euegroot is an F G; Maer soo wy vooren gheseyt hebben, A is swaerder dan het uytgegoten water; T'vlackvat dan F G sal van A dieper sincken alst van t'water F G dede duer de 3^e begheerte. Wy hebben dan bethoont dattet lichaem A sincken sal. Daer rest noch bewesen te worden dattet oock sincken sal tot op den grondt E C, aldus: En latet (soot mughelick waer) niet sincken tot E C, maer op den wech tusschen beyden bliuen als daer I is, ende laet ons t'stijflichaem datter in t'vlackvat I steekt, weeren, ende vullen dat met water, t'selue sal duer het 1^e voorstel op die plaets bliuen: Maer dit water is lichter als dat lichaem, een swaerder dan ende een lichter, sullen op een selfde plaets bliuen, t'welck ongheschickt ende teghen de 3^e begheerte is. T'lichaem A dan, en can tusschen t'oppervlack B D ende den grondt E C niet bliuen, t'moet dan nootfakelick sincken tot dattet op den grondt



GF , and GF is equal to A , GF is heavier than A . Let us now pour out the water GF that is in the surface vessel EF , and lay therein the body A , which will just fill that place, since by the preliminary A is similar and equal to GF . But, as has been said above, the body A is lighter than the water poured out. The surface vessel EF therefore will not sink as deep under the influence of A as it did under the influence of the water GF , by the 3rd postulate. But by as much as the surface vessel EF sinks less deep, by so much must the body A necessarily stick out of the water. CONCLUSION. A solid body of greater specific levity than water therefore does not sink completely below the upper surface, but a part continues to stick out of it, which we had to prove.

THEOREM III.

PROPOSITION III.

A solid body of greater specific gravity than water sinks to the bottom.

SUPPOSITION. Let A be a solid body of greater specific gravity than the water BC , whose upper surface is BD and its base EC . WHAT IS REQUIRED TO PROVE. We have to prove that A , when laid in the water BC , will sink to the base EC . PRELIMINARY. Let FG be a surface vessel filled with water, equal and similar to A , whose upper surface FH shall be in the plane BD . PROOF. Since by the supposition, A is of greater specific gravity than the water FG , and FG is equal to A , A is heavier than FG . Let us now pour out the water FG which is in the surface vessel FG , and lay therein the body A , which will just fill that place, since by the supposition A is similar and equal to FG . But as we have said before, A is heavier than the water poured out. The surface vessel FG therefore will sink deeper under the influence of A than under the influence of the water FG , by the 3rd postulate. We have therefore shown that the body A will sink. It remains to be proved that it will also sink to the base EC , thus: Let it (if this were possible) not sink to EC , but remain somewhere between the two, as where I is, and let us take away the solid body in the surface vessel I , and fill it with water; it will remain in that place by the 1st proposition. But this water is lighter than that body. A heavier and a lighter body will therefore remain in the same place, which is absurd, and contrary to the 3rd postulate. The body A therefore cannot remain between the upper surface BD and the base EC ; it must therefore necessarily sink

gront E C rust. **T'beslyyt.** Een stijflichaem dan stoffwaerder als water, sinckt tot den grondt, t'welck wy bewyfen moesten.

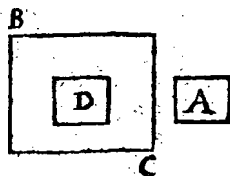
IIII VERTOOGH.

IIII VOORSTEL.

EEN stijflichaem euestoffwaer an water, houdt alle plaets diemen hem binnen water gheeft.

T'ghegheven. Laet het stijflichaem A, euestoffwaer sijn metter t'water B C. **T'begheerde.** Wy moeten bewyfen dat A in t'water B C gheleydt, alle plaets houdt diemen hem daer gheeft.

T'bereyt sel. Laet D een vlackvat vol waters sijn, euegroot ende ghelijck an A. **T'bewys.** Angheffen A euestoffwaer is door t'ghegheuen an t'water D, ende dat D euegroot is met A, soo is D oock euestoffwaer met A; Laet ons nu t'water D dat in t'vlackvat D is, uytghieten, ende legghen daerin sijn euewichtich lichaem A, t'welck die plaets effen vullen sal, ouermits A door t'ghestelde ghelijck en euegroot is an G F; T'vlackvat dan D en sal van A niet dieper sincken noch hooger rijfen dan van t'water D, duer de 3^e begheerte: Maer t'water D hielt in B C alle plaets diemen hem gaf duer het 1^e voorstel, t'stijflichaem A dan, houdt in t'water B C alle plaets diemen hem gheeft.



T'beslyyt. Een stijflichaem dan euestoffwaer an water, houdt alle plaets diemen hem binnen water gheeft, t'welck wy bewyfen moesten.

V VERTOOGH.

V VOORSTEL.

EEN stijflichaem stofflichter dan water daert inlight, is euewichtich an t'water euegroot met sijn deel dat binnen t'water is.

T'ghegheven. Laet A B een stofflichter stijflichaem sijn dan t'water C D daert in light, ende sijn vlackvat sy A B, ende sijn deel binnen t'water sy E B. **T'begheerde.** Wy moeten bewyfen dat het stijflichaem A B, euewichtich is an t'water dat euegroot is met het deel E B dat binnen t'water C D is. **T'bewys.** Laet ons t'stijflichaem A B trecken uyt het vlackvat A B, ende vullen t'vlackvat weder met water, tot dattet soo diep inghesoncken is alst eerst metter lichaem was. T'welck soo sijnde, t'water E B datter in t'vlackvat A B is, sal (want t'oppervlack van alle water des vlackvats met een deel buyten t'water stekende

until it rests on the base EC . **CONCLUSION.** A solid body of greater specific gravity than water therefore sinks to the bottom, which we had to prove.

THEOREM IV.

PROPOSITION IV.

A solid body of equal specific gravity to water keeps any place given to it in water.

SUPPOSITION. Let the solid body A be of equal specific gravity to the water BC . **WHAT IS REQUIRED TO PROVE.** We have to prove that A , when laid in the water BC , keeps any place given to it therein. **PRELIMINARY.** Let D be a surface vessel full of water, equal and similar to A . **PROOF.** Since, by the supposition, A is of equal specific gravity to the water D , and D is equal to A , D is also of equal gravity to A . Let us now pour out the water D which is in the surface vessel D , and lay therein the body A having the same weight, which will just fill that place, since by the supposition A is similar and equal to GF . The surface vessel D therefore will neither sink deeper nor rise higher under the influence of A than under the influence of the water D , by the 3rd postulate. But the water D kept in BC any place given to it, by the 1st proposition. The solid body A therefore keeps in the water BC any place given to it. **CONCLUSION.** A solid body of equal specific gravity to water therefore keeps any place given to it in water, which we had to prove.

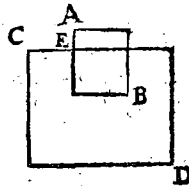
THEOREM V.

PROPOSITION V.

A solid body of greater specific levity than the water in which it lies is of equal weight to the water having the same volume as its part in the water.

SUPPOSITION. Let AB be a solid body of greater specific levity than the water CD in which it lies, and its surface vessel shall be AB , and its part in the water shall be EB . **WHAT IS REQUIRED TO PROVE.** We have to prove that the solid body AB is of equal weight to the water having the same volume as the part EB which is in the water CD . **PROOF.** Let us pull the solid body AB from the surface vessel AB , and fill the surface vessel again with water until it has sunk to the same depth at which it was first when filled with the body. This being so, the water EB which is in the surface vessel AB will (because the upper surface of any water in the surface vessel, a part of which is sticking out of the

kende, is altijd in 't oppervlack des omuangen- den waters, ouermids 't vlackvat niet en weeght) euewichtich sijn an 't ghegheuen lichaem A B, Re- den, dat twee ghewichten die een vat euediep doen sincken oock euefwaer sijn; duer de 3^e be- gheerte. **T B E S L V Y T.** Een stijf lichaem dan stoflichter als water daert in light, is euewichtich an 't water euegroot met sijn deel dat binnen 't wa- ter is, 't welck wy bewyfen moesten.



I EYsch.

VI Voerstel.

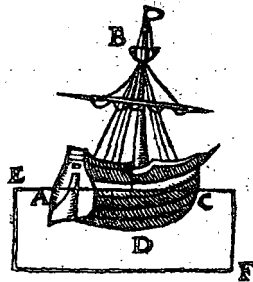
L I G G H E N D E t'een deel des stijf lichaems bekender grootheyt, in water bekender swaer- heyt, ende t'ander deel daer buyten: Te vinden t'ghewicht des heelen lichaems.

T G H E G H E V E N. Laet ABCD een stijf lichaem wesen van forma soot valt ende E F een water van 't welck een teerlincksche voet weeght 65 lb (soo veel weeght naer d'ruaring een Delfsche voet Delfs water, ende daerop sullen wyse inde volghende voorbeelden altijd schatten) ende des lichaems deel binnen 't water sy ACD, wiens grootheyt sy van 10000 teerlincksche voeten. **T B E G H E E R D E.** Wy moeten vinden hoe swaer t'heel lichaem A B C D sy, met al datter in ende op is.

T W E R C K. Men sal 10000 menichvuldighen met de 65 lb come 650000 lb voor t'begheerde.

T B E W Y S. Het heel lichaem A B C D is euewichtich an 't water euegroot met ACD duer het 3^e voorstel, maer 't wa- ter euegroot an ACD weeght 650000 lb, het heel lichaem dan A B C D weeght 650000 lb, t welc wy bewyfen moesten.

T B E S L V Y T. Ligghende dan t'een deel des stijf lichaems bekender groot- heyt, in water bekender swaerheyt, en- de t'ander deel daer buyten; wy hebben t'ghewicht des heelen lichaems ghe- vonden naer den eyfch.



water, is always in the upper surface of the surrounding water, since the surface vessel has no weight) be of equal weight to the given body AB , because two weights which cause a vessel to sink to the same depth are also of equal gravity, by the 3rd postulate. **CONCLUSION.** A solid body of greater specific levity than the water in which it lies is therefore of equal weight to the water having the same volume as its part in the water, which we had to prove.

PROBLEM I.

PROPOSITION VI.

One part of a solid body of known volume lying in water of known gravity, and the other outside it: to find the weight of the whole body.

SUPPOSITION. Let $ABCD$ be a solid body of any form, and EF a water one cubic foot¹⁾ of which weighs 65 lbs (that is, by experience, the weight of a Delft foot of Delft water, and this weight we will always assume for it in the following examples), and the part of the body in the water shall be ACD , whose volume shall be 10,000 cubic feet. **WHAT IS REQUIRED TO FIND.** We have to find the weight of the whole body $ABCD$, with all that is in or on it. **CONSTRUCTION.** Multiply 10,000 by the 65 lbs, then the required weight will be 650,000 lbs. **PROOF.** The whole body $ABCD$ is of equal weight to the water having the same volume as ACD , by the 5th proposition, but the water having the same volume as ACD weighs 650,000 lbs; the whole body $ABCD$ therefore weighs 650,000 lbs, which we had to prove. **CONCLUSION.** One part of a solid body of known volume therefore lying in water of known gravity, and the other part outside it: we have found the weight of the whole body, as required.

¹⁾ It is only here that Stevin uses the correct term „cubic foot“. Elsewhere „foot“ means the unit of length as well as the corresponding units of area (square foot) and volume (cubic foot).

VI VERTOCH.

VII VOORSTEL.

W E S E N D E twee oneuſtoffware wateren, ende een ſtiſſichaem ſtoſſichter dan eenich van dien: Ghelijck de ſtoſſwaerheyt des ſwaerſten waters, tot de ſtoſſwaerheyt der lichtſten, alſo de grootheyt diens ſtiſſichaems binnen t'water in t'lichtſte water gheleyt, tot ſijn grootheyt binnen t'water in t'ſwaerſte gheleyt.

T G H E G H E V E N. Laet A B een water ſijn, ſtoſſwaerder dan t'water C D, ende E F ſy een ſtiſſichaem ſtoſſichter dan eenich dier twee wateren, t'welck eerst gheleyt in t'water A B, ſoo daelter onder t'water het deel G F, maert t'ſelue lichaem E F gheleyt in t'water C D, t'welck daer ſy H I, ſoo ſinckter onder t'water het deel K I. T B E G H E E R D E. Wy moeten bewyſen dat ghelijck de ſtoſſwaerheyt des waters A B, tot de ſtoſſwaerheyt des waters C D; alſoo de grootheyt K I, tot G F.

T B E W Y S. T'water des waters A B euegroot an G F, is eueſwaer metter lichaem E F, ende t'water des waters C D euegroot an K I, is eueſwaer metter lichaem H I duer het 5^e voorſtel, maert t'lichaem E F ofte H I is al een ſelfde lichaem duer t'ghegheuen, daerom t'water des waters A B euegroot met G F, is eueſwaer an t'water des waters C D euegroot met K I; Maer weſende twee eueſware waterē, ghelijc haer grootheyt tot grootheyt, alſo ouerandert haer ſtoſſwaerheyt tot ſtoſſwaerheyt, als nootſakelick volght uyt de toeghelatē 5^e bepaling, daerom ghelijck de ſtoſſwaerheyt des waters A B, tot de ſtoſſwaerheyt des waters C D, alſoo de grootheyt K I, tot de grootheyt G F.

T B E S L V Y T. Weſende dan twee oneuſtoffware wateren ende een ſtiſſichaem, &c.

VII VERTOCH.

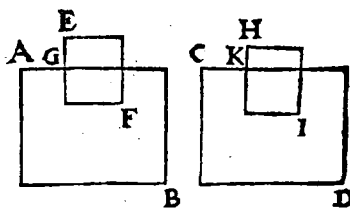
VIII VOORSTEL.

Y D E R ſtiſſichaems ſwaerheyt is ſo veel lichter in t'water dan inde locht, als de ſwaerheyt des vvaters met hem euegroot.

T G H E G H E V E N. Laet A een ſtiſſichaem ſijn, ende B C een water.

C c

T B E G H E E R D E.



THEOREM VI.

Given two waters of unequal specific gravity, and a solid body of greater specific levity than either of these: as is the specific gravity of the heavier water to the specific gravity of the lighter, so is the volume of that solid body within the water when laid in the lighter water to its volume within the water when laid in the heavier.

PROPOSITION VII.

SUPPOSITION. Let AB be a water of greater specific gravity than the water CD , and EF shall be a solid body of greater specific levity than either of those two waters; when this body is first laid in the water AB , the part GF will sink below the water, but when the same body EF , which now shall be HI , is laid in the water CD , the part KI will sink below the water. WHAT IS REQUIRED TO PROVE. We have to prove that as the specific gravity of the water AB is to the specific gravity of the water CD , so is the volume KI to GF . PROOF. The water of the water AB which has the same volume as GF is of equal gravity to the body EF , and the water of the water CD which has the same volume as KI is of equal gravity to the body HI , by the 5th proposition, but the body EF or HI is one and the same body, by the supposition. Therefore the water of the water AB which has the same volume as GF is of equal gravity to the water of the water CD which has the same volume as KI . But given two bodies of water of equal gravity, as their volumes are to each other, so are their specific gravities inversely to each other, as follows necessarily from the 5th definition that has been granted. Therefore, as the specific gravity of the water AB is to the specific gravity of the water CD , so is the volume KI to the volume GF .

CONCLUSION. Given therefore two waters of unequal specific gravity, and a solid body, etc.

THEOREM VII.

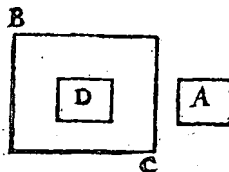
PROPOSITION VIII.

The gravity of any solid body is as much lighter in water than in air as is the gravity of the water having the same volume ¹⁾.

SUPPOSITION. Let A be a solid body, and BC a water. WHAT IS REQUIRED

¹⁾ This proposition contains the famous principle of Archimedes (*On Floating Bodies* I, Props 6 and 7). Here, just as in his demonstration of the law of the inclined plane (*Art of Weighing* I, Prop. 19), Stevin shows his remarkable gift for demonstrations which immediately appeal to common sense and hence may be understood without any previous knowledge.

T'BEGHEERDE. Wy moeten bewyfen dat A in t'water B C gheleyt, aldaer soo veel lichter sal sijn dan inde locht, als de swaerheyt des waters met hem euegroot. **T'BEREYTSSEL.** Laet D een vlackvat vol waters sijn, euen ende ghelijck an A. **T'BEWYS.** T'vlackvat D vol waters, en is in t'water B C licht noch swaer, want het daer in alle ghestalt houdt diemen hem gheeft, duer het 1^o voorstel, daerom t'water D uytghegoten, t'vlackvat sal t'ghewicht des waters lichter sijn dant in sijn eerste ghedaente was, dat is, van soo veel volcommentlick licht: Laet ons nu daer in legghen t'lichaem A, t'selue sal daerin effen passien, om dat sy euen ende ghelijck sijn duer t'ghestelde. Ende t'vlackvat metter lichaem A alsoo daer in, sal weghen t'ghewicht van A met sijn voornoemde lichticheyt, dat is t'ghewicht van A min t'ghewicht des waters datter eerst uytghegoten was, maer dat water is euegroot an A. Daerom A in t'water B C gheleyt, is daer in soo veel lichter dan inde locht, als de swaerheyt des waters met hē euegroot.



T'BESLUYT. Yder styflichaems swaerheyt dan, is soo veel lichter in t'water dan inde locht, als de swaerheyt des waters met hem euegroot, t'welck wy bewyfen moesten.

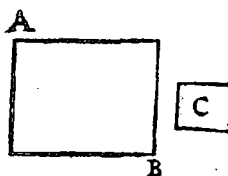
II EYSCH.

IX VOORSTEL.

W E S E N D E ghegheuen de reden der stofsvaerheyt des vvaters, ende eens stijflichaems, ende des stijflichaems svvaerheyt: Sijn staltvicht in t'vvater te vinden.

1^o VOORBEELT *atwaer t'stijflichaem stoflichter is dan water.*

T'GHEGHEVEN. Laet A B een water sijn, ende C een stijflichaem weghende 2 lb, ende de stofswaerheyt des waters, tot de stofswaerheyt des stijflichaems sy als van 5 tot 1. **T'BEGHEERDE.** Wy moeten des stijflichaems C staltvicht in t'water A B vinden. **T'WERCK.** Men sal sien hoe veel een lichaem waters euegroot met C, weghen soude, wort beuonden 5 mael 2 lb, dat is 10 lb, de selue ghetrocken van 2 lb des stijflichaems C, rest min 8 lb, dat is licht ofte ryfendwicht 8 lb voor C in t'water A B.



Om t'welck

TO PROVE. We have to prove that A , when laid in the water BC , will there be as much lighter than in air as is the gravity of the water having the same volume.

PRELIMINARY. Let D be a surface vessel full of water, equal and similar to A .

PROOF. The surface vessel D full of water is in the water BC neither light nor heavy, for it there keeps any place given to it, by the 1st proposition. Therefore, the water D being poured out, the surface vessel will be lighter by the weight of the water than it was in the first position, i.e. it will be so light absolutely. Let us now lay therein the body A ; this will just fit therein, because they are equal and similar by the supposition. Then the surface vessel with the body A therein will weigh as much as the weight of A with its aforesaid levity, i.e. the weight of A minus the weight of the water that was first poured out of it, but this water has the same volume as A . Therefore A , when laid in the water BC , is therein as much lighter than in air as is the gravity of the water having the same volume.

CONCLUSION. The gravity of any solid body therefore is as much lighter in water than in air as is the gravity of the water having the same volume, which we had to prove.

PROBLEM II.

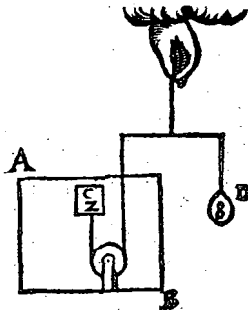
PROPOSITION IX.

Given the ratio between the specific gravity of the water and that of a solid body, and the gravity of the solid body: to find its apparent weight in the water.

EXAMPLE I, where the solid body is of greater specific levity than water.

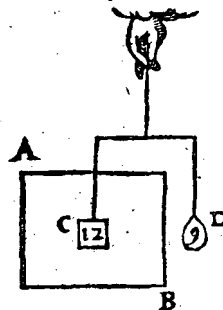
SUPPOSITION. Let AB be a water, and C a solid body weighing 2 lbs, and the specific gravity of the water shall be to the specific gravity of the solid body as 5 to 1. WHAT IS REQUIRED TO FIND. We have to find the apparent weight of the solid body C in the water AB . CONSTRUCTION. It shall be ascertained what would be the weight of a body of water having the same volume as C ; this is found to be 5 times 2 lbs, that is 10 lbs. This being subtracted from the 2 lbs of the solid body C , there is left minus 8 lbs, that is light or rising weight of 8 lbs for C in the water AB .

Om t'welck opentlicker te verclaren, soo neemt dat C in t'water A B ghesteken sy, ende daerteghen ghehanghen t'ghewicht D van 8 lb, als hier neuens, ende D sal met C euestalwichich sij.



11° VOORBEELT, alwaer t'styflichaem stoffwaerder is dan t'water, diens wercking ghelyck is ande voorgaende.

T'GHEGHEVEN. Laet de reden der stoffwaerheyt des waters A B hier bouen, tot het styflichaem C, nu sijn als van 1 tot 4, ende laet C weghen 12 lb. T'BEGHEERDE. Wy moeten des styflichaems staltwicht in t'water A B vinden. T'WERCK. Men sal sien hoe veel een lichaem waters euegroor met C weghen soude, wort beuonden het $\frac{1}{4}$ van C 12 lb, dat is 3 lb, de selue ghetrocken van 12 lb des styflichaems C, rest 9 lb voor t'ghewicht van C in t'water A B.



Om t'welck breeder te verclaren, so neemt dat C in t'water A B ghesteken sy, ende daer teghen hanghe t'ghewicht D van 9 lb, als hier neuens, ende D sal met C euestalwichich sij.

Wy souden oock mueghen een derde voorbeelt setten, alwaer de reden der stoffwaerheyt des waters ende styflichaems euen waer; maer t'is blyckelick dat (oock volghende de reghel der voorgaender wercking) sulcken styflichaem in t'water licht noch swaer sijn en sal, van alle welcke t'bewys openbaer is duer t'boueschreuen 8° voorstel.

T'BESELYT. Wesende dan ghegheuen de reden der stoffwaerheyt des waters, ende eens styflichaems, ende des styflichaems swaerheyt: Wy hebben sijn staltwicht in t'water ghevonden naer den eysch.

In order to explain this more clearly, assume that C be put in the water AB , and that against this there be suspended the weight D of 8 lbs, as in the figure opposite; then D will be of equal apparent weight to C .

EXAMPLE II, where the solid body is of greater specific gravity than the water, the procedure of which is similar to the preceding.

SUPPOSITION. Now let the ratio of the specific gravity of the water AB above to that of the solid body C be as 1 to 4, and let C weigh 12 lbs. WHAT IS REQUIRED TO FIND. We have to find the apparent weight of the solid body in the water AB . CONSTRUCTION. It shall be ascertained what would be the weight of a body of water having the same volume as C . This is found to be $\frac{1}{4}$ of C (12 lbs), that is 3 lbs. The latter being subtracted from the 12 lbs of the solid body C , there is left 9 lbs for the weight of C in the water AB .

In order to explain this more fully, assume that C be put in the water AB , and that against this there be suspended the weight D of 9 lbs, as in the figure opposite. Then D will be of equal apparent weight to C .

We might also give a third example, where the ratio between the specific gravity of the water and that of the solid body should be equal; but it is evident that (also according to the rule of the preceding procedure) such a solid body will be neither light nor heavy in water, of all of which the proof is manifest from the 8th proposition described above.

CONCLUSION. Given therefore the ratio of the specific gravity of water to that of a solid body, and the gravity of the solid body: we have found its apparent weight in water, as required.

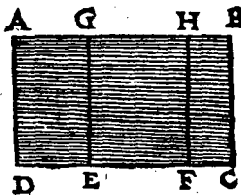
*Parallelē cū
Horizonte.*

*Perpendicu-
laris.
Plane.*

OP yder bodem des vvaters * euevvydich sijnde vanden sichteinder, rust een ghevricht euen ande svvaerheyt vvaters die euegroot is met den pilaer, vviens grondt dien bodem is, ende hoochde, de * hanghende lini van * t'plat duer t'vvaters oppervlack tot den grondt.

T'GHEGHEVEN. Laet $ABCD$ een water sijn, van form een lichamelick rechthouck, diens oppervlack AB is, ende eenighen bodem daer in EF , euewydich vanden sichteinder; Laet oock GE de hanghende lini sijn van t'plat duer t'waters oppervlack totten grondt EF , ende den pilaer begrepen onder den bodem EF en hoochde EG , sy $GHFE$.

T'BEGHEERDE. Wy moeten bewysen dat op den bodem EF , rust het ghewicht euen ande swaerheyt waters des pilaers $CHFE$. T'BEWYS. Soo op den bodem EF meer ghewicht rust dan des waters $GHFE$, dat sal moeten comen van wegghen t'neuenstaende water; Laet t'sijn soot mueghelick waer, van t'water $AGED$ ende $HBCF$; Maer dat soo ghenomen, daer sal op den bodem DE , van wegghen t'water $GHFE$, om dat de reden de selue is, oock meer ghewichts rusten dan des waters $AGED$; ende op den bodem FC , oock meer ghewichts dan des waters $HBCF$, ende veruolghens op den heelen bodem DC sal meer ghewichts rusten, dan des heelen waters $ABCD$, t'welck (ghemerckt $ABCD$ een lichamelick rechthouck is) ongheschickt waer. S'ghelijcx salmen oock bethoopen dat op den bodem EF niet min en rust dan t'water $GHFE$, daer rust dan nootfelick op t'ghewicht euen ande swaerheyt waters des pilaers $GHFE$.



I. VERVOLGH.

Laet ons nu in t'water $ABCD$ des 10^{en} voorstels, legghen een styf-lichaem $IKLM$, stoflichter dan water, dat is driuende op t'water, met t' deel $NOLM$ daer binnen, ende met t' deel $NOKI$ daer buyten, welker ghestalt dan sy als hier onder. Dit soo sijnde, t'styflichaem $IKLM$ is euewichtich metter water euegroot an $NOLM$ duer het 5^e voorstel, waer duer t'lichaem $IKLM$, met de rest des waters rondom hem, euewichtich is an een lichaem waters euegroot an $ABCD$: daerom segghen wy noch naer luyt des voorstels, dat teghend en bodem EF een ghe-

THEOREM VIII.

PROPOSITION X.

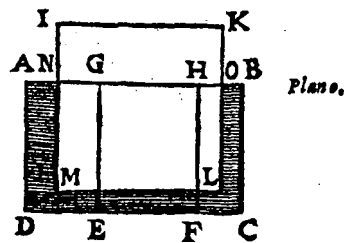
On any bottom of the water being parallel to the horizon there rests a weight equal to the gravity of the water the volume of which is equal to that of the prism whose base is that bottom and whose height is the vertical from the plane through the water's upper surface to the base.

SUPPOSITION. Let $ABCD$ be a water, whose form be a corporeal rectangle, whose upper surface be AB and a bottom therein EF , parallel to the horizon. Let also GE be the vertical from the plane through the water's upper surface to the bottom EF , and the prism comprehended by the bottom EF and the height EG shall be $GHFE$. WHAT IS REQUIRED TO PROVE. We have to prove that on the bottom EF there rests a weight equal to the gravity of the water of the prism $GHFE$. PROOF. If there rests on the bottom EF more weight than that of the water $GHFE$, this will have to be due to the water beside it. Let this, if it were possible, be due to the water $AGED$ and $HBCF$. But this being assumed, there will also rest on the bottom DE , owing to the water $GHFE$, because the reason is the same, more weight than that of the water $AGED$; and on the bottom FC also more weight than that of the water $HBCF$; and consequently on the entire bottom DC there will rest more weight than that of the whole water $ABCD$, which (in view of $ABCD$ being a corporeal rectangle) would be absurd. In the same way it can also be shown that on the bottom EF there does not rest less than the water $GHFE$. Therefore, on it there necessarily rests a weight equal to the gravity of the water of the prism $GHFE$.

COROLLARY I.

Let us now lay in the water $ABCD$ of the 10th proposition a solid body $IKLM$, of greater specific levity than water, i.e. floating on the water, with the part $NOLM$ within the water and with the part $NOKI$ above it, the position then being as shown below. This being so, the solid body $IKLM$ is of equal weight to the water having the same volume as $NOLM$, by the 5th proposition, owing to which the body $IKLM$, with the remainder of the water surrounding it, is of equal weight to a body of water having the same volume as $ABCD$. Therefore we still say, according to the proposition, that against the bottom EF there rests a weight

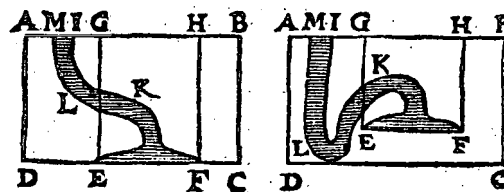
een ghewicht rust, euen ande swaerheyt waters die euegroot is met den pilaer, diens grondt E F is, ende hoochde de hanghende lini G E, van *t'plat A B, duer t'waters oppervlack A N totten grondt E F: Waer nyt blijktt dat eenighe drijende stof in t'water gheleyt, sy en verwaert noch en verlicht (welverstaende als t'water inde selfde hoochde blijft) den grondt niet.



II. VERVOLGH.

Laet andermael int water A B C D, legghen een styf lichaem, ofte verscheyden styf lichamen euestoffwaer metter water, ick neem alsoo, datter maer water en blijft als t'begrepen binnen I K F E L M; Twelck so sijnde, dese lichamen en beswaren noch en verlichten den grondt E F niet meer dan t'water eerst en dede: Daerom segghen wy noch naer luyt des voorstels, dat te-

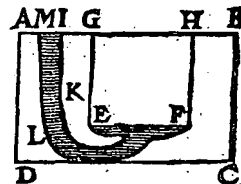
ghē den bodē E F, een ghewicht rust euen ande swaerheyt waters die euegroot is metten pilaer, wies grondt E F is, en hoochde de hanghende lini G E, van t'plat A B



duer t'waters oppervlack M I, totten grondt E F.

III. VERVOLGH.

Laet wederom A B C D t'eenemael water sijn, ende E F een bodem daer in euewydich vanden sichteindet. Twelck soo wesende, t'water onder den bodem E F, stoot euen soo stijf daer teghen opwaert, als t'water bouen den bodem E F, daer teghen neerwaert stoot: Want by aldient soo niet en waer, t'cranckste soude voor t'sterckste wycken, t'welck niet en ghebuert, want yder houdt sijn ghegheuen plaets duer het 1^e voorstel. Laet nu eenighe styf lichamen euestoffwaer metter water, alsoo gheleyt worden, dattet water I K E F L M, van onder anstoot teghen E F, als hier neuens. Dit soo sijnde, t'water onder den bodem E F, stoot nu so stijf teghen E F, dat is teghen t'styf lichaem,



Cc 3 alst te

equal to the gravity of the water having the same volume as the prism whose base is EF and whose height is the vertical GE , from the plane AB through the water's upper surface AN to the base EF . From which it appears that if any floating substance is laid in the water, it does not weight or lighten the bottom (provided the water remain at the same level).

COROLLARY II.

Let there again be put in the water $ABCD$ a solid body, or several solid bodies of equal specific gravity to the water. I take this to be done in such a way that the only water left is that enclosed by $IKFELM$. This being so, these bodies do not weight or lighten the base EF any more than the water first did. Therefore we still say, according to the proposition, that against the bottom EF there rests a weight equal to the gravity of the water having the same volume as the prism whose base is EF and whose height is the vertical GE , from the plane AB through the water's upper surface MI to the base EF .

COROLLARY III.

Let again $ABCD$ be a water, and EF a bottom therein, parallel to the horizon. This being so, the water below the bottom EF exerts an upward thrust against it as great as the downward thrust which the water above the bottom EF exerts against it. For if this were not so, the weakest would give way to the strongest, which does not happen, for each keeps its appointed place, by the 1st proposition. Now let a number of solid bodies of equal specific gravity to the water be laid therein in such a way that the water $IKFELM$ thrusts against EF from below, as in the figure opposite. This being so, the water below the bottom EF exerts the same thrust against EF , i.e. against the solid body, as it did before against the

alst te vooren teghen t'water dede; maer t'stac daer teghen soo stijf als t'bouenste teghen E F stiet, soo vooren gheseyt is, ende t'bouenste stiet teghen E F naer luydt deses voorstels, daerom t'onderste stoot oock teghen E F naer luydt deses voorstels, dat is soo wy bouen gheseyt hebben, dat teghen den bodem E F noch een ghewicht rust, euen ande swaerheyt waters die euegroot is metten pilaer, diens grondt E F is, ende hoochde de hanghende lini G E, van t'plat A B duer t'waters oppervlack M I totten grondt E F.

IIII VEROOLCH.

Laet ons nu de styfichamen des 2^{en} ende 3^{en} vervolghs tot haer plaets hechten, ende t'water uytghieten, ende daer sal een ledighe plaets I K F E L M bliuen, ende den grondt E F en sal gheen ghewicht draghen; waer uyt blijktt, dat met die cleyne ledighe plaets weder vol waters te ghieten, so salmen den grondt E F euen so seer beswaren; als of t'gheheele vat A B C D (de ingheleyde styfichamen gheweert sijnde) vol waters waer.

V VEROOLCH.

Maer anghesien de ingheleyde styfichamen des 2^{en} ende 3^{en} veruolghs t'haerder plaets ghehecht sijn, soo en gheeft noch en neemt haer uytste stof tot de bewaring ofte verlichting des grondts E F, daerom laet ons de stof der seluer rondtom afcorten, alsoo datter bliuen de inwendighe ongheschickte vormen oft vaten met water ghevult M I K F E L, als hier onder.



Ende sullen noch segghen naer luyt des voorstels, dat teghen den bodem E F een ghewicht rust, euen ande swaerheyt waters die euegroot is metten pilaer, wiens grondt E F is, ende hoochde de hanghende lini van t'plat duer t'waters oppervlack M I, totten grondt E F. Ende dit alsoo om de selue reden van alle ander vormen diens bodems in een plat sijn euewydich vanden sichteinder. T B E S L Y T. Op yder bodem dan des waters euewydich sijnde, &c.

Leest d'eruaringhen hier af breeder inden Anuang der Waterwichtdaet.

M E R C K T

water. But it exerted against the latter the same thrust as the upper part against EF , as has been said above, and the upper part exerted a thrust against EF according to the present proposition. Therefore the lower part also exerts a thrust against EF according to the present proposition, that is, as we have said above, that against the bottom EF there still rests a weight equal to the gravity of the water having the same volume as the prism whose base is EF and whose height is the vertical GE , from the plane AB through the water's upper surface MI to the base EF .

COROLLARY IV.

Let us now put the solid bodies of the 2nd and the 3rd corollary in their places, and pour out the water. Then there will be left an empty space $IKFELM$, and the base EF will not bear any weight; from which it is apparent that by pouring that small empty space full of water again, the base EF will be weighted as much as if the whole vessel $ABCD$ (the solid bodies laid therein being taken away) were full of water.

COROLLARY V.

But since the solid bodies of the 2nd and the 3rd corollaries are put in their places, their outward matter neither adds to nor subtracts from the weighting or lightening of the base EF . Therefore let us cut away the matter thereof all round, in such a way that the interior irregular forms or vessels filled with water, $MIKFEL$, are left, as shown below.

And we shall still say, according to the proposition, that against the bottom EF there rests a weight equal to the gravity of the water having the same volume as the prism, whose base is EF and whose height is the vertical from the plane through the water's upper surface MI to the base EF . And this applies for the same reason to any other forms whose bottoms are in a plane parallel to the horizon. **CONCLUSION.** On any bottom of the water therefore being parallel, etc.

Read the experiences hereof more amply in the Preamble of the Practice of Hydrostatics.

MERCKT. Wy fouden t'boveschreuen 10^e voorstel eyghentlicker aldus uytgheproken hebben:

OP yder bodem des waters in een weereltvlack sijnde, rust een ghewicht euen ande swaerheyt waters die euegroot is metter clootsdeel begrepen tusschen den bodem ende tweereltvlack duer t'waters hoochste punt, ende t'vlack tusschen die twee vlacken, beschreuen met de oneindelicke rechte lini vast in t'weerelts middelpunt, ende ghedraeyt duer des bodems * omtreck.

Circumferentiam.

Daer af bewysende sulcx als bouen bewesen is, maer om de redenen onder de 7^e begheerte verclaert, son ist beter ghelaten.

IX. VERTOOC.

XI. VOORSTEL.

W E S E N D E een gheschickt bodem diens hoochste punt in t'waters oppervlack is: T'ghewicht daer reghen rustende is euen anden helft des pilaers waters, diens grondt euen an dien bodem is, ende hoochde, de * hanghende lini van des bodems hoochste punt, tot het * plat euewydich vandē sichteinder duer des bodems leeghste punt.

*perpendicularis.
Plano.*

Horizonte.

1^e VOORBEELT.

T G H E G H E V E N. Laet A B een vat waters wesen, ende den bodem A C D E sy ten eersten een euewydich vierhouck, oneuewydich vanden sichteinder, daer op rechthouckich, diens hoochste sijde A C in t'waters oppervlack A C F G is, ende A E sy de hanghende lini van des bodems hoochste punt, tot het plat euewydich vanden sichteinder duer des bodems leeghste punt, dat is duer E D, ende A G sy so lanck alst valt. Laet oock de lini D B euewydich sijn vanden sichteinder, ende daer in gheteeckent H, alsoodat D H euen sy an D G, oock ghetrocken worden C H, ende met A C H D E sy beteeckent den helft des pilaers diens grondt A C D E, ende hoochde D H euen an A E.

T B E G H E E R D E. Wy moeten bewysen dattet ghewicht waters teghen den bodem A C D E rustende, euen is anden voornoemden haluen pilaer A C H D E; Dat is (om t'selue opentlicker te verclaren) ghenomen dat

men dat

NOTE.

It would have been more appropriate to have worded the above 10th proposition as follows:

On any bottom of the water being in a world surface there rests a weight equal to the gravity of the water having the same volume as the part of a sphere comprehended by the bottom in question and the world surface through the highest point of the water, and the surface between these two surfaces, described by the infinite straight line fixed in the centre of the world, and revolved through the circumference of the bottom in question.

We might prove this as above, but for the reasons explained in the 7th postulate it is better to omit it.

THEOREM IX.

PROPOSITION XI.

Given a regular bottom whose highest point is in the water's upper surface: the weight resting against it is equal to the half of the prism of water whose base is equal to that bottom and whose height is the vertical from the highest point of the bottom to the plane parallel to the horizon through the lowest point of the bottom.

EXAMPLE I.

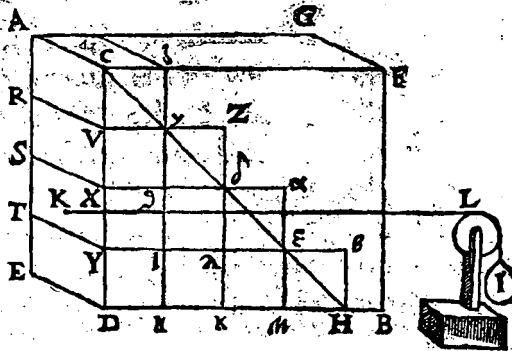
SUPPOSITION. Let AB be a vessel with water, and the bottom $ACDE$ shall first be a parallelogram¹⁾, not parallel to the horizon, but at right angles thereto, whose highest side AC is in the water's upper surface $ACFG$, and AE shall be the vertical from the highest point of the bottom to the plane parallel to the horizon through the lowest point of the bottom, i.e. through ED , and AG shall have any given length. Let also the line DB be parallel to the horizon²⁾, and let H be marked therein in such a way that DH shall be equal to DC , and let CH also be drawn, and $ACHDE$ shall denote the half of the prism whose base is $ACDE$ and whose height DH is equal to AE . WHAT IS REQUIRED TO PROVE. We have to prove that the weight of the water resting against the bottom $ACDE$ is equal to the aforesaid half prism $ACHDE$; i.e. (to explain it more clearly), assuming that I be an oblique weight³⁾ of equal gravity to $ACHDE$, whose drawing line KL is parallel to DH , and that K

¹⁾ The subsequent reasoning is valid only if $ACDE$ is a rectangle.

²⁾ and at right angles to AC .

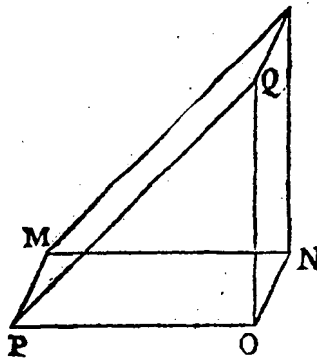
³⁾ oblique, because the drawing line KL is not vertical.

men dat I een scheefwicht sy, euefwaer met A C H D E, diens trecklini K L, euewydich is met D H, ende dat K swaerheys middelpunt sy vande macht des gheprangs vergaert indē bodē (wiens middelpunt vinding duer rvolghende 18^e voorstel bekenr wōrt) t'ghewicht I staet tegen t'gheprang des waters euewichtich, houdende dē bodem A C D E (ghenomen datse beueeghlick waer) in die standt.



Homologa.

Ofte tor meerder clærheyt, laet M N O P een bodem sijn, euen ende ghelick an A C D E, te weten de sijde M P * lijkstandighe met A C, ende M N met A E, op welken bodem M N O P, light een stijfichaem M N O P Q, euen, ghelijck, ende euefwaer met den haluen pilaer A C H D E, ende de lini Q O euen an D H, sy rechthouckich op den sichteinder. Ick seg dat alsulcken gheprang als dat stijfichaem M N O P Q, doet teghen den bodem M N O P, te weten meer pranghende naer N O dan naer M P, om datter aldaer dicker en swaerder is dan alhier, euen soodanighen gheprang doet t'water A B, oock teghen den bodem A C D E, meer pranghende naer E D dan naer A C. T's E R E Y T S E L. Laet de sijde A E ghedeelt worden in vier euen deelen, met de punten R, S, T, en daer uyt ghetrocken worden R V, S X, T Y, euewydighe met A C; Laet oock ghetrocken worden V Z, X α, Y ε euewydighe met D H, ende sniende C H inde punten γ, δ, ε, ende also, dat yder der linien γ Z, δ α, ε ε, euen sy an Y γ; Laet daernaer duer t'punt γ ghetrocken worden de lini ζ α, euewydighe met C D, sniende χ α in τ, ende Y β in ι, sghelijcx de lini Z κ duer δ, sniende Y β in λ, sghelijcx de lini α μ duer ε, ende ten laetsten ε H.



T's W Y S. Teghen den bodem A C V R rust meer ghewichts dan niet,

be centre of gravity of the force of the total pressure on the surface (the finding of whose centre of gravity becomes known from the 18th proposition hereinafter), the weight I is in equilibrium with the pressure of the water, keeping the bottom $ACDE$ (assuming it to be movable) in that position.

Or, to make it clearer, let $MNOP$ be a bottom, equal and similar to $ACDE$, i.e. the side MP being homologous to AC , and MN to AE , on which bottom $MNOP$ lies a solid body $MNOPQ$, equal, similar, and of equal gravity to the half prism $ACHDE$, and the line QO , equal to DH , shall be at right angles to the horizon. I say that the same pressure as that exerted by the solid body $MNOPQ$ against the bottom $MNOP$, to wit thrusting more heavily adjacent to NO than adjacent to MP , because it is thicker and heavier in the former place than in the latter, is also exerted by the water AB against the bottom $ACDE$, thrusting more heavily adjacent to ED than adjacent to AC . PRELIMINARY. Let the side AE be divided into four equal parts by the points R, S, T , and from these let there be drawn RV, SX, TY parallel to AC . Let there also be drawn $VZ, X\alpha, Y\beta$, parallel to DH and intersecting CH in the points γ, δ, ϵ and in such a way that each of the lines $\gamma Z, \delta\alpha, \epsilon\beta$ be equal to $V\gamma$. Thereafter let there be drawn through the point γ the line $\zeta\eta$, parallel to CD , intersecting $X\alpha$ in ϑ and $Y\beta$ in i . In the same way the line $Z\kappa$ through δ , intersecting $Y\beta$ in λ , in the same way the line $\alpha\mu$ through ϵ , and finally βH . PROOF. Against the bottom $ACVR$ there rests more weight than nothing, for if that bottom were in the

niet, want waer dien bodem in t'waters oppervlack, soo souder niet teghen rusten, maer sy comt nu leegher, daer rust dan meer teghen als niet: Ten anderen seg ick datter min teghen rust dan t'lichaem waters $AC\zeta\gamma VR$, want waer sy euewydich vanden sichteinder duer RV , so souder dat lichaem $AC\zeta\gamma VR$ teghen rusten, duer het 10^o voorstel, maer sy comt nu hoogher, daer rust dan min teghen. S'ghelijcx seg ick dat teghen den bodem RVS , meer ghewichts rust dan des lichaems $AC\zeta\gamma VR$, want waer dien bodem euewydich vanden sichteinder duer RV , daer soude dat lichaem teghen rusten duer het 10^o voorstel, maer sy comt nu leegher, daer rust dan meer teghen, maer t'lichaem $RV\gamma\delta XS$ is euen an t'lichaem $AC\zeta\gamma VR$, daerom teghen den bodem RVS , rust meer ghewichts dan des lichaems $RV\gamma\delta XS$. Ten anderen seg ick datter min teghen rust dan t'lichaem $AC\zeta\delta XS$, want waer dien bodem euewydich vanden sichteinder duer SX , soo souder dat lichaem $AC\zeta\delta XS$ teghen rusten duer het 10^o voorstel, maer sy comt nu hoogher, daer rust dan min teghen, maer t'lichaem $RVZ\delta XS$ is euen an t'lichaem $AC\zeta\delta XS$, daerom rust teghen den bodem RVS min als t'lichaem $RVZ\delta XS$. S'ghelijcx seg ick dat teghen den bodem $SXYT$ meer ghewichts rust dan des lichaems $AC\zeta\delta XS$, want waer dien bodem euewydich vanden sichteinder duer SX , daer soude dat lichaem teghen rusten duer het 10^o voorstel, maer sy comt nu leegher, daer rust dan meer teghen, maer t'lichaem $SX\lambda YT$ is euen an t'lichaem $AC\zeta\delta XS$, daerom teghen den bodem $SXYT$ rust meer ghewichts dan des lichaems $SX\lambda YT$. Ten anderen seg ick datter min teghen rust dan t'lichaem $AC\zeta\gamma YT$, want waer dien bodem euewydich vanden sichteinder duer TY , soo souder dat lichaem $AC\zeta\gamma YT$ teghen rusten, duer het 10^o voorstel, maer sy comt nu hoogher, daer rust dan min teghen, maer t'lichaem $SX\alpha YT$ is euen an t'lichaem $AC\zeta\gamma YT$, daerom rust teghen den bodem $SXYT$ min als t'lichaem $SX\alpha YT$. S'ghelijcx seg ick dat teghen den bodem $TYDE$, meer ghewichts rust dan des lichaems $AC\zeta\gamma YT$, want waer dien bodem euewydich vanden sichteinder duer TY , daer soude dat lichaem teghen rusten duer het 10^o voorstel, maer sy comt nu leegher, daer rust dan meer teghen, maer t'lichaem $TY\mu DE$ is euen an t'lichaem $AC\zeta\gamma YT$, daerom teghen den bodem $TYDE$ rust meer ghewichts dan des lichaems $TY\mu DE$. Ten anderen seg ick datter min teghen rust dan t'lichaem $AC\zeta\mu DE$, want waer dien bodem euewydich vanden sichteinder duer ED , so souder dat lichaem $AC\zeta\mu DE$ teghen rusten, duer het 10^o voorstel, maer sy comt nu hoogher, daer rust dan min teghen, maer t'lichaem $TY\epsilon HDE$, is euen an t'lichaem $AC\zeta\mu DE$, daerom rust teghen den bodem $TYDE$ min als t'lichaem $TY\epsilon HDE$. Nu anghesien als vooren bewesen is, dat teghen den bodem $ACVR$ meer rust dan niet, ende te-

D d

ghen

water's upper surface, nothing would rest against it; but it comes lower, so there rests more than nothing against it. On the other hand I say that there rests against it less than the body of water $AC\zeta\gamma VR$, for if it were parallel to the horizon through RV , the body $AC\zeta\gamma VR$ would rest against it, by the 10th proposition; but it comes higher, so there rests less against it. In the same way I say that there rests against the bottom $RVXS$ more weight than that of the body $AC\zeta\gamma VR$, for if that bottom were parallel to the horizon through RV , that body would rest against it by the 10th proposition; but it comes lower, so there rests more against it. But the body $RV\gamma\delta XS$ is equal to the body $AC\zeta\gamma VR$, so there rests against the bottom $RVXS$ more weight than that of the body $RV\gamma\delta XS$. On the other hand I say that there rests against it less than the body $AC\zeta\delta XS$, for if that bottom were parallel to the horizon through SX , that body $AC\zeta\delta XS$ would rest against it by the 10th proposition; but it comes higher, so there rests less against it. But the body $RV\delta XS$ is equal to the body $AC\zeta\delta XS$, so there rests against the bottom $RVXS$ less than the body $RV\delta XS$. In the same way I say that there rests against the bottom $SXYT$ more weight than that of the body $AC\zeta\delta XS$, for if that bottom were parallel to the horizon through SX , that body would rest against it by the 10th proposition. But it comes lower, so there rests more against it. But the body $SX\delta\lambda YT$ is equal to the body $AC\zeta\delta XS$, so there rests against the bottom $SXYT$ more weight than that of the body $SX\delta\lambda YT$. On the other hand I say that there rests less against it than the body $AC\zeta\lambda YT$, for if that bottom were parallel to the horizon through TY , that body $AC\zeta\lambda YT$ would rest against it, by the 10th proposition. But it comes higher, so there rests less against it. But the body $SX\alpha\epsilon YT$ is equal to the body $AC\zeta\lambda YT$, so there rests against the bottom $SXYT$ less than the body $SX\alpha\epsilon YT$. In the same way I say that there rests against the bottom $TYDE$ more weight than that of the body $AC\zeta\lambda YT$, for if that bottom were parallel to the horizon through TY , that body would rest against it, by the 10th proposition. But it comes lower, so there rests more against it. But the body $TY\epsilon\mu DE$ is equal to the body $AC\zeta\lambda YT$, so there rests against the bottom $TYDE$ more weight than that of the body $TY\epsilon\mu DE$. On the other hand I say that there rests less against it than the body $AC\zeta\eta DE$, for if that bottom were parallel to the horizon through ED , that body $AC\zeta\eta DE$ would rest against it, by the 10th proposition. But it comes higher, so there rests less against it. But the body $TY\beta HDE$ is equal to the body $AC\zeta\eta DE$, so there rests against the bottom $TYDE$ less than the body $TY\beta HDE$. Now since, as has been proved above, against the bottom $ACVR$ there rests more than nothing, and against the bottom $RVXS$ more than the body

ghen den bodem $R V X S$ meer als t'lichaem $R V \gamma \delta X S$, ende teghen den bodem $S X Y T$ meer dan t'lichaem $S X \delta \lambda Y T$, ende teghen den bodem $T Y D E$ meer als t'lichaem $T Y \epsilon \mu D E$, soo rust teghen den heelen bodem $A C D E$ meer dan t'ghewicht van alle die lichamen t'samen, t'welck is t'binneschreuen lichaem $R V \gamma \delta \delta \lambda \epsilon \mu D E$ inden haluen pilaer $A C H D E$: Tis oock bewesen dat teghen den bodem $A C V R$ min rust dan t'lichaem $A C \zeta \gamma V R$, ende teghen den bodem $R V X S$ min als t'lichaem $R V Z \delta X S$, ende teghen den bodem $S X Y T$ min dan t'lichaem $S X \alpha \epsilon Y T$, ende teghen den bodem $T Y D E$ min als t'lichaem $T Y \epsilon H D E$, daerom rust teghen den heelen bodem $A C D E$ min dan t'ghewicht van alle die lichamen t'samen, dat is t'omschreuen lichaem $A C \zeta \gamma Z \delta \alpha \epsilon H D E$. Maer datmen nu den bodem $A C D E$ welcke hier bouen ghedeelt is in vier euen deelen, alsoo deelde in acht euen deelen, tis kennelick dat het binneschreuen lichaem inden haluen pilaer $A C H D E$, ende het omschreuen, alsdan van dien haluen pilaer maer den helft soo veel verschillen soudens als sy nu doen, tis dan openbaer duer sulcke oneindelicke deeling des bodems, datter gheen ghewicht soo cleen ghegheuen en can worden, oft men sal bethoonen dattet verschil (foder eenich waer) des ghewichts teghen den bodem $A C D E$ rustende, tot het ghewicht des haluen pilaers $A C H D E$ noch minder is, waer uyt ick aldus strije:

- A. *Alle swaerheyt die min verschilt van t'ghewicht teghen den bodem $A C D E$ rustende dan ghegheuen can worden, is euen mettet ghewicht teghen den bodem $A C D E$ rustende;*
- I. *T'ghewicht des haluen pilaers $A C H D E$, is een swaerheyt die min verschilt van t'ghewicht teghen den bodem $A C D E$ rustende dan ghegheuen can worden;*
- L. *T'ghewicht dan des haluen pilaers $A C H D E$, is euen mettet ghewicht teghen den bodem $A C D E$ rustende.*

11^e VOORBEELT.

T'GHEGHEVEN. Laet $A B$ andermael een vat waters wesen, ende den bodem $A C D E$ sy een euewydich vierhouck des selfden, oneuewydich vanden sichteinder, ende daerop scheefhouckich, diens hoochste sijde $A C$ in t'waters oppervlack $A C F G$ is; T'selue water ende bodem sy alsoo ghedeelt ende gheteekent als t'water des 1^{en} voorbeelts, ende $A \gamma$ sy hanghende lini van des bodems hoochste sijde, tot het plat euewydich vanden sichteinder duer des bodems leegste sijde $E D$.

T'BEGHEERDE. Wy moeten bewysen dattet ghewicht waters teghen den bodem $A C D E$ rustende, euen is anden helft des pilaers diens bodem $A C D E$, ende hoochde $A \gamma$.

T'BERBYT-

$RV\gamma\theta XS$, and against the bottom $SXYT$ more than the body $SX\delta\lambda YT$, and against the bottom $TYDE$ more than the body $TY\epsilon\mu DE$, there rests against the whole bottom $ACDE$ more than the weight of all those bodies together, which is the inscribed body $RV\gamma\theta\delta\lambda\epsilon\mu DE$ in the half prism $ACHDE$. It has also been proved that there rests against the bottom $ACVR$ less than the body $AC\zeta\gamma VR$, and against the bottom $RVXS$ less than the body $RVZ\delta XS$, and against the bottom $SXYT$ less than the body $SX\alpha\epsilon YT$, and against the bottom $TYDE$ less than the body $TY\beta\eta DE$. So there rests against the whole bottom $ACDE$ less than the weight of all those bodies together, i.e. the circumscribed body $AC\zeta\gamma Z\delta\alpha\epsilon\beta\eta DE$. But if the bottom $ACDE$, which is divided above into four equal parts, were thus divided into eight equal parts, it is evident that the inscribed body in the half prism $ACHDE$, and the circumscribed body, would differ from that half prism by only half as much as they do now. It is therefore manifest ¹⁾, through this infinite division of the bottom, that no weight so small can be given but it can be shown that the difference (if there were any) between the weight resting against the bottom $ACDE$ and the weight of the half prism $ACHDE$ is even less, from which I argue as follows: ²⁾

- A. *Any gravity which differs less from the weight resting against the bottom $ACDE$ than can be given is equal to the weight resting against the bottom $ACDE$;*
- I. *The weight of the half prism $ACHDE$ is a gravity which differs less from the weight resting against the bottom $ACDE$ than can be given;*
- I. *Therefore the weight of the half prism $ACHDE$ is equal to the weight resting against the bottom $ACDE$.*

EXAMPLE II.

SUPPOSITION. Let AB again be a vessel with water, and the bottom $ACDE$ shall be a parallelogram ³⁾ therein, not parallel to the horizon, and at oblique angles thereto, whose highest side AC is in the water's upper surface $ACFG$. This water and bottom shall be divided and marked in the same way as the water of the 1st example, and $A\gamma$ shall be the vertical from the highest side of the bottom to the plane parallel to the horizon through the lowest side ED of the bottom. WHAT IS REQUIRED TO PROVE. We have to prove that the weight of the water resting against the bottom $ACDE$ is equal to the half of the prism, whose base is $ACDE$ and whose height is $A\gamma$. PRELIMINARY. Let the side $A\gamma$ be divided

¹⁾ Euclid X 1, porism.

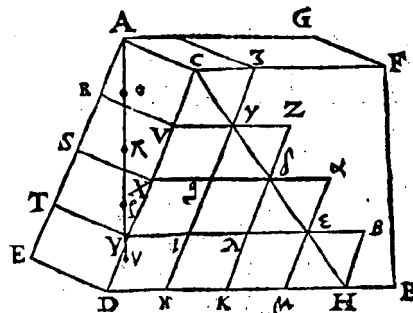
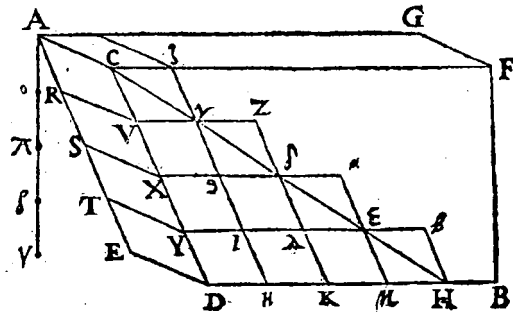
²⁾ See note 2 to p. 143.

³⁾ Again the parallelogram is supposed to be a rectangle.

T B E R E Y T S E L. Laet de sijde A γ ghedeelt worden in vier euen deelen met de punten α, π, ρ . T B E W Y S. Teghen den bodem A C V R, rust meer ghe-

rust meer ghe-
wichts dā niet,
want waer dier
bodem in rwa-
ters oppervlac,
soo fonder niet
teghen rusten,
maer sy comt
nu leeger, daer
rust dan meer
teghen als niet:
Ten anderen
seg ick datter
min teghē rust

dan dē pilaer diens gront
A C V R is, ende hooch-
de A o, want waer sy eue-
wydich vanden sichtein-
der duer R V, soo souder
dien pilaer teghen rusten
duer het 10^e voorstel, maer
sy comt nu hoogher, daer
rust dan min teghen, maer
A C ζ γ V R. is euen an
dien pilaer, daerom teg-
ghen dē bodem A C V R
rust min ghewicht dan
des pilaers A C ζ γ V R.
euen soo sy in t eerste voor-
den dattet ghewicht teghe-
r lichaem A C H D E, maer
diens bodem A C D E, en-
ghende van H rechtthouck-
wicht teghen den bodem A
laers waters diens grondt e



31.V.11.B.E

III. VOORBEELT.

T'GHEGHEVEN. Laet A B eenich gheschickt bodem sijn; Ick neem een *scheefrondt, diens hoochste punt A in t'waters oppervlack *Ellipf.* is, ende B sy rleeghste punt, ende A C de hanghende lini van t'hoochste Dd 2 punt.

into four equal parts by the points σ , π , ρ . PROOF. Against the bottom $ACVR$ there rests more weight than nothing, for if that bottom were in the water's upper surface, nothing would rest against it. But it comes lower, so there rests more than nothing against it. On the other hand I say that there rests less against it than the prism whose base is $ACVR$ and whose height is $A\sigma$, for if it were parallel to the horizon through RV , that prism would rest against it, by the 10th proposition. But it comes higher, so there rests less against it. But $AC\xi\gamma VR$ is equal to that prism, so there rests against the bottom $ACVR$ less weight than that of the prism $AC\xi\gamma VR$. In the same way all the rest shall be proved just as it was in the first example, from which it shall be concluded that the weight resting against the bottom $ACDE$ is equal to the body $ACHDE$. But that body is equal to the half of the prism whose base is $ACDE$ and whose height is $A\nu$ (for $A\nu$ is equal to the vertical ¹⁾ from H at right angles to the plane through $ACDE$). Therefore the weight resting against the bottom $ACDE$ is equal to the half of the prism of water whose base is equal to $ACDE$ and whose height is $A\nu$.

EXAMPLE III.

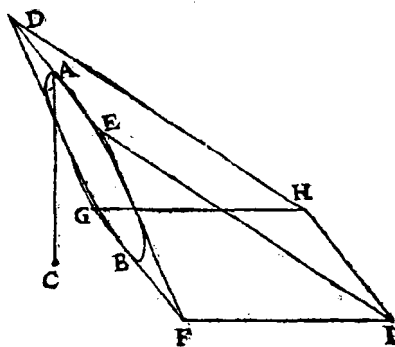
SUPPOSITION. Let AB be any regular bottom. I take it to be an ellipse, whose highest point A is in the water's upper surface, and B shall be the lowest point; and AC the vertical from the highest point A to the plane parallel to the horizon

¹⁾ The use of the term *vertical* (hanghende) is perhaps to be explained by considering $ACDE$ as horizontal.

punt A, tot het plateuewydich vanden sichteinder duer B.

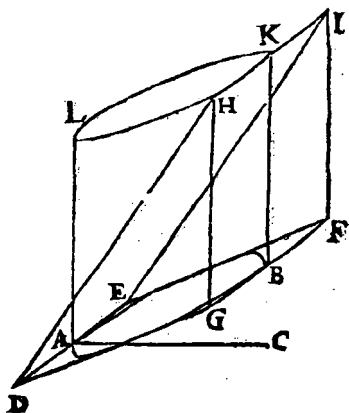
T'BEGHEERDE. Wy moeten bewyſen dattet ghewicht waters teghen den bodem AB ruſtende, euen is anden helft des pilaers dieos grondt den bodem AB is, ende hoochde AC.

T'BEREYTSSEL. Laet ghetrocken ſijn een euewydich vierhouck DEFG, in wiens plat begrepē ſy t'scheefrondt AB, alſoo dat DE in t'waters oppervlack ſijnde, naecke an t'punt A, ende dat GF naecke an t'punt B; Laet daernaer ghetrocken worden FI euen an AF ende rechthouckich op FG maer euewydich vanden ſichteinde, rende uyt GF ende FI ſy beſchreuen den rechthouck Fghi, voorts de linien E I ende D H.



Laet daer naer een ander form gheſtelt ſijn euen, ghelijck, ende eueſwaer met de voorgaende, maer alſoo dat FI rechthouckich ſy op den ſichteinder ghelijck hier neuen. Ende laet in deſe tweede form t'lichaem DEFGHI een ſtijflichaem weſen, ruſtende op den bodem DEFG.

T'BEWYS. Alſulcken druckſel als t'ſtijflichaem DEFGHI der tweeder form, veroirſaect teghen den bodem DEFG, euen ſoodanighen druckſel veroirſaect het water des eerſten forms teghen ſijn bodem DEFG, ſoo bouen bewezen is; ende veruolghens alſulcken druckſel alſſer valt teghen t'scheefrondt AB der tweede form, euen ſoodanighen druckſel valter oock teghen t'scheefrondt AB der eerſte form, maer het druckſel op t'scheefrondt der tweede form is den helft des pilaers (ſoo wy hier onder verclaren ſullen) diens grondt dat ſcheefrondt is, ende hoochde euen an AC (want ghetrocken een hanghende lini van K rechthouckich op t'plat duer t'scheefrondt AB, ſy is euen



through B . WHAT IS REQUIRED TO PROVE. We have to prove that the weight of the water resting against the bottom AB is equal to the half of the prism whose base is the bottom AB and whose height is AC . PRELIMINARY. Let there be drawn a parallelogram ¹⁾ $DEFG$, in whose plane shall be comprehended the ellipse AB in such a way that DE , being in the water's upper surface, shall touch at the point A and GF shall touch at the point B . Thereafter let there be drawn FI , equal to EF and at right angles to FG , but parallel to the horizon, and from GF and FI there shall be drawn the rectangle $FGHI$, further the lines EI and DH .

Thereafter let there be drawn another figure, equal, similar, and of equal gravity to the preceding one, but such that FI shall be at right angles to the horizon, as shown opposite. And in this second form let the body $DEFGHI$ be a solid body resting on the bottom $DEFG$. PROOF. The same pressure as is exerted by the solid body $DEFGHI$ of the second figure against the bottom $DEFG$ is exerted by the water of the first figure against its bottom $DEFG$, as has been proved above, and consequently the same pressure as is exerted against the ellipse AB of the second figure is also exerted against the ellipse AB of the first figure. But the pressure on the ellipse of the second figure is the half of the prism (as we shall explain below) whose base is that ellipse and whose height is equal to AC (for if a vertical ²⁾ be drawn from K at right angles to the plane through

¹⁾ rectangle.

²⁾ See note 1 to p. 429.

euen an A C) daerom het druckfel des waters teghen t'scheefrondt A B der eerste form, is euen anden helft des pilaers wiens grondt dat scheefrondt is, ende hoochde A C.

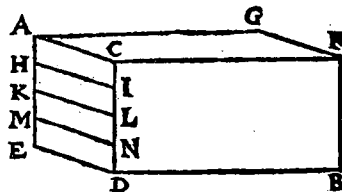
Maer dattet ghewicht rustende in dese tweede form teghen t'scheefrondt A B, euen is anden helft des pilaers diens grondt dat scheefrondt is, ende hoochde euen an A C, wort aldus bethoont: Laet ghetrocken worden de lini B K euen ende euewydighe met F I; Laet nu het onderste B der schuer lini B K, ghedraeyt worden inden omtreck des scheefrondts A B, tot dat sy weder ter plaets comt daer sy begon te roeren, ende blijuende int roeren altydt euewydich van F I, de selue sal tusschen de twee bodems een pilaer A B K L beschrijven, welke mettet plat D E I H, ghesneen wort duer twee lijkstandighe punten A, K, schuens teghen ouer malcander staende inde omtrecken der bodems; Maer alle pilaer diens grondt een gheschickt bodem sijnde, ghesneen wort met een plat duer twee lijkstandighe punten inde omtrecken der bodems schuens teghen ouer malcander staende, die pilaer wort van dat plat in twee euen deelen ghedeelt, daerom het deel diens pilaers onder t'plat D E I H, is den helft des heelen pilaers A B K L rustende op t'scheefrondt A B, maer dat den pilaer A B K L euen is anden pilaer diens grondt A B ende hoochde A C, blijkt daer an, dat sijn hoochde euen is an A C, daerom t'ghewicht rustende teghen t'scheefrondt A B, is euen anden helft des pilaers diens grondt dat scheefrondt is, ende hoochde euen an A C.

IIII^e VOORBEELT.

Wy hebben hier bouen drie voorbeelden ghegheuen. met * Wiscon- *Mathemati-*
stich bewys, t'welck, wel is waer, den grondt volcommentliker verclaert *ca demon-*
als ander; doch anghesien t'bewys duer ghetalen tot opentlicker kennis *strations.*
van alles niet en verachtet, sullen dit 4^e voorbeelt duer ghetalen stellen.

T G H E G H E V E N. Laet A B een vat waters sijn, diens bodem wy nemen te wesen een rechthouckich vierhouck, rechthouckich op den sichteinder, ende d'hoochste sijde A C doende een voet, sy int waters oppervlack A C F G, ende A E doe oock een voet, maer A G sy soolanck alst valt.

T B E G H E E R D E. Wy moeten duer ghetalen bewyfen, dattet ghewicht waters rustende teghen den bodem A C D E, euen is anden helft des pilaers waters, wiens grondt euen is an dien bodem, ende hoochde de hanghende lini A E: Maer dien pilaer is een teerlinck doende een



voet, wy moeten dan bethoonen dat teghen den bodem A C D E rust
D d 3 het ghe-

the ellipse AB , it is equal to AC). Therefore the pressure of the water against the ellipse AB of the first figure is equal to the half of the prism whose base is that ellipse and whose height is AC .

But that the weight resting in this second figure against the ellipse AB is equal to the half of the prism whose base is that ellipse and whose height is equal to AC , is shown as follows: Let there be drawn the line BK , equal and parallel to FI . Now let the lowest point B of this line BK be revolved in the circumference of the ellipse AB until it reaches again the place from which its motion started, and if during the motion it always remain parallel to FI , it will describe between the two surfaces a prism $ABKL$, which is cut by the plane $DEIH$ in two homologous points A, K , diametrically opposite to each other in the circumferences of the bases. But if any prism whose base is a regular surface is cut by a plane in two homologous points in the circumferences of the bases, diametrically opposite to each other, it is divided by that plane into two equal parts. Therefore the part of that prism below the plane $DEIH$ is the half of the whole prism $ABKL$ resting on the ellipse AB . But that the prism $ABKL$ is equal to the prism whose base is AB and whose height is AC , is evident from the fact that its height is equal to AC . Therefore the weight resting against the ellipse AB is equal to the half of the prism whose base is that ellipse and whose height is equal to AC .

EXAMPLE IV.

We have given above three examples with a mathematical proof, which indeed explains the cause more perfectly than any other, but since there is no harm in giving the proof by means of numbers in order to make everything clearer, we shall give this 4th example by means of numbers.

SUPPOSITION. Let AB be a vessel of water, the bottom of which we take to be a right-angled quadrilateral, at right angles to the horizon, and the highest side AC , being one foot, shall be in the water's upper surface $ACFG$, and AE shall also be one foot, but AG shall have any length. **WHAT IS REQUIRED TO PROVE.** We have to prove by means of numbers that the weight of the water resting against the bottom $ACDE$ is equal to the half of the prism of water whose base is equal to that bottom and whose height is the vertical AE . But that prism is a cube of one foot. We therefore have to show that against the bottom

het ghewicht van een halve voet waters. **TBEREYTSSEL.** Laet duer den bodem ghetrocken worden drie euewydighe linien met A C als H I, K L, M N, alsoo dat A H euen sy an H K, ende an K M, ende an M E.

TBEWYS. Tis blijkelyck dat op den bodem A I meer rust dan o, want alwaer sulcken bodem duer A C euewydich vanden sichteinder, so souder o, op rusten, maer sy comt nu leegher daer rust dan meer op als o. Ten anderen seg ick datter min op rust dan $\frac{1}{16}$, voets, want al waer sulcken bodem duer H I euewydich vanden sichteinder, soo souder $\frac{1}{16}$ voets op rusten, maer sy comt nu hoogher, daer rust dan min op als $\frac{1}{16}$, ende om der ghelijcke reden ist oock openbaer, dat op den bodem H L meer rust dan $\frac{1}{16}$, ende min als $\frac{2}{16}$, ende op den bodem K N meer dan $\frac{2}{16}$, ende min als $\frac{3}{16}$; maer op den bodem M D meer dan $\frac{3}{16}$ ende min als $\frac{4}{16}$. Nu dan vergaert de vier ghewichten (ghenomen dat o ghewicht waer) die lichter sijn d'ander op elcken bodem rust, als o. $\frac{1}{16}$. $\frac{2}{16}$. $\frac{3}{16}$. maken t'samen $\frac{6}{16}$. Inghelijcx vergaert de vier ghewichten die swaerder sijn d'ander op elcken bodem rust, als $\frac{1}{16}$. $\frac{2}{16}$. $\frac{3}{16}$. $\frac{4}{16}$. maken t'samen $\frac{10}{16}$. Tis dan openbaer dat op den heelen bodem A C D E meer rust dan $\frac{6}{16}$ voets, ende min als $\frac{10}{16}$ voets, tusschen welke twee den $\frac{1}{2}$ voet is, die wy noch bewysen moeten op den bodem A C D E te rusten.

Nu ghelijck den bodem hier bouen duer de drie euewydighe linien ghedeelt is in vieren, alsoe muegghen wyse deelen in soo veel deelen alst ons beliest, later sijn in thienen, en om de voorgaende redenen, de thien ghewichten die lichter sijn d'ander op elcken bodem rust, sullen sijn o. $\frac{1}{100}$. $\frac{2}{100}$. $\frac{3}{100}$. $\frac{4}{100}$. $\frac{5}{100}$. $\frac{6}{100}$. $\frac{7}{100}$. $\frac{8}{100}$. $\frac{9}{100}$. t'samen $\frac{45}{100}$. Inghelijcx de thien ghewichten die swaerder sijn d'ander op elcken bodem rust, als $\frac{1}{100}$. $\frac{2}{100}$. $\frac{3}{100}$. $\frac{4}{100}$. $\frac{5}{100}$. $\frac{6}{100}$. $\frac{7}{100}$. $\frac{8}{100}$. $\frac{9}{100}$. $\frac{10}{100}$. maken t'samen $\frac{55}{100}$; tis dan kennelick dat op den bodem A C D E meer rust dan $\frac{45}{100}$ voets, en min als $\frac{55}{100}$ voets, tusschen welke twee den haluen voet is die wy noch bewysen moeten op den bodem A C D E te rusten: Maer dese twee *pa-
Termini. len sijn naerder den haluen voet dan d'eerste twee, want min verschilt $\frac{45}{100}$ van $\frac{1}{2}$, dan $\frac{55}{100}$, alsoo oock verschilt $\frac{45}{100}$ min van $\frac{1}{2}$, dan $\frac{55}{100}$; Waer uyt blijkt dat hoe wy den bodem A C D E in meer sulcke euen deelen snien, hoe dat wy den haluen voet altijd naerder commen.

Twelck so verstaen sijnde, laet op den bodem A C D E min of meer rusten $\frac{1}{1000}$ voets (waert mueghelick) dan een halve voet, ende laet ons de waerheyt daer af onder soucken, de elende den boden duer de ghedacht in 1000 euen deelen alsvooren. Ende om de voorgaende redenen, de duyfent ghewichten die lichter sijn d'ander op elcken bodem rust, sullen sijn o. $\frac{1}{1000000}$. $\frac{2}{1000000}$. ende so voorts tot het laetste, dat sijn sal van $\frac{999}{1000000}$, alle welke ghetalen t'samen, sullen maken (wiens corte manier om te vergaren wy hier onder verhalen sullen) $\frac{499500}{1000000}$. Sghelijcx de duyfent ghewichten die swaerder sijn d'ander op elcken bodem rust als

ACDE there rests the weight of half a foot of water. PRELIMINARY. Let there be drawn through the bottom three lines parallel to *AC*, as *HI*, *KL*, *MN*, in such a way that *AH* be equal to *HK*, and to *KM*, and to *ME*. PROOF. It is evident that on the bottom *AI* there rests more than 0, for if this bottom were parallel to the horizon through *AC*, 0 would rest thereon; but it comes lower, so there rests more than 0 on it. On the other hand I say that there rests on it less than $\frac{1}{16}$ foot, for if this bottom were parallel to the horizon through *HI*, $\frac{1}{16}$ foot would rest on it. But it comes higher, so there rests less than $\frac{1}{16}$ on it. And for similar reasons it is also manifest that on the bottom *HL* there rests more than $\frac{1}{16}$ and less than $\frac{2}{16}$, and on the bottom *KN* more than $\frac{2}{16}$ and less than $\frac{3}{16}$; but on the bottom *MD* more than $\frac{3}{16}$ and less than $\frac{4}{16}$. Now adding together the four weights (assuming 0 to be a weight) which are lighter than what rests on each bottom, viz. 0, $\frac{1}{16}$, $\frac{2}{16}$, $\frac{3}{16}$, this makes $\frac{6}{16}$. Adding together likewise the four weights which are heavier than what rests on each bottom, viz. $\frac{1}{16}$, $\frac{2}{16}$, $\frac{3}{16}$, $\frac{4}{16}$, this makes $\frac{10}{16}$. It is therefore manifest that on the whole bottom *ACDE* there rests more than $\frac{6}{16}$ foot, and less than $\frac{10}{16}$ foot, between which two is the $\frac{1}{2}$ foot which we still have to prove rests on the bottom *ACDE*.

Now just as the bottom above is divided into four parts by the three parallel lines, we may divide it into as many parts as we like, say into ten parts. Then, for the above reasons, the ten weights which are lighter than what rests on each bottom will be 0, $\frac{1}{100}$, $\frac{2}{100}$, $\frac{3}{100}$, $\frac{4}{100}$, $\frac{5}{100}$, $\frac{6}{100}$, $\frac{7}{100}$, $\frac{8}{100}$, $\frac{9}{100}$, together $\frac{45}{100}$. In the same way, the ten weights which are heavier than what rests on each bottom, viz. $\frac{1}{100}$, $\frac{2}{100}$, $\frac{3}{100}$, $\frac{4}{100}$, $\frac{5}{100}$, $\frac{6}{100}$, $\frac{7}{100}$, $\frac{8}{100}$, $\frac{9}{100}$, $\frac{10}{100}$, making together $\frac{55}{100}$. It is therefore evident that on the bottom *ACDE* there rests more than $\frac{45}{100}$ foot and less than $\frac{55}{100}$ foot, between which two is the half foot which we still have to prove rests on the bottom *ACDE*. But these two terms are nearer to the half foot than the first two, for $\frac{45}{100}$ differs less from $\frac{1}{2}$ than $\frac{6}{16}$, and likewise $\frac{55}{100}$ differs less from $\frac{1}{2}$ than $\frac{10}{16}$. From which it is evident that the greater the number of such equal parts into which we divide the bottom *ACDE*, the more we shall approximate to the half foot.

This being grasped, let there rest on the bottom *ACDE* $\frac{1}{1000}$ foot (if this were possible) less or more than one half foot, and let us examine the truth thereof, dividing the bottom in thought into 1000 equal parts as before. Then, for the above reasons, the thousand weights which are lighter than what rests on each bottom will be 0, $\frac{1}{1,000,000}$, $\frac{2}{1,000,000}$, and so on to the last, which will be $\frac{999}{1,000,000}$, all of which numbers together will make (the short method for adding up such numbers is to be related below) $\frac{499,500}{1,000,000}$. In the same way the thousand weights

als $\frac{3}{1000000}$, $\frac{2}{1000000}$, $\frac{1}{1000000}$, ende so voorts tot het laetste, dat sijn sal van $\frac{1}{1000000}$, maken t'samen $\frac{3000000}{1000000}$, daer rust dan meer op den bodē als $\frac{3000000}{1000000}$ voets, ende min dan $\frac{3000000}{1000000}$ voets; Maer $\frac{499000}{1000000}$ en is maer $\frac{1}{1000}$ minder dan $\frac{1}{2}$, daer en rust dan gheen $\frac{1}{1000}$ voets min op den bodem dan $\frac{1}{2}$ voet. Alsoo en is $\frac{1000000}{1000000}$ maer $\frac{1}{2000}$ meerder dan $\frac{1}{2}$, daer en rust dan gheen $\frac{1}{2000}$ meer op den bodem dan $\frac{1}{2}$ voet. Ende alsoo salmen diergheijcke bethoonen over alle ghestelt deel hoe cleen het sy. Het blijktt dan, dat het verschil (sooder eenich waer) tusschen t'water op den bodem A C D E rustende, ende een halue voet waters, minder soude moeten sijn dan mueghelick is ghestelt te worden, waer uyt ick aldus strije:

- A. *Neuen yder ghewicht dat met een halue voet Waters verschil heeft, can een ghewicht ghestelt worden daer af min verschillende;*
- O. *Neuen t'ghewicht Waters op den bodem A C D E rustende, en can gheen ghewicht ghestelt worden van een halue voet Waters min verschillende;*
- O. *T'ghewicht Waters dan op den bodem A C D E rustende, een heeft met een halue voet Waters gheen verschil.*

TRESLVYT. Wefende dan een gheschied bodem diens hoochste punt int waters, &c.

DE reden waerom het half hier bouen, altijd blijft tusschen de twee ghetalen, welcke an het half oneindelick naederen, maer nummermeer daer toe en gheraken, is begrepen in sulcken * vertooch:

Theorema.

W E S E N D E een voortganck van ghetalen malcanderen in eenheydt te bouen gaende, ende beghinnende vande eenheydt. Den helft des viercants van t'laetste, is meerder dan de somme van al de ghetalen, maer minder dan de somme van al de ghetalen min t'laetste.

MAER, om te verclaren (als bouen beloof is) de manier om duer cortheyt te vergaren die groote menichte der ghetalen; Soo is ten eersten kennelick, dat haer noemers al euen sijn, waerduer wy alleene-lick op der ghetalen telders te letten hebben, de selue sijn in oirdenlicke * voortganck beghinnende vande * eenheydt, ende met eenheydt malcanderen te bouengaende, daerom vermenichvuldicht t'laetste met sijn helft, ende an t'uytbreng noch ghevoucht sijn helft, gheeft de begheerde somme. By voorbeelt ick wil weten hoe veel de somme is van 1, 2, 3, 4, 5, 6; Ick seg 6 mael 3 is 18, met 3 maeckt 21 voor de begheerde somme

*Progressione.
Vniuers.*

which are heavier than what rests on each bottom, viz. $\frac{1}{1,000,000}$, $\frac{2}{1,000,000}$, $\frac{3}{1,000,000}$, and so on to the last, which will be $\frac{1,000}{1,000,000}$, make together $\frac{500,500}{1,000,000}$. So there rests on the bottom more than $\frac{499,500}{1,000,000}$ foot and less than $\frac{500,500}{1,000,000}$ foot. But $\frac{499,500}{1,000,000}$ is only $\frac{1}{2,000}$ less than $\frac{1}{2}$; so there does not rest $\frac{1}{1,000}$ foot less than $\frac{1}{2}$ foot on the bottom. In the same way $\frac{500,500}{1,000,000}$ is only $\frac{1}{2,000}$ more than $\frac{1}{2}$, so there does not rest $\frac{1}{1,000}$ more than $\frac{1}{2}$ foot on the bottom. And in the same way such a thing can be proved with regard to any given part, however small. It therefore appears that the difference (if there were any) between the water resting on the bottom ACDE and half a foot of water would have to be less than is possible to be assumed, from which I argue as follows: ¹⁾

- A. *Beside any weight differing from half a foot of water there can be placed a weight differing less therefrom;*
- O. *Beside the weight of the water resting on the bottom ACDE there cannot be placed any weight differing less from half a foot of water;*
- O. *Therefore the weight of the water resting on the bottom ACDE does not differ from half a foot of water.*

CONCLUSION. Given therefore a regular bottom whose highest point is in the water's upper surface, etc.

The reason why the half referred to above always remains between the two numbers infinitely approximating to the half without ever reaching it, is given in the following theorem:

Given a progression of numbers exceeding one another by unity and starting from unity: the half of the square of the last number is less than the sum of all the numbers, but more than the sum of all the numbers minus the last.

Now in order to explain (as promised above) the short method for adding up that great multitude of numbers, it is firstly evident that their denominators are all equal, so that we only have to heed the numerators of the numbers; these are in a regular progression, starting from unity and exceeding one another by unity, therefore when the last number is multiplied by its half, and to the product is added the half thereof, the required sum is obtained. For example, I wish to know what is the sum of 1, 2, 3, 4, 5, 6. I say: 6 times 3 are 18, plus 3 makes 21

¹⁾ See note 2 to p. 143.

somme. Laet het laetste nu oneuen ghetal sijn, als 1, 2, 3, 4, 5, 6, 7; Ick seg 7 mael $3\frac{1}{2}$ is $24\frac{1}{2}$, met $3\frac{1}{2}$ maeckt 28, voor de begheerde somme. Maer als t'laetste aldus oneuen is, soo vallet lichter om duer gheen ghebroken te wercken, datmen t'laetste menichvuldicht duer den helft der somme van t'laetste met 1, als andermael willende weten de somme van 1, 2, 3, 4, 5, 6, 7; Ick doe 1 tot 7, maeckt 8, sijn helft is 4, die vermenichvuldicht duer 7, comt alsbouen 28, voor de begheerde somme, ende alsoo met allen anderen.

M E R C K T.

Angheken de boueschreuen helft des pilaers euen is anden heelen pilaer diens grondt den ghegeuen boden is, ende hoochde den helft der hanghende lini van des bodems hoochste punt, tottet plat euewydich vanden sichteinder duer des bodems leeghste punt, men soude t'boueschreuen 11^e voorstel oock mueghen aldus uyten:

W E S E N D E een gheschickt bodem diens hoochste punt in t'waters oppervlack is: T'ghewicht daer tegheu rustende is euen anden pilaer waters diens grondt euen an dien bodem is, ende hoochde den helft der hanghende lini van des bodems hoochste punt, tottet plat euewydich vanden sichteinder duer des bodems leeghste punt.

Ende na sulcke wyse sullen wy t'laetste deel deses 12^{en} voorstels vormen.

X. VERTOCH.

XII. VOORSTEL.

*Perpendicu-
laris.
Plano.*

Horizonte.

W E S E N D E een gheschickt bodem diens hoochste punt onder t'waters oppervlack is: T'ghewicht daer teghen rustende is euen anden pilaer waters diens grondt euen is an dien bodem, ende hoochde de * hanghende lini van * t'plat duer t'waters oppervlack, tot des bodems hoochste punt, ende bouen dien den helft der hanghende lini van des bodems hoochste punt, tottet plat euewydich vanden * sichteinder duer des bodems leeghste punt.

I VOORBEELT.

T'GHEGHEVEN. Laet ABCD een gheschickt bodem sijn, als ten
eersten

for the required sum. Now let the last be an odd number, viz. 1, 2, 3, 4, 5, 6, 7. I say: 7 times $3\frac{1}{2}$ are $24\frac{1}{2}$, plus $3\frac{1}{2}$ makes 28 for the required sum. But if the last is thus an odd number, it is easier, in order not to operate with fractions, to multiply the last by the half of the sum of the last and 1. Thus, if again I wish to know the sum of 1, 2, 3, 4, 5, 6, 7, I add 1 to 7, which makes 8; the half of that is 4, which when multiplied by 7 makes 28, as above, for the required sum. And the same with all others ¹⁾).

NOTE.

Since the above-mentioned half of the prism is equal to the complete prism whose base is the given bottom and whose height is the half of the vertical from the highest point of the bottom to the plane parallel to the horizon through the lowest point of the bottom, the 11th proposition described above might also be worded as follows:

Given a regular bottom whose highest point is in the water's upper surface: the weight resting against it is equal to the prism of water whose base is equal to that bottom and whose height is the half of the vertical from the highest point of the bottom to the plane parallel to the horizon through the lowest point of the bottom.

And in this way we will word the last part of the 12th proposition.

THEOREM X.

PROPOSITION XII.

Given a regular bottom whose highest point is below the water's upper surface: the weight resting against it is equal to the prism of water whose base is equal to that bottom and whose height is the vertical from the plane through the water's upper surface to the highest point of the bottom, and in addition thereto the half of the vertical from the highest point of the bottom to the plane parallel to the horizon through the lowest point of the bottom.

EXAMPLE I.

SUPPOSITION. Let $ABCD$ be a regular bottom, viz. first a parallelogram ²⁾

¹⁾ Obviously Stevin here applies one of the numerous rules of arithmetic, which were communicated as recipes, without any demonstration.

²⁾ rectangle.

eersten een cweydych vierhouck diens hoochste sijde A B onder 'twaters oppervlack is, cweydych neem ick, vanden sichteinder, ende E A sy de hanghende lini van 'twaters oppervlack, tot des bodem hoochste punt A, ende A F de hanghende lini van A, tot het plat cweydych Vanden sichteinder duer D C, ende A G sy den helft van A F.

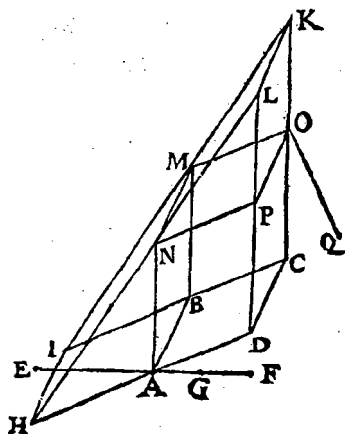
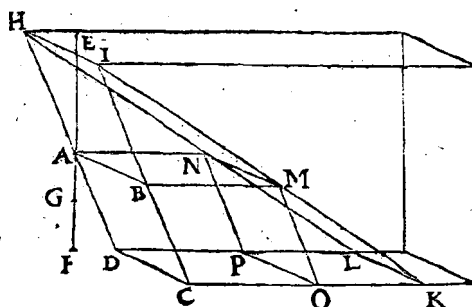
T'EGHEERDE. Wy moeten bewyfen datter ghewicht waters
teghen den bodem
A B C D rustende,
euen is anden pilaer
diens grondt dien
bodem is eñ hooch-
de G E.

The diagram shows a rectangular prism with a base labeled A B C D. A vertical line segment GE is drawn from the base to the top surface, representing the height. A diagonal line segment HE is drawn from the top-left corner H to the base, representing the weight of water. The text explains that the weight of water is equal to the height of the prism.

TBEREYTSSEL.
Laet D A ende C B
voortghetrockē wor-
den tot H ende I,
beyde in 'twaters op-
pervlack; laet oock
ghetrocken sijn H I,
daer naer C K eue-
wydich vanden licht-
einder, ende euen an C I, maer rechthouckich op D C, (ghelijcx D L
euen ende euewydighe met C K, voort L K, daer naer I K ende H L,
voort B M euewydighe met C K, ende also dar M inde lini I K sy, daer
naer A N euen ende euewydighe met B M, voort M O ende N P beyde
euen ende euewydighe met B C.

Laet daer na een ander form
gheselt worden, een, ghelijck, en
euewichtich metter water een
ande voorgaende CDHIKL,
maer also dat CK rechthouckich
sy op dē sichteinder als hier neuen.

THEWYS. Alfulcken druck-
fel als r'stijflichaem CDHIKI
der tweede form, veroirfaecht te-
ghen den bodem CDHI, enen
loodanigen druckfel veroirfaecht
t'water des eerften forms teghen
sijn bodem CDHI fo bewelen is
int 11^o voorstfel ende veruolghens
fulcken druckfel alffer valt teghen
het deel A B C D der tweede



Ec **form,**

whose highest side AB is below the water's upper surface, parallel — I assume — to the horizon, and EA shall be the vertical from the water's upper surface to the highest point A of the bottom, and AF the vertical from A to the plane parallel to the horizon through DC , and AG shall be the half of AF . WHAT IS REQUIRED TO PROVE. We have to prove that the weight of the water resting against the bottom $ABCD$ is equal to the prism whose base is that bottom and whose height is GE . PRELIMINARY. Let DA and CB be produced to H and I , both in the water's upper surface. Let there also be drawn HI , thereafter CK parallel to the horizon and equal to CI , but at right angles to DC ; in the same way DL equal and parallel to CK , further LK ; thereafter IK and HL , further BM parallel to CK and in such a way that M shall be on the line IK . Thereafter AN equal and parallel to BM , further MO and NP both equal and parallel to BC .

Thereafter let there be drawn another figure, equal, similar, and of equal weight to the water having the same volume as the above $CDHIKL$, but in such a way that CK be at right angles to the horizon, as shown opposite. PROOF. The same pressure as is exerted by the solid body $CDHIKL$ of the second figure against the bottom $CDHI$ is also exerted by the water of the first figure against its bottom $CDHI$, as has been proved in the 11th proposition. And consequently the same pressure as is exerted against the part $ABCD$ of the second figure is also exerted

form, euen soodanighen druckfel valter oock teghen het deel $A B C D$ der eerste form, maer het druckfel teghen $A B C D$ der tweede form is t'lichaem $A B C D L K M N$, t'welck euen is anden pilaer diens bodem $A B C D$ ende hoochde $G E$, ghelijck wy terstont segghen sullen, daerom t'ghewicht des waters rustende teghen $A B C D$ der eerste form, is euen anden pilaer diens grondt $A B C D$, ende hoochde $G E$. Maer dattet lichaem $A B C D L K N M$ euen is anden pilaer diens bodē $A B C D$ ende hoochde $G E$ blijkt aldus: Ghetrocken $O Q$ rechthouckich op t'plat duer $A B C D$, de selfde $O Q$ is d'hoochde des pilaers $A B C D P O M N$, daerom dat lichaem is euen anden pilaer diens grondt $A B C D$ ende hoochde $O Q$: Maer anghesien $A H$ euen is an $O C$, ende den houck $H A E$ euen anden houck $C O Q$, ende dat $A E$ rechthouckich is op t'plat duer de punten H, E , t'ghelijcx $O Q$ rechthouckich op t'plat duer de punten C, Q , soois $A E$ euen an $O Q$, daerom t'lichaem $A B C D P O M N$ is euen anden pilaer diens grondt $A B C D$, ende hoochde $A E$. Maer t'lichaem $M N P O K L$ is euen anden pilaer diens grondt $A B C D$ ende hoochde $A G$ duer t'vervolg des 11^{en} voorstels, daerom die twee lichaemen makende t'samen t'lichaem $A B C D L K N M$, sijn euen anden pilaer diens bodem $A B C D$ ende hoochde $G E$.

ANDER BEWYS.

Ghenomen datter in t'water der eerste form hier bouen een bodem sy, euen ende ghelijck met $A B C D$, maer euewydich vanden sichteinder in t'plat daer $A B$ in is: Teghen den seluen bodem sal rusten t'ghewicht euen anden pilaer waters diens grondt euen is an $A B C D$, ende hoochde $A E$ duer het 10^e voorstel, t'selue ghewicht rust oock teghen alle bodem die euen is an dien bodem ende leegher; Daer rust dan voor al teghen $A B C D$, een pilaer diens grondt euen is an $A B C D$, ende hoochde $A E$: Nu gheweert al t'water datter bouen den voornomden bodem is, die wy euen stelden an $A B C D$, alsoo dat $A B$ in t'waters oppervlack sy soo rustet teghen $A B C D$ duer t'vervolg des 11^{en} voorstels, den pilaer diens grondt euen is an $A B C D$, ende hoochde $A G$, welcke twee pilaeren maken t'samen den pilaer diens grondt $A B C D$, ende hoochde $E G$, voor t'ghewicht rustende teghen den bodem $A B C D$ als vooren.

11 VOORBEELT.

Laet A eenich gheschickt bodem wesen, diens hoochste punt onder t'waters oppervlack sijnde, is A , ende t'leeghste B , ende de hanghende lini van t'waters oppervlack tot des bodems hoochste punt sy $C A$, ende van des bodems hoochste punt tot het plat euewydich vandē sichteinder duer des bodems leeghste punt B , sy $A D$, diens helft $A E$. Ick seg dattet ghewicht

against the part $ABCD$ of the first figure. But the pressure against $ABCD$ of the second figure is the body $ABCDLKMN$, which is equal to the prism whose base is $ABCD$ and whose height is GE , as we shall say presently. Therefore the weight of the water resting against $ABCD$ of the first figure is equal to the prism whose base is $ABCD$ and whose height is GE . But that the body $ABCDLKMN$ is equal to the prism whose base is $ABCD$ and whose height is GE becomes evident as follows: OQ being drawn at right angles to the plane through $ABCD$, this OQ is the height of the prism $ABCDPOMN$; therefore that body is equal to the prism whose base is $ABCD$ and whose height is OQ . But since AH is equal to OC , and the angle HAE is equal to the angle COQ , and AE is at right angles to the plane through the points H, E , ¹⁾ and likewise OQ at right angles to the plane through the points C, Q , ²⁾ AE is equal to OQ . Therefore the body $ABCDPOMN$ is equal to the prism whose base is $ABCD$ and whose height is AE . But the body $MNPOKL$ is equal to the prism whose base is $ABCD$ and whose height is AG , by the corollary of the 11th proposition ³⁾. Therefore those two bodies, together making up the body $ABCDLKMN$, are equal to the prism whose base is $ABCD$ and whose height is GE .

OTHER PROOF.

Assuming that there be in the water of the first figure above a bottom, equal and similar to $ABCD$, but parallel to the horizon in the plane in which is AB : against this bottom there will rest the weight that is equal to the prism of water whose base is equal to $ABCD$ and whose height is AE , by the 10th proposition. This weight also rests against any bottom that is equal to that bottom and lower. So in any case there rests against $ABCD$ a prism whose base is equal to $ABCD$ and whose height is AE . Now if all the water is taken away which is above the aforesaid bottom, which we assumed to be equal to $ABCD$, in such a way that AB be in the water's upper surface, there rests against $ABCD$, by the corollary of the 11th proposition, the prism whose base is equal to $ABCD$ and whose height is AG . Which two prisms together make the prism whose base is $ABCD$ and whose height is EG , for the weight resting against the bottom $ABCD$ as before.

EXAMPLE II.

Let AB be any regular bottom, whose highest point being below the water's upper surface is A and the lowest B , and the vertical from the water's upper surface to the highest point of the bottom shall be CA , and from the highest point of the bottom to the plane parallel to the horizon through the lowest point B of the bottom shall be AD , the half of which is AE . I say that the weight of the

¹⁾ the plane HEI .

²⁾ the plane of $ABCD$.

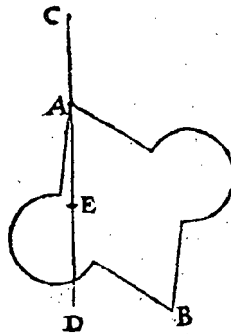
³⁾ There is no corollary to Prop. 11. Stevin evidently refers to the Note, and applies the proposition in the form given there, assuming AB to be in the water's upper surface.

ghewicht waters teghen den bodem A B rustende, euen is anden pilaer waters diens bodem euen is ande selue A B, ende hoochde E C, waer af t'bewys sijn sal als van t'voorgaende.

T'BESELYT. Wefende dan een gheschickt bodem diens hoochste punt, &c.

MERCKT.

Wy hebben hier bouen beboont t'ghewicht teghen een gheschickt bodem rustende, mettet behulp der hanghende lini duer des bodems hoochste punt; Maer alst een ongheschickt bodem is, soo en wort dat ghewicht duer die hanghende lini niet bekent: Tu wel waer, datter altyt vooral op rust t'ghewicht euen an den pilaer waters diens grondt den bodem is, ende hoochde de hanghende lini van t'waters oppervlack tot des bodems hoochste punt, maer t'ander deel en is niet euen anden helft des pilaers wiens grondt dien bodem is, ende hoochde de hanghende lini van des bodems hoochste punt, tottet plat cunwydich vanden sichteinder duer des bodems leeghste punt, waer af d'orsaeck is, dat den pilaer met een ongheschickt bodem niet nootsakelick in twee euen deelen (ghelyck den pilaer met een gheschickt bodem) ghesdelt en wort, met een plat, duer tweelyckstandighe punten schoens teghen ouer malcander staende inde omtrecken der bodems. Maer op dat wy t'ghewicht teghen alle ongeschickt plat bodem oock bekent maken, sullen daer af soodanighen eysch beschrijven.



III EYSCH.

XIII VOORSTEL.

WESSENDE in t'water een platte bodem van form soot valt: Te vinden een lichaem waters cufswaer an t'ghewicht teghen dien bodem rustende.

T'GHEGHEVEN. Laet A B een platte bodem in t'water sijn, gheschickt ofte ongheschickt soot valt. T'BEGHEERDE. Wy moeten een lichaem waters vinden cufswaer an t'ghewicht rustende teghen A B.

T'WERCK. Ick treck het plat A B ouer allen sijden oneindelick voort, diens ghemeen sine mettet waters oppervlack sy C, uyt de selue sine C trek ick een lini duer t'plat A B als C D, maer alsoo datter plat rechthouckich opden sichteinder duer C D, oock rechthouckich sy op oneindelick plat duer den ghegheuen bodem; Daernaer treck ick de lini D E, euen ande lini D C, maer cunwydich vanden sichteinder, ende

E c 2

recht-

water resting against the bottom AB is equal to the prism of water, whose base is equal to this AB and whose height is EC , the proof of which will be the same as that of the preceding example.

CONCLUSION. Given therefore a regular bottom whose highest point, etc.

NOTE.

We have shown above the weight resting against a regular bottom, with the aid of the vertical through the highest point of the bottom. But if it is an irregular bottom, that weight does not become known through that vertical. It is true indeed that in any case there always rests on it the weight that is equal to the prism of water whose base is the bottom and whose height is the vertical from the water's upper surface to the highest point of the bottom, but the other part is not equal to the half of the prism whose base is that bottom and whose height is the vertical from the highest point of the bottom to the plane parallel to the horizon through the lowest point of the bottom, the cause of which is that the prism with an irregular base is not necessarily divided into two equal parts (like the prism with a regular base) by a plane through two homologous points diametrically opposite to each other in the circumferences of the bases. But in order to make also known the weight against any irregular plane bottom, we shall describe a problem with regard to this.

PROBLEM III.

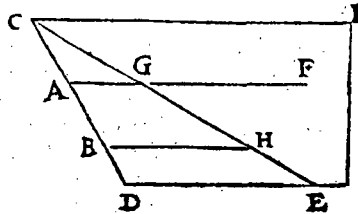
PROPOSITION XIII.

Given in the water a plane bottom of any form: to find a body of water of equal weight to the weight resting against that bottom.

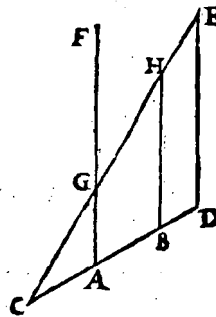
SUPPOSITION. Let AB be a plane bottom in the water, regular or irregular, as the case may be. WHAT IS REQUIRED TO FIND. We have to find a body of water of equal weight to the weight resting against AB . CONSTRUCTION. I produce the plane AB indefinitely on all sides, whose common intersection with the water's upper surface shall be C . From this intersection C I draw a line through the plane AB , as CD , but in such a way that the plane at right angles to the horizon through CD be also at right angles to the infinite plane through the given bottom¹⁾. Thereafter I draw the line DE , equal to the line CD , but parallel to the horizon and at right angles to the line which in the infinite plane

¹⁾ This means that the plane of the drawing is perpendicular to the line of intersection of the plane of the surface under consideration with a horizontal plane.

rechthouckich op de lini die in 'oneindelic plat duer D oock ewwydich vanden fichteinder is; Daer naer treck ick duerde ghemeen sne C, ende duer E, een oueraloneindelic plat C E, voort, nyt eenich punt vanden omtreck des ghegheuen bodems als ulyt A, een oneindelicke lini A F, draeyende de selue mettet punt A in des bodems A B omtreckt, tot datse weder ter plaats comt daerse begon te roeren, maer alsoo datse int roeren altyt ewwydich blijft met de lini D E, beschrijvende alsoo een lichaem begrepen tusschen de twee oneindelicke platten, ende 'vlack van die roerlicke lini beschreuen, alst lichaem A G H B. Ick seg dat een lichaem waters ewegroot an 'lichaem A G H B, eweswaer is an 'ghewicht rustende teghen den ghegheuen bodem.



T'BEREYTSSEL. Laet beschreuen sijn dese tweede form euen ende ghelijck an d'eerste, ende eweswaer an water, maer also dat de lini D E rechthouckich sy op den fichteinder. **T'BEWYS.** Alsulcken ghewicht alser rust teghen den bodem A B der tweede form, euen soodanighen rust oock teghen den bodem A B van d'eerste, als vooren bewesen is, maer teghen A B der tweede form, rust het ghewicht des lichaems A G H B, daerom teghen den bodem A B der eerste form, rust oock een ghewicht eue an 'lichaem waters A G H B, welck wy bewysen moesten.



T'BESELYT. Wesende dan int water een platte bodem van form soor valt, wy hebben een lichaem waters gheuonden eweswaer an 'ghewicht teghen dien bodem rustende, naer den eyfch.

XI. VERTOOGH.

XIIII. VOORSTEL.

W E S E N D E twee ewwydighe vierhouckighe bodems van euen breeden, ende euediep int water, ende haer hoochste sijden int waters oppervlack: Ghelijck der bodems langde tot langde, alsoo haer gheprang des waters, tot gheprang des waters.

T G H I.

through D ¹⁾ is also parallel to the horizon. Thereafter I draw through the common intersection C and through E a plane CE which is infinite in every direction; further, from some point of the boundary of the given bottom, as from A , an infinite line AF , revolving this with the point A in the boundary of the bottom AB until it again reaches the place where it started its motion, but in such a way that during its motion it always remains parallel to the line DE , thus describing a body comprehended by the two infinite surfaces and the plane described by that moving line, viz. the body $AGHB$. I say that a body of water having the same volume as the body $AGHB$ is of equal weight to the weight resting against the given bottom.

PRELIMINARY. Let there be described this second figure, equal and similar to the first and being of equal weight to water, but in such a way that the line DE be at right angles to the horizon. PROOF. The same weight as rests against the bottom AB of the second figure also rests against the bottom AB of the first figure, as has been proved before, but against AB of the second figure rests the weight of the body $AGHB$; therefore against the bottom AB of the first figure rests also a weight equal to the body of water $AGHB$, which we had to prove.

CONCLUSION. Given therefore in the water a plane bottom of any form, we have found a body of water of equal weight to the weight resting against that bottom, as required.

THEOREM XI.

PROPOSITION XIV.

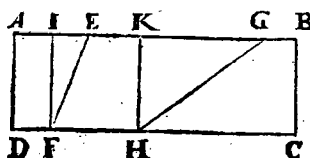
Given two bottoms in the form of parallelograms ²⁾, of equal breadths and at equal depths in the water, their highest sides being in the water's upper surface: as the length of one bottom to that of the other, so is the pressure of the water against one to that against the other.

¹⁾ the plane of the surface.

²⁾ rectangles.

T'GHEGHEVEN. Laet $ABCD$ een water sijn, daer in twee euewydighe vierhouckighe bodems wese, EF ende GH , van euen breedten, ende euediep int water, te weten, dat de * hanghende IF , euen sy ande hanghende KH , ende haer hoogste sijden E en G , sijn int waters oppervlack. **T'BEWYSEN.** Wy moeten bewysen dat ghelijck de langde EF , tot de langde GH , also t'gheprang des waters teghen den bodem EF , tottet gheprang des waters teghen den bodem GH .

Perpendicularu.



T'BEWYS. T'ghewicht des waters teghen den bodem EF rustende, is euen anden helft des pilaers waters diens hoogde IF , ende grondt het plat EF , duer het 11^e voorstel; Sghelijcx is t'ghewicht des waters teghen den bodem GH rustende, euen anden helft des pilaers waters diens hoogde KH , ende grondt het plat GH : maer dit sijn twee pilaren met euen hoogden * daerom sijn ic inde reden haerder gronden; maer ghelijck de langde EF , totte langde GH , also den grondt EF , totten grondt GH , want sy * duer t'ghestelde van euen breedten sijn, daerom ghelijck de langde EF totte langde GH , also diens pilaer tot defens pilaer, ende wyder also diens haluen pilaer tot defens haluen pilaer, ende veruolghens also diens ghewicht des waters teghen haer rustende, tot defens ghewicht des waters teghen haer rustende.

31. v. 11. B. E

Per hypothesin.

T'BESELYT. Wese dan twee euewydighe vierhouckighe bodems, van euen breedten, ende euediep int water, ende haer hoogste sijden int waters oppervlack: Ghelijck der bodems langde tot langde, also haer gheprang des waters, tot gheprang des waters, t'welck wy bewysen moesten.

IIII EYSC.

XV VOORSTEL.

WESENDE den bodem des waters een euewydich vierhouck onuewydich vanden * sichteinder, met sijn bekende hoogste sijde in t'waters oppervlack, ende bekend wese de lini vande hoogste sijde rechthouckich op de voortgetrocken leeghste, oock de * hangende vande hoogste sijde tot het * plat euewydich vanden sichteinder duer de leeghste sijde: Te vinden t'ghewicht waters daer teghen rustende.

Horizonte.

*Perpendicularu.
Planu.*

Ee 3

MERCKT.

SUPPOSITION. Let $ABCD$ be a water, in which there be two bottoms in the form of parallelograms, EF and GH , of equal breadths and at equal depths in the water, to wit that the vertical IF be equal to the vertical KH , and their highest sides E and G be in the water's upper surface. **WHAT IS REQUIRED TO PROVE.** We have to prove that as the length EF is to the length GH , so is the pressure of the water against the bottom EF to the pressure of the water against the bottom GH . **PROOF.** The weight of the water resting against the bottom EF is equal to the half of the prism of water whose height is IF and whose base is the plane EF , by the 11th proposition. In the same way the weight of the water resting against the bottom GH is equal to the half of the prism of water whose height is KH and whose base is the plane GH . But these are two prisms with equal heights; therefore they are in the same ratio as their bases. But as the length EF is to the length GH , so is the base EF to the base GH , for by the supposition they have equal breadths. Therefore, as the length EF is to the length GH , so is the former's prism to the latter's prism, and further so is the former's half prism to the latter's half prism, and consequently so is the weight of the water of the former resting against it to the weight of the water of the latter resting against it.

CONCLUSION. Given therefore two bottoms in the form of parallelograms, of equal breadths and at equal depths in the water, their highest sides being in the water's upper surface: as the length of one bottom is to that of the other, so is the pressure of the water against one to that against the other, which we had to prove.

PROBLEM IV.

PROPOSITION XV.

The bottom in the water being a parallelogram non-parallel to the horizon, with its known highest side in the water's upper surface, and the line from the highest side at right angles to the lowest side produced being known, as also the vertical from the highest side to the plane parallel to the horizon through the lowest side: to find the weight of the water resting against it.

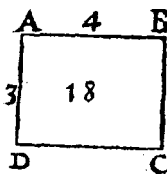
M E R C K T.

ALLE euewydich vierhouck euewydich vanden fichteinder met sin hoochste fide int waters oppervlack, is of rechthouckich of scheefhouckich, ende elck van desen is op den fichteinder rechthouckich oft scheefhouckich, daerom vallender vier verscheyden ghestalten, daer wy so wel inde volghende twee voorstellen als indit, vier voorbeelden af beschryuen sullen: T eerste van een rechthouck opden fichteinder rechthouckich, wiens drie linien, als de fide oneuewydich vanden fichteinder, ende de lini uyt het uytterste vande hoochste fide rechthouckich opde voortghetrocken leeghste fide, ende de hanghende uyt het uytterste vande hoochste fide tottet plat euewydich vanden fichteinder duer de leeghste fide, al een selfde lini sin: Het tweede voorbeelt sal sin van een euewydich scheefhouckich vierhouck opden fichteinder rechthouckich, diens twee linien als de lini vande hoochste fide rechthouckich opde leeghste fide, ende de hanghende vande hoochste fide tottet plat euewydich vanden fichteinder duer de leeghste fide, beyde een selue sin: Het derde voorbeelt sal sin van een rechthouck scheefhouckich opden fichteinder, diens twee linien, als de fide oneuewydich vanden fichteinder, ende de lini van t'uyterste der hoochste fide rechthouckich op de leeghste fide, beyde een selue sin: T vierde voorbeelt van een euewydich scheefhouckich vierhouck opden fichteinder scheefhouckich, diens voornoemde drie linien al verscheyden sin.

I^e VOORBEELT.

T'GHEGHEVEN. Laet ABCD een rechthouck wesen rechthouckich opden fichteinder, diens sijde AB in t'waters oppervlack doe 4 voeten, ende AD 3 voeten. T'BEGHEERDE. Wy moeten t'ghewicht waters vinden rustende teghen ABCD.

T'WERCK. Ick menichvuldighe 3 van AD duer 4 van AB, maeckt 12, die andermael ghemenichvuldicht duer 3 van AD comt 36 voeten, diens helft voor t'begheerde 18 voeten. Ofte andersins ick menichvuldighe t'vircant der 3 van AD, duer den helft der 4 van AB, comt als vooren 18 voeten. nu ghenomen den voet te weghen 65 lb, soo rustet 1170 lb teghen.

II^e VOORBEELT.

T'GHEGHEVEN. Laet ABCD een euewydich scheefhouckich vierhouck wesen, rechthouckich op den fichteinder, diens sijde AB in t'waters oppervlack doe 4 voeten, ende AE hanghende lini vande hoochste sijde A B, tot inde voortghetrocken CD, sy van 3 voeten.

T'BEGHEERDE. Wy moeten t'ghewicht waters vinden rustende teghen ABCD.

T'WERCK

NOTE.

Any parallelogram non-parallel to the horizon with its highest side in the water's upper surface is either right-angled or oblique-angled, and each of them is at right angles or oblique angles to the horizon. Therefore there are four different cases, of which we shall describe four examples both in the following two propositions and in the present: the first of a rectangle at right angles to the horizon, three lines of which, viz. the side non-parallel to the horizon, and the line from the end of the highest side at right angles to the lowest side produced, and the vertical from the end of the highest side to the plane parallel to the horizon through the lowest side, are all one and the same line. The second example is to be of an oblique-angled parallelogram at right angles to the horizon, two lines of which, viz. the line from the highest side at right angles to the lowest side, and the vertical from the highest side to the plane parallel to the horizon through the lowest side, are one and the same line. The third example is to be of a rectangle at oblique angles to the horizon, two lines of which, viz. the side non-parallel to the horizon, and the line from the end of the highest side at right angles to the lowest side, are one and the same line. The fourth example is to be of an oblique-angled parallelogram at oblique angles to the horizon, the aforesaid three lines of which are all different.

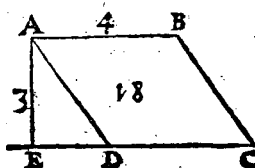
EXAMPLE I.

SUPPOSITION. Let $ABCD$ be a rectangle at right angles to the horizon, whose side AB in the water's upper surface be 4 feet, and AD 3 feet. WHAT IS REQUIRED TO FIND. We have to find the weight of the water resting against $ABCD$. CONSTRUCTION. I multiply 3 of AD by 4 of AB , which makes 12, which being multiplied again by 3 of AD makes 36 feet, whose half gives what was required: 18 feet. Or else I multiply the square of the 3 of AD by the half of the 4 of AB , which works out at 18 feet, as above. If I now take one foot to weigh 65 lbs, there rest against it 1,170 lbs.

EXAMPLE II.

SUPPOSITION. Let $ABCD$ be an oblique-angled parallelogram, at right angles to the horizon, whose side AB in the water's upper surface be 4 feet, and AE , the vertical from the highest side AB to CD produced, be 3 feet. WHAT IS REQUIRED TO FIND. We have to find the weight of the water resting against

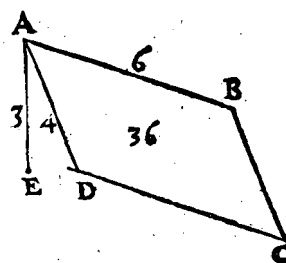
TWERCK. Ick menichvuldighe 3 van A E duer 4 van A B, maeckt 12, die andermael ghemenichvuldicht duer 3 van A E comt 36 voeten, diens helft voor t'begheerde 18 voeten. Oft andersins ick menichvuldighe als bouen t'viercant der 3 met den helft der 4 van A B.



III. VOORBEELT.

T'GHEGHEVEN. Laet ABCD een rechthouck wesen scheefhouckich op den sichteinder, diens sijde AB in t'waters oppervlack sijn, de doet 6 voeten, ende AD 4 voeten, maer A E hanghende van A tot in t'plat euewydich vanden sichteinder duer D C doe 3 voeten.

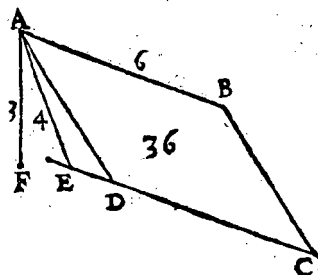
T'BEGHEERDE. Wy moeten t'ghewicht waters vinden teghen A B C D rustende. **TWERCK.** Ick menichvuldighe 4 duer 6 comt 24, de selue duer 3 maeckt 72 voeten, diens helft voor t'begheerde 36 voeten. Ofte andersins ick menichvuldighe den uytbreng van 3 met 4, duer den helft van 6, comt als vooren 36 voeten.



IIII. VOORBEELT.

T'GHEGHEVEN. Laet ABCD een euewydich scheefhouckich vierhouck sijn, scheefhouckich op den sichteinder, diens sijde AB in t'waters oppervlack sijnde doet 6 voeten, ende A E rechthouckich op de voortghetrocken C D doet 4 voeten, ende A F hanghende van A tot het plat euewydich vanden sichteinder duer D C doet 3 voeten.

T'BEGHEERDE. Wy moeten t'ghewicht waters vinden teghen A B C D rustende. **TWERCK.** Ick menichvuldighe 4 van A E, met 6 van A B comt 24, tselue met 3 van A F, comt 72 voeten, diens helft voor t'begheerde 36 voeten. Ofte andersins, ick menichvuldighe, als vooren, den uytbreng van 3 met 4, duer den helft van 6, comt oock 36 voeten.



T'BEWYS.

ABCD. CONSTRUCTION. I multiply 3 of *AE* by 4 of *AB*, which makes 12, which being multiplied again by 3 of *AE* makes 36 feet, whose half gives what was required: 18 feet. Or else I multiply, as above, the square of 3 by the half of the 4 of *AB*¹⁾.

EXAMPLE III.

SUPPOSITION. Let *ABCD* be a rectangle at oblique angles to the horizon, whose side *AB* being in the water's upper surface be 6 feet, and *AD* 4 feet, but *AE*, the vertical from *A* to the plane parallel to the horizon through *DC*, be 3 feet. WHAT IS REQUIRED TO FIND. We have to find the weight of the water resting against *ABCD*. CONSTRUCTION. I multiply 4 by 6, which makes 24; this being multiplied by 3 makes 72 feet, whose half gives what was required: 36 feet. Or else I multiply the product of 3 and 4 by the half of 6, which makes 36 feet, as before.

EXAMPLE IV.

SUPPOSITION. Let *ABCD* be an oblique-angled parallelogram, at oblique angles to the horizon, whose side *AB* being in the water's upper surface be 6 feet, and *AE*, at right angles to *CD* produced, be 4 feet, and *AF*, the vertical from *A* to the plane parallel to the horizon through *DC*, be 3 feet. WHAT IS REQUIRED TO FIND. We have to find the weight of the water resting against *ABCD*. CONSTRUCTION. I multiply 4 of *AE* by 6 of *AB*, which makes 26; this, being multiplied by 3 of *AF*, makes 72 feet, whose half gives what was required: 36 feet. Or else I multiply, as before, the product of 3 and 4 by the half of 6, which

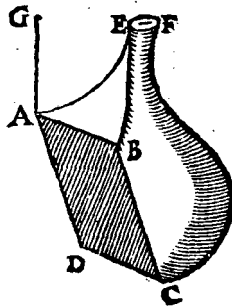
¹⁾ Here and in Example 4, the 11th proposition is applied to an oblique-angled parallelogram, though the demonstration given applied only to a rectangle.

T'BEWYS. Wefende een pilaer diens grondt 12 voeten, ende hoochde 3 voeten, den helft van dien doet 18 voeten; maer sulcken lichaem rustet teghen den bodem A B C D des 1^{en} voorbeelts, duer het 11^e voorstel, daer rust dan t'ghewicht van 18 voeten waters teghen. Sghelijcx sal oock t'bewys sijn van d'ander voorbeelden. T'BESELVYT. Wefende dan den bodem des waters een euewydich vierhouck, &c.

I VERVOLGH.

Uyt het boueschreuen is blijckelick, hoemen vinden sal t'ghewicht waters teghen een euewydich vierhouck rustende, wefende d'hoochste sijde des ghegheuen vierhouck onder t'waters oppervlack, want tot het ghewicht gheuonden alvooren, noch vergaert den pilaer diens grondt dien bodem is, ende hoochde de hanghende lini van t'plat duer t'waters oppervlack tot de hoochste sijde des bodems, de somme sal t'begheerde sijn.

Laet by voorbeelt A B C D een euewydich vierhouck sijn onuewydich vanden sichteinder, diens hoochste sijde A B onder t'waters oppervlack E F is, ende G A doende drie voeten sy de hanghende lini van t'plat duer E F tot de sijde A B, ende t'plat A B C D sy groot 20 voeten, ende als A B in t'waters oppervlack waer, soo souder op rusten (twelck ick neem gheuonden te sijne duer de voorgaende leering) 40 voeten waters: Vraegh hoe veel datter nu op rusten? Ick menichvuldighe 20 des plats van A B C D, duer 3 van G A, comt een pilaer van 60 voeten, die tot de 40 maeckt 100 voeten dieder teghen A B C D rusten.



II. VERVOLGH.

Soo den ghegheuen platten bodem ongheschickt waer, men sal vinden een lichaem waters euefwaer an t'ghewicht teghen dien bodem rustende duer het 13^e voorstel, t'selue lichaem ghemeten sal de begheerde swaerheyt bekent maken.

V EYSCH.

XVI VOORSTEL.

Horizonte. W E S E N D E den bodem des waters een euewydich vierhouck, onuewydich vanden sichteinder, met sijn hoochste sijde int waters oppervlack, ende bekent sijde t'ghewicht daer teghen rustende,

also makes 36 feet. PROOF. Given a prism, whose base is 12 feet and whose height is 3 feet; the half of that makes 18 feet. But such a body rests against the bottom $ABCD$ of the 1st example, by the 11th proposition; so there rests against it the weight of 18 feet of water. The same proof will also be true of the other examples. CONCLUSION. The bottom in the water therefore being a parallelogram, etc.

COROLLARY I.

From the above it appears how one is to find the weight of the water resting against a parallelogram, if the highest side of the given quadrilateral is below the water's upper surface, for if to the weight found as before there be added the prism whose base is that bottom and whose height is the vertical from the plane through the water's upper surface to the highest side of the bottom, the sum will be the required weight.

For example, let $ABCD$ be a parallelogram non-parallel to the horizon, whose highest side AB is below the water's upper surface EF , and GA , being three feet, shall be the vertical from the plane through EF to the side AB , and the plane $ABCD$ shall be 20 feet. If AB were in the water's upper surface, there would rest on it 40 feet of water (which I take to have been found by means of the preceding theory). It is asked how much there now rests on it. I multiply 20 of the area of $ABCD$ by 3 of GA . The product is a prism of 60 feet; these being added to the 40 feet, there will rest against $ABCD$ 100 feet.

COROLLARY II.

If the given plane bottom is irregular, a body of water shall be found which is of equal weight to the weight resting against that bottom by the 13th proposition. When this body is measured, the required gravity will be known.

PROBLEM V.

PROPOSITION XVI.

The bottom in the water being a parallelogram, non-parallel to the horizon, with its highest side in the water's upper surface, and the weight resting against it

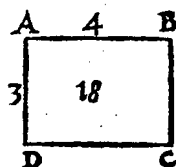
rustende, oock de lini vande hoochste sijde recht-
houckich opde voortghetrocken leeghste sijde,
mette *hanghende vande hoochste sijde, tottet
*plat euewydich vandē sichteinder duer de leegh-
ste sijde: D' hoochste sijde bekennt te maken.

*Perpendicu-
lari.
Planum.*

I VOORBEELT.

TGHEGHEVEN. Laet ABCD een rechthouck wesen re chthouckich opden sichteinder, daer teghen rustende rghewicht van 18 voeten waters, ende d' hoochste sijde AB in rwaters oppervlack sy onbekent, maer AD doet 3 voeten. TBEGHEERDE. Wy moeten de sijde AB bekennt maken.

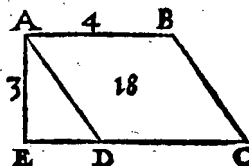
TWERCK. Ick deel de 18 duer r'viercant der 3 van AD comt 2 voeten, diens dobbel voor AB 4 voeten.



II VOORBEELT.

TGHEGHEVEN. Laet ABCD een euewydich scheefhouckich vierhouck wesen, rechthouckich opden sichteinder, daer teghen rustende rghewicht van 18 voeten waters, ende de hoochste sijde AB in rwaters oppervlack sy onbekent, maer de lini AE vande hoochste sijde rechthouckich opde voortghetrocken leeghste sijde doet 3 voeten.

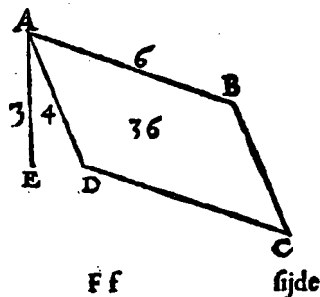
TBEGHEERDE. Wy moeten de sijde AB bekennt maken. TWERCK. Ick deel de 18 duer r'viercant der 3 van AE comt 2 voeten, diens dobbel voor AB 4 voeten.



III VOORBEELT.

TGHE. Laet ABCD een rechthouck wesen scheefhouckich opden sichteinder, daer teghen rustende rghewicht van 36 voeten waters, ende d' hoochste sijde AB in rwaters oppervlack sy onbekent, maer de lini AD doet 4 voeten, ende AE hanghende vande hoochste sijde tot het plat euewydich vanden sichteinder duer de leeghste sijde doet 3 voeten.

TBEGHEERDE. Wy moeten de



being known, as also the line from the highest side at right angles to the lowest side produced, with the vertical from the highest side to the plane parallel to the horizon through the lowest side: to make known the highest side.

EXAMPLE I.

SUPPOSITION. Let $ABCD$ be a rectangle at right angles to the horizon, against which there rests the weight of 18 feet of water, and the highest side AB in the water's upper surface shall be unknown, but AD is 3 feet. WHAT IS REQUIRED TO FIND. We have to make known the side AB . CONSTRUCTION. I divide the 18 by the square of the 3 of AD , which makes 2 feet; the double of this for AB is 4 feet.

EXAMPLE II.

SUPPOSITION. Let $ABCD$ be an oblique-angled parallelogram, at right angles to the horizon, against which there rests the weight of 18 feet of water, and the highest side AB in the water's upper surface shall be unknown, but the line AE from the highest side at right angles to the lowest side produced is 3 feet. WHAT IS REQUIRED TO FIND. We have to make known the side AB . CONSTRUCTION. I divide the 18 by the square of the 3 of AE , which makes 2 feet; the double of this for AB is 4 feet.

EXAMPLE III.

SUPPOSITION. Let $ABCD$ be a rectangle at oblique angles to the horizon, against which there rests the weight of 36 feet of water, and the highest side AB in the water's upper surface shall be unknown, but the line AD is 4 feet, and AE , the vertical from the highest side to the plane parallel to the horizon through the lowest side, is 3 feet. WHAT IS REQUIRED TO FIND. We have to make

sijde A B bekend maken. T'WERCK. Ick menichvuldighe 3 van A E duer 4 van A D comt 12, daer duer ghedeelt de 36 comt 3 voeten, diens dobbel voor A B 6 voeten.

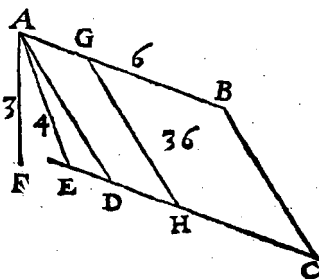
IIII^e VOORBEELT.

T'GHE. Laet A B C D een euewydich scheefhouckich vierhouck sijn scheefhouckich opden sichteinder, daer teghen rustende t'ghewicht van 36 voeten waters, ende de hoochste sijde A B in t'waters oppervlack sy onbekent, maer A E lini vande hoochste sijde rechthouckich op de voortghetrocken leeghte sijde C D, doet 4 voeten, ende A F hanghende vande hoochste sijde tot het plat euewydich vanden sichteinder duer de leeghte sijde, doet 3 voeten.

T'BEGHEERDE. Wy moeten de sijde A B bekend maken.

T'WERCK. Ick menichvuldighe 3 van A F, met 4 van A E, comt 12, daer duer ghedeelt de 36, comt 3 voeten, diens dobbel voor A B 6 voeten. T'BEWYS. Soo A B des 1^{en} voorbeelts langher of corter waer als 4 voeten, t'ghewicht waters teghen den bodem rustende soude moeten meerder of minder sijn dan 18 voeten, t'welck teghen t'ghestelde waer, daerom A B is van 4 voeten. Sghelijcx sal oock t'bewys sijn van d'ander voorbeelden.

T'BESLUYT. Wefende dan den bodem des waters een euewydich vierhouck oneuewydich, &c.



I VERVOLGH.

Uyt het voorgaende is blijckelick, hoemen d'hoochste sijde bekend sal maken, als sy onder t'waters oppervlack is, want van t'gheheel ghewicht waters teghen den bodem rustende, ghetrocken den pilaer diens grondt dien bodem is, ende hoochde de hanghende lini van t'plat duer t'waters oppervlack tot de hoochste sijde des bodems, daer sal resten t'ghewicht waters opden bodem rustende als haer hoochste sijde in t'waters oppervlack is, waer duer sy aldan bekend sal worden als vooren gheleert is.

II VERVOLGH.

Soomen inden bodem een lini wilde trecken euewydich met de sijde die vanden sichteinder oneuewydich is, de noodighe langde der hoochste sijde can bekend worden. Laet by voorbeelt inde form des boueschreuen 4^{en} voorbeelts, te trecken sijn een lini als G H, euewydich met A D,

known the side AB . CONSTRUCTION. I multiply 3 of AE by 4 of AD , which makes 12. The 36, divided by this, makes 3 feet; the double of this for AB is 6 feet.

EXAMPLE IV.

SUPPOSITION. Let $ABCD$ be an oblique-angled parallelogram at oblique angles to the horizon, against which there rests the weight of 36 feet of water, and the highest side AB in the water's upper surface shall be unknown, but AE , the line from the highest side at right angles to the lowest side CD produced, is 4 feet, and AF , the vertical from the highest side to the plane parallel to the horizon through the lowest side, is 3 feet. WHAT IS REQUIRED TO FIND. We have to make known the side AB . CONSTRUCTION. I multiply 3 of AF by 4 of AE , which makes 12. The 36, divided by this, makes 3 feet; the double of this for AB is 6 feet. PROOF. If AB of the 1st example were longer or shorter than 4 feet, the weight of the water resting against the bottom would have to be more or less than 18 feet, which would be contrary to the supposition. Therefore AB is 4 feet. The same proof will also be true of the other examples. CONCLUSION. The bottom in the water therefore being a parallelogram non-parallel, etc.

COROLLARY I.

From the foregoing it appears how one is to make known the highest side when it is below the water's upper surface, for if from the complete weight of the water resting against the bottom there be subtracted the prism whose base is that bottom and whose height is the vertical from the plane through the water's upper surface to the highest side of the bottom, there will be left the weight of the water resting on the bottom when its highest side is in the water's upper surface, from which it will then become known, as has been taught before.

COROLLARY II.

If a line were to be drawn in the bottom, parallel to the side which is non-parallel to the horizon, the necessary length of the highest side can become known. For example, in the figure of the 4th example described above let there be drawn a line, as GH , parallel to AD , in such a way that there shall rest on $AGHD$ the

AD, alsoodat op AGHD ruste 'ghewicht van 12 voeten waters. Ick sie wat deel dese 12 sijn vande 36 die der teghen rusten, wort beuonden het derdedeel, daerom oock sal $AG = \frac{1}{3}$ wesen van AB dat sijn 2 voeten.

VI. EYSCH.

XVII. VOORSTEL.

W E S E N D E den bodem des waters een euewydich vierhouck oneuewydich vanden * sichte-einder, met sijn bekende hoogste sijde in t'waters oppervlack, ende bekend sijnde 'ghewicht daer teghen rustende, oock * de hanghende lini vande hoogste sijde tot het * plat euewydich vanden sichteinder duer de leeghte sijde: De lini vande hoogste sijde rechthouckich op de voortghetrocken leeghte sijde bekend te maken.

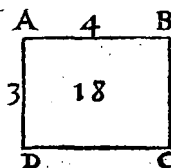
Horizonte.

Perpendic-
laris.
Planim.

I VOORBEELT.

T G H E G H E V E N. Laet ABCD een rechthouck wesen, rechthouckich op den sichteinder, daer teghen rustende 'ghewicht van 18 voeten waters, ende d'hoogste sijde AB in t'waters oppervlack sijnde, doet 4 voeten.

T B E G H E E R D E. Wy moeten de sijde AD bekend maken. T W E R C K. Ick deel de 18 duer 2, helft van AB, comt 9, diens viercantighe sijde voor AD doet 3 voeten.

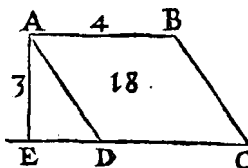


II VOORBEELT.

T G H E G H E V E N. Laet ABCD een euewydich scheefhouckich vierhouck wesen, rechthouckich op den sichteinder, daer teghen rustende 'ghewicht van 18 voeten waters, ende d'hoogste sijde AB in t'waters oppervlack sijnde doet vier voeten.

T B E G H E E R D E. Wy moeten de lini AE bekend maken.

T W E R C K. Ick deel de 18 duer 2 helft van AB, comt 9, diens viercantighe sijde voor AE is 3 voeten.



III VOORBEELT.

T G H E G H E V E N. Laet ABCD een rechthouck wesen scheefhouckich

Ff 2

kich

weight of 12 feet of water. I ascertain what part these 12 are of the 36 resting against it. This is found to be one-third. Therefore also AG will be $\frac{1}{3}$ of AB , that is 2 feet.

PROBLEM VI.

PROPOSITION XVII.

The bottom in the water being a parallelogram non-parallel to the horizon, with its known highest side in the water's upper surface, and the weight resting against it being known, as also the vertical from the highest side to the plane parallel to the horizon through the lowest side: to make known the line from the highest side at right angles to the lowest side produced.

EXAMPLE I.

SUPPOSITION. Let $ABCD$ be a rectangle, at right angles to the horizon, against which there rests the weight of 18 feet of water, and the highest side AB , being in the water's upper surface, is 4 feet. WHAT IS REQUIRED TO FIND. We have to make known the side AD . CONSTRUCTION. I divide the 18 by 2, the half of AB , which makes 9, being the square of the side AD , which is thus 3 feet.

EXAMPLE II.

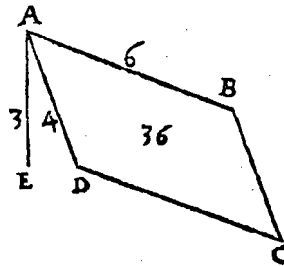
SUPPOSITION. Let $ABCD$ be an oblique-angled parallelogram, at right angles to the horizon, against which there rests the weight of 18 feet of water, and the highest side AB , being in the water's upper surface, is four feet. WHAT IS REQUIRED TO FIND. We have to make known the line AE . CONSTRUCTION. I divide the 18 by 2, the half of AB , which makes 9, being the square of the side AE , which is thus 3 feet.

EXAMPLE III.

SUPPOSITION. Let $ABCD$ be a rectangle at oblique angles to the horizon, against

kich opden sichteinder, daer teghen rustende t'ghewicht van 36 voeten waters, ende de hoochste sijde AB in t'waters oppervlack sijnde doet 6 voeten, ende AE doende 3 voeten, sy de hanghende lini vande hoochste sijde tot het plat euewydich vanden sichteinder duer de leeghte sijde.

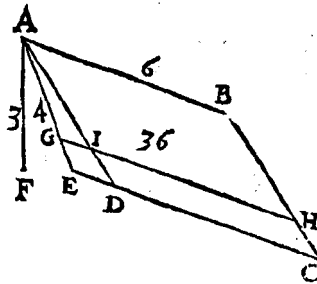
T'BEGHEERDE. Wy moeten AD bekent maken. T'WERCK. Ick deel de 36 duer 3 helft van AB, comt 12, de selue ghedeelt duer 3 van AE, comt 4 voeten voor AD.



IIII VOORBEELT.

T'GHEGHEVEN. Laet ABCD een euewydich scheefhouckich vierhouck sijn scheefhouckich opden sichteinder, daerop rustende t'ghewicht van 36 voeten waters, ende de hoochste sijde AB in t'waters oppervlack sijnde doet 6 voeten, ende AE sy de lini van d'hoochste sijde rechthouckich opde voortghetrocken leeghte sijde, ende AF doende 3 voeten, is de hanghende vande hoochste sijde tot het plat euewydich vanden sichteinder duer de leeghte sijde. T'BEGHEERDE.

Wy moeten de lini AE bekent maken. T'WERCK. Ick deel de 36 duer 3 helft der 6 van AB, comt 12, de selue duer 3 van AF comt 4 voeten, voor AE. T'BEWYS. So AD



des 1^{en} voorstels langher of corter waer als 3 voeten, t'ghewicht waters teghen den bodem rustende soude moeten meerder of minder sijn dan 18 voeten, t'welck teghen ghestelde is, AD dan is van 3 voeten. Sghelijcx sal oock t'bewys sijn van d'ander voorbeelden. T'BESELYT. Wesende dan den bodem des waters een euewydich vierhouck oneuewydich vanden sichteinder, &c.

I VERVOLGH.

Uyt het voorgaende is blijkelyck hoemen de lini vande hoochste sijde rechthouckich opde leeghte sijde, bekent sal maken, als de hoochste sijde onder t'waters oppervlack is, want van t'gheheel ghewicht waters teghen den bodem rustende, ghetrocken den pilaer diens grondt dien bodem is, ende hoochde de hanghende lini van t'plat duer t'waters oppervlack,

which there rests the weight of 36 feet of water, and the highest side AB , being in the water's upper surface, is 6 feet, and AE , which is 3 feet, shall be the vertical from the highest side to the plane parallel to the horizon through the lowest side. WHAT IS REQUIRED TO FIND. We have to make known AD . CONSTRUCTION. I divide the 36 by 3, the half of AB , which makes 12; this being divided by 3 of AE makes 4 feet for AD .

EXAMPLE IV.

SUPPOSITION. Let $ABCD$ be an oblique-angled parallelogram at oblique angles to the horizon, against which there rests the weight of 36 feet of water, and the highest side AB , being in the water's upper surface, is 6 feet, and AE shall be the line from the highest side at right angles to the lowest side produced, and AF , which is 3 feet, is the vertical from the highest side to the plane parallel to the horizon through the lowest side. WHAT IS REQUIRED TO FIND. We have to make known the line AE . CONSTRUCTION. I divide the 36 by 3, the half of the 6 of AB , which makes 12; this being divided by 3 of AF makes 4 feet for AE . PROOF. If AD of the 1st proposition were longer or shorter than 3 feet, the weight of the water resting against the bottom would have to be more or less than 18 feet, which is contrary to the supposition; AD therefore is 3 feet. The same proof will also be true of the other examples. CONCLUSION. The bottom in the water therefore being a parallelogram non-parallel to the horizon, etc.

COROLLARY I.

From the foregoing it appears how one is to make known the line from the highest side at right angles to the lowest side when the highest side is below the water's upper surface, for if from the complete weight of the water resting against the bottom there be subtracted the prism whose base is that bottom and whose height is the vertical from the plane through the water's upper surface to the

oppervlack, tot de hoochste sijde des bodems, daer sal resten 'ghewicht waters opden bodem rustende als haer hoochste sijde in 'waters oppervlack is, waer duer sy alsdan bekend sal worden als voren gheleert is.

II VERVOLGH.

Soomen inde ghegheuen bodem een lini wilde trecken euewydich met de hoochste sijde, alsoo datse affine een deel des bodems daer een be- gheert ghewicht teghen ruste, de noodighe langde der lini vande hooch- ste sijde rechthouckich opde voortghetrocken leeghste sijde can bekend worden. Laet by voorbeeld inde form des bouesckreuen 4^e voorbeelds, te trecken sijn een lini als G H, sniende A D in I, euewydich met A B, alsoo dat op A B H I rust 'ghewicht van 24 voeten waters; Ick deel die 24 duer 3, helft van A B, comt 8, daer naer vinde ick twee ghetalen tot malcanderen in sulcken reden als 3 van A F, tot 4 van A E, ende dat haer uytbreng de voornomde 8 make, die ghetalen sijn $\sqrt{6}$ ende $\sqrt{10\frac{2}{3}}$, t'laetste is voor A G, want uyt G ghetrocken G H euewydighe met A B, daer sal teghen A B H I rusten 'ghewicht van 24 voeten waters duer her 15 voorstel.

MERCKT.

Wy moeten nu naer luyt des Coribegrijps, inde volghende 18^e, 19^e, 20^e, voorstellen, schrijuen vande swaerheys middelpunten der gheprangelen des wa- ters in bodems vergaert; alwaer niet onbillichlick eerst soude mueghen gheseyt worden, vande bodems euewydich sinde vanden sichteinder, maer ouermids der seluer swaerheys middelpunten (welcke gheuonden worden na de leering des 2^{de} boucx vande beghinselen der Weeghconst) oock de swaerheys middelpun- ten sijn der voornoemde baer gheprangelen, soo en beschrijuen wy daer af ons cortheys wil, gheen besonder voorstel. Sullen dan beghinnen ande bodems on- euewydich vanden sichteinder als volght.

XII VERTOCH.

XVIII VOORSTEL.

W E S E N D E den bodem des waters een eue- wydich vierhouck oneuewydich vanden * ficht- Horizonte. einder, diens hoochste sijde in 'waters oppervlack is, uyt welcke sijdens middel een lini ghetrocken is, tot in 't middel vande leeghste sijde: * T'swaer- Centrum gravitatu. heys middelpunt des gheprangs inden bodem vergaert, deelt die lini alsoo, dat haer opperste stuck dobbel is an ronderste.

highest side of the bottom, there will be left the weight of the water resting against the bottom when its highest side is in the water's upper surface, from which it will then become known, as has been taught before.

COROLLARY II.

If a line were to be drawn in the given bottom, parallel to the highest side, in such a way that it cut off a part of the bottom against which there should rest a desired weight, the necessary length of the line from the highest side at right angles to the lowest side produced can become known. For example, in the figure of the 4th example described above let there be drawn a line, as GH , intersecting AD in I , parallel to AB , in such a way that there rests on $ABHI$ the weight of 24 feet of water. I divide those 24 by 3, the half of AB , which makes 8. Thereafter I find two numbers in the proportion 3 (of AF) to 4 (of AE) and so that their product be the aforesaid 8. Those numbers are $\sqrt{6}$ and $\sqrt{10\frac{2}{3}}$; the latter is AG , for when from G there be drawn GH , parallel to AB , there will rest against $ABHI$ the weight of 24 feet of water, by the 15th proposition.

NOTE.

As announced in the Argument, we now have to write, in the 18th, 19th, and 20th propositions, about the centres of gravity of the total pressure of the water on bottoms; here it would not be inappropriate to speak first of the bottom being parallel to the horizon, but since the latter's centres of gravity (which are found by the theory of the 2nd book of the elements of the Art of Weighing) are also the centres of gravity of their aforesaid pressure, we will, for brevity's sake, not describe any separate proposition about this. We shall therefore start with the bottoms which are non-parallel to the horizon, as follows.

THEOREM XII.

PROPOSITION XVIII.

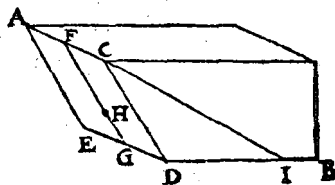
The bottom in the water being a parallelogram non-parallel to the horizon, whose highest side is in the water's upper surface, from the middle point of which side is drawn a line to the middle point of the lowest side: the centre of gravity of the total pressure on the bottom so divides that line that its upper part is double of the lower.

I VOORBEELT.

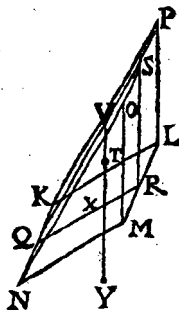
T'GHEGHEVEN. Laet AB een water sijn, ende den bodem $ACDE$ sy een euewydich vierhouck oneuewydich vanden sichteinder, diens hoochste sijde AC in 'twaters oppervlack is, ende F sy 'tmiddel van AC , ende G 'tmiddel van ED , ende tusschen de punten F G sy ghetrocken de lini FG , welke in H alsoo ghedeelt is, dat FH dobbel is tot HG .

T'BEGHEERDE. Wy moeten bewysen dat H 'tswaerheys middelpunt is des gheprangs inden bodem vergaert.

T'BEREYTSSEL. Laet ghetrocken worden de lini CI , alsoo dat DI euen sy an DC , ende mettet lichaem $ACIDE$ sy beteeckent den helft des pilaers diens grondt $ACDE$, ende hoochde de hanghende van A tot in 't'plat euewydich vanden sichteinder duer E D .



Laet daer naer ghetrocken worden 'tstijf lichaem $KLMNOP$ euen ende ghelijck ende eufwaer an 't'licham $ACIDE$, te weten $KLMN$ *lijckstandich plat met $ACDE$, ende MO rechthouckich opden sichteinder, sy lijckstandighe lini met DI ende QR sy lijckstandighe lini met FG , ende van S in 'tmiddel van OP , sy ghetrocken de lini SQ , ende SR , ende des driehoucx QSR swaerheys middelpunt sy T , duer 'twelck ghetrocken is de lini VX rechthouckich opden sichteinder.



T'BEWYS. Alsulcken gheprang als 't'licham $KLMNOP$ doet teghen den bodem $KLMN$, euen sulcken gheprang doet 't'water AB teghen de bodem $ACDE$ duer het 1^o voorstel, daerom ghelijck 'tswaerheys middelpunt des gheprangs inden bodem $KLMN$ valt, alsoo salt oock vallen inden bodem $ACDE$. Om dan totter bewys te comen, soo is ten eersten blijklick dat T , welke duer *'r'ghestelde swaerheys middelpunt is des driehoucx QSR , oock swaerheys middelpunt is (duer het 1^o voorstel des 2^o boucx der beghinselen vande Weeghconst) des lichaems $KLMNOP$, maer VX is duer T rechthouckich opden sichteinder, VX dan is des lichaems swaerheys middellini, daerom soo wy de

*Homologum
planum.*

Hypothesin.

EXAMPLE I.

SUPPOSITION. Let AB be a water, and the bottom $ACDE$ shall be a parallelogram non-parallel to the horizon, whose highest side AC is in the water's upper surface, and F shall be the middle point of AC , and G the middle point of ED , and between the points F and G there shall be drawn the line FG , which is so divided in H that FH is double of HG . **WHAT IS REQUIRED TO PROVE.** We have to prove that H is the centre of gravity of the total pressure on the bottom. **PRELIMINARY.** Let there be drawn the line CI , in such a way that DI shall be equal to DC , and by the body $ACIDE$ there shall be denoted the half of the prism whose base is $ACDE$, and whose height is the vertical from A to the plane parallel to the horizon through ED .

Thereafter let there be drawn the solid body $KLMNOP$, equal, similar, and of equal weight to the body $ACIDE$, to wit $KLMN$ being a plane homologous to $ACDE$, and MO , at right angles to the horizon, shall be a line homologous to DI , and QR shall be a line homologous to FG , and from S in the middle point of OP there shall be drawn the line SQ , and SR , and the centre of gravity of the triangle QSR shall be T , through which is drawn the line VX at right angles to the horizon. **PROOF.** The same pressure as is exerted by the body $KLMNOP$ against the bottom $KLMN$ is also exerted by the water AB against the bottom $ACDE$, by the 11th proposition. Therefore, just as the centre of gravity of the pressure falls in the bottom $KLMN$, so it will also fall in the bottom $ACDE$. Now to arrive at the proof, firstly it is evident that T , which by the supposition is centre of gravity of the triangle QSR , is also centre of gravity (by the 15th proposition of the 2nd book of the elements of the Art of Weighing) of the body $KLMNOP$. But VX is through T at right angles to the horizon, therefore VX is the centre line of gravity of the body. If therefore we produce the line XY downwards, the body $KLMNOP$ will, with the point X on the line XY , keep its given position in the mathematical sense; therefore X is centre of gravity of the total pressure of the body on the bottom $KLMN$. But VX is through the centre of gravity T at right angles to the horizon, and thus also parallel to SR . And consequently it intersects QR (by the 5th proposition of the 2nd book of the elements of the Art of Weighing), in such a way that QX is double of XR . But as has been said above, the centre of gravity falls in the bottom $ACDE$ in the same way as it does in the bottom $KLMN$; therefore it falls in it in such a way that the upper part of the line FG is double of the lower. But that is in H ; therefore H is the centre of gravity of the total pressure of the water on the bottom $ACDE$.

wy de lini XY neerwaert trecken, t'lichaem KLMNOP sal metiet punt X op de lini XY, * Wisconstlick verstaen, sijn ghegheuen standt houden, daerom X is swaerheys middelpunt van des lichaems gheprang, vergaert inden bodem KLMN; maer VX is duer t'swaerheys middelpunt T rechthouckich op den sichteinder, daerom oock euewydich met SR, ende vervolghens sy snijt QR (duer het 5^e voorstel des 2^d boucx vande beghinfelen der Weeghconst) alsoo dat QX dobbel is an XR; Maer so bouen gheseyt is, t'swaerheys middelpunt valt inden bodem ACDE, in sulcken ghestalt ghelijck inden bodem KLMN doet, het valter dan alsoo in, dattet bouenste deel der lini FG, dobbel is an ronderste, maer dat is in H, daerom H is t'swaerheys middelpunt van r'gheprangh des waters inden bodem ACDE vergaert.

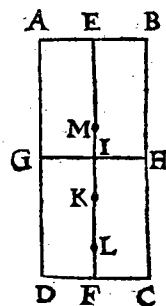
*Mathemati-
cæ.*

II VOORBEELT.

Om alfulcke redenen als int 4^e voorbeelt des 11^{en} voorstels gheseyt sijn,ullen wy hier bouen t'voorgaende * Wisconstlich bewys, noch een voorbeelt duer ghetalen stellen, aldus:

*Mathemati-
cæ demon-
strationem.*

Laet ABCD een bodem sijn, daer in ghetrocken is de lini EF, tusschen de middelen van AB, ende DC, deelende dien bodem in ettelicke euen deelen (die wy maten noemen) met linien euewydich van AB, ick neem ten eersten in twee, mette lini GH, snijende EF in I, ende t'punt K sy alsoo, dat EK dobbel is an KF, welcke K wy bewysen moeten t'swaerheys middelpunt des gheprangs te sijn aldus: Ghenomen dat teghen ABHG, ruste 1 pondt, ofte ghewicht waters, soo salder teghen GHCD sücke 3 ghewichten rusten: Dit so sijnde, ick acht ten eersten al oft t'swaerheys middelpunt des gheprangs van ABHG, waer in I, ende van GHCD in F (tis seker dat sy hoogher sijn) so sal IF balck wesen, welcke ghedeelt in haer ermen tot malcanderen in sulcken reden als de voornoemde ghewichten van 3 tot 1, t'welck in t'punt L valle, so sal FL doen $\frac{1}{4}$ eender maet, dat is $\frac{1}{4}$ van IF. Ten tweeden so achtick, al oft t'swaerheys middelpunt des gheprangs van ABHG waer in E, ende van GHCD in I (tis seker dat sy leegher sijn) soo sal haer ghemeen swaerheys middelpunt vallen een maet bouen L als in M. Tis dan blijkcllick dattet ware begheerde swaerheys middelpunt is tusschen M ende L. Maer ghelijck wy den bodem hier bouen ghedeelt hebben in twee, alsoo canmense deelen in oneindelicke stucken, daer af vinden de twee swaerheys middelpunten als bouen, tusschen de welcke altijdt is, het ware begheerde swaerheys middelpunt. Wy connen dan duer sulcke



EXAMPLE II.

For all such reasons as mentioned in the 4th example of the 11th proposition, we will, in addition to the foregoing mathematical proof, also give an example by means of numbers, as follows:

Let $ABCD$ be a bottom in which there is drawn the line EF , joining the middle points of AB and DC , dividing that bottom into several equal parts (which we call measures) by lines parallel to AB . I take the bottom first to be divided in two, by the line GH , intersecting EF in I , and the point K shall be such that EK is double of KF , which K we have to prove to be the centre of gravity of the pressure, as follows. Assuming that there rest against $ABHG$ 1 pound or one weight of water, there will rest 3 such weights against $GHCD$. This being so, I first imagine the centre of gravity of the pressure of $ABHG$ to be in I and that of $GHCD$ in F (it is certain that they are higher); then IF will be beam, and if this is divided into its arms having to each other the same ratio as the aforesaid weights, i.e. 3 to 1, which point of division shall fall in L , FL will be $\frac{1}{4}$ of a measure, i.e. $\frac{1}{4}$ of IF . Secondly I imagine the centre of gravity of the pressure of $ABHG$ to be in E , and that of $GHCD$ in I (it is certain that they are lower); then their common centre of gravity will fall one measure above L , viz. in M . It is therefore evident that the true required centre of gravity is between M and L . But just as above we divided the bottom in two, it is also possible to divide it into an infinite number of parts, and find two centres of gravity thereof ¹⁾, as above, between which is always the true required centre of gravity. We can therefore, by this means, always approximate infinitely. If therefore we find by this experience that the point L never reaches K , but remains very near to it and always below it; in the same way that the point M never reaches K , but always remains above it, we conclude from this that K is the true required centre of gravity. But because it would be a difficult calculation to find in this way the common centre of gravity of all those bottoms, we shall explain a short method for doing this, as follows. I write down a progression, as 1.3.5.7.9 and so on, always ascending by 2, for in such a progression and proportion are the pressures of the equal parts of a bottom $ABCD$ by the 15th proposition. Thereafter I place $\frac{1}{4}$ (which has been found above for FL) above the second number 3, as below:

$$\begin{array}{ccccccc} & \frac{1}{4} & & & & & \\ 1 & . & 3 & . & 5 & . & 7 & . & 9 & . & 11 \end{array}$$

Thereafter I add up 4, the denominator of $\frac{1}{4}$, and 5 (the third term of the progression), which makes 9; I place that as denominator above the 5, and above the 9 I place 5, i.e. the sum of the denominator and the numerator of $\frac{1}{4}$, so that the scheme is then as follows:

$$\begin{array}{ccccccc} & \frac{1}{4} & & \frac{5}{9} & & & \\ 1 & . & 3 & . & 5 & . & 7 & . & 9 & . & 11 \end{array}$$

In the same way I also find all the others, for in order to have the number that is to be placed above 7, I add up the denominator 9 and 7, which makes 16. Above this I place the sum of 9 and 5 (which are the denominator and the

¹⁾ To wit one centre of gravity, when the centre of each strip is taken to be in its lowest side, and one, when it is taken to be in the highest side.

fulcke middel altijt oneindelick naerderen, daerom als wy duer dese er-
uaring beuinden, dattet punt als L nummermeer tot K en comt, maer
seer by ende altijt daer onder blijft; Sghelijcx dattet punt als M nummer-
meer tot K en comt, maer altijt daer bouen blijft, wy besluyten uyt sulcx,
dat K het ware begheerde swaerheyt middelpunt is. Maer want het moy-
licke rekeninghe soude sijn t'ghemeene swaerheys middelpunt van alle
die bodems also te vinden, wy sullen daer af een corte manier verclaren
Progreffionē. aldus, ick schrijf een * voortganck als 1. 3. 5. 7. 9. ende so voort altijt met
tween opclimmende, want in sulcken voortganck ende reden sijn de
prangelen der euen deelen eens bodems A B C D duer het 1st voorstel,
daer naer stel ick $\frac{1}{4}$, (welck hier bouen beuonden is voor F L) bouen
het tweede ghetal 3, als hier onder:

$$1. \quad \frac{1}{4} \quad 3. \quad 5. \quad 7. \quad 9. \quad 11.$$

Daer naer vergaet ick 4, noemer van $\frac{1}{4}$, met de 5 derde in d'oirden,
comt 9, die stel ick als noemer bouen de 5, ende bouen de 9 set ick 5, dat
is de somme des noemers en telders van het $\frac{1}{4}$ welcker gheftalt dan al-
dus is:

$$1. \quad \frac{1}{4} \quad \frac{5}{9} \quad 3. \quad 5. \quad 7. \quad 9. \quad 11.$$

Sghelijcx vinde ick oock alle d'ander, want om t'ghetal te hebben dat
bouen 7 comen sal, ick vergaet den telder 9 ende 7, maect 16, daer bo-
uen stelick de somme van 9 ende 5 (die noemer ende telder sijn vande
 $\frac{5}{9}$) maect 14. Inder voughen dat bouen de 7 comen sal $\frac{14}{16}$, wiens ghe-
ftalt dan aldus sijn sal:

$$1. \quad \frac{1}{4} \quad \frac{5}{9} \quad \frac{14}{16} \quad 3. \quad 5. \quad 7. \quad 9. \quad 11.$$

Ende soo voortgaende, bouen de 9 ende 11 sullen ghetalen comen als
hier onder:

$$1. \quad \frac{1}{4} \quad \frac{5}{9} \quad \frac{14}{16} \quad \frac{20}{25} \quad \frac{35}{36} \quad 3. \quad 5. \quad 7. \quad 9. \quad 11.$$

Dit soo verstaen sijnde, men wilt weten neem ick, waer t'punt als L
vallen sal, wanneer den bodem ghedeelt is in vijf euen deelen: Ick sien
wat ghetal datter bouen t'vijfde in d'oirden staet, dat is bouen de 9, ende
beuinde $\frac{14}{16}$, diens eerste ghebroken doet $\frac{6}{7}$, daer uyt besluyt ick dat de
lini als L F van sulcken bodem in vijuen ghedeelt, sijn sal van $\frac{6}{7}$ een-
der maet, der maten daer den bodem in ghedeelt is, maer dat die min sijn
dan $\frac{1}{5}$ van E F, ende dat haer uysterste als L vallen sal onder K, wort al-
dus bethoont: De $\frac{6}{7}$ eender maet der maten daer den bodem in ghedeelt
is, dat is $\frac{6}{7}$ van $\frac{1}{5}$ doen $\frac{6}{35}$, vande heele lini als E F, welcke $\frac{6}{35}$ minder
sijn als

numerator of $\frac{5}{9}$), which makes 14. In such a way that above the 7 there shall be placed $\frac{14}{16}$, so that the scheme will then be as follows:

$$\begin{array}{ccccccc} & \frac{1}{4} & \frac{5}{9} & \frac{14}{16} & & & \\ 1 & \cdot & 3 & \cdot & 5 & \cdot & 7 \cdot 9 \cdot 11 \end{array}$$

And proceeding in this way, above the 9 and 11 there will come the numbers shown below:

$$\begin{array}{ccccccc} & \frac{1}{4} & \frac{5}{9} & \frac{14}{16} & \frac{30}{25} & \frac{55}{36} & \\ 1 & \cdot & 3 & \cdot & 5 & \cdot & 7 \cdot 9 \cdot 11 \end{array}$$

This being understood, I assume that it is desired to know where the point L will fall when the bottom is divided into five equal parts. I ascertain what number is above the fifth term of the progression, i.e. above the 9, and find $\frac{30}{25}$, which in its lowest terms is $\frac{6}{5}$; from this I conclude that the line LF of this bottom divided into five parts will be $\frac{6}{5}$ of a measure, of the measures into which the bottom is divided. But that this is less than $\frac{1}{3}$ of EF , and that its extremity, viz. L , will fall below K , is proved as follows: The $\frac{6}{5}$ of a measure of the measures into which the bottom is divided, i.e. $\frac{6}{5}$ of $\frac{1}{5}$, make $\frac{6}{25}$ of the complete line EF , which $\frac{6}{25}$ is less than $\frac{1}{3}$ FK , for if $\frac{6}{25}$ be subtracted from $\frac{1}{3}$, there is left $\frac{7}{75}$ of the line EF , and the point L will be at this distance from K . But in order to find the point M , I add one measure to the $\frac{6}{5}$ measures, which makes $\frac{11}{5}$ of a measure. This is $\frac{11}{25}$ of the complete line EF , which $\frac{11}{25}$ is more than $\frac{1}{3}$ of FK . For if $\frac{1}{3}$ be subtracted from $\frac{11}{25}$, there is left $\frac{8}{75}$ of the line EF , and the point M will be at this distance from K , that is $\frac{1}{75}$ further away from it than L . And the same with all the others, for if the bottom $ABCD$ were divided into 40 equal parts, the line FL would be found to be $\frac{20,550}{1,600}$ of a measure, that is of one fortieth part of the line EF , through which the points L and M would be found to be much nearer than above, though they will never reach it, the necessity of which has been proved mathematically in the 1st example described above. The ground of the above short method for finding the common centre of gravity of the various pressures will be easily understood by those who seek for it at full length according to the theory of 2nd proposition of the 1st book of the elements of the Art of Weighing 1). CON-

1) The procedure may be explained as follows. Dividing the surface into n strips, putting the pressure on the highest strip k_n , and the height of each strip p_n , we have: Pressures on successive strips (starting from the highest):

$$k, 3k, 5k, \dots, (2n-1)k$$

Distances from the centres of pressure, supposed to be in the lowest sides of the strips, to F :

$$(n-1)p_n, (n-2)p_n, \dots, p_n, 0$$

Moments of the pressures with respect to F :

$$(n-1)p_n k_n, 3(n-2)p_n k_n, \dots, (2n-3)p_n k_n.$$

Total pressure

$$(1+3+5+\dots+2n-1)k_n = n^2 \cdot k_n$$

We now put the total moment $p_n k_n [(n-1) + 3(n-2) + \dots + (2n-3)] = p_n k_n T_n$

sijn als $\frac{1}{3}$ F K, want ghetrocken $\frac{6}{21}$ van $\frac{1}{3}$, blijft $\frac{7}{21}$ der lini E F, ende so verre sal dan t'punt als L van K vallen. Maer om t'punt als M te vinden, ick doe een maet tot de $\frac{6}{7}$ maets, comt $\frac{1}{7}$ eender maet, de selue doen $\frac{1}{21}$ vande heele lini E F, welcke $\frac{1}{21}$ meerder sijn dan $\frac{1}{3}$ van F K, want ghetrocken $\frac{1}{3}$ van $\frac{1}{21}$ blijft $\frac{8}{21}$ der lini E F, ende so verre sal dan t'punt als M van K vallen, dat is $\frac{1}{7}$ verder dander L afviel, ende alsoo met allen anderen, want soomen den bodem A B C D deelde in 40 euen deelen, de lini als F L soude beuonden worden van $\frac{20 \frac{1}{2} \frac{1}{20}}{1600}$ eender maet, dat is eens veertichstendeels der lini E F, duer t'welcke men de punten als L M veel naerder soude beuinden dan bouen, maer nummermeer daer toe comen, waer af de nootzakelicheyt int bouenscreuen 1^e voorbeelt Wiskonstelick betoocht is. De reden vande bouescreuen corte manier der vindingh des ghemeen swaerheys middelpunts van die verscheyden prangfelen, sal den ghenen lichtelick connen bemercken, die se in t'langhe souckt naer de leering des 2^{de} voorstels van her 1^e bouck der beghinfelen vande Weeghconst. T'BESELYT. Wefende dan den bodem des waters een euewydich vierhouck oneuewydich, &c.

XIII VERTOCH.

XIX VOORSTEL.

W E S E N D E den bodem des waters een euewydich vierhouck oneuewydich vanden ^{Horizonte.} sichteinder, diens hoogste sijde onder t'waters oppervlack is, maer euewydich vanden sichteinder, uyt welcke sijdens middel een lini ghetrocken is, tot in t'middel vande leeghste sijde: T'swaerheys middelpunt des gheprangs inden bodem vergaert, is inde lini tusschen t'middelpunt des bodems, ende t'punt dat het onderste derdendeel dier lini afsnijt; ende tusschen die twee punten in foodanighen punt, t'welck t'onderste deel alsoo afsnijt, dattet sulcken reden heeft totter bouenste, ghelijck de ^{Perpendic-} hanghende lini van t'waters oppervlack ^{laris.} in des bodems leeghste sijde, tot den helft der hanghende lini van des bodems hoogste sijde, totter ^{Planum.} plat euewydich vanden sichteinder duer des bodems leeghste sijde.

Gg

T'GHE-

CLUSION. The bottom in the water therefore being a parallelogram non-parallel, etc.

THEOREM XIII.

PROPOSITION XIX.

The bottom in the water being a parallelogram non-parallel to the horizon, whose highest side is below the water's upper surface, but parallel to the horizon, from the middle point of which side there is drawn a line to the middle point of the lowest side: the centre of gravity of the total pressure on the bottom is in the line joining the middle point of the bottom and the point cutting off the lower third part of that line, and between those two points in a point such as cuts off the lower part in such a way that it has to the upper part the same ratio as the vertical from the water's upper surface in the highest²⁾ side of the bottom to the half of the vertical from the highest side of the bottom to the plane parallel to the horizon through the lowest side of the bottom.

The distance from the resulting centre of pressure to F is

$$\frac{p_n k_n \cdot T_n}{n^2 \cdot k_n} = \frac{p_n}{n^2} T_n.$$

If we now replace n by $(n+1)$, the total pressure becomes $(n+1)^2 \cdot k_{n+1}$.

The sum of the moments is now:

$$n \cdot p_{n+1} \cdot k_{n+1} + 3(n-1) p_{n+1} \cdot k_{n+1} + \dots + (2n-1) p_{n+1} \cdot k_{n+1} = p_{n+1} \cdot k_{n+1} [T_n + \{1 + 3 + \dots + (2n-1)\}] = p_{n+1} \cdot k_{n+1} [T_n + n^2] = p_{n+1} \cdot k_{n+1} \cdot T_{n+1}.$$

The distance $LF = \frac{T_n + 1}{(n+1)^2} \cdot p_{n+1}$, where $T_{n+1} = T_n + n^2$.

This recurrent relation, together with the initial value 0 for $n=1$, determines the successive values of the coefficient of p_n . This gives indeed Stevin's series for $\frac{T_n}{n^2}$

$$\frac{0}{1} \frac{1}{4} \frac{5}{9} \frac{14}{16} \frac{30}{25}$$

For $n=5$ we find with Stevin for the distance LF , if $EF = l$, $\frac{30}{25} \cdot \frac{l}{5} = \frac{6}{25} l$ and so for

$$LK \left(\frac{1}{3} - \frac{6}{25} \right) l = \frac{7}{75} l.$$

If the centres are all taken in the highest sides of the strips, the recurrent relation is

$$T_{n+1} = T_n + (n+1)^2,$$

and the series of coefficients becomes $\frac{1}{1} \frac{5}{4} \frac{14}{9} \frac{30}{16} \frac{55}{25}$.

For the distance MK from the resultant centre to K we now find with Stevin

$$\left(\frac{55}{25} \cdot \frac{1}{5} - \frac{1}{3} \right) l = \frac{8}{75} l.$$

The general expression for FL proves to be: $FL = \frac{1}{6} \cdot \frac{n(n-1)(2n-1)}{n^2} \cdot \frac{l}{n}$,

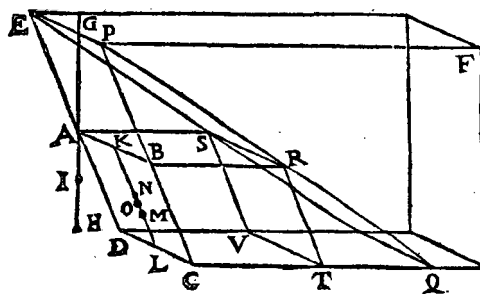
which gives for $n=40$

$$FL = \frac{1}{6} \cdot \frac{40 \cdot 39 \cdot 79}{1,600} \cdot \frac{l}{40} = \frac{20,540}{1,600} \cdot \frac{l}{40} \text{ and which for } n \rightarrow \infty \text{ converges towards } \frac{l}{3}.$$

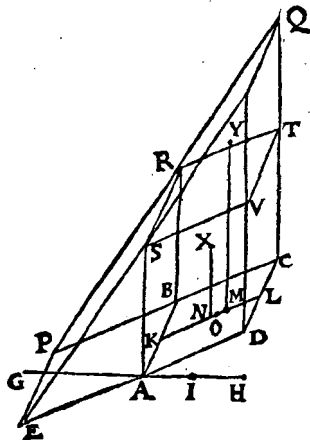
Stevin finds 20,550 for the nominator.

2) Stevin's text has „leeghste", which obviously is an error.

T'GHEGHEVEN. Laet $ABCD$ een bodem sijn oneuwydich vanden sichteinder diens hoochste sijde AB onder 'twaters oppervlack EF is, maer euwydich vanden sichteinder, ende GA sy de hanghende lini van 'twaters oppervlack tot de hoochste sijde AB , ende AH de hanghende lini van A , tottet plat euwydich vanden sichteinder duer DC , ende AI sy den helft van AH , ende KL sy de lini ghetrocken tusschen de middelen van AB ende DC , ende LM sy het derdendeel vande lini LK , ende N 'tmiddelpunt des bodems $ABCD$, ende O een punt tusschen M ende N , alsoo dat OM sulcken reden heeft tot ON , ghelijck AG tot AI . **T'BEGHEERDE.** Wy moeten bewyfen dat O 'tswaerheys middelpunt is van 't'gheprang des waters inden bodem $ABCD$ vergaert. **T'BEREYTSSEL.** Laet CB ende DA voortghetrocken worden tot in 'twaters oppervlack, als tot P en E , daer naer CQ en $an CP$, maer euwydich vanden sichteinder, ende rechthouckich op CD , daer naer BR euwydighe met CQ , wetende R inde lini PQ : Sghelijcx AS euen ende euwydighe met BR , voort RT , en SV euen ende euwydighe met BC .



Laet daer naer een ander form ghestelt worden, euen ghelijck ende euwichtich ande voorgaende $EPCDQ$, maer also dat CQ rechthouckich sy op de sichteinder, ende X sy swaerheys middelpunt des pilaers $ABCDRSVT$, ende Y swaerheys middelpunt des li-



chaems

SUPPOSITION. Let $ABCD$ be a bottom non-parallel to the horizon, whose highest side AB is below the water's upper surface EF , but parallel to the horizon, and GA shall be the vertical from the water's upper surface to the highest side AB , and AH the vertical from A to the plane parallel to the horizon through DC , and AI shall be the half of AH , and KL shall be the line joining the middle points of AB and DC , and LM shall be the third part of the line LK , and N the centre of the bottom $ABCD$, and O a point between M and N , in such a way that OM has to ON the same ratio as AG to AI . WHAT IS REQUIRED TO PROVE. We have to prove that O is the centre of gravity of the total pressure of the water on the bottom $ABCD$.

PRELIMINARY. Let CB and DA be produced to the water's upper surface, viz. to P and E ; thereafter let CQ be made equal to CP , but parallel to the horizon and at right angles to CD ; thereafter BR parallel to CQ , R being in the line PQ . In the same way AS equal and parallel to BR ; further RT and SV equal and parallel to BC .

Thereafter let there be drawn another figure, equal, similar, and of equal weight to the preceding $EPCDQ$, but in such a way that CQ be at right angles to the horizon, and X shall be centre of gravity of the prism $ABCDRSVT$ and

289 1438
473 1416

chaems $R S V T Q$, Laet oock ghetrocken worden de linien $X N$ ende $Y M$. **TBEWYS.** Angheſien in deſe tweede form, X ſwaerheys middelpunt is des pilaers $A B C D R S V T$, ende N ſwaerheys middelpunt haers gronds $A B C D$, ende dat $C T$ rechthouckich is op den ſichteinder, ſoo is $X N$ haer euewydighe, oock rechthouckich op den ſichteinder, ende veruolghens huer ſwaerheys middellini, daerom oock is N ſwaerheys middelpunt des gheprangs diens pilaers; Maer M ſwaerheys middelpunt te weſen des gheprangs van t'lichaem $S R T V Q$ dat is int 18° voorſtel betoocht: T'welck ſo ſijnde, $M N$ is Weeghconſtighen balck, die in O alſoo ghedeelt is, dat ghelijck $A G$ tot $A I$, alſoo $O M$ tot $O N$ duer t'ghegheuen, maer ghelijck $A G$ tot $A I$, alſoo den pilaer $A B C D R S V T$, totter lichaem $S R T V Q$, daerom ghelijck den pilaer $A B C D R S V T$, totter lichaem $S R T V Q$, alſoo $O M$ tot $O N$, waer duer O t'ſwaerheys middelpunt is deſer tweede form, duer het 1° voorſtel des eerſten boucx vande beghinſelen der Weeghconſt, maer t'ſwaerheys middelpunt van d'eerſte form, om de redenen alſvooren, valt aldaer ghelijck in de tweede, O dan der eerſte form, is t'begheerde ſwaerheys middelpunt. **T'BESELVYT.** Weſende dan den bodem des waters een euewydich vierhouck onuewydich vanden ſichteinder, &c.

VII EYSC.

XX VOORSTEL.

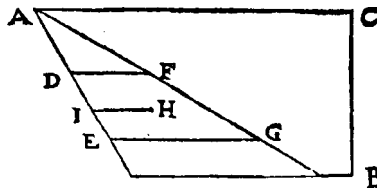
WESENDE den bodem in t'water een rechtlinich* plat van form ſoot valt: Te vinden het *Planum.* ſwaerheys middelpunt des gheprangs inden bodem vergaert.

T'GHEGHEVEN. Laet $A B$ een water weſen, diens oppervlack $A C$, ende $D E$ een bodem, welcke een rechtlinich plat ſy.

T'BEGHEERDE. Wy moeten het ſwaerheys middelpunt vinden van t'gheprang des waters in dien bodem vergaert.

T'WERCK. Men ſaleerſt vinden een lichaem waters eueſwaer an t'gheprang teghen den bodem $D E$, naer de leering des 13° voorſtels, t'ſelue ſy $D E F G$, vindende daer naer ſijn ſwaerheys middelpunt duer het 21° voorſtel des tweeden boucx vande beghinſelen der Weeghconſt,

t'welck H ſy, daer naer ghetrocken $H I$ euewydighe met $G E$, diens *G g 2* *uyterſte*



Y centre of gravity of the body $RSVTQ$. Let there also be drawn the lines XN and YM . PROOF. Since in this second figure X is centre of gravity of the prism $ABCDRSVT$, and N centre of gravity of its base $ABCD$, and CT is at right angles to the horizon, XN — which is parallel to it — is also at right angles to the horizon, and consequently also its centre line of gravity. Therefore N is centre of gravity of the pressure of that prism. But that M is centre of gravity of the pressure of the body $SRTVQ$ has been shown in the 18th proposition. Which being so, MN is a mathematical beam, which is so divided in O that as AG is to AI , so is OM to ON by the supposition. But as AG is to AI , so is the prism $ABCDRSVT$ to the body $SRTVQ$; therefore, as the prism $ABCDRSVT$ is to the body $SRTVQ$, so is OM to ON , owing to which O is the centre of gravity of this second figure, by the 1st proposition of the first book of the elements of the Art of Weighing. But, for the reasons mentioned above, the centre of gravity of the first figure falls there as in the second. O of the first figure therefore is the required centre of gravity. CONCLUSION. The bottom in the water therefore being a parallelogram non-parallel to the horizon, etc.

PROBLEM VII.

PROPOSITION XX.

The bottom in the water being a rectilinear plane figure of any form: to find the centre of gravity of the total pressure on the bottom.

SUPPOSITION. Let AB be a water, whose upper surface is AC , and DE a bottom which shall be a rectilinear plane figure. WHAT IS REQUIRED TO FIND. We have to find the centre of gravity of the total pressure of the water on that bottom. CONSTRUCTION. There shall first be found a body of water of equal weight to the pressure against the bottom DE , according to the theory of the 13th proposition. This shall be $DEFG$. Thereafter its centre of gravity shall be found, by the 21st proposition of the second book of the elements of the Art of Weighing, which shall be H , and if then HI be drawn parallel to GE , whose extremity I

uyterste punt I inden bodem D E sy; Ick seg t'selue punt I te wesen t'be-
gheerde swaerheys middelpunt, waer af t'bewys ghelijck sal sijn ande
bewyfen des voorgaenden 18^{en} ende 19^{en} voorstels.

T'BESELYT. Wefende dan den bodem int water een rechtli-
nich plat, &c.

VIII EYSCH.

XXI VOORSTEL.

W E S E N D E ghegheuen een water onbeken-
der grootheyt, maer bekender swaerheyt: Sijn
grootheyt duer sijn eyghenwicht te vinden.

Geometrick.

M E R C K T. Men soude des waters grootheyt mueghen* Meetconf-
lick vinden naer de ghemeene reghel van dien, maer want het in cley-
ne menichvuldicheyt, Weeghconflick ghreeder ende sekerder wer-
king is, voornamelick inde ongheschickte formen, wy sullense daer duer
beschrijuen.

T'GHEGHEVEN. Laet A een water sijn diens grootheyt onbekent
is, maer tis bekender swaerheyt, dat is (duer de 1^e bepaling deses boucx)
dat sijn bekende grootheyt duer bekent ghewicht can gheuytet worden;
ick neem dat een voet des selfden weghe 65 lb.

T'BEGHEERDE. Wy moeten de grootheyt van A duer haer ey-
ghenwicht vinden.

T'WERCK. Men sal t'water A weghen, t'welck
ick neem beuonden te worden van 5 lb, die ghedeelt
duer de voornomde 65 lb, comt $\frac{1}{13}$; dat is $\frac{1}{13}$ voets
voor de begheerte grootte van A.

T'BEWYS. Anghesien t'water A 5 lb weeght, ende
dat een voet des selfden weeght 65 lb, ende dattet
oueral eenvaerdigher swaerheyt is duer de 2^e be-
gheerte, soo heeft sijn ghewicht sulcken reden tot 65
lb, als sijn grootheyt tot een voet, maer 5 lb heeft tot
65 lb, de reden van 1 tot 13, daerom sijn grootheyt
heeft sulcken reden tot 1 voet, als 1 tot 13, de grootheyt dan des waters
A is $\frac{1}{13}$ voets, t'welck wy bewyfen moesten.

T'BESELYT. Wefende dan ghegheuen een water onbekender
grootheyt maer bekender swaerheyt, wy hebben sijn grootheyt duer sijn
eyghenwicht gheuonden, naer den eysch.

IX EYSCH.

XXII VOORSTEL.

W E S E N D E ghegheuen tweer lichamen redenen
der grootheyt, en stoffswaerheyt, en t'ghewicht vā
t'een lichaem: T'ghewicht van t'ander te vinden.

T'GHE-



shall be in the bottom *DE*, I say that this point *I* is the required centre of gravity, the proof of which will be identical with those of the 18th and 19th propositions hereinbefore. CONCLUSION. The bottom in the water therefore being a rectilinear plane figure, etc.

PROBLEM VIII.

PROPOSITION XXI.

Given a water of unknown volume, but known gravity ¹⁾: to find its volume from its proper weight.

NOTE.

One might find the water's volume geometrically according to the common rule about this, but because with a small quantity the weighing method is quicker and surer, especially with irregular forms, we shall describe them by this latter method.

SUPPOSITION. Let *A* be a water whose volume is unknown, but whose gravity is known, i.e. (by the 1st definition of this book) its known volume can be expressed by the known weight. I assume that one foot of it weighs 65 lbs. WHAT IS REQUIRED TO FIND. We have to find the volume of *A* from its proper weight. CONSTRUCTION. The water *A* shall be weighed, which I take to be

found 5 lbs; the latter, divided by the aforesaid 65 lbs, makes $\frac{1}{13}$, i.e. the required volume of *A* is $\frac{1}{13}$ foot. PROOF. Since the water *A* weighs 5 lbs, and one foot of it weighs 65 lbs, while it has uniform gravity throughout, by the 2nd postulate, its weight has to 65 lbs the same ratio as its volume to one foot. But 5 lbs has to 65 lbs the ratio of 1 to 13, therefore its volume has to 1 foot the ratio of 1 to 13. The volume of the water *A* therefore is $\frac{1}{13}$ foot, which we had to prove. CONCLUSION. Given therefore a water of unknown volume, but known gravity, we have found its volume from its proper weight, as required.

PROBLEM IX.

PROPOSITION XXII.

Given the ratio of the volumes and that of the specific gravities of two bodies, and the weight of the one body: to find the weight of the other.

¹⁾ This naturally means: specific gravity.

T'GHEGHEVEN. Laet A B t'een lichaem wesen, ende C t'ander, ende de reden der grootheyt van A B tot C, sy van 3 tot 1 ende der stoffwaerheyt van 1 tot 2, ende A B weeghe 6 lb.

T'BEGHEERDE. Wy moeten t'ghewicht des lichaems C vinden.

T'WERCK. Ick teecken D B euegroot met C, de selue D B dan is het derdendeel van A B 6 lb, daerom D B weeght 2 lb, maer de stoffwaerheyt van D B tot C, is als van 1 tot 2, daerom soo weeght C 4 lb.

T'BEWYS. Laet C (soot mueghelick waer) meer dan 4 lb weghen; T'welck soo ghenomen huer swaerheyt sal meerder dan dobbel reden hebben tot de swaerheyt van D B, want D B weeght 2 lb, ende veruolghens de stoffwaerheyt van C (anghesien C ende D B euen groot sijn) sal in meerder dan dobbel reden sijn tot D B, t'welck teghen t'ghestelde is, daerom en weeght C niet meer dan 4 lb. Sghelijcx salmen oock bethoonen dat sy niet min en weeght, sy weeght dan nootsakelick 4 lb t'welck wy bewylen moesten.

T'BESLUYT. Wefende dan ghegheuen t'weer lichamen redenen der grootheyt, ende stoffwaerheyt, ende t'ghewicht van t'een lichaem, wy hebben t'ghewicht van t'ander lichaem gheuonden na den cyfch.

VERVOLGH. Tis uyt het voorgaende openbaer dat,

Ghetrocken reden der grootheyt, van reden des ghewichts, rest reden der stoffwaerheyt.

Ghetrocken reden der stoffwaerheyt, van redendes ghewichts, rest reden der grootheyt.

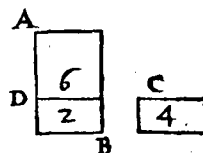
Vergaert reden der stoffwaerheyt, tot reden der grootheyt, comt reden des ghewichts.

WAER uyt blijktt dat een ghebrekende *pael der ses, duer de *Terminus* vijf ghegheuen palen alijt bekend can worden. Maer om t'selue by voorbeeld te verclaren, laet A weghen 6 lb, ende groot sijn 5 voeten;

ende t'ghewicht van B sy onbekent, maer huer grootheyt is van 2 voeten, ende de reden der stoffwaerheyt van A tot B, sy van 4 tot 7.

Nu om t'onbekende ghewicht van B te vinden, ick vergaer redē der stoffwaerheyt, dat is Reden $\frac{4}{7}$ tot reden der grootheyt, dat is Reden $\frac{1}{2}$, comt reden des ghewichts Reden $\frac{2}{7}$,

t'ghewicht dan van A heeft sulcken reden totter ghewicht van B, als 10 tot 7



	A	B
Ghewichten.	6 lb.	4 $\frac{2}{7}$ lb
Grootheden.	5 voet.	2 voet
Stoffwaerheden.	4	7

G g 3 tot 7

SUPPOSITION. Let AB be the one body and C the other, and the ratio of the volume of AB to that of C shall be 3 to 1, and that of the specific gravities 1 to 2, and AB shall weigh 6 lbs. **WHAT IS REQUIRED TO FIND.** We have to find the weight of the body C . **CONSTRUCTION.** I draw DB as having the same volume as C ; this DB is therefore one-third of AB (6 lbs), so DB weighs 2 lbs. But the specific gravity of DB to that of C is 1 to 2, therefore C weighs 4 lbs. **PROOF.** Let C (if this were possible) weigh more than 4 lbs. This being assumed, its gravity will be more than double of the gravity of DB , for DB weighs 2 lbs. And consequently the specific gravity of C (seeing that C and DB have the same volume) will be more than double of DB , which is contrary to the supposition. Therefore C does not weigh more than 4 lbs. In the same way it can also be shown that it does not weigh less. Therefore it necessarily weighs 4 lbs, which we had to prove.

CONCLUSION. Given therefore the ratio of the volumes and of the specific gravities of two bodies, and the weight of the one body, we have found the weight of the other body, as required.

COROLLARY.

It is manifest from the foregoing that

The ratio of the volumes being subtracted¹⁾ from the ratio of the weights, there remains the ratio of the specific gravities. The ratio of the specific gravities being subtracted from the ratio of the weights, there remains the ratio of the volumes. The ratio of the specific gravities being added to the ratio of the volumes, there comes the ratio of the weights.

From this it is clear that if one of the six terms is unknown, it can always be made known from the five given terms. But in order to explain this with an example, let A weigh 6 lbs and be 5 feet, and the weight of B shall be unknown, but its volume is 2 feet, and the ratio of the specific gravities of A to B shall be 4 to 7. Now in order to find the unknown weight of B , I add the ratio of the specific gravities, i.e. ratio $\frac{4}{7}$, to the ratio of the volumes, i.e. ratio $\frac{5}{2}$, which yields the ratio of the weights: ratio $\frac{10}{7}$. The weight of A therefore has to the weight of B the ratio 10 to 7. But A weighs 6 lbs. Therefore I say: 10 gives 7, what 6 lbs? the weight of B becomes $4\frac{1}{5}$ lbs.

¹⁾ In the theory of ratios current in Stevin's days, subtraction of ratios meant division of the corresponding fractions, addition of ratios meant multiplication of the fractions. By subtracting the ratio $a : b$ from the ratio $c : d$ we therefore obtain the ratio $cb : ad$; by adding it to $c : d$, we get $ac : bd$.

tot 7, maer A weeght 6 lb, daerom seg ick 10 gheeft 7, wat 6 lb? comt voor t'ghewicht van B $4\frac{1}{2}$ lb.

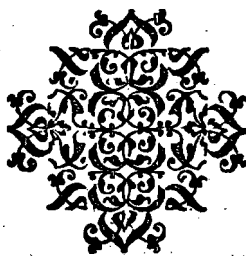
LAET ten tweeden de grootheyt van B onbekent sijn, welcke wy duer d'ander vijf palen vinden willen. Ick treck reden der stofswaerheyt, dat is *Reden* $\frac{4}{7}$, van reden des ghewichts, dat is *Reden* $\frac{10}{7}$, rest reden der grootheyt *Reden* $\frac{1}{2}$; de grootheyt dan van A, heeft sulcken reden tot de grootheyt van B, als 5 tot 1, maer A is groot 5 voeten, daerom seg ick 5 gheeft 2 wat 5 voeten? comt voor B 2 voeten.

LAET ten laetsten de reden der stofswaerheyt onbekent sijn, welcke wy door d'ander twe ghegeuen redenen bekend willen maken. Ick treck reden der grootheyt, dat is *Reden* $\frac{1}{2}$, van reden des ghewichts, dat is *Reden* $\frac{10}{7}$, rest reden der stofswaerheyt van 4 tot 7.

*Aquaticis
questionibus.*

Dit voorstel is ghemeen ouer alle stoffen, doch schijnt sijn grootste ghebruyck in * watersche verschillen te bestaen.

TEINDE DER BEGHINSELEN DES WATERWICHTS.



Secondly, let the volume of B be unknown, which we wish to find from the other five terms. I subtract the ratio of the specific gravities, i.e. ratio $\frac{4}{7}$, from the ratio of the weights, i.e. ratio $\frac{10}{7}$; the remainder is the ratio of the volumes: ratio $\frac{5}{2}$. The volume of A therefore has to the volume of B the ratio 5 to 2. But A is 5 feet. Therefore I say: 5 gives 2, what 5 feet? B becomes 2 feet.

Lastly, let the ratio of the specific gravities be unknown, which we wish to make known from the other two given ratios. I subtract the ratio of the volumes, i.e. ratio $\frac{5}{2}$, from the ratio of the weights, i.e. ratio $\frac{10}{7}$; the remainder is the ratio of the specific gravities: 4 to 7.

This proposition is common to all substances, but it seems to be used most in questions relating to waters.

THE END OF THE ELEMENTS OF HYDROSTATICS.



ANVANG DER
WATERWICHTDAET,
BESCHREVEN DVER
SIMON STEVIN
van Brugghe.

A N D E N L E S E R.

NADIEN hier vooren beschreuen sijn de Beghinselen des Waterwichts, soo soudet beta-
melick sijn, dat bekenick, de Waterwicht-
daet te volghen, van sulcx als wy daer af
connen verclaren; maer hebben om seker re-
denen gheschiët, dat voor t eerste niet schrif-
telick, maer werckelick te laten gheschien: Alleenlick sullen
h er drie voorstellen setten, die opentlick uyt het voorgaende
volghen, welke ons niet weerdich dunckende den naem van
Waterwichtdaet te verstrecken, doch ghemeenschap daer
mede hebbende, wy noemensē Anvang van dien. De selue be-
minde Leser belieue v int goede te nemen, ende de rest t sijnder
tijdt te verwachten.

Hoe

PREAMBLE OF THE PRACTICE OF HYDROSTATICS

Described by Simon Stevin of Brugghe.

TO THE READER.

Since in the foregoing there have been described the Elements of Hydrostatics, it would, I confess, be appropriate for the Practice of Hydrostatics to follow, in as far as we can explain it. But for certain reasons we have arranged this not to be done in writing for the present, but in actual fact. We shall only give three propositions which follow manifestly from the foregoing, and because we do not deem them worthy of the name of Practice of Hydrostatics, though they are connected therewith, we call them Preamble thereof. Dear reader, do not take this amiss, and expect the rest in due time.

Hoe t'ghewicht van een schip met al datter in ende op is, oft van eenich lichaem int water driuende, bekend wort duer de bekende grootheyt des deels int water liggende, sulcx is uyt het 6^e voorstel openbaer ghenouch, daerom sullen wy dat ouerslaen, ende wat segghen van t'ghene uyt het 7^e volght, aldus.

I VOORSTEL.

TE vinden hoe veel een selfde lichaem dat stoffichter is als water, in t'een water dieper sijncken sal als int ander dat stoffwaerder is.

LAET by voorbeelt een schip ligghen inden Rhijn te Leyen, ende men wil weten hoe veel datter daerin dieper sincken sal dan in See voor Catwyck. Men sal onderfoucken de reden der stoffwaertheyt van dat water tot dit, welke sy als van 42 tot 43, soo heb ickse in Hoymaent duer d'eruaring beuonden, want nemende twee euegroote lichamen, dat vanden Rhijn wouch 4260 azen maer t'Seewater 4362 azen, t'welck na ghenouch is als van 42 tot 43.

Geometria.

Mathematica.

Daerom salmen segghen, de grootheyt des deels van dat schip onder water in den Rhijn is tot de grootheyt van sulck deel onder water in See voor Catwyck, als van 43 tot 42, waer uyt den * Meter naer gheleghentheydt der form des voorghestelden schips, dese diepte tot die sal connen oordeelen. waer af de nootsaeklicheyt * Wilconstlick blijct int 7^e voorstel der beghinselen des Waterwichts.

II VOORSTEL.

DVER daetlicke voorbeelden te verclaren het 10^e voorstel der beghinselen des Waterwichts.

Wy hebben int 10^e voorstel der beghinselen des Waterwichts int 5^e vervolgh Wilconstlick bewesen, dat den bodem des watersaldaer E F, duer een grooter water (d'hoochde de selfde blijuende) niet meer beswaerten wort dan duer een cleinder, ende weder verkeert, datse duer een cleinder water soo seer beswaert wort, als duer een grooter: Maer want den menighen datvoor onnatuerlick mocht achten, sullen bouen t'voorgaende Wilconstich bewys, daer af vijf daetlicke voorbeelden beschrijven, welke yghelijck versoucken, ende ooghen schijnlick sien mach.

I^e VOORBEELT.

LAET den bodem A B euen ende ghelijck sijn anden bodem C D, ende de hoochde des waters op A B als E F, sy euen ande hoochde des waters

How the weight of a ship with all that is in or on it, or of any body floating in the water, becomes known from the known volume of the part lying in the water, this is sufficiently manifest from the 6th proposition. We will therefore omit that, and say something about what follows from the 7th, as follows.

PROPOSITION I.

To find how much deeper the same body, which is of greater specific levity than water, will sink in one kind of water than in another, which is of greater specific gravity.

For example, let a ship lie in the Rhine at Leyden, and it is required to know how much deeper it will sink therein than it does in the sea off Katwijk. The ratio of the specific gravity of the former water to the latter shall be ascertained, which shall be 42 to 43; this is the ratio I have found in Hay Month ¹⁾ by experience, for taking two bodies of equal volume, that of the Rhine weighed 4,260 azen ²⁾, but the seawater 4,362 azen, which is substantially 42 to 43.

Therefore it shall be said that the volume of the part of the ship under water in the Rhine is to the volume of such part under water in the sea off Katwijk as 43 to 42. From this the geometer will be able to judge, according to the form of the ship in question, the ratio of the latter depth to the former, the necessity of which is proved mathematically in the 7th proposition of the elements of Hydrostatics.

PROPOSITION II.

To explain by practical examples the 10th proposition of the elements of Hydrostatics.

We have proved mathematically in the 10th proposition of the elements of Hydrostatics, in the 5th corollary, that the bottom in the water there, *EF*, is not subject to any heavier pressure from a larger quantity of water (the height remaining the same) than from a smaller, and also the reverse: that it is subject to the same pressure from a smaller quantity of water as from a larger. But because many people may consider this unnatural, we will, in addition to the foregoing mathematical proof, describe five practical examples thereof, which anyone may test and see with his own eyes.

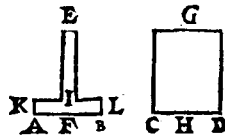
EXAMPLE I.

Let the bottom *AB* be equal and similar to the bottom *CD*, and the height of the water on *AB*, viz. *EF*, shall be equal to the height of the water on *CD*, viz.

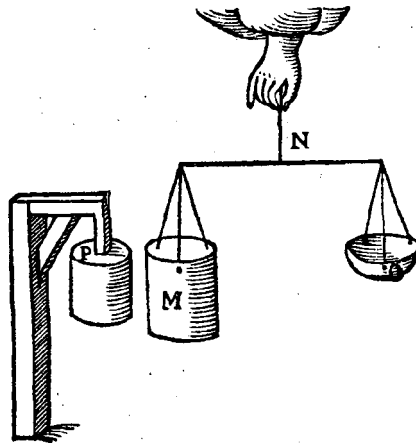
¹⁾ Haymonth is August.

²⁾ The „aas” is an ancient unit of weight. The word is derived from Latin *as*, unit of a system. The value of an „aas” is 46 or 48 mg. See K. M. C. Zevenboom and Dr D. A. Wittop Koning, *Nederlandse gewichten. Stelsels, ijkezezen, vormen, makers en merken*. Leiden 1953, p. 147.

waters op C D, als G H; maer het deel I E bouen t'water K L B A staende, sy cleender dan alsulck deel des lichaems G C D, ende t'water van E A B weghe 1 lb, ende van G C D 10 lb, ende de form van G C D sy een ronds pilaer, ende t'water G C D sal thienmael grooter en swaerder sijn dan t'water E A B, nochtans segghen wy t'ghewicht des waters E A B, euen soo stijf te drucken opden grondt A B, als t'ghewicht des waters G C D opden bodem C D. Twelck aldus daetlick bewesen wort :



Laet M N O een waegh sijn, diens schalen M, O, welcker schalen M van de form eens pilaers sy, euen ende ghelijck an t'vat hier bouen G C D, ende sal houden 10 lb waters; Laet oock P een houten lichaem wesen, vast staende als hier neuen, en lijckformich ande schael M, maer soo veel cleender datment daerin steken can sonder erghens ande schael te ghenaken.



Laet nu t'lichaem P ghesteken worden inde schael M, als in dees tweede form, ende inde schael O sy gheleyt t'ghewicht Q van 10 lb, ende den bodem der schael M sal soo stijf ghenaken teghen t'onderste des lichaems P, als de 10 lb van Q veroirsaken. Ick neem nu dat de ledighe plaets tusschen t'lichaem P ende de schael M, ghevult can worden met 1 lb waters, dat is met een lichaem waters euegroot an t'lichaem E A B; Daerom 1 lb waters in die ledighe plaets ghegoten, sal de schael M doen dalen, en O doen rijzen, so deruaring dat betuyghen sal, ende ghelijck de redenen daer af oock openbaer sijn duer t'boueschreuen 10^e voorstel. Dat 1 lb waters dan in die schael M gheleyt, sal daer in so grooten macht doen, als 10 lb ghewichts van loot yser ofie eenighe ander stijue stof ande schael M ghehecht. Ende om de selue reden sal 1 lb waters, also connen meer ghewelts doen dan duyfent ponden ander ghewicht. Dit soo sijnde daer is water tusschen den bodē der schael M en t'onderste des lichaems P,

H h

teghen

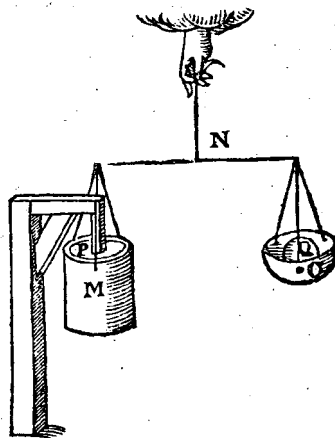
GH. But the part *EI* standing above the water *KLBA* shall be smaller than such part of the body *GCD*, and the water of *EAB* shall weigh 1 lb and that of *GCD* 10 lbs, and the form of *GCD* shall be a circular prism, and the water *GCD* shall be ten times greater and heavier than the water *EAB*. Nevertheless we say that the weight of the water *EAB* exerts the same pressure on the bottom *AB* as the weight of the water *GCD* on the bottom *CD*. This is proved in practice as follows:

Let *MNO* be a balance, whose pans are *M* and *O*, of which pans *M* shall have the form of a prism, equal and similar to the vessel *GCD* above, and it shall contain 10 lbs of water. Let *P* also be a wooden body, fixed as shown opposite and similar to the pan *M*, but so much smaller that it can be put therein without touching the pan anywhere.

Now let the body *P* be put in the pan *M*, as shown in the second figure, and in the pan *O* there shall be laid the weight *Q* of 10 lbs, and the bottom of the pan *M* shall touch the bottom of the body *P* as strongly as is caused by the 10 lbs of *Q*. I now assume that the empty space between the body *P* and the pan *M* can be filled with 1 lb of water, i.e. with a body of water having the same volume as the body *EAB*. Therefore, if 1 lb of water be poured into that empty space, this will cause the pan *M* to descend and *O* to ascend ¹⁾, as experience will show, and as the reasons thereof are also manifest from the 10th proposition described above. The 1 lb of water put in the pan *M* will therefore exert thereon the same force as 10 lbs of lead, iron or some other solid material attached to the pan *M*. And for the same reason 1 lb of water will thus be able to exert a greater force than one thousand pounds of weight in another form. This being so, there is water between the bottom of the pan *M* and the bottom of the body *P*, against

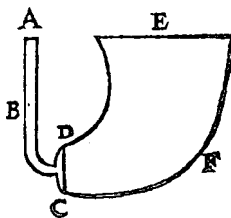
¹⁾ It is evident that if the bottom of *M* is completely covered with water, one pound of water in *M* will balance 10 pounds in *Q*, but not that *M* will descend and *O* ascend.

teghen welck water den bodem van M nu soo stijf druck, als sy eerst teghen t'onderste des lichaems P stack, want t'selue ghewicht Q ligt noch in d'ander schael O; Maer sy stack eerst soo stijf daer teghen als 10 lb van Q veroirsaecten, daerom den bodem van M steeckt soo stijf teghen t'water als de 10 lb van Q veroirsaken, ende, weder verkeert, t'water steeckt soo stijf teghen den bodem M, als die 10 lb van Q veroirsaecten. Laet ons nu nemen dattet water op den bodem M ligghe, euegroot sy an t'water K L B A, ende de rest rondtom t'licham P staende, euegroot mettet water I E; t'water dan E A B, druckt euen soo stijf teghen den grondt A B, als dit water tegen den grondt M, maer dit druckt soo stijf als 10 lb, soo bouen berhoont is, dat water E A B dan, druckt oock soo stijf teghen den grondt A B als 10 lb, ende so stijf druck oock t'water G C D teghen den grondt C D: Daerom soo wy voorghenomen hadden daetlick te bewyfen, t'water E A B weghende 1 lb, druckt euen soo stijf teghen sijn grondt A B, als t'water G C D weghende 10 lb, teghen sijn grondt C D. Ende ghelijck wy hier bewesen hebben 1 lb so stijf te drucken als 10 lb, alsoo salmen oock bewyfen 1 lb stijuer te connen drucken als duyfent ponden.



II VOORBEELT.

Laet A B C D een cleen dun buyken sijn, en C D E F een groot dick vat afghesondert van t'buyken, met een ghemeene bodem C D, ende beyde vol waters, alsoo dat der wateren oppervlacken in een selfde weereltvlack sijn. Nu dat het groot water des vats C D E F, niet stijuer en druckt teghen den bodem C D, dan t'cleyne water der cleyne buy, blijkt daetlick aldus: Laet gheweert worden den bodem D C, en t'groot water sal op die plaets teghen t'cleyenste stooten: Nu soo t'water C D E F, van te voren stijuer ghestooten had teghen den bodem D C, dan t'water A B C D, soo salt nu oock stijuer stooten teghen dat



which water the bottom of M now exerts as strong a pressure as it first did against the bottom of the body P , for the same weight Q still lies in the other pan O . But it first exerted against it a pressure such as was caused by 10 lbs of Q ; therefore the bottom of M exerts against the water a pressure such as is caused by the 10 lbs of Q , and also the reverse: the water exerts against the bottom M a pressure such as is caused by the 10 lbs of Q . Let us now assume that the water lying on the bottom M be equal in volume to the water $KLBA$, and the remainder surrounding the body P be equal in volume to the water IE . The water EAB therefore exerts against the bottom AB the same pressure as this water against the bottom M . But the latter exerts a pressure of 10 lbs, as has been shown above. The water EAB therefore also exerts a pressure of 10 lbs against the bottom AB , and the water GCD exerts the same pressure against the bottom CD . Therefore, as we had proposed to prove by practical examples, the water EAB weighing 1 lb exerts against its bottom AB the same pressure as the water GCD weighing 10 lbs against its bottom CD . And just as we have here proved 1 lb to exert the same pressure as 10 lbs, it can also be proved that 1 lb can exert a greater pressure than a thousand pounds.

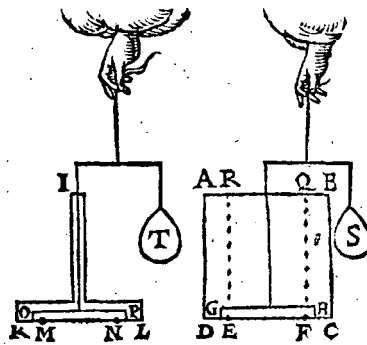
EXAMPLE II.

Let $ABCD$ be a small, thin tube, and $CDEF$ a large, wide vessel, separated from the tube, with a common bottom CD and both full of water, in such a way that the upper surfaces of the waters are in the same world surface. Now that the large water of the vessel $CDEF$ does not exert any greater pressure against the bottom CD than does the small water of the small tube, appears in practice as follows: Let the bottom DC be taken away; then the large water will thrust against the smaller in that place. Now if the water $CDEF$ had previously exerted a greater pressure against the bottom DC than the water $ABCD$, it will now also

dat water dan dat teghen dit: waer duer t'eranckste voor t'sterckste sal moeten wycken, dat is, t'water A B C D sal rijsen, ende van C D E F sal dalen; Maer dit so wesende, haer oppervlacken en sullen niet euen hooch sijn t'welck opentlick teghen d'eruaring is. Daerom t'cleinste water A B C D druckt euen soo stijf teghen den bodem C D, als t'grootste water C D E F.

III VOORBEELT.

Laet A B C D een vat vol waters sijn, in wiens bodem D C euewighdich liggende vanden sichteinder, een rondt gat E F is waer op ligt een ronde houten schijf G H, stoflichter dan water, ende dat gat E F bedekkende, en rondtom dicht sluytende teghen den bodem D C. Laet oock I K L een ander vat vol waters sijn, euenhooch mettet vat A B C D, maer cleinder, in wiens bodem K L oock een rondt gat M N sy, euen an t'gat E F, waer op licht een schijf O P, euegroot ende euefwaer ande schijf G H: T'welck soo wesende, d'eruaring sal bethoonen dat de schijf G H niet rijsen en sal, naer de ghemeen en aert des hauts in t'water, maer sal so stijf op t'gat E F drucken, als een ghewicht euefwaer an t'water dat euegroot is anden pilaer E F Q R, min t'verschil des ghewichts der houten schijf G H, tot het ghewicht des waters an die schijf euegroot. Maer om sulcx duer de daet oock te sien, men mach ande schijf G H een waegh voughen, diens ghewicht S euefwaer sy an dat voornomde ghewicht, ende de schijf G H sal daer teghen euewighdich bliuen. Laet nu insghelijcx ande schijf O P oock een waegh voughen, diens ghewicht T euefwaer sy an S, ende de schijf O P sal daer teghen oock euewighdich bliuen. Maer soomen S ende T yet swaerder maeckt, sy sullen haer schijuen doen rijsen, inder voughen dat de schijuen G H, O P, duer sulcke euewichten beuonden worden euestijf teghen haer bodems te drucken, waer uyt het voornemen blijktt, te weten het cleinder water I K L, euen so stijf teghen sijn grondt te drucken, als t'groter A B C D.



M E R C K T.

Tis kennelick dat so t'verschil des ghewichts der schijf als G H, tot het ghewicht des waters an haer euegroot, meerder waer dan t'ghewicht des

exert a greater pressure against the former water than that against this, as a result of which that which is weaker will have to yield to that which is stronger, i.e. the water $ABCD$ will ascend and that of $CDEF$ will descend. But this being so, their upper surfaces will not be on a level, which is manifestly contrary to experience. The smaller water $ABCD$ therefore exerts the same pressure against the bottom CD as the larger water $CDEF$.

EXAMPLE III.

Let $ABCD$ be a vessel full of water, in whose bottom DC , which is parallel to the horizon, there is a round hole EF , on which there lies a round wooden disc GH of greater specific levity than water and covering that hole EF and closely fitting all round the bottom DC . Let also IKL be another vessel full of water, of the same height as the vessel $ABCD$, but smaller, in whose bottom KL there be also a round hole MN , equal to the hole EF , on which there lies a disc OP of the same volume and weight as the disc GH . This being so, experience will show that the disc GH will not rise, in accordance with the common nature of wood in water, but will exert on the hole EF the same pressure as a weight of equal gravity to the water which is equal in volume to the prism $EFQR$, minus the difference between the weight of the wooden disc GH and the weight of the water having the same volume as that disc. But in order to see this also in practice, there can be applied to the disc GH a balance whose weight S shall be of equal gravity to the aforesaid weight, and the disc GH will be in equilibrium therewith. Let there now, in the same way, also be applied to the disc OP a balance whose weight T shall be of equal gravity to S , and the disc OP will also be in equilibrium therewith. But if S and T be made a little heavier, they will cause their discs to rise, in such a way that the discs GH , OP are found by such equilibria to exert the same pressure against their bottoms, from which the fact intended to be proved is evident, to wit that the smaller water KL exerts the same pressure against its bottom as the larger $ABCD$.

NOTE.

It is evident that if the difference between the weight of the disc as GH and the weight of an equal volume of water were greater than the weight of the

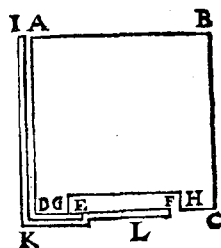
waters euegroot anden pilaer, als $E F Q R$, sulcken schijf en soude op t'gat als $E F$ niet connen rusten, maer soude nootakelick oprijfen.

Tis oock blijkkelick dat soo de schijf al $G H$ stoffwaerder waer dan water, als van loot, yser, &c. datse dan op t'gat $E F$ soo stijf drucken soude, als een ghewicht des waters euegroot anden pilaer $E F Q R$, meer t'verschil des ghewichts der schijf, tottet ghewicht des waters an haer euegroot.

Maer waer de schijf $G H$ euestoffwaer an t'water, tis openbaer datse dan effen so stijf op t'gat $E F$ drucken soude, als een ghewicht des waters euegroot anden pilaer $E F Q R$.

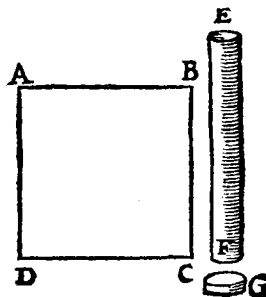
IIII^e VOORBEELT.

Laet $A B C D$ een vat vol waters sijn, met een gat $E F$ inden grondt $C D$, daerop een schijf $G H$ light, stoffichter dan t'water, de selue sal op t'gat $E F$ so stijf pranghen als vooren bewesen is. Laet oock, $I K L$ een cleen dun buycken wesen, diens opperste gat I inde selfde hoochde van $A B$ sy, ende t'onderste gat sy $E F$: Daer naer dit buycken vol waters ghegoren, dat cleen water sal soo groot ghewelt doen teghen den grondt des schijfs $G H$, als al t'water dat in t'vat $A B C D$ is, want de schijf $G H$ sal rijfen. Inder voughen dat i t'water (duer t'welck ick neem de buy $I K L$ te mueghen ghevult worden) meer ghewelts sal connen doen teghen de schijf $G H$, dan hondert duyfent ponden als S hier vooren, t'welckmen der naturen verborghenheyt soude mueghen noemen dat d'oirsaken onbekent waren.



V VOORBEELT.

Om nu oock werckelick betooch te gheuen ouer de voorbeelden alwaer t'water opwaert teghen den bodem steeckt, als int 3^e veruolgh des voornomden 10^{en} voorstels, so laet $A B C D$ een water sijn, ende $E F$ een dichte buy, ende G een schijf stoffwaerder dan water, ick neem van loot, als in dese eerste form.



Laet

water having the same volume as the prism $EFQR$, this disc could not rest on the hole EF , but would of necessity ascend.

It is also evident that if the disc GH were of greater specific gravity than water, for example if it were lead, iron, etc., it would then exert on the hole EF the same pressure as the weight of the water having the same volume as the prism $EFQR$, plus the difference between the weight of the disc and the weight of the water having the same volume.

But if the disc GH were of equal specific gravity to the water, it is manifest that it would then exert on the hole EF the same pressure as the weight of the water having the same volume as the prism $EFQR$.

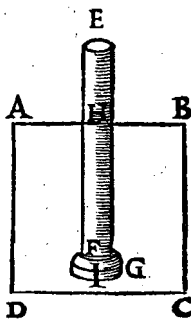
EXAMPLE IV.

Let $ABCD$ be a vessel full of water, with a hole EF in the bottom CD , on which there lies a disc GH , of greater specific levity than the water; this will exert on the hole EF the pressure that has been proved before. Let also IKL be a small, thin tube, whose upper opening I shall be on a level with AB , and the lower opening shall be EF . If thereafter this tube is filled with water, the small water will exert the same pressure on the base of the disc GH as all the water that is in the vessel $ABCD$, for the disc GH will ascend. In such a way that 1 lb of water (with which I take that the tube IKL can be filled) will be able to exert a greater pressure against the disc GH than a hundred thousand pounds, as S above, which might be called one of the secrets of Nature, if the cause were unknown.

EXAMPLE V.

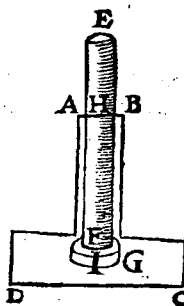
In order to give a practical explanation about the examples in which the water exerts an upward thrust against the bottom, as in the 3rd corollary of the afore-said 10th proposition, let $ABCD$ be a water, and EF a closed tube, and G a disc of greater specific gravity than water, say of lead, as in the first figure.

Laet dese schijf G gheleyt worden teghen 't gat F, alsoo datse dicht daer op pas, ende de buys met de schijf dan alsoo t'samen in 't water A B C D ghesteken, ick neem tot H toe, als hier onder, de schijf G en sal naer den ghemeen aert des loots, in 't water niet sincken, maer ande buys blijen hanghen, ende daer teghen soo stijf drucken, als een ghewicht eue swaer an 't water dat eue groot is anden pilaer, diens grondt de groote des gats F, ende hoochde H I is, min 't verschil des ghewichts der schijf G, tot 't ghewicht des waters an die schijf eue groot. Maer soo de schijf G niet dicht ghenouch teghen de buys en sloet, ende datter eenich water indrong, soo sal de schijf G daer soo langhe anhanghen, tot dat sulck inghedronghen water 't voornomde ghewicht ouerwint.



Maer want nu yemandt dencken mocht, het groot swaer water rondom de buys staende, stijuer drucking der schijf teghen de buys te veroirsaken, dan een cleender water van de selfde hoochde, soo laet ons 't water neuen de buys rondom afcorten, dat is, dat de reste water sy in een vat van form als hier neuen, ende d'eruaring sal bewysen (versouckende de macht des gheprangs in 't een en 't ander water, duer euewichten inde buys op G rustende) dit cleen water euen soo stijf te drucken als 't voornomde grooter, waer af de reden bouen grondelick beschreuen is.

T B E S L V Y T. Wy hebben dan duer daetlicke voorbeelden het 10^e voorstel der beghinselen des Waterwichts verclaert, naer 't voornemen.



M E R C K T.

Wat het 11^e voorstel belangt, daer uys is onder anderen kennelick, wat ghewicht waters datter druct, teghen elcke siide der duer van een sluys, ende dierghelijcke: Oock dattet water ouer d'een siide alleenlick een stroobreet, daer teghen soo stijf prangt als 't water diens breede de Zee van Oceane ouer d'ander siide; Welverstaende als sy euenhooghe siin. Van welcke dinghen wy om haer voornoemde claeerbeys hier gheen besonder voorstellen en maken.

Let this disc G be laid against the hole F , in such a way that it fits closely thereto, and if then the tube together with the disc be put in the water $ABCD$ — I take as far as H , as shown below —, the disc G will not, in accordance with the common nature of lead, sink into the water, but cling to the tube and exert against it the same pressure as a weight of equal gravity to the water having the same volume as the prism whose base has the size of the hole F and whose height is HI , minus the difference between the weight of the disc G and the weight of the water having the same volume as that disc. But if the disc G should not fit closely enough against the tube, and some water should enter into it, the disc G will cling to it until this entering water shall overcome the aforesaid weight.

But because someone might suppose that the large, heavy water surrounding the tube would cause a greater pressure of the disc against the tube than a smaller water of the same height, let us take away the water all around the tube, i.e. so that the remainder of the water be in a vessel of a form as shown opposite. Then experience will prove (testing the force of the pressure in either water by means of equal weights in the tube resting on G) that this small water exerts the same pressure as the aforesaid larger water, the cause of which has been thoroughly described above. **CONCLUSION.** We have thus explained, by means of practical examples, the 10th proposition of the elements of Hydrostatics, as intended.

NOTE.

As regards the 11th proposition, from that it is evident, among other things, what is the weight of the water pressing against either side of the gate of a lock and the like. Also that the water on one side, even if it were only the width of a straw, exerts the same pressure against it as the water having the breadth of the Ocean on the other side, provided they are on the same level. Of these matters we do not draw up any special propositions, in view of their aforesaid clearness.

D'O I R S A E C K te verclaren waerom een mensch diep onder t'water swemmende, niet doot gheprangt en wort, van t'groot ghewicht des waters op hem ligghende.

Laet een mensch 20 voeten diep onder water ligghen, weghende elcke voet waters 65 lb, ende t'gheheel vlack sijns lichaems sy groot 10 voeten. Dit soo wesende, daer sal teghen sijn lijf perffen byde 13000 ponden ghewichts, duer het 10^e ofte 11^e voorstel vande beghinselen des Waterwichts. T'welck soo sijnde, hoe ist mueghelick, sal ymant segghen, dat sulcken ghewicht den mensch niet doot en druct? D'antwoort is daerop foodanich:

- A. *Alle duwing die t'lichaem weedom andoet, verset eenich deel des lichaems uyt sijn natuerlicke plaets;*
- O. *Deze duwing des waters en verset gheen deel des lichaems uyt sijn natuerlicke plaets;*
- O. *Deze duwing des waters dan, en doet het lichaem gheen weedom an.*

*Syllogismi
minor.*

Des *bewysfredens tweede voorstel is openbaer duer de daet, waer af de reden dese is: Soo eenich deel als vleesch, bloet, ydichheit, watter sy, uyt sijn natuerlicke plaets verset wierde, t'soude moeten plaets hebben daert in ghinghe, die plaets en is buyten t'lichaem niet, ouermidts t'water oueral euestijf anstoot (Angaende t'onderste deel een weynich stijuer gheprangt wort dan t'opperste, duer het 11^e voorstel der Beghinselen des Waterwichts, dat en is in desen gheuallē van gheender acht, want sulck verschil gheen deel uyt sijn natuerlicke plaets versetten en can) sy en is oock binnen t'lichaem niet, wantret daer soo vol lichamelicheyts is als daer buyten, waer duer yder dit deel, soo stijf stoot teghen yder dat deel, als yder dat, teghen yder dit, ouermits t'water rondom t'lichaem tot allen sijden met een selue reden staet. Die plaets dan en is buyten t'lichaem niet, noch daer binnen, daerom nergens, waerduer het onmeughelick is, dat eenich deel uyt sijn natuerlicke plaets ghebrocht worde, ende vervolghens t'lichaem en can daer af gheen weedom ontsaen.

Maer om t'selue metter daet noch merckelicker te bewysen, laet A B C D een water sijn, hebbende inden grondt D C een gat, ghesloten met den tap E, ende op den seluen grondt ligghe een man F, met sijn rug op E: T'welck soo sijnde, daer en can van wegghen t'ghewicht des waters op hem ligghende, gheen deel des lichaems uyt sijn natuerlicke plaets verset worden,

PROPOSITION III.

To explain the cause why a man, swimming deep below the water, is not crushed to death by the great weight of the water lying on him.

Let a man lie 20 feet below the water's surface, each foot of water weighing 65 lbs, and the whole area of his body shall be 10 feet. This being so, there will be exerted against his body a pressure of about 13,000 pounds, by the 10th or 11th proposition of the elements of Hydrostatics. Which being so, how is it possible, it may be said, that this weight does not crush the man to death? The answer to this as follows ¹⁾:

- A. *Any thrust which hurts the body moves some part of the body from its natural place;*
- O. *This thrust of the water does not move any part of the body from its natural place;*
- O. *Therefore this thrust of the water does not hurt the body.*

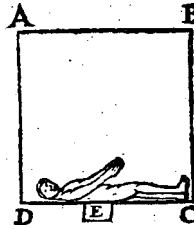
The second proposition of the syllogism is clear from practice, the reason of which is as follows: If any part, such as flesh, blood, fluid or whatever it be were moved from its natural place, it would have to find a place into which it might enter. That place is not outside the body, since the water exerts the same pressure against it on all sides (as to the fact that the lower part is subject to a somewhat greater pressure than the upper part, by the 11th proposition of the Elements of Hydrostatics, that is of no account in this case, for this difference cannot move any part from its natural place); nor is that place inside the body, for there it is as full of corporeity as outside, so that one part exerts the same pressure against the other as that against this, since the water round the body has the same position on all sides. That place therefore is neither outside the body nor inside it; therefore it is nowhere, owing to which it is impossible that any part should be moved from its natural place, and consequently the body cannot be hurt by it.

But in order to prove this even more clearly by experience, let *ABCD* be a water, having in the bottom *DC* a hole closed with the plug *E*, and on this bottom there shall lie a man *F* with his back on *E*. Which being so, it is not possible for any part of the body to be moved from its natural place by the weight of the water

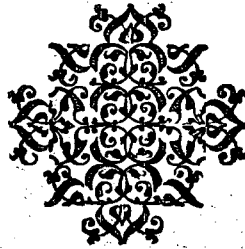
¹⁾ See note 2 to p. 143.

worden, ouermits t'water an allen sijden euestijf anstoot, als vooren gheseyt is.

Maer wildi nu daetlick sien dit de waerachtighe oirfaeck te wesen, so treect den tap E uyt, ende dan en sal teghen sijn rug an E gheen stoortel sijn, als an alle d'ander plaetsen sijns lichaems, daerom oock sal het lichaem daer prangsel lijden, ende dat soo stijf als int derde voorbeeld des 2^{en} voorstels van desen betoocht is; te weten soo stijf, als veroirfaect wort duer t'ghewicht des pilaers waters, diens gront het gat E is, ende hoochde, de hoochde des waters bouen hem, waermede t'voornemen opentlick bewesen is.



TEINDE DES ANVANGS DER WATERWICHTDAET.



lying on him, since the water exerts the same pressure on all sides, as has been said before.

But if you desire to see by experience that this is the true cause, pull out the plug *E*, then there will be no pressure against his back at *E* such as there is at all the other places of his body. Therefore the body will there be subjected to pressure, and this as strongly as has been shown in the third example of the 2nd proposition, i.e. to a pressure such as is caused by the weight of the prism of water whose base is the hole *E* and whose height is the height of the water above him, with which the fact intended to be proved is clearly evinced ¹⁾.

THE END OF THE PREAMBLE OF THE PRACTICE OF HYDROSTATICS

¹⁾ This experiment calls forth the same sceptical comment that was made by Boyle on similar experiments of Pascal: more ingenious than practicable. R. Boyle, *Hydrostatical Paradoxes Made Out by New Experiments*. Works. London 1772. II 732-797.

ANHANG VAN
DE WEEGHCONST

APPENDIX
TO THE ART OF WEIGHING

INTRODUCTION

It is one of Stevin's firm principles never to mingle his scientific argument with polemics. On the other hand there are several controversial questions on which he likes to express his opinion. Some of these are dealt with in the following *Appendix to the Art of Weighing*.

In the first chapter he combats the so-called principle of virtual displacements, which enables statics to be founded on dynamical principles. In the second he refutes various erroneous opinions on the motion of falling bodies in resistant media. In the third it is contended that the Art of Weighing is entitled to be considered an independent branch of mathematics on the same footing with Arithmetic and Geometry. In the fourth chapter certain objections to the use of numbers in proofs of statical propositions are refuted, and the final chapter contains some further elucidations on Prop. VIII of the *Elements of Hydrostatics* (Archimedes' principle).

A N H A N G,
INDE WELCKE ONDER
ANDEREN WEERLEYDT WORDEN
ETLICKE DWALINGHEN DER
wichtighe ghedaenten.

A N D E N L E S E R.

Argumentu.

M

Y ghedenckende van t' misshagen dat ick somwy-
len ghehad heb inde * strijtedens ettelicker schrij-
uers, welcke ghedreuen van haer ghemoet, ander
persoonens dwalinghen in consten so verachtelick
berispten, dat sy daermede een ghetuych gauen,

Materia.

van haer veel slimmer dwalinghen inde seden; ende dat my daer
beneuen ouervloedighe stof ontmoet was, om te connen weerleg-
ghen veel dolinghen vande wichtighe ghedaenten duer sommi-
ghe beschreuen: Heb ghevreest int verclaren der seluer, den Le-
ser van my een vermoeden te mueghen gheuen, van sulx als
my in anderen misuiel. Nochtans achtende hier beneuen, dat-
tet gantschelick verswyghen (want wy met voorset daer af in-
de voorgaende boucken niet en hebben willen reren, om de lee-
ring met gheen * strijding te verduysteren) den sommighen

Argumenta-
tion.

eenich misverstant ende achterdeel mocht veroirsaken, heb my
ghepoocht naer t'middel te trachten, ende inde plaets van veel
besonder dwalinghen, alleen haer ghemeene oirspronck duer de
twee eerstvolghende * hoofdsticken te verclaren niet tot vermin-
dering des naems van so weerdigen schrijuers, maer veel eer om
die met danckbaerheyt te helpen vermeerderen, als van bedwe-
ghende oirsaken haerder nacommers, sonder welcke veel beson-
derheden dickmael ongheroert souden ghebleuen hebben.

Capita.

I HOOFT-

APPENDIX

IN WHICH, AMONG OTHER THINGS, MANY ERRORS ABOUT THE
QUALITIES OF WEIGHTS ARE REFUTED.

TO THE READER.

Bearing in mind the displeasure I have sometimes had in the arguments of many writers who, moved by their feelings, censured the scientific errors of other persons so contemptuously that by this they testified to their own — much worse — errors in manners, and further having found abundant material to refute many errors about the qualities of weights described by some writers, I feared lest in pointing them out I might give the Reader cause to suspect me of the same thing that had displeased me in others. Further considering nevertheless that complete silence (for we intentionally refrained from referring thereto in the preceding books, in order not to obscure the theory with argumentations) might cause misunderstanding and disadvantage to some people, I have tried to steer a middle course, and instead of many particular errors point out only their common origin in the two following chapters, not in order to throw discredit upon the reputation of such worthy writers, but rather in order to aggrandize it by gratitude, because they are the moving causes for their successors, without which many particulars might often not have been referred to at all.

CHAPTER I, that the cause of bodies being of equal apparent weight does not reside in the circles described by the extremities of the arms.

The reasons why equal gravities at equal arms are of equal apparent weight are known by common knowledge, but not so the cause of the equality of apparent weight of unequal gravities at unequal arms proportional ¹) thereto, which cause the Ancients, when they inquired into it, considered to reside in the circles de-

¹ Read: inversely proportional.

I^e HOOFTSTIC, DAT DER EVESTALT- *Caput 1.*

WICHTIGHEN OIRSAECK NIET EN SCHVYLT

onder de ronden beschreuen met d'uytersten der ermen.

DE redenen waerom een swaerheden an een ermen euestaltwichtich sijn, is duer ghemene wetenschap bekent, maer niet also d'oirsaeck der euestaltwichticheyt van oneuen swaerheden an oneuen ermen met haer * euerednich, welcke oirsaeck d'ouden onderfouckende, hebben *Proportionales.* die gheacht te schuylen onder de ronden beschreuen duer d'uytersten der ermen, als blijct by Aristoteles *in Mechanicis* met sijn nauolghers; Twelck wy ontkennen ende reden daer af aldus gheuen:

E. *Dat stil hangt en beschrijft gheen rondt;*A. *Twee euestaltwichtige hanghen stil;*E. *Twee euestaltwichtige dan en beschrijven gheen rondt.*

Ende veruolghens soo en isser gheen rondt; Maer alwaer gheen rondt en is daer en can t' rondt het ghene niet wesen daer eenige oirsaeck onder schuylt, waer duer de ronden hier t'ghene niet en sijn, daer d'oirsaeck der euestaltwichticheyt onder bestaet. Angaende (op dat wy des * Bewysfrens tweede voorstel verclaren) t'roetsel ofte te beschrijving der ronden *Syllogismi minorem.* welcke haer oogghenschijnlick mach vertooghen, die en is niet eyghen der euestaltwichtigen, maer by gheualle, als duer windt, hurting, oft eenighe ander beweghing, met welcke niet alleen dese, maer oock d'oneuestaltwichtige ronden connen beschrijven. Tis dan openbaer dat dese oirsaeck in gheen ronden en bestaet, maer onder t'ghene int 1^e voorstel des 1^{en} boucx vande beghinfelen der Weeghconst, daer af * Wisconstlick *Mathematic.* bethoocht is. Daerom die sulcke dwaling voor seker grondt namen, ten is gheen wonder dat sy, sonder te comen tot kennis der oirsaecken, oock sonder te krijghen form van Weeghconst, hemlien in veel valsche voorstellen oeffenden, die wy hier int besonder souden connen weerlegghen, maer sulcx laten om de redenen hier bouen verhaelt, te meer dat sy duer haer contrari, als t'voornoemde warachtighe, ghenouch bekent sijn.

Men soude hier oock mueghen weerlegghen ettelicke voorstellen van scheefwichten, beschreuen duer Cardanus *lib. 5. De proportionib.* daer hys raemt uyt seker houcken van linien oft platten, maer dat de selue ghemist sijn, is openbaer ghenouch duer het Wisconstich bewys van ander * eueredenheyt, int 19^e voorstel des 1^{en} boucx vande beghinfelen der *Proportione.* Weeghconst.

II^e HOOFTSTICK, DAT DE GHE-

ROERDEN MET HAER BELETSELEN IN

*gheen * eueredenheyt en beslaen.**Proportione.*

WY hebben inde voorreden der Weeghdaet anden Leser, gheseyt, dat de gheroerden met haer beletselen niet euerednich en sijn, oock

I i aldaer

scribed by the extremities of the arms, as appears in Aristotle's *In Mechanicis* ¹⁾ and his successors ²⁾).

This we deny, and we give the following reason therefor ³⁾

E. *That which hangs still does not describe a circle;*

A. *Two gravities of equal apparent weight hang still;*

E. *Therefore two gravities of equal apparent weight do not describe circles.*

And consequently there is no circle. But where there is no circle, the circle cannot be that in which resides any cause, so that the circles are not here that in which resides the cause of the equality of apparent weight. As regards (in order to explain the second proposition of the syllogism) the motion or the description of the circles which may appear to present itself, this is not peculiar to gravities of equal apparent weight, but accidental, as by wind, pushing or some other movement by which not only these, but also gravities of unequal apparent weight can describe circles. It is therefore manifest that this cause does not reside in circles, but in that which has been proved about it mathematically in the 1st proposition of the 1st book of the elements of the Art of Weighing. Therefore it is no wonder that those who took this error as a sure foundation, without becoming acquainted with the causes and also without attaining some sort of Art of Weighing, practised many false propositions, which we might refute here in particular but that we refrain from doing so, for the reasons mentioned above, the more so because they are sufficiently known from their contraries, being the aforesaid truth ⁴⁾).

It would also be possible to refute here many propositions on oblique weights, described by Cardanus lib. 5, *De proportionib.*, where he estimates them from certain angles of lines or planes, but that these are wrong is sufficiently manifest from the mathematical proof of another proportion, in the 19th proposition of the 1st book of the elements of the Art of Weighing ⁵⁾).

CHAPTER II, that bodies in motion are not proportional to their impediments.

We have said in the preface to the Practice of Weighing to the Reader that bodies in motion are not proportional to their impediments, and also promised there to give a more appropriate proof thereof elsewhere, which we have arranged to do here, where are to be refuted the arguments of those who are of the opposite opinion, as follows. Aristotle — in the 4th book of Physics in the chapter on vacuum ⁶⁾ — and his followers hold that if two similar bodies fall

¹⁾ Stevin refers to the pseudo-Aristotelian treatise *Quaestiones Mechanicae*.

²⁾ Abundant documentation on these successors is to be found in Ernest A. Moody and Marshall Clagett, *The Medieval Science of Weights*. Madison 1952.

³⁾ See Note 2 to p. 143.

⁴⁾ The above passage refutes the assertion not uncommon in books on the history of science that Stevin was the initiator of the principle of virtual displacements or of virtual work. This principle, which originates in the pseudo-Aristotelian work *Quaestiones Mechanicae*, had been applied in the Middle Ages by Jordanus Nemorarius and his followers.

⁵⁾ Cardanus, *Opus novum de proportionibus* etc. Basileae 1578, V, 72. Cardano contends, that the force required to hold a body at rest on an inclined plane is proportional to the angle of inclination.

⁶⁾ Stevin obviously means *Physica* IV 8; 215a-216a. It may here be remarked that Aristotle considers separately the influence of the density of the medium acting as an impediment (215a 29-215b 12) and that of the gravity which furthers the motion (216a 12-21). He nowhere asserts the proportionality of gravity and impediment.

Argumenta.

aldaer beloofst elders van dies eyghentlicker bewys te doen, 'welck wy hier veroirdent hebben, alwaer weerleyt sullen worden de * strijtrede van de ghene die de contrarie meynen, aldus: Aristoteles int 4^e bouck der Natuer int hoofstuc des ydels met sijn nauolghers, willen, dat vallende twee lijkformighe lichamen duer de locht, ghelijck de swaerheyt van een tot die van t'ander, also diens tijt des duerlijdens tot defens, dat is, ghelijck swaerheyt tot swaerheyt, also belet tot belet. 'Welck sijn meyning soo te wesen hy in versheyden boucken opentlicker verclaert als lib. 6. *Phys.* oock lib. 1. 2. 3. 4. *de Caelo*, tot veel plaetsen. Hier in heeft teghen Aristoteles gheschreuen *Ioannes Taisnier Hannonius*, willende oock eueredenheyt, doch so, dat die twee voornomde lichamen in eenen tijden duer eenen langden des lochts vallen; In welcke meyning Cardanus oock is lib. 5. *de Proportionib. prop.* 110. Maer d'een noch d'ander en heeft de saeck ghetroffen, 'welck wy eerst met daetlicke eruaring verclaren sullen, ende daer naer d'oirsaek bethoonen. D'eruaring teghen Aristoteles is dese: Laet nemen (soo den hoochgheleerden H. IAN CORNETS DE GROOT vlietichste onderfoucker der Naturens verborghentheiden, ende ick ghedaen hebben) twee loyen clooten d'een thienmael grooter en swaerder als d'ander, die laet t'samen vallen van 30 voeten hooch, op een bart oft yet daer sy merckelick gheluyt tegen gheuen, ende sal blijcken, dat de lichtste gheen thienmael langher op wech en blijft dan de swaerste, maer datse t'samen so ghelijck opt bart vallen, dat haer beyde gheluyden een selue clop schijnt te wesen. S'ghelijcx beuint hem daetlick oock also, met twee euegrootelichamen in thienvoudighe reden der swaerheyt, daerom Aristoteles voornomde eueredenheyt is onrecht. D'eruaring teghen Taisnier is dusdanich: Neemt een cleen ynckel cort haerken boomwols, en een paxken des selfden stijf in een ghebonden, weghende een pondt, ende van ghelijcke form mettet haerken, dese laet t'samen neervallen van vijf of te ses voeten hooch, ende d'eruaring sal betoogen datter haerken (niet teghenstaende sijn stof veel dichter in een ghesloten is, dan des pax, waer in veel ledighe plaets ofte locht is) wel vijuentwintich mael langher op wech blijft dan t'paxken, daerom sy en vallen na sijn meyning op gheen eenen tijden duer eenen langden des lochts. Ander eruaring blijft oock teghen Taisnier int rijfendwicht, als in een lanck claer glas vol waters, 'welck gheroert, also datter veel locht-clooten ofte lochtbellen in comen, en daerna stil ghehouden, de grootste bellen sullen snellick in een ooghenblick opcomen, de cleender niet soo ras, maer de minste als sandekens, soo traechlick als een flecke cruypt; 'welck verre is van eenen tijden. Dit is van d'eruaring gheseyt. Daer rest nu noch d'oirsaek te verclaren, waerom hier gheen eueredenheyt en is, aldus: Yder roerende lichaem heeft eenich belet sijns roersels, dat van een vallende lichaem duer de locht, is t'ghenaecsel sijns * vlaex teghen de

Superfici.

through the air, as the gravity of the one is to that of the other, so is the time of passage of the latter to that of the former, i.e. as gravity is to gravity, so is impediment to impediment¹⁾. Which in several books he more openly declares to be his opinion, as in lib. 6 *Phys* 2), also lib. 1.2.3.4. *de Caelo*, in many places. In this, Johannes Taisnier Hannonius³⁾ has written against Aristotle, also maintaining proportionality, but in such a way that the two aforesaid bodies fall in equal times through equal distances of the air. Which opinion is also held by Cardanus, lib. 5 *de Proportionib. prop* 110⁴⁾. But neither the one nor the other has hit on the truth, which we will first expound by means of practical experience, after which we will set forth the cause. The experience against Aristotle is the following: Let us take (as the very learned Mr. Jan Cornets de Groot, most industrious investigator of the secrets of Nature, and myself have done) two spheres of lead, the one ten times larger and heavier than the other, and drop them together from a height of 30 feet on to a board or something on which they give a perceptible sound. Then it will be found that the lighter will not be ten times longer on its way than the heavier, but that they fall together on to the board so simultaneously that their two sounds seem to be one and the same rap⁵⁾. The same is found also to happen in practice with two equally large bodies whose gravities are in the ratio of one to ten; therefore Aristotle's aforesaid proportion is incorrect. The experience against Taisnier is as follows: Take a small, single, short hair of cotton and a packet of the same, tightly tied together, weighing one pound and of similar form to the hair; drop these together from a height of five or six feet, and experience will show that the hair (in spite of the fact that its material is much more compact than that of the packet, in which there is much empty space or air) is at least twenty-five times longer on its way than the packet; therefore they do not, as was his opinion, fall in equal times through equal distances of the air. Another experience also appears to be against Taisnier, with ascending weight; for example, in a high, clear glass of water, which is stirred in such a way that many spheres of air or air bubbles are produced therein, and thereafter kept still, the largest bubbles will ascend rapidly, in a moment, the smaller ones not so rapidly, but the smallest, like grains of sand, as slowly as a snail creeps; which is far from equal times. This is what is said about the experience. It now remains to set forth the cause why there is no proportionality here, as follows. Every body in motion has some impediment to its motion, that of a body falling through the air is the friction of its surface against the air.

¹⁾ It is by no means clear, how this inference is drawn.

²⁾ This must be a mistake for 7. The reference is to *Physica* VII 5; 250 a.

³⁾ *Opusculum perpetua memoria dignissimum de natura magnetis, et eius effectibus. Item De motu continuo. Demonstratio proportionum motuum localium contra Aristotelem et alios Philosophos. De motu celerrimo, hactenus incognito Authore Ioanne Taisnerio Hannonio. Coloniae 1562.* Jean Taisnier was born at Ath (Hainault) in 1509; the year of his death is unknown. After various peregrinations he became master of the archiepiscopal church choir at Cologne. The name of his birth place accounts for the adjective *Athenensis* which is sometimes added to his name. For further information the reader may consult: L. Thorndike, *A History of Magic and Experimental Science*. V 580-588.

⁴⁾ Hieronymi Cardani Mediolanensis... *Opus Novum de Proportionibus*... Basileae 1578. Prop. 110. p. 104.

⁵⁾ This experiment is usually ascribed to Galileo, who is said to have performed it during his professorship at Pisa (1589-1592). The story is rather suspect. At all events he was anticipated by Stevin and De Groot by more than three years.

teghen de locht, daerom ontfangt t'meeſte der ghelijcke lichamen wel t'meeſte beletſel, maer ouermids der lichamen grootheden met haer vlacken ſelfs niet euerednich en ſijn (want twee teerlinghen in achtvoudighe reden, hebben haer vlacken alleen in viervoudighe) ſo en connen ſy met die beletſelen niet euerednich weſen, ende daerom iſt dat de minſtelichamen meer belet ontfanghen, int anſien der eueredenheyt, dan de meeſte, waerduer ſy oock traechlicker neervallen.

Ende of de vlacken ſchoon inde reden haerder grootheden waren, ſo iſt t'middel daer de lichamen duer vallen, alleen oock een oirſaeck die ſulcke eueredenheyt weert, t'welck opentlick blijct in twee lichamen, t'een int water ſinckende, t'ander daer op driuende, wiens beletſelen der vlacken eenighe reden hebben, maer de tijden gheen, daerom en ſijnſe niet euerednich. Ymant ſal hier toe mueghen ſegghen t'ghemeen woort * *D'ander parich*, dat is, hem ſulcx alleen te verſtaen van lichamen die beyde duer t'water ſincken. Ick ſeg dat de voornomde eueredenheyt in ſulcke oock niet en beſtaet: Om t'welck te bewyſen ſo laet twee lichamen ſijn, A t'ſwaerſte, B t'lichtſte, die beyde int water ſincken, ende tuſſchen hun beſta de voornomde eueredenheyt. Dit ſoo weſende, tis kennelick datmen neuen A, ander oneindelicke menichte van lichamen voughen can, t'een lichter als t'ander, ende elck lichter als B, die alle daer in ſincken. Nu yder van deſe verſieken met A, men ſal allencx naerder en t'ghene bouen gheſeyt is gheen eueredenheyt te weſen, dat is men ſal naken de verlijcking eens lichaems dat ſinct, met een dat niet en ſinct: Maer dit ſoo naerderende, ende in A, B, de begheerde eueredenheyt beſtaende, ſeker gheen dier oneindelicke menichte der lichamen met A verſieken, en ſullen die eueredenheyt hebben; want ſooſer in waer, ſy en ſouden niet naerder en; t'welck teghen t'gheſtelde is. Daerom ſoo wy voorghenomen hadden te verclaren, t'middel daer de lichamen duer vallen, is oock een oirſaeck die de voornomde eueredenheyt weert. *Ceteris paribus.*

Maer hier aldus bethoont hebbende, gheen eueredenheyt te beſtaen tuſſchen de ghetoerden met haer beletſelen inde aldergheschieſte voorbeelden, alwaer maer een eenvoudich ſtrijſel der vlacken en is teghen de locht, oft teghen t'water, ſoo en ſalder uyt noch ſtercker reden, gheen eueredenheyt weſen in ongheschieſter voorbeelden van verſcheyden ſtoſſen, als in reetſchappen van haut, ijſer, en dier ghelijcke, want dit wort beolijt, dat beſmeert, t'een can met een vochtich weer opſwellen, t'ander verroeſten, alle welcke (ick laet veel ander varen) de roerſelen der reetſchappen verlichten of beſwaren. Daerom ſoo gheſeyt is inde boueſchreuen voorreden der Weeghdaet, men ſal hem op deſe ſchijn van eueredenheyt niet verlaten, maer t'ghene Cardanus *lib. 5. de Proportionibus* in veel verſcheyden voorſtellen, met meer ander Schrijuers daer af beſluyten, voor dwalinghen houden, ſich vernoughende met de Wiſ-

Therefore the largest of such bodies indeed meets with the greatest impediment, but since the volumes of bodies are not proportional to their surfaces (for two cubes in the ratio of one to eight have their surfaces only in the ratio of one to four), they cannot be proportional to those impediments, and that is why the smaller bodies meet with a relatively greater impediment than the greater, owing to which they also fall more slowly.

And even if the surfaces were proportional to their volumes, the medium alone through which the bodies fall is also a cause preventing such proportionality, which is clearly apparent with two bodies, one sinking in the water and the other floating thereon, the impediments of whose surfaces have a certain ratio, but the times have not; therefore they are not proportional. Someone might here add the common term: other things being equal, i.e. that this applies only to bodies which both sink in the water. I say that the aforesaid proportionality does not exist between such bodies either. In order to prove this, let there be two bodies, *A* the heavier, *B* the lighter ¹⁾, which both sink in the water, and let them be in the aforesaid ratio. This being so, it is evident that besides *A* there can be added an infinite number of bodies, one lighter than the other, and each lighter than *B*, which all sink therein. Now if each of these is compared with *A*, we shall gradually approach the situation in which, as has been said before, there is no proportion, i.e. we shall approximate to a comparison of a body that sinks with one that does not sink. But with this approximation, and the desired proportionality existing between *A*, *B*, certainly none from that infinite number of bodies compared with *A* will have that proportionality; for if it were there, they would not approximate, which is against the supposition. Therefore, as we intended to set forth, the medium through which the bodies fall is also a cause preventing the aforesaid proportionality.

But since we have thus proved that there is no proportionality between bodies in motion and their impediments in the most obvious examples, where there is only simple friction of the surfaces against the air or against the water, there will *a fortiori* be no proportionality in less obvious examples of several materials, such as tools of wood, iron, and the like, for the former is oiled, the latter greased; the one can swell in moist weather, the other rust; all of which things (I omit many others) lighten or weight the motions of the tools. Therefore, as has been said in the above-mentioned preface to the Practice of Weighing, we should not rely on this appearance of proportionality, but consider the conclusions reached by Cardanus, lib. 5 *de Proportionibus* in a great many different propositions, and by other writers, to be errors, being satisfied with the mathematical knowledge of

¹⁾ It is obvious that „heavier” and „lighter” here refer to specific gravity. The weights of *A* and *B* are supposed to be equal. Stevin considers a series of bodies of this same weight *G* with a gradually decreasing specific gravity *S*. If the resistance *R* exerted by the water were proportional to the volume, *R* would be inversely proportional to *S*, and consequently *S* would be proportional to the time required for the motion through a given distance. If *S* approaches the specific gravity *S*₁ of the water, the time tends to a certain limit, whereas, on the other hand, if *S* = *S*₁, there is no motion at all.

constighe kennis der euestatwichticheyt van 'roerende ende het te roeren, als totter voornemen ghenouch doende.

III. HOOFSTICK, DAT DE WEEGH

Art Mathematica.

CONST EEN BESONDER VRIE * WISCONST IS.

Quaestiones.

Materia.

Arithmetica Geometria.

Accidentia.

Tis wel waer, dat van der dinghen namen die de saeck niet en verduysteren, dickwils onnoodighe * verschillen sijn, onder de welcke ick niet en ontken dit derde hoofsttick te mueghen gherekent worden, doch anghesien wy de Weeghconst daert te pas ghecommen heeft, een vrye Wisconst ghenoeemt hebben, soo moeten wy met corte woorden daer af wat redens gheuen, aldus: Ouermits de * stof des ghetals al een ander is dan die der grootheyt, soo sijn de leeringhen haerder eyghenschappen te recht vanden anderen ghescheyden, ende elck voor een besonder Const ghehouden, als * Telconst ende * Meetconst, op dat elcke alsoo oirdentlicker, eyghentlicker, ende verstaenlicker soude mueghen beschreuen worden. Ten anderen, want haer diepsinnighe * ancleuinghen ons niet uyter natuer bekent en sijn, maer dat wy die leeren uyt de vergaerde schriften der ghene die duer besondervliet hun daer in gheoeffent hebben, ia dickmael by gheualle ter kennis van yet besonders ghe-rocht sijn, ende dat haer wetenschap den menschen daerenbouen seer nut is, soo worden se met recht vrye consten ghenoeemt. Ten derden, nadien de sekerheyt in haer bestaende, de ghewisheyt van d'ander Const ver-re te bouen gaet, soo worden se billichlick daerbeneuen Wisconsten ghe-heeten. T'selue is om der ghelijcke redenen vande Weeghconst oock te oirdeelen; want anghesien haer stof, te weten swaerheyt, al een ander is dan ghetal ofte grootheyt; oock dat de nutte eyghenschappen van dese, in diepsinnicheyt an d'eyghenschappen van die niet en wycken (t'welck daerin blijft, dat sy om sulcx laetst tot smenschen kennis ghecomen sijn, ende of sy v schoon licht dochten, dat muecht ghy d'onbegrijpelicke volmaetheyt der Duytsche praeck dancken) Voorts dat sy duer haer uyterste beghinselen, in sulcken ghewisheyt bestaet als die, soo sal sy om haer ghemeene reden, een besonder vrye Wisconst ghenoeemt worden.

Species.

Theoremas.

Yemant sal hier teghen mueghen segghen, dat de Meetconst tot haer bewysen dickmael ghebruyct wort, ende die daerom als * afcomst der Meetconst stellen. Ick anwoord de Telconst sulcx oock te ghebueren, nochtans een besonder vrye Wisconst blijuende. Want wat voornamelicke * Verdoogen heeftse, diens grondelicke kennis duer de Meetconstighe formen niet vercreghen en wort? Ia die Meetconst seluer hoe soude se sonder ghetalen bestaen? Siet haer beghinselen als die van Euclides, hoe dickmael d'een form des anders dobbel, dese drie platten euen an die twee gheseyt worden. T'blijct dan die voorstellen sonder t'behulp van ghetalen

the equality of apparent weight of the moving body and the body to be moved as being sufficient for the purpose.

CHAPTER III, that the Art of Weighing is a distinct, free branch of mathematics.

It is true that there are often unnecessary points of controversy about the names of things, which do not give rise to obscurity, among which I cannot deny but this third chapter may be reckoned, but since we have, where relevant, called the Art of Weighing a free branch of mathematics, we briefly have to account for this, as follows. Since the subject matter of number is quite different from that of magnitude, the theories of its properties are justly dissociated from the other, and each is considered a distinct art, viz. arithmetic and geometry, so that each might thus be described in a more orderly, appropriate, and comprehensible way. Secondly, because their profound attributes are not known to us by nature, but are learned by us from the collected writings of those who have studied them with special zeal, nay, have often quite accidentally become acquainted with some special feature, and because their knowledge is moreover very useful to mankind, they are rightly termed free arts. Thirdly, since the certainty residing in them far exceeds that of the other arts, they are also on that account rightly termed "Wisconsten"¹⁾. The same can for similar reasons also be said of the Art of Weighing. For since its subject matter, to wit gravity, is quite different from number or magnitude; also because the useful properties of the latter are not inferior in profundity to the properties of the former (which is evident from the fact that for this reason they were the last to come to man's knowledge, and though they may seem easy to you, you owe that to the incomprehensible perfection of the Dutch language); further because in its fundamental principles it is of equal certainty to the former, it is, on account of this common reason, to be termed a distinct, free branch of mathematics.

Someone may object to this that geometry is often used for its proofs, and therefore may call the Art of Weighing a species of geometry. I reply that this also happens with arithmetic, which nevertheless remains a distinct, free branch of mathematics. For what important theorems does it have, thorough knowledge of which is not acquired by means of geometrical figures? Nay, how could even geometry itself exist without numbers? Consider its elements, for example, those of Euclid, how often one figure is said to be double of another, these three plane surfaces are said to be equal to those two. It is therefore found that those propositions cannot be proved without the aid of numbers, without, however, one

¹⁾ „Wisconst” literally means a sure, certain art. The pun could not be preserved in the translation.

ghetalen onbewyslick te wesen, nochtans d'een des anders afcomst niet sijnde, ende alsoo oock met de Weeghconst.

Angaende dat de * Duerlichtighe ende * Spieghelconst voor gheen besonder vrye Wisconsten, maer als afcomsten der Meetconst gheacht sijn, by welcke yemandt de Weeghconst mocht willen verlijcken; hun redens sijn seer verscheyden, ouermids de stof van dese, te weten swaerheyt, sulcx is, dat sy ghelijck de grootheyt, bestaet in alle voorghestelde wesentlicke saeck, met t'menschen groote nutbaerheyt; maer niet also de stof van die. Wy besluyten dan te recht, de Weeghconst een besonder vrye Wisconst te sijne, ghelijck ons voornemen was te bethoonen.

*Perspectiva
Catoptrica*

IIII. HOOFSTICK, DAT SOMMIGHE

VOORGAENDE BEWYSEN DVER T'BEHVLP

der ghetalen * Wisconstich sijn.

DE gheleerden maken onderscheyt tusschen Wisconstich ende * Werckelick bewys: T'welck niet sonder reden en is, want dat is ghemeen ouer allen, oock grondelick d'oirsaek verclarende, dit besonder alleenlick op r'ghegheuen, sonder kennis der reden waerom dat also gheschiet. Als by voorbeeld, yemant om te bewysen dattet viercant der langste sijde eens rechthouckich driehoucx, euen is ande twee viercanten van d'ander sijden, neemt een driehouck, diens cortste sijde van 3 voeten, d'ander van 4, de derde van 5 voeten is, de selue wort rechthouckich beuonden; Bethoont daer mede dattet viercant der langste sijde 25, euen is ande viercanten van d'ander twee sijden, als 16 ende 9. Maer dit is alleenlick bewys van dat voorghestelde; waer uyt niet en volght sulcx ouer alle rechthouckighe driehoucken soo te moeten wesen, oock en sietmen daer duer de oirsaek niet, waerom dat also ghebuert. ende ouermits dit aldus werckelick gheschiet, so wordet daerom oock werckelick bewys gheheeten. Maer t'betooch van sulcx duer Euclides ghedaen int 47^e voorstel des 1^{en} boucx, is ghemeen ouer allen, anwysende duer d'uyterste beginselen, de reden waerom dat so is, ende niet anders sijn en can; t'selue wort om sulcke ghewisheyt Wisconstich ghenoeemt, t'welck de * Wisconstnaers om de redenen hier vooren verhaelt, lieuer ghebruycken dant werckelick duer ghetalen. Yemandt mocht nu segghen; Dit so sijnde, waerom hebby dan de bewysen der 4^e, 11^e, 12^e, 18^e, voorstellen des 2^{en} boucx vande beginselē der Weeghconst, duer ghetalen ghedaen? D'antwoort valt daer op, dat de ghetalen der bewysen ons op tweederley manieren ontmoeten, d'eene die als * palen alleenlick de * redenen ende * eueredenheden der deelen des voorghestelden forms verclaren, d'ander de * menichvuldicheyt; T'bewys duer die is Wisconstich, wanttet hem op

Mathematica.

Mechanicam

Mathematicis

*Termini.
Rationes.
Proportiones.
Quantitates.*

being a species of the other; and the same is also true of the Art of Weighing.

As to the fact that perspective and catoptrics are considered not to be distinct, free branches of mathematics, but species of geometry, to which someone might wish to compare the Art of Weighing, the reasons are quite different, since the subject matter of the latter, to wit gravity, is such that, like magnitude, it resides in every real thing under consideration, to the great advantage of man; but not so the subject matter of the former. We therefore rightly conclude that the Art of Weighing is a distinct, free branch of mathematics, as we intended to prove.

CHAPTER IV, that some of the preceding proofs in which numbers were used are mathematical.

Scholars make a distinction between a mathematical and a practical proof; which is not without reason, for the former applies to all cases and also thoroughly sets forth the cause, and the latter only applies to the particular given case, without the cause why it thus happens being known. For example, in order to prove that the square of the longest side of a rightangled triangle is equal to the two squares of the other sides, a man takes a triangle whose shortest side is 3 feet, the second 4, and the third 5 feet; this is found to be rightangled. Therewith he proves that the square of the longest side, 25, is equal to the squares of the other two sides, viz. 16 and 9. But this is only a proof of the case under consideration, from which it does not follow that this must be so with all rightangled triangles. Nor do we thus see the cause why it happens in this way, and since this is done by practical means, it is called a practical proof. But the proof thereof given by Euclid in the 47th proposition of the 1st book applies to all cases, and shows, by means of the fundamental principles, the reason why this is so and cannot be otherwise; because of such certainty, this proof is called "Wisconstich", which mathematicians prefer, for the reasons given above, to the practical proof by means of numbers. Now someone might say: if this is so, why then have you given the proofs of the 4th, 11th, 12th, 18th propositions of the 2nd book of the elements of the Art of Weighing by means of numbers? The reply to this is that we meet with numbers in the proofs in two different ways, one in which as terms they only set forth the ratios and proportions of the parts of the figure under consideration, the other the quantity¹⁾. The proof by means of the former is mathematical, for it applies to all species of the figure under consideration and sets forth the causes, but the proof by means of the latter is not, for the opposite reason. Which Eutocius, the commentator of Apollonius, in the 11th proposition of the 1st book

¹⁾ The meaning of this passage is: In the propositions quoted, the line segments that occur therein have certain definite ratios to each other, which are expressed by numbers, e.g. in a triangle the centre of gravity divides a median into parts which are in the ratio 1 : 2. This way of using numbers is quite different from the one where we suppose the median to be of a given length. It appears that in Stevin's day the explicit mention of numbers gave rise to objections, and that a distinction was made between this and the Greek method of expressing ratios in words. This difference is, of course, quite inessential.

Species. alle * afcomften des voorgheftelden forms verftaet, ende d'oirfaken verclaert, maer defe niet om de contrarie redenen. Twelck Eutochius

Comentator. * uytlegghe van Appollonius int 11^e voorftel des 1^{en} bouck alfo mede verftaet iegghende: Niemand en beroer hem hier in dat dit duer de ghetalen bethoont is, want d'ouden pleghen fulcke bewyffinghen te ghebruycken, fo, doch beter Wifconftich fin dan Telconftich, om de eueredenheys wil; Merft oock datter begheerde Telconftich is, want de eueredenheden, ende de menichvuldigheden der eueredenheden, oock de * menichvuldighinghen, fin ten eerften inghetalen, ten anderen duer ghetalen inde grootheden, na t'ordeel van hem die aldus gheschreuen heeft: ταῦτα γὰρ τὰ μαθηµατὰ δοκουντιον ἀλλοφά. dat is, defe Conften alle een moets kinderen te fchynen. Nu foud ymant mueghen voortbrenghe, dat Prolæmus, Archimedes, Appollonius, Commandinus, Regiomontanus, ende meer ander in der ghelijcke voorftellen, alle palen met gheen fo uytghedruckte ghetalen en beteekenen, als hier ghedaen is. Daerop antwoord ick, dat met alfulcke reden als fy segghen van der palen tweevoudicheyt, drievoudicheyt, mette felue falmen oock mueghen fpreken daer te pas comt, van haer t'welfvoudicheyt, als A D tot R D int voornomde 23^e voorftel, ende van haer reden als 37 tot 23, ghelijck A R tot R D des bouefchreuen 11^e voorftels, ende alfo met allen anderen, want fulcke linien in yder voorgheftelde form dier afcomft, gheen ander reden en hebben. Nu anghesien datmen int onderfoucken der eyghenfchap fulcker formen, defe ghetalen ghebruyct, die ons als feker anwys, met lichticheyt tot claer verftant der faeck brenghe, fo is int befchrijuen der feluer, die ghetalen daer oock by te fetten, om voor anderen niet duyfter na te laten, t'ghene den * Vinders felfs licht en openbaer was. Want fulcx is t'recht Wifconftich bewys, t'voorgheftelde duer d'oirfaken verclarende; Twelck ons voornemen was te bethoonen.

Inuentoribus

Angaende sommighe bewyfen des eerften bouck vande beghinfelen der Weeghconst, oock des Waterwichts, inde welcke de fwaerheden duer ghetal en bekent ghewicht, als ponden, beteekent fin, t'welck yemant voorghehen Wifconftighe, maer voor werckelicke handeling mocht achten; die fal weten, dat beneuen foodanighe, oock mede ghestelt fin haer Wifconftighe bewyfen, als int 1^e voorbeelt des eerften voorftels van reerste bouck, alwaer duer ghetalen ende bekent ghewicht, des voorftels inhoudt bethoont is, maer int tweede voorbeelt, is t'felue oock Wifconftelick bewyfen, ende alfo met d'ander. Inder voughen datter werckelick bewys tot meerder claerheyt fomwylen by t'Wifconftich ghevoucht is.

V. HOOFTSTICK, WELCK VERCLARING IS OP HET VIII^e VOORSTEL DER beghinfelen des Waterwichts.

DAER is int bouefchreuen 8^e voorftel aldus gheleyt: *Y der ftyflichaems fwaerheyt is fo veel lichtet int water dan inde locht, als de fwaerheyt des waters*

also understands in this way saying ²⁾: *Let not one become agitated because this has been proved by means of numbers, for the Ancients are accustomed to use such proofs, since they are better versed in mathematics than in arithmetic, for the sake of the proportionality. Note, too, that what is required to prove is of an arithmetical nature, for ratios, and quantities of ratios* ¹⁾, and multiplications firstly reside in numbers, and secondly through numbers in magnitudes, according to the opinion of him who has written: ²⁾ ταῦτα γὰρ τὰ μαθηήματα δοκοῦντι εἶμεν ἀδελφά: ³⁾, i.e. all these arts seem to be children of one mother. Now someone might advance that Ptolemy, Archimedes, Apollonius, Commandinus, Regiomontanus, and many others do not designate in such propositions all the terms with such explicit numbers as has been done here. To this I reply that with the same right with which they speak of the double and the treble of the terms, we may also speak of their twelvefold, wherever it is relevant, as *AD* to *RD* in the aforesaid 23rd proposition ⁴⁾, and of their ratio 37 to 23, as *AR* to *RD* in the above-mentioned 11th proposition ⁵⁾, and in this way with all others, for such lines do not have any different ratio in each figure of this species. Now since in an examination of the property of such figures these numbers are used, which are a certain indication easily leading to a clear understanding of the matter, it is useful in a description thereof to add also those numbers, in order not to leave obscure for others what was light and manifest for the inventors. For such is the true mathematical proof, explaining the matter under consideration by means of the causes, as we intended to show.

As to some proofs of the first book of the elements of the Art of Weighing, and also of Hydrostatics, in which the gravities are designated by numbers and known weights, such as pounds: if someone should hold these not to be mathematical, but practical proofs, he should know that side by side with these are also given the mathematical proofs, as in the 1st example of the first proposition of the first book, where the contents of the proposition have been shown by means of numbers and known weights, but in the second example it has also been proved mathematically, and similarly with the others. In such a way that the practical proof has sometimes been added to the mathematical one, for the sake of greater clarity.

CHAPTER V, which is a commentary on Proposition VIII of the elements of Hydrostatics.

It has been said as follows in the above-mentioned 8th proposition: *The gravity of any solid body is as much lighter in water than in air as is the gravity of*

¹⁾ The translation here follows the Greek text, not Stevin's rendering, which is defective. The quantity of a ratio (ἡ πηλικότης) is the number after which the ratio is called (οὗ παρόντος ἐστὶν ὁ λόγος).

²⁾ *Apollonii Pergaei quae graece exstant cum commentariis antiquis*, ed. J. L. Heiberg. Leipzig 1891-1893. II 220. The reference is not quite to the point in that Eutocius does not speak of cases where a certain numerical ratio occurs, but of the consideration of ratios, whether rational or irrational, as numbers. After giving a demonstration in which ratios are dealt with in this way, he refutes, in the words quoted by Stevin, in advance any possible objections to this method, which indeed is at variance with the style of classical Greek mathematics.

³⁾ Archytas. Diels, *Fragmente der Vorsokratiker*, 35 B.

⁴⁾ Prop. 23 of Book II of the *Art of Weighing*. For *AD* : *RD* read *AD* : *RE*.

⁵⁾ Prop. 11 of Book II of the *Art of Weighing*.

Waters met hem euegroot. Waer uyt ymant sulcken veruolgh mocht willen maken: *I der stijflichaems swaerheyt is so veel lichter int quicfiluer dan int water, als de swaerheyt des quicfiluers met hem euegroot.* Ofte aldus: *I der stijflichaems swaerheyt is so veel lichter int water dan inde olie, als de swaerheyt des waters met hem euegroot.* ende soo met dierghelicke. Welck nootlick veruolgh, de saeck eenuoudichlick anghesien, d'eruaring teghen schijnt, want een pont loot en sal na de ghemeene ghebruyck van wegen, int water niet so veel lichter sijn dan in de olie, als de swaerheyt des waters inet hem euegroot, maer alleenlick soo veel lichter, als t'verschil tweer lichamen van water en olie met dat voornomde loot euegroot. Doch den grondt dieper inghesien, ende * d'ander parich ghestelt, so bestaet alles in d'uyterste volmaectheyt. Om t'welck te bewysen, so is t'anmercken, dat inde 1^e begheerte der beghinselen des Waterwichts versocht is, *Der lichamen ghewicht inde locht eyghen ghenoecht te worden*, ende inde 5^e begheerte, *T'ylas vat vol waters uytghegoren sinde, ledich te bliuen*, dat is vol lochts te wesen duer de 11^e bepaling, daerom ghenomen dat de twee middelen quicfiluer en water sijn, alwaer nu water inde plaets des lochts ghestelt is, ende datmen hier sghelijcx begheere, *Der lichamen ghewicht int water eyghen ghenoecht te worden*. Oock, *T'ylas vat vol quicfiluers uytghegoren sinde, vol waters te bliuen*, so is t'voornomde voorstel (*I der stijflichaems swaerheyt is so veel lichter int quicfiluer dan int water, als de swaerheyt des quicfiluers met hem euegroot*) waerachtich. Om t'welck duer ghelijcknis noch opentlicker te verclaren, so neemt dat een man gantsch onder t'water sy, aldaer by hem hebbende quicfiluer, gout, met een waegh, en houdende t'water als voor locht: ick seg datter gaudt aldaer so veel lichter sal sijn int quicfiluer dan int water, als de swaerheyt des quicfiluers metter gaudt euegroot; t'welck openbaer is. Tis wel waer dat soomen naem, *Der lichamen ghewicht int ydel eyghen ghenoecht te worden*, soot in eenuoudich an sien oock is, men soude naer sulcke eyghenheyt mueghen segghen, *I der stijflichaems swaerheyt is so veel lichter int water dan int ydel, als de swaerheyt des waters met hem euegroot*. Maer anghemerct d'omstaende, te weten dat ons ghemeene daetlicke weghinghen (naer welcke de * *Spic- Theoria* gheling alijt opsicht behoort te nemen) niet int ydel en gheschien, maer inde locht, soo ist beter na d'eerste wyse, der lichamen eyghenwicht inde locht te stellen; Int an sien van welcken, t'boueschreuen 8^e voorstel met dieder uytvolghen, in haer uyterste volcommenheyt sijn, soo wy voorghenomen hadden te verclaren.

the water having the same volume. From which someone might wish to make the following corollary: *The gravity of any solid body is as much lighter in quicksilver than in water as is the gravity of the quicksilver having the same volume.* Or as follows: *The gravity of any solid body is as much lighter in water than in oil as is the gravity of the water having the same volume,* and thus with similar cases. Which necessary corollary, when looking at the matter in a simple way, seems to be against experience, for a pound of lead will not, according to the common weighing practice, be as much lighter in water than in oil as is the gravity of the water having the same volume, but only as much lighter as is the difference between two bodies of water and oil having the same volume as the aforesaid lead. But when we look more deeply into the matter, and other things are taken to be equal, everything is quite perfect. In order to prove this, it is to be noted that in the 1st postulate of the elements of Hydrostatics it has been asked to grant *The weights of bodies in air to be called their proper weights,* and in the 5th postulate, *The surface vessel full of water, the latter being poured out, to be left empty,* i.e. to be full of air, according to the 11th definition; therefore, taking the two media to be quicksilver and water, where now water is substituted for air, and postulating similarly *The weights of bodies in water to be called their proper weights;* also *The surface vessel full of quicksilver, the latter being poured out, to be left full of water,* the aforesaid proposition (*The gravity of any solid body is as much lighter in quicksilver than in water as is the gravity of the quicksilver having the same volume*¹⁾) is true. In order to explain this even more clearly by means of comparison, assume a man to be completely under water, having there with him quicksilver, gold, with a balance, and let the water be taken for air: I say that the gold will there be as much lighter in quicksilver than in water as is the gravity of the quicksilver having the same volume as the gold; which is manifest. It is indeed true that if we took *The weights of bodies in a vacuum to be called their proper weights,* as is the case when looking at it in a simple way, it might be said in accordance therewith: *The gravity of any solid body is as much lighter in water than in a vacuum as is the gravity of the water having the same volume*²⁾. But considering the circumstances, to wit that our common practical weighings (at which the theory should always be directed) do not take place in a vacuum, but in air, it is better to postulate in the first manner the proper weights of bodies in air; in view of which the above-mentioned 8th proposition with those following therefrom is quite perfect, as we intended to set forth.

THE END OF THE APPENDIX

¹⁾ When weighed in water.

²⁾ When weighed in a vacuum.

BYVOUGH DER WEEGHCONST

SUPPLEMENT TO THE ART
OF WEIGHING

INTRODUCTION TO THE SUPPLEMENT TO THE ART OF WEIGHING

The second edition of the *Art of Weighing*, which forms part of the *Wisconstighe Ghedachtenissen* (Work XI) contains a Supplement consisting of four treatises out of six that had been planned for it.

The first, which bears the title *Of the Cord Weight*, deals with *Spartostatics*, i.e. statical problems relating to systems of stretched cords carrying weights. The solutions of these problems are based on Theorem 27 of Book I of the *Art of Weighing*, which is equivalent to the parallelogram (or triangle) of forces.

In the second treatise, *Of the Pulley Weight*, the movable pulley and the block-and-tackle are discussed. The cords carrying the lowest pulley, and consequently the weight to be raised, are first supposed to be vertical, afterwards oblique. The mechanical advantage is found in the first case by considering the number of ropes bearing the weight, and in the second case by applying the above-mentioned Prop. 27.

The third treatise, *Of the Floating Top-heaviness*, is based on a practical military problem. For the assault of a town or fortress use was occasionally made of ladders on boats, to be ascended by the soldiers carrying out the assault. In order to avoid the risk of the boats capsizing, a test ascension would be made. Stevin now endeavours to render this superfluous by means of calculation, without, however, succeeding in the enterprise.

The Supplement is concluded with a treatise entitled *Of the Pressure of the Bridle* and dealing with practical matters of horsemanship. The principal object is to understand the action of the bridle on the basis of the statical principles exposed in the *Art of Weighing*. Present-day experts on equestrian mechanics no longer accept Stevin's method. It is nevertheless a remarkable symptom of the scientific attitude which he and his princely pupil took towards the affairs of practical life that they felt impelled to study from a mechanical point of view a device which was in general use. Moreover, we learn from this treatise that they constructed an adjustable test bridle in order to verify their theoretical conclusions.

The Summary of the Supplement mentions two more titles: *Of the Drawing of Water* and *Of the Weight of the Air*. We do not know whether the first was to have been concerned with dredging problems, water wheels or marsh mills. In the latter case it may perhaps have been identical with the treatise on mills to be published in our Volume V.

The treatise *Of the Weight of the Air*, if written at all, appears to be irretrievably lost, and we can do no more than guess at its possible contents. The title suggests that Stevin attributed weight to the air, which is in agreement with some passages of his *Hydrostatics* (Defs X and XI; Postulate I) and the *Appendix to the Art of Weighing* (Ch. V). In how far he would have dealt with aerostatics on the same principles as hydrostatics, thus again forestalling Pascal, it is, however, impossible to decide.

C O R T B E G R Y P

deses byvoughs der VVeeghconst.

Theoria
quàm praxi.

MY sijn na d eerste beschrijving der Weeghconst, verscheyden stoffen der wichtighe ghedaenten voorghecommen, soo in * spiegheling als daet, diemen in dese tweede druck elck t'haerder plaets van d'eerste oirden soude hebben meugen schicken, om daer afeen verknocht lichaem te maken: Maer in siende dattet gheschapen staet, na dese meer ander te connen volghen, die om de selve reden dan dergelijcke schickking soudén vereyffschen, sulcx datter elckemael een verandering van t'voorgaende mocht vallen, so en souder des veranderens geen eynde sijn: En hoe wel dat in sijn selven best mocht wesen, nochtans belet van ander noodigher dinghen en latet my niet toe: Inder voughen dat ick d'eerste beschrijving der Weeghconst (veranderende alleen de veranderlicke) in haer form ghelaten heb, daer by voughende de voorschreven na ghecommen stoffen, die ick int geheel **B Y V O U G H** noem, inhoudende ses deelen:

Het eerste van het Tauwicht.

Het tweede van het Catrolwicht.

Het derde vande vlietende Topsvaerheyt.

Het vierde vande Toomprang.

Het vijfde vande Watertrecking.

Het sesste vant Lochtlicht.

SUPPLEMENT TO THE ART OF WEIGHING

ARGUMENT OF THIS SUPPLEMENT TO THE ART OF WEIGHING

After the original description of the Art of Weighing there have occurred to me several matters concerning static properties, both in theory and in practice, which in this second edition ¹⁾ might each have been arranged in its place in the first edition, so as to make a whole of it. But seeing that the position is such that, after these, others may follow, which for the same reason would then require to be similarly arranged, so that each time the preceding edition would be changed, there would be no end of such changes. And though this might in itself quite well be done, I am prevented by more necessary things from doing so, so that I have left the original description of the Art of Weighing as it was (only changing the things that had to be changed), adding thereto the above-mentioned matters that afterwards occurred to me, the total of which I call SUPPLEMENT, containing six parts:

- The first of the Cord Weight.
- The second of the Pulley Weight.
- The third of the Floating Top-heaviness.
- The fourth of the Pressure of the Bridle.
- The fifth of the Drawing of Water.
- The sixth of the Weight of the Air ²⁾.

¹⁾ This *Supplement* first appeared in the *Wisconstighe Ghedachtenissen* (XI), in which the *Art of Weighing* (VI) was reprinted.

²⁾ As has been remarked in the Introduction, the last-mentioned two treatises are missing.

E E R S T E D E E L
D E S B Y V O V G H S
D E R W E E G H C O N S T ,
V A N H E T
T A V W I C H T .

FIRST PART
OF THE SUPPLEMENT
TO THE ART OF WEIGHING,
OF THE CORD WEIGHT

CORTBEGRYP DES TAVWICHTS.



Y hebben inde drie laetste voorstellen des 1 boucx der VVeeghconst, beschreven de wichtige ghedaenten van swaerheden hangende an twee linien, gehecht ant lichaem tot twee verscheyden plaetsen. Maer want de swaerheden op meer ander vyfien an linien connen hanghen, waer afmen oock begheert te weten wat ghevvelt op yder lini ancomt, soo hebben wy daer af dese besonder handel ghemaect: Ende om dat in plaets van sulcke linien metter daet onder ander stoffen meest tauwvenghebruyct worden, so noemen wy dit na gemeenste gebruyck TAVWICHT; waermen by verstaen mach, een handel verclarende wat ghevvelt datter ancomt op yder tau, van verscheyden tauwen daer een bekende swaerheyt an hangt. De somme des inhouts is dusdanich: Int 27 voorstel des 1 boucx der VVeeghconst, is bevvesen dat hangende een pylaer evest alswichtich teghen twee scheefhefwichten: Gelijck als dan scheefheslijn tot rechtheslijn, also scheefhefwich tot sijn rechtheswich: Hier wyt sullen wy in dit 1 deel verscheyden vervolghen trecken, in wiens plaets men wel soude hebben meughen maken gheformde* voorstellen, doch is dat ghelaten, eensdeels om cortheyt, ten anderen dat dese vervolghen wyt het voorgaende aldus claer ghenouch schijnen.

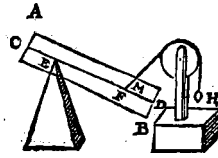
ARGUMENT OF THE CORD WEIGHT

In the three last propositions of the 1st book of the Art of Weighing we have described the static properties of gravities hanging on two lines, attached to the body in two different places. But because gravities can hang on lines in several other ways, with regard to which it is also desired to know what force acts on each line, we have made thereof the following special treatise. And because instead of such lines, among other things, cords are mostly used in practice, according to the most common usage we call this cord weight; by which is to be understood a treatise setting forth what force acts on each cord, among several cords on which a known gravity is hanging. The gist of the contents is as follows: In the 27th proposition of the 1st book of the Art of Weighing it has been proved that if a prism is hanging in equality of apparent weight with two oblique lifting weights, as the oblique lifting line then is to the vertical lifting line, so is the oblique lifting weight to its vertical lifting weight. From this we will in this 1st part draw different corollaries, instead of which regular propositions might have been made, but this has been omitted, in the first place for brevity's sake, in the second place because these corollaries appear thus to be clear enough from what precedes.

E E R S T E V E R V O L G H

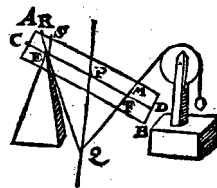
des 27 voorstels vant 1 bouck der VVeeghconst.

SOOMEN inde form des 27 voorstels vant 1 bouck, an t'punt E, in plaets van het scheefwicht G, vervoughde een vastpunt als hier nevens, tis kennelick dat teghen dit vastpunt een persing soude sijn, even an t'ghewicht G, en dat met sulcken scheefheyt teghen t'selve punt E ancommende, als de scheeflijn LE anwijst.



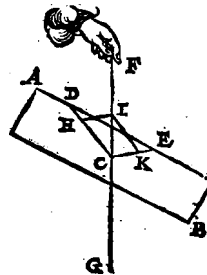
2 V E R V O L G H.

Soomen int boveschreven 27 voorstel de twee scheeflijnen LE, MF, voorttreckt tot datse versamen, tis kennelick deur het 23 voorstel, datter punt der saming comt inde hanghende swaerheys middellijn des lichaems: Daerom somen wilde weten wat scheve persing datter ancomt, opt vastpunt E des 1 vervolghs, men sal aldus meughen doen: Ick treck deur des pylaers swaerheys middelpunt als P hier nevens, de oneyndelicke swaerheys middellijn, welke vande voortgetrocken MF, ontmoet wort in Q; daer na van Q deur E de lini QR, vallende R in A M. T'welck soo sijnde, de persing op E ancommende, is als van R na E, en om te weten hoe groot, men ghebruyckt int werck ER voor scheefheflijn, en ES voor rechthefflijn, waer me men openbaerlick tottet begheerde comt.



3 V E R V O L G H.

Maer om nu te comen tot verclaring vande ghedaenten der gewichten an tauwen hanghende, soo laet AB een pylaer sijn, diens middelpunt C, en hanghende ande twee vastpunten D, E, mer twee linien C D, C E, commende uyt het swaerheys middelpunt C; de selve C D en C E sijn swaerheys middellijnen des pylaers deur des bepaling: Daerom tusschen D C en C F, ghetrocken H I ewewijdeghe met C E, soo is deur de 13 bepaling C I rechthefflijn, C H scheefhefflijn, waer me wy segghen, dat ghelijck C I tot C H, also diens rechthefwicht, tot desens scheefheflicht: Maer t'rechthefwicht van C I, is even ant ghewicht des heelen pylaers: Daerom ghelijck C I tot C H, alsoo t'ghewicht des heelen pylaers, tottet ghewicht op D ancommende: Ende inder selver voughen vintmen oock t'ghewicht op E ancommende, midts te trecken van I tot in C E, de lini I K, ewewijdeghe met D C, en meughen dan segghen, ghelijck rechthefflijn C I, tot scheefhefflijn C K, also t'ghewicht des heelen pylaers, tottet ghewicht op E ancommende.



Q 3

Maer

1st COROLLARY

of the 27th proposition of the 1st book of the Art of Weighing

If in the figure of the 27th proposition of the 1st book there were attached at the point E , instead of the oblique weight G , a fixed point as shown opposite, it is obvious that against this fixed point a pressure equal to the weight G would be exerted, this pressure acting on the said point E with such obliqueness as is indicated by the oblique line LE ¹⁾.

2nd COROLLARY

If in the above-mentioned 27th proposition the two oblique lines LE , MF are produced until they meet, it is obvious by the 25th proposition that the meeting point comes in the vertical centre line of gravity of the body. Therefore, if one should wish to know what oblique pressure acts on the fixed point E of the 1st corollary, one can do as follows: I draw through the prism's centre of gravity, viz. P opposite, the infinite vertical centre line of gravity, which MF produced meets in Q ; thereafter from Q through E the line QR , R falling in AM . This being so, the pressure acting on E is as from R to E ²⁾, and in order to know how great it is, ER is used as oblique lifting line in the construction, and ES as vertical lifting line, by means of which the required quantity is manifestly found ³⁾.

3rd COROLLARY

But in order to set forth the properties of weights hanging on cords, let AB be a prism, whose centre be C and which be hanging in the two fixed points D , E , with two lines CD , CE coming from the centre of gravity C ; these lines CD and CE are centre lines of gravity of the prism by the 5th definition ⁴⁾. Therefore, if HI be drawn between DC and CF ⁵⁾, parallel to CE , by the 13th definition CI is vertical lifting line, CH oblique lifting line, with which we say that as CI is to CH , so is the former's vertical lifting weight to the latter's oblique

¹⁾ The letter L only occurs in the figure of Prop. 27 of Book 1 of the *Art of Weighing*.

²⁾ This means that the pressure on E is directed along RE .

³⁾ With the aid of Prop. 17 of Book 1 of the *Art of Weighing* the required vertical force is found, from which by means of Prop. 20 the oblique force may be derived.

⁴⁾ The reader is reminded that, as has been said in note 3 to p. 101 of the present volume, the meaning of „centre line of gravity” in the second edition of the *Art of Weighing* is not the same as in the first. In the second edition, and consequently also in this *Supplement*, centre line of gravity is any line through the centre of gravity.

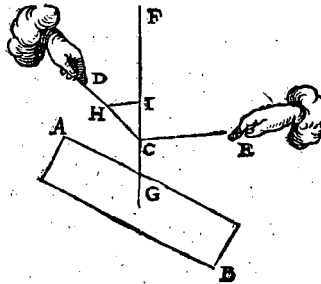
⁵⁾ CF is the vertical through C .

Maer CK valt altijd even an HI, daerom en ist niet noodich te trecken dese laetste lini IK, maer hebben alle noodighe bekende palen inde drie sijden des driehouck HIC, met welke wy menighen aldus segghen:

Ghelijck CI tot CH, alsoo t'ghewicht des pylaers, tottet ghewicht op D ancommende. Voort ghelijck CI tot IH, alsoo t'ghewicht des pylaers, tottet ghewicht op E ancommende. Weerom ghelijck CH tot HI, alsoo t'ghewicht op D ancommende, tottet ghewicht op E ancommende.

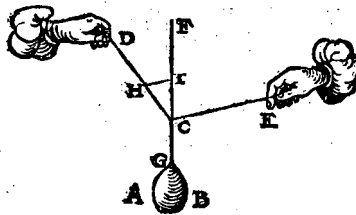
4 V E R V O L G H.

Maer op dat wy ons voorghenomen verclaring der ghedaente van ghewichten an tauwen hanghende noch naerder commen, so laet de pylaer AB neerwaert ghetrocken worden, alsoo datse nu sy ter plaats als hier onder, en deur de 3 begeerte, soo en veroirsaecte an t'ghene daerse an hangt, gheen ander ghewicht danse eerst en dede hoogher hanghende. Daerom de voorgaende everedenheyt des 3 vervolghs is noch inde form des 4 vervolghs.



5 V E R V O L G H.

Laet ons nu in plaats des pylaers AB int 4 vervolgh, hanghen een ander lichaem der selfde swaerte, maer van form en stofwaerheyt soot valt, als AB in dit 5 vervolgh. Ende is noch openbaer dat ghelijck CI tot CH, alsoo t'ghewicht AB, tottet ghewicht op D ancommende. Voort gelijck CI tot IH, alsoo t'ghewicht AB, tottet ghewicht op E ancommende. Weerom ghelijck CH tot HI, alsoo t'ghewicht op D ancommende, tottet ghewicht op E ancommende.



Hier uyt is openbaer, dat sooder aen een lini DCE als coorde, hinghe een bekend ghewicht AB, en de houcken FCD, FCE, oock bekend sijnde, darmen can segghen hoe veel gewelt elck deel DC, CE te draghen heeft.

6 V E R V O L G H.

Maer by aldien an een lini alsoo hinghen twee of meer ghewichten, als inde volghende form de lini ABCDEF, diens uytterste vastpunten sijn A en F, an welke lini hanghen vier bekende ghewichten, als G, H, I, K: Tis openbaer darmen can segghen hoe veel gewelt datter comt an elck der vijf linien AB, BC, CD, DE, EF: Want treckende, by voorbeeld gheseyt, de lini GB voorwaert, alst tot L, daer na MN ewewijdeghe met BC: Ick seggh BN gheeft BM, wat t'ghewicht G: T'ghene daer uyt volghet is voor t'ghewelt op A ancommende.

Wedes

lifting weight. But the vertical lifting weight of CI is equal to the weight of the whole prism. Therefore as CI is to CH , so is the weight of the whole prism to the weight acting on D . And in the same way the weight acting on E is also found, provided there be drawn from I to CE the line IK , parallel to DC ; we can then say: as the vertical lifting line CI is to the oblique lifting line CK , so is the weight of the whole prism to the weight acting on E .

But CK is always equal to HI , therefore it is not necessary to draw this latter line IK , but we have all the requisite known terms in the three sides of the triangle HIC , so that we can say as follows:

As CI is to CH , so is the weight of the prism to the weight acting on D . Further as CI is to IH , so is the weight of the prism to the weight acting on E . Again, as CH is to HI , so is the weight acting on D to the weight acting on E ¹⁾.

4th COROLLARY

But in order that we may make our proposed explanation of the property of weights hanging on cords even clearer, let the prism AB be pulled down in such a way that it be now in the place shown below ²⁾; then by the 3rd postulate it does not cause on that from which it is hanging any different weight from that it first did, when hanging higher. Therefore the foregoing proportion of the 3rd corollary still holds in the figure of the 4th corollary.

5th COROLLARY

Now let us hang instead of the prism AB in the 4th corollary another body of equal gravity, but of any form and specific gravity, viz. AB in this 5th corollary. Then it is also manifest that as CI is to CH , so is the weight AB to the weight acting on D . Further, as CI is to IH , so is the weight AB to the weight acting on E . Again, as CH is to HI , so is the weight acting on D to the weight acting on E .

From this it is manifest that if from a line DCE as cord there were hanging a known weight AB , and if the angles FCD ; FCE were also known, it can be said how much weight each part DC , CE has to carry.

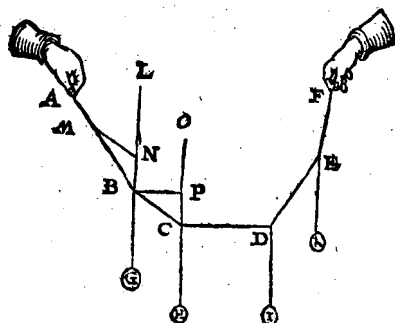
6th COROLLARY

But if there were thus hanging on a line two or more weights, as in the following figure the line $ABCDEF$, whose extreme fixed points are A and F , on which line there are hanging four known weights, viz. G , H , I , K , it is manifest that it can be said how much force acts on each of the five lines AB , BC , CD , DE , EF . For if, for example, the line GB be produced to L , and MN be then drawn parallel to BC : I say BN gives BM , what the weight G ? What follows therefrom is the force acting on AB ³⁾.

¹⁾ Here, once again, it is seen that Stevin was fully acquainted with the parallelogram (or triangle) of forces.

²⁾ Actually this figure is found opposite these words.

³⁾ Up to this point Stevin has always spoken of the weight to be carried by a fixed point or of the force acting on that point. Here there is some doubt whether „force acting on AB “ means „force on A acting along AB “ or „force on B acting along BA “ It is probable that the first meaning was intended, but Stevin is fully aware of the fact that the second force is equal and opposite to the first.

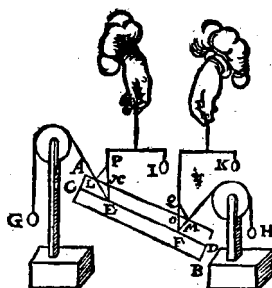


Weerom B N gheeft M N, wat T'ghewicht G: T'ghene daeruyt volgt is voor t'ghewelt op B C ancommende.

Laet andermael H C voorwaert ghetrocken worden tot O, en B P ewewijdeghe met C D: Ick segg C P gheeft C B, wat t'ghewicht H: T'gene daer uyt comt is voor t'ghewelt op B C ancommende. Waer uyt blijkt datmen aldan t'selve sal moeten vinden, datmen te vooren van B C vant: En soo voort met al d'ander. Hier af en van meer ander heeft sijn VORSTELICKE GHENADE dadelicke proef ghedaen, en bevonden die gantschelick t'overcomen mette * spiegheling.

Theorio.

De everedenheyt des 27 voorstels candeur ander manier uyt gesproken worden dan daer gedaen is, waer uyt lichter wercking volght. Om t'welck by voorbeeld te verclaren, laet hier andermael gestelt wordē de form des selven 27 voorstels, alwaermen seght, ghelijck scheefhefwicht tot rechthefwicht, also elck scheefhefwicht tot sijn rechthefwicht. Maer om dit deur ander manier uyt tespreken, waer uyt lichter wercking volght; ick treck tusschen rechtheflijn en scheefheflijn, een lini als L P ewewijdeghe met F M: T'welck soo wefende, ick segg nu, ghelijck rechtheflijn tot scheefheflijn, alsoo t'ghewicht des heelen pylaers, tot haer scheefhefwicht, dat is, ghelijck E P tot E L, alsoo t'ghewicht des pylaers tot G. Wederom ghelijck E P tot P L, alsoo t'ghewicht des pylaers tot H: Na welcke manier het vinden der onbekende palen openbaerlick corter valt, als na d'ander. Merckt noch datmen in plaets van L P, oock soude hebben meugen trecken een lini tusschen d'ander rechtheflijn en haer scheefheflijn, als hier M Q ewewijdeghe met E L, waer me men dergelijcke soude meugen doen als met L F gedaen is, en tot een selve besluyt commen: Want ghelijck P E tot E L, alsoo Q F tot F M, uyt oirsaeck dat den driehouck F M Q, even en gelijk is met L P E, deur dien Q F ewewijdeghe is met P E, en M F met L P.



Again BN gives MN , what the weight G ? What follows therefrom is the force acting on BC .

Let HC again be produced to O , and BP drawn parallel to CD . I say: CP gives CB , what the weight H ? What follows therefrom is the force acting on BC . From which it is evident that the same will then have to be found that was previously found of BC . And so on with all the others. This and several other things his PRINCELY GRACE tested in practice and found to be entirely in agreement with the theory.

The proportion of the 27th proposition can be expressed differently from the way in which it has there been done, through which the construction is easier. In order to explain this by means of an example, let there be taken again the figure of this 27th proposition, where it is said: as the oblique lifting weight is to the vertical lifting weight, so is each oblique lifting weight to its vertical lifting weight ¹⁾. But in order to express this differently, through which the construction is easier, I draw between the vertical lifting line and the oblique lifting line a line, viz. LP , parallel to FM . This being so, I now say: as the vertical lifting line is to the oblique lifting line, so is the weight of the whole prism to its oblique lifting weight, that is: as EP is to EL , so is the weight of the prism to G . Again, as EP is to PL , so is the weight of the prism to H ²⁾. By which method the finding of the unknown terms is obviously shorter than by the other. It should also be noted that instead of LP one might also have drawn a line between the other vertical lifting line and its oblique lifting line, such as here MQ parallel to EL , with which one might do the same as has been done with LF , and come to the same conclusion. For as PE is to EL , so is QF to FM , because the triangle FMQ is equal and similar ³⁾ to LPE , since QF is parallel to PE , and MF to LP .

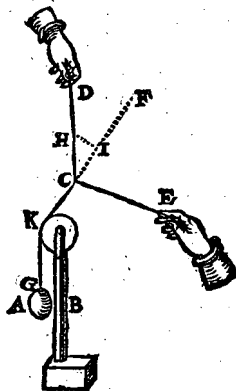
¹⁾ Stevin naturally means to say: as the oblique lifting *line* is to the vertical lifting *line*, etc.

²⁾ This of course is true, but it is quite a different thing from what was taught in Prop. 27. The object of the latter was to find the ratio of the vertical and the oblique forces which have to act on a given point in order to keep a body in equilibrium, if at another point a vertical or an oblique force is also acting on it. Stevin here determines the ratio of each of the aforesaid forces to the weight of the whole prism. It is no longer the ratio $NE : LE$ which matters, but $PE : LE$, where PL is parallel to FM .

³⁾ Only similar.

8 V E R V O L G H.

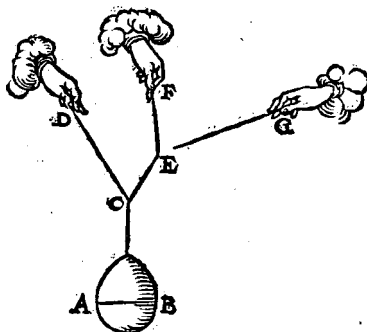
Tot hier toe is gheseyt van ghewichten hanghende an twee linien: Int volghende willen wy handelen van ghewichten an meer dan twee linien hanghende: Tot desen eynde segg ick aldus: Laet ons andermael nemen de form des 5 vervolghs, welcke sy de onderschreven deses 8 vervolghs, alleenelick daer in verschillende, dat de lini C G hier comt over een catrol an K, sulcx dat hoewel K C F een rechte lini is, nochtans comtse scheefhoutklich op den sichteinder: Voort sy dit ghewicht A B t'selve, en de twee houcken D C F, F C E oock de selve. Dit soo wesende, tis kennelick dat wy hier meughen seggen als int 5 vervolgh, ghelijck C I, tot C H, alsoo t'ghewicht A B tottet ghewicht op D ancommende: Voort ghelijck C I, tot I H, alsoo t'ghewicht A B tottet ghewicht op E ancommende: Weerom ghelijck C H, tot H I, alsoo t'ghewicht op D ancommende, tottet ghewicht op E ancommende.



Hier uyt is openbaer dat sooder an een lini D C E als coorde, hinghe een ghewicht A B, datmen can segghen hoe veel ghewelt elck deel D C, C E, te doen heeft.

9 V E R V O L G H.

Soo een ghewicht hinghe an drie linien, alst hier onder, t'ghewicht A B hanghende ande twee linien C D, C E, maer de selve C E ande twee linien E F, F G, sulcx dattet ghewicht A B hangt ande drie linien C D, E F, E G, men can weten hoe veel ghewelt datter op elcke der selve drie linien ancomt. Want deur het 5 vervolgh is openbaer watter op C D en C E ancomt: Maer bekent wesende wat ghewelt op C E ancomt, soo wort deur het 8 vervolgh gheweten watter op elcke der twee linien E F, E G ancomt.



Maer sooder ande lini C D hier boven ghehecht waren sulcke twee treckende linien als an C E, ghelijck hier onder D H, D I, tis openbaer dattet ghewicht an yder dier twee linien, alsoo bekent soude worden ghelijck over d'ander sijde, en vervolghens dat bekent soude sijn hoe veel ghewelt op yder der vier linien E F, E G, D H, D I ancomt, t'sy oock dat de linien

8th COROLLARY

So far weights hanging on two lines have been discussed. In the following we will deal with weights hanging on more than two lines. To this end I say as follows: Let us take once more the figure of the 5th corollary, which shall be the one below, of the 8th corollary, differing only in that the line CG here passes across a pulley at K , in such a way that though KCF is a straight line, it comes at oblique angles to the horizon. Further this weight AB shall be the same, and the two angles DCF , FCE shall also be the same. This being so, it is obvious that we can here say, as in the 5th corollary: as CI is to CH , so is the weight AB to the weight acting on D . Further, as CI is to IH , so is the weight AB to the weight acting on E . Again, as CH is to HI , so is the weight acting on D to the weight acting on E .

From this it is manifest that if on a line DCE as cord there were hanging a weight AB , it can be said how much force each part DC , CE has to carry.

9th COROLLARY

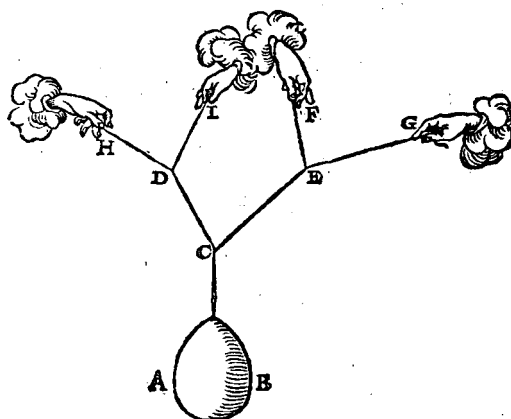
If a weight were hanging on three lines, as below, the weight AB hanging on the two lines CD , CE , but this CE on the two lines EF , FG , in such a way that the weight AB is hanging on the three lines CD , EF , EG , it can be known how much force acts on each of these three lines. For by the 5th corollary it is manifest what force acts on CD and CE . But if it is known what force acts on CE , it is known by the 8th corollary what force acts on each of the two lines EF , EG .

But if to the line CD above there were attached two such drawing lines as at CE , as below DH , DI , it is manifest that the weight on each of those two lines would become known in the same way as on the other side, and that consequently it would be known how much force acts on each of the four lines EF , EG , DH , DI , no matter whether the lines as DH and EF and the like come in the same plane or not.

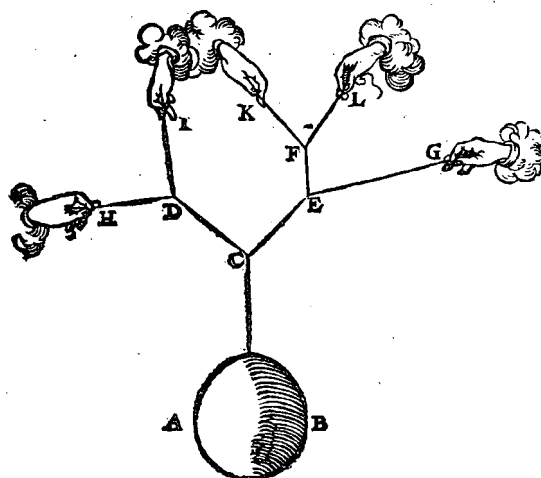
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linien als DH en EF met dierghelijcke, commen in een selve plat of niet.

Merckt noch openbaer te sijn dat de linien als CEG, CEF en dierghelijcke, niet recht en connen wesen, maer moeten een houck hebben an E, want de lini



E F eenighe ghewelt doende deur t'ghestelde, moet de lini CEG nootfakelick eenighe cromte gheven an E, alsoo oock moet de lini E G ande lini CEF.



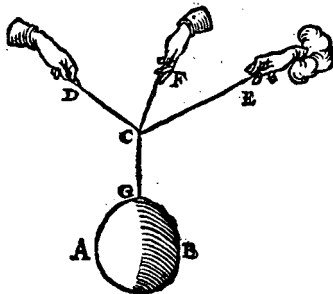
Maer so ande lini EF hier boven, ghehecht waren sulcke twee treckende linien als FK, FL hier onder, men can weten om de voorgaende redenen hoe veel ghewelt datter ankomt op elck der twee linien FK, FL: En vervolgens hoe veel an elck der vijf linien DH, DI, FK, FL, EG. En soo voort int oneyndelick met allen anderen dierghelijcke.

It should also be noted that it is manifest that the lines as CEG , CEF and the like cannot be straight, but must have an angle at E , for if the line EF exerts some force, by the supposition, it must necessarily impart some curvature to the line CEG at E ; the same must also be done by the line EG to the line CEF .

But if on the line EF above there were attached two such drawing lines as FK , FL below, it can be known for the above reasons how much force acts on each of the two lines FK , FL . And consequently, how much force acts on each of the five lines DH , DI , FK , FL , EG . And so on *ad infinitum* with all other similar cases.

Theorie.

Tot hier toe is gheseyt van een ghewicht als A B, hanghende an een lini die streckt tot C, en commende vande selve C twee ander linien C D, C E. Maer soder van die C sulcke drie linien quamen, de *spiegelingen vallen anders. Om hier af met onderscheyt te spreken ick segh aldus: De voorschreven drie linien sijn of in een selve plat, of niet: In een selve wesende, het voorstel en heeft geen eenich seker besluyt. Laet tot voorbeeld A B een ghewicht sijn, en de drie linien



daert an hangt sijn C D, C E, C F: De lini van C tottet ghewicht sy C G: Laet daer na de lini C F deursneen of ghebroken worden, sulcx dattet ghewicht A B blijve hanghen ande twee linien C D, C E; t welck soo sijnde, t ghewicht A B blijft op sijn selve plaats, en de twee houcken D C G, E C G blijven oock de selve sonder verandering; hoewel nochtans op de twee linien C D, C E, nu meer gewelt ancomt dan eer de lini C F deursneen was, wantse d'ander twee so veel verlichte, als heur ghewelt veroirsaeckte: Maer de ghewelt can an C F ghestelt worden van oneyndelicke verscheydenheden, deen grooter als d'ander, waer uyt openbaerlick blijktt sulck voorstel gheen eenich seker besluyt te hebben, ghelijck het voornemen was te verklaren.

II V E R V O L G H.

Maer soo de boveschreven drie linien in twee verscheyden platten waren, het voorstel en heeft maer een besluyt, en dat bekend. Laet by voorbeeld t'ghewicht A B hier onder genome worden te hangen ande drie linien C D, C E, C F. Maer soo datse nu niet al in een selve plat en sijn, voort is C G de lini van C tottet ghewicht. Om nu te vinden t'ghewicht op een der drie linien ancommende, als op C F, ick neem de ghemeene sine des plats daer C D, C E in sijn, en des plats daer C F in sijn, welcke ghemeene sine sy de lini C H: De selve ghenomen voor lini daer t'ghewicht A B an hangt, en d'ander twee C D, C E doorsneen, of ghebroken sijnde, sulcx dattet alleenelick blijft hanghen ande twee linien C H, C F, tis kennelick dat den houck G C F de selve blijft, diefe was voor de deursnijding der twee linien C D, C E; en de ghewelt die eerst op C F an quam, blijft na de doorsnijding oock de selve: Daerom ghenomen t'ghewicht A B te hangen ande voorschreven twee linien C F, C H, soo is deur het ; vervolg bekent wat ghewelt

10th COROLLARY

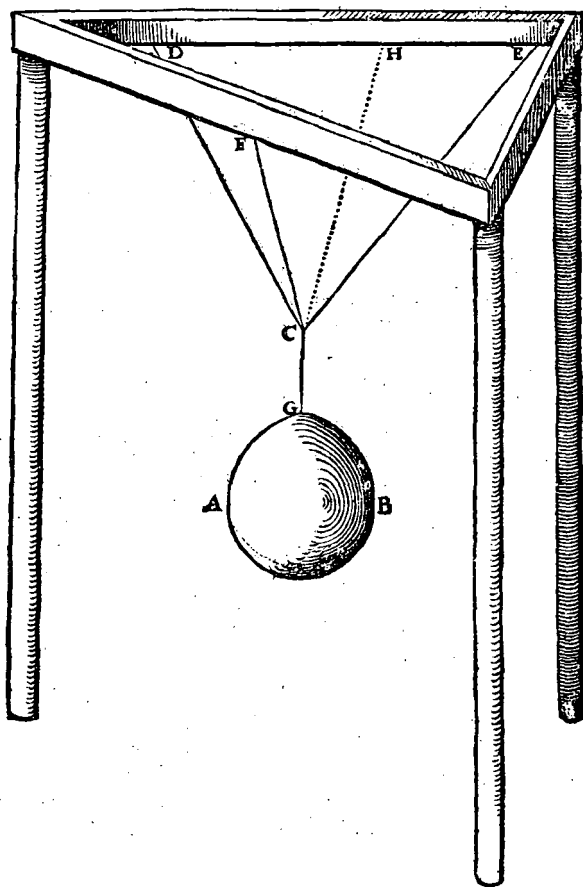
So far a weight has been discussed, such as AB , hanging on a line extending to C , with two other lines CD , CE coming from this C . But if from this C there came three such lines, the theory is different. In order to make a distinction, I say as follows: The above-mentioned three lines are either in the same plane or not. If they are in the same plane, the proposition does not have any single definite conclusion. By way of example, let AB be a weight, and let the three lines on which it is hanging be CD , CE , CF . The line from C to the weight shall be CG . Thereafter let the line CF be intersected or broken, in such a way that the weight AB continue to hang on the two lines CD , CE . This being so, the weight AB remains in the same place, and the two angles DCG , ECG also remain the same, without any change, though nevertheless there now acts more force on the two lines CD , CE than before the line CF was intersected, because it relieved the other as much as was caused by its own force. But the force on CF can be taken of infinite variety, one greater than the other, from which it is manifest that this proposition does not have any single definite conclusion, as was intended to be set forth.

11th COROLLARY

But if the above-mentioned three lines are in two different planes, the proposition has only one conclusion, and this is known. For example, let the weight AB below be taken to be hanging on the three lines CD , CE , CF , but in such a way that now they are not all in the same plane. Further CG is the line from C to the weight. Now in order to find the weight acting on one of the three lines, viz. on CF , I take the common intersection of the plane in which are CD , CE , and the plane in which are GC , CF , which common intersection shall be the line CH . If this is taken for the line on which the weight AB is hanging, and the other two CD , CE are intersected or broken, in such a way that it continues to hang only on the two lines CH , CF , it is obvious that the angle GCF remains the same that it was before the intersection of the two lines CD , CE , and the force which first acted on CF also remains the same after the intersection. If therefore the weight AB is taken to be hanging on the above-mentioned two lines CF , CH ,

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welt op C Fancomt. En alsoo sal oock bekend worden wat ghewelt op elck der twee ander linien CD, CE ancomt.



Tis oock openbaer dat by al dien an eenige, of an elcke deser drie treckende linien noch ander treckende linien quamen, na de manier des 9 vervolghs, dat bekend soude worden wat ghewelt op yder ancomt.

12 V E R V O L G H.

By al dien een ghewicht hinghe an sulcke vier linien, ghelijckt int 31 vervolgh an drie hangt, t'voorstelen heeft gheen seker eenich besluyt. Laet tot voorbeeld A, B, C, D, als in grontteyckening, sijn vier uysterste bovenste punten der vier linien daer an deur t'ghedacht het ghewicht hangt: De hanghende swaerheits middellijn des selfden sal comen of inde lini A D, of daer buyten binnen den driehouck A D B, of binnen den driehouck A D C. (want buyten den driehouck A B C D, of in sijn omtreck te vallen is onmeugelick) Maer inde lini

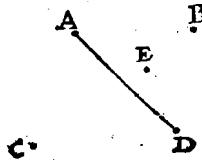
it is known by the 5th corollary what force acts on CF . And in this way it will also become known what force acts on each of the two other lines CD , CE .

It is also manifest that if to any or each of these three drawing lines there were added more drawing lines, in the manner of the 9th corollary, it would become known what force acts on each.

12th COROLLARY

If a weight be hanging on four such lines, as it is hanging on three lines in the 11th corollary, the proposition does not have any single definite conclusion. For example, let A , B , C , D , as in the plan, be four upper extremes of the four lines on which the weight is conceived to be hanging. The vertical centre line of gravity of the latter will come either in the line AD , or outside it within the triangle ADB , or within the triangle ADC (for it is impossible that it should fall outside the quadrilateral $ABCD$ or in its perimeter). But if it falls in the line

lini A D vallende, tis kennelick dat de ghewelt der twee linien onder B en C commende, wel meughen verlichten de ghewelt der twee linien onder A en D commende, maer de ghestalt des driehouckx dier twee linien, te weten de twee onder A en D, mette derde A D, en crijcht gheen verandering: En daerom meughen oneyndelicke verscheiden grooter en cleender ghewelden ande linien onder C, B, vervought worden, die de ghewelden op A en D ancommende veranderen, blijvende nochtans de form van t'ghegheven de selve, sulcx datter gheen seker eenich besluyt en is. Maer vallende de hanghende swaerheys middellijn in een der driehoucken, ick neem inden driehouck A D B an t'punt E, tis kennelick dat alsdan de ghewelt opt punt C ancommende, gheen verandering en geeft ande ghestalt der drie linien commende onder A, B, D, waer uyt her selve alvoreen volght, te weten sulck voorstel gheen seker eenich besluyt te hebben.



Noch valt hier dit te bedencken: Anghesien t'voorstel met een gewicht hanghende an vier linien als in dit 12 vervolgh, gheen seker eenich besluyt en heeft, soo en sal uyt noch stercker reden, t'voorstel met meer dan vier linien gheen seker eenich besluyt hebben. Voort anghesien een ghewicht hanghende an drie linien die in een selve plat sijn, als int 10 vervolgh, gheen seker eenich besluyt en hebben, soo en sal uyt noch stercker reden een ghewicht hanghende an vier of meer linien die in een selve plat sijn, gheen seker eenich besluyt hebben.

M E R C K T.

Een lichaem can noch hanghen an drie linien op een ander wijsse dan de voorgaende des 11 vervolghs, te weten soo dat de linien ant lichaem self tot verscheiden plaetsen ghehecht sijn, in sulcker voughen datse voorgetrocken nergens in een selve punt en vergaren ghelijck nootakelick gebeurt alst lichaem alleenelick an twee linien hangt deur het 5 voorstel des 1 boucx. Maer hoe gevonden sal worden t'ghewicht op yder van sulcke drie linien ancommende, daer heb ick op gedacht, maer int beschrijven van desen en is t'begeerde my niet verschenen, watter een ander mael of commen wil, of wat ymant anders daer in sal doen of niet, dat wert den tijt leeren.

13. V E R V O L G H.

Tot hier toe is gheseyt van ghewichten hanghende an een lini, uyt een punt van welcke twee of drie ander linien na verscheiden oirten strecken: Waer deurt openbaer sijn derghelijcke wichughe ghedaenten, van swaerheden hanghende an twee of drie linien, die ande selve swaerheyt ghehecht en opwaert voortstreckende, vergaren inde hanghende swaerheys middellijn in een selve punt. Laet by voorbeeld A B een swaerheyt sijn, hanghende ande twee linien D C, E C, versamende in C, en hanghende ande swaerheys middellijn C F. Om hier af te vinden de ghewelt op elck der twee linien D C, E C ancommende, men treckt F C voorwaert na G, en uyt eenich punt in D C, ick neem H, een lini tot in C G, als H I, ewewijdich met C E. T welck soo sijnde, ick seggh dat ghelijck

C I

AD , it is obvious that the forces acting on the two lines represented by B and C can indeed relieve the force acting on the two lines represented by A and D , but the form of the triangle of those two lines, to wit the two represented by A and D , with the third AD , is not changed. And therefore an infinite variety of larger and smaller forces can be applied to the lines represented by C , B , which alter the forces acting on A and D , the form of the given figure nevertheless remaining the same, so that there is no single definite conclusion. But if the vertical centre line of gravity falls within one of the triangles—I assume in the triangle ADB in the point E —it is obvious that in this case the force acting on the point C does not change the position of the three lines represented by A , B , D , from which follows the same as before, to wit that this proposition does not have any single definite conclusion.

In addition the following should be borne in mind: Since the proposition with a weight hanging on four lines, as in this 12th corollary, does not have any single definite conclusion, *a fortiori* the proposition with more than four lines will not have any single definite conclusion. Further, since a weight hanging on three lines which are in the same plane, as in the 10th corollary, does not have any single definite conclusion, *a fortiori* a weight hanging on four or more lines which are in the same plane will not have any single definite conclusion.

NOTE

A body can also be hanging on three lines in a different way from the foregoing one of the 11th corollary, to wit in such a way that the lines are attached to the body itself in different places, so that, when produced, they never meet in the same point, as necessarily happens when the body is hanging only on two lines, by the 25th proposition of the 1st book. But as to how the weight acting on each of such three lines is to be found: I have thought about this, but I have not been able to find the required construction; time will show whether I shall succeed another time, or what someone else will find in this matter or not ¹⁾.

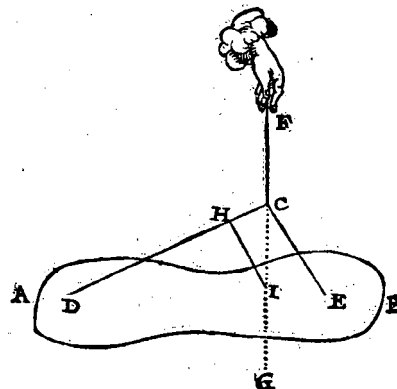
13th COROLLARY

So far weights have been discussed which are hanging on a line, from a point from which two or three other lines extend in different directions. From which are manifest such static properties of gravities hanging on two or three lines which, attached to this gravity and extending upwardly, meet in the vertical centre line of gravity in one and the same point. For example, let AB be a gravity hanging on the two lines DC , EC , meeting in C , and hanging on the centre line of gravity CF . In order to find from this the force acting on each of the two lines DC and EC , FC is produced to G , and from some point in DC —I take H —a line is drawn to CG , viz. HI , parallel to CE . This being so, I say that

¹⁾ Stevin here comes up against the problem how to resolve a system of forces acting on a rigid body along skew lines, a problem which was not adequately solved until the beginning of the 19th century.

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CI tot CH, alsoo 't ghewicht A B tottet ghewicht op D ancommende. Voort ghelijck CI tot IH, alsoo ghewicht A B, tottet ghewicht op E ancommende.



Wederom ghelijck CH tot HI, alsoo 't ghewicht op D ancommende, tot het ghewicht op E ancommende, waer af 't bewijs blijktt int 5 vervolg.

Tis oock openbaer dat sulcke eyghenschappen als gheseyt sijn te vallen inde formen van ghedaente des 9, 10, 11 en 12 vervolghs, derghelijcke eyghenschappen oock te vallen in derghelijcke formen van ghedaente deses 13 vervolghs.

TAVWICHTS
EY N D E.



R

as CI is to CH , so is the weight AB to the weight acting on D . Further, as CI is to IH , so is the weight AB to the weight acting on E . Again, as CH is to HI , so is the weight acting on D to the weight acting on E , the proof of which appears from the 5th corollary.

It is also manifest that such properties as have been said to be present in the figures of the 9th, 10th, 11th, and 12th corollaries also exist in similar figures of the 13th corollary.

END OF THE CORD WEIGHT

**TWEEDE DEEL
DES BYVOVGHS
DER WEEGHCONST,
VANT
CATROLWICHT.**

SECOND PART OF THE
SUPPLEMENT
TO THE ART OF WEIGHING,
OF THE PULLEY WEIGHT

CORTBEGRYP

DES CATROLWICHTS.



Alſoo ſijn VORSTELICKE GHENADE
deurſien hadde het bouck Delle fortificationi
di Buonaiuto Lorini, en daer in overleſen een
handel van catrollen, waer in gheſeyt wort
van ghevichten alleenlick recht opgaende, deur
treckende crachten recht neerwaert ſtrecken-
de: En dat nochtans metter daet die wils de ſelue niet recht op en
neer en gaen, ſo is hy begheerich geweeſt oock te verſtaen de crach-
ten, reden en oirſaken der ſcheewe, om alſoo van deſen handel vol-
commen kennis te hebben, wetcke gheneghentheyt oock ingenouch-
ſaem reden gegront ſchijnt, ghemerct catrollen dadelick ſeer ghe-
bruyckt worden, tot optrecking van groote ghevichten, en dat-
tet ſomwylen oirboir can ſijn, van te vooren te weten wat
macht datter behouft om een voorgheſtelde ſwaerheyt op te trec-
ken. Nu alſoo hy hem gheoeffent hadde inde voorgaende VVeegh-
conſt, mettet eerſte deel des Byvoughs; waer deur de vvichtighe
ghedaenten des Catrolvvichts grondelick connen verſtaen wor-
den, en dat hy hem dadelick daer toe begaf, ſoo hebick t'ghene daer
af ghedaen vviert onder ſijn vvifconſtighe ghedachteniſſen ver-
vough, als volgt.

V O O R.

ARGUMENT OF THE PULLEY WEIGHT

As his PRINCELY GRACE had looked through the book *Delle Fortificationi di Buonaiuto Lorini*¹⁾, and had read therein a treatise about pulleys, in which weights moving only upwards through drawing forces tending straight downwards are discussed, and because nevertheless in practice they frequently do not move straight up and down, he was anxious to understand also the forces, reasons, and causes of the oblique weights, in order thus to have perfect knowledge of this matter, a desire which also seems to be founded on sufficient reasons, seeing that pulleys are frequently used in practice for pulling up large weights, and that it may sometimes be useful to know beforehand what force is required to pull up a given gravity. Now having exercised himself in the preceding Art of Weighing, with the first part of the Supplement, through which the static properties of the pulley weight can be thoroughly understood, and because he applied himself to it in practice, I have included the matter referring thereto among his mathematical memoirs, as follows.

¹⁾ *Delle fortificationi di Buonaiuto Lorini, Nobile Fiorentino, Libri Cinque. Venetia 1592, 1597, 1609.* We do not know which of these editions it was that Maurice consulted.

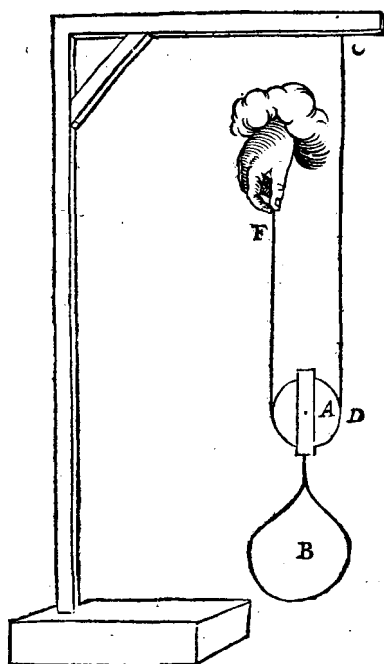
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VOORSTEL.

T'onderfoucken de ghedaenteder ghevichten opghe-
trocken met catrollen.

EER wy totte faeck commen sullen int ghemeen dit segghen: Als wy spre-
ken van een ghegeven ghewicht; men mach sich int ghedacht beelden,
om vande faeck met volcommenheyt claerlicker te handelen, dattet ghe-
wicht des ondersten catrols, mettet ghewicht daer an hangende, t'samen maken
t'ghegeven gewicht; voort dattet verschil der swaerheyt veroirsaect deur de tau,
hier voor gheen verschil ghenomen en wort.

1 Voorbeelt met rechtwichticheyt.

Laet in dees eerste form A een catrol sijn, hanghende daer an t'ghewicht B,
de tau sy C D E F, wiens twee deelen C D, F E, ewewijt van malcander sijn, of
beyde rechthou: kich op den *lichteinder. Dit aldus wesende, en het heel ghe- *Horizontem.*
wicht B alsoo hanghende ande twee deelen C D, F E, en op yder deel eveveel
ghewelts ancomende, soo hangt om de dracyende beweeghlickheyt der schijf



an yder deel den helft van B: Daerom soo ymant sijn hant stelde ant punt F,
houdende t'ghewicht in die standt, op sijn handt soude commen den helft der

R 3 swaer-

PROPOSITION

To investigate the properties of weights pulled up by means of pulleys.

Before coming to the point, we shall say this in general: When we speak of a given weight, in order to deal more clearly and completely with the matter it is to be imagined that the weight of the lowest pulley together with the weight hanging thereon constitutes the given weight; further that the difference in gravity caused by the cord is here taken to be no difference.

1st Example, with vertical weights

In this first figure let A be a pulley, on which be hanging the weight B , the cord be $CDEF$, whose two parts CD , FE , are equidistant from each other or both at right angles to the horizon. This being so, and the whole weight B thus hanging on the two parts CD , FE , and an equal force acting on each part, there hangs, owing to the rotatability of the disc, on each part the half of B . Therefore, if a man applied his hand at the point F , keeping the weight in that position, his hand

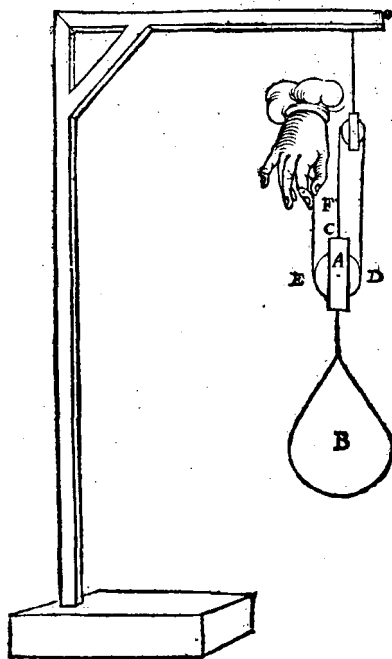
swaerheit van B, waer uyt de oirfaeck blijkt, waerom de ghewichten alſoo met een catrol lichter opgetrocken worden dan ſonder catrol. Merckt noch datmen hier ſiet plaets te houden deſe ghemeene weeghconſtighe regel:

Ghelijck wech des doenders, tot wech des lijdens,

Alſoo ghewelt des lijdens, tot ghewelt des doenders.

Want de hant an F, welcke hier doender is, opgaende 2 voeten, t'ghewicht B, dat hier lijder, en gaet maer op 1 voet, en dat om bekende oirfaken.

Deur t'ghene tot hier toe verclaert is vande eerſte form, alwaer t'ghewicht opghetrocken wort over een ſchijf, canmen verſtaen derghelijcke ghedaente wanneerment treckt over twee ſchijven, als in deſe tweede form, alwaer C



weerom tander uiterſte der tau bereyckent: Want het ghewicht B dan hangende an drie tauwen, die elck een derdendeel draghen, ſoo en heeft de hant an F dan maer de ghewelt te doen van een derdendeel des ghegheven ghewichts.

Ende

would have to carry the half of the gravity of *B*, from which appears the cause why weights are thus pulled up more easily with a pulley than without a pulley. It is also to be noted that the following common rule of statics is here seen to hold:

As is the path of the doer to the path of the sufferer,

So is the force of the sufferer to the force of the doer ¹⁾.

For the hand at *F*, which here is the doer, rising 2 feet, the weight *B*, which here is the sufferer, rises only 1 foot, such through well-known causes.

From that which has so far been set forth about the first figure, where the weight is pulled up over one disc, similar properties can be understood when the weight is pulled over two discs, as in this second figure, where *C* again designates the other end of the cord. For the weight *B* then hanging on three cords, each of which carries one-third, the hand at *F* then need only exert the force of one-third of the given weight.

¹⁾ This is a literal translation of Stevin's Dutch rendering of the rule which in Latin reads as follows:

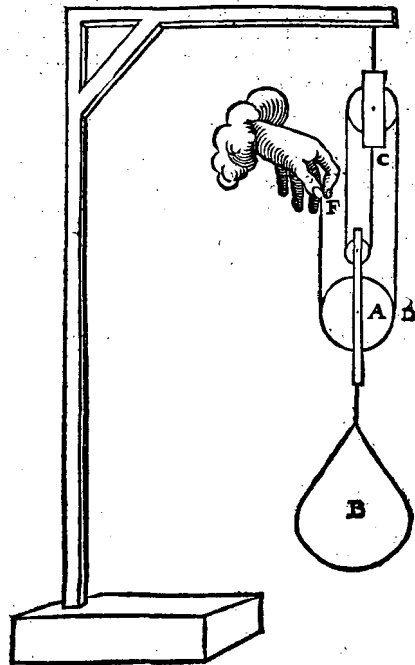
Ut spatium agentis ad spatium patientis

Sic potentia patientis ad potentiam agentis.

Stevin translates *agens* by *doender* (doer) and *patiens* by *lijder* (sufferer).

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Ende over noch een schijf meer loopende als in dees 3 form, want het ghewicht B dan hanghende an vier tauwen die elck een vierendeel draghen van B, soo en heeft de hant an F dan maer een vierendeel des ghewichts B ghewelt te



doen. Waer me bekend is de gemeene reghel van ghewichten over meer schijven ghetrocken sijnde.

Hier staet noch te dedencken datmen nietter daet selden alsoo an F opwaert treckt, ghelijck wy om claerder bewijs wille inde boveschreven drie formen by voorbeeld ghestelt hebben, maer men doet ghemeenlick de tau loopen over noch een schijf meer, om van boven neerwaert te trecken als in dese 4 form: Doch soo is te weten dat sulcke vierde oft laetste schijf, ande hant F gheen verlichting noch verandering des ghewichts en brengt, om dattet gewicht B maer an vier tauwen en hangt ghelijck inde 3 form, want dese laetste tau een vijfde tau schijnende, en is eyghentlick mette vierde al maer een selve. Waer by te verstaen is, dat al liepe die tau over noch hondert sulcke catrollen, dat den trecker daer me gheen verlichting en crijcht.

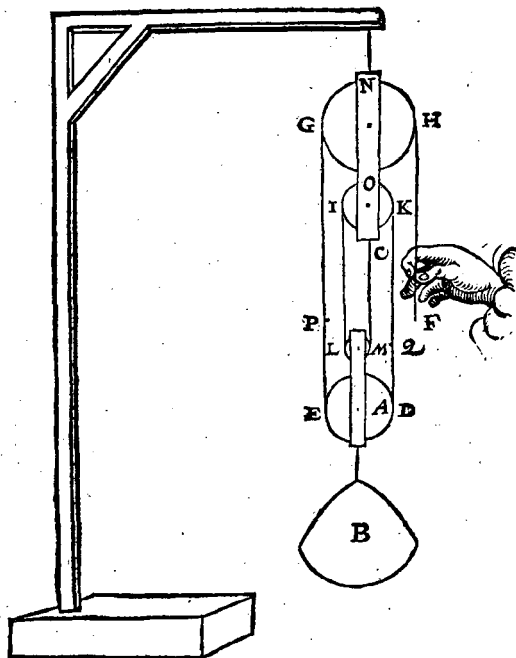
Maer soomen van t'voornomde dadelicke proef wilde sien, men sal an F deser vierde form, in plaets des hants hanghen een ghewicht als doender, wesende t'vierendeel van het optreckelick ghewicht, en sullen teghen malcander soo int werck gheen faute en is, evefaltwichtich bevonden worden. Maer om dat optreckelick ghewicht heel volcommelick uyt te spreken, het is de somme deser drie, te weten t'ghewicht B, t'onderste catrol A, en t'ghewicht veroirsaeckt deur de swaerheyt der tau. Maer om de selve swaerheyt der tau breeder te verclaren, soo laet D en E sijn de uytterste gheraeckfelen der tau teghen de schijf A, en G H

And when the cord passes over one more disc, as in this 3rd figure, because the weight *B* then hangs on four cords, each of which carries one-fourth of *B*, the hand at *F* then need only exert the force of one-fourth of the weight *B*. With which is known the common rule of weights pulled over several discs.

It should be borne in mind that in practice one will seldom pull upwards at *F* in this way, as we have assumed by way of example in the above three figures, for the sake of a clearer proof, but the cord is usually passed over one more disc, in order to pull downwards from above, as in this 4th figure. But it should be known that such fourth or last disc does not cause any relief or change of the weight on the hand at *F*, because the weight *B* only hangs on four cords, as in the 3rd figure, for this last cord, which seems to be a fifth cord, is in reality one and the same with the fourth. By this it is understood that even if that cord ran over a hundred more such pulleys, the puller would not receive any relief therefrom.

But if it were desired to see a practical proof of the foregoing, at *F* of this fourth figure there shall be hung instead of the hand a weight as doer, being one-fourth of the weight to be pulled up, and if there is no error in the construction, they will be found to be of equal apparent weight with each other. But to describe that weight to be pulled up quite completely: it is the sum of these three, to wit the weight *B*, the lower pulley *A*, and the weight caused by the gravity of the cord. But in order to set forth this gravity of the cord more fully, let *D* and *E* be the extreme points of contact of the cord against the disc *A*, and *G* and *H* the extreme points of contact of the cord against the upper disc of the

de uiterste gheraackfelen der tau teghen de bovenste schijf des bovenste catrols,
 L M de uiterste gheraackfelen der tau teghen de bovenste schijf des ondersten
 catrols; voort sy N t' middelste punt der tau tusschen G en H, en O t' middelste



punt der tau tusschen I en K, en C t' ander uiterste der tau: Laet voort gheteyckent worden in G E t' punt P, alsoo dat G P even sy met H F: Daer na in K D t' punt Q, alsoo dat K Q even sy met I L. Dit so wesende, N G P is even en evenwichtich met N H F, en O I L met O K Q: Maer C M en brengt lichticheyt noch swaerheyt by. Sulcx dattet ghegeven gewicht mettet catrol, noch beswaert worden, so veel als veroiraken dedrie sticken taus, te weten des halffronts L M, des halffronts D E, en het recht stick Q D.

Merckt noch dat als men met catrollen dadelick yet optrecket, alsoo dattet eynde der voortghetrocken tau inde locht blijft hanghen, sonder vloer te gheraken, soo veel dat voortghetrocken deel taus weeght, soo veel sal openbaerlick den trecker min ghewelt behouven te doen.

2 Voorbeelt met scheefwichticheyt.

Laet dese eerste form sijn als insghelijck d' eerste des eersten voorbeelts, uytghenomen dat de hant hier an F niet recht op en treckt, maer scheef ter sijde waert uyt, t'welck soo sijnde, t'ghewicht op elcke tau ancommende, wort bekend deur het s vervolg des 1 deels des byvoughs der Weeghconst. Maer om daer af met een wat verclaring te doen; ick treck de lini daer t'ghewicht B an hangt opwaert tot G, als B G, en F E voorwaert, tot datse de oneyndelicke door B G ontmoet, t'welck sy in H: Daer na uyt eenich punt der lini H F als I, een
 lini

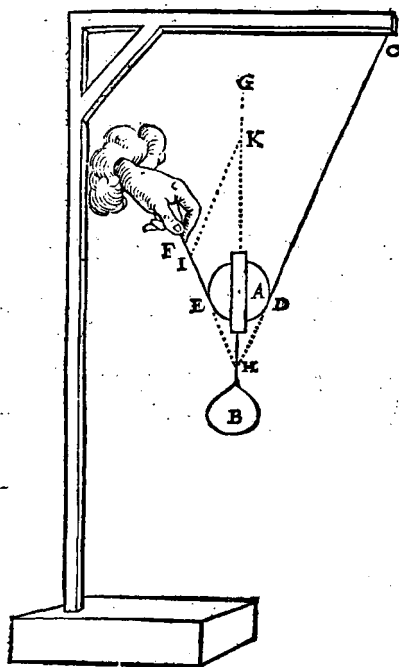
upper pulley, L and M the extreme points of contact of the cord against the upper disc of the lower pulley; further N shall be the middle point of the cord between G and H , and O the middle point of the cord between I and K , and C the other end of the cord. Further let there be marked in GE the point P so that GP be equal to HF ; thereafter in KD the point Q so that KQ be equal to IL . This being so, NGP is equal and of equal weight to NHF , and OIL to OKQ . But CM adds neither levity nor gravity, so that the given weight with the pulley is weighted by the weight of the three pieces of cord, to wit that of the semicircle LM , that of the semicircle DE , and the straight piece QD .

It is also to be noted that if in practice something is pulled up by means of pulleys, so that the end of the cord that is being pulled hangs in the air, without touching the floor, it is manifest that the puller will need to exert so much less force as is the weight of the piece of cord that is being pulled.

2nd Example, with oblique weights

Let this first figure be in every respect identical with the first of the first example, except that the hand here does not pull vertically upwards at F , but obliquely sidelong, which being so, the weight acting on each cord becomes known by the 5th corollary of the 1st part of this Supplement to the Art of Weighing. But in order to give at once some explanation of this: I produce the line on which the weight B is hanging upwards to G , viz. BG , and FE forwards until it meets the vertical through BG , which shall be in H . Thereafter I draw from some point of the line HF , viz. I , a line meeting BG in K , viz. IK parallel to DC . This being

lini gherakende BG in K, als I K ewijdeghe met DC. T welck soo sijnde, ick segh ghelijck I K tot K H, alsoo t'ghewicht deur de hant F ghetrocken, totter ghegheven ghewicht B: Voort ghelijck H I tot I K (die in voorbeelden met een schijf als dit altijt evelanck moeten sijn, want CD voortghetrocken wefende moet comen in H, en den houck G H I, valt om bekende redenen altijt even anden houck G H C) alsoo t'ghewelt op de hant F ancommende, totter ghewelt op C ancommende, welcke twee machten in voorbeelden met een schijf als dit,



altijt even moeten sijn, doende elck den helft eens ghewichts, dat in sulcken reden is totter ghegheven ghewicht, als H K tot H I deur het voorschreven 3 vervolgh des 1 deels deses Byvoughs der Weeghconst.

Maer by aldien de scheefstreckende tauwe liepe over twee of meer schijven, alles wort oock bekend. Laet by voorbeelt dese tweede form sijn alsins ghelijck de tweede des eersten voorbeelts, uytghenomen dat de hant hier an F niet recht op en treckt, maer scheef ter sijde waert uyt, t welck soo sijnde, t'ghewicht op elke tau ancommende, wort oock bekend deur het boveschreven 3 vervolgh. Maer om daer af met een wat verclaring te doen, ick treck de lini daer t'gewicht an hangt opwaert tot G, als BG, en FE voorwaert tot datse de oneyndelicke door BG ontmoet, t welck sy in H, teyckenende daer na t'bovenste punt daert bovenste catrol an hangt met I, en treck H I, daer na wt eenich punt der lini H F als K, een lini gherakende H G in L, als K L ewijdighe met H I: T welck soo sijnde, ick segh ghelijck K H tot L H, also t'ghewelt op de hant ancommende, totter gegeven ghewicht: Maer K H is in alle voorbeelden met twee schijven als dit, altijt even an den helft van K L, daerom t'ghewelt op F ancommende, is den

so, I say: as IH is to KH , so is the weight pulled by the hand F to the given weight B . Further, as HI is to IK (which in examples with a disc like this one should always be equally long, because CD being produced must come in H , and for known reasons¹⁾ the angle GHI is always equal to the angle GHC), so is the force acting on the hand F to the force acting on C , which two forces in examples with a disc like this one should always be equal, each being the half of a weight which has the same ratio to the given weight as HK to HI ²⁾, by the above-mentioned 5th corollary of the 1st part of this Supplement to the Art of Weighing.

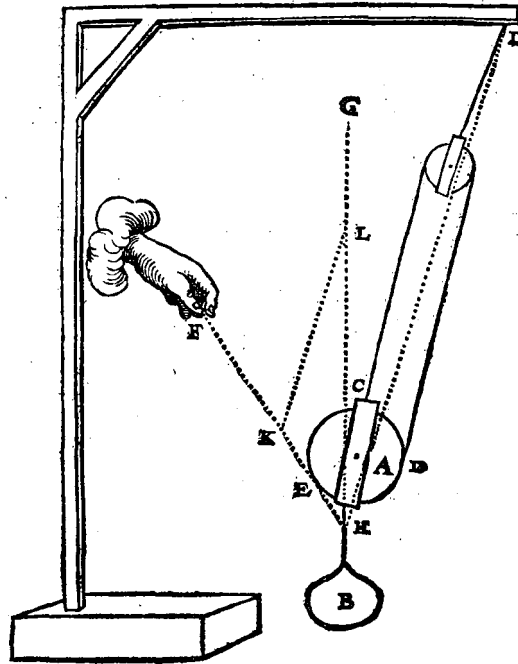
But if the oblique cords pass over two or more discs, everything also becomes known. For example, let this second figure be in every respect identical with the second of the first example, except that the hand here does not pull vertically upwards at F , but obliquely sidelong, which being so, the weight acting on each cord also becomes known by the above-mentioned 5th corollary. But in order to give at once some explanation of this, I produce the line on which the weight is hanging upwards to G , viz. BG , and I draw FE forwards until it meets the infinite vertical through BG , which shall be in H , marking thereafter the highest point on which the upper pulley is hanging by I , and I draw HI , thereafter from some point of the line HF , viz. K , a line meeting HG in L , viz. KL parallel to HI . This being so, I say: as KH is to LH , so is the force acting on the hand to the given weight. But KH is, in all examples with two discs like this one, always equal to the half of KL ; therefore the force acting on F is the half of the force acting

¹⁾ The angle EHD being bisected by AH .

²⁾ We are at a loss to understand this. If we take this passage as it stands, we find Stevin asserting that the force X acting along HF is the half of a force Y determined by the proportion $\frac{Y}{G} = \frac{HK}{HI}$

Now we also have the proportion $\frac{G}{X} = \frac{HK}{HI}$; hence $G^2 = X.Y = 2.X^2$, and since $HI = IK$, the triangle KHI would have to be isosceles and rectangular. However, not only does the figure show nothing of the kind, but also: why should the angle FHK have to be 45° ?

den helft des gewelts op I ancommende, waer deur op elck der drie tauwen evenveel ghewelts comt, te weten het derdendeel eens ghewichts, dat in sulcken reden is tottet gheheven ghewicht, als L H tot H K, daerom segghende in alle



sulcke voorbeelden, K H gheeft H L, wat t'gheheven ghewicht? het derdendeel van t'ghene daer uyt comt is voor de ghewelt op de hant F ancommende, en oock op elcke van d'ander twee tauwen.

Maer alstier alsoo drie schijven sijn, soo ist kennelick datmen dan moet nemen het vierendeel van dat uytcommende ghewicht, en soo voort met allen anderen.

De reden waerom K L hier boven meer ewijdeghe moest sijn met H I, dan met eenighe der tauwen, is kennelick deur t'ghene wy van d'ergelijcke gheseyt hebben int 2 en 3 vervolg van 1 deel des Byvoughs der Weeghconst, want de hanghende swaerheys middellijn des gheheels, streckt deur t'punt H, van welck punt openbaerlick de twee linien moeten comen daer wy ons rekening op maken. **T B E S L V Y T.** Wy hebben dan ondersocht de ghedaente der ghewichten opghetrocken met catrollen, na den eysch.

C A T R O L W I C H T S
E Y N D E.

on I , as a result of which an equal force acts on each of the three cords, to wit one-third of a weight which has the same ratio to the given weight as LH to HK . Therefore we say in all such examples: KH gives HL , what the given weight? the third part of what follows therefrom is the force acting on the hand F , and also on each of the other two cords ¹⁾).

But if in this way there are three discs, it is obvious that the fourth part of the resulting weight should then be taken, and so on with all others ²⁾).

The reason why KL above had to be parallel to HI rather than to anyone of the cords is obvious from what we have said about similar things in the 2nd and the 3rd corollary of the 1st part of the Supplement to the Art of Weighing, for the vertical centre line of gravity of the whole passes through the point H , from which point must manifestly proceed the two lines which are used in the calculation. CONCLUSION. We have therefore examined the properties of weights pulled up by means of pulleys, as required.

END OF THE PULLEY WEIGHT

¹⁾ Here the same difficulty arises as was pointed out in Note 2.

$$\text{If really } X = \frac{1}{3} Y, \text{ and } \frac{Y}{G} = \frac{LH}{KH},$$

$$\text{while at the same time } \frac{G}{X} = \frac{LH}{KH},$$

then $G^2 = 3 X^2$ or $HL^2 = 3 KH^2$. Now according to Stevin, $KH = \frac{1}{2} KL$, thence $KL^2 = 4 KH^2 = HL^2 + KH^2$, so that the angle HKL would have to be a right angle, and consequently KH a horizontal line.

²⁾ When we follow up Stevin's reasoning, we here again arrive at impossible results.

D E R D E D E E L
D E S B Y V O V G H S
D E R W E E G H C O N S T ,
V A N D E
V L I E T E N D E T O P .
S W A E R H E Y T .

THIRD PART OF THE
SUPPLEMENT TO THE ART
OF WEIGHING, OF THE
FLOATING TOP-HEAVINESS

CORTBEGRYP

der vlietende Topsvaerheyt.



Is ghebeurt dat men wilde bereyden seker schuyten, met leeren daer in overeynde staende, ontrent 20 voeten hoog, om crijchsvolck daer op te gaen: Maer alsoot in twijfel stont oft niet te groote topsvaerheyt by en soude brengen, sulcx dat de schuyt mocht ommeslaen, en t volck int water vallen, men bereyde, om verskerder te syn, een schuyte met haer leere en toebehoorten; daer na versochtment dadelick. Dit veroirsaeckte my te overdencken, oft niet meughelick en soude syn sulcx te vveten deur vveeghconstighe rekeninghen op ghestelde formen en svaerheden, scnder de saeck eerst int groot te moeten maken, en daer na dadelick te versoucken. Tot dien eynde vonden en beschreven wy het volgende* vertooch: T welck als ment een onderscheyden naem wilde gheven, nagheleghentheyt vant voornaemste eynde daert toe streckt, men soudet meugen heeten Vertooch der vlietende Topsvaerheyt, dat is van topsvaerheyt der stoffen die opt water vlieten, of drijven, vant van ander topsvaerheyt der lichamen opt vast lant, die omvallen als des lichaems svaerheys middelpunt is buyten de hanghende svaerheys middellijn, en is ons voornemen niet hier te handelen.

Theorema.

VER.

ARGUMENT OF THE FLOATING TOP-HEAVINESS

It has sometimes happened that it was desired to make certain vessels, with ladders standing upright therein, about 20 feet high, for soldiers to ascend them. But since it was doubted whether this would not cause too great top-heaviness, so that the vessel might capsize and the soldiers fall into the water, a vessel was made, in order to be surer, with its ladder and accessories; thereafter it was tested in practice. This set me thinking whether it would not be possible to know this through static calculations of assumed forms and gravities, without first having to make the thing on a large scale and then testing it in practice. To this end we found and described the following theorem: which, if one wished to give it a distinct name, might be called, because of the chief end it serves, *Theorem of the Floating Top-heaviness*, i.e. of the top-heaviness of materials floating on water, for we do not intend to deal here with other top-heaviness, of bodies on firm land, which turn over when the body's centre of gravity is outside the vertical centre line of gravity ¹⁾).

¹⁾ Since the vertical centre line of gravity is defined as the vertical through the centre of gravity of the body, this situation cannot arise. Stevin probably means to say: when the vertical centre line of gravity meets the floor outside the perimeter of the base.

VANDE VLIETENDE TOPSWAERHEYT. 201
VERTOOCH.

Een lichaem vlietende opt vvater, neemt daer in altyt sulcken ghefalt, dat sijn svvaerheys middelpunt, is in des vvaterhols hanghende svvaerheys middellijn.

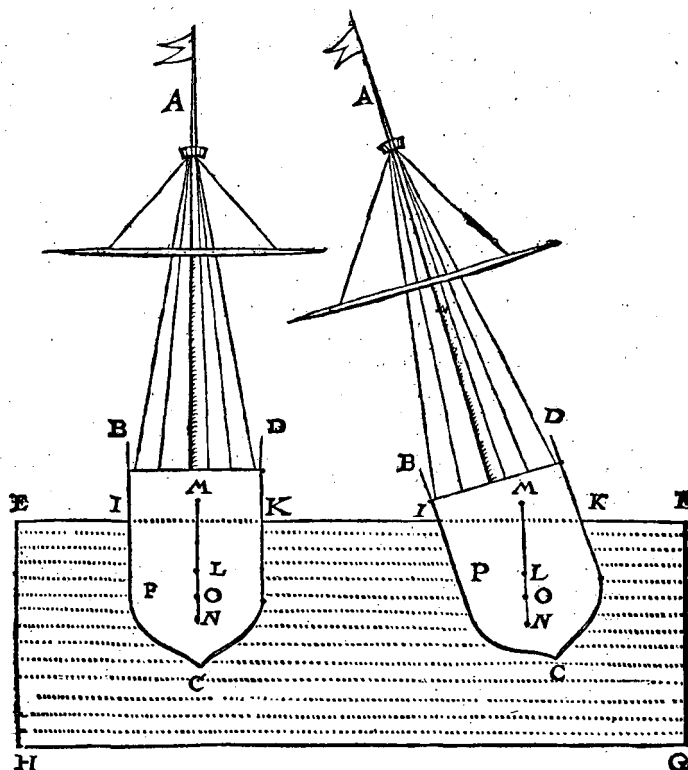


I GHEGHEVEN. Laet $ABCD$ een lichaem sijn, drijvende opt water $EFGH$, diens oppervlack EF , en steekt daer in tot IK toe, sulcx dat ICK het waterhol beteykent, diens swaerheys middelpunt L , sijn hanghende swaerheys middellijn MLN , en des lichaems $ABCD$ swaerheys middelpunt O .

T BEGEERDE. Wy moeten bewijfen dattet lichaems $ABCD$ swaerheys middelpunt O , is in des waterhols ICK hangende swaerheys middellijn MLN .

T B E W Y S.

Laet ons t'lichaem $ABCD$ uyt het water trecken, en nemen door t'gedacht dattet waterhol ICK in sijn selve form blijve: En tot noch meerder clactheyt,



dattet selve waterhol sy een vlackvat, na de wijze der 7 bepaling des waterwichts. Dit vlackvat aldus gheledicht wesende van sijn lichaem, latet vol waters ghegoten worden: En want water in water alle ghefalt hout diemen hem gheeft, deur het 1 voorstel des waterwichts, so sal t'vlackvat in die ghefalt blijven, sulcx
S dat

THEOREM

A body floating on the water always takes therein such a position that its centre of gravity is in the vertical centre line of gravity of the body of displaced fluid.

SUPPOSITION. Let $ABCD$ be a body, floating on the water $EFGH$, whose upper surface is EF , and it is submerged therein as far as IK , so that ICK designates the body of displaced fluid, whose centre of gravity is L , its vertical centre line of gravity being MLN , while the centre of gravity of the body $ABCD$ is O .

WHAT IS REQUIRED TO PROVE. We have to prove that the centre of gravity O of the body $ABCD$ is in the centre line of gravity MN of the body of displaced fluid ICK .

THE PROOF

Let us pull the body $ABCD$ out of the water and conceive that the body of displaced fluid ICK keeps the same form, and for greater clarity, that this body of displaced fluid be a surface vessel, in the manner of the 7th definition of hydrostatics. This surface vessel thus being emptied of its body, let it be poured full of water. And because water keeps in water any place given to it, by the 1st proposition of hydrostatics, the surface vessel will keep that place, so that it keeps the same place, whether it be filled with water or with the body $ABCD$. But the centre of gravity of this water poured in is also the centre of gravity of the body of displaced fluid or surface vessel, to wit L ; and therefore the centre of gravity of the body $ABCD$ must be in the vertical centre line of gravity MN of the surface vessel. For let it, if it were possible, be outside it, I assume in the point P . But this cannot happen without change of the form of the body of displaced fluid ICK , for since it had this form when the centre of gravity of the body was in O by the supposition ¹⁾, through displacement of the material of the body in such a way that the centre of gravity should come in P , B would then have to sink, D to rise, and C to turn towards K , which would be contrary to the supposition, and the body of displaced fluid would be another than was assumed. There-

¹⁾ The argument is far from being convincing. It has to be proved that the centre of gravity of the floating body, viz. O , is in the vertical through L , the centre of gravity of the displaced fluid. Here, however, O is said to be in this vertical by the supposition. It is to be noted that Stevin nowhere gives an equivalent of the statement that the upthrust experienced by the floating body acts along the vertical through the centre of gravity L of the displaced fluid, and that O therefore has to be in the vertical of L in order that the upthrust may balance the weight of the body.

dattet so wel met water ghelaen, als mettet lichaem ABCD, al een selve ghestalt hout: Maer dit inghegoten waters swaerheys middelpunt is oock des waterhols of vlackvats swaerheys middelpunt, te weten L; en daerom moet des lichaems ABCD swaerheys middelpunt sijn in des vlackvats hangende swaerheys middellijn MN: Want latet soot meughelick waer daer buyten wesen, ick neem ant punt P: Maer dat en can niet gheschien sonder verandering vande form des waterhols ICK, want nadient dese ghestalt hadde wesende des lichaems swaerheys middelpunt an O deur t'ghestelde, so soude deur verlegging der stof des lichaems, sulcx dattet swaerheys middelpunt quaem an P, alsdan B moeten dalen, D oprijfen, en C keeren na K toe, t'welck teghen t'gestelde waer, en een ander waterhol soude sijn dan daer verschil af is: Daerom des lichaems swaerheys middelpunt is in MN, te weten of onder des waterhols swaerheys middelpunt L, of daer boven, of daer in. T B E S L V Y T. Een lichaem dan drijvende opt water, neemt daer in alijt sulcken ghestalt, dat sijn swaerheys middelpunt is in des waterhols hanghende swaerheys middellijn, t'welck wy bewijfen moesten.

1 V E R V O L G H.

Tis kennelick dat als des lichaems swaerheys middelpunt, is boven des waterhols swaerheys middelpunt, so heeftet sulcken topswaerheyt dat alles omkeert, (midts welverstaende dattet niet onderhouden en worde) tot dat des lichaems swaerheyt middellijn, is in des waterhols hanghende swaerheys middellijn, onder des waterhols swaerheys middelpunt. Als by voorbeeld een cromme stock opt water vlietende, sy hout daer in een seker ghestalt, sulcx dat al keertmen opwaert t'gene onder was, ten wil so niet blijven, maer neemt weerom d'eerste ghestalt, uyt oirfaec dat des stock swaerheys middelpunt, dan niet en is in des waterhols hangende swaerheys middellijn, onder des selfden swaerheyt middelpunt.

2 V E R V O L G H.

Tis kennelick dat eenich gewicht in een schip of ander vat verleyt sijnde, sulcx dat de form des waterhols verandert, dat daer me oock verandert de plaets van des waterhols swaerheys middelpunt.

3 V E R V O L G H.

Tis openbaer dat alle ghewicht geleyt onder des waterhols swaerheys middelpunt, dat ewewijlich is metten sichteinder, streckt tot vaster ganck des schips de topswaerheyt min onderworpen sijnde: Maer alle ghewicht datmen daer boven leght, streckt tot meerder topswaerheyt.

M E R C K T.

Soo de twee swaerheys middelpunten, te weten des waterhols en des schips, mer al de lichamelicke stof dieder in en op is, licht om vinden waer, tis kennelick datmen soude connen segghen deur weeghconstige wercking sonder dadelicke ervaring te doen, wat scheefheyt of ghestalt een verdocht gheladen schip int water nemen sal; en oft water over de canten soude commen of niet, gelijk mijn voornemen was te willen weten: Maer want die foucking der swaerheys middelpunten van soo veel verscheyden stoffen als ghemeenlick in een schip sijn te moeyelick soude vallen, soo en dienet niet om in sulck voorbeeld hem daer me te behelpen. Nochtans insiende dat kennis der oirfaken van topswaerheyt, en der ghestalt eens vlietende lichaems int water elders can te pas commen: Oock me dat de ghene die moeyte mocht doen van dat te foucken, hier me geholpen can worden, soo heb ick dit by ghedachtis ghestelt alsboven.

D E R V L I E T E N D E T O P S W A E R H E Y T S

fore the body's centre of gravity is in MN , to wit either below the centre of gravity L of the body of displaced fluid, or above it, or in it. **CONCLUSION.** A body therefore, floating on the water, always takes therein such a place that its centre of gravity is in the vertical centre line of gravity of the body of displaced fluid, which we had to prove.

1st COROLLARY

It is obvious that if the body's centre of gravity is above the centre of gravity of the body of displaced fluid, it has such top-heaviness that everything turns over ¹⁾ (provided, however, it be not supported) until the body's centre line of gravity ²⁾ is in the vertical centre line of gravity of the body of displaced fluid, below the centre of gravity of the body of displaced fluid. For example, if a crooked stick floats on the water, it keeps therein a certain place, in such a way that even if it is turned upside down, it will not remain thus, but resumes its first place, because the stick's centre of gravity is then not in the vertical centre line of gravity of the body of displaced fluid, below the latter's centre of gravity.

2nd COROLLARY

It is obvious that if some weight in a ship or other vessel is displaced, in such a way that the form of the body of displaced fluid changes, the position of the centre of gravity of the body of displaced fluid also changes accordingly.

3rd COROLLARY

It is manifest that any weight laid below the centre plane of gravity of the body of displaced fluid, which is parallel to the horizon, tends to steady the course of the ship, which is then less subject to top-heaviness. But any weight laid above the said plane tends to increase the top-heaviness.

NOTE

If it were easy to find the two centres of gravity, to wit of the body of displaced fluid and of the ship with all the physical material that is in and on it, it is obvious that it could be told by static construction without practical experience what obliqueness or place an imaginary loaded ship will assume in the water; and whether the water would wash over the sides or not, as I wished to know. But because it would be too difficult to find the centres of gravity of the many varied materials that are usually present in a ship, it is no use managing therewith in this example. Realizing, however, that knowledge of the causes of top-heaviness and of the place of a floating body in the water may be convenient elsewhere, and also that this may be of use to the man who should make an attempt to find it, I have written the above *pro memoria*.

END OF THE FLOATING TOP-HEAVINESS

¹⁾ It is now common knowledge that this is not generally true. The condition for stable equilibrium is that the centre of gravity of the body shall be below the metacentre. The conception of metacentre, however, was not introduced until Bouguer (1698-1758)

²⁾ Read: centre of gravity.

VIERDE DEEL
DES BYVOVGHS
DER WEEGHCONST,
VANDE
TOOMPRANG.

FOURTH PART OF THE
SUPPLEMENT TO THE ART
OF WEIGHING, OF THE
PRESSURE OF THE BRIDLE

CORTBEGRYP

DES TOOMPRANGS.

Hebbende syn VORSTELICKE GHENADE van kintſche daghen af tot noch toe, hem gheduerlick met grooten yver ſeer vlietich gheoeffent inde Ruyterconſt, (ſoo vwort der Italianen Cavallarizzo, in Duytſch gheenoemt, deur den Schrijver L. B. C. Stalmeeſter des Keyſers) en benevens mondelicke tſaemſpraec mette ervarenſte die hem in deſe ſtof ontmoeteden, noch deurleſen veel verſcheyden Schrijvers daer af handelende, ſoo veel nieuwytcommende als ouden: En heeft nochtans deur vvoorden noch ſchriften, noyt connen geraken tot grondelicke kennis der reden van tſgeprang der toomen, tſwelc deur cleyne wercoring, verlanging en cromming der toomdeelen, haest groote onſeker veranderinghen crycht int regieren des peerts. Sulcx dat onder anderen oock dit, hem ſeer begheerich maeckte te verſtaen de voorgaende VVeegheconſt, verhopende daer deur tot grondelicke kennis dier ſaeck te commen: Tſwelck tot ſyn vernougen oock ghebeurde, ſulcx dat hy nu toomen doet maken, niet onſekerlick taſtende ghelijck te vooren, maer met kennis der reden. Al tſwelck op * wiſconſtighe gront gebout ſynde, my heeft behoirlick ghedocht tſelve (dat hier om de voorgaende redenen int gemeen TOOMPRANG gheenoemt vwort) by ſyn wiſconſtighe ghedachteniſſen te vervougen: Te meer dat anderen dit ter hant commende, noch meer daer in fuller meughen mercken tot vordering deſer ſtof ſtreckende.

Subiecto Mathematico.

BEPAN

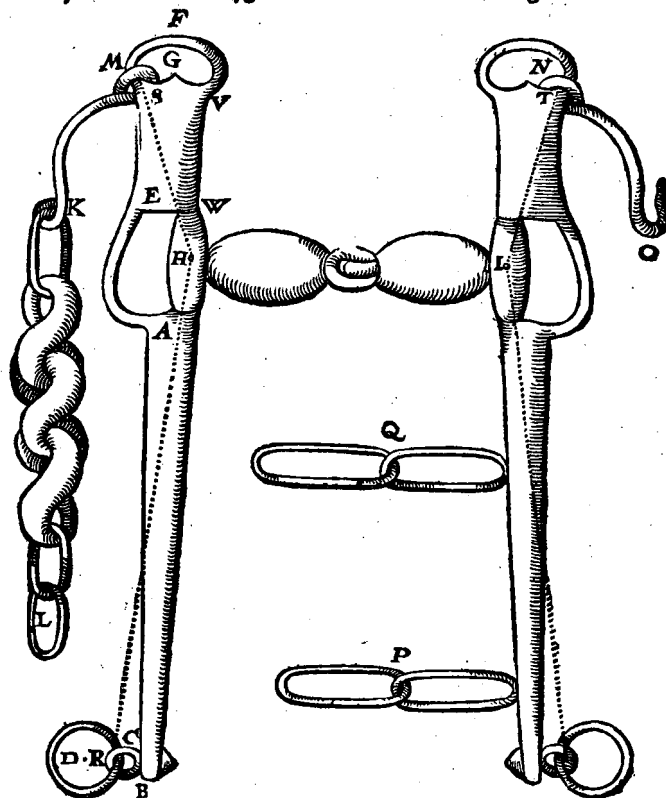
ARGUMENT OF THE PRESSURE OF THE BRIDLE

His PRINCELY GRACE having from early childhood to this day continually practised the Art of Riding (that is how the Italians' *Cavallarizzo* is called in Dutch by the writer L. B. C., the Emperor's master of the horse¹⁾) with great zeal and diligence, and having, besides oral conversations with the greatest experts he has met with in this field, also read through many different writers dealing therewith, both newly appearing and old, nevertheless he never succeeded in gaining, either through words or writings, thorough knowledge of the reason of the pressure of bridles, which through slight shortening, lengthening, and twisting of the parts of the bridle may soon bring about great uncertain changes in managing the horse. So that this, among other things, also made him very anxious to understand the foregoing Art of Weighing, hoping thereby to gain thorough knowledge of that matter. Which to his pleasure happened, so that he now causes bridles to be made, not groping uncertainly, as before, but with knowledge of the reason. All this being founded on a mathematical basis, it seemed appropriate to me to include this (which, for the above reasons, is here generally called *Pressure of the Bridle*) among his mathematical memoris. The more so that others, when it comes to their notice, may discover more about it, which will tend to advance this matter.

¹⁾ We do not know who L.B.C. was. Probably he was a German, but *Ruyterconst* in Stevin's text is a Dutch word. Here again it is evident that Stevin does not distinguish sharply between the two languages.

BEPALINGHEN.

D E ghewoonlicke namen vande deelen des tooms tot dit voornemen noodich, worden deur de byghestelde forma verclaert als volght.



1 BEPALING.

AB Stang.

2 BEPALING.

C Stangbout.

3 BEPALING.

D Teughelrinck.

4 BEPALING.

EF Stangsbovedeel.

S,

5 BEPA-

DEFINITIONS

The usual names of the parts of the bridle, required for this purpose, are set forth as follows by the accompanying figure.

1st DEFINITION

AB Cheek

2nd DEFINITION

C Bit bolt.

3rd DEFINITION

D Bit ring.

4th DEFINITION

EF Upper cheek.

G Oogh.

6 BEPALING.

HI Montstick.

7 BEPALING.

KL Kinketen.

8 BEPALING.

KM De es.

9 BEPALING.

NO Kinketenhaeck.

10 BEPALING.

P,Q Tyvee tuffcheketens.

11 BEPALING.

Wreetoom, of vree deelen der felve, fijn die t'mont-
stick ftijs teghen het onderfte tantvlees en de kinketen te-
gen de kin doen drücken. Slappe, die ter facht tegen doen
drücken.

VERCLARING.

Hoe wel een ghetrocken toom verfcheyden druckingen veroufaecht, als be-
neven de bovefchreven teghen het tantvlees, en kin noch vande tuffcheketen
teghen de borft: En vanden teughelrinck teghen de ftangbout: Nochtans foo
verfaemen metter woort wreetheyt, alleenelick de ftijs drucking des mont-
stick teghen het onderfte tantvlees en des kinketens teghen de kin, als wefende
de drucking daer t'peert deur beweeght wort, en die hem wee doet, fulcx dattet
om die weedom te verfachten, de kin na fijn borft brengt, en den hals cromt:
Want ghenomen dat de kin deur de tughel een palm verre na t'peert gheuoc-
ken worde, het can deur de buyging vanden hals, maken dat de drucking onver-
meerdert blijve. Tis oock defe perfing die hem doet achterwaert deysen, mey-
nende de felve alfoo t'ontcomen of verminderen, en vreefende deur voor-
waert te gaen die te vermeerren. Dit dan wreetheyt fijnde, foo worden die too-
men of deelen der felve, welke alfo het montstick ftijs of facht teghen het tant-
vlees en kinketen teghen de kin doen drücken, ghefeyt wreet, of flap te fijn, als
wreet toom, flappe toom, wreet ftang, flappe ftang, wreet bovedeel, flap bovedeel.

5th DEFINITION

G Eye.

6th DEFINITION

HI Bit.

7th DEFINITION

KL Curb chain.

8th DEFINITION

KM The S¹).

9th DEFINITION

NO Curb hook.

10th DEFINITION

P, Q Two intermediate chains.

11th DEFINITION

Severe bridle, or severe parts thereof, are those parts which force the bit tightly against the lower gums and the curb chain against the chin. Gentle are those parts which press them gently against the gums and the chin.

EXPLANATION

Although a bridle that is being pulled causes different pressures, viz. besides those described above against the gums and the chin also that of the intermediate chain against the breast, and of the bit ring against the bit bolt, nevertheless by the word severity is only meant the tight pressure of the bit against the lower gums and of the curb chain against the chin, this being the pressure by which the horse is constrained and which hurts it, so that in order to relieve this pain it approaches its chin to its breast and bends its neck. For if it is assumed that the chin is pulled a palm further towards the horse by the rein, it can, by bending its neck, cause the pressure to remain unchanged. It is also this pressure which makes it start back, thinking that it can thus escape from it or decrease it and fearing that by going forward it may increase it. This therefore being severity, those bridles or parts thereof which thus force the bit tightly or gently against the gums and the curb chain against the chin are said to be severe or gentle, viz. severe bridle, gentle bridle, severe cheek, gentle cheek, severe upper cheek, gentle upper cheek.

¹) Stevin writes *es*, and since the part *KM* actually is shaped like the letter *S* in its long form, it is probable that *es* merely stands for the pronunciation of this letter. Hence it may be rendered in English by *es* as well.

12 BEPALING.

De cromme bochten der stanghen vvorden * keeren ghenoeemt.

*Int hooch-
duyts wron-
ghen.
Int Francken
condes.*

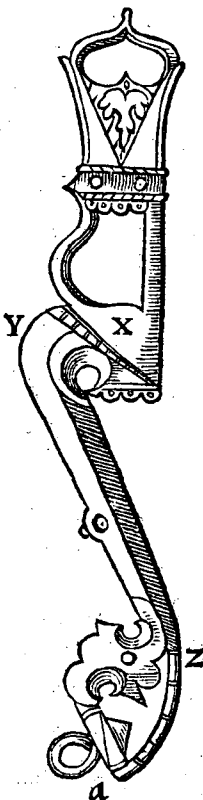
VERCLARING.

De stanghen worden recht en crom ghemaeckt, recht als in d'eerste form, crom als in dese tweede, met een bocht keerende van X na Y, van Y na Z, en van Z na a, welke men daerom deses stangs keeren noemt.

DE VOLGHENDE BEPALINGHEN SYN NIEV.

13 BEPALING.

T'middelste punt R des raeckfels vanden teughelrinck D teghen den bout C, als t'peert ghetoomt sijnde de teughels ghespannen staen, noemen vvy Teugelrijncx raeckpunt.



14 BEPALING.

T'middelste punt S des raeckfels van de es teghen het oogh, oock het middelste punt T des raeckfels vanden haeck teghen het oogh als t'paert ghetoomt sijnde de teughels ghespannen staen, noemen vvy ooghraeckpunt.

15 BEPALING.

Het punt H vanden asdes montsticx int middel vande olive commende daerden as in draeyt, noemen vvy Montsticxaspunt.

16 BEPALING.

Den houck R H S begrepen tusschen tvvee linien, d'eene van des teughelrinck raeckpunt R, tot des montsticx aspunt H, d'ander vant montsticx aspunt H, tottet ooghraeckpunt S, noemen vvy Raeckpunthouck.

12th DEFINITION

The curved bends of the cheeks are called twists.

EXPLANATION

The cheeks are made straight and curved, straight as in the first figure, curved as in this second figure, with a bend twisting from *X* to *Y*, from *Y* to *Z*, and from *Z* to *a*, which are therefore called the twists of this cheek.

THE FOLLOWING DEFINITIONS ARE NEW.

13th DEFINITION

The middle point *R* of the area of contact of the bit ring *D* against the bolt *C* when, the horse being bridled, the reins are tight, we call the point of contact of the bit ring.

14th DEFINITION

The middle point *S* of the area of contact of the *S* against the eye, also the middle point *T* of the area of contact of the hook against the eye when, the horse being bridled, the reins are tight, we call point of contact of the eye.

15th DEFINITION

The point *H* of the axis of the bit, coming in the middle of the olive in which the axis turns, we call axial point of the bit.

16th DEFINITION

The angle *RHS* contained between two lines, one from the point of contact *R* of the bit ring to the axial point *H* of the bit, and the other from the axial point *H* of the bit to the point of contact *S* of the eye, we call angle at the point of contact.

Prouftoom noem ick, een toom dienende om an alle peerden te prouven vvat ghebruyckelicke toom hun bequaemst sal sijn, en die met sekerheyt ten eersten vvelpaffende te maken.

Vande form en omstandighen deses prouftooms sal int volghende t'sijnder plaets gheseyt worden.

1 VOORSTEL.

De keeren an een slang meerder noch minder vvreetheyt te veroirfaken.

Sijn VORSTELICKE GHENADE voorseker wetende, dattet ghemeen ghevoelen van velen onrecht is, gheloovende de keeren der slang tot vvreetheyt of slapheyt te helpen, blijvende nochtans de drie punten als R, H, S, t'haerder plaets, seght daer teghen aldus: Laet op de rechte slang AB hier vooren, gheschrouft of ghehecht worden yser stucken, die de slang een form gheven als met groote keeren ghemaectte sijn: Soomen nu seght uyt die anhechting eenighe verandering der vvreetheyt te volghen, het is soo veel al of men seyde dat de selve aenghehechte yfers eenighe verborghen treckende of stekende cracht in haer hadden, ghelijck de seylsteen heeft, of dierghelijcke: T'welck ongeschickt waer. Belanghende sy segghen verandering metter daet te blijcken, dat wort weerleyt met te seggen dat sulcx metter daet niet en blijckt. Angaende Pyqueurs, toommakers, en ander met desen handel dadelick omgaende, sullen voortbrenghe de ghemeene spreuck, *Mien maer yghelick in sijn const ghelooven*: Daer wort op gheantwoort sulcx teghen hemlien te strijden, om dat sy oordeelen vande wichtige ghedaenten sonder in Weeghconst ervaren te wesen, waer in men verstaet datter verandering gheschien can deur verandering der boveschreven drie punten R, H, S: Maer die blijvende, en vervolghens oock de twee verdachte linien R, H, H, S, mercken houck R, H, S, soo blijft de vvreetheyt oock de selve, uytghenomen, om heel eyghentlick te spreken, t'verschil dattet ghewicht des bygevoughden yfers mocht veroirfaken, t'welck tot dese saeck niet en ghelt: En alsmender immers op letten wilde, t'can soo wel tot achterdeel strecken van t'ghene sy drijven, als tot voordeel.

Merckt noch wijder, dat de lini des bovedeels der slang als hier vooren VW, tot gheen seker ghemeene gront en can verstrecken om daer uyt de bocht der slang te veroirdenen, ghelijck gemeenlick ghedaen wort, maer welde lini HS, want d'een slangs bovedeel een breeder ooghe hebbende als d'ander, t'gheeft verandering en onsekerheyt inde saeck. **T B E S L Y T.** De keeren dan en veroirfaken meerder noch minder vvreetheyt an een slang, t'welck wy bewijzen moesten.

2 VOORSTEL.

De cortste slanghen de vvreetste te sijn.

De reden is hieraf tweederley: D'ene, dat met eueveel optrecking der tethels

17th DEFINITION

Test bridle I call a bridle serving to test on all horses what actual bridle will be most suitable for them, and to make it first of all fit with certainty.

The form and conditions of this test bridle will be discussed in the following in its appropriate place.

1st PROPOSITION

The twists in a cheek cause neither more nor less severity.

His PRINCELY GRACE, knowing positively that the common opinion of many people is wrong, who believe that the twists of the cheek are conducive to severity or gentleness, the three points *R*, *H*, *S* remaining nevertheless in their places, argues against it as follows:

Let there be screwed or fastened on the straight cheek *AB* hereinbefore some iron pieces which give to the cheek a form as if it were made with large twists. If one should now say that from this addition there follows a change of severity, this is as if one should say that these added irons had some hidden attractive or repellent force in them, as the magnet has, or something of the kind; which would be absurd. As regards their saying that the change becomes manifest in practice, this is refuted by saying that this does not become manifest in practice. As to the fact that riding-masters, bridle-makers, and other people practically engaged in these matters will advance the common saying: *Everyone is to be trusted in his own art*, to this it is replied that this argues against them, because they judge of the properties of weights without being versed in the Art of Weighing, in which it is understood that a change may be brought about by a change of the above-mentioned three points *R*, *H*, *S*. But if the latter remain, and consequently also the two imaginary lines *RH*, *HS*, with the angle *RHS*, the severity also remains the same, except—to speak quite accurately—for the difference which the weight of the added iron may cause, which is of no account in this matter. And even if it were to be taken into account, it may be to the detriment as well as the advantage of that which they argue.

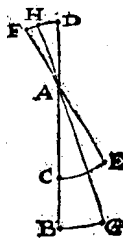
It is further to be noted that the line of the upper cheek hereinbefore *VW* cannot serve as a certain common basis on which to prescribe the bend of the cheek, as is usually done, but rather the line *HS*, for if one upper cheek has a wider eye than the other, this produces change and uncertainty in the matter. CONCLUSION. The twists therefore cause neither more nor less severity in a cheek, which we had to prove.

2nd PROPOSITION

The shortest cheeks are the most severe.

The reason hereof is twofold. One is that with the same amount of pulling of the reins the curb chain is moved more by short than by long cheeks. In order to explain this, let *AB* signify a long cheek, *AC* a shorter one, having the same upper cheek *AD*, whose point of contact of the eye is *D*; further, through the pulling of the reins, the point of contact *C* of the bit ring of the shorter cheek *AC* shall have reached *E*, having described the arc *CE*. And the point of contact

ghels, meerder beweeghnis des kinketens ghemaect wott deur corte stanghen dan deur lange. Om van t'welck verclaring te doen; Laet A B een langhe stang beteycken, A C een coter, hebbende een self bovedeel der stang A D, diens ooghraeckpunt D is, voort sy deur optrecking der teughels, des teughelrinx raeckpunt C vande corte stang A C gecommen tot E, beschreven hebbende de booch C E: En het ooghraeckpunt D sal ghecontinuen wesen tot F, beschreven hebbende de booch D F: Laet daer na deur der teughels even soo veel optrecking als d'eerste, des teughelrinx raeckpunt B vande langste stang, ghecommen sijn tot G, te weten dat de booch B G, even sy an de booch C E, en het ooghraeckpunt D, sal ghecommen wesen tot H, beschreven hebbende de booch D H. Maer de booch D F is meerder dan D H, en daer teghen in sulcken reden als de langste stang A B, totte cortste A C: Daerom de kinketen ant oogh vast sijnde, krijcht mer eveel optrecking der teughels, meerder beweeghnis deur corte stanghen dan deur lange. Maer de meeste beweging of opganck des kinketens druckt sijver teghen de kin, en veroirsaeckt oock de stijfste drucking des montsticx teghen het tantvlees: Daerom de corte stangen veroirsaken de meeste wreetheyt, en vervolgghens sijn daerom de wreetste.



D'ander reden is de bochtighe form van t'peerts hals, welke maect dat de tusscheketen der corte stang, verder vande borst staet dan vande langher, waer uyt volghet datmen de teughels van een corte stang, verder can voorttrecken eer de tusscheketen de borst gheraeckt, dan de teugels van een langhe stang, t'welck soo ghebeurt openbaerlick oock meerder wreetheyt mebrengt.

M E R C K T.

Ymant mocht nu twijfelen, en dencken hoe dit overcomt mette weeghconstighe reghelen, die leeren dat de langste steerten de grootste gewelt doen, want ansiende B D voor stock die de timmerlien waegh noemen, wiens langste steert daer den * Doender an treckt A B is, en A vastpunt, soo schijnt hier t'verkeerde besloten te worden: Men antwoort hier op aldus: De vraegh en is niet na de gewelt die den rijder metter hant int trecken doet, want hy an een corte stang, om het ooghraeckpunt eveel bewegingh te gheven, sijver moet trecken dan an een langher: Maer sijf ghenouch ghetrocken wesende, men vraecht welke trecking aldan de meeste wreetheyt mebrengt. T B E S L V Y T. De corte stanghen dan sijn de wreetste, t'welck wy bewijfen moesten.

3 V O O R S T E L.

De langste bovedeelen der stang de vvreetste te sijn.

De reden is dat met eveel optrecking der teughels, meerder beweeghnis des kinketens ghemaect wott deur langhe bovedeelen der stang dan deur corte: Om van t'welck verclaring te doen; Laet A B een lanck bovedeel beteycken, diens ooghraeckpunt B, en A C een coter, diens ooghraeckpunt C, en hebbende beyde een selve stang A D. Voort sy deur optrecking der teugels, des teughelrinx raeckpunt D, ghecommen tot E, en het ooghraeckpunt B sal ghecommen sijn tot F, beschreven hebbende den booch B F: Maer het ooghraeckpunt C tot G, beschreven hebbende de booch C G, cleender dan B F, want ghe-

lijck

D of the eye shall have reached F , having described the arc DF . Thereafter, by pulling the reins to the same extent as in the first case, let the point of contact B of the bit ring of the longer cheek have reached G , to wit that the arc BG be equal to the arc CE , and the point of contact D of the eye shall have reached H , having described the arc DH . But the arc DF is greater than DH and has thereto the same ratio as the longer cheek AB to the shorter AC . Therefore the curb chain, if attached to the eye, by the same amount of pulling of the reins is moved more by short than by long cheeks. But the greatest movement or rise of the curb chain forces it more tightly against the chin and also causes the tightest pressure of the bit against the gums. Therefore shorter cheeks cause the greatest severity, and consequently are the most severe ¹⁾.

The other reason is the curved form of the horse's neck, which causes the intermediate chain of the shorter cheek to be further away from the breast than that of the longer cheek, from which it follows that the reins of a short cheek can be pulled further before the intermediate chain touches the breast than the reins of a long cheek, which when it thus happens manifestly also involves greater severity.

NOTE

Someone might now be in doubt and think how this is in accordance with the rules of statics which teach that the longest levers exert the greatest force, for if we look upon BD as the stick which the carpenters call "waegh", whose longest lever, at which the doer pulls, is AB and A the fixed point, it seems that the opposite is concluded here. To this the following reply is given: The question is not what is the force which the rider exerts with his hand in pulling, for in order to give the same movement to the point of contact of the eye, he has to pull more firmly at a shorter than at a longer cheek. But when the pulling is firm enough, it is asked which pulling then involves the greatest severity. CONCLUSION. The shortest cheeks therefore are the most severe, which we had to prove.

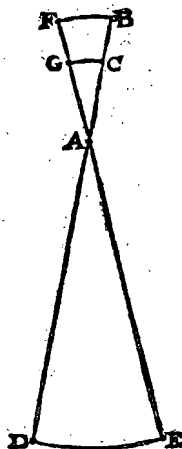
3rd PROPOSITION

The longest upper cheeks are the most severe.

The reason is that with the same amount of pulling of the reins the curb chain is moved more by long than by short upper cheeks. In order to explain this, let AB signify a long upper cheek, whose point of contact of the eye is B , and AC

¹⁾ The gist of Stevin's reasoning consists in the assumption that the cheek AB and the upper cheek EF may be considered as the two arms of a lever of the first kind, the fulcrum being in A . This assumption, however, is untenable. The device indeed constitutes a lever, but it is one of the second kind (load between force and fulcrum), in which the pressure to be exerted on the gums acts as load, the point of contact of the eye is the fulcrum, and the force is applied at the bit ring. According to J. H. Anderhub, who pointed out Stevin's error in a paper *Hier irrt Simon Stevin* (Deutsche Mathematik 7, 2-3, 1943; p. 299-304) the first to interpret the cheek as a lever of the second kind was G. O. d'Aquino, *Disciplina del Cavallo*, Udini 1636; p. 204. His words are: "... già che la guardia tutta altro non è, che una leva, la cui forza mediante le redini è posta nel pedicino e il sostegno nell'altra estremità dell'occhio dove v'è il porta morso, e il cui peso è l'incastro dove opera l'imboccatura.

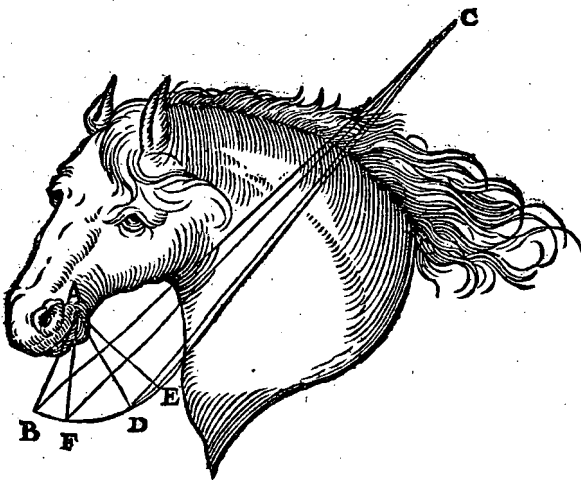
lijck AC tot AB , also CG tot BF : Daerom de kinketen ant oogh B des langste bovedeels der stang vast sijnde, krijcht mer eveveel optrecking der teughels, meerder beweeghnis dan ant oogh C des conste bovedeels vast sijnde: Maer de meeste beweging of opganck der kinketen druckt stijver teghen de kin, en veroirfaecht oock de stijfste drucking des montslicx teghen het tantvlees, daerom de langste bovedeelen sijn de wreestse. Angaende ymant twijfelen mocht wacrom den Doender an D , meer ghewelt doet op des waeghs langer eynde AB , dan op het conter AC , schijnende teghen de Weeghconstighe reghelen te strijden: De reden daer af machmen verstaen deur t'ghene van dergelijcke gheseyt is int Merck des 2 voorstels. **T B E S L V Y T.** Langhe bovedeelen dan sijn de wreestse, t'welck wy bewijfen moesten.



4 VOORSTEL.

Teughelrincx raecpunt verder vant peerts borst, geeft meerder vvreetheyt.

T G H E G H E V E N. Laet A den as des montslicx beteyckenen, AB een stang, BC den teughel, B des teughelrincx raecpunt, $A D$ een ander stang even an AB , en DC sijn teughel, D des teughelrincx raecpunt: Ende het teughelrincx



raecpunt B , sy verder vant peerts borst dan het teughelrincx raecpunt D .

T B E G H E E R D E. Wy moeten bewijfen datter teughelrincx raecpunt B , meerder wreetheyt geeft dan D . **T B E R E Y T S E L.** Laet opt punt A als middelpunt, mette halfmiddellijn AB , beschreven worden de booch BDE : Daer na sy

a shorter, whose point of contact of the eye is *C*, both having the same cheek *AD*. Further, through pulling of the reins, the point of contact *D* of the bit ring shall have reached *E*, and the point of contact *B* of the eye shall have reached *F*, having described the arc *BF*. But the point of contact *C* of the eye shall have reached *G*, having described the arc *CG*, smaller than *BF*, for as *AC* is to *AB*, so is *CG* to *BF*. Therefore, if the curb chain is attached to the eye *B* of the longest upper cheek, by the same amount of pulling of the reins the curb chain is moved more than if it is attached to the eye *C* of the shortest upper cheek. But the greatest movement or rise of the curb chain forces it more tightly against the chin and also causes the tightest pressure of the bit against the gums, therefore the longest upper cheeks are the most severe. If anyone should doubt why the doer at *D* exerts more force on the longer end *AB* of the "waegh" than on the shorter *AC*, which seems to be contrary to the rules of statics: The reason thereof can be understood from what has been said about a similar point in the Note to the 2nd proposition. **CONCLUSION.** Long upper cheeks therefore are the most severe, which we had to prove.

4th PROPOSITION

When the point of contact of the bit ring is further away from the horse's breast, this causes greater severity.

SUPPOSITION. Let *A* signify the axis of the bit, *AB* a cheek, *BC* the rein, *B* the point of contact of the bit ring, *AD* another cheek, equal to *AB*, and *DC* its rein, *D* the point of contact of the bit ring; and the point of contact *B* of the bit ring shall be further away from the horse's breast than the point of contact *D* of the bit ring. **WHAT IS REQUIRED TO PROVE.** We have to prove that the point of contact *B* of the bit ring causes greater severity than *D*. **PRELIMINARY.** Let there be described on the point *A* as centre, with the semi-diameter *AB*, the arc *BDF*. Thereafter the point of contact *B* of the bit ring shall, through pulling

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na sy des teughelrinx raeckpunt B, deur optrecking des teughels ghecommen tot F, en des teughelrinx raeckpunt D tot E, sulcx dat de booch D E, even sy an de booch B F.

T B E W Y S.

Tis daer voor te houden, dat soo veel de lini B C langher is dan F C, soo veel heeft de treckende hant by C, hoogher moeten sijn wesende des teughelrinx raeckpunt an F, dan doent was an B. Sghelijcx dat soo veel de lini D C langer is dan E C, so veel heeft de treckende hant by C, hoogher moeten sijn wesende des teughelrinx raeckpunt an E dan doent was an D: Maer E C verschilt meer van D C, dan F C van B C: En daerom soo veel t'verschil dier twee verschillen bedraecht, soo veel gaet de hant hoogher metter roersel des teughelrinx raeckpunt van D tot E, dan metter roersel van B tot F: Maer t'roersel of de booch B F, is even an t'roersel of de booch D E deur t'bereytsel, daerom de hant an C, gaet op evegroote roersels van B en D, hoogher metter roersel van D, dan metter roersel van B: En vervolghens by aldien de hant an d'een en d'ander even hooch ginghe, soo soudet t'roersel van B na F, grooter moeten sijn dan t'roersel van D na E: Maert'grootter roersel van B na F, veroirsaeckt oock grootter roersel des ooghs, en vervolghens des kinketens, dan het cleender roersel van D na E: Daerom de hant an d'een en d'ander even hooch ghegaen hebbende, soo sal t'roersel des kinketens veroirsaeckt deur trecking van B na F, grooter sijn dan deur t'roersel des kinketens veroirsaeckt deur trecking van D na E: Maert'grootter roersel of grootter opganck des kinketens, druckt stijverteghen des peerts kin, ende vervolghens doedet montstick stijver drucken teghen het tantvlees dan een cleender opganck des kinketens: Daerom met evenhooghe trecking des hants an C, doetmen het peert meer weedom, wesende des teughelrinx raeckpunt an B der stang A B, dan an D der stang A D: En vervolghens het teughelrinx raeckpunt B verder vant peerts borst, geeft meerder wreetheyt dan D.

1 M E R C K.

Anghesien den houck A D C, naerder den rechthouck is dan den houck A B C, die veel scherper is, soo doet de macht des hants by C, meerder ghewelt an de stang A D, dan de selve macht des hants by C, an de stang A B deur t'vervolgh des 24 voorstels vant 1 bouck der Weeghconst. Maer want ymant dencken mocht dit te strijden teghen t'voorgaende bewijs, soo segghen wy daer op ghelijck int merck des 2 voorstels gheantwoort wiert, te weten dat de vraegh niet en is wat macht de hant an C doet, maer de hant op den houck A B C, soo veel stijver treckende dan op den houck A D C, datse op d'een en d'ander eveveel verhoocht, men vraeght welcke trecking aldan de meeste wreetheyt mebrengt.

2 M E R C K.

Beneffens de voorgaende oirsaeck der wreetheyt, vervought heur somwijlen noch een tweede, in deser voughen: Hoe het teughelrinx raeckpunt naerder des peerts borst comt, hoe de tusscheketen oock meer de borst naerdert, volgende de ghemeene manier diemen int toommaken ghebruyckt: Maer die tusscheketen soo na commende, datse int trecken des tooms de borst gheraeckt, soo is de wreetheyt daer ten eynde; want al treckmen dan veel stijver, dat comt al opt
peerts

of the reins, have reached *F*, and the point of contact *D* of the bit ring shall have reached *E*, so that the arc *DE* shall be equal to the arc *BF*.

PROOF

It is to be assumed that by so much as the line *BC* is longer than *FC*, by so much the pulling hand at *C* had to be higher when the point of contact of the bit ring was in *F* than when it was in *B*. In the same way that by so much as the line *DC* is longer than *EC*, by so much the pulling hand at *C* had to be higher when the point of contact of the bit ring was in *E* than when it was in *D*. But *EC* differs more from *DC* than *FC* does from *BC*. And therefore, by so much as the difference of these two differences amounts to, by so much the hand rises higher with the displacement of the point of contact of the bit ring from *D* to *E* than with the displacement from *B* to *F*. But the displacement or the arc *BF* is equal to the displacement or the arc *DE* by the preliminary; therefore, if the displacements of *B* and *D* are equal, the hand at *C* rises higher with the displacement of *D* than with the displacement of *B*. And consequently, if the hand rose to the same height with both, the displacement from *B* to *F* would have to be greater than the displacement from *D* to *E*. But the greater displacement from *B* to *F* also causes greater displacement of the eye, and consequently of the curb chain, than the smaller displacement from *D* to *E*. Therefore, the hand having risen to the same height with both, the displacement of the curb chain caused by pulling from *B* to *F* will be greater than that of the curb chain caused by pulling from *D* to *E*. But the greater displacement or greater rise of the curb chain presses more tightly against the horse's chin, and consequently causes the bit to press more tightly against the gums than a smaller rise of the curb chain. Therefore, when the hand at *C* pulls to the same height, the rider hurts the horse more if the point of contact of the bit ring is in *B* of the cheek *AB* than in *D* of the cheek *AD*. And consequently, when the point of contact *B* of the bit ring is further away from the horse's breast, this causes greater severity than *D*.

1st NOTE

Since the angle *ADC* is nearer to a right angle than the angle *ABC*, which is much more acute, the power of the hand at *C* exerts greater force on the cheek *AD* than the same power of the hand at *C* does on the cheek *AB*, by the corollary of the 24th proposition of the 1st book of the Art of Weighing. But because someone might think this to be contrary to the foregoing proof, we give to this the same reply as in the note to the 2nd proposition, to wit that the question is not what force the hand at *C* exerts, but if the hand pulls so much more tightly on the angle *ABC* than on the angle *ADC* that it rises equally in both cases, it is asked which pulling then involves the greatest severity.

2nd NOTE

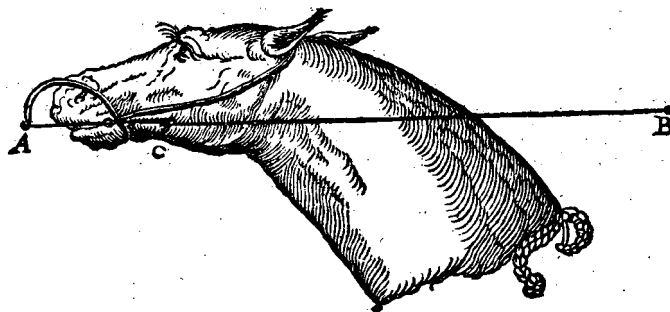
In addition to the foregoing cause of the severity, there is sometimes a second cause, as follows: The closer the point of contact of the bit ring comes to the horse's breast, the closer the intermediate chain also comes to the breast, if the common manner used in bridle-making is followed. But if that intermediate chain

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peerts borst an, sonder teghen kin of tant vlees meerder persing te maken: Maer een ander teughelrinx raackpunt verder vande borst sijnde, en de tusscheketen daerom oock verder, soo volght daer uyt datmen die stanghen verder achterwaert na de borst sal connen trecken als d'ander, eer de tusscheketen de borst gheraect, waer uyt oock openbaerlick meerder wreetheyt moet volgen. Doch en valt daer af niet te segghen als de tusscheketen na d'een en d'ander wijze de borst niet en raect.

3 MERCK.

T'ghebeurt ettelicke peerden datse hun selfs van t'gheprang des tooms verlossen, mette mont om hooch te steken, ghelijck de byghévoughde form anwijst: Sulcx dat hun aldan den Ruyter niet dwingen en can, maer loopen daerse willen: Nochtans mocht ymant segghen, is dan des teughelrinx raackpunt verder van des peerts borst, als in ander ghestalt, inder voughen dat daer mē den toom wreeder behoort te wesen, t'welck teghen de regel deses voorstels schijnt



te strijden. Hier op wort gheseyt, dat wanneer de ghespannen teughelriem A B, ewewijdich is mette verdochte rechte lini van des teughelrinx raackpunt A, tot des montsticx aspunt C, ghelijck dese ghestalt mebrengt, aldan en can stijver trecking ant bovedeel gheen roetsel gheven, noch de kinketen doen opgaen, en vervolghensen isser gheen wreetheyt, want hoe wel het montsticx stijver achterwaert ghetrocken wort, dat en veroirsaect het boveschreven wreet geprang niet. Maer soo de ghespannen teughelriem noch hoogher waer alsvooten gheseyt is, hoemen dan stijver treckt, hoe openbaerlick de kinketen slapper wort. Sulcx dat dit een uytneeming is in bekende oirsaken bestaende.

5 VOORSTEL.

De cortste kinketens gheven de meeste vvreetheyt.

Tis daer voor te houden, datter gheprang des montsticx eerst begint als de kinketen reghen de kin gheraect: Maer tot een langhe kinketen moet de hant verder opgaen eerse de kin gheraect dan tot een corte, en daerom doetmen met eveveel beweeghnis des hants, meer geprang met corte kinketens dan met langhe. T B E S L Y T. De cortste kinketens dan gheven de meeste wreetheyt, t welck wy bewijzen moesten.

MERCKT.

comes so close that in the pulling of the bridle it touches the breast, the severity is at an end there; for even if one then pulls much more tightly, all this pressure will be exerted on the horse's breast, without producing more pressure against the chin or gums. But if another point of contact of the bit ring is further away from the breast, and the intermediate chain therefore also further away, it follows therefrom that it will be possible to pull those cheeks further back towards the breast than the others before the intermediate chain touches the breast, from which there must also evidently follow greater severity. But nothing can be said thereof if the intermediate chain in any way does not touch the breast.

3rd NOTE

It happens with many horses that they relieve themselves of the pressure of the bridle by raising their mouths, as the accompanying figure shows, so that the rider cannot then force them, but they run as they like. Nevertheless, someone might say: the point of contact of the bit ring is then further away from the horse's breast than in the other position, in such a way that therewith the bridle ought to be more severe, which seems to be contrary to the rule of this proposition. To this it is said that when the tight rein *AB* is parallel to the imaginary straight line from the point of contact *A* of the bit ring to the axial point *C* of the bit, as this position involves, then a tighter pulling at the upper cheek cannot cause any displacement or cause the curb chain to rise, and consequently there is no severity, for though the bit is pulled more tightly backwards, that does not cause the severe pressure described above. But if the tight rein is even higher than has been said above, the more tightly one pulls, the gentler the curb chain manifestly becomes. So this is an exception which can be understood from known causes.

5th PROPOSITION

The shortest curb chains cause the greatest severity.

It is to be assumed that the pressure of the bit does not begin until the curb chain touches the chin. But with a long curb chain the hand has to rise further before it touches the chin than with a short one, and therefore with the same movement of the hand greater pressure is exerted with short than with long curb chains. CONCLUSION. The shortest curb chains therefore cause the greatest severity, which we had to prove.

M E R C K T.

Wy hebben hier boven gheseyt daer voor te houden te sijn, dattet gheprang des montsticx eerst begint als de kinkeren teghen de kin gheraecht: doch ghebeuret wel dat de peerden eenich gheprang ghevoelen voor sulck gheraeckel, ja met een toom sonder kinkeren, t'een peert eer als t'ander, na datse teer of hart van monde sijn: Oock na dat d'een toom van stijver of slapper stof, lossen of sluytender mocht ghemaect sijn als d'ander: Doch soo cleyn onseker en onghelijck gheprang, en schijnt gheen dieper ondersoucking noch beschrijving der omstandighen te vereyschen, als van gheender acht wesende.

6 V O O R S T E L.

Een prouftoom te maken, en daer uyt een ghebruyc-kelicke toom.

Wat prouftoom is hebben wy verclaert inde 17 bepaling. Om hier van het maecksel te segghen, dat mach aldus gheschien: De ghestalt is ghelijck de volghende form acnwijst, alwaer A B twee stanghen beteyckenen, die verlangt en vercoert connen worden deur de schuyvende sticken als C B, welcke ter begeerde langde connen vast gheecht worden mette schrouven als D. Dese stanghen draeyen elck op een bout als E, makende metter bovestick sulcken houck of cromte als men begheert, en worden alsoo vast gheecht mette schrouven F. De bovedeelen G H sijn eenvaerdigher dichte, soo lanck als de langste diemen behouft. De ooghen als I sijn daer aen schuyvende ghemaect, en worden met schrouven als K vast gheecht ter plaets daermen se begheert. Inder voughen dat hier mede soo wel het bovedeel als onderdeel sulcken langde gegeven wort als men wil.

T

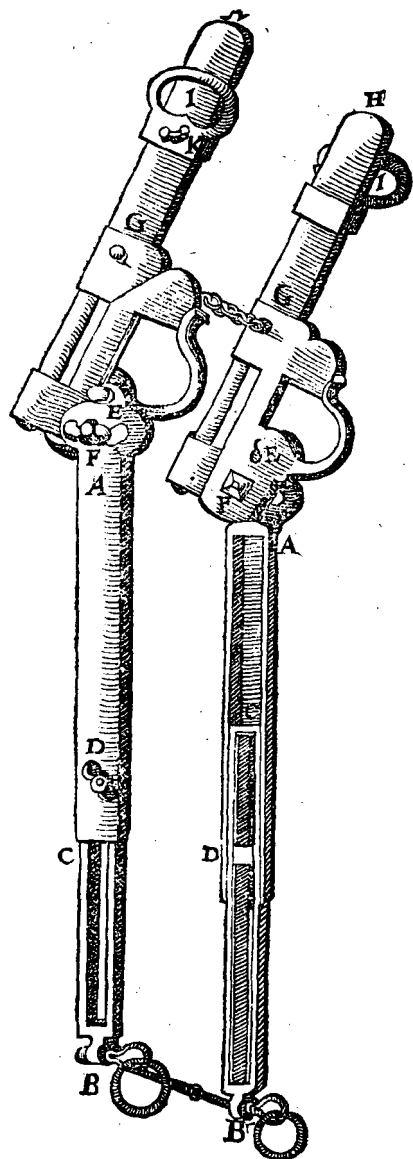
NOTE

We have said above that it is to be assumed that the pressure of the bit does not begin until the curb chain touches the chin. Yet it sometimes happens that horses feel some pressure before the curb chain touches the chin, nay, even with a bridle without a curb chain, according as a horse is tender or hard in the mouth. Also according as one bridle may be made of stiffer or softer material, more or less tightly fitting, than the other. But such a slight, uncertain, and dissimilar pressure does not seem to call for any deeper study or description of the circumstances, as being of no account.

6th PROPOSITION

To make a test bridle, and from that an actual bridle.

What a test bridle is, we have set forth in the 17th definition. As regards the construction, that may be effected as follows. The form is as shown in the following figure, where *AB* signifies two cheeks which can be lengthened and shortened by means of the sliding members *CB*, which can be fastened at the desired length with the screws *D*. Each of these cheeks pivots about a bolt *E*, including with the upper cheek such an angle or bend as is desired, and they are thus fastened with the screws *F*. The upper cheeks *GH* are of uniform thickness and as long as the longest that are required. The eyes *I* have been adapted to slide thereon, and are fastened with screws *K* in the place where they are desired, in such a way that thus both the upper and the lower cheek are given the desired length.

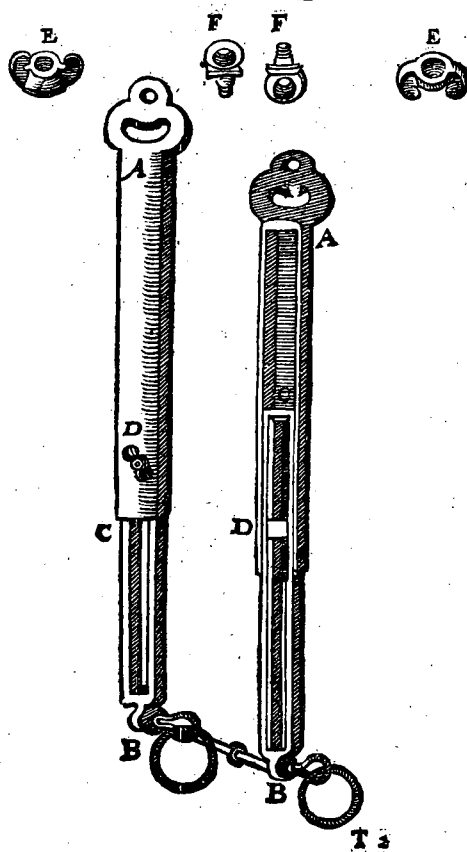
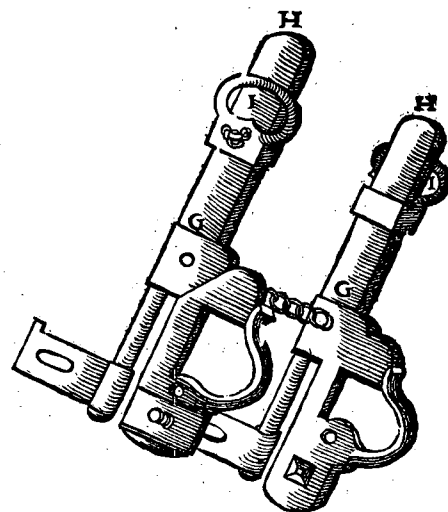


Tot hier toe is beschreven de maniere des prouffooms int gheheel, de stikken by malcander vervoucht : Maer om noch breeder verclaring te doen vande form der stucken int besonder, soo sullen wy die hier nu verscheyden stellen, alwaer de letteren andermael van beteyckening sijn als vooren.

Dit is

So far the construction of the test bridle as a whole has been described, the pieces when assembled. But in order to give a more detailed exposition of the form of the pieces in particular, we shall now describe them here separately, the letters designating the same parts as before.

Dit is de form die sijn
VORSTELICKEGE-
NADE alsoo heeft doen
maken, en metter daet
bequaem bevint: doch
alsmen daer in betering
merckt, 't sal billich sijn
die 't sijnen voordeele te
nemen.



This is the form which his PRINCELY GRACE has had made in this manner, and which he finds suitable in practice; but if anyone sees a means of improving on it, he may take advantage thereof.

N V V A N T M A K E N D E S G H E -

*bruyckelicken tooms deur t'behulp
des prouftooms.*

An de prouftoom een montflick vervought sijnde na den eyfch van t'peert, men fal deur t'behulp der fchuyverkens, de langde der ftanghen en bovedeelen, oock den raackpunthouck, voor t'eerfte Stellen na t'ghene het voorgheftelt peert fchijnt te vereyffchen: Maer t'felve an t'peert dadelick verfocht sijnde, en bevonden wefende datter verandering moet gedaen fijn an een der vier faken, of an altemael, te weten verlangingh of vercortingh der ftanghen, verlanging of vercorting der bovedeelen, vermeerdering of vermindering des raackpunthouck, of verlanging of vercorting des kinketens, dat can van elck met luttel moeyte, groote fekerheyt, en feer haest ghefchlen; la sonder den toom telcken af te moeten doen, oock sonder dat den Rijder behouft af te ftijghen. Nu de prouftoom foo gheftelt hebbende, datse voor dat peert paff, men falfe af doen, en een ghebruyckelicke toom doen maken, met fulcke keeren, form, en cyraet als men begheert, mits wel verftaende, dat de drie punten des raackpunthouck, even comen fulcken houck te maken als die des prouftooms, en de twee rechte verdochte linien dien houck begrijpende, oock vande felve langde als d'andere: Dat voort de tuffchenketen, kinketen, en montflick, mede comen op dergelijke gheftalt en form: T welck foo sijnde, dees ghebruyckelicke toom moet het peert paffen, en fal daer mede ter handt fijn, even als mette prouftoom, ghelijck fijn V O R S T E L I C K E . G H E N A D E dat oock dadelick bevint.

Ettelicke van defe ftoffchrijvende, hebben gemaect toomen daermen verfcheyden ftanghen in mach fteken met onghelijke keeren, d'een crommer als d'ander: Maer het teughelrijncx raackpunt op een selve plaets commende, foo en gheeft meerder noch minder cromheyt der keeren totte faeck niet, ghelijck int eerfte Voorftel verclaert is: Of anders gheseyt, commende het teughelrijncx raackpunt op een ander plaets, fo en is meerder of minder cromheyt des ftangs, de oirfaeck niet der veranderingh diemen inde regteringhe des peerts ghewac wort, ghemerckt fulcx comt uyt verandering van plaets des teughelrijncx raackpunt: Waer deur fulcke foucking sonder kennis der oirfaken foo moeylick en onfecker valt, datter hun weynigh begheven tot deur foodanighe middel welpaffende toomen te maken. T B E S L V Y T. Wy hebben dan een prouftoom gemaect, en daer uyt een ghebruyckelicke toom na den eyfch.

M E R C K T.

Ymant overdenckende de ghemeene reghel der wichtighe ghedaenten van alle ruych daermen ghewelt mede doet, mocht segghen, dat wannecmen met even voorttrekkingen des handts, de kinketen evevel voortgancx geeft, t'mach mette langde der bovedeelen en ftanghen fijn hoe't wil, daer volghet een selve gheprang uyt. Om hier af by voorbeelt te fpreken, gemaect sijnde twee toomen op even raackpunthoucken, en de kinketen in d'een, met fulcken lofheyt of verheyt vande kin als in d'ander, voort de ftangen en bovedeelen *everedenich, doch van d'een kleender als van d'ander, de kinketen crijcht dan met evevel voorttrekking des handts evevel beweeghnis, en vervolghens een selve gheprang, t welck ick deur een form breeder verclaren fal.

Proportionale.

T G H E.

NOW AS TO THE CONSTRUCTION OF THE ACTUAL BRIDLE WITH THE AID OF THE TEST BRIDLE

A bit having been attached to the test bridle according to the requirements of the horse, the length of the cheeks and the upper cheeks, and also the angle at the point of contact, shall first be adjusted by means of the sliding members in the way the horse in question seems to require. But when this has been tested in practice on the horse and it has been found that a change has to be made in one of the four members or all of them, to wit lengthening or shortening of the cheeks, lengthening or shortening of the upper cheeks, increasing or decreasing of the angle at the point of contact, or lengthening or shortening of the curb chain, this can be done with each of them with little trouble, great certainty, and very quickly, nay, even without having to take off the bridle every time, and also without the rider having to dismount. When the test bridle has been so adjusted that it fits the horse, it shall be taken off, and an actual bridle shall be caused to be made, with such twists, form, and adornments as may be desired, provided the three points of the angle at the point of contact make the same angle as that of the test bridle and the two straight imaginary lines comprehending that angle be also of the same length as the others, while further the intermediate chain, the curb chain, and the bit also have a similar position and form. This being so, this actual bridle is bound to fit the horse, and it will be held easily in hand, just as with the test bridle, as his PRINCELY GRACE indeed finds in practice.

Many writers dealing with this matter have made bridles into which may be mounted different cheeks with dissimilar twists, one more curved than the other. But if the point of contact of the bit ring comes in the same place, greater or lesser curvature is of no account in the matter, as has been set forth in the first Proposition. Or in other words: if the point of contact of the bit ring comes in another place, greater or lesser curvature of the cheek does not cause the change observed in the governing of the horse, seeing that this is due to a change of place of the point of contact of the bit ring. Owing to which such an examination without knowledge of the causes is so difficult and uncertain that few people try to make fitting bridles by such means. CONCLUSION. We have therefore made a test bridle, and from that an actual bridle, as required.

NOTE

Someone, reflecting on the common rule of the static properties of all devices with which force is exerted, might say that if with an equal amount of pulling of the hand the curb chain is equally advanced, no matter what the length of the upper cheeks and the cheeks, the same pressure will result therefrom. To give an example of this: two bridles being made with equal angles at the point of contact and the curb chain of one being just as loose or remote from the chin as the other, the cheeks and upper cheeks further being proportional, but those of the one smaller than those of the other, the curb chain is then displaced the same distance with an equal amount of pulling of the hand, and consequently it undergoes the same pressure, which I will explain more in detail by means of a figure.

Holomogaster
kyra.

Ghelijck A K tot A M, alsoo K L tot M N.

Ghelijck A F tot A G, alsoo F H tot G I.

Ghelijck A K tot A M, alsoo F H tot G I.

Ghelijck A K tot A M, alsoo K L tot G I.

T 3

Nu dan

SUPPOSITION. Let AB signify a long cheek, AC its long upper part in BA produced. Further AD shall be a short cheek, AE its short upper part, having the same ratio to AD as AC to AB . Further there shall be drawn AF equal to AB , and AG in FA produced, equal to AC , and from F the line FH at right angles to BC , also GI at right angles to this same BC , further AK equal to AD , also so that KL , at right angles to BC , be equal to FH , and AM , in KA produced, equal to AE , and MN at right angles to BC . This being so, let us now assume the bit ring B of the long cheek to have been pulled from B to F , so that its advance be HF , and the short cheek from D to K , so that its advance be LK . Then the eye C of the longest upper cheek will have reached G , its advance being IG , and the eye E of the shortest upper cheek will have reached M , its advance being NM . But the advance HF and LK is to be considered the hand's pulling at the rein, because they are equal thereto, and IG with MN the advance of the curb chains, because they are equal to IG , from which, as was intended to be proved, there must result the same pressure.

PROOF

The triangle AKL is similar to the triangle AMN , in consequence of which their homologous sides are proportional, to wit

As AK is to AM , so is KL to MN .

The triangle AFH is similar to the triangle AIG , in consequence of which their homologous sides are proportional, to wit

As AF is to AG , so is FH to GI .

But as AF is to AG , so is AK to AM , therefore

As AK is to AM , so is FH to GI .

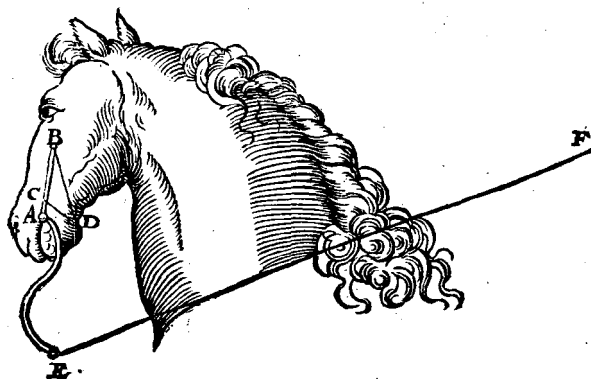
But FH is equal to KL by the supposition, therefore

As AK is to AM , so is KL to GI .

So that GI and MN are each the fourth proportional to the same three terms, to wit MN in the first proportion and GI in this last, in consequence of which they must be equal.

Nu dan de kinketen van d'een en d'ander toom aldus eveveel voortganck crijghende, waer uyt ymant dencken mocht sulcx een selve gheprang te geven, en datter nochtans groot verschil in valt, so sullen wy daer af wat breeder seggē.

D'ervaring leert, soo ettelicke oock schrijven, dat langer bovedeelen aen sommighe peerden het hooft hoogher doen verheffen als corter: Waer af sijn VORSTELICKE GENADE d'oirsaeck hout dusdanich te wesen: Laet A B een lanck bovedeel beteycken, A C een cort, B D de kinketen ant lang bovedeel, en C D de kinketen ant cort bovedeel. De langhe kinketen B D maect opt bovedeel een scherper houck dan de corter kinketen C D, want scherper is den



houck A B D, dan A C D. Hier me sietmen dat deur trecking des teugheltiens E F, het bovedeel A B beweeghnis crijghende, soo perst de kinketen C D platter teghens t'peerts kin, dan de kinketen B D, welke daer teghen meer opwaert druct: En t'peert om die opwaert persing te versachtē, verheft het hooft hooger.

Effectus.

Ymant soude hier op meughen segghen, dat by aldien sulcx de eyghenschap waer van langher bovedeelen, dat de * daet daer af niet alleen blijcken en soude an sommighe peerden, ghelijck boven gheseyt is, maer an allen, t'welck nochtans teghen d'ervaring te strijden by verscheyden betuycht wort, en onder anderen deur *le Sieur de la Brouë* int 3 bouck onder dit opschrift.

Occasions pour lesquelles on doit faire l'ail de la branche plus haut ou plus bas que la mesure ordinaire.

Ick heb oock sijn VORSTELICKE GENADE hooren bevestighen dadelick bevonden te hebben, dat verlanging van bovedeelen an sommighe peerden het hooft dede dalen, an ettelicke verheffen: T'welck hy doen, ghelijck ander, met verwonderen ansach: Maer daer na hier op met kennis der Weeghconst lettende, heeft voor ghewis gehouden dit d'oirsaeck te wesen. Verlanging des bovedeels, t'welck meerder wreetheyt mebrengt op het tantvlees en teghen de kin deur het 3 voorstel, werckt twee verkeerde saken t'seffens, want deur de stijver persing des montsticx teghen het tantvlees, is t'peert geneycht het hooft neerwaert te buyghen, om die weedom te versachten, maer deur de stijver opwaert persing des kinketens teghen de kin, ist om die smerte te verminderen gheneycht het hooft opwaert te verheffen, gelijk wy boven verclaert hebben: Dese twee t'seffens ancommende, het souckt hem dadelick meest t'ontlasten

Now therefore the curb chain of one bridle as well as the other thus making the same advance, from which someone might conclude that this produces the same pressure, while nevertheless there is great difference between them, we will discuss this a little more in detail.

Experience teaches, as many writers affirm, that longer upper cheeks make some horses lift their heads higher than shorter ones, the cause of which is considered by his PRINCELY GRACE to be as follows: Let *AB* signify a long upper cheek, *AC* a short one, *BD* the curb chain on the long upper cheek and *CD* the curb chain on the short upper cheek. The long curb chain *BD* makes a more acute angle with the upper cheek than the shorter curb chain *CD*, for the angle *ABD* is more acute than *ACD*. Thus it is seen that when by the pulling of the rein *EF* the upper cheek *AB* is moved, the curb chain *CD* presses more flatly against the horse's chin than the curb chain *BD*, which presses against it in a more upward direction. And the horse, in order to relieve that upward pressure, will lift its head higher.

Someone might say to this that if this were the property of longer upper cheeks, the effect would be apparent not only with some horses, as has been said above, but with all, which is nevertheless stated by different people to be contrary to experience, among others by *le Sieur de la Brouë*¹⁾, in the 3rd book with the following heading:

Occasions pour lesquelles on doit faire l'oeil de la branche plus haut ou plus bas que la mesure ordinaire.

I have also heard it affirmed by his PRINCELY GRACE that he has found in practice that the lengthening of the upper cheeks caused some horses to lower and many to lift their heads, which he watched, just like others, with surprise. But when he afterwards noted this with knowledge of the Art of Weighing, he held it for certain that this is the cause. Lengthening of the upper cheek, which involves greater severity to the gums and against the chin, by the 3rd proposition, simultaneously produces two contrary effects, for owing to the stiffer pressure of the bit against the gums the horse is inclined to bend its head downwards, in order to relieve that pain, but owing to the stiffer upward pressure of the curb chain against the chin it tends, in order to relieve that pain, to lift its head upwards, as we have set forth above. When these two tendencies act simultaneously, it tries in practice to relieve itself most of that which causes it the greatest pain.

¹⁾ The work referred to is: *Le cavalerie françois composé par Salomon de la Broue*. Paris 1602. Livre III, ch. 25; p. 62.

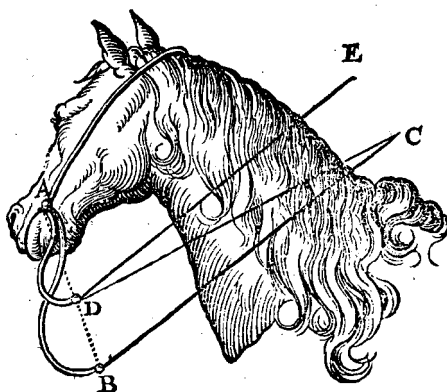
van t'ghene hem de meeste weedom aendoet: Maer sommige peerden sijn teer van tantvlees en hart van kin, ander verkeert, hart van tantvlees en teer van kin, waer uyt volght dattet een peert deur verlanging des bovedeels het hoofd leger buycht, het ander hoogher verheft: Maer om int ghemeen daer af te spreken, alle langher bovedeelen intansien der opwaert persing des kinketens alleen, veroirsaken eenige genegentheyte des peerts tot verheffing des hoofts, hoe wel het nochtans om d'ander meerder sinerte t'verkeerde wel mocht te werc stellē.

uyt het voorgaende valt te besluyten, datmen tot peerden die uytter natuer het hoofd hooch genouch dragen, en het tantvlees niet te teer en hebben, soude meughen ghebruycken corter bovedeelen met een sluytender kinketen, te meer dat langer bovedeelen en lossen kinketens met een stercke snack ghetrocken wesende, het montstick en kinketen veel harder, als met een slach ancommende, de peerden den mont bederven, meer als corte bovedeelen, en sluytender kinketens, die sachter ankommen, en nochtans daer na eveveel persing gheven. Ten anderen dat al te langhe kinketens als B D, lichtelick over de kin slijberen, sonder dat den Ruyter het peert dan regieren can, welck onghewal de kinketens, als C D niet onderworpen en sijn.

Merckt noch dat als men niet ghedronghen en is langhe bovedeelen te nemen om t'peert sijn hoofte te doen verheffen, (t'welck ghebeurt als de teerheyt des tantvlees niet en overtreft de teerheyt des kins) soo mach men een seer cort bovedeel ghebruycken, en de stanghen van langde soose best vougen: Daer na vermeerderen of verminderen de wreetheyt na sijn wille, met verlanging of vercorting der kinketen.

Maer want sijn VORSTELICKE GHENADE dese eygenscapen seer nauwe deurgont heeft, soo sal ick hier stellen noch wat ander onghelijckheyt, tusschen de boveschreven toomen met everedenighe stangen en bovedeelen:

Laet tot dien eynde A B een lange sijn, diens teughelriem B C, en A D een corte, diens teughelriem D E, en met haer bovedeelen neem ick everedenich. Alwaert nu dat dese twee stanghen om die everedenheyt een selve gheprang gaven, soo ist nochtans kennelick dat de treckende hant niet tot een selve plaets en soude moeten blijvē, maer sose op B treckende, is an C, sy sal op D treckende, moeten sijn by E, sulcx dat



D E * ewewijdeghe is met B C, want treckende de teughelriem van D tot C, sy *Parallelle* maeckt op de rechte lini A B een ander houck dan D E, t'welck openbaerlick verandering moet mebrenghen, te weten minder wreetheyt an C, dan an E.

But some horses have tender gums and a hard chin, others on the contrary have hard gums and a tender chin, from which it follows that in consequence of lengthening of the upper cheek one horse will bend its head lower, and another will lift it higher. But to speak of this in a general way: all longer upper cheeks, as regards the upward pressure of the curb chain alone, cause some inclination in the horse to lift its head, though nevertheless, because of the other greater pain, it might do the contrary.

From the foregoing it may be concluded that for horses which by nature carry their heads high enough and do not have too tender gums one might use shorter upper cheeks with a more closely fitting curb chain, the more so because longer upper cheeks and looser curb chains, being pulled too tightly, the bit and the curb chain acting much harder, abruptly, spoil the horses' mouths more than short upper cheeks and more closely fitting curb chains, which act more gently and nevertheless cause further the same amount of pressure. Secondly, too long curb chains, such as *BD*, will hang too loosely on the chin, the rider then being unable to govern the horse, to which inconvenience curb chains such as *CD* are not subject.

It should also be noted that if it is not necessary to take long upper cheeks in order to make the horse lift its head (which happens when the tenderness of the gums does not exceed the tenderness of the chin), one may use a very short upper cheek and cheeks of the length that is most suitable, and then increase or decrease the severity at will, by lengthening or shortening the curb chain.

But because his *PRINCELY GRACE* has studied these properties very closely, I will here describe another dissimilarity between the bridles described above with proportional cheeks and upper cheeks.

To this end let *AB* be a long cheek, whose rein is *BC*, and *AD* a short cheek, whose rein is *DE*, and I take them to be proportional to their upper cheeks. Even if these two cheeks, because of that proportionality, produced the same pressure, it is nevertheless obvious that the pulling hand would not have to remain in the same place; but if, pulling at *B*, it is in *C*, when pulling at *D*, it will have to be in *E*, so that *DE* is parallel to *BC*, for when the rein is pulled from *D* to *C*, it makes a different angle with the straight line *AB* than *DE*, which manifestly is bound to cause a change, to wit less severity at *C* than at *E*.

END OF THE PRESSURE OF THE BRIDLE.

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