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A Viability Analysis for a Bio-Economic Model

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Abstract: This paper presents a simple dynamic model dealing with the management of a renewable resource. But instead of studying the ecological and economic interactions in terms of equilibrium or optimal control, we pay much attention to the viability of the system or, in a symmetric way, to crisis situations. These viability/crisis situations are defined by a set of economic and biological state constraints. The analytical study focus on the compatibility between the state constraints and the controlled dynamics. Using the mathematical concept of viability kernel, we reveal the situations and, if possible, management options to guarantee a perennial system. Going further, we define "overexploitation" indicators by the time of crisis function. In particular, we point out irreversible overexploitation configurations related to the resource extinction.

Key-words: renewable resource, bio-economic model, overexploitation, viable control.

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1 Introduction

Economics of natural resources relies mainly on interactions between economic and biological (or ecological) dynamics, which makes the problem both interesting and difficult. Over the past decades, the topic has received growing attention because of the recent emergence of a common "environmental issue consciousness" and also because natural resource exploitation has been shown to be characterized by very frequent "market failures" (Pierce-Warford, 1993; Tisdell, 1991). This paper concentrates on the example of marine renewable resources and their economic exploitation through fisheries.

Most economic models addressing the problem of renewable resource exploitation are built up on the frame of a biological model. Such models may account for the demographic structure (age or size classes) of the exploited stock (trees, fish population) or may attempt to deal with the inter-species dimension of the exploited (eco)system. However, biologists have often found it necessary to introduce various degrees of simplifications to reduce the complexity of the analysis and one of the simplest models used in population dynamics is the "logistic model". In such a model, the stock, measured by its biomass, is considered globally, as one single unit, without any consideration for the structure population, and its growth is materialized through the logistic equation:

$$\dot{x}(t) = \frac{dx}{dt}(t) = rx(t)(1 - \frac{x(t)}{l}) = f(x(t)),$$

where $x(t)$ stands for the resource biomass, l is the limit carrying capacity of the ecosystem and r is the intrinsic growth rate of the resource.

When fishing activities are included, the model becomes the Schaefer (1954) model

$$\dot{x}(t) = f(x(t)) - h(t),$$

where $h(t)$ is the harvesting flow at time t. It is frequently assumed that the catch rate h is proportional to both biomass and extraction effort, namely $h = qex$, where e stands for the extraction effort (or fishing effort, an index related for instance to the number of boats involved in the activity) and q is a constant parameter usually referred as the catchability coefficient. At equilibrium, when the exploitation rate equals the population growth, one obtains the so-called sustainable yield associated to the fishing effort by inverting the relation

$$e = \frac{r}{q}(1 - \frac{x}{l}) = s(x).$$

The economic model which is directly derived from the Schaefer model is the Gordon model (Gordon, 1954) which integrates the economic aspects of the fishing activity through the fish price p and the fishing costs c per unit of effort. Assuming that the profit is defined by $\Pi = pqex - ce$, one can compute the effort $\overline{e}$ maximizing Π. The Gordon model prediction, when open access situation occurs (i.e. when no limitation is imposed on the fishery's effort), is that the rent generated by the activity is dissipated as the effort expands beyond $\overline{e}$ and the economy converges toward the so-called "open access bionomic equilibrium".

Although it suffers from a large number of unrealistic assumptions (some of them ensuing directly from the Schaefer model limitations), the Gordon model displays a certain degree of concordance with empirical fisheries histories (Wilen, 1976). This is probably the reason why, along with its indisputable normative character, it has been regularly used as the underlying framework by optimal control theory since the latter has been introduced in fisheries sciences (see for instance Charles, 1983; Clark, 1976; Clark-Kirkwood, 1979; Cohen, 1987; Goh, 1979; McKelvey, 1985).

In optimal control theory, assuming a fixed production structure (i.e. constant capital and labor), the problem can be stated as the inter-temporal maximization of the profit with respect to the fishing effort $e(t)$

$$\max_{e(\cdot)} \int_0^{\infty} e^{-\delta t}(pqe(t)x(t) - ce(t))dt,$$

where δ represents the social discount rate and $x(\cdot)$ is the solution of the control system:

$$\begin{cases} \dot{x}(t) = f(x(t)) - qe(t)x(t), & 0 \leq e(t) \leq e_{\max} \\ x(t) \geq 0. \end{cases}$$

with $e_{\max}$ the effort limit resulting from the fixed production capacity (number of boats and of fishermen). Using the maximum principle, it can be shown that the optimal strategy is to reach the steady state $x^{\star}$ following a "bang" (or most rapid) solution (c.f. Clark, 1976). Several extensions have then been proposed to improve the realism of this basic model: for instance accounting for the multi-species or multi-objectives that are known to characterize most fishery systems (Charles, 1989; Diaz-Seijo, 1992; Healey, 1984); or integrating the irreversibility or the non-malleability of the capital dynamics (Charles, 1989; Clark, 1976; Clark-Munro, 1975); or introducing uncertainty (Charles, 1983; Clark-Kirkwood, 1986).

These various developments however did not prevent optimal control theory to suffer criticisms regarding its applicability to fisheries management. One has observed that fishermen' decisions are seldom driven by the search for direct optimality (Bockstael, 1983; Hilborn-Ledbetter, 1979). In order to maximize the objective function, one may also have in some cases to follow optimal solutions that bring undesirable outcomes. Two typical examples of these undesirable consequences are (i) negative profits: the optimal strategy requires to close the fishery for a while (which, practically, implies some fixed costs not taken into account) and (ii) resource extinction: Clark (1973) showed that it would be optimal to "liquidate" the resource and to re-invest the capital in another activity when the resource growth rate is lower than the discount rate.

Optimal control theory has not only been criticized for the outcomes that it may bring about. It also suffers some limitations inherent to the optimization principle itself. On one hand, the optimal solution path is generally unique which does not allow for possible alternate strategies. In addition, it is strongly related to the choice of the discount factor δ, which is itself rather arbitrary.

All these remarks show the interest to address the problem from a different perspective attempting, in particular, at reconciling ecological and economical issues. This is the way we choose in the present paper by looking for some instantaneous and simultaneous "criteria" that materialize the "good health" of the bio-economic system, i.e. its viability . The viability approach (Aubin, 1991) deals with dynamic systems under state constraints (see also Clarke et al, 1995). The aim of this method is to analyze the compatibility between the (possibly uncertain) dynamics of a system and state constraints and to determine the set of controls (or decisions) than would prevent this system from entering crises i.e. from violating its viability constraints. The present study is an attempt to apply this approach to the case of the management of renewable resources and especially to fisheries. We refer to Béné-Doyen (2000), Bonneuil (1994), Doyen et al. (1996) in others contexts and to Toth et al (1998) for a similar approach untitled the Tolerable Windows Approach. Here, we define the viability from the viewpoint of a government that aims at maintaining the fishery in a sustainable way. To achieve this, a condition (or constraint) of net benefits (positive balance sheets) of the fishing activity is imposed at any time on the resource and the effort levels. Whenever one assumes some fixed costs, it appears that this economic constraint induces a minimal threshold for the renewable resource which is interesting if the government policy aims at reconciling ecological and economics objectives. Then, using the mathematical concept of viability kernel (Aubin, 1991), we make clear the need to anticipate the dynamics of the system to maintain its viability. This analysis allows us to identify situations of overexploitation and the adaptive regulations required to prevent deficits and/or overexploitation situations from occurring. A second mathematical concept, the time of crisis function (Doyen-StPierre, 1997), is then used to refine the overexploitation analysis. In particular, this overexploitation indicator allows us to identify two different types of crises, the reversible crises and the irreversible ones. It shows how the irreversible crisis situations are linked to the resource extinction.

2 The model

2.1 The dynamics

The natural growth rate of the resource x is represented by a logistic law f depending on the intrinsic growth parameter $r > 0$ and the limit capacity l i.e.

$$f(x) = rx(1 - \frac{x}{l}).$$

Taking into account that harvesting is proportional both to the biomass x and to the fishing effort e, we consider the following dynamics for the renewable resource

$$\dot{x}(t) = f(x(t)) - qe(t)x(t),$$

where q stands for a catchability coefficient.

We furthermore assume that the time variation of the effort is bounded:

$$\dot{e}(t) = u(t), \quad u(t) \in U = [u^-, u^+]. \tag{1}$$

This assumption is a way to model the rigidity of the decision since it implies the continuity of effort with respect to time and thus rules out jumps of harvesting. For instance, it means that a decrease or an increase in number of used boats requires time to be completed. We furthermore suppose that $u^- \leq 0 \leq u^+$ which means that the effort can be kept constant (case $\dot{e} = 0$).

We obtain the following control differential system where the state variables are (x, e) and the control variable is u:

$$\begin{cases} \dot{x}(t) = f(x(t)) - qe(t)x(t), \\ \dot{e}(t) = u(t), \quad u^- \leq u(t) \leq u^+. \end{cases} \tag{2}$$

2.2 The viability constraints

A second step is to express state variable constraints from a regulating (say a government) agency viewpoint.

First, we impose an ecological constraint in the sense that the government policy requires a minimum level $x_{\min} > 0$ for the resource:

$$x_{\min} \leq x(t), \ \forall t \geq 0. \tag{3}$$

The second constraint concerns the fishing effort $e(t)$. We assume that capital and labor involved in the production process remain constant; therefore $e(t)$ is constrained by a fixed production capacity. We denote this limit capacity $e_{\max}$ and we thus identify a second constraint:

$$0 \leq e(t) \leq e_{\max}, \ \forall t \geq 0. \tag{4}$$

Furthermore, we consider that the government agency seeks to guarantee the sustainability of the activity by maintaining a global positive net benefit in the sector at any time. Let us denote by $R(x(t), e(t))$ the sector balance sheet, defined as the difference between the income and the harvesting costs. Therefore the benefit constraint reads as follows :

$$R(x(t), e(t)) = pqe(t)x(t) - ce(t) - C \geq 0, \ \forall t \geq 0, \tag{5}$$

where $p > 0$ represents the (exogenous) unit fish price, $c > 0$ denotes the cost per unit of effort and $C > 0$ is a fixed cost - which may include, for instance, wages (assumed constant since labor is fixed) or a fixed amortization of the capital stock (boats,..).

Note first that the net benefit constraint (5) induces the resource biomass to be strictly greater than c/pq, namely an ecological constraint. For sake of simplicity, we assume that :

$$x_{\min} \leq \frac{c}{pq} < l.$$

Note also that this same constraint (5) yields a strictly positive minimal effort (and harvesting) since

$$e(t) \geq \frac{C}{pqx(t) - c} > 0.$$

This can be related, for instance, to an employment requirement. Thus one can see this net benefit constraint as a way to reconcile the ecological and economic points of view. Notice also that constraint (5) materializes a difference between the viability approach and the optimal control approach. In the optimal control approach, the value to optimize would be the integral over time of the discounted profit $\int_0^{\infty} e^{-\delta t} R(x(t), e(t))dt$. The optimal solution[1] may require the shutdown of the activity for a while ($e(t) = 0$) which induces a negative balance for the period and thus leads to the violation of the constraint (5). Furthermore, we assign an equal weight to every time period, thereby avoiding the discussion with respect to the choice of the discount factor.

To summarize the viability constraints, we denote by K the constraint (or target) set represented on figure 1 and defined by

$$K = \{(x, e) \in \mathbb{R}^2,\ 0 \leq e \leq e_{\max},\ R(x, e) = pqex - ce - C \geq 0\}. \quad (6)$$

We shall say that a trajectory $(x(.), e(.))$ is viable in K if and only if we have

$$(x(t), e(t)) \in K,\ \forall t \geq 0.$$

3 A viability analysis

A first question that arises now is to determine whether the evolution (2) is compatible with the set of constraints K defined by (6). In other words, we aim at revealing levels of resource and effort of the constraint domain K that are associated with a viable trajectory in K and thus with a viable regulation (open-loop) $t \to u(t)$. To achieve this, we proceed in several steps: identification of viable stationary points, of viability niches, of the viability kernel and computation of the viable regulations associated with it. In order to restrict the mathematical content of the paper, we omit the proofs of the different results presented below; these proofs can be found in Béné et al, 1998.

[1]At this stage, let us emphasize that viability and optimality are not exclusive approaches. Indeed, some optimal solution can be viable (in the sense we have just defined), while some are not (according to the existence of a fixed cost). Furthermore, there are several ways to reconcile optimal and viability approaches. One consists in solving the inter-temporal optimality problem under the viability constraint. This means that the economic motivations of the agents operating in the system could be to optimize the inter-temporal profit under the positive profit constraint. Another direction is to consider a myopic (non inter-temporal) viable optimization namely to find the feedback control(s) that maximizes profit among the viable options (i.e in the so-called viable regulation map).

3.1 Viable stationary points

The first step of the analysis concerns the viable stationary points of the system, which correspond to $\dot{x} = 0$, $\dot{e} = 0$. Because of the constraint $x(t) \geq x_{\min} > 0$, the solution $x = 0$ is clearly not a viable equilibrium. The others solutions, referred as the "sustainable yield levels" in the classical approach, are defined by:

$$e = s(x) = \frac{r}{q}(1 - \frac{x}{l}) \text{ and } u = 0.$$

These equilibria are viable whenever $(x, s(x)) \in K$. It can be shown (Béné et al., 1998, Appendix A1) that:

Proposition 3.1 *A necessary and sufficient condition for the existence of viable equilibria for the system (2) is that*

$$\begin{cases} r \geq r^* = \frac{4pq^2Cl}{(pql-c)^2} \\ e_{\max} \geq e^+ = \frac{r}{2q}(1 - \frac{c+\sqrt{\Delta}}{pql}), \end{cases} \tag{7}$$

where

$$\Delta = (pql + c)^2 - 4pql(\frac{Cq}{r} + c) = (pql - c)^2 - 4\frac{pq^2lC}{r}.$$

In that case, the set of viable equilibria corresponds to the segment

$$EQ = \{(x, e),\ e = s(x),\ \max(x_\diamond, x^-) \leq x \leq x^+\}$$

defined by

$$\begin{cases} x^- = \frac{l}{2} + \frac{c-\sqrt{\Delta}}{2pq}, \\ x^+ = \frac{l}{2} + \frac{c+\sqrt{\Delta}}{2pq}, \\ x_\diamond = s^{-1}(e_{\max}). \end{cases}$$

Remark — The first condition (7) reveals **a minimum intrinsic growth rate** value r^* necessary to ensure the existence of a viable equilibria while the second condition emphasizes the necessity of a **minimum production capacity** e^+ induced, in particular, by the occurrence of the fixed cost C. These thresholds will play an important role for the computation of the viability kernel. □

3.2 Viability niches

As a second step of the analysis, we identify what we call the viability niches. These viability niches correspond to initial resource level $x(0) = x_0$ such that the resulting evolution, with a permanent policy $e(t) = e_0$, remains viable. They correspond to the most favorable situation since no regulation is needed and the effort does not have to be changed to guarantee viability. The niches $N(e_0)$ are thus defined by

$$\mathrm{N}(e_0) = \left\{ x_0 \;\middle|\; \begin{array}{l} \text{the solution } (x(\cdot), e_0) \text{ of (2) with } u(t) = 0, \\ \text{starting from } (x_0, e_0), \text{ is viable in } K \end{array} \right\}.$$

Clearly, viable equilibria are part of viability niches. As illustrated on figure 2, it turns out that the niches $N(e_0)$ exist whenever $e_0 \in [e^+, e^-]$ where we denote by e^- the effort value

$$e^- = s(x^-) = \tfrac{r}{2q}(1 - \tfrac{c-\sqrt{\Delta}}{pql}).$$

This requires condition (7) to hold. In the present case, the niches are defined by

$$N(e_0) = [\frac{(\frac{C}{e_0} + c)}{pq}, +\infty[.$$

3.3 Viability kernel and overexploitation

The next step is to study the whole viability of the system using the concept of viability kernel. The viability kernel, denoted by Viab(K), corresponds to the set of all initial conditions (x_0, e_0) such that there exists at least one trajectory starting from (x_0, e_0) that stays in the set of constraints K. In other words:

$$\mathrm{Viab}(K) = \left\{ (x_0, e_0) \in K \;\middle|\; \begin{array}{l} \exists u(\cdot) \text{ such that the solution } (x(\cdot), e(\cdot)) \text{ of (2),} \\ \text{starting from } (x_0, e_0), \text{ is viable in } K \end{array} \right\}.$$

The viability kernel differs from the niches in that, for the kernel, regulations through changes in effort can take place, thus allowing the viability to be enlarged. To focus on over-exploitation issues, we assume that:

$$u^+ \geq \max_{x > x^+} \gamma'(x)(f(x) - q\gamma(x)x), \tag{8}$$

where $\gamma(x) = \frac{C}{pqx-c}$. This condition (8) is related to the maximal rigidity of effort and guarantee[2] the possibility to increase sufficiently the effort for high levels of resource and low levels of effort.

We can distinguish three qualitative configurations for the viability kernel, depending mainly on the value of the resource parameter r. These three cases are illustrated on Figure 3.

Case 1: Global viability (figure 3(d)): The most favorable case takes place when the viability kernel equals the whole set of constraints K, i.e., $\mathrm{Viab}(K) = K$. This situation occurs mainly if the intrinsic growth rate r is high enough (see proof in Béné et al., 1998, Appendix A3).

Proposition 3.2 *If the conditions (7) and (8) hold true and if*

$$e_{\max} \leq e^{-} \tag{9}$$

then,

$$\mathrm{Viab}(K) = K,$$

namely, for every $(x_0, e_0) \in K$, there exists a control $u(.)$ such that the solution $(x(.), e(.))$ of dynamics (2) starting from (x_0, e_0) is viable in K.

Remark — This statement means that whenever the effort $e_{\max}$ is sufficiently low (still assuming that the second condition of (7) is not violated) or, in a symmetric way, whenever the resource growth rate r is high enough, viability holds everywhere and no over-exploitation occurs. In such a case, every viable situation turns out to correspond to a viability niche and the government agency does not need to regulate the fishery. In such a "scenario", the maximal fleet size is too moderate to induce any dangerous mortality on the exploited resource. □

Case 2: Partial viability (figures 3(b-c)): The most significant and interesting case occurs when the viability kernel $\mathrm{Viab}(K)$ is a strict and non-empty subset[3] of K. This case occurs mainly when condition (9) does not hold true i.e. $e_{\max} > e^{-}$.

[2] This condition makes sense since it can be checked that $\max_{x > x^+} \gamma'(x)(f(x) - q\gamma(x)x) < +\infty$

[3] Namely $\mathrm{Viab}(K) \subset K$ with $\emptyset \neq \mathrm{Viab}(K)$ and $\mathrm{Viab}(K) \neq K$.

To neglect under-exploitation issues, we still assume that (8) holds. In that case, one can prove (Béné et al., 1998, Appendix A2) that the viability kernel is defined as follows :

Proposition 3.3 *Under assumptions (7), (8) and if* $e_{\max} > e^-$ *, the viability kernel* $\mathrm{Viab}(K)$ *is a strict and non-empty subset of* K *and it is defined by*

$$\mathrm{Viab}(K) = \{(x, e) \in K, \;\; x \geq g(e) \;\; \textit{if} \; e \geq e^-\},$$

where $g(x)$ *is the solution of the differential equation on* $[e^-, e_{\max}[$

$$\begin{cases} \frac{dg}{de}(e) = \frac{f(g(e)) - qeg(e)}{u^-}, \; \forall e \geq e^-, \\ g(e^-) = x^-. \end{cases}$$

Remark — The curve g, which defines the upper boundary of the viability kernel, corresponds to a trajectory satisfying $\dot{e} = u^-$ and reaching the viable equilibrium point (x^-, e^-) (see Figure 3(b) and 3(c)). In fact, the curve g represents the states of the system where it is necessary to change the control and thus the effort in order to prevent the system from leaving the domain of constraint K. In other words, this is the "last" zone and consequently the "last" moment where it is still possible to change the extraction effort in order to preserve economic viability. The existence of this boundary thus makes clear the need to regulate the fishery and to anticipate the dynamics of the system in order to avoid a crisis (here a negative cash-flow). In particular, it emphasizes the need to reduce as drastically as possible the effort e with respect to the capital rigidity (1) when attaining this curve. Note that part of this curve g (especially the portion around the equilibrium point (x^-, e^-)) is located in a zone characterized by a "low" resource and a "high" effort levels. This is quite coherent with general expectations since much of the problems encountered in renewable resource management relates to the combination: low resource level - high extraction rate.

Along with this, we can identify the **over-exploitation zone** denoted by OZ as the set

$$\mathrm{OZ} = K \backslash \mathrm{Viab}(K) = \{x \in K, \; x \notin \mathrm{Viab}(K)\}.$$

This set OZ materializes the situations where the extraction effort is too high and would drive the system to a crisis (through a decrease of the resource) despite admissible regulations having been applied. □

Case 3: No viability (figure 3(a)): It turns out that the viability kernel is empty whenever condition (7) related to the existence of a viable equilibrium is not satisfied. Indeed, an equilibrium located in K is always a state of the viability kernel.

Proposition 3.4 *If condition (7) is not satisfied, then* $\text{Viab}(K) = \emptyset$.

Remark — In particular, this means that, if the intrinsic growth rate value r is smaller than the threshold r^* defined in (7), then the system is not sustainable; in other words, we face an overexploitation situation in every case, i.e. $\text{OZ} = K$. □

3.4 Viable regulations: how to avoid over-exploitation.

The viability kernel revealed the states biomass-effort compatible with the constraints. The present step is to compute the viable management options (decision or control) associated with it. For this purpose, we introduce the viable regulation map $U(x, e)$ which represents the controls $u = \dot{e}$ that can maintain viability for a state (x, e). For any point (x, e) in the viability kernel $\text{Viab}(K)$, we know that this regulation set is not empty. We consider the case of partial viability, namely

$$\text{Viab}(K) = \{(x, e) \in K, \ \ x \geq g(e) \text{ if } e \geq e^-\}.$$

Under the assumptions (7) and (8), we can prove (Béné et al., 1998, Appendix A4) that $U(x, e)$ is then defined by:

$$U(x,e) = \begin{cases} 0 & \text{if } e = e^-, \ x = x^- \\ u_- & \text{if } x = g(e), \ e^- < e \leq e_{\max} \\ [u_-, 0] & \text{if } x > g(e), e = e_{\max} \\ [\alpha(x), \ u^+] & \text{if } R(x,e) = 0, \ x > x^- \\ [u_-, u^+] & \text{otherwise,} \end{cases}$$

where $\alpha(x) = \max(u^-, \gamma'(x)(f(x) - q\gamma(x)x))$.

Remark — From the calculation of this regulation map U, it appears that the only mandatory unique regulation occurs when the anticipation of a profit crisis is required i.e., when $x = g(e)$. In that case, the choice in $U(x, e)$ reduces to u^-. For any other situations within the viability kernel, several viable regulations and viable policies are possible. This means that policy-makers are offered different viable alternatives which extends their flexibility with respect to the multiple, evolving (or sometimes conflicting) objectives they attempt to achieve. This result may represent an improvement with respect to others approaches where only one solution is usually proposed. □

4 Time of crisis and irreversibility.

In this section, we go a step further in the analysis of the system viability by using the concept of time of crisis (Doyen-StPierre, 1997). We have pointed out above the possible existence of an "overexploitation" area OZ where the dynamics leads the system to leave the domain of constraints and in particular to violate the benefit constraint. Negative profits happen in lots of fisheries (at least for some finite period) everywhere in the world. Indeed it is not unusual that fishermen have to face periods where the value of the catch does not cover the total operating costs. However these negative cash-flows do not necessarily induce the definitive shutdown of the activity provided that they do not last for too long. Consequently it is relevant to determine what would happen in this situation, to evaluate the corresponding level of overexploitation and, in particular, to study its irreversibility feature. So, for a given trajectory $(x(.), e(.))$, we measure[4] the length of the period of negative profit $(t,\ R(x(t), e(t)) < 0)$. Then, we compute the minimal length of cash-flow crisis:

$$V(x,e) = \inf_{x(\cdot),e(.),u(\cdot)} \text{measure}(t,\ R(x(t),e(t)) < 0),$$

under the conditions

$$\begin{cases} x(\cdot), e(.), u(\cdot) \text{ solution of system (2)} \\ x(0) = x,\ e(0) = e, \\ (x(t), e(t)) \in H \end{cases}$$

[4]The measure is taken in the sense of

$$\text{measure}(t,\ R(x(t),e(t)) < 0) = \int_{\{t \geq 0 \ \mid \ R(x(t),e(t))<0\}} dt.$$

where H ($H \supset K$) stands for the subset of constraints consisting simply of the ecological minimal threshold and the maximal effort capacity as follows

$$H = \{(x, e) \in \mathbb{R}^2,\ x_{\min} \leq x,\ \ 0 \leq e \leq e_{\max}\}.$$

The crisis function V provides an indicator of overexploitation and shows that three qualitative areas can be distinguished:

- **No overexploitation**: $V(x, e) = 0$. Inside the viability kernel $\mathrm{Viab}(K)$, the value $V(x, e)$ equals 0. This emphasizes again the existence of a viable control and a solution that does not violate the state constraints K. This case has already been fully discussed above.
- **Reversible overexploitation**: $0 < V(x, e) < +\infty$: The overexploitation crisis $(x, e) \in \mathrm{OZ} = K \backslash \mathrm{Viab}(K)$ can be solved in finite time. As illustrated in Figure 4, the cash flow crisis ($R < 0$) can be long, and there is a period of time during which, even with a drastic reduction of the activity ($\dot{e} = u^-$), the resource level will continue to decrease until it reaches a level where it can be rebuilt. In that case, the strategy is then to let the resource grow thanks to a "weak" effort in order to return into the viability kernel.
- **Irreversible overexploitation**: $V(x, e) = +\infty$. The crisis ($R < 0$) induced by the overexploitation becomes an irreversible crisis since it leads to the "extinction" ($x < x_{\min}$) of the resource and therefore to the definitive shutdown of the economic activity. From Figure 4, it appears that this situation may happen if the effort level is set high. But it can also happen, in moderate harvesting effort situations, when the change in strategy is decided too late i.e. when the profit $R(x, e)$ becomes negative. This can occur for instance if fishermen behaviors are completely rigid: the reduction of effort is not applied ($u = 0$) until the direct feasibility (to be in K) is at stake. In that case, if one starts above the viability curve g, the resource decreases until one reaches a point $(\widehat{x}, e_0)$ where the viability is at stake ($R(\widehat{x}, e_0) = 0$). If $V(\widehat{x}, e_0) = +\infty$, then, whatever the later change of strategy and regulation, the resource and thus the economic activity will collapse.

5 Conclusion

In this study, we have addressed the problem of the management of natural resource exploitation systems. We re-visit the classical dynamic fishery model within a new framework based on the concept of viability. The main purpose of this new approach is not to maximize an objective function, but to analyze the compatibility between the dynamics of the system and its constraints and to determine the set of controls (or decisions) that prevent the system from violating these viability constraints.

In the present case of a fishery model, management options are identified, assuming a deterministic dynamics (no uncertainty) and a net benefit constraint with fixed cost. This benefit constraint induces effort and biomass minimal thresholds and thus aims at reconciling ecological and economics requirements. The viability kernel analysis highlights the need to anticipate the system dynamics to prevent sector deficits that we relate with overexploitation issues. Then, using the time of crisis concept, we evaluate different types of overexploitation. In particular, we distinguish a reversible overexploitation zone, where the system can recover from crisis and come back into the viable domain in finite time, and an irreversible overexploitation situation which leads to the extinction of the resource and to the definitive shutdown of the activity.

It is clear that the model adopted in this study is quite stylized and built up on simplistic assumptions. Future research is needed to relax some of these assumptions. In particular, we aim at including resource uncertainties, capital dynamics through investment, price dynamics and market demand to make the model more realistic. We also hope to incorporate and analyze behavior mechanisms such as cooperation with respect to the resource access issue. More generally, the viability approach may provide an interesting analytical framework to address some of the issues encountered in natural resource management and sustainable development.

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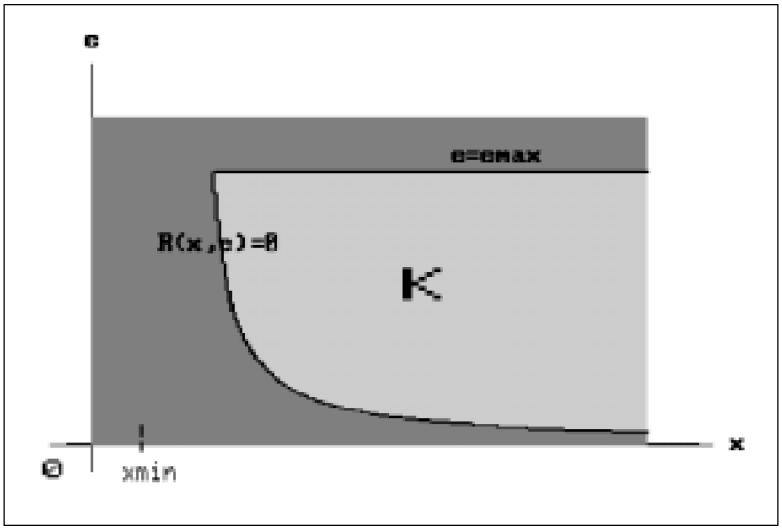


Figure 1: In grey, the domain of viability constraints K delimited by the two constraints $0 \leq R(x, e)$ and $e \leq e_{\max}$.

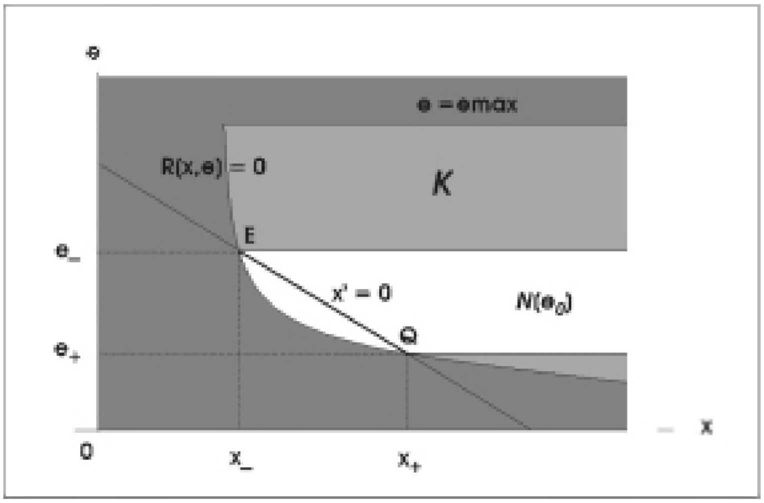
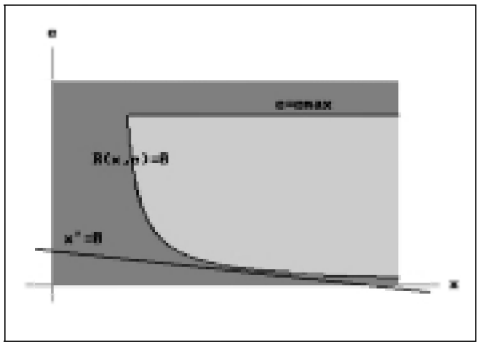
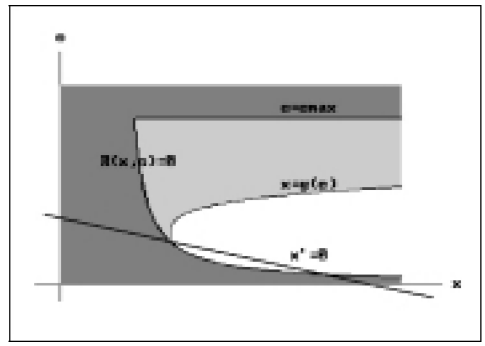


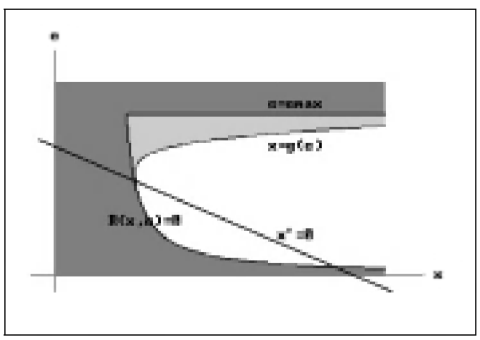
Figure 2: In white, the viability niches.



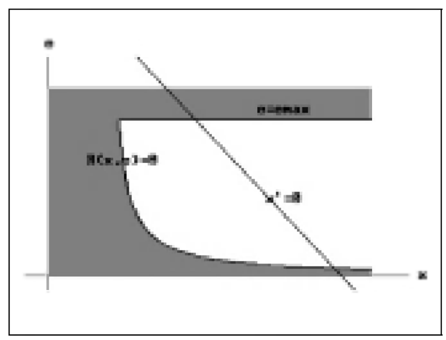
(a) No Viability



(b) Partial Viability

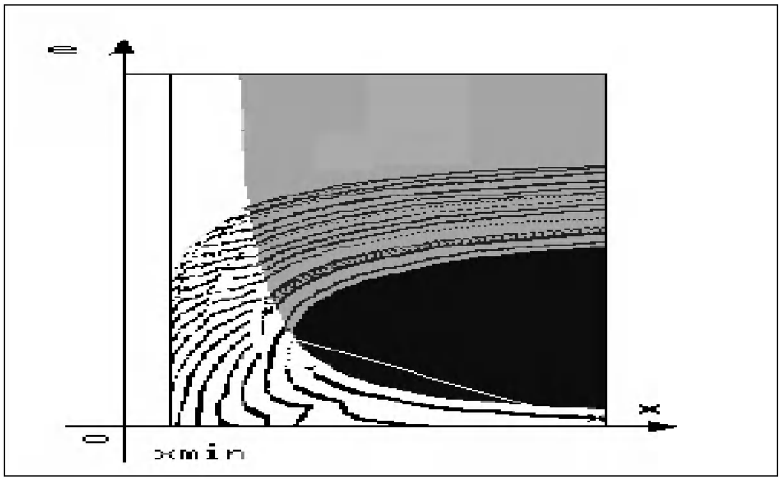


(c) Partial Viability

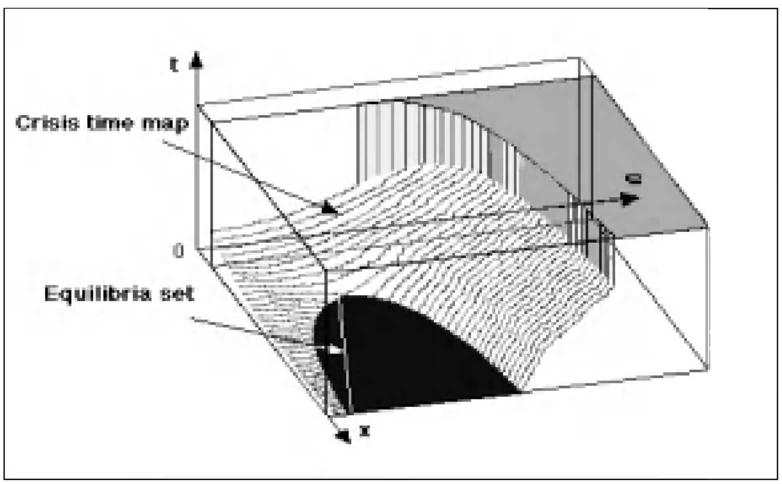


(d) Global Viability

Figure 3: Viability kernels for different values of the parameter r. In white, the viability kernel $\mathrm{Viab}(K)$, in grey the overexploitation zone, $\mathrm{OZ} = K \backslash \mathrm{Viab}(K)$.



(a) Level curves of the minimal time crisis function.



(b) Graph of the minimal time crisis function. The grey area indicates infinite time crisis i.e., irreversible overexploitation situations. The black area stands for the viability kernel.

Figure 4: Approximation of the crisis function $V(x, e)$.