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A Stochastic Model for Purse Seining in a Two-species Fishery

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Purse seine fishing in a two-species fishery is viewed as a semi-Markov process. Activities of a vessel during a fishing day are assigned to five states: searching, successfully setting on either species, and unsuccessfully setting on either species. Searching for fish schools is assumed to be a Poisson process. Transition probabilities are defined in terms of species densities in the fishing area, the chance that a sighted school is captured, and the chance of relocating an escaped school. Waiting time in the search state is determined by school density in the fishing area and search rate of the vessel. Waiting times in the remaining states depend on numerous factors such as vessel characteristics and weather. With results from renewal theory, expectations of the number of successful sets on each species during a time interval of arbitrary length are approximated. Numerical comparison with exact results from a simpler fishing model indicates the approximation from renewal theory for the expectations to be excellent. Several examples are given to demonstrate the model's utility. It can be used to develop abundance measures for the two species which account for temporal changes in efficiency of the vessels, dead time after a school is encountered while the vessel is not searching, and the fact that two species are being exploited simultaneously.

1. Introduction

1.1. THE PURSE SEINING PROCESS FOR TUNA

The purse seine fishery in the eastern tropical Pacific Ocean primarily exploits the yellowfin and skipjack populations. These tunas occur in schools near and at the ocean surface in at least a coastal zone extending from the Baja California peninsula to Peru and Chile. Densities of the two species vary temporally and geographically from time-and-area strata containing one species alone, to others containing mixtures of the species.

A purse seiner seeks strata in which tuna are expected to be abundant. Upon entering an area the vessel travels along searching for fish, usually during only the daylight hours. When a school of significant size is sighted, the vessel attempts to encircle it with a net. The school may be captured by the set or escape. The term 'set' used in this paper refers to the activity of a vessel when attempting to capture a sighted school of fish. The time spent in a set depends, among other things, on success or failure of the set, size of the school captured (Bayliff & Orange, 1967), and probably on multifarious factors such as weather, ocean conditions, the vessel crew and vessel characteristics such as speed, net length and size. If the school escapes, the vessel retrieves the net and pursues the school, provided it is sighted again. Otherwise the search for a new school begins unless there is insufficient light to continue. If a school is captured, the vessel remains in the set until the catch is on board even though nightfall is past. Purse seining in this tuna fishery is conducted, roughly speaking, in the manner described above but has many variations not mentioned. The actual fishing process probably does not deviate sufficiently from the idealized version to invalidate the stochastic model of purse seining developed on the basis of this conception of the process.

1.2. MOTIVATION FOR THE MODEL

In a general sense motivation for development of a stochastic model of the fishing process known as purse seining stems from two observations. First, there is a need to develop improved indices of the density of fish stocks, for fisheries in which gear is inactive for significant portions of the fishing day while engaged with the catch. The usual density measure is catch per unit time, where time comprises both active and inactive periods. Such measures do not index truly the abundance of fish, since proportions of active and inactive time change with density. Second, there is increasing awareness that in multiple-species fisheries currently-employed abundance measures are inadequate. Rothschild (1967) has developed improved population density measures for a multiple-species long line fishery, but little else has been accomplished in this direction.

Specifically, this model was developed because of a need for improved measures of yellowfin and skipjack abundance in the eastern tropical Pacific Ocean. The monitoring of these tunas' population densities from data of the baitboat and purse seine fleet shares the above-mentioned difficulties. This study was precipitated, however, on discovering that the efficiency of purse seine vessels generally appeared to be increasing. Improvements in the purse seine fleet which have come to our attention are two: a trend to reduction of time spent in sets, i.e. inactive time; and a trend to increasing probability for vessels to capture schools of tuna once sighted.

As evidence for the reduction of time in sets, we present here two figures taken from a report by Bayliff & Orange (1967). They sampled from log book records of IATTC (Inter-American Tropical Tuna Commission) paired observations of time spent in a set on either species, and the catch size. Time spent in a set for either species seemed to be a linear function of the catch size. Hence the linear regression equations illustrated in Figs 1 and 2

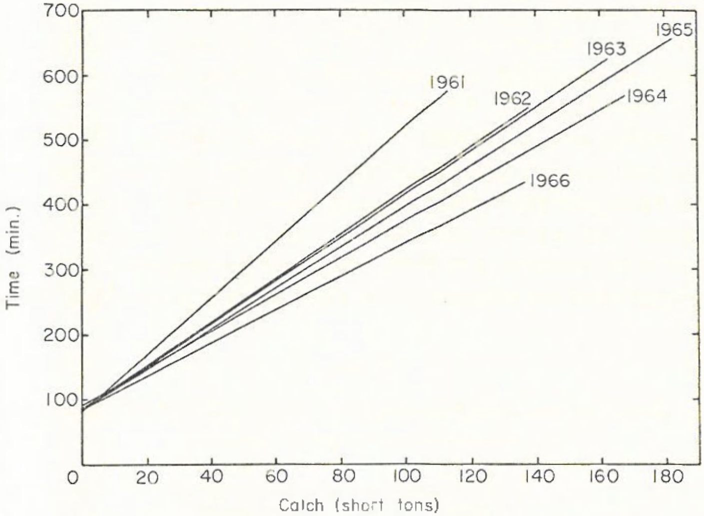

FIG. 1. Regressions for the years 1961–1966 of the durations of the sets on the quantities caught in them for skipjack.

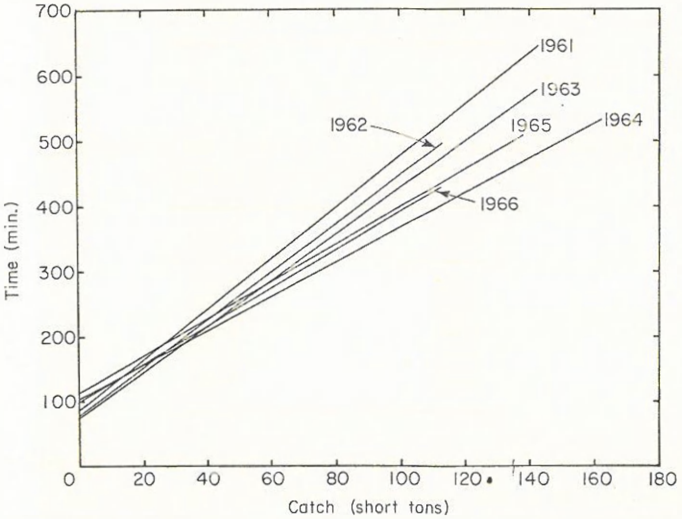

FIG. 2. Regressions for the years 1961–1966 for the durations of the sets on the quantities caught in them for yellowfin.

were computed for the two species. Analysis of covariance indicated the apparent differences between years were highly significant ($P < 0{\cdot}01$) for both species. In general, time in sets appears to have decreased in recent years for at least large catches.

The trend toward higher probabilities of setting successfully on a sighted school of tuna is indicated by IATTC log book records which include data on numbers of successful and attempted sets. These data can be stratified by numerous classifications. We have chosen, for illustration, two portions: first, for fishing area 2 (defined by Shimada & Schaefer, 1956) the ratio of successful sets to total sets attempted, regardless of species, has been computed for each year from 1962 through 1966 (Table 1); and second, for all fishing areas, the ratio of successful to total sets attempted on yellowfin associated with porpoise has been computed for the same years (Table 2). Improvement in vessel efficiency over these years is patent from these ratios.

TABLE 1

Ratios of successful to total sets in area 2 regardless of species

Year	Ratio
1962	0·515
1963	0·516
1964	0·566
1965	0·607
1966	0·626

TABLE 2

Ratios of successful to total sets over the entire fishery for yellowfin associated with porpoise

Year	Ratio
1962	0·455
1963	0·485
1964	0·533
1965	0·655
1966	0·655

In this paper abundance measures are developed which take into consideration improvements in the purse seine fleet, the fact that this is a multiple-species fishery, and the observation that purse seiners spend a significant amount of time in sets.

2. A Semi-Markov Process Model for Purse Seining

Activities of a tuna purse seine vessel may be divided into five operations or states: S_0 = searching, S_1 = being in a successful set on yellowfin tuna, S_2 = being in an unsuccessful set on yellowfin tuna, S_3 = being in a successful set on skipjack tuna, and S_4 = being in an unsuccessful set on skipjack tuna. Possible transitions between states which a vessel may assume during a day are indicated by arrows in Fig. 3. Every possible sequence of fishing activities can be obtained from Fig. 3. For instance, a vessel could begin searching, discover a yellowfin tuna school and set unsuccessfully, be unable to relocate the escaped school, return to the search state, discover and capture a skipjack school, and thereafter continue searching without finding any more schools. This particular sequence of activities can be represented symbolically:

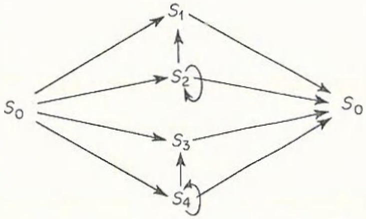

FIG. 3. A schematic representation of the two-species purse seining process. (S_0 = search state, S_1 = state of successfully setting on yellowfin tuna, S_2 = state of unsuccessfully setting on yellowfin tuna, S_3 = state of successfully setting on skipjack tuna, S_4 = state of unsuccessfully setting on skipjack tuna.)

$$S_0 \to S_2 \to S_0 \to S_3 \to S_0.$$

2.1. WAITING TIMES IN THE STATES

Time spent in a state is considered a random variable with distribution depending on the state occupied and, in some cases, the next state entered. In the model, time spent in S_2 and S_4 may depend not only on these states but also on whether the process proceeds to the search state, to the successful set state, or even remains in the unsuccessful set state, thus allowing for changes in behavior of either fish or fishermen after entry into an unsuccessful set state. Fishermen may decide, for instance, to make a more cautious and time-consuming approach to the next set on a relocated school, following a failure.

The density function, mean, and variance of time spent in the search state, denoted by f, μ_0 and σ_0^2, respectively, are obtained by the following theoretical considerations. Consider a large area of size A in which there are K schools of fish distributed randomly, i.e. if a vessel can search through an area of size a of A per unit of time and finds all schools in a, we say that schools are randomly distributed if in searching time t the probability that k schools are discovered, is given by the familiar binomial density function,

$$p_k(t) = \binom{K}{k}\left(\frac{a}{A}t\right)^k \left[1 - \left(\frac{a}{A}t\right)\right]^{K-k}, \tag{1}$$

for $0 \leq k \leq K$. If K is large and $(a/A)t$ is small,

$$p_k(t) \cong \frac{(\lambda t)^k}{k!} \mathrm{e}^{-\lambda t}, \tag{2}$$

where $\lambda = (a/A)K$. From well-known results of the Poisson process,

$$E(k) = \lambda t \tag{3}$$

$$\mathrm{Var}(k) = \lambda t. \tag{4}$$

Waiting time between discoveries, u, is a random variable with the negative exponential distribution. Again from studies of the Poisson process, it is known that

$$f(u) = \lambda\, \mathrm{e}^{-\lambda u} \qquad u \geq 0 \tag{5}$$

$$\mu_0 = \frac{1}{\lambda} \tag{6}$$

$$\sigma_0^2 = \frac{1}{\lambda^2}. \tag{7}$$

Thus if schools are randomly distributed in the area of size A, distribution of waiting time in the search state is negative exponential with mean and variance determined by school density in the area and the vessel search rate.

Distributions, means and variances of waiting times in the set states for a given time-and-area stratum seem to depend on manifold factors such as ocean conditions, weather, vessel properties, and the fish themselves. These distributions and their moments cannot be determined from theoretical considerations but may be determined by sampling duration of sets from vessel log book records. To complete the notation on waiting times, let the density functions, means, and variances of waiting times spent in S_1 and S_3 be denoted by g_1, g_3, μ_1, μ_3, σ_1^2 and σ_3^2, respectively. Further, we denote the density function of waiting time spent in S_2 given S_i is the next state, by g_{2i} with mean μ_{2i} and variance σ_{2i}^2 for $i = 0$, 1 and 2. Similarly the density function of the waiting time in S_4 given S_i is next visited is g_{4i} which has mean μ_{4i} and variance σ_{4i}^2 for $i = 0$, 3 and 4.

2.2. TRANSITION PROBABILITIES

To complete the model we need the probabilities of passing from one state to another given a transition has occurred. For unsuccessful set states, a transition may occur without the process having left these states. We shall denote the transition probability of passing from state i to state j by π_{ij}. If we let

ν_z = probability of successfully setting on a school of species z once sighted,

K_z = number of schools of species z in the area of size A,

$z = 1, 2$

and $K = K_1 + K_2$, then provided the purse seine vessels are not selective for either species

$$\begin{aligned} \pi_{01} &= \frac{K_1}{K}\nu_1, \\ \pi_{02} &= \frac{K_1}{K}(1-\nu_1), \\ \pi_{03} &= \frac{K_2}{K}\nu_2, \\ \pi_{04} &= \frac{K_2}{K}(1-\nu_2). \end{aligned} \tag{8}$$

If a school of species z escapes a set and is relocated with probability η_z,

$$\begin{aligned} \pi_{21} &= \eta_1\nu_1, \\ \pi_{43} &= \eta_2\nu_2, \\ \pi_{22} &= \eta_1(1-\nu_1), \\ \pi_{44} &= \eta_2(1-\nu_2), \\ \pi_{20} &= 1-\eta_1, \\ \pi_{40} &= 1-\eta_2. \end{aligned} \tag{9}$$

The remainder of the transition probabilities all have a value of zero.

For a given density combination of the two species, values for the parameters of the waiting time distributions, and values for the transition probabilities, we should like to compute the expected catch of the two species for a given amount of fishing time. We shall denote by $N_i(t)$ the random number of entries into S_i during a time interval of length t given that the process starts in the search state. Now $N_1(t)$ and $N_3(t)$ are of particular interest since they represent the number of schools captured by the seiner during the interval. It seems difficult to derive exact expressions for the expectations of $N_1(t)$ and $N_3(t)$. However, if one considers the sequence of recurrence times for S_i, $[T_{ii}^{(1)}, T_{ii}^{(2)}, \ldots]$ where $T_{ii}^{(m)}$ is the random length of time between the mth and $(m+1)$st entry into state i, these random variables form a renewal process (Smith, 1958). If we augment the sequence with the first passage time from S_0 to S_i, $[T_{0i}, T_{ii}^{(1)}, T_{ii}^{(2)}, \ldots]$ we have a general renewal process as Perrin & Sheps (1964) noted for a somewhat similar process. From the study of the cumulants of the general renewal process by Murthy (1961), the mean of $N_i(t)$ given the process begins in S_0 is given by the approximation,

$$E[N_i(t)] \cong \frac{t}{\tau_{ii}} + \frac{\tau_{ii}^{(2)}}{2\tau_{ii}^2} - \frac{\tau_{0i}}{\tau_{ii}}, \tag{10}$$

where τ_{ii} is the mean recurrence time to S_i,

τ_{0i} is the mean first passage time from S_0 to S_i

and $\tau_{ii}^{(2)}$ is the second moment about the origin of T_{ii}.

The task of computing the expected number of entries into S_i given the process begins in S_0 is reduced to determining τ_{ii}, τ_{0i} and $\tau_{ii}^{(2)}$. Pyke's method (1961*a*,*b*) is used to compute these expectations. We begin by displaying Pyke's **Q**-matrix for this problem.

$$\mathbf{Q} = \begin{bmatrix} 0 & \pi_{01}(1-e^{-\lambda t}) & \pi_{02}(1-e^{-\lambda t}) & \pi_{03}(1-e^{-\lambda t}) & \pi_{04}(1-e^{-\lambda t}) \\ \int_0^t g_1(u)\,du & 0 & 0 & 0 & 0 \\ \pi_{20}\int_0^t g_{20}(u)\,du & \pi_{21}\int_0^t g_{21}(u)\,du & \pi_{22}\int_0^t g_{22}(u)\,du & 0 & 0 \\ \int_0^t g_3(u)\,du & 0 & 0 & 0 & 0 \\ \pi_{40}\int_0^t g_{40}(u)\,du & 0 & 0 & \pi_{43}\int_0^t g_{43}(u)\,du & \pi_{44}\int_0^t g_{44}(u)\,du \end{bmatrix}, \tag{11}$$

where q_{ij} = probability that, given the purse seiner is in state $i-1$, it will pass directly into state $j-1$ within t units of time after entering state $i-1$ for $i = 1, \ldots, 5$ and $j = 1, \ldots, 5$.

Let us denote by G_1 and G_3 the Laplace–Stieltjes transforms of g_1 and g_3 and by G_{20}, G_{21}, G_{22}, G_{40}, G_{43} and G_{44}, the Laplace–Stieltjes transforms of g_{20}, g_{21}, g_{22}, g_{40}, g_{43} and g_{44}. The Laplace–Stieltjes transform of the negative exponential distribution with parameter λ is designated by F and equals $\lambda/(\lambda+s)$ where s is the variable of the transform. We now define the matrix **R** (script $\mathscr{Q}$ in Pyke's notation) as follows:

$$\mathbf{R} = \begin{bmatrix} 0 & \pi_{01}F & \pi_{02}F & \pi_{03}F & \pi_{04}F \\ G_1 & 0 & 0 & 0 & 0 \\ \pi_{20}G_{20} & \pi_{21}G_{21} & \pi_{22}G_{22} & 0 & 0 \\ G_3 & 0 & 0 & 0 & 0 \\ \pi_{40}G_{40} & 0 & 0 & \pi_{43}G_{43} & \pi_{44}G_{44} \end{bmatrix}. \qquad (12)$$

Pyke has shown that the Laplace–Stieltjes transforms of the probability distributions of the first passage times from S_{i-1} to S_{j-1}, and of the recurrence times for S_{i-1} are given by the elements h_{ij} and h_{ii}, respectively, of the matrix $\mathbf{H}$ (his matrix script g) where

$$\mathbf{H} = \mathbf{R}(I-R)^{-1}[{}_d(I-\mathbf{R})^{-1}]^{-1} \qquad i = 1,\ldots,5; \quad j = 1,\ldots,5, \qquad (13)$$

where ${}_dA = (\delta_{ij}a_{ij})$ and δ_{ij} is the Kronecker delta. So, for example, h_{11} is the Laplace–Stieltjes transform of the probability distribution of the recurrence time for S_0, while h_{12} is the Laplace–Stieltjes transform of the probability distribution of the first passage time from S_0 to S_1. From the properties of the transforms we can compute the moments of the recurrence and first passage times as follows:

$$\begin{aligned} \tau_{ij} &= -\frac{\partial}{\partial s} h_{ij}\bigg|_{s=0} \\ \tau_{ij}^{(2)} &= \frac{\partial^2}{\partial s^2} h_{ij}\bigg|_{s=0}. \end{aligned} \qquad (14)$$

These expressions are somewhat lengthy but are presented in an appendix as a computer program. A dictionary is included to translate FORTRAN names into symbols used here.

2.3. THE NEYMAN MODEL AS A SPECIAL CASE

Neyman (1949) developed a stochastic model for purse seine fishing for a single-species fishery. He assumed that each sighted school which was set on was captured, and that time spent in the set state was a constant. The searching process was assumed to be a Poisson process. Our five state model collapses to a two state model: S_0 = search state, S_1 = set state. The process occurs as follows: $S_0 \to S_1 \to S_0$. Density for waiting time in S_0 is given by equation (5) where λ is defined in equation (2) with K representing the number of schools of the single species in the area of size A. The waiting time probability density for S_1 is degenerate at μ, the constant set time. Transition probabilities are: $\pi_{00} = 0$, $\pi_{01} = 1$, $\pi_{10} = 1$ and $\pi_{11} = 0$. The Laplace–Stieltjes transforms of waiting time distributions in the two states are: for S_0, $\lambda/(\lambda+s)$ and for S_1, $e^{-\mu s}$. Then we have

$$\mathbf{Q} = \begin{bmatrix} 0 & 1-e^{-\lambda t} \\ \int_0^t g(u)\,du & 0 \end{bmatrix},$$

where $g(u) = \begin{cases} 1 & u = \mu \\ 0 & \text{elsewhere} \end{cases}$ is the density for waiting time in the set state.

$$\mathbf{R} = \begin{bmatrix} 0 & \dfrac{\lambda}{\lambda+s} \\ e^{-\mu s} & 0 \end{bmatrix},$$

$$\mathbf{H} = \begin{bmatrix} \dfrac{\lambda}{\lambda+s}\, e^{-\mu s} & \dfrac{\lambda}{\lambda+s} \\ e^{-\mu s} & \dfrac{\lambda}{\lambda+s}\, e^{-\mu s} \end{bmatrix},$$

$$\tau_{11} = \frac{1}{\lambda} + \mu,$$

$$\tau_{11}^{(2)} = \frac{2}{\lambda^2} + \frac{2\mu}{\lambda} + \mu^2,$$

$$\tau_{01} = \frac{1}{\lambda}.$$

When mean first passage and recurrence times are substituted into (10) we can compute the expected number of schools captured by time t for any pair of parameter values by the resultant equation,

$$E[N_1(t)] \cong \frac{t}{\dfrac{1}{\lambda} + \mu} + \frac{\mu^2}{2\left(\dfrac{1}{\lambda} + \mu\right)^2}.$$

We recall that with no interruption in searching, average time spent between school sightings is $1/\lambda$. Time spent in the inactive set state at each sighting is μ. One might guess intuitively the expectation of the number of entries into the set state, to be equal to the first term on the right-hand side of the equation. The second term is in fact insignificant when considering the expected number of successful sets per day with practical values for λ and μ.

Neyman developed an exact expression for the expectation of this simpler model. Several numerical comparisons, based on the assumption that a set lasts exactly two hours, were made between the results from Neyman's formulations and those from the semi-Markov model. The parameter λ was allowed to vary between 0·050 and 0·300, a realistic range for the tuna fishery in the sense that the expected number of sets during a fishing day computed from the model covers fairly well the range of sets per fishing day in the actual fishery. Expected catches computed by the exact formula of Neyman and those computed by the approximation from general renewal process theory agree almost perfectly (Fig. 4). Values computed by Neyman's formula oscillate about the lines obtained from the semi-Markov model. Naturally we are not interested in the comparison much beyond 12 hours, the length of the fishing day. It appears that the approximate formulas for the expected number of schools captured are satisfactory.

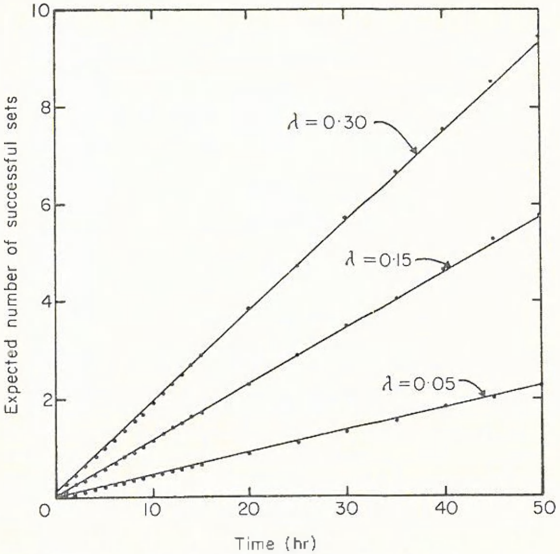

FIG. 4. Numerical comparison of the expected number of successful sets predicted by Neyman's model (dots) and the semi-Markov process model (lines) at three densities. The fixed waiting time in the set state was 2 hr.

3. Utility of the Model

The model lends itself nicely to theoretical studies of changes in parameters of the process. We consider several examples to illustrate information which can be derived from the model.

3.1. EXAMPLE 1

We examined the influence of changes in setting time (μ_1) and probability of making a successful set (ν_1) on the first species, when the second species is absent from the fishing area or ignored by the fishing vessels. Setting times were assumed to be constants rather than random variables. Setting time in the unsuccessful set state was taken to be 30 minutes. The probability of relocating an escaped school was taken as zero. The expected number of successful sets for a 12-hour fishing day at a large number of parameter pairs (μ_1, ν_1) was computed by the computer program. The loci of points in the (μ_1, ν_1)-plane at which equal values for the expected number of successful sets would be obtained, are indicated for several levels of the expected value by the lines in Fig. 5. The increasing degree of upward tilting of the lines with increasing values for ν_1, and the virtual absence of tilting at low values of ν_1 indicate that the time in a set is relatively unimportant when the probability of making a successful set is low, but becomes critical when that probability increases.

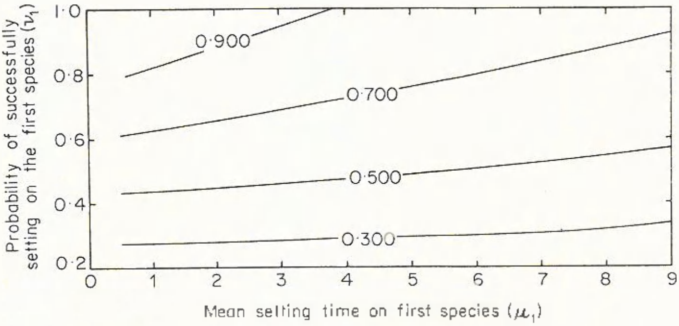

FIG. 5. Expected number of successful sets on the first species in 12 hr, as a function of the mean setting time in the successful set state and the probability of successfully setting on the first species. Additional parameter values chosen in the computation are given in the text.

3.2. EXAMPLE 2

We examined the effect of changes in the probability of setting successfully on the first species (ν_1), on the expected number of successful sets in a fishing day of both species. The expected number of successful sets on both species as a function of ν_1 was computed under two density combinations: $(\lambda_1, \lambda_2) = (0{\cdot}100, 0{\cdot}225)$ and $(0{\cdot}100, 0{\cdot}100)$. Setting time in the successful set state for either species was taken to be a constant of two hours, and the setting time in the unsuccessful set states was taken to be a constant of one hour. Expectations were computed assuming that an escaped school of either species would not be relocated. The expected number of successful sets in a 12-hour day are plotted in Fig. 6. Although the lines appear nearly linear, they are slightly curved—those for the first species are convex upward while those of the second species are concave downward. The expected number of successful sets on the first species increases nearly in proportion to changes in the probability of successfully setting on that species. The

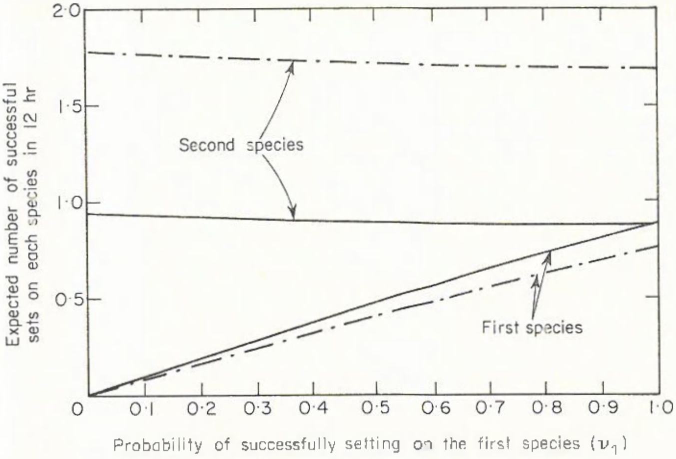

FIG. 6. Expected number of successful sets on each species in 12 hr at two density combinations, as a function of the probability of successfully setting on the first species, with the probability of sitting successfully on the second species fixed at one. Additional parameter values chosen in the computation are given in the text. —-—-— $\lambda_1 = 0{\cdot}100$, $\lambda_2 = 0{\cdot}225$; ——— $\lambda_1 = \lambda_2 = 0{\cdot}100$.

important feature is, however, the influence which the density of each species has on the expected number of successful sets of the other. The apparent abundance (schools captured per day) of the first species for any particular value of v_1 is lower when the second species is more abundant, i.e. when $\lambda_2 = 0{\cdot}225$. Further, as the probability of successfully setting on the first species increases, the apparent abundance of the second species declines.

3.3. EXAMPLE 3

The primary use of the model by IATTC will be the development of improved measures of yellowfin and skipjack abundance. The basis for such indices can be introduced best by a third example. We computed the expected number of successful sets of each species for many density combinations (λ_1, λ_2), taking the probability of making a successful set on either species as one, and the waiting time in the successful set states as a constant of two hours for either species (Fig. 7). The lines indicate the loci of points in the (λ_1, λ_2)-plane at which the expected number of successful sets is 0·5, 1·0 and 1·5 for a 12-hour period. One can interpolate between them for other levels of expectation. Now for any pair of densities (λ_1, λ_2), it is simple to determine the corresponding expected number of successful sets of each species. On the other hand, if the expected number of succesfsul sets of each species was known, we could, say from Fig. 7, determine the densities of the two species, provided the means and variances of waiting times in the set states and the transition probabilities, were those used in computing the figure.

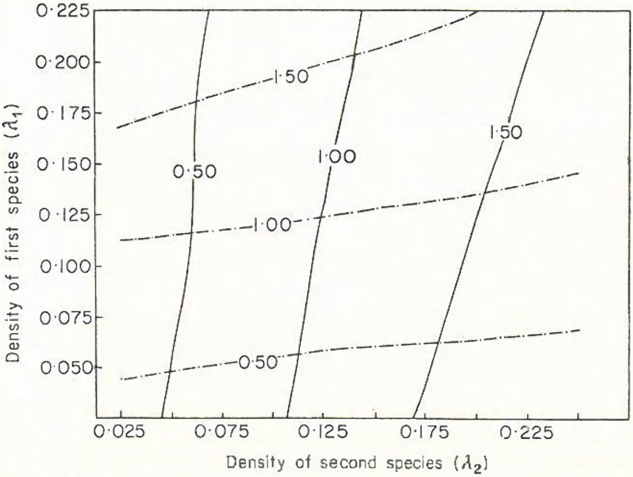

FIG. 7. Expected number of successful sets on each species in 12 hr for various combinations of densities (λ_1, λ_2) of the two species. Additional parameter values chosen in the computation are given in the text. — - — - — = species 1; ——— = species 2.

The above procedure is essentially that which can be used to develop indices of abundance for the two species assuming the vessels operate independently. Estimates of the means and variances of waiting times and of transition probabilities for area-time strata can be determined from log book records, and these estimates can be set equal to corresponding parameters in the model. The observed average numbers of successful sets in a fishing day per vessel in the strata may be set equal to their expectations defined in equation (10). The two equations for the two species in each stratum can then be solved for λ_1 and λ_2 by an iterative scheme suitable for a high-speed electronic computer, rather than by the inverse graphical approach. These resultant density estimates account for the time spent in sets, probability of making a successful set on each species, and competition for the gear by the two species—the sort of indices were originally intended to develop.

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Appendix

List of symbols used in the text and their FORTRAN equivalents†

Text symbol(s)	FORTRAN name(s)
λ	LAMDA
$\frac{a}{A}K_1, \frac{a}{A}K_2$	LAMDA1, LAMDA2
ν_1, ν_2	NU1, NU2
μ_0, μ_1, μ_3	MU0, MU1, MU3
$\mu_{20}, \ldots, \mu_{44}$	MU20, . . ., MU44
$\sigma_0, \sigma_1, \sigma_3$	SGM0, SGM1, SGM3
$\sigma_{20}, \ldots, \sigma_{44}$	SGM20, . . ., SGM44
$\pi_{01}, \ldots, \pi_{44}$	PI01, . . ., PI44
$F, G_1, \ldots, G_{44}$	F, G1, . . ., G44
$\frac{\partial}{\partial s} F\Big\|_0, \frac{\partial}{\partial s} G_1\Big\|_0, \ldots$	DF, DG1, . . .
$\frac{\partial}{\partial s} G_{44}\Big\|_0$	DG44
$\frac{\partial^2}{\partial s^2} F\Big\|_0, \frac{\partial^2}{\partial s^2} G_1\Big\|_0, \ldots$	D2F, D2G1, . . .,
$\frac{\partial^2}{\partial s^2} G_{44}\Big\|_0$	D2G44
$\frac{\partial}{\partial s} h_{12}\Big\|_0, \ldots, \frac{\partial}{\partial s} h_{44}\Big\|_0$	DH12, . . ., DH44
$\frac{\partial^2}{\partial s^2} h_{22}\Big\|_0, \frac{\partial^2}{\partial s^2} h_{44}\Big\|_0$	D2H22, D2H44
$E[N_1(12)], E[N_2(12)]$	EN1, EN2

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† All other FORTRAN names are intermediate steps in determining the expected number of schools of each species captured and are defined in terms of the above variables.

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C
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$JB9$PELLA$T/2T$P/80T$SCP/COOPSIM$.
$SET,WIDEOUT.
       PROGRAM SEINER
       REAL LAMDA1.LAMDA2.LAMDA.NU1.NU2,MU0,MU1.MU20,MU3.MU40,
      1  MU21,MU43,MU22.MU44
       TYPE DOUBLE AA121.AA221.AA222.AA141.AA441;AA442.A22,A32
      1  ,A42,A52,A24,A34,A44,A54,A12,A14,DAA121,DAA141,DAA221
      2  ,DAA441.DAA222.DAA442,D2A121.D2A141,D2A221.D2A441,D2A
      3  222.D2A442,DA12.DA14,DA22.DA44,D2A22,D2A44.D2A12,D2A1
      4  4,DH22,D2H22,DH44,D2H44,DA32,DA42.DA52,DA24.DA34,DA54
      5  ,DH12.DH14
C      STATEMENTS FOR READING IN PARAMETER VALUES
       READ(50.1)PI21,PI43,PI22,PI44
       READ(50.1) MU1,MU3,MU20,MU21,MU22.MU40.MU43,MU44
       READ(50.1)SGM1.SGM3,SGM20,SGM21,SGM22,SGM40,SGM43,SGM44
       NU2=0.99999
  20   READ(50,1)LAMDA1.LAMDA2
       DO 10 I=1.11
       READ(50,1)NU1
C      EVALUATION OF LAPLACE-STIELTJES TRANSFORMS OF
C      DISTRIBUTIONS OF FIRST PASSAGE TIMES BETWEEN STATES
       LAMDA=LAMDA1+LAMDA2
       MU0=1./LAMDA
       DF=-MU0
       D2F=2./LAMDA**2
       DG1=-MU1
       D2G1=SGM1**2+MU1**2
       DG3=-MU3
       D2G3=SGM3**2+MU3**2
       DG20=-MU20
       D2G20=SGM20**2+MU20**2
       DG21=-MU21
       D2G21=SGM21**2+MU21**2
       DG22=-MU22
       D2G22=SGM22**2+MU22**2
       DG40=-MU40
       D2G40=SGM40**2+MU40**2
       DG44=-MU44
       D2G44=SGM44**2+MU44**2
       DG43=-MU43
       D2G43=SGM43**2+MU43**2
       PI01=LAMDA1*NU1/LAMDA
       PI02=LAMDA1*(1.-NU1)/LAMDA
       PI03=LAMDA2*NU2/LAMDA
       PI04=LAMDA2*(1.-NU2)/LAMDA
       PI20=1.-PI22-PI21
       PI40=1.-PI44-PI43
       AA121=PI01*(1.-PI22)+PI02*PI21
       AA221=(1.-PI22)*(1.-PI03)-PI02*PI20
       AA222=PI04*(PI43+PI40)
       AA141=PI03*(1.-PI44)+PI04*PI43
       AA441=(1.-PI44)*(1.-PI01)-PI04*PI40
       AA442=PI02*(PI21+PI20)
       A22=AA221*(1.-PI44)-AA222*(1.-PI22)
       A32=(-PI03*PI21+PI21+PI01*PI20)*(1.-PI44)-
      1PI21*PI04*PI43-PI04*PI40*PI21
       A42=(PI01*(1.-PI22)+PI02*PI21)*(1.-PI44)
       A52=(PI43+PI40)*(PI01*(1.-PI22)+PI02*PI21)
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      A24=(PI03*(1.-PI44)+PI04*PI43)*(1.-PI22)
      A34=(PI21+PI20)*(PI03*(1.-PI44)+PI04*PI43)
      A44-AA441*(1.-PI22)-AA442*(1.-PI44)
      A54=(-PI01*PI43+PI43+PI03*PI40)*(1.-PI22)-
     1PI43*PI02*PI21-PI02*PI20*PI43
      A12=AA121*(1.-PI44)
      A14-AA141*(1.-PI22)
      DAA121=-PI01*PI22*DG22+PI01*(1.-PI22)*DF+
     1PI02*PI21*DG21+PI02*PI21*DF
      DAA141=-PI03*PI44*DG44+PI03*(1.-PI44)*DF+
     1PI04*PI43*DG43+PI04*PI43*DF
      DAA221=(1.-PI22)*(-PI03*DG3-PI03*DF)+(1.-PI03)*
     1(-PI22*DG22)-PI02*PI20*DG20-PI02*PI20*DF
      DAA441=(1.-PI44)*(-PI01*DG1-PI01*DF)+(1.-PI01)*
     1(-PI44*DG44)-PI04*PI40*DG40-PI04*PI40*DF
      DAA222=PI04*(PI43*DG43+DG3*PI43+PI40*DG40)+
     1(PI43+PI40)*PI04*DF
      DAA442=PI02*(PI21*DG21+DG1*PI21+PI20*DG20)+
     1(PI21+PI20)*PI02*DF
      D2A121=-PI01*PI22*D2G22-PI01*PI22*DG22*DF
     1+PI01*(1.-PI22)*D2F-PI01*DF*PI22*DG22+PI02*
     2PI21*D2G21+PI02*PI21*DG21*DF+PI02*PI21*D2F+
     3PI02*PI21*DF*DG21
      D2A141=-PI03*PI44*D2G44-PI03*PI44*DG44*DF
     1+PI03*(1.-PI44)*D2F-PI03*DF*PI44*DG44+PI04*
     2PI43*D2G43+PI04*PI43*DG43*DF+PI04*PI43*D2F+
     3PI04*PI43*DF*DG43
      D2A221=(1.-PI22)*(-PI03*D2G3-PI03*DG3*DF
     1-PI03*D2F-PI03*DF*DG3)+(-PI03*DG3-PI03*DF)*
     2(-PI22*DG22)+(1.-PI03)*(-PI22*D2G22)+(-PI22*
     3DG22)*(-PI03*DG3-PI03*DF)-PI02*PI20*D2G20
     4-PI02*PI20*DG20*DF-PI02*PI20*D2F-PI02*
     5PI20*DF*DG20
      D2A441=(1.-PI44)*(-PI01*D2G1-PI01*DG1*DF
     1-PI01*D2F-PI01*DF*DG1)+(-PI01*DG1-PI01*DF)*
     2(-PI44*DG44)+(1.-PI01)*(-PI44*D2G44)+(-PI44*
     3DG44)*(-PI01*DG1-PI01*DF)-PI04*PI40*D2G40
     4-PI04*PI40*DG40*DF-PI04*PI40*D2F-PI04*
     5PI40*DF*DG40
      D2A222=PI04*(PI43*D2G43+PI43*DG43*DG3+DG3*
     1PI43*DG43+D2G3*PI43+PI40*D2G40)+(PI43*DG43
     2+DG3*PI43+PI40*DG40)*PI04*DF+(PI43+PI40)*
     3PI04*D2F+PI04*DF*(PI43*DG43+PI43*DG3+
     4PI40*DG40)
      D2A442=PI02*(PI21*D2G21+PI21*DG21*DG1+DG1*
     1PI21*DG21+D2G1*PI21+PI20*D2G20)+(PI21*DG21
     2+DG1*PI21+PI20*DG20)*PI02*DF+(PI21+PI20)*
     3PI02*D2F+PI02*DF*(PI21*DG21+PI21*DG1+
     4PI20*DG20)
      DA12=-PI44*DG44*AA121+(1.-PI44)*DAA121
      DA14=-PI22*DG22*AA141+(1.-PI22)*DAA141
      DA22=-PI44*DG44*AA221+(1.-PI44)*DAA221+PI22*DG22*AA222-
     1  (1.-PI22)*DAA222
      DA44=-PI22*DG22*AA441+(1.-PI22)*DAA441+PI44*DG44*AA442-
     1  (1.-PI44)*DAA442
      D2A22=-PI44*DG44*DAA221-PI44*AA221*D2G44+(1.-PI44)*D2A2
     1  21-DAA221*PI44*DG44+PI22*DG22*DAA222+PI22*AA222*D2G22
     2  -(1.-PI22)*D2A222+DAA222*PI22*DG22
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      D2A44=-PI22*DG22*DAA441-PI22*AA441*D2G22+(1.-PI22)*D2A4
     1  41-DAA441*PI22*DG22+PI44*DG44*DAA442+PI44*AA442*D2G44
     2  -(1.-PI44)*D2A442+DAA442*PI44*DG44
      D2A12=-PI44*D2G44*AA121-PI44*DG44*DAA121+(1.-PI44)*
     1D2A121-PI44*DG44*DAA121
      D2A14=-PI22*D2G22*AA141-PI22*DG22*DAA141+(1.-PI22)*
     1D2A141-PI22*DG22*DAA141
      DH22=(1./A22)*(A12*DG1+DA12-A12*DA22*(1./A22))
      D2H22=(A12/A22)*D2G1+DG1*(1./A22**2)*(A22*
     1DA12-A12*DA22)+(1./A22)*D2A12+DA12*(1./A22**2)
     2*(A22*DG1-1.*DA22)-(1./A22**2)*(A12*D2A22+
     3DA22*DA12)-A12*DA22*(1./A22**4)*(A22**2*
     4DG1-2.*1.*A22*DA22)
      DH44=(1./A44)*(A14*DG3+DA14-A14*DA44*(1./A44))
      D2H44=(A14/A44)*D2G3+DG3*(1./A44**2)*(A44*
     1DA14-A14*DA44)+(1./A44)*D2A14+DA14*(1./A44**2)
     2*(A44*DG3-1.*DA44)-(1./A44**2)*(A14*D2A44+
     3DA44*DA14)-A14*DA44*(1./A44**4)*(A44**2*
     4DG3-2.*1.*A44*DA44)
      DA32=(-PI03*PI21*DG21-PI03*PI21*DG3-PI03*PI21*
     1DF+PI21*DG21+PI01*PI20*DG20+PI01*PI20*DF)*
     2(1.-PI44)+(-PI44*DG44)*(-PI03*PI21+PI21+PI01*PI20)
     3-PI21*PI04*PI43*DG43-PI21*PI04*PI43*DG3-PI21*PI04*PI43
     4*DF-PI04*PI40*PI21*DG21-PI04*PI40*PI21*DG40-PI04*PI40*
     5PI21*DF-PI21*PI04*PI43*DG21
      DA42=(1.-PI44)*(PI01*(-PI22*DG22)+(1.-PI22)*PI01*
     1DF+PI02*PI21*DG21+PI02*PI21*DF)+(PI01*(1.-PI22)
     2+PI02*PI21)*(DG3-PI44*DG44-PI44*DG3)
      DA52=(PI43+PI40)*(PI01*(-PI22*DG22)+(1.-PI22)*
     1PI01*DF+PI02*PI21*DG21+PI02*PI21*DF)+(PI01*(
     21.-PI22)+PI02*PI21)*(PI43*DG43+PI43*DG3+PI40*DG40)
      DA24=(1.-PI22)*(PI03*(-PI44*DG44)+(1.-PI44)*PI03*
     1DF+PI04*PI43*DG43+PI04*PI43*DF)+(PI03*(1.-PI44)
     2+PI04*PI43)*(DG1-PI22*DG22-PI22*DG1)
      DA34=(PI21+PI20)*(PI03*(-PI44*DG44)+(1.-PI44)*
     1PI03*DF+PI04*PI43*DG43+PI04*PI43*DF)+(PI03*(
     21.-PI44)+PI04*PI43)*(PI21*DG21+PI21*DG1+PI20*DG20)
      DA54=(-PI01*PI43*DG43-PI01*PI43*DG1-PI01*PI43*
     1DF+PI43*DG43+PI03*PI40*DG40+PI03*PI40*DF)*
     2(1.-PI22)+(-PI22*DG22)*(-PI01*PI43+PI43+PI03*PI40)
     3-PI43*PI02*PI21*DG21-PI43*PI02*PI21*DG1-PI43*PI02*PI21*
     4DF-PI02*PI20*PI43*DG43-PI02*PI20*PI43*DG20-PI02*PI20*
     5PI43*DF-PI43*PI02*PI21*DG43
      DH12=PI01*DF+(1./A22)*(PI02*DA32+PI02*A32*DF
     1+PI03*DA42+PI03*A42*DF+PI04*DA52+PI04*A52*DF)
     2-(PI02*A32+PI03*A42+PI04*A52)*(1./A22**2)*DA22
      DH14=PI03*DF+(1./A44)*(PI01*DA24+PI01*A24*DF
     1+PI02*DA34+PI02*A34*DF+PI04*DA54+PI04*A54*DF)
     2-(PI01*A24+PI02*A34+PI04*A54)*(1./A44**2)*DA44
C      EVALUATION OF THE EXPECTED NUMBER OF SUCCESSFUL SETS
C      FOR EACH SPECIES
       EN1=-12./DH22+D2H22/(2.*DH22**2)-DH12/DH22
       EN2=-12./DH44+D2H44/(2.*DH44**2)-DH14/DH44
C      STATEMENTS FOR PRINTING RESULTS
       WRITE(51,2)EN1,EN2,MU1,NU1
 10    CONTINUE
       IF(EOF,50)30,20
  1    FORMAT(8F10.0)
C
C

                        C
                        C
                          2   FORMAT(4F10.6)
                           30 END
                                 FINIS
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