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Non-Linear Theory of an Equatorial Flow, with Special Application to the Cromwell Current*

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Abstract: A method is given for computing the velocity of equatorial flow at and around the Equator, taking into account the effect of field accelerations in addition to the pressure gradients and Coriolis forces. Applied to the observational data, this method gives a result which agrees with the observed flow pattern in the equatorial Pacific quite well. The Cromwell Current (Equatorial Undercurrent) and the Equatorial Countercurrent are noticed quite distinctly. We can also show from the result that effect of inertia terms is negligible except at or within a few degrees from the Equator.

1. Introduction

In this paper will be discussed a method to compute a steady, oceanic flow in which the velocity components and pressure gradients have no variation in east-west direction. In the equatorial zones of the oceans, particularly in the Pacific and Indian Oceans, this condition is approximately fulfilled.

In a paper by the author and Y. NAGATA (HIDAKA and NAGATA, 1958), a dynamical computation of ocean currents was given in a meridional section of the Pacific, taking into account the Coriolis forces, pressure gradients, and the friction due to both vertical and lateral mixing. In that computation the two authors could locate an eastward flow comparable in magnitude with the Equatorial Undercurrent (Cromwell Current). The next year Robert S. ARTHUR (ARTHUR, 1960) pointed out that the term $v\dfrac{\partial u}{\partial y}$ will amount to 10^{-4} on both sides of the Equator and will no longer be negligible. Moreover, it will be interesting to check the effect of those inertia terms on the equatorial flow pattern. For this reason the author has tried to recompute the problem ever since, taking the effect of these non-linear terms into consideration.

In another of the recent papers by the author (HIDAKA, 1961), it was discussed to compute the velocity of flow at the Equator, taking into account the inertia terms:

$$-v\frac{\partial u}{\partial y} \quad \text{and} \quad -v\frac{\partial v}{\partial y}$$

respectively, and the result was compared with that obtained from the method proposed by HIDAKA (HIDAKA, 1955) and M. TSUCHIYA (TSUCHIYA, 1955) almost simultaneously for computing the velocity at the Equator itself. By this comparison, we could have an idea on the influence of the inertia terms on the magnitudes of an equatorial flow.

This paper is the result of one of the attempts to compute the effect of inertia terms on the velocity distribution in a meridional dynamical section.

2. The problem

Suppose a meridional section passing through the Equator. Take x-, y-and z-axis positive eastward, northward and downward, and let the velocity components in these directions be u, v and w respectively. Assume that the east-west variation of velocity components are small compared with the meridional, and neglect the terms $u\dfrac{\partial u}{\partial x}$ and $u\dfrac{\partial v}{\partial x}$.

Since it is not easy to solve non-linear equations as they stand, and because it is well-known that the ocean current is in most cases represented by geostrophic flows except at or close to the Equator, we shall neglect the frictional forces due to mixing in dynamical equations. Then the steady state equations for a planetary flow become

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$$\left.\begin{aligned} 2\omega\sin\varphi v - v\frac{\partial u}{\partial y} - w\frac{\partial u}{\partial z} &= \frac{\partial D}{\partial x}\ ; \\ -2\omega\sin\varphi u - v\frac{\partial v}{\partial z} - w\frac{\partial v}{\partial z} &= \frac{\partial D}{\partial y} \end{aligned}\right\} \qquad (1)$$

where D is a geopotential distance of an isobaric surface counted above a sufficiently deep reference level, ω the angular velocity of the Earth and φ the geographical latitude.

The general principle of solving the equations (1) is first to neglect the terms:

$$-w\frac{\partial u}{\partial z} \quad \text{and} \quad -w\frac{\partial v}{\partial z}, \qquad (2)$$

and to solve the ordinary non-linear simultaneous equations:

$$\left.\begin{aligned} 2\omega\sin\varphi v - v\frac{\partial u}{\partial y} &= \frac{\partial D}{\partial x}\ ; \\ -2\omega\sin\varphi u - v\frac{\partial v}{\partial y} &= \frac{\partial D}{\partial y}. \end{aligned}\right\} \qquad (3)$$

This solution will be possible if the quantities (2) are small compared with $\partial D/\partial x$ and $\partial D/\partial y$ respectively. The principal part of this paper is devoted to the solution of the equations (3).

When the approximate values of the velocity components are computed along the meridional section, it will be possible to approximately compute the vertical velocity w from the equation of continuity. This enables us to compute the quantities (2). By subtracting these from $\partial D/\partial x$ and $\partial D/\partial y$, we obtain a pair of equations of the type (3), there being slight changes in the magnitudes of $\partial D/\partial x$ and $\partial D/\partial y$. Repetition of this process will give the final solution of the problem.

For the processes of obtaining w and improving the solution, refer to the sections 8 and 9.

3. On the balance of acting forces at the Equator

It has been customary in most cases to neglect the terms of field accelerations or the non-linear terms. This seems to be possible except for narrow equatorial zones. However, the Coriolis forces vanish or become vanishingly small exactly at or very close to the Equator, so that these non-linear terms are considered to balance the pressure gradients and are no longer negligible. We may suppose that $-w(\partial u/\partial z)$ and $-w(\partial v/\partial z)$ are small compared with the terms $-v(\partial u/\partial y)$ and $-v(\partial v/\partial y)$ which have to balance the pressure gradients in the absence of the Coriolis forces at the Equator.

In a geostrophic computation, it is assumed that several terms in the dynamical equations except the Coriolis forces and the pressure gradients are negligibly small. Thus we have

$$\left.\begin{aligned} 2\omega\sin\varphi v &= \frac{\partial D}{\partial x}\ ; \\ -2\omega\sin\varphi u &= \frac{\partial D}{\partial y}. \end{aligned}\right\}$$

This balance evidently fails at the Equator where the Coriolis forces vanish. For this reason we have to consider a factor f which would balance the pressure gradients at and near the Equator, but is much smaller than the Coriolis terms at higher latitudes. Thus, *in a meridional direction*, we must have

$$-2\omega\sin\varphi u + f = \frac{\partial D}{\partial y} \qquad (4)$$

This factor f will not vanish even if the Coriolis term vanishes at the Equator, thus balancing the pressure gradient $\partial D/\partial y$ there.

There is another requirement imposed on the factor f. According to HIDAKA (HIDAKA, 1958) and M. TSUCHIYA (TSUCHIYA, 1958), the equatorial velocity u can be computed by differentiating the geostrophic equations with respect to y and putting $\varphi = y = 0$. The approximate validity of the resulting formula

$$u_0 = \frac{R}{2\omega}\left(\frac{\partial^2 D}{\partial y^2}\right)_{\varphi=0} \qquad (5)$$

where R is the average radius of the Earth, is now widely accepted. In order that this requirement be fulfilled, however, the quantity $\partial f/\partial y$ should become much smaller than the term $(\partial/\partial y)(-2\omega\sin\varphi u)$ at the Equator. This is another necessary condition which the factor f has to satisfy.

From these considerations, the factor f in (4) should be in a balance with the term $\partial D/\partial y$ at the Equator, and its derivative $\partial f/\partial y$ must either small or vanish there.

It appears from observations that the zonal velocity component u is distributed symmetrically around the Equator. This can be seen from the meridional section of the Equatorial Undercurrent (Cromwell Current) recently published by KNAUSS (KNAUSS, 1959). On the other hand, we have a very little knowledge on the meridional distribution of the meridional component v. We have not had an exact image on this question as yet. However, it can be anticipated that v as well as $\partial v/\partial y$ will not vanish even at the Equator itself. Thus it seems reasonable for us to suppose that $-v(\partial v/\partial y)$ corresponds with or with a part of the quantity f which we considered above. It will be a small quantity at or very near the Equator, being in balance with the term $\partial D/\partial y$ which in turn appears quite small there. Thus in a region very close to the Equator, the term $-2\omega\sin\varphi u$ is small, but its meridional gradient:

$$\left|\frac{\partial}{\partial y}(-2\omega\sin\varphi u)_{\varphi=0}\right|=\frac{2\omega}{R}\left|u_0\right|$$

can be much larger than

$$\left|\frac{\partial}{\partial y}\left(-v\frac{\partial v}{\partial y}\right)_{\varphi=0}\right|=\left|\left(\frac{1}{2}\cdot\frac{\partial^2 v^2}{\partial y^2}\right)_{\varphi=0}\right|$$

which is the meridional gradient of an inertia term.

It is also quite hard to know the distribution of meridional gradient $\partial D/\partial y$ at the Equator. But various estimations show that $\partial D/\partial y$ never vanishes there, though it is a small quantity (Ca. $10^{-5}{}_{c.g.s.}$).

These prudent considerations lead us to suppose that, in the equation:

$$-2\omega\sin\varphi u-v\frac{\partial v}{\partial y}=\frac{\partial D}{\partial y},$$

$-2\omega\sin\varphi u$ vanishes at the Equator, thus a balance having to exist primarily between $-v(\partial v/\partial y)$ and $\partial D/\partial y$ there, whereas the derivative of the Coriolis force $-2\omega\sin\varphi u$ is much larger than that of the inertia term $-v(\partial v/\partial y)$, thus an approximate balance existing between $\partial/\partial y(-2\omega\sin\varphi u)$ and $(\partial/\partial y)(\partial D/\partial y)$ at the Equator.

In this paper, the terms of both horizontal and vertical mixing were neglected, though some of them appear to act as a part of the factor f in the equation (4). The importance of these terms will be left for later studies.

4. Solution of the non-linear equations for equatorial flows

Neglecting $-w(\partial u/\partial z)$ and $-w(\partial v/\partial z)$ as small, we have from (1)

$$\left.\begin{aligned}2\omega\sin\varphi v-v\frac{\partial u}{\partial y}&=\frac{\partial D}{\partial x};\\-2\omega\sin\varphi u-v\frac{\partial v}{\partial y}&=\frac{\partial D}{\partial y}.\end{aligned}\right\}\qquad(3)$$

Consider that $\partial D/\partial x$, $\partial D/\partial y$ and u, v can be expanded in series of y around the Equator $\varphi=0$ or $y=0$. Thus assume

$$\frac{\partial D}{\partial x}=d_0+d_1y+d_2y^2+\cdots,\qquad(6)$$

$$\frac{\partial D}{\partial y}=\delta_0+\delta_1y+\delta_2y^2+\cdots,\qquad(7)$$

where, d_0, d_1, $d_2,\cdots$; δ_0, δ_1, $\delta_2,\cdots$, can be obtained from the meridional distribution of the geopotential distance D. Further let u and v be given by

$$u=u_0+u'y+\frac{u''}{2!}y^2+\cdots+\frac{u^{(n)}}{n!}y^n+\cdots;\qquad(8)$$

$$v=v_0+v'y+\frac{v''}{2!}y^2+\cdots+\frac{v^{(n)}}{n!}y^n+\cdots,\qquad(9)$$

where u_0, v_0 are the east and north component of the current velocity at the Equator, whereas u', u'',$\cdots$; $u^{(n)}$,$\cdots$; v', v'',$\cdots$, $v^{(n)}$,$\cdots$ are the 1st, 2nd,$\cdots$, nth derivatives of u and v at $y=0$, which are to be determined by solving the equations (3).

Substituting (6)……(9) in (3), we have

$$\begin{aligned}&\left(\frac{2\omega}{R}y-2\omega\frac{y^3}{3!R^3}+\cdots\right)\left(v_0+v'y+\frac{v''}{2!}y^2+\cdots\right.\\&\quad\left.+\frac{v^{(n)}}{n!}y^n+\cdots\right)-\left(v_0+v'y+\frac{v''}{2!}y^2\cdots\right.\\&\quad\left.+\frac{v^{(n)}}{n!}y^n+\cdots\right)\cdot\left\{u'+u''y+\frac{u'''}{2!}y^2+\cdots\right.\\&\quad\left.+\frac{u^{(n-1)}}{(n-2)!}y^{n-2}+\frac{u^{(n)}}{(n-1)!}y^{n-1}+\cdots\right\}\\&\quad=d_0+d_1y+d_2y^2+\cdots\end{aligned}\qquad(10)$$

or

$$\begin{aligned}&2\omega\sin\varphi v-v\frac{\partial u}{\partial y}-\frac{\partial D}{\partial x}=\\&(-u'v_0-d_0)+\left(\frac{2\omega}{R}v_0-u'v'-u''v_0-d_1\right)y\\&+\left(\frac{2\omega}{R}v'-\frac{1}{2!}u'v''-u''v'-\frac{1}{2!}u'''v_0-d_2\right)y^2\\&+\left(\frac{1}{2!}\cdot\frac{2\omega}{R}v''-\frac{2\omega}{R}\cdot\frac{1}{3!R^2}v_0-\frac{1}{3!}u'v'''-u''v''\right.\end{aligned}$$

$$-\frac{1}{2!}u'''v'-\frac{1}{3!}u^{(4)}v_0-d_3\Big)y^3+\cdots$$
$$+\Big(\frac{2\omega}{R}v^{(n-1)}-\frac{2\omega}{R}\cdot\frac{1}{3!R^2}v^{(n-3)}-\frac{1}{n!}u'v^{(n)}-\cdots$$
$$-\frac{1}{(n-1)!}u^{(n)}v'-\frac{1}{n!}u^{(n+1)}v_0-d_n\Big)y^n+\cdots=0. \tag{11}$$

Equating to zero the coefficients of y^0, y, y^2, $y^3,\cdots$, $y^n,\cdots$, we have

$$\left.\begin{aligned}
&-u'v_0-d_0=0,\\
&\frac{2\omega}{R}v_0-u'v'-u''v_0-d_1=0,\\
&\frac{2\omega}{R}v'-\frac{1}{2!}u'v''-u''v'-\frac{1}{2!}u'''v_0-d_2=0,\\
&\frac{1}{2!}\frac{2\omega}{R}v''-\frac{2\omega}{R}\cdot\frac{1}{3!R^2}v_0-\frac{1}{3!}u'v'''-u''v''\\
&\qquad-\frac{1}{2!}u'''v'-\frac{1}{3!}u^{(4)}v_0-d_3=0,\\
&\qquad\cdots\cdots\qquad\cdots\cdots,\\
&\frac{1}{(n-1)!}\cdot\frac{2\omega}{R}v^{(n-1)}-\frac{1}{(n-2)!}\frac{2\omega}{R}v^{(n-3)}-\cdots\\
&\quad-\frac{1}{n!}u'v^{(n)}-\cdots\cdots-\frac{1}{(n-1)!}u^{(n)}v'\\
&\quad-\frac{1}{n!}u^{(n+1)}v_0-d_n=0.
\end{aligned}\right\} \tag{12}$$

In a similar manner, we have from the second equation of (3),

$$\left.\begin{aligned}
&-v_0v'-\delta_0=0,\\
&-\frac{2\omega}{R}u_0-v_0v''-v'-\delta_1=0,\\
&-\frac{2\omega}{R}u'-\frac{3}{2}v'v''-\frac{1}{2}v_0v'''-\delta_2=0,\\
&-\frac{1}{2!}\cdot\frac{2\omega}{R}u''+\frac{2\omega}{R}\cdot\frac{1}{3!R^2}u_0-\frac{1}{2}(v'')^2\\
&-\frac{2}{3}v'v'''-\frac{1}{6}v_0v^{(4)}-\delta_3=0,\\
&\qquad\cdots\cdots\qquad\cdots\cdots,\\
&-\frac{1}{(n-1)!}\cdot\frac{2\omega}{R}\cdot u^{(n-1)}+\frac{1}{(n-3)!}\frac{2\omega}{R}\\
&\times\frac{1}{6R^2}u^{(n-3)}-+\cdots\cdots-\frac{1}{n!}v_0v^{(n-1)}-\cdots\cdots\\
&-\frac{1}{(n-1)!}v_0v^{(n)}-\frac{1}{n!}v_0v^{(n+1)}-\delta_n=0.
\end{aligned}\right\} \tag{13}$$

Two sets of equations (12) and (13) are non-linear and simultaneous. They satisfy the expansions of two equations (3) to y^n. Here we notice, however, that the number of the equations in (12) and (13) is $2n+2$, while we have $2n+4$ unknowns. We have therefore a larger number of unknowns than that of the equations. For this reason, these equations cannot be solved in their forms as they stand.

A feasible countermeasure for this situation will be to obtain approximate roots by neglecting the terms:

$$-\frac{1}{n!}u^{(n+1)}v_0 \quad\text{and}\quad -\frac{1}{n!}v_0v^{(n+1)}$$

in the last equations of (12) and (13). By neglecting these terms, the number of the unknowns just equals that of the equations. It is therefore possible to solve the equations provided the expansions are satisfied only up to y^n, having the residues

$$-\frac{1}{n!}u^{(n+1)}v_0y^{n+1} \quad\text{and}\quad -\frac{1}{n!}v_0v^{(n+1)}y^{n+1} \tag{14}$$

respectively. Actually there is a good reason for us to neglect these two terms, because these quantities become smaller and smaller as n increases. The result will be therefore more improved as we adopt a larger number of equations. Thus we have to try several approximations, of which we shall mention cases $n=2$ and 3.

i) Case $n=2$

Take $n=2$. Then we have

$$\left.\begin{aligned}
&-u'v_0-d_0=0,\\
&\frac{2\omega}{R}v_0-u'v'-u''v_0-d_1=0,\\
&\frac{2\omega}{R}v'-\frac{1}{2}u'v''-u''v'-d_2=0,\\
&-v_0v'-\delta_0=0,\\
&-\frac{2\omega}{R}u_0-(v')^2-v_0v''-\delta_1=0,\\
&-\frac{2\omega}{R}u'-\frac{3}{2}v'v''-\delta_2=0.
\end{aligned}\right\} \tag{15}$$

Eliminating v', v'' and u', u'' out of the five equations which do not involve u_0, we have a quartic equations for v_0, or

$$(3d_2\delta_0-d_0\delta_2)v_0^4+\frac{2\omega}{R}d_0^2v_0^3+3d_1\delta_0^2v_0^2+3d_0\delta_0^3=0 \tag{16}$$

This equation will give the possible value of v_0 as one of its four roots. Substituting v_0 thus obtained in the remaining equation of (15) which involves u_0, we have

$$u_0 = -\frac{R}{2\omega}\left(\frac{2d_2}{d_0}v_0^3 + \frac{2d_1\delta_0}{d_0}v_0 + \delta_1 + 3\frac{\delta_0^2}{v_0^2}\right) \tag{17}$$

In this case, the expansions of the equations (3) will be satisfied up to y and the residues will be

$$-\frac{1}{2}u''v_0y^2 \text{ and } -\frac{1}{2}v_0v''y^2$$

respectively.

ii) Case $n=3$

Take $n=3$. The equations to be solved are

$$\left.\begin{aligned}
&-u'v_0 - d_0 = 0,\\
&\frac{2\omega}{R}v_0 - u'v' - u''v_0 - d_1 = 0,\\
&\frac{2\omega}{R}v' - \frac{1}{2}u'v'' - u''v' - \frac{1}{2}u'''v_0 - d_2 = 0,\\
&\frac{1}{2}\cdot\frac{2\omega}{R}v'' - \frac{2\omega}{R}\cdot\frac{1}{6R^2}v_0 - \frac{1}{6}u'v'''\\
&\qquad -u''v'' - \frac{1}{2}u'''v' - d_3 = 0\,;\\
&-v_0v' - \delta_0 = 0,\\
&-\frac{2\omega}{R}u_0 - v_0v'' - (v')^2 - \delta_1 = 0,\\
&-\frac{2\omega}{R}u' - \frac{3}{2}v'v'' - \frac{1}{2}v_0v''' - \delta_2 = 0,\\
&-\frac{1}{2}\cdot\frac{2\omega}{R}u'' - \frac{2\omega}{R}\cdot\frac{1}{6R^2}u_0 - \frac{2}{3}v'v'''\\
&\qquad -\frac{1}{2}(v'')^2 = 0.
\end{aligned}\right\} \tag{18}$$

In this case the expansions of the equations (3) will be satisfied up to y^2 and the residues will be

$$-\frac{1}{2}u'''v_0y^3 \text{ and } -u_0v'''y^3$$

respectively. The resulting equation giving v_0 is

$$\left.\begin{aligned}
&-\frac{1}{288R^4}\cdot\left(\frac{2\omega}{R}\right)^2v_0^{14} - \frac{2\omega}{R}\left(\frac{d_1}{288R^4}\right.\\
&\left.+\frac{d_3}{24R^2}\right)v_0^{13} - \left(\frac{d_1d_3}{48R^2} + \frac{d_3^2}{8}\right)v_0^{12}\\
&-\frac{2\omega}{R}\left(\frac{d_0\delta_0}{96R^4} + \frac{d_0\delta_2}{72R^2} + \frac{d_2\delta_0}{24R^2}\right)v_0^{11}\\
&+\left\{\left(\frac{2\omega}{R}\right)^2\cdot\left(\frac{d_0^2}{72R^2} - \frac{d_1^2}{32}\right) - \left(\frac{d_0d_3\delta_0}{16R^2}\right.\right.\\
&+\frac{d_0d_1\delta_2}{144R^2} + \frac{d_1d_2\delta_0}{48R^2} + \frac{d_1^2\delta_1}{96R^2} + \frac{d_1^2\delta_3}{16}\\
&\left.\left.+\frac{d_2d_3\delta_0}{4} + \frac{d_0d_3\delta_2}{12}\right)\right\}v_0^{10} + \frac{2\omega}{R}\left(\frac{d_0^2d_1}{144R^2}\right.\\
&\left.+\frac{d_1^3}{32} + \frac{d_0^2d_3}{12}\right)v_0^9 + \left\{-\frac{3d_0d_1\delta_0}{16}\cdot\left(\frac{2\omega}{R}\right)^2\right.\\
&-\frac{d_0^2\delta_0\delta_2}{48R^2} - \frac{d_0d_2\delta_0^2}{16R^2} - \frac{d_1^2\delta_0^2}{32R^2} - \frac{d_0d_1\delta_0\delta_1}{16R^2}\\
&-\frac{d_1^2\delta_0\delta_2}{12} - \frac{3d_0d_1\delta_0\delta_3}{8} - \frac{d_0^2\delta_2^2}{72} - \frac{d_2^2\delta_0^2}{8}\\
&\left.-\frac{d_0d_2\delta_0\delta_2}{12}\right\}v_0^8 + \frac{2\omega}{R}\left(\frac{d_0^3\delta_0}{48R^2} + \frac{d_0\delta_0^3}{12R^2}\right.\\
&\left.+\frac{29d_0d_1^2\delta_0}{96} + \frac{d_0^2d_2\delta_0}{12} + \frac{d_0^3\delta_2}{36}\right)v_0^7\\
&+\left\{\left(\frac{2\omega}{R}\right)^2\cdot\left(-\frac{9d_0^2\delta_0^2}{32} - \frac{d_0^4}{72}\right) - \frac{9d_0^2\delta_0^2\delta_3}{16}\right.\\
&-\frac{3d_0^2\delta_0^2\delta_1}{32R^2} - \frac{d_0d_1\delta_0^3}{12R^2} - \frac{d_0d_1\delta_0^2\delta_2}{12}\\
&\left.+\frac{d_0d_3\delta_0^3}{2}\right\}v_0^6 + \frac{2\omega}{R}\cdot\frac{31}{32}d_0^2d_1\delta_0^2v_0^5\\
&+\left(-\frac{5d_0^2\delta_0^4}{32R^2} + \frac{d_1^2\delta_0^4}{4} - \frac{7d_0^2\delta_0^3\delta_2}{12}\right.\\
&\left.+\frac{d_0d_2\delta_0^4}{2}\right)v_0^4 + \frac{2\omega}{R}\cdot\frac{83}{96}d_0^3\delta_0^3v_0^3\\
&+\frac{3d_0d_1\delta_0^5}{4}v_0^2 + \frac{5d_0^2\delta_0^6}{8} = 0
\end{aligned}\right\} \tag{19}$$

If we evaluate v_0 by solving the equation (19), u_0 will be given by

$$\begin{aligned}
u_0 = &-\frac{R}{2\omega}\cdot\frac{1}{3d_0\delta_0v_0^2 + d_1v_0^4}\left\{\frac{1}{3R^2}\cdot\frac{2\omega}{R}\cdot v_0^7 - 2d_3\cdot v_0^6\right.\\
&+\left(\frac{2}{3}d_0\delta_2 + d_1\delta_1 + 2d_2\delta_0\right)v_0^4 - \frac{2}{3}\cdot\frac{2\omega}{R}\cdot d_0^2\cdot v_0^3\\
&\left.+(3d_1\delta_0^2 + 3d_0\delta_0\delta_1)v_0^2 + 5d_0\delta_0^3\right\}.
\end{aligned} \tag{20}$$

Further approximations are of course possible theoretically. But the derivation of the equation giving v_0 will be extremely complicated then, so that we have to satisfy with this degree of approximation. The accuracy of the solution will be estimated by a comparison of the results of the cases $n=2$ and $n=3$.

5. Extension of the computation to higher latitudes

Once the velocity components u and v at the Equator are known, the next question is how to compute those at higher latitudes. For this purpose, let us start again with the equations (3).

From the first equation, we have

$$\frac{\partial u}{\partial y} = 2\omega\sin\varphi - \frac{1}{v}\frac{\partial D}{\partial x}.$$

Integrating this equation from 0 to y, we have

$$u-u_0=2\omega\int_0^y \sin\frac{\eta}{R}d\eta-\int_0^y\left(\frac{1}{v}\frac{\partial D}{\partial x}\right)_{y=\eta}\cdot d\eta$$
$$=2\omega R\left(1-\cos\frac{y}{R}\right)-\int_0^y\left(\frac{1}{v}\frac{\partial D}{\partial x}\right)_{y=\eta}\cdot d\eta \qquad (21)$$

where u_0 is the value of u at the Equator and both v and $\partial D/\partial x$ are functions of y only.

From the second equation of (3), we have

$$\frac{d}{dy}\left(\frac{v^2}{2}+D\right)=-2\omega\sin\varphi u. \qquad (22)$$

Substituting from (21), we have

$$\frac{d}{dy}\left(\frac{v^2}{2}+D\right)=-2\omega\sin\frac{y}{R}\left\{u_0+2\omega R\left(1-\cos\frac{y}{R}\right)-\int_0^y\left(\frac{1}{v}\frac{\partial D}{\partial x}\right)_{y=\eta}\cdot d\eta\right\}.$$

Integrating this equation with respect to y, we have

$$\frac{v^2}{2}+D=+\left(\frac{v_0^2}{2}+D_0\right)-2\omega R\cdot u_0\int_0^y\sin\frac{\eta}{R}d\left(\frac{\eta}{R}\right)-(2\omega R)^2\cdot\int_0^y\sin\frac{\eta}{R}\left(1-\cos\frac{\eta}{R}\right)d\left(\frac{\eta}{R}\right)+2\omega R\cdot\int_0^y\sin\frac{\eta}{R}\cdot\left\{\int_0^\eta\left(\frac{1}{v}\frac{\partial D}{\partial x}\right)_{y=s}\cdot ds\right\}d\left(\frac{\eta}{R}\right) \qquad (23)$$

where v_0 and D are the values of v and D at the Equator.

If we confine the integration limit within a few degrees from the Equator and change the order of integration, we have

$$\frac{v^2}{2}+D=\frac{v_0^2}{2}+D_0-2\omega Ru_0\cdot\left\{\frac{1}{2}\left(\frac{y}{R}\right)^2-\frac{1}{4!}\left(\frac{y}{R}\right)^4+\frac{1}{6!}\left(\frac{y}{R}\right)^6\right\}-(2\omega R)^2\left\{\frac{1}{8}\left(\frac{y}{R}\right)^4-\frac{1}{48}\left(\frac{y}{R}\right)^6+\frac{1}{640}\left(\frac{y}{R}\right)^8\right\}+2\omega R\cdot\int_0^y\left(\frac{1}{v}\frac{\partial D}{\partial x}\right)_{y=s}\cdot\left(\cos\frac{s}{R}-\cos\frac{y}{R}\right)ds \qquad (24)$$

This equation gives the meridional component v of current in terms of the horizontal distance of a point y from the Equator, and geopotential distance D at the point y.

Suppose we have the observational data at the Equator and the integral degrees of meridional arc, or at 0°($y=0$), 1°N($y=+\Delta$), 2°N($y=+2\Delta$) and 3°N($y=+3\Delta$). Since the length Δ of one degree of meridional arc is

$$\Delta=\frac{\pi R}{180}=1.112\times10^7\text{cm},$$

we have to compute the velocity component v at the above latitudes, use being made of the corresponding values of D. Let the velocity component v at 1°N, 2°N and 3°N be denoted by v_1, v_2 and v_3 respectively.

It is essential now to express the integrals in the right-hand members in terms of v_1, v_2 and v_3. For this purpose, it will be recommended to apply Newton-Côtes' mean value method. (RUNGE und KÖNIG 1924) In evaluating these integrals, we have several formulas resulting from the Newton-Côtes' rule, or

$$\int_0^\Delta F(s)ds=\left\{\frac{1}{2}F(0)+\frac{1}{2}F(\Delta)\right\}\Delta \qquad (25)$$

$$\int_0^{2\Delta} F(s)ds=\left\{\frac{1}{6}F(0)+\frac{4}{6}F(\Delta)+\frac{1}{6}F(2\Delta)\right\}\cdot 2\Delta \qquad (26)$$

$$\int_0^{3\Delta} F(s)ds=\left\{\frac{1}{8}F(0)+\frac{3}{8}F(\Delta)+\frac{3}{8}F(2\Delta)+\frac{1}{8}F(3\Delta)\right\}\cdot 3\Delta \qquad (27)$$

The formula (25) gives the so-called trapezoidal rule, while (26) is the well-known Simpson's rule. The formula (27) gives a rule applicable to the interval ($0\leqq s\leqq 3\Delta$). The accuracies of (26) and (27) are nearly the same, but the trapezoidal rule which replaces the curve by a straight line is much less accurate than two other formulas. In order to remove this difficulty, we have to replace the right-hand member of (25) by a difference of the rules (26) and (27), or

$$\int_0^\Delta F(s)ds=\int_0^{3\Delta}F(s)ds-\int_\Delta^{3\Delta}F(s)ds$$
$$=\left\{\frac{1}{8}F(0)+\frac{3}{8}F(\Delta)+\frac{3}{8}F(2\Delta)+\frac{1}{8}F(3\Delta)\right\}\cdot 3\Delta-\left\{\frac{1}{6}F(\Delta)+\frac{4}{6}(2\Delta)+\frac{1}{6}F(3\Delta)\right\}\cdot 2\Delta$$
$$=\left\{\frac{3}{8}F(0)+\frac{19}{24}F(\Delta)-\frac{5}{24}F(2\Delta)+\frac{1}{24}F(3\Delta)\right\}\cdot\Delta \qquad (28)$$

Applying the rules (28), (26) and (27) to the integrals at the right hand-members of (23), we have after necessary numerical computations,

$$\left.\begin{aligned} v_1^2+Q_1&=\frac{a_{12}}{v_2}+\frac{a_{13}}{v_3},\\ v_2^2+Q_2&=\frac{a_{21}}{v_1},\\ v_3^2+Q_3&=\frac{a_{31}}{v_1}+\frac{a_{32}}{v_2}\end{aligned}\right\} \tag{29}$$

where

$$\left.\begin{aligned} Q_1=&-v_0^2+2(D_1-D_0)+28.2957u_0\\ &+200.2734-1180.26\times10^5\frac{\left(\frac{\partial D}{\partial x}\right)_0}{v_0},\\ Q_2=&-v_0^2+2(D_2-D_0)+113.0793u_0\\ &+3203.886-4196.16\times10^5\frac{\left(\frac{\partial D}{\partial x}\right)_0}{v_0},\\ Q_3=&-v_0^2+2(D_3-D_0)+254.0404u_0\\ &16215.556-10620.62\times10^5\frac{\left(\frac{\partial D}{\partial x}\right)_0}{v_0},\end{aligned}\right\} \tag{30}$$

and

$$\left.\begin{aligned} a_{12}&=1966.90\times10^5\left(\frac{\partial D}{\partial x}\right)_2,\\ a_{13}&=-1048.89\times10^5\left(\frac{\partial D}{\partial x}\right)_3,\\ a_{21}&=12588.16\times10^5\left(\frac{\partial D}{\partial x}\right)_1,\\ a_{31}&=28320.00\times10^5\left(\frac{\partial D}{\partial x}\right)_1,\\ a_{32}&=17698.74\times10^5\left(\frac{\partial D}{\partial x}\right)_2\end{aligned}\right\} \tag{31}$$

It is possible to eliminate v_1 and v_2 from the three equations in (27). First we have, from the second equation,

$$v_1=\frac{a_{21}}{v_2^2+Q_2}. \tag{32}$$

Substituting this equation in the first equation of (29), we have

$$v_3=\frac{a_{12}}{\left(\frac{a_{21}}{v_2^2+Q_2}\right)^2+Q_1-\frac{a_{12}}{v_2}}. \tag{33}$$

Substituting (32) and (33) in the third equation of (29), we finally have

$$\frac{a_{12}^2}{\left\{\left(\frac{a_{21}}{v_2^2+Q_2}\right)^2+Q_1-\frac{a_{12}}{v_2}\right\}^2}+Q_3-\frac{a_{31}}{a_{21}}(v_2^2+Q_2)-\frac{a_{32}}{a_2}=0 \tag{34}$$

or

$$\left\{Q_3v_2-\frac{a_{31}}{a_{21}}v_2(v_2^2+Q_2)-a_{32}\right\}\cdot\{a_{21}^2v_2-a_{12}(v_2^2+Q_2)^2+Q_1v_2(v_2^2+Q_2)^2\}^2+a_{12}^2v_2^3(v_2^2+Q_2)^4=0 \tag{35}$$

This is an equation of 13th order and determines the possible value of v_2, or the meridional velocity at 2°N. Substituting the value of v_2 thus obtained in (32) and (33), we shall have v_1 and v_3, or the meridional velocities at 1°N and 3°N respectively.

A simpler way of solving (29) is to neglect the squares v_1^2, v_2^2 and v_3^2 in the left-hand members of (29). Then we have, from the second equation,

$$v_1=\frac{a_{21}}{Q_2}. \tag{36}$$

Substituting in the third equation of (29), we have

$$v_2=\frac{a_{32}}{Q_3-\frac{a_{31}}{a_{21}}Q_2}. \tag{37}$$

Again inserting this in the first equation of (29), we have

$$v_3=\frac{a_{12}}{Q_1-\frac{a_{12}}{a_{32}}\left(Q_3-\frac{a_{31}}{a_{21}}Q_2\right)}. \tag{38}$$

These are approximate values. Inserting them in the left-hand members of (27) gives more accurate roots. Repetition of this process results in an accurate set of roots of (29).

In order to obtain the values of u_1 and u_2, we have only to transform the second equation of (3) as

$$u=-\frac{\frac{d}{dy}\left(\frac{v^2}{2}+D\right)}{2\omega\sin\varphi} \tag{39}$$

Since the quantity $\frac{v^2}{2}+D$ is given for 1°, 2° and 3°N, we have from (39)

$$\left.\begin{aligned} u_1&=-\frac{\frac{v_2^2}{2}-\frac{v_0^2}{2}+(D_2-D_0)}{2\omega\sin1^\circ\cdot2\Delta},\\ u_2&=-\frac{\frac{v_3^2}{2}-\frac{v_1^2}{2}+(D_3-D_1)}{3\omega\sin2^\circ\cdot2\Delta}.\end{aligned}\right\} \tag{40}$$

In this process we cannot compute u_3, though we already have v_3 from (33).

In the expressions (40), $v_2^2/2-v_0^2/2$ and $v_3^2/2-v_1^2/2$ are in most cases larger than D_2-D_0 and D_3-D_1 respectively. This is in very close agreement with R.B. MONTGOMERY's conclusion on his dynamical computation which showed that the zonal component of velocity in the Equatorial Pacific is in an approximate geostrophic balance even in the 1° intervals next to the Equator. (MONTGOMERY, 1961) This conclusion was also supported by John A. KNAUSS (KNAUSS, 1960) who showed that the Coriolis force and the pressure gradient are in a balance in the north-south direction except very near the Equator.

For still higher latitudes ($|\varphi|>2°$), we can compute these components by simpler successive approximations, transforming the equations (3) into the form:

$$\left.\begin{aligned} u&=\frac{-\dfrac{\partial D}{\partial y}-v\dfrac{\partial v}{vy}}{2\omega\sin\varphi}\,, \\ v&=\frac{\dfrac{\partial D}{\partial x}+v\dfrac{\partial u}{\partial y}}{2\omega\sin\varphi} \end{aligned}\right\} \tag{41}$$

with initial guesses $u=v=0$, using the values computed already at 2°N and S and assuming that the flow at stations very far from the boundaries are purely geostrophic, or

$$\left.\begin{aligned} u&=-\frac{\dfrac{\partial D}{\partial y}}{2\omega\sin\varphi}\,; \\ v&=\frac{\dfrac{\partial D}{\partial x}}{2\omega\sin\varphi}\,. \end{aligned}\right\} \tag{42}$$

This process fails at as low a latitude as 1°N and 1°S or in between, because the Coriolis parameter becomes too small. So we had better confine it for the latitudes higher than 3°N and 3°S.

The result of practical application shows that at latitudes higher than 3°N or S, the flow is actually geostrophic.

6. Southern hemisphere

In order to compute u and v for the southern hemisphere, we have, to solve

$$\left.\begin{aligned} v_{-1}^2+Q_{-1}&=-\frac{a_{12}}{v_{-2}}-\frac{a_{13}}{v_{-3}}, \\ v_{-2}^2+Q_{-2}&=-\frac{a_{21}}{v_{-1}}, \\ v_{-3}^2+Q_{-3}&=-\frac{a_{31}}{v_{-1}}-\frac{a_{32}}{v_{-2}} \end{aligned}\right\} \tag{43}$$

where

$$\left.\begin{aligned} Q_{-1}&=-v_0^2+2(D_{-1}-D_0)+28.2957u_0 \\ &\quad+200.2734+1180.26\times10^5\frac{\left(\dfrac{\partial D}{\partial x}\right)_0}{v_0}, \\ Q_{-2}&=-v_0^2+2(D_{-2}-D_0)+113.079u_0 \\ &\quad+3203.886+4196.16\times10^5\frac{\left(\dfrac{\partial D}{\partial x}\right)_0}{v_0}, \\ Q_{-3}&=-v_0^2+2(D_{-3}-D_0)+254.040u_0 \\ &\quad+16215.556+10620.62\times10^5\frac{\left(\dfrac{\partial D}{\partial x}\right)_0}{v_0}, \end{aligned}\right\} \tag{44}$$

and a_{12}, a_{13}, a_{21}, a_{31} and a_{32} are the quantities given by (31).

The east-west components u_{-1} and u_{-2} can be computed by

$$\left.\begin{aligned} u_{-1}&=\frac{\dfrac{v_0^2}{2}-\dfrac{v_{-2}^2}{2}+(D_0-D_{-2})}{2\omega\sin1°\cdot2\Delta} \\ u_{-2}&=\frac{\dfrac{v_{-1}^2}{2}-\dfrac{v_{-3}^2}{2}+(D_{-1}-D_{-3})}{2\omega\sin2°\cdot2\Delta}. \end{aligned}\right\} \tag{45}$$

This process gives the components at 1°S and 2°S. For higher southern latitudes, successive approximation used for the northern latitudes will also apply without an alteration.

7. Upwelling velocity and the vertical circulation in a meridional section

The equation of continuity is, since $\partial u/\partial x=0$,

$$\frac{1}{\cos\varphi}\frac{\partial}{\partial y}(v\cos\varphi)+\frac{\partial w}{\partial z}=0 \tag{46}$$

where w is the vertical velocity counted positive downward. If we integrate this equation, we have

$$w(z)-w(0)=-\frac{1}{\cos\varphi}\frac{\partial}{\partial y}\left(\cos\varphi\int_0^z vdz\right).$$

But, since there will be no vertical velocity at the sea surface ($z=0$), we have

$$w(z)=-\frac{1}{\cos\varphi}\frac{\partial}{\partial y}\left(\cos\varphi\int_0^z vdz\right). \tag{47}$$

This is an expression of vertical component of velocity in terms of v.

It is evident that the equation of continuity

(46) will result in a stream function $\psi(y, z)$ such that

$$v=\frac{\partial\psi}{\partial z};\ w=-\frac{\partial\psi}{\partial y}. \tag{48}$$

A constant value of this function, or the equation:

$$\psi(y, z)=\text{a constant} \tag{49}$$

gives stream lines of vertical circulation in a meridional section. The numerical values of stream function will be computed as

$$\psi(y, z)=\cos\varphi\cdot\int_0^z v dz, \tag{50}$$

because there is a reason to suppose that the free sea surface is a stream line $\psi=0$.

8. Practical computation of ocean currents in a meridional section

In the expressions (6) and (7) for $\partial D/\partial x$ and $\partial D/\partial y$, we have

$$\left.\begin{aligned} d_0&=\left(\frac{\partial D}{\partial x}\right)_{\varphi=0},\quad d_1=\left(\frac{\partial^2 D}{\partial x\partial y}\right)_{\varphi=0},\ \text{etc.} \\ \delta_0&=\left(\frac{\partial D}{\partial y}\right)_{\varphi=0},\quad \delta_1=\left(\frac{\partial^2 D}{\partial y^2}\right)_{\varphi=0},\ \text{etc.} \end{aligned}\right\} \tag{51}$$

The theory was applied to the hydrographic data obtained by Townsend CROMWELL (CROMWELL, 1951, 1954), Thomas S. AUSTIN (AUSTIN, 1954) and E.D. STROUP (STROUP, 1954) in the Mid-Pacific Equatorial Waters. Their reports give the data occupied at stations along several meridional sections across the Equator, each station being located at 0° (Equator), 1°N and S, 2°N and S, etc., or at a constant spacing 1° of a meridional arc.

The data of the geopotential distances were computed from the observations of the following cruises of the M/V Hugh M. Smith of the Pacific Oceanic Fishery Investigations, Honolulu, and published in the Special Scientific Reports, United States Department of the Interior Fish and Wildlife Service.

Cruises	Occupied in	Published in the special scientific report-fisheries No.	Latitudes
I	Jan.-Mar. 1950	54	158°W, 172°W
V	June-Aug. 1950	131	158°W, 172°W
VIII	Jan.-Mar. 1951	131	158°W, 172°W
XIV	Jan.-Mar. 1952	135	155°W, 168°W, 180°

In order to remove small fluctuations involved in the meridional variation of D and $\partial D/\partial y$, the anomalies of D over 1000 decibar surface in these several meridional sections were averaged for the same latitudes. They extend from 5°S to 15°N and are given in Table 4.

The meridional gradients of D, or $\partial D/\partial y$ at a latitude φ were computed by making difference $D(\varphi+1°)-D(\varphi-1°)$ in a meridional direction, and then dividing them by twice the length of 1° of meridional arc, or

$$\left(\frac{\partial D}{\partial y}\right)_\varphi=\frac{D(\varphi+1°)-D(\varphi-1°)}{2\times 1.112\times 10^7} \tag{52}$$

The meridional variation of $\partial D/\partial y$ near the Equator was then determined, assuming

$$\frac{\partial D}{\partial y}=\delta_0+\delta_1\cdot y \tag{53}$$

or $\delta_2=\delta_3=\cdots=0$.

The values of $\partial D/\partial x$ were computed by NAGATA (HIDAKA and NAGATA, 1958) only for the Equator also from Hawaiian observational data. In their computation, this was assumed common to all latitudes, or

$$\frac{\partial D}{\partial x}=d_0 \tag{54}$$

and $d_1=d_2=\cdots=0$. In Table 5, are compiled the values of δ_0, δ_1 and d_0 at the Equator.

First we are to compute the equatorial velocity components u_0 and v_0 for each depth from the formulas (16), (17), (19) and (20). They are also shown in Table 5, the case $n=3$ giving of course the more accurate values. The case $n=2$ is given to estimate the accuracy of the case $n=3$. Table 1 gives the velocity components at all latitudes between 4.5°S and 14.5°N. Those for the Equator in this table are the result of the case $n=3$ above. The values of velocity components at 1° and 2°N and S respectively were computed by the equations (29), (34) or (35) and (40) for the northern hemisphere and (43), (44) and (45) for the southern. The result is given in the corresponding columns in Table 1.

For higher latitudes, the iteration process given at the end of 5 enables us to compute the velocity components. Since the initial

Table 1. Velocity components, computed taking inertia terms into consideration. (cm/sec)

	4° S		3°		2°		1° S		0°		1° N		2°		3°		4°	
	u	v	u	v	u	v	u	v	u	v	u	v	u	v	u	v	u	v
0 m	−27.52	3.89	−22.87	5.66	−41.77	25.10	−63.44	3.14	39.46	−15.91	−32.34	−4.85	−43.22	−10.59	−19.41	−6.42	−10.75	−4.42
20 m	−26.54	3.57	−22.94	5.16	−40.00	24.53	−57.82	2.97	41.55	−16.16	−21.21	−4.88	−37.91	−7.77	−18.21	−5.78	−7.82	−4.08
40 m	−26.54	3.11	−21.82	4.41	−37.31	20.14	−48.48	2.32	68.04	−17.91	−4.46	−3.73	−35.30	−5.03	−15.68	−5.00	−5.87	−3.57
60 m	−24.57	2.77	−19.43	3.94	−35.45	12.46	−42.66	2.25	63.01	−18.13	8.07	−4.05	−27.34	−3.92	−13.09	−4.28	−3.90	−3.17
80 m	−22.61	2.47	−20.80	3.50	−32.77	9.96	−38.16	2.02	67.13	−19.55	13.89	−3.64	−23.81	−3.37	−10.47	−3.77	−1.95	−2.82
100 m	−22.61	2.29	−19.56	3.21	−30.98	7.45	−30.68	1.78	82.33	−19.62	28.08	−3.32	−17.63	−2.75	−9.17	−3.38	−0.96	−2.61
120 m	−21.61	1.76	−16.98	2.36	−23.00	5.26	−22.95	1.56	73.32	−20.76	39.13	−3.36	−8.79	−1.98	−5.24	−2.46	−0.97	−1.94
140 m	−20.64	1.39	−14.41	1.61	−18.57	2.74	−10.18	1.10	84.86	−21.34	49.98	−2.26	−1.75	−1.31	−3.93	−1.64	0	−1.32
160 m	−19.66	0.80	−13.09	1.01	−9.73	1.89	−4.56	0.78	81.62	−22.09	53.80	−1.65	8.85	−0.95	1.31	−1.01	−0.98	−0.82
200 m	−16.71	0.24	−2.62	0.28	6.19	0.44	16.79	0.31	75.18	−17.32	57.45	−0.59	20.33	−0.30	9.17	−0.28	−1.97	−0.23
240 m	−10.81	−0.30	5.24	−0.41	12.38	−0.73	10.52	−0.56	43.25	13.82	33.51	0.93	16.80	0.50	14.41	0.37	0.98	0.29
280 m	−5.90	−0.60	9.17	−0.80	14.15	−1.68	5.40	−1.49	21.16	14.63	23.09	2.73	12.41	1.22	13.10	0.75	0.98	0.56
320 m	−4.91	−0.67	7.86	−0.81	9.75	−1.96	−2.21	−2.61	−3.68	12.09	11.88	7.09	9.95	1.40	10.48	0.85	0.98	0.63
400 m	−3.93	−0.65	5.25	−0.83	4.53	−2.07	−12.07	−5.03	−29.72	6.27	0.33	−13.09	8.71	1.59	5.24	0.81	−1.97	0.63
480 m	−3.93	−0.61	3.93	−0.81	3.18	−1.76	−14.06	−18.05	−45.25	3.45	−5.22	−3.76	6.25	1.60	2.62	0.77	−2.95	0.60
560 m	−4.91	−0.52	1.31	−0.67	1.08	−1.32	−8.48	6.75	−42.53	−6.48	0.36	−5.12	8.07	1.12	1.31	0.65	−3.93	0.51
640 m	−3.93	−0.42	0	−0.55	0.92	−0.87	−1.21	2.82	−44.50	−8.00	4.10	−2.62	9.75	0.86	0	0.52	−3.93	0.42
800 m	−2.95	−0.16	−1.31	−0.21	0.06	−0.30	2.62	3.60	−30.05	−9.83	4.39	−3.23	7.12	0.31	0	0.20	−1.97	0.16
960 m	−0.98	−0.03	0	−0.04	0	−0.06	0.15	−0.18	−9.43	−4.18	0.15	0.18	1.77	0.06	0	0.04	0	0.03
1120 m	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0

Table 1. (Continued)

	5°		6°		7°		8°		9°		10°		11°		12°		13°		14°N	
	u	v	u	v	u	v	u	v	u	v	u	v	u	v	u	v	u	v	u	v
0 m	7.89	−3.60	27.57	−2.82	34.89	−2.21	24.14	−1.84	7.33	−1.71	−7.11	−1.51	−13.66	−1.41	−11.87	−1.32	−8.84	−1.21	−11.90	−1.12
20 m	10.24	−3.30	30.85	−2.58	36.57	−2.01	24.64	−1.68	6.42	−1.56	−7.90	−1.38	−14.74	−1.29	−12.86	−1.21	−9.45	−1.11	−12.19	−1.03
40 m	13.39	−2.89	33.46	−2.24	38.26	−1.74	24.14	−1.45	5.50	−1.36	−8.29	−1.20	−15.09	−1.13	−13.19	−1.05	−9.75	−0.96	−12.47	−0.90
60 m	15.74	−2.57	35.43	−1.98	38.26	−1.54	23.16	−1.29	4.13	−1.21	−8.69	−1.08	−14.74	−1.01	−13.52	−0.94	−10.06	−0.86	−12.75	−0.80
80 m	18.10	−2.30	38.05	−1.74	36.57	−1.35	19.22	−1.15	2.29	−1.09	−8.29	−0.96	−14.02	−0.89	−13.19	−0.83	−10.06	−0.77	−12.19	−0.71
100 m	18.89	−2.10	34.77	−1.59	31.51	−1.25	15.77	−1.07	1.38	−1.02	−6.71	−0.90	−12.22	−0.83	−12.53	−0.77	−10.06	−0.71	−11.90	−0.66
120 m	15.73	−1.59	30.83	−1.21	25.88	−0.96	9.36	−0.83	0.46	−0.80	−3.55	−0.70	−9.70	−0.65	−11.54	−0.59	−9.45	−0.55	−10.77	−0.51
140 m	12.59	−1.06	23.62	−0.83	18.00	−0.67	5.91	−0.59	0	−0.56	−2.76	−0.48	−8.63	−0.44	−9.89	−0.41	−7.92	−0.38	−9.35	−0.35
160 m	8.65	−0.68	13.78	−0.54	10.13	−0.44	3.94	−0.39	0	−0.37	−2.37	−0.32	−6.83	−0.29	−8.25	−0.27	−7.01	−0.25	−7.94	−0.23
200 m	−0.79	−0.18	3.94	−0.16	3.38	−0.14	1.48	−0.12	0	−0.11	−1.18	−0.09	−4.31	−0.09	−5.61	−0.08	−4.27	−0.07	−4.82	−0.07
240 m	−3.93	0.24	0.66	0.21	1.69	0.17	0.99	0.15	0.46	0.14	−0.39	0.12	−2.88	0.11	−3.63	0.10	−2.44	0.09	−3.12	0.09
280 m	−3.93	0.48	0	0.41	0.56	0.34	0.49	0.30	0.46	0.28	0.39	0.24	−2.16	0.22	−2.31	0.20	−1.22	0.19	−2.27	0.17
320 m	−3.15	0.53	−0.66	0.45	0.56	0.38	0.49	0.34	0.46	0.31	0.39	0.27	−1.80	0.24	−1.98	0.22	−0.61	0.21	−1.13	0.19
400 m	−3.15	0.52	0	0.44	0.56	0.37	1.48	0.33	0.92	0.30	0.39	0.26	−1.08	0.24	−1.32	0.22	0.30	0.20	0.28	0.19
480 m	−2.36	0.50	0	0.41	0.56	0.35	1.48	0.31	0.46	0.28	0	0.24	−1.08	0.22	−0.99	0.20	0.61	0.19	0.57	0.18
560 m	−1.57	0.42	0	0.35	0.56	0.30	1.48	0.26	0.46	0.24	0.39	0.21	−0.72	0.19	−0.99	0.17	0.61	0.16	0.57	0.15
640 m	−1.57	0.35	0	0.28	0.56	0.24	0.99	0.21	0.46	0.20	0.39	0.17	−0.36	0.15	−0.66	0.14	0.30	0.13	0.28	0.12
800 m	−0.79	0.13	0	0.10	0	0.09	0.49	0.08	0.46	0.07	0	0.06	−0.36	0.06	−0.33	0.05	0	0.05	0	0.05
960 m	0	0.02	0	0.02	0	0.02	0	0.01	0	0.01	0	0.01	0	0.01	0	0.01	0	0.01	0	0.01
1120 m	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0

guesses are $u=v=0$ except at the Equator ($\varphi=0$), 1°, 2° and 3°N and S, and the outer boundaries (14.5°N, 4.5°S), where the flow is considered to be geostrophic, the first approximations will give geostrophic flows for all latitudes in between.

As a second step,

$$\left(\frac{\partial u}{\partial y}\right)_{\varphi}=\frac{u(\varphi+1°)-u(\varphi-1°)}{2\times1.112\times10^7},$$

$$\left(\frac{\partial v}{\partial y}\right)_{\varphi}=\frac{v(\varphi+1°)-v(\varphi-1°)}{2\times1.112\times10^7}$$

were computed and the results multiplied by $-v$, thus evaluating the quantities:

$$-v\frac{\partial u}{\partial y} \text{ and } -v\frac{\partial v}{\partial y}$$

These two terms were added to $\partial D/\partial x$ and $\partial D/\partial y$ respectively, thus the result, divided by $2\omega\sin\varphi$, giving second approximations for u and v. This process was hardly repeated three or four times before the final, steady values were attained for the successive pairs. The convergence of iteration was better as we go away from the Equator.

This process failed for the interval $2°\text{S}\leqq\varphi\leqq2°\text{N}$, because the factor $2\omega\sin\varphi$ becomes too small there. 2°N and S seem to be the limits of internal boundaries for this iteration process. This is the reason why the author had to compute the flow at and around the Equator in spite of a big labor. The contents of Table 1, for the latitudes higher than 3°N and S, were exclusively computed by this process.

In Table 7 are given the percentage differences between the geostrophic currents computed from the equations (41) and the corresponding components in Table 1 computed taking the inertia terms into account. A satisfactory agreement at latitudes more than a few degrees away from the Equator means that the inertia terms have very little influence on the computation of the ocean currents except at and close to the Equator.

9. Discussions of the result

i) *Equatorial Undercurrent (Cromwell Current) and Equatorial Countercurrent*

The velocity components computed from the meridional distribution of pressure gradients and compiled in Table 1 are pronounced in the east-west direction, remarkable meridional flow being seen only within a few degrees from the Equator.

The diagrams in Figs. 1 and 2 give the meridional distribution of the components u and v. The variation of u is particularly remarkable in the layers above 500 m. We can see the Equatorial Undercurrent or Cromwell Current exactly at the Equator and the Equatorial Countercurrent around 6.5°N.

The Equatorial Undercurrent has a core around 100 m to 140 m attaining a speed about 85 cm/sec. This seems to be a little below observed values. It is only seen in a zone between 1°S and 1.30°N. The result that the surface also moves to the east at the Equator seems contrary to observed facts in connection with the equatorial flow. This may be because we have neglected the wind stresses at the sea surface. Thus we may have to add westward drift currents of EKMAN's type which, however, is very hard to either estimate or compute.

Below the Equatorial Undercurrent we notice a slower westward flow less than 50cm/sec. This flow centers around 400–600m layers. The speed of this westward flow seems a little higher than observed values.

The computed Equatorial Countercurrent of a velocity a little more than 40 cm/sec is located at the latitudes 5° to 8°N, the core of this current being seen at 6.5°N and 50m layer. This location and speed are in good agreement with observations. No distinct connection is seen between the Equatorial Countercurrent and Equatorial Undercurrent.

The westward flows in the South Equatorial Current has velocity of 45 and 70 cm/sec on both sides of the Equator respectively. These are regarded as geostrophic in most cases.

ii) *Upwelling and vertical circulation in a meridional section*

In Table 2, are compiled the upwelling velocities computed by the formula (42) for several latitueds and depths. The upwelling is most intense in layers 200–250 m the velocity exceeding -14×10^{-3} cm/sec. Intense vertical motion down in deeper layers are

Table 2. Computed upwelling and sinking. (vertical velocity component w in 10^{-3} cm/sec)

	3°S	2°	1°S	0°	1°N	2°	3°	4°	5°	6°	7°	8°	9°	10°	11°	12°	13°N
0 m	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
20 m	-1.896	0.212	3.674	0.712	-0.616	0.111	-0.443	-0.238	-0.139	-0.121	-0.085	-0.043	-0.028	-0.026	-0.016	-0.017	-0.017
40 m	-3.604	0.404	7.214	1.337	-1.573	0.209	-0.675	-0.445	-0.267	-0.230	-0.161	-0.080	-0.053	-0.048	-0.031	-0.033	-0.032
60 m	-4.806	0.574	10.300	1.893	-2.791	0.276	-0.774	-0.616	-0.380	-0.328	-0.227	-0.112	-0.074	-0.067	-0.044	-0.047	-0.045
80 m	-5.578	0.717	13.003	2.430	-4.157	0.292	-0.833	-0.759	-0.482	-0.417	-0.285	-0.138	-0.092	-0.165	-0.056	0.021	-0.057
100 m	-6.147	0.848	15.547	2.914	-5.643	0.301	-0.864	-0.883	-0.576	-0.498	-0.335	-0.161	-0.108	-0.183	-0.067	0.010	-0.067
120 m	-6.536	0.948	17.934	3.365	-7.246	0.263	-0.872	-0.980	-0.655	-0.565	-0.375	-0.178	-0.121	-0.198	-0.078	0	-0.076
140 m	-6.754	1.007	20.187	3.737	-8.991	0.195	-0.873	-1.045	-0.710	-0.611	-0.403	-0.190	-0.132	-0.210	-0.086	-0.007	-0.032
160 m	-6.864	1.040	22.348	3.997	-10.843	0.138	-0.879	-1.086	-0.745	-0.639	-0.420	-0.198	-0.140	-0.219	-0.092	-0.012	-0.086
200 m	-6.980	1.058	26.101	4.297	-14.274	0.053	-0.897	-1.125	-0.776	-0.664	-0.438	-0.207	-0.149	-0.228	-0.097	-0.017	-0.091
240 m	-6.960	1.069	26.390	4.234	-14.607	0.075	-0.884	-1.122	-0.775	-0.661	-0.436	-0.207	-0.149	-0.228	-0.096	-0.017	-0.091
280 m	-6.824	1.144	23.615	3.730	-12.203	0.304	-0.806	-1.036	-0.755	-0.643	-0.420	-0.199	-0.141	-0.219	-0.091	-0.013	-0.087
320 m	-6.611	1.368	20.884	2.478	-10.036	1.043	-0.677	-1.033	-0.725	-0.616	-0.401	-0.188	-0.174	-0.208	-0.083	-0.007	-0.082
400 m	-6.123	2.447	16.857	2.183	-7.272	-0.335	-0.366	-0.923	-0.658	-0.563	-0.361	-0.162	-0.104	-0.184	-0.061	0.005	-0.071
480 m	-5.661	6.304	14.420	1.063	-6.097	-3.650	-0.013	-0.822	-0.590	-0.509	-0.323	-0.137	-0.079	-0.163	-0.052	0.018	-0.062
560 m	-5.310	8.070	14.411	0.628	-7.131	-5.502	0.276	-0.732	-0.527	-0.460	-0.289	-0.114	-0.058	-0.143	-0.038	0.029	-0.055
640 m	-5.085	6.129	16.622	3.741	-10.092	-7.105	0.465	-0.661	-0.473	-0.419	-0.260	-0.096	-0.041	-0.125	-0.025	0.038	-0.048
800 m	-4.873	3.546	22.615	8.155	-16.926	-9.468	0.677	-0.574	-0.401	-0.365	-0.228	-0.074	-0.020	-0.103	-0.011	0.049	-0.040
960 m	-4.812	2.226	27.525	10.482	-22.099	-10.652	0.742	-0.542	-0.376	-0.350	-0.217	-0.063	-0.013	-0.100	-0.007	0.052	-0.040
1120 m	-4.801	2.277	29.007	10.353	-23.624	-10.601	0.753	-0.535	-0.372	-0.350	-0.214	-0.060	-0.013	-0.100	-0.007	0.052	-0.040

Table 3. Computed stream function ψ (y, z) in cm^2/sec

	4° S	3°	2°	1° S	0°	1° N	2°	3°	4°	5°	6°	7°	8°	9°	10°	11°	12°	13°	14° N
0 m	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
20 m	7460	10820	49630	6110	−32070	−9730	−18360	−12200	−8500	−6900	−5400	−4220	−3520	−3270	−2890	−2700	−2530	−2320	−2150
40 m	14140	20390	94300	11400	−66140	−18340	−31160	−22980	−16150	−13090	−10220	−7970	−6650	−6190	−5470	−5120	−4790	−4390	−4080
60 m	20020	28740	126900	15970	−102180	−26120	−40110	−32260	−22890	−18550	−14440	−11250	−9390	−8760	−7750	−7260	−6780	−6210	−5780
80 m	25260	36180	149320	20240	−139860	−33810	−47400	−40310	−28880	−23420	−18160	−14140	−11830	−11060	−9790	−7380	−8550	−7840	−7290
100 m	30020	42890	166730	24040	−179030	−40770	−53520	−47460	−34310	−27820	−21490	−16740	−14050	−13170	−11650	−9100	−10150	−9320	−8660
120 m	34070	48460	179440	27380	−219410	−47450	−58250	−53300	−38860	−31510	−24290	−18950	−15950	−14990	−13250	−10580	−11510	−10580	−9830
140 m	37220	52430	187440	30040	−261510	−53070	−61540	−57400	−42120	−34160	−26330	−20580	−17370	−16350	−14430	−11670	−12510	−11510	−10690
160 m	39410	55050	192070	31920	−304940	−56980	−63800	−60050	−44260	−35900	−27700	−21690	−18350	−17280	−15230	−12400	−13190	−12140	−11270
200 m	41490	57630	196730	34100	−383760	−61460	−66300	−62630	−46360	−37620	−29100	−22850	−19370	−18240	−16050	−13160	−13890	−12780	−11870
240 m	41370	57370	196150	33600	−390760	−60780	−65900	−62450	−46240	−37500	−29000	−22790	−19310	−18180	−15990	−13120	−13850	−12740	−11830
280 m	39570	54950	191330	29500	−333860	−53460	−62460	−60210	−44540	−36060	−27760	−21770	−18410	−17340	−15270	−12460	−13250	−12180	−11310
320 m	37030	51730	184050	21300	−280420	−33820	−57220	−57010	−42160	−34040	−26040	−20330	−17130	−16160	−14250	−11540	−12410	−11380	−10590
400 m	31750	45170	167930	−9260	−206980	−57820	−45260	−50370	−37120	−29840	−22480	−17330	−14450	−13720	−12130	−9620	−10650	−9740	−9070
480 m	26710	38610	152610	−101580	−168100	−125220	−32500	−44050	−32200	−25760	−19080	−14450	−11830	−11400	−10130	−7780	−8970	−8180	−7590
560 m	22190	32690	140290	−146780	−180220	−160740	−21620	−38370	−27760	−22080	−16040	−11850	−9610	−9320	−8330	−6140	−7490	−6780	−6270
640 m	18430	27810	131530	−108500	−238140	−191700	−13700	−33690	−24040	−19000	−13520	−9690	−7730	−7560	−6810	−4780	−6250	−5620	−5190
800 m	13790	21730	122170	−57140	−380780	−238500	−4340	−27930	−19400	−15160	−10480	−7050	−5410	−5400	−4970	−3100	−4730	−4180	−3830
960 m	12270	19730	119290	−29780	−492860	−262900	−1380	−26010	−17880	−13960	−9520	−6170	−4690	−4760	−4410	−2540	−4250	−3700	−3350
1120 m	12030	19410	118810	−31220	−526300	−261460	−900	−25690	−17640	−13800	−9360	−6010	−4610	−4680	−4330	−2460	−4170	−3620	−3270

quite doubtful.

Table 3 gives the stream function computed from the formula (50). The contents of this table are more visualized in the diagram of Fig. 3, indicated by stream lines in the meridional section. We notice, against observations, that the water is sinking around the Equator, giving equatorial convergence. This will be because we neglected the wind drift at and around the Equator. We may thus conclude that wind will be responsible for most part of the actually observed equatorial divergence.

Fig. 1. East-west component. (cm/s)

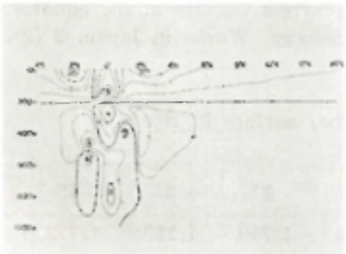

Fig. 2. North-south component. (cm/s)

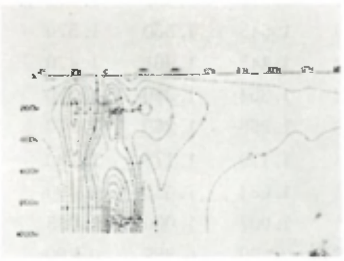

Fig. 3. Meridional circulation (stream lines).

iii) *Merit of inertia terms in interpreting the east-west flow*

So far it seems to the author that the inertia terms

$$-v\frac{\partial u}{\partial y} \quad \text{and} \quad -v\frac{\partial v}{\partial y}$$

greatly improves the explanation of the equatorial flow, thus giving us an idea that field accelerations are necessary factors for interpreting the Cromwell Current or the Equatorial Undercurrent in the Pacific. There are some other factors in hydrodynamical equations of the ocean currents, which may balance the pressure gradients at the Equator. However, there seem to exist many difficulties for us in treating the problem by regarding them as the probable agencies for the explanation of the equatorial flow. Thus the discussions for them will be left for future studies.

10. Summary and conclusions

1. A simple model was considered for the equatorial current system on the assumption that pressure gradients and current velocity do not change in the east-west direction.

2. A balance between the Coriolis forces, pressure gradients and field accelerations was assumed, the terms $-w(\partial u/\partial z)$ and $-w(\partial v/\partial z)$ being neglected as small compared with $-v(\partial u/\partial y)$ and $-v(\partial v/\partial y)$ respectively. The friction due to mixing and thermodynamic process were disregarded.

3. The solution was carried out for a dynamic meridional section, first computing the velocity components at the Equator, then extending calculation to higher latitudes of both hemispheres by different procedures.

4. An attempt was made to interpret the significance of HIDAKA-TSUCHIYA's formula of computing the east-west equatorial velocity.

5. The Equatorial Undercurrent can be located as an equatorial jet stream of a speed about 85 cm/sec confind to a narrow band between 1°S to 1.5°N.

6. The location and size of Equatorial Countercurrent agree with observations quite well.

7. It can be shown theoretically that the east-west component of velocity is approximately geostrophic even in the 1° intervals next to the Equator.

8. Upwelling velocities were determined and found to be an order of 10^{-3} cm/sec.

9. Vertical circulation in a meridional section was determined by constructing the stream function between v and w.

10. A brief discussion was made as to the merit of the inertia terms in hydrodynamical equations of an equational flow.

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Table 4. Anomalies of geopotential distances D over 1000 decibar surface in 10^5c.g.s. (Data from POFI S.S.R. No. 54, 131, 135 averaged)

dep. \ lat.	5° S	4°	3°	2°	1° S	0°	1° N	2°	3°	4°
0 m	1.822	1.791	1.760	1.755	1.713	1.721	1.704	1.740	1.753	1.772
20 m	1.727	1.697	1.667	1.661	1.622	1.630	1.613	1.643	1.656	1.672
40 m	1.632	1.600	1.572	1.568	1.530	1.541	1.520	1.545	1.560	1.570
60 m	1.536	1.506	1.480	1.475	1.440	1.450	1.430	1.447	1.461	1.470
80 m	1.439	1.414	1.387	1.383	1.350	1.360	1.339	1.354	1.366	1.370
100 m	1.346	1.320	1.295	1.291	1.260	1.272	1.249	1.258	1.269	1.273
120 m	1.255	1.230	1.206	1.205	1.180	1.190	1.168	1.170	1.178	1.180
140 m	1.167	1.140	1.120	1.118	1.099	1.110	1.086	1.084	1.088	1.090
160 m	1.080	1.060	1.036	1.040	1.025	1.035	1.010	1.007	1.000	1.005
200 m	0.950	0.923	0.913	0.922	0.920	0.930	0.908	0.899	0.885	0.880
240 m	0.854	0.831	0.831	0.846	0.845	0.851	0.838	0.833	0.819	0.804
280 m	0.779	0.762	0.765	0.782	0.781	0.784	0.774	0.772	0.760	0.745
320 m	0.722	0.707	0.711	0.724	0.722	0.722	0.717	0.716	0.706	0.695
400 m	0.612	0.599	0.604	0.613	0.609	0.606	0.607	0.606	0.598	0.593
480 m	0.514	0.501	0.505	0.510	0.507	0.502	0.506	0.505	0.499	0.498
560 m	0.422	0.410	0.412	0.415	0.413	0.410	0.415	0.410	0.406	0.408
640 m	0.335	0.324	0.326	0.326	0.327	0.325	0.330	0.323	0.319	0.323
800 m	0.177	0.170	0.170	0.169	0.170	0.170	0.174	0.168	0.166	0.168
960 m	0.035	0.034	0.034	0.034	0.034	0.034	0.035	0.034	0.033	0.034
1120 m	0	0	0	0	0	0	0	0	0	0
1280 m	0	0	0	0	0	0	0	0	0	0

Table 4. (Continued)

dep. \ lat.	5°	6°	7°	8°	9°	10°	11°	12°	13°	14°	15°N
0 m	1.784	1.754	1.686	1.605	1.570	1.570	1.613	1.663	1.693	1.721	1.787
20 m	1.681	1.646	1.573	1.491	1.457	1.460	1.504	1.557	1.591	1.619	1.686
40 m	1.579	1.538	1.460	1.375	1.346	1.350	1.394	1.450	1.487	1.514	1.585
60 m	1.476	1.430	1.349	1.267	1.240	1.250	1.291	1.347	1.385	1.412	1.485
80 m	1.376	1.324	1.238	1.168	1.150	1.160	1.198	1.253	1.290	1.320	1.385
100 m	1.277	1.223	1.150	1.090	1.078	1.087	1.115	1.168	1.204	1.235	1.297
120 m	1.184	1.138	1.069	1.027	1.030	1.029	1.045	1.094	1.127	1.159	1.210
140 m	1.093	1.055	1.004	0.979	0.980	0.980	0.990	1.037	1.062	1.090	1.135
160 m	1.008	0.980	0.957	0.940	0.940	0.942	0.950	0.987	1.009	1.033	1.070
200 m	0.893	0.885	0.876	0.872	0.871	0.872	0.874	0.900	0.914	0.927	0.952
240 m	0.817	0.818	0.814	0.812	0.811	0.811	0.810	0.831	0.838	0.847	0.863
280 m	0.758	0.759	0.757	0.756	0.755	0.755	0.751	0.770	0.770	0.774	0.786
320 m	0.705	0.706	0.705	0.704	0.702	0.702	0.697	0.714	0.714	0.715	0.722
400 m	0.604	0.603	0.603	0.601	0.597	0.598	0.592	0.606	0.604	0.601	0.602
480 m	0.508	0.505	0.508	0.504	0.499	0.502	0.496	0.508	0.505	0.502	0.501
560 m	0.416	0.413	0.416	0.412	0.408	0.411	0.405	0.416	0.413	0.411	0.410
640 m	0.329	0.327	0.328	0.326	0.323	0.325	0.321	0.329	0.327	0.326	0.326
800 m	0.172	0.170	0.172	0.171	0.169	0.170	0.169	0.171	0.172	0.171	0.172
960 m	0.034	0.034	0.034	0.034	0.034	0.034	0.034	0.034	0.034	0.034	0.034
1120 m	0	0	0	0	0	0	0	0	0	0	0
1280 m	0	0	0	0	0	0	0	0	0	0	0

Table 5. Computation of the equatorial velocity components.

depth	d_0, δ_0 and δ_1 at the Equator			Equatorial velocity components			
				case $n=2$		case $n=3$	
	d_0	δ_0	δ_1	u_0	v_0	u_0	v_0
0 m	-3.96×10^{-5}	-4.050×10^{-5}	-0.2025×10^{-10}	79.25	−27.90	39.46	−15.91
20 m	−3.63 〃	−4.050 〃	−0.2025 〃	78.41	−26.71	41.55	−16.16
40 m	−3.16 〃	−4.500 〃	−0.2592 〃	100.63	−26.51	68.04	−17.91
60 m	−2.82 〃	−4.500 〃	−0.2430 〃	91.57	−24.64	63.01	−18.13
80 m	−2.51 〃	−4.950 〃	−0.2511 〃	90.20	−23.44	67.13	−19.55
100 m	−2.33 〃	−4.950 〃	−0.2835 〃	104.82	−23.28	82.33	−19.62
120 m	−1.79 〃	−5.400 〃	−0.2592 〃	81.22	−19.95	73.32	−20.76
140 m	−1.24 〃	−5.850 〃	−0.2835 〃	72.00	−16.98	84.86	−21.34
160 m	−0.81 〃	−6.749 〃	−0.2835 〃	18.19	−13.73	81.62	−22.09
200 m	−0.24 〃	−5.400 〃	−0.2592 〃	−125.41	− 7.31	75.18	−17.32
240 m	0.31 〃	−3.150 〃	−0.1539 〃	− 14.36	− 7.29	43.25	13.82
280 m	0.61 〃	−3.150 〃	−0.1053 〃	− 4.13	− 9.30	21.16	14.63
320 m	0.68 〃	−2.250 〃	−0.0405 〃	− 19.30	− 7.73	− 3.68	12.09
400 m	0.66 〃	−0.900 〃	0.0324 〃	—	—	−29.72	6.27
480 m	0.62 〃	−0.450 〃	0.0729 〃	—	—	−45.25	− 3.45
560 m	0.53 〃	0.900 〃	0.0648 〃	—	—	−42.53	− 6.48
640 m	0.43 〃	1.350 〃	0.0567 〃	—	—	−44.50	− 8.00
800 m	0.16 〃	1.800 〃	0.0324 〃	—	—	−30.05	− 9.83
960 m	0.03 〃	0.450 〃	0.0081 〃	—	—	− 9.43	− 4.18

Table 6. Horizontal pressure gradients $\partial D/\partial x$, $-\partial D/\partial y$. ($\times 10^{-5}$gr. cm/sec)

	$\frac{\partial D}{\partial x}$	4° S	3°	2°	1° S	0°	1° N	2°	3°	4°
0 m	−3.96	28	16.20	21.15	15.30	4.05	−8.55	−22.05	−14.40	−11
20 m	−3.63	27	16.20	20.25	13.95	4.05	−5.85	−19.35	−13.05	− 8
40 m	−3.16	27	14.40	18.90	12.15	4.45	−1.80	−18.00	−11.25	− 6
60 m	−2.82	25	13.95	18.00	11.25	4.45	1.35	−13.95	−10.35	− 4
80 m	−2.51	23	13.95	16.65	10.35	4.95	2.70	−12.15	− 7.20	− 2
100 m	−2.33	23	13.05	15.75	8.55	4.95	6.30	− 9.00	− 6.75	− 1
120 m	−1.79	22	11.25	11.70	6.75	5.40	9.00	− 4.50	− 4.50	− 1
140 m	−1.24	21	9.90	9.45	3.60	5.85	11.70	0.90	− 2.70	0
160 m	−0.81	20	9.00	4.95	2.25	6.75	12.60	4.50	0.90	− 1
200 m	−0.24	17	0.45	−3.15	−3.60	5.40	13.95	10.35	8.55	− 2
240 m	0.31	11	−6.75	−6.30	−2.25	3.15	8.10	8.55	13.05	1
280 m	0.61	6	−9.00	−7.20	−0.90	3.15	5.40	6.30	12.15	1
320 m	0.68	5	−7.65	−4.95	0.90	2.25	2.70	4.95	9.45	1
400 m	0.66	4	−6.30	−2.25	3.15	0.90	0	4.05	5.85	− 2
480 m	0.62	4	−4.05	−0.90	3.60	0.45	−1.35	3.15	3.15	− 3
560 m	0.53	5	−2.25	−0.45	2.25	−0.90	0	4.05	0.90	− 4
640 m	0.43	4	−0.90	−0.45	0.45	−1.35	0.90	4.95	0	− 4
800 m	0.16	3	0.45	0	0.45	−1.80	0.90	3.60	0	− 2
960 m	0.03	1	0	0	0	−0.45	0	0.90	0	0
1120 m	0	0	0	0	0	0	0	0	0	0

	5°	6°	7°	8°	9°	10°	11°	12°	13°	14° N
0 m	10	42	62	49	16	−18	−38	−36	−29	−42
20 m	13	47	65	50	14	−20	−41	−39	−31	−43
40 m	17	51	68	49	12	−21	−42	−40	−32	−44
60 m	20	54	68	47	9	−22	−41	−41	−33	−45
80 m	23	58	65	39	5	−21	−39	−40	−33	−43
100 m	24	53	56	32	3	−17	−34	−38	−33	−42
120 m	20	47	46	19	1	− 9	−27	−35	−31	−38
140 m	16	36	32	12	0	− 7	−24	−30	−26	−33
160 m	11	21	18	8	0	− 6	−19	−25	−23	−28
200 m	− 1	6	6	3	0	− 3	−12	−17	−14	−17
240 m	− 5	1	3	2	1	− 1	− 8	−11	− 8	−11
280 m	− 5	0	1	1	1	1	− 6	− 7	− 4	− 8
320 m	− 4	− 1	1	1	1	1	− 5	− 6	− 2	− 4
400 m	− 4	0	1	3	2	1	− 3	− 4	1	1
480 m	− 3	0	1	3	1	0	− 3	− 3	2	2
560 m	− 2	0	1	3	1	1	− 2	− 3	2	2
640 m	− 2	0	1	2	1	1	− 1	− 2	1	1
800 m	− 1	0	0	1	1	0	− 1	− 1	0	0
960 m	0	0	0	0	0	0	0	0	0	0
1120 m	0	0	0	0	0	0	0	0	0	0

Table 7. Percentage differences between the geostrophic and present computation of the east-west velocity components.

	3° S	2°	1° S	0°	1° N	2°	3°	4°	5°	6°	7° N
0 m	−3.0	6.3	7.6	—	− 8.5	15.8	−1.2	−0.6	0.3	0.1	0
20 m	−2.7	7.2	5.1	—	− 5.8	13.5	−0.7	−0.1	0.1	0.1	0
40 m	−2.0	5.5	2.8	—	(−62.1)*	19.8	−0.3	−0.5	0.1	0	0
60 m	−1.1	−12.8	− 1.3	—	0	15.9	−0.1	−0.8	0.1	0	0
80 m	−0.8	− 4.2	−11.7	—	(253.4)*	21.2	−0.1	−0.1	0.1	0	0
100 m	−0.5	5.1	−13.2	—	78.6	28.2	0	−2.0	0.1	0	0
120 m	−0.3	6.4	−16.6	—	42.2	49.2	0	−1.0	0	0	0
140 m	0	5.0	−48.2	—	27.2	0	0	0	0	0	0
160 m	0.1	− 0.9	(−61.3)*	—	24.5	9.9	0	0	0	0	0
200 m	0	5.1	(113.6)*	—	12.5	− 5.9	0	0	0	0	0
240 m	0	5.0	33.8	—	6.6	− 5.0	0	0	0	0	0
280 m	0	2.9	37.4	—	17.5	− 9.7	0	0	0	0	0
320 m	0	− 0.7	0	—	0.8	1.3	0	0	0	0	0
400 m	0.2	15.3	2.4	—	91.6	10.8	0	0	0	0	0
480 m	0	0	−19.3	—	32.8	59.0	0	0	0	0	0
560 m	0	0	7.9	—	0	37.0	0	0	0	0	0
640 m	0	0	0	—	4.3	24.0	0	0	0	0	0
800 m	0	0	33.3	—	11.7	20.9	0	0	0	0	0
960 m	0	0	0	—	0	− 9.7	0	0	0	0	0

* Figures in brackets seem uncertain, because of low speeds.