II. *The Principal Constituent of the Tides in the English and Irish Channels.*

By A. T. DOODSON *and* R. H. CORKAN (*Liverpool Observatory and Tidal Institute*).

Communicated by J. PROUDMAN, *F.R.S.*

(Received December 31, 1931.—Read April 21, 1932.)

1. A chart of co-tidal and co-range lines for the North Sea* was prepared at the Tidal Institute in the year 1923, and the methods then used have now been further developed and improved and applied to the English Channel, the Irish Sea, and their approaches.†

The methods used depend largely upon the known dynamical equations connecting the currents with the gradients of the elevations. If any assumption be made as to the values of the range of tide and the relative phases of the elevation and currents, then we can deduce from this information at one station, not only the directions of the co-tidal and co-range lines at that point, but also the degree of separation of the lines for any given unit of phase or range. This criterion, applied to speculative charts hitherto published, suffices at once either to verify or to condemn the charts. Again, without any assumptions at all, from the gradients at a number of stations on a line of small curvature, starting at a point at which the elevation is known, the elevations at all points along the line can be computed by simple methods of numerical integration.

For the North Sea there were available the results of harmonic analyses of tidal currents, observed over periods of a fortnight, at 15 stations, but for the English and Irish Channels much inferior data have had to be used, and most of this information was obtained many years ago. The Hydrographer ordered special observations to be taken across three lines at the entrances to the Channels, and the results were in approximate agreement with the older observations at such points where comparisons could be made. As most of the observations probably extended over only 24 hours, it became necessary to criticise the data, and to reject anomalies wherever such existed. Since the methods of integration tend to diminish the importance of individual errors the results of integration have fitted in with the coastal observations so well as to

* PROUDMAN and DOODSON, 'Phil. Trans.,' A, vol. 224, p. 185 (1923).

† The preparation of the new tidal charts was ordered by the Hydrographer of the Navy, and the cost of investigation and preparation defrayed by the Hydrographic Department of the Admiralty. The new tidal charts have been combined with the former North Sea Tidal Chart, and the combined chart is on sale to the general public as Admiralty Chart No. 301.

warrant the confident construction of co-tidal and co-range lines for these Channels. It will be found that the charts differ very considerably from the speculations previously published in that the latter are dominated by the idea of a tidal wave travelling more quickly in the middle of the Channel than at the sides, so that the charts show great curvature of the co-tidal lines inward to the Channel. This assumption is entirely wrong, and the new charts, on the contrary, show that many of the lines are actually curved (slightly) in the opposite direction.

2. *Definitions of Tidal Elevations.*

We shall deal only with the principal lunar constituent M_2, and the tidal elevation, at any point, due to this constituent will be expressed as

$$\zeta = \mathrm{H} \cos(\sigma t - g - 30^\circ) = \mathrm{H} \cos(\sigma t - d) = \zeta_1 \cos \sigma t + \zeta_2 \sin \sigma t,$$

where σ is the speed of the harmonic motion (*i.e.*, 2π/period), t is the time, and H, g are functions of position required for the co-range and co-tidal charts respectively. The origin of the time chosen for this work has been the time of high water (of M_2) at Dover; this choice was made because most of the data concerning tidal currents are referred to the time of high water at Dover. We define g to be the lag in phase of the elevation behind that of the corresponding equilibrium constituent on the meridian of Greenwich, and at Dover the value of g has been taken as 330°. If t is expressed in hours we have $\sigma = 28 \cdot 984^\circ$.

3. *Definitions of Tidal Currents.*

As a general rule tidal currents are rotatory and the vector-diagram of velocity is an ellipse. The velocity-components in the directions of the major and minor axes of this ellipse are expressed respectively by

$$u_0 \cos(\sigma t - g' - 30^\circ)$$

and

$$v_0 \sin(\sigma t - g' - 30^\circ),$$

i.e., by $u_0 \cos(\sigma t - d')$ and $v_0 \sin(\sigma t - d')$. It is necessary to specify the direction (θ) in which the maximum current is $+ u_0$, and this angle will be measured from the east round by the north. The positive minor axis, by a convention, is taken as 90° in advance of the positive major axis, and thus v_0 can be either positive or negative: in the former case the currents rotate in the positive sense, mathematically.

Taking Cartesian axes Ox, Oy on the mean surface of the sea, to the east and to the north respectively, then we define the corresponding current components as

$$u = u_1 \cos \sigma t + u_2 \sin \sigma t,$$
$$v = v_1 \cos \sigma t + v_2 \sin \sigma t.$$

4. *Currents in the Open Sea.*

The principal source of information of currents is the Admiralty publication, "Tides and Tidal Streams."* Details of streams, between 6 hours before to 6 hours after the time of high water at Dover, are given for a large number of stations, and these are tabulated, with values of current components for M_2, and the elevation-gradients, in Table I. The notation S 263 means that the station is the third station on p. 26 of the publication referred to. Figs. 1 and 2 indicate the situations of the stations.

TABLE I.—Current Data.

Station.	Latitude.	Longitude.	Depth.	u_1.	v_1.	u_2.	v_2.	$\partial\zeta_1/\partial x$.	$\partial\zeta_1/\partial y$.	$\partial\zeta_2/\partial x$.	$\partial\zeta_2/\partial y$.
S.	° ′	° ′	fathoms								
222	49 16N	8 11W	70	−15	5	30	25	− 36	− 19	4	− 29
223	49 52	7 58	70	−10	10	25	5	− 25	4	− 9	− 15
224	49 02	6 46	70	−25	− 5	5	40	− 12	− 29	9	15
231	49 40	6 28	57	−25	− 5	0	25	− 5	− 8	− 8	− 8
232	49 15	6 03	61	−30	−25	10	20	− 40	6	− 20	− 48
233	49 02	5 39	61	−10	20	40	25	− 34	− 26	10	− 18
234	49 25	5 20	55	−51	−17	32	10	− 59	48	− 69	− 60
235	49 45	4 35	45	0	0	− 50	− 13	72	19	− 9	57
236	50 00	3 45	40	−38	−10	− 67	− 18	89	69	− 67	62
251	49 35	5 41	54	−30	− 5	10	10	− 18	20	− 32	− 18
252	49 49	5 28	45	−35	20	15	25	4	2	− 23	9
253	49 30	5 03	52	−42	− 6	54	18	− 79	23	− 47	− 72
254	49 29	4 15	48	−20	10	15	30	− 9	− 20	4	− 5
255	49 49	3 19	50	−20	−10	− 15	− 15	11	44	− 45	4
256	48 48	4 45	56	−21	−15	27	12	− 55	31	− 19	− 52
261	50 05	2 20	35	−20	0	− 80	− 15	117	45	− 29	94
262	50 19	1 10	39	0	0	−125	5	179	− 7	45	141
263	50 15	0 00	26	25	5	− 80	− 10	123	− 26	68	113
271	50 11	1 17	39	0	0	− 90	− 10	150	15	25	125
272	49 50	2 20	91	− 5	−10	− 70	− 35	111	56	− 43	96
273	49 45	1 16	25	45	−20	−175	80	193	−150	288	109
341	49 35	1 00	15	−25	30	− 80	85	167	−117	125	64
342	49 35	0 30	17	0	0	−105	10	150	− 14	72	112
343	49 40	0 00	14	20	10	− 90	− 55	128	48	33	158
361	50 43	0 27E	7	25	5	− 80	− 20	92	− 3	115	121
362	50 54	1 00	15	25	10	15	15	− 15	− 55	49	− 7
363	50 55½	1 06	15	30	30	15	20	5	− 74	62	19
364	50 49	1 20	20	20	40	25	35	5	− 84	64	18
365	50 57	1 16	15	60	45	30	25	− 32	−127	100	16
366	50 54	1 27	20	30	60	45	90	− 12	−196	122	− 12
367	51 03	1 22	15	45	50	65	45	− 65	−145	75	− 31
371	51 09	1 28	15	40	70	45	60	− 11	−180	99	12
372	51 13	1 36	15	40	70	80	135	− 78	−317	125	−143
373	51 16	1 29	5	35	55	60	80	−101	−278	7	−202
391	49 45	6 35W	54	−15	−15	− 5	15	− 9	− 4	− 5	− 17

* "Tides and Tidal Streams," H.M. Stationery Office, 1st Ed. (1909).

TABLE I.—(continued).

Station.	Latitude.	Longitude.	Depth.	u_1.	v_1.	u_2.	v_2.	$\partial\zeta_1/\partial x$.	$\partial\zeta_1/\partial y$.	$\partial\zeta_2/\partial x$.	$\partial\zeta_2/\partial y$.
S.	° ′	° ′	fathoms								
392	49 52N	6 35W	48	−25	10	10	35	−2	−22	3	1
393	50 05	6 26	45	−15	−5	10	20	−20	−12	1	−19
401	50 03	6 04	40	−15	5	10	35	−8	−33	18	−6
402	49 56	6 06	40	−30	−5	10	35	−17	−16	−4	−22
403	49 52	6 12	45	−30	−10	10	40	−23	−22	1	−29
421	49 50	5 52	45	−20	10	15	20	−9	−6	−7	−4
422	49 57	5 48	40	−45	−5	0	40	0	−6	−20	−12
491	50 32½	2 29½	20	−20	25	10	40	18	−40	15	15
493	50 31	2 20	13	−25	−15	−50	−35	67	85	−55	52
501	50 27	2 34	27	0	0	−61	20	87	29	37	66
502	30 31	2 25	10	−20	−60	−15	−35	−31	116	−59	−44
541	—	—	15	−30	0	−70	15	115	11	7	71
542	—	—	35	−12	−6	−175	−29	250	51	39	212
543	—	—	15	10	0	−90	−30	180	46	79	187
551	—	—	10	−15	5	−70	−30	117	60	−4	107
552	50 37½	0 54	15	0	10	−95	−10	147	14	44	126
571	50 42½	0 59½	10	−30	20	−40	−20	96	56	−43	84
572	50 44	1 04	10	−25	30	−30	30	93	−32	14	59
1611	51 46	11 00	100	−10	20	20	35	−6	−40	26	5
1612	51 30	10 18	55	5	35	25	−15	5	13	−12	22
1613	51 28	8 41	45	−45	−30	−5	5	−23	49	−59	−37
1614	51 39	8 11	35	−50	−30	−5	5	−20	56	−66	−37
1615	51 43	8 15	15	−35	−20	10	35	−29	−7	−13	−53
1621	52 02	6 39	30	5	−25	−70	−30	71	41	−9	54
1622	52 07	6 24	25	−35	−10	−95	−35	142	93	−49	113
1731	50 19	5 52	32	−20	0	20	30	−27	−20	3	−26
1811	51 31	5 01	30	−40	−5	50	−5	−71	54	−72	−65
1812	51 30½	4 25	20	−35	20	60	−30	−49	75	−108	−30
1813	51 27	3 56	15	−50	15	70	−20	−53	76	−145	−45
A_1	52 00	5 36	57	−10	−10	−40	−75	47	120	−85	39
A_2	52 06	5 54	52	5	−5	−40	−80	51	108	−75	46
B_1	50 25	6 38	51	−15	−20	0	10	−22	3	−11	−29
B_2	50 48	7 35	58	−15	−5	10	5	−20	10	−16	−18
C_1	48 49	5 23	49	−45	−15	25	30	−48	8	−34	−53
C_2	49 14	5 28	58	−45	−20	−5	30	−11	9	−31	−25
C_3	49 41	5 32½	58	−45	0	0	30	3	8	−31	−2
A	48 33	8 27	80	−25	−15	5	40	−23	−28	9	−29
B	48 46	10 04	90	−25	5	10	20	−7	−1	−14	−5
C	49 47	8 19	70	−25	−15	5	30	−23	−14	−2	−29
D	49 32	9 59	70	−25	10	20	30	−16	−15	−4	−10
E	50 37	8 57	60	−25	−5	10	20	−19	0	−14	−19
F	50 37	10 16	70	−30	5	15	25	−13	−2	−16	−11
G	50 50	5 30	40	−30	−20	20	10	−49	25	−33	−53
1951	52 21	6 10	25	−5	−65	−30	−65	−35	119	−73	−37
1952	52 31	6 04	16	−10	−50	−25	−110	−23	204	−130	34
1953	52 41	5 56	20	−15	−45	−50	−110	27	200	−123	62
1954	52 54	5 50	15	−20	−50	−50	−120	28	240	−127	97
1955	53 04	5 45	10	−25	−75	−45	−100	3	255	−104	53
1956	53 19½	5 54	15	0	−45	0	−80	−54	148	−96	−8
1961	52 24	5 00	35	10	25	−30	−85	70	104	−80	92
1962	53 06	4 44	30	0	35	0	−75	42	98	−89	73

The Hydrographer undertook to obtain information of currents at a number of places on three sectional lines :—

A—From Tuskar Rock to St. David's Head.

B—From C. Clear to Land's End.

C—From Land's End to Ushant.

Deductions from these tabulations are also tabulated in Table I. Certain unpublished data for stations in the approaches (A to G) are also given in this table. The Hydrographic Department provided corrected data for stations 1951 to 1956.

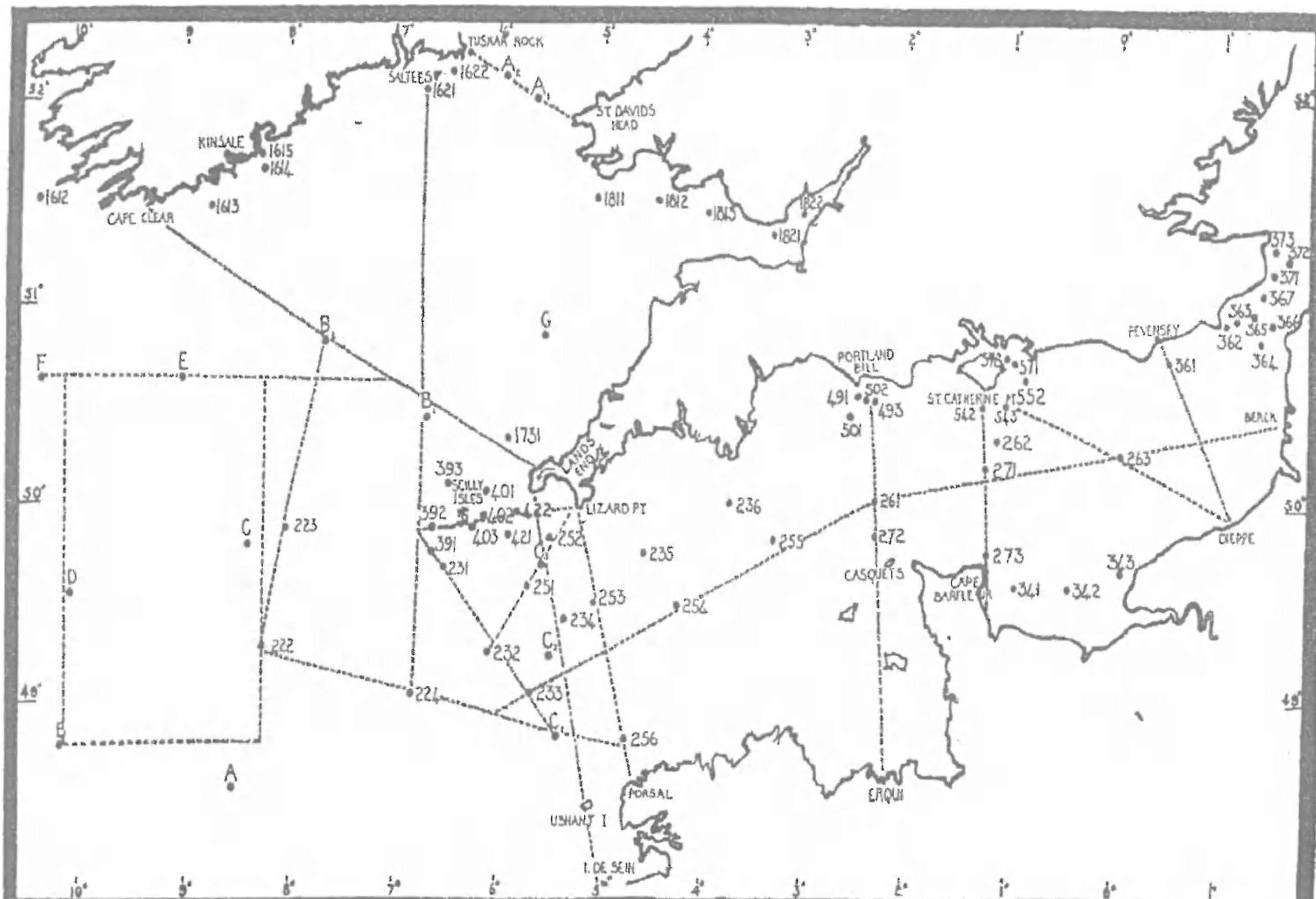

FIG. 1.—Current-stations and sectional lines in the English and Bristol Channels.

The velocity of current is recorded as for spring tides, and the appropriate reduction factors have been assumed to be the same as for the elevations at neighbouring coastal stations (see § 9). The corresponding values of u, v (in directions E and N) were then computed, and u_1, u_2, v_1, v_2, were deduced as follows :—

Since

$$u(t) = u = u_1 \cos \sigma t + u_2 \sin \sigma t,$$

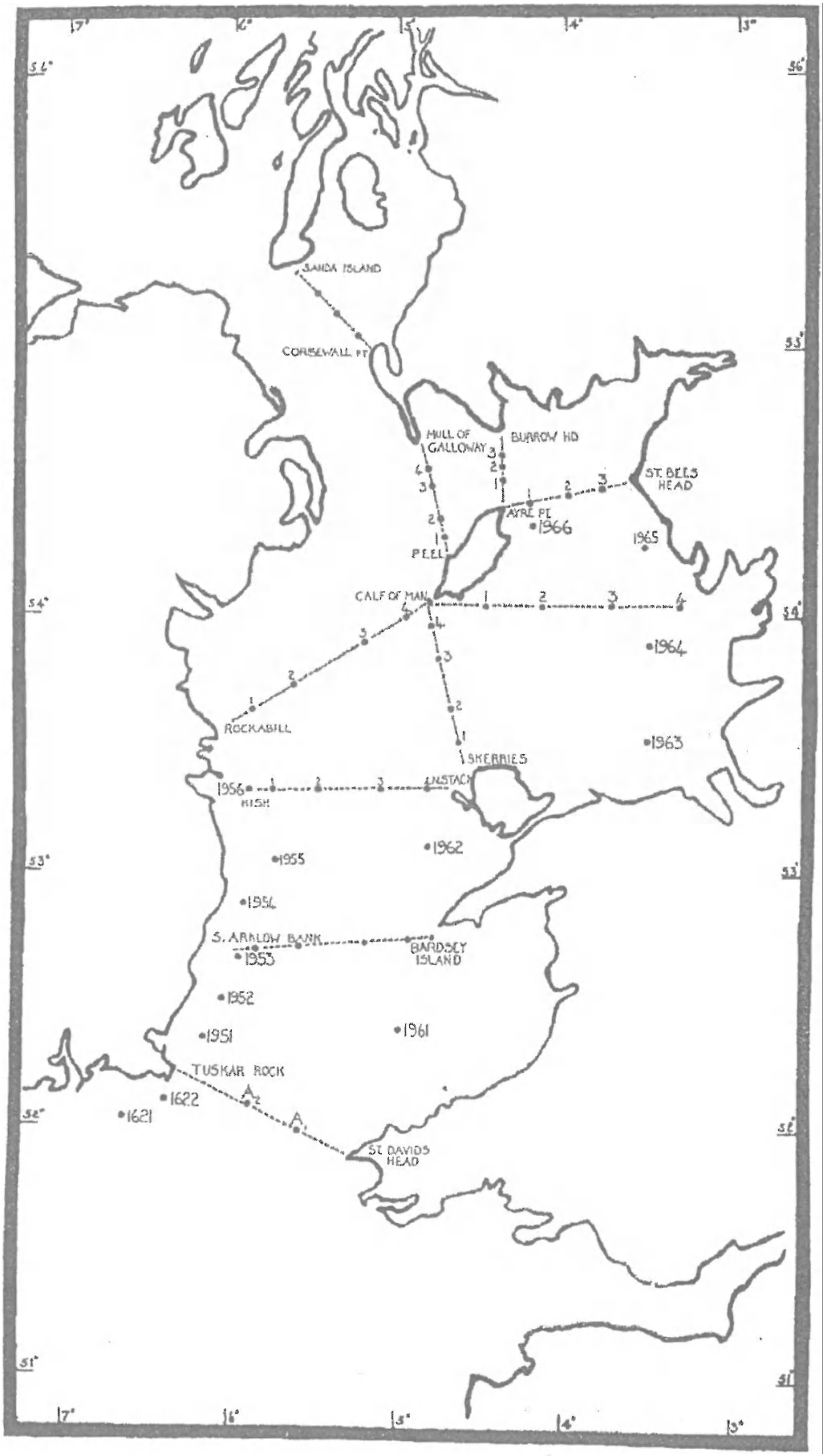

FIG. 2.—Current-stations and sectional lines in the Irish Channels.

then if t is taken at intervals of 1 hour from -6 to $+6$, we can eliminate u_2 by taking values of $\frac{1}{2}\{u(t) + u(-t)\}$, and this will be equal to $u_1 \cos \sigma t +$ a "constant," if there be any permanent current or M_4. It is a simple matter to deduce from the maximum and minimum currents, at $t = 0$ and $t = 6$, the values of u_1 and the "constant." Similarly, from $\frac{1}{2}\{u(t) - u(-t)\}$ we obtain values of $u_2 \sin \sigma t$ and the maximum value at $t = 3$ gives u_2. A simple graph enables a proper estimation, and an allowance for errors of observation, to be made. As a rule, no greater accuracy has been attempted than to give velocities to the nearest multiple of 5 cm. per second.

5. *Dynamical Equations.*

The relations between the currents and the elevations at and near a point are expressed in the form (PROUDMAN and DOODSON, *loc. cit.*) :—

$$\frac{\partial \zeta_1}{\partial x} = -au_2 + bv_1 - c\mathrm{F}_1$$

$$\frac{\partial \zeta_1}{\partial y} = -av_2 - bu_1 - c\mathrm{G}_1$$

$$\frac{\partial \zeta_2}{\partial x} = au_1 + bv_2 - c\mathrm{F}_2$$

$$\frac{\partial \zeta_2}{\partial y} = av_1 - bu_2 - c\mathrm{G}_2$$

where x, y are measured in units of 10^7 cm., u, v are measured in centimetres per second and $a = 1\cdot432$, $b = 1\cdot487 \sin \lambda$, $c = 1\cdot019 \div h/10000 = 56/f$, $\lambda =$ latitude, $h =$ depth in centimetres, $f =$ depth in fathoms, and F_1, G_1, F_2, G_2, are components of the frictional forces at the bottom of the sea. We define F_1, F_2 to be the coefficients of $\cos \sigma t$, $\sin \sigma t$, respectively, in $0\cdot002u\sqrt{u^2+v^2}$ and G_1, G_2 to be similar coefficients derived from $0\cdot002v\sqrt{u^2+v^2}$. These coefficients have generally been obtained by direct harmonic analysis of the synthesised expressions for $u\sqrt{u^2+v^2}$, $v\sqrt{u^2+v^2}$ at intervals of $\sigma t = 0°, 30°, 60°, \ldots, 330°$, but sufficiently accurate values, in view of the roughness of the data, can be obtained from the formulæ :

$$\mathrm{F}_1 = 0\cdot0017u_1\sqrt{u_0^2+v_0^2}, \quad \mathrm{F}_2 = 0\cdot0017u_2\sqrt{u_0^2+v_0^2}$$
$$\mathrm{G}_1 = 0\cdot0017v_1\sqrt{u_0^2+v_0^2}, \quad \mathrm{G}_2 = 0\cdot0017v_2\sqrt{u_0^2+v_0^2}.$$

The dynamical equations are expressed in terms of the mean currents in the vertical line below the point, from top to bottom, but in practically all cases we only know the surface currents. Fortunately, it is known that the surface current has often been found to be very approximately equal to the mean current.

6. *Rates of Change of Amplitude and Phase-lag.*

From the equations $H^2 = \zeta_1^2 + \zeta_2^2$ and $\tan d = \zeta_2/\zeta_1$, using the special time origin, we immediately deduce the rates of change of H and of d in any direction (s) to be given by

$$\frac{\partial H}{\partial s} = \cos d \frac{\partial \zeta_1}{\partial s} + \sin d \frac{\partial \zeta_2}{\partial s}, \qquad H\frac{\partial d}{\partial s} = \cos d \frac{\partial \zeta_2}{\partial s} - \sin d \frac{\partial \zeta_1}{\partial s} \quad . \quad . \quad . \quad . \quad (6.1)$$

Let ψ, ψ' = angles made by co-tidal and co-range lines with Ox, respectively;

R, R′ = positive values of $H\frac{\partial d}{\partial s}, \frac{\partial H}{\partial s}$ in directions $\psi + 90°$, $\psi' + 90°$, respectively.

For a co-tidal line g (and therefore d) is a constant, and for a co-range line H is constant; hence

$$\left.\begin{aligned} R\cos(\psi + 90°) &= H\frac{\partial d}{\partial x}, & R\sin(\psi + 90°) &= H\frac{\partial d}{\partial y}, \\ R'\cos(\psi' + 90°) &= \frac{\partial H}{\partial x}, & R'\sin(\psi' + 90°) &= \frac{\partial H}{\partial y}, \end{aligned}\right\} \quad . \quad . \quad . \quad . \quad (6.2)$$

whence R, R′, ψ, ψ' can be uniquely determined. By this convention the phases and amplitudes increase positively in the directions 90° in advance of the directions of the co-tidal and co-range lines respectively.

7. *Deductions at a Single Station.*

If the currents have been observed at a single station then the direction of the co-tidal and co-range lines are functions of g only and not of H. Possible limits of g may be suggested by the coastal data, and generally it is sufficient to compute ψ, ψ' for two (or at most three) values at intervals of 30° in g. Approximate values of H can then be obtained and the distances between neighbouring co-tidal (and co-range) lines can thus be obtained. This method is very valuable, even if no other is available.

If integrations of $\partial\zeta/\partial s$ are possible from coast to coast then the values of H and g at stations on the line become known, but it frequently happens that the current-stations are not well placed for this purpose; the terminal points of the line may be among shoals or eddies, involving difficulties as to the proper terminal values of H and g, and these terminal values in any case may be subject to criticism; hence it is desirable to compute the directions of the co-tidal and co-range lines at each station, however the values of g may have been obtained (Table VIII).

8. *Deductions from Coastal Currents.*

Near the coasts the currents move backwards and forwards in one direction, generally parallel with the coast; hence $v_0 = 0$. If we take the axis of x to be along the direction of the current (θ) and take the origin of time so that the current is represented by $-u_0 \sin \sigma t$, then, using zero suffixes to denote quantities referred to this orientation of the axes, we can write for equations (6.1), (6.2)

$$\frac{\mathrm{H}}{bu_0}\frac{\partial g}{\partial x} = -\frac{a}{b}\sin\phi + e\cos\phi = \frac{\mathrm{R}}{bu_0}\cos(\psi_0 + 90^\circ),$$

$$\frac{1}{bu_0}\frac{\partial \mathrm{H}}{\partial x} = e\sin\phi + \frac{a}{b}\cos\phi = \frac{\mathrm{R}'}{bu_0}\cos(\psi_0' + 90^\circ),$$

$$\frac{\mathrm{H}}{bu_0}\frac{\partial g}{\partial y} = \cos\phi = \frac{\mathrm{R}}{bu_0}\sin(\psi_0 + 90^\circ),$$

$$\frac{1}{bu_0}\frac{\partial \mathrm{H}}{\partial y} = \sin\phi = \frac{\mathrm{R}'}{bu_0}\sin(\psi_0' + 90^\circ),$$

where $\phi = d - d' - 90^\circ$ and $e = 0\cdot0017cu_0/b$.

The computations required by these formulæ to give ψ_0, ψ_0', and thence ψ, ψ' by adding θ, the direction of the axis $\mathrm{O}x_0$, are very simple. The value of g is obtained from the coastal data. The angle $d' + 90^\circ$ is obtained taking the time at which the current changes from the direction θ to the opposite direction, in hours after high water at Dover, and multiplying that number by 29.

The directions of the co-tidal lines and co-range lines for a large number of points along the coasts have been computed and utilised.

9. *Coastal Elevations.*

Harmonic constants for M_2 are only known at a few stations round the coasts, and the tidal data at coastal stations consist of :—

H.W.F.C. = the time-interval between the local common transit of sun and moon at syzygy and the succeeding high water at the place.

L.W.F.C. = the corresponding low water interval.

M.H.W.S. = Height of mean high-water springs, relatively to chart datum, which is approximately low-water springs.

M.H.W.N. = Ditto for mean high-water neaps.

M.L. = mean level.

For only a small fraction of the places do we find all the above quantities tabulated, and often only the first and third are given. The simplest procedure for the deduction of M_2-constants is to compare values of the known harmonic constants g with the values of $k = $ H.W.F.C. (in hours) $\times$ 28·984, and this method is of value only if both g and H.W.F.C. are well established. It is necessary, of course, to avoid places like Portland (with double low-waters), Southampton (with double high-waters), and Havre (in a shallow estuary), and ultimately we chose Liverpool, Dover, and Brest as suitable places. Curiously enough, we find that at these places $g - k$ is only $\pm 1^\circ$ and we have accordingly taken $g = k$ throughout, for simplicity. The theoretical relation between H.W.F.C.

and g is not, in any case, very simple as it depends upon the relations of S_2 to M_2, and other constituents such as M_4, MS_4, .., all have influence.

With regard to the coastal elevations H, greater difficulties occur because chart-datum does not necessarily mean low-water springs. Thus, at Penzance (very near Newlyn, for which $H = 5 \cdot 6$ feet) we have $S = $ M.H.W.S. $= 18 \cdot 5$ feet, $N = $ M.H.W.N. $= 14 \cdot 8$ feet, $L = $ M.L. $= 10 \cdot 6$ feet, and we see at once that chart datum is about $2 \cdot 5$ feet below the level of low-water springs, whence values of S, N, L, referred to M.L.W.S. as datum, are $16 \cdot 0$ feet, $12 \cdot 3$ feet, $8 \cdot 1$ feet, respectively; of course, in this reduction we have taken the mean level at about half the value of S. The ratios of H to S, N, L (with $L = \frac{1}{2}S$) at Newlyn thus become $0 \cdot 35$, $0 \cdot 46$, $0 \cdot 71$ respectively; the latter value is, of course, the ratio of H to $S - L$ if $L = \frac{1}{2}S$.

In general, wherever S and L are tabulated we obtain H from the difference, using an appropriate factor.

An alternative method is to compare H with $\frac{1}{2}S + \frac{1}{2}N - L$, since the latter quantity is approximately itself equal to M_2, modified by shallow-water constituents. In practice we found that there is nothing to be gained by this method. With proper reductions, we get the following values :—

	$H/(S-L)$.	H/S.	H/N.	N/S.	$H/(\frac{1}{2}S+\frac{1}{2}N-L)$.
Liverpool	0·74	0·37	0·47	0·78	0·95
Dover	0·76	0·38	0·49	0·78	0·97
Brest	0·68	0·34	0·46	0·74	0·92

These values agree very well with deductions from the known values of M_2 and S_2. We have used interpolated values at intermediate places.

In Table II we give values of H and g, deduced from the Admiralty Tide Tables as explained above. Two columns are given for H: in the first column the values are

TABLE II.—Values of H and g for Coastal Stations.

(*a*)

	H.	H.	g.		H.	H.	g.		H.	H.	g.
Scilly Isles—St. Agnes	—	172	130	Salcombe	154	—	165	Ventnor	104	—	321
,, St. Mary's	171	—	124	Start Point	156	—	165	Nab Tower	138	—	330
,, Tresco .	173	—	127	Dartmouth	152	—	182	Selsea Bill	—	187	341
Land's End	—	244	123	Torquay	147	—	174	Littlehampton . . .	183	—	329
Newlyn*	170	—	136	Exmouth	—	122	187	Shoreham	201	—	332
Penzance	171	—	130	Lyme Regis	122	—	184	Brighton	—	240	326
Lizard Point	149	—	131	Bridport	127	—	175	Newhaven	209	—	326
Coverack	158	—	133	Portland Bill	—	95	191	Eastbourne	239	—	325
Falmouth	164	—	143	Portland*	64	—	194	Hastings	289	—	316
Mevagissey	162	—	147	Needles	—	78?	282	Rye Bay	—	277	329
Fowey	156	—	151	Portsmouth	140	—	334	Dungeness	259	—	328
East Lowe	165	—	157	Ryde	—	146	329	Dover*	214	—	331
Plymouth	—	171	157	Bembridge Point . .	—	150	329	Deal	—	186	327
Devonport	165	—	160	Chichester	—	161	334	Ramsgate*	184	—	341

TABLE II—(continued).

(b)

	H.	H.	g.		H.	H.	g.		H.	H.	g.
Calais	239	—	340	Cherbourg*	188	—	228	Portrieux	350	—	168
Wissant	253	—	338	Omonville	190	—	218	Paimpol	355	—	172
Boulogne	300	—	329	Goury	229	—	206	Ile de Brehat	325	—	174
Le Touquet	—	345?	326	Dielette	280	—	192	La Croix Light	315	—	170
Berck	—	342?	325	Cartaret	335	—	191	Lizardreiux	330	—	170
St. Valery-en-Somme	—	335?	334	St. Germain	—	430?	181	Heaux de Bréhat	305	—	169
Cayeux	345	—	326	Regnéville	—	440?	181	Plougrescan	286	—	160
Le Tréport	345	—	324	Granville	435	—	183	Treguier	304	—	158
Dieppe	314	—	320	Cancale	430	—	182	Ploumaneal	269	—	156
St. Valery-en-Caux	276	—	313	St. Malo*	376	—	178	Ile Lovet	260	—	142
Fecamp	261	—	311	Iles des Ebbiens	—	435?	180	Ile de Bas	243	—	139
Havre*	263	—	286	Braye	187	—	196	L'Abervrach	250	—	132
Honfleur	239	—	258	Casquets	—	200?	196	Porsal	231	—	120
Trouville	240	—	272	St. Peter Port	281	—	192	Ushant	215	—	105
Dives	230	—	264	St. Helier	356	—	186	Ile de Molène	210	—	114
Ouistreham	232	—	275	Rozel	329	—	181	Le Couquet	202	—	107
Courseulles	212	—	258	Les Ecrehoux	341	—	189	Brest*	207	—	108
Port-en-Bessin	220	—	266	Les Minquiers	356	—	185	Camaret	204	—	104
St. Vaast	206	—	258	Iles Chausey	415	—	181	Douarnenez	193	—	104
Barfleur	189	—	252	Erqui	370	—	174	Ile de Sein	180	—	103

(c)

	H.	H.	g.		H.	H.	g.		H.	H.	g.
Crookhaven	106	—	120	Youghal	138	—	151	Wexford	—	53?	212
Skull	106	—	117	Dungarvan	132	—	150	Polldulf	—	41?	202
Cape Clear	—	93	116	Dunmore	134	—	158	Courtown	—	38?	217
Baltimore	108	—	127	Duncannon Fort	134	—	154	Kilmichael Point	—	41?	231
Castletownsend	123	—	126	,, Bridge	148	—	177	Arklow	—	43?	231
Clonakilty Bay	—	122	130	New Ross	139	—	176	Arklow Bank	—	40?	231
Courtmacshessy	119	—	133	Saltees	—	144	164	Wicklow	—	97?	304
Kinsale	132	—	136	Tuskar	—	99?	166	Brayhead	—	134	311
Queenstown*	128	—	150	Carnsore Point	—	99?	174	Dalky Island	—	148	311
Cork	—	143	142	Rosslare Harbour	—	63?	164	Kingstown*	137	—	325
Ballycottin	134	—	142	,, Point	—	53?	188				

TABLE II.—(continued).

(*d*)

	H.	H.	*g*.		H.	H.	*g*.		H.	H.	*g*.
Land's End	—	244	123	Chepstow	—	413	217	Fishguard	—	132	202
Gurnard Head	—	251	130	Newport	428	—	208	Newport	—	130	203
St. Ives	225	—	137	Cardiff	409	—	203	Cardigan	—	130	203
New Quay	—	231	136	Flatholm	390	—	200	New Quay	152	—	217
Padstow	223	—	151	Barry	385	—	196	Aberystwith	148	—	220
Boscastle	—	242	152	Nashpoint	—	361	186	Aberdovey	161	—	227
Bude Haven	—	247	167	Porthcawl	306	—	178	Sarn-y-bach	—	154?	222
Lundy	—	290	152	Port Talbot	—	313	175	Barmouth	161	—	225
Appledore	258	—	173	Swansea	300	—	175	Sarn Badrig	—	143?	217
Morte Bay	—	298	165	Worms Head	—	286	174	Port Madoc Bar	153	—	220
Ilfracombe	292	—	165	Llanelly	264	—	177	Pwllheli	150	—	227
Lynmouth	—	323	175	Burry Port	283	—	175	St. Tudwall Road	146	—	224
Porlock	—	346	183	Ferryside	262	—	168	Bardsey	131	—	222
Minehead	360	—	189	Carmarthen Bar	—	282	166	Port Dinlleyn	126	—	251
Watchet	—	365	190	Tenby	279	—	163	Holyhead	173	—	295
Bridgewater Bar	—	382	198	Caldy Road	—	277	162	Amlwch	—	220	297
Weston	408	—	200	Stackpole Quay	257	—	155	Llanddwyn Island	142	—	268
Walton Bay	—	434	204	Pembroke Dock	218	—	177	Carnarvon	146	—	275
Portishead	440	—	208	St. Anne's Head	208	—	173	Trwyn du	246	—	300
Bristol	—	347	212	Skomar	215	—	171				
Avonmouth*	421	—	201	Smalls Lighthouse	—	230?	174				
Sharpness	—	308	231	Ramsey Sound	—	186?	174				

(*e*)

	H.	H.	*g*.		H.	H.	*g*.		H.	H.	*g*.
Culdaff Bay	—	91	171	Small Isles	—	35	146	Ardrishaig	97	—	349
Londonderry	88	—	233	Loch Crinan	68	—	139	Burnt Isles	—	112	343
Warren Point	—	72	183	Feolin Ferry	74	—	136	Loch Striven Head	—	68?	345
Coleraine	72	—	186	West Loch Tarbert	—	45?	73	Rothsay Bay	112	—	346
Port Rush*	51	—	190	Port Ellen	—	57	145	Greenock*	134	—	346
Skerries	—	49	181	Gigha Sound	—	40	68	Port Glasgow	115	—	26
Ballycastle Bay	—	31	186	Mull of Cantyre	—	45?	306	Largs	—	115?	344
Red Bay	54	—	305	Sanda Island	—	90	338	Mill Port	—	116	344
Carnlough	—	65?	326	Cambeltown	—	91	340	Ardrossan	102	—	344
Maiden Rocks	—	81	310	Lamlash	—	105	342	Troon	101	—	344
Larne	74	—	314	Brodick Bay	—	102	343	Ayre	97	—	344
Sealasaig	—	112	153	Garrock Head	—	108	343	Stranraer	110	—	339
Glengarrisdale	—	117	153	Skipness	—	93	343				
Noamh Island	—	113	146	East Loch Tarbert*	109	—	340				

TABLE II.—(continued).

(*f*)

	H.	H.	*g*.		H.	H.	*g*.		H.	H.	*g*.
Point of Air	273	—	316	Port Carlisle	—	210	352	Donaghadee	122	—	325
Hilbre Island*	298	—	319	Douglas*	226	—	326	South Rock	147	—	318
Liverpool*	305	—	326	Ramsey	215	—	324	Killard Point	149	—	316
,, N.W. Light	—	296	319	Castletown	226	—	323	Ardglass	—	174	319
Formby Point	—	302	307	Port St. Mary	226	—	323	Newcastle	162	—	321
Preston	—	221	329	Calf Sound	—	183	327	Cranfield Point	156	—	319
Ribble Light	300	—	314	Peel	180	—	322	Soldiers Point	158	—	330
Wyre Light	310	—	324	Ayre Point	—	226	324	Pile Light	149	—	316
Lancaster	—	337	326	Annan Foot	—	300	350	Boyne River Entrance	—	129	319
Glasson Dock	—	338	326	Southerness	—	292	344	Balbriggan	—	130	310
Morecambe	308	—	326	Abbey Head	—	270	323	Skerries	—	143	319
Barrow Docks	310	—	324	Kirkcudbright	—	250	323	Rogerstown	—	114	327
Tarn Point	267	—	329	Wigton	—	—	333	Malahide	—	112	327
Whitehaven	262	—	326	Garliestown	—	230	328	Howth	—	143	323
Harrington	—	273	326	Port William	—	207	324	Dublin (Poolbeg Light)	—	143	325
Workington	—	286	328	Mull of Galloway	—	169	326	,, (North Wall)*	134	—	326
Maryport	255	—	332	Port Patrick	—	169	324				
Silloth	278	—	338	Belfast	106	—	307				

obtained from (S − L) and in the second column from the average values of H/S, H/N provided that the values of N/S are approximately correct. If there is any doubt about the values they are marked (?), and all values obtained from S alone are so marked. For places marked * actual harmonic constants at the place are given.

10. *Western Approaches to the English and Irish Channels.*

From the beginning, great difficulty was experienced in integrating from Land's End to Ushant, and it was not until a first approximation to the co-tidal lines had been made that it became possible to criticise the coastal data. These criticisms are discussed in the following account of the basic work done in this region. It was possible to avoid Land's End by integrating from the Scilly Isles to the Lizard; from a suitable point on this line, integrations were made on a line running south to Ushant and the Ile de Sein. Other sectional lines provided cross checks, and the processes of fitting together the various pieces of the puzzle were rather elaborate. In the end a satisfactory network of points was completed, sufficing to describe the tides over the whole region.

We shall denote by s the distance in units of 10^7 cm. along a sectional line. Gradients of ζ_1 and ζ_2 along the line are derived from the components to the East and to the North, given in Table I. We denote by letters $P_1, P_2, \ldots$, various special points (not current-stations) along the lines. We thus get the lines, distances, gradients, etc., as in Tables III to VII; the values of ζ_1 and ζ_2, equal to $H \cos (g + 30°)$, $H \sin (g + 30°)$ respectively, are deductions except where they are obtained from the coastal data in Table II.

TABLE III.—Sectional Lines at Entrance to English Channel.

Line.	Point.	s.	$\partial\zeta_1/\partial s$.	$\partial\zeta_2/\partial s$.	ζ_1.	ζ_2.	H.	g.	Remarks.
I	P_1	0·00	—	—	—152	73	168	124	Junction line IV.
	S 392 . . .	0·11	— 5	3	—153	74	170	124	—
	St. Mary's .	0·31	—	—	—154	75	171	124	From Table II.
	S 402 . . .	0·46	—22	— 7	—160	74	176	125	—
	S 422 . . .	0·67	0	—24	—162	69	176	127	—
	P_2	0·80	—	—	—161	61	172	129	—
	Lizard . . .	1·11	—	—	—141	48	149	131	From Table II.
II	Land's End	0·00	—	—	—160	61	171	129	Cp. Table II.
	P_2	0·11	—	—	—161	60	172	129	From line I.
	P_3	0·36	— 3	—15	—160	57	170	130	—
	P_4	0·86	—22	32	—165	66	178	128	—
	P_5	1·40	—30	52	—176	101	203	120	—
	Ushant . . .	1·80	—	—	—176	120	212	116	Cp. Table II.
	Ile de Molène	1·90	—	—	—170	123	210	114	From Table II.
	Ile de Sein .	2·26	—	—	—123	134	180	103	From Table II.
III	Lizard . . .	0·00	—	—	—141	48	149	131	From Table II.
	S 252 . . .	0·25	— 6	8	—141	51	150	130	—
	C_2	0·41	— 9	26	—142	60	154	127	—
	S 251 . . .	0·56	— 6	40	—143	66	157	125	—
	S 232 . . .	0·94	23	58	—143	83	165	120	See line IV.
IV	P_1	0·00	—	—	—152	73	168	124	From line I.
	S 391 . . .	0·10	— 3	12	—152	75	169	124	—
	S 231 . . .	0·34	3	1	—153	79	172	123	—
	S 232 . . .	0·52	—35	27	—155	83	176	122	See line III.
	S 233 . . .	0·63	— 3	23	—158	88	180	121	—
	C_1	0·85	—44	21	—165	93	188	121	—
V	Lizard . . .	0·00	—	—	—141	48	149	131	From Table II.
	S 253 . . .	0·55	—48	69	—165	48	172	134	—
	S 256 . . .	1·34	—24	55	—195	97	215	124	—
	Porsal . . .	1·59	—	—	—200	115	231	120	From Table II.

TABLE IV.—Sectional Lines West of the English and Bristol Channels.

Line.	Point.	s.	$\partial\zeta_1/\partial s$.	$\partial\zeta_2/\partial s$.	ζ_1.	ζ_2.	H.	g.	Remarks.
VI	S 224 . . .	0·00	—34	17	—140	77	159	121	See line IX.
	P_1	0·86	—15	—10?	—153	74	170	124	From lines I, IV
	B_1	1·54	4	—34	—157	50	165	132	—
	P_6	2·13	—	—	—156	14	157	145	—
	P_7	2·71	—	—	—153	—25	154	159	—
	S 1621 . . .	3·30	41	54	—143	—40	148	166	—
	Saltees . .	3·40	—	—	—139	—35	144	164	—
VII	C. Clear . .	0·00	—	—	— 94	64	112	117	Estimated.
	P_8	0·20	—27	—26	—107	56	121	122	—
	B_2	1·52	—24	— 2	—134	48	142	130	—
	B_1	2·30	—25	5	—157	50	165	132	From line VI.
	P_9	2·84	—11	15	—163	52	171	132	—
	Land's End	3·03	—	—	—164	57	173	131	Cp. II and Table II.

TABLE IV—(continued).

Line.	Point.	s.	$\partial\zeta_1/\partial s$.	$\partial\zeta_2/\partial s$.	ζ_1.	ζ_2.	H.	g.	Remarks.
VIII	B_2	0·00	— 7	25	—134	48	142	130	From line VII.
	S 223 . . .	1·09	1	20	—139	72	145	123	—
	S 222 . . .	1·76	30	32	—125	88	152	115	See line IX.
IX	S 256 . . .	0·00	64	10	—195	97	215	124	From line V.
	P_6	0·38	—	—	—176	101	203	120	From line II.
	C_1	0·53	57	25	—168	103	196	119	See line IV.
	S 224 . . .	1·55	7	—14	—144	88	168	119	See line VI.
	S 222 . . .	2·60	36	—12	—125	70	143	121	See line VIII.

TABLE V.—Sectional Lines Across the English Channel.

Line.	Point.	s.	$\partial\zeta_1/\partial s$.	$\partial\zeta_2/\partial s$.	ζ_1.	ζ_2.	H.	g.	Remarks.
X	S 493 . . .	0·00	— 85	— 52	— 72	— 67	98	193	—
	S 261 . . .	0·42	— 45	— 94	— 98	—100	140	196	—
	S 272 . . .	0·69	— 56	— 96	—117	—122	169	196	—
	Casquets . .	0·88	—	—	—135	—140	195	196	—
	Inter-	1·02	—	—	—165	—159	—	—	—
	polated	1·07	—	—	—180	—193	—	—	—
	values;	1·15	—	—	—209	—188	—	—	—
	approx.	1·24	—	—	—229	—198	—	—	—
	only	1·27	—	—	—230	—193	—	—	—
		1·61	—	—	—296	—178	—	—	—
		1·75	—	—	—296	—178	—	—	—
	Erqui . . .	2·06	—	—	—339	—151	370	174	Table II.
XI	S 542 . . .	0·00	— 51	—212	70	— 32	76	305	See text.
	—	0·21	—	—	56	— 70	89	291	—
	S 271 . . .	0·41	— 15	—141	46	—101	111	265	—
	—	0·65	—	—	39	—137	141	256	—
	S 273 . . .	0·89	150?	—109?	37	—172	176	252	—
	Barfleur . .	1·00	—	—	39	—185	189	252	Table II.
	St. Vaast .	1·11	—	—	64	—196	206	258	Table II.
XII	S 543 . . .	0·00	143	— 9	118	— 23	119	319	See text.
	—	0·28	—	—	156	— 31	159	319	—
	—	0·56	—	—	190	— 39	194	318	—
	S 263 . . .	0·83	108	8?	222	— 45	226	319	—
	—	1·11	—	—	253	— 48	257	319	—
	—	1·39	—	—	283	— 50	286	320	—
	Dieppe . .	1·67	82	— 16	310	— 54	314	320	Table II.
XIII	Pevensey . .	0·00	—	—	259	— 23	260	325	See text.
	S 361 . . .	0·08	45	— 53	263	— 26	264	324	—
	—	0·33	—	—	273	— 32	275	323	—
	P_{10}	0·59	—	—	285	— 31	287	323	—
	—	0·85	—	—	300	— 40	303	322	—
	Dieppe . .	1·11	8	— 90	310	— 54	314	320	Table II.

TABLE VI.—Sectional Lines down the Centre of the English Channel.

Line.	Point.	s.	$\partial\zeta_1/\partial s$.	$\partial\zeta_2/\partial s$.	ζ_1.	ζ_2.	H.	g.	Remarks.
XIV	P_{11}	−2·88	− 36	− 32	−154	90	179	120	Line IX.
	S 233 . . .	−2·68	− 49	1	−158	86	180	121	Line IV.
	P_{12}	−2·36	− 29	−54	−169	80	186	125	Line II.
	P_{13}	−2·10	− 58	−120	−175	64	186	129	Line V.
	S 254 . . .	−1·50	?	?	−188	− 25	190	158	—
	—	−1·11	—	—	−185	− 75	200	172	—
	S 255 . . .	−0·72	35	− 44	−170	− 99	196	180	—
	—	−0·36	—	—	−138	−105	174	187	—
	S 261 . . .	0·00	111	16	− 98	−100	140	196	Line X.
XV	S 261 . . .	0·00	115	− 19	− 98	−100	140	196	Line X.
	—	0·42	—	—	− 26	−109	112	227	—
	S 271 . . .	0·83	176	60	46	−101	111	265	Line XI.
	—	1·26	—	—	136	− 72	154	302	—
	S 263 . . .	1·68	119	67	222	− 45	226	319	Line XII.
	P_{10}	2·27	—	—	285	− 34	287	323	Line XIII.
	Berck . . .	2·81	—	—	304	− 27	305	325	See Table II.

TABLE VII.—Sectional Lines in Irish Sea.

Line.	Point.	s.	$\partial\zeta_1/\partial s$.	$\partial\zeta_2/\partial s$.	ζ_1.	ζ_2.	H.	g.	ψ.	ψ'.
1	Skerries	0·00	—	—	173	−100	200	300	—	—
	1	0·09	58	129	179	− 86	199	304	46	316
	2	0·24	29	84	190	− 68	201	310	40	310
	3	0·49	19	66	201	− 46	206	317	37	306
	4	0·64	−12	99	205	− 34	207	320	9	282
	Calf of Man	0·73	—	—	203	− 25	205	323	—	—
2	Rockabill	0·00	—	—	149	− 25	150	320	—	—
	1	0·09	21	−42	152	− 29	154	319	103	18
	2	0·29	18	− 3	160	− 32	162	319	45	354
	3	0·63	59	7	176	− 32	178	320	35	310
	4	0·83	72	29	191	− 29	192	321	18	291
	Calf of Man	0·92	—	—	203	− 25	205	323	—	—
3	Peel	0·00	—	—	176	− 21	178	323	—	—
	1	0·09	9	46	—	—	—	—	19	295
	2	0·17	− 5	57	—	—	—	—	7	283
	3	0·32	−31	95	—	—	—	—	357	273
	4	0·40	−56	127	—	—	—	—	352	267
	Mull of Galloway . .	0·49	—	—	147	0	170	330	—	—
4	Ayre Point	0·00	—	—	203	− 21	205	323	—	—
	1	0·09	−17	137	—	—	—	—	347	264
	2	0·15	37	141	—	—	—	—	6	283
	3	0·21	69	178	—	—	—	—	12	289
	Burrow Head . . .	0·30	—	—	225	0	225	330	—	—

TABLE VII—(continued).

Line.	Point.	s.	$\partial\zeta_1/\partial s$.	$\partial\zeta_2/\partial s$.	ζ_1.	ζ_2.	H.	g.	ψ.	ψ'.
5	Ayre Point	0·00	—	—	203	— 25	205	323	—	—
	1	0·09	96	—11	215	— 26	217	323	324	215
	2	0·26	97	— 5	234	— 28	236	323	328	250
	3	0·41	54	6	248	— 28	250	323	341	260
	St. Bees Head . . .	0·50	—	—	257	— 27	260	324	—	—
6	Calf of Man	0·00	—	—	204	— 25	205	323	—	—
	Langness	0·13	—	—	229	— 28	230	323	—	—
	1	0·22	172	—35	237	— 31	238	322	19	295
	2	0·44	98	—23	256	— 37	257	322	21	295
	3	0·75	80	15	280	— 39	281	322	353	270
	4	1·01	93	50	299	— 30	300	324	333	248
	Walney Island . . .	1·10	—	—	309	— 24	310	326	—	—

TABLE VIII.—Values of ψ and ψ' at Current Stations.

Station.	ψ.	ψ'.	Station.	ψ.	ψ'.	Station.	ψ.	ψ'.	Station.	ψ.	ψ'.
222	338	273	361	325	270	543	330	275	A_1	306	198
223	299	267	362	263	165	551	356	293	A_2	302	193
224	4	314	363	287	184	552	335	271	B_1	329	60
231	318	324	364	285	183	571	27	300	B_2	311	47
232	318	225	365	289	166	572	347	251	C_1	308	217
233	332	280	366	264	176	1611	54	303	C_2	301	176
234	303	219	367	248	156	1612	137	72	C_3	262	122
235	189	105	371	277	177	1613	290	187	A	353	286
236	239	135	372	221	166	1614	292	188	B	288	135
251	286	193	373	182	160	1615	341	266	C	338	264
252	216	72	391	335	252	1621	218	131	D	325	304
253	315	240	392	12	352	1622	232	137	E	314	245
254	39	336	393	356	227	1731	352	294	F	331	233
255	283	187	401	53	347	1811	40	315	G	302	200
256	310	220	402	339	295	1812	293	224			
261	257	152	403	349	295	1813	300	241			
262	303	204	421	301	310	1951	346	233			
263	320	248	422	304	74	1952	334	218			
271	292	201	491	123	42	1953	342	225			
272	250	151	493	292	180	1954	9	286			
273	298	118	501	261	135	1955	18	332			
341	246	149	502	354	250	1956	75	11			
342	279	172	541	300	230	1961	305	203			
343	314	197	542	328	262	1962	4	262			

The gradients at points P_3 to P_5 are obtained by interpolation between pairs of stations lying close to the line.

The integrations along line I are very simple, and quite definite agreement with terminal values is easily obtained. The range is a maximum about S 422, corresponding to the rapid increase of H round the coast near Land's End. Though P_2 is comparatively near to Land's End, both ζ_1 and ζ_2 are considerably less than the apparent local values at Land's End, as given in Table II.

The values of ζ_1 and ζ_2 along line II are obtained graphically; the curves were drawn to fit the gradients and the terminal data, with such slight modifications as were suggested by line IX. Note the differences between the values of H and *g* deduced for Ushant and those given in Table II.

No great importance is attached to lines III and IV as the observations do not appear to be very good, but both lines agree at S 232, and the values of H and *g* at C_1 are in quite good accordance with those deduced from line IX.

Two very important lines are those running from C. Clear to Land's End and northwards from S 224 to Saltees, on the Irish coast. The latter yields the unimpeachable result (indicated also by the coastal currents) that the co-tidal line through Saltees passes westwards, turns south, and later runs east—that is, the co-tidal line has a considerable curvature outward towards the Atlantic Ocean. At C. Clear we ignore the Tide Tables, for the coastal currents indicate that at C. Clear the value of *g* should be about equal to that for Skull and rather greater than for Crookhaven, while the value of H should be a little greater than at these other points. An estimate of the currents at points P_8 and P_9 has been made from stations S 1611, S 1612, S 1613, S 1614 and from stations S 1731, S 401, S 402, S 403, respectively. Lines VIII and IX yield results at S 222, a station well out at sea. There are certain small discordances between two derived values of H and *g* at one and the same station, but these are of the order of uncertainty in the coastal data. On the whole, the results of the integrations may be considered as very satisfactory.

11. *The English Channel.*

A number of stations lie on a line running approximately down the centre of the English Channel, but it is desirable first of all to use such cross-sectional lines as can possibly be obtained. The usual coastal difficulties are encountered, but information can be obtained along four of these lines, Nos. X to XIII.

A line due north (true) through the Casquets ends at station S 493, just east of Portland Bill, and a number of current-stations are found in this neighbourhood. By interpolating between current-stations data are obtainable for integration along the line from Portland Bill to S 493, with increments 0 and -5 in ζ_1 and ζ_2 respectively. Hence we deduce from Table II values of ζ_1 and ζ_2 at S 493 as given, with some measure of uncertainty, because of the peculiarities of the tides between Portland and Southampton. South of the Casquets, the line passes between places on or near the Channel Islands, and by interpolation between these stations a number of values of ζ_1 and ζ_2 have been deduced. Plotting gradients as usual indicates rapid changes in ζ_1 and ζ_2, principally affecting H rather than *g*, near the Casquets. The value of H deduced from the Casquets is greater than would be indicated even by the incomplete data of the Tide Tables, but an independent justification for this conclusion can be obtained by a rough integration westwards from Braye. The currents are rather strong but we obtain, apart from friction, $\partial\zeta_1/\partial s = 0 \cdot 5$ U, $\partial\zeta_2/\partial s = 0 \cdot 2$ U, where U is the maximum current, which we can take to be about 150 cm. per second (3 knots). With $\delta s = 0 \cdot 13$ we find the values of ζ_1, ζ_2 at Casquets respectively less than those at Braye by 9 and 4, whence, from Table II (b), we obtain $H = 196$, $g = 195$ at Casquets ; this agrees very well with line X.

For sectional lines XI and XII it is necessary to deduce terminal values near the Isle of Wight. At Nab Tower we have $\zeta_1 = 138$, $\zeta_2 = 0$, and along the line to S 543 ($s = 0 \cdot 15$) we compute gradients of ζ_1, ζ_2 at S 543 to be 118, 202, whence $\zeta_1 = 119$, $\zeta_2 = -29$ at S 543 ; another estimate may be obtained from the line Ventnor to S 543, gradients 143, -9, and $s = 0 \cdot 10$, whence $\zeta_1 = 117$, $\zeta_2 = -17$; the average values yield $\zeta_1 = 118$, $\zeta_2 = -23$ at S 543. Also, along the line due east from S 542 to S 543 we have average gradients 215, 59, and with $s = 0 \cdot 15$ we deduce increments of ζ_1, ζ_2 to be 42 and 9 respectively ; whence $\zeta_1 = 76$, $\zeta_2 = -32$ at S 542, corresponding to $H = 82$, $g = 307$; the results of sectional line XI indicate the slightly smaller values of $H = 76$, $g = 305$, at S 542.

Sectional line XI is drawn almost due south to C. Barfleur, but we have deviated it slightly to pass through Barfleur and St. Vaast, for the terminal data are very indefinite because of the strong currents off C. Barfleur, and the conditions can only be satisfied by a rapid change in ζ_1, near C. Barfleur.

For sectional lines XII and XIII we use Dieppe as a terminal point, and the coastal currents off the straight coast near Dieppe are of value. By considering the rates of increase of ζ_1 and ζ_2 along the coast a mean current was deduced from the dynamical equations, with a maximum velocity of 80 cm. per second and turning from an easterly direction to a westerly one at about 10 minutes before H.W.D. ; this agrees fairly well with the data in "Tides and Tidal Streams," if we consider the stretch of coast between Tréport and Valery-en-Caux.

Line XIII terminates at Pevensey between Eastbourne and Hastings, and interpolation gives $H = 260$, $g = 325$ at Pevensey ; we ignore the data for Hastings, for the coastal currents indicate that g progressively increases from Eastbourne to Dover.

Finally, two lines down the centre of the Channel have been used as a check upon the results of the sectional lines across the Channel. Line XIV is drawn through S 233, S 254, S 255, to S 261, and line XV is drawn from S 261 through S 271, S 263 to Berck. After slight modifications of the first approximations yielded by the cross-lines, the values of ζ_1 and ζ_2 along XIV and XV are in almost perfect agreement with the gradients indicated by the currents.

12. *St. George's Channel : Tuskar Rock to St. David's Head.*

At a number of lightships in this Channel the current data have been tabulated at every hour from 6 hours before to 6 hours after high water at Dover, but for cross-sectional integrations the data are not so accurate; the directions of linear currents, and the maximum velocities only being given, the currents are stated to be slack throughout the region between $\frac{1}{4}$ hour before until $\frac{3}{4}$ hour after high water at Dover. On the "first sectional line,"* from Tuskar Rock to St. David's Head, four stations are given, which we shall denote by a_1 to a_4, and the information has been supplemented by special observations at stations A_1 and A_2. These later observations are generally consistent with the former if we take the earlier limit of $\frac{1}{4}$ hour before H.W.D. for the time of turning. We then deduce results as follows :—

Point.	s.	$\partial\zeta_1/\partial s$.	$\partial\zeta_2/\partial s$.
Tuskar	0·00	−121	− 50
a_4	0·09	− 21	−127
A_2	0·16	− 12	− 88
a_3	0·24	− 9	−117
A_1	0·43	− 21	− 93
a_2	0·45	− 32	− 90
a_1	0·58	− 29	−162
St. David's Head	0·67	− 16	− 75

The gradients given for St. David's Head are, of course, due to coastal currents, data from Bais Bank; the currents near Tuskar turn about 2 hours before high water Dover, hence the somewhat exceptional values of the gradients there. The difficulties in dealing with this line arise from the poor coastal observations at or near the terminal points. From Waterford, on the South Coast of Ireland to Kingstown, on the East Coast, only the values of M.H.W.S. and M.H.W.N. are given, and most of these seem to be known only to the nearest foot or 0·5 foot, while the times are given to the nearest hour or half-hour in some instances. On the Welsh side, there is no really trustworthy information (except possibly at Fishguard) between New Quay and Skomar. At St. David's Head, for instance, H.W.F.C. must be between 06:00 (Ramsey Sound) and 06:58 (Fishguard), with M.H.W.S. lying between 17·0 feet and 12·4 feet.

Important results arise from the consideration of the directions of the co-tidal lines and co-range lines. The currents at the stations a_1, a_2, a_3, a_4 are so similar in direction and magnitude that we can take average values; similarly for A_1 and A_2; whence we deduce :—

g.	Stations.	ψ.	ψ'.	R.	R'.
°		°	°		
180	A	281	178	96	120
	a	279			
210	A	313	205	112	103
	a	310			

The sectional line is at an angle of about 330°.

* "Tides and Tidal Streams," p. 186.

The numerical values of $\partial\zeta_1/\partial s$, $\partial\zeta_2/\partial s$ are such that we can at once assert that ζ_1 and ζ_2 change, for all practical purposes, linearly with s, and their increments for Tuskar to St. David's Head will be -14 and -66 respectively. Let the values at the centre of the line be H_1 and H_2. Then we have to find by trial such values of H_1 and H_2 that they will give values of H and g, on the coast, reasonably in conformity with the Admiralty Tide Tables, and such that the direction of the co-tidal line and co-range line through the mid point are correct. We obtain results as follows :—

Mid Point.				Tuskar.		St. David's Head.	
H.	g.	H_1.	H_2.	H.	g.	H.	g.
100	° 180	-87	-50	82	162	125	191
	210	-50	-87	69	201	132	214
150	180	-130	-75	129	168	174	188
	210	-75	-130	118	205	182	213

Variation in H makes only little difference in the resulting values of g. It is clear that the co-tidal line through the centre of the sectional line must strike the Irish coast to the north of Tuskar, and the Welsh coast to the south of St. David's Head ; and since the sectional line is along $\theta = 330°$, then ψ is a little, and only a little, less than 330°. The latter argument would point to $g = 210°$ approximately, at the centre of the line, but this appears to be undoubtedly too high for the coastal data ; the highest value we can allow for the centre of the line is about 195°. The co-range lines definitely indicate $H = 110$, very nearly, at the centre of the sectional line, whence we deduce :—

Tuskar	$H = 85$	$g = 183$
St. David's Head	$H = 138$	$g = 202$

The directions of the co-tidal lines are not altogether satisfactory by this compromise, but examination of a few observations at A_1 and A_2 which were made near the bottom shows that the currents at the bottom are a little to the left of those at the top and turn a little later, and the corrections due to each tend to make ψ larger and more in conformity with the run of the co-tidal lines.

It has already been stated that on the Irish coast the tidal data do not appear, at first sight, to be worthy of much confidence, but the values deduced for Tuskar are in some agreement, as it happens, with the value of Table II (c). On the Welsh coast, we have ample evidence that the values of H for Smalls Lighthouse and Ramsey Sound in Table II (d) are much too high, for the coastal currents show beyond all question that the range of tide steadily decreases from Swansea to Fishguard, and a first approximation to the co-range lines shows that the range is a minimum near Fishguard. It appears, therefore, that the values deduced for St. David's Head are reasonably correct.

13. *St. George's Channel: S. Arklow to Bardsey.*

The four current-stations on this line yield the values of gradients as follows, ψ and ψ' being computed for $g = 240°$, $d = 270°$:—

Point.	s.	$\partial\zeta_1/\partial s$.	$\partial\zeta_2/\partial s$.	ψ.	ψ'.	H.	g
S. Arklow	0·00	—	—	—	—	35	228
	0·09	26	−150	354	252	48	235
	0·28	38	−128	351	251	73	241
	0·53	17	−117	357	260	106	244
	0·72	−28	−112	15	282	125	243
Bardsey	0·81	—	—	—	—	135	242

The values of H, g are deduced with the help of the following considerations. If H_1, H_2, are the values of ζ_1 and ζ_2 at the centre of the line then the terminal values are $H_1 - 12$, $H_2 + 55$ at S. Arklow and H_1, $H_2 - 46$ at Bardsey; and since the gradients of H in this instance are those of ζ_2 we should get an increment of 101 in H from Arklow to Bardsey. The coastal values of H, g, at S. Arklow and Bardsey are given in Tables II (c), II (d), as 40, 231° and 131, 222° respectively. By taking various values of H_1 and H_2 we deduce that the most appropriate terminal values are 35, 228° and 135, 242° respectively. On the Irish side these values fit quite well with the general data; on the Welsh side $g = 222°$ is very unlikely and the revised value of 242° is in much better accordance with the coastal values as a whole.

14. *St. George's Channel: Kish to Holyhead.*

On this sectional line, we similarly deduce :—

Point.	s.	$\partial\zeta_1/\partial s$.	$\partial\zeta_2/\partial s$.	ζ_1.	ζ_2.	H.	g. °	ψ.	ψ'.
Kish	0·00	—	—	140	− 17	140	323	—	—
	0·09	5	− 82	140	− 24	141	320	63	348
	0·28	9	−100	142	− 42	140	313	55	341
	0·53	37	− 98	145	− 68	152	304	34	318
	0·72	23	−123	148	− 89	170	298	33	310
North Stack .	0·81	—	—	150	−100	180	295	—	—

The values for Kish are obtained from the general run of the co-tidal and co-range lines from Kingstown. There is an excellent agreement between coastal elevations and gradients.

15. *Irish Sea.*

A number of sectional lines have been dealt with in this region, and the results are given in Table VII. Apart from small amendations of coastal data no difficulties have been experienced in deducing the elevations from coast to coast.

16. *North Channel.*

An amphidromic point has been located in this channel. Its position was located in the first place by assuming the values of ζ_1 and ζ_2 to be on two planes, whence the zero lines were deduced, of which the intersection gave the required point. By studying the directions of the co-tidal lines for assumed values of g it was possible to satisfy all the conditions.

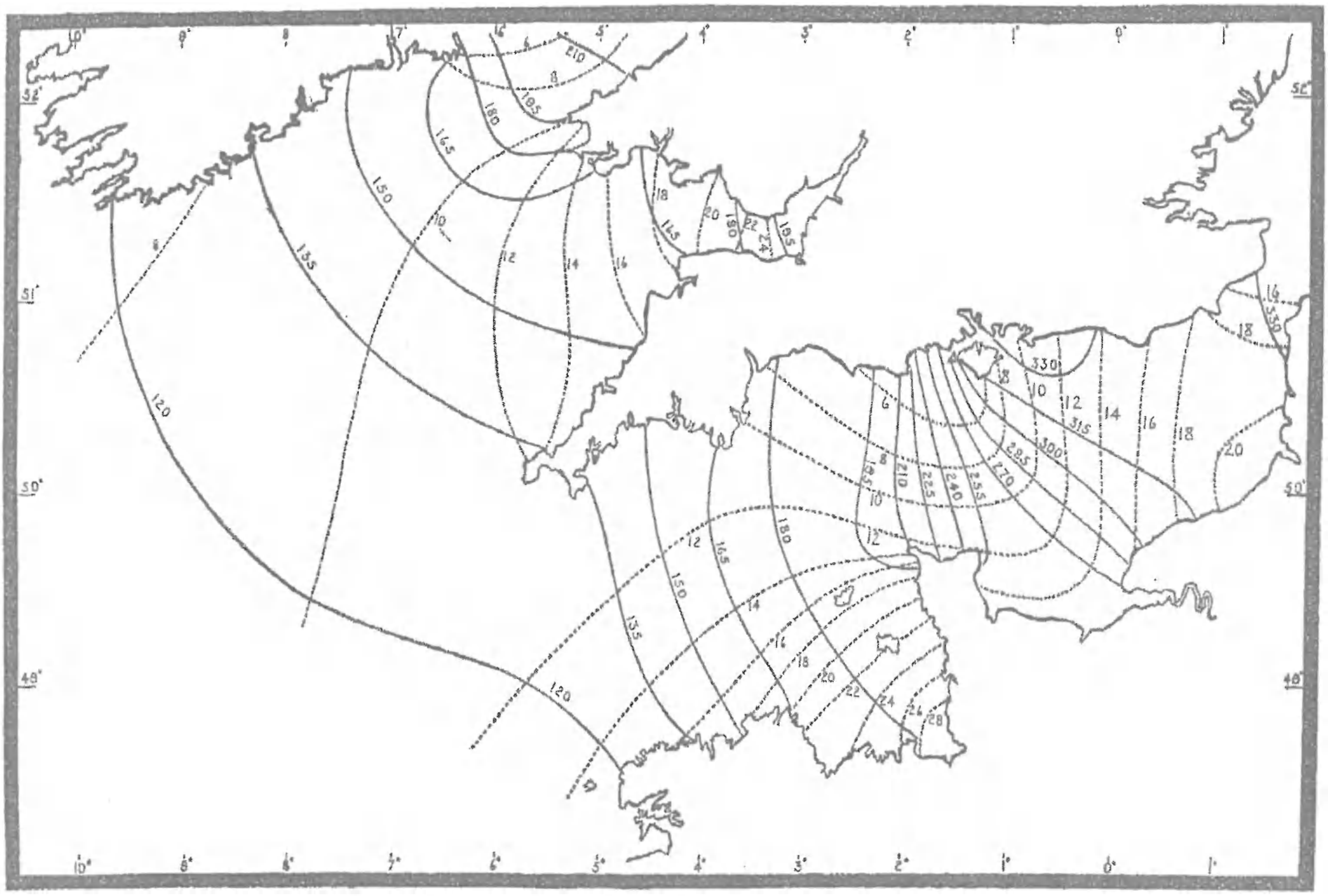

Fig. 3.—Co-tidal and co-range lines in the English Channel. (Reproduced from Admiralty Chart No. 301 by permission of the Controller of H.M. Stationery Office and the Hydrographer of the Navy.)

17. *The Final Charts.*

Having obtained all the data along the sectional lines and having drawn in all the appropriate directions of the co-tidal and co-range lines, it was a simple matter to construct the charts, though it is perhaps needful to say that a large number of considerations have had to be borne in mind during the process, and that it is impossible to discuss all the details. In the Straits of Dover the co-tidal line for 330° differs slightly from the line given on the chart for the North Sea, owing to improved treatment of the data. Co-tidal lines are represented by full lines, and co-range lines by broken lines; the phases are given in degrees and the amplitudes or semi-ranges in feet.

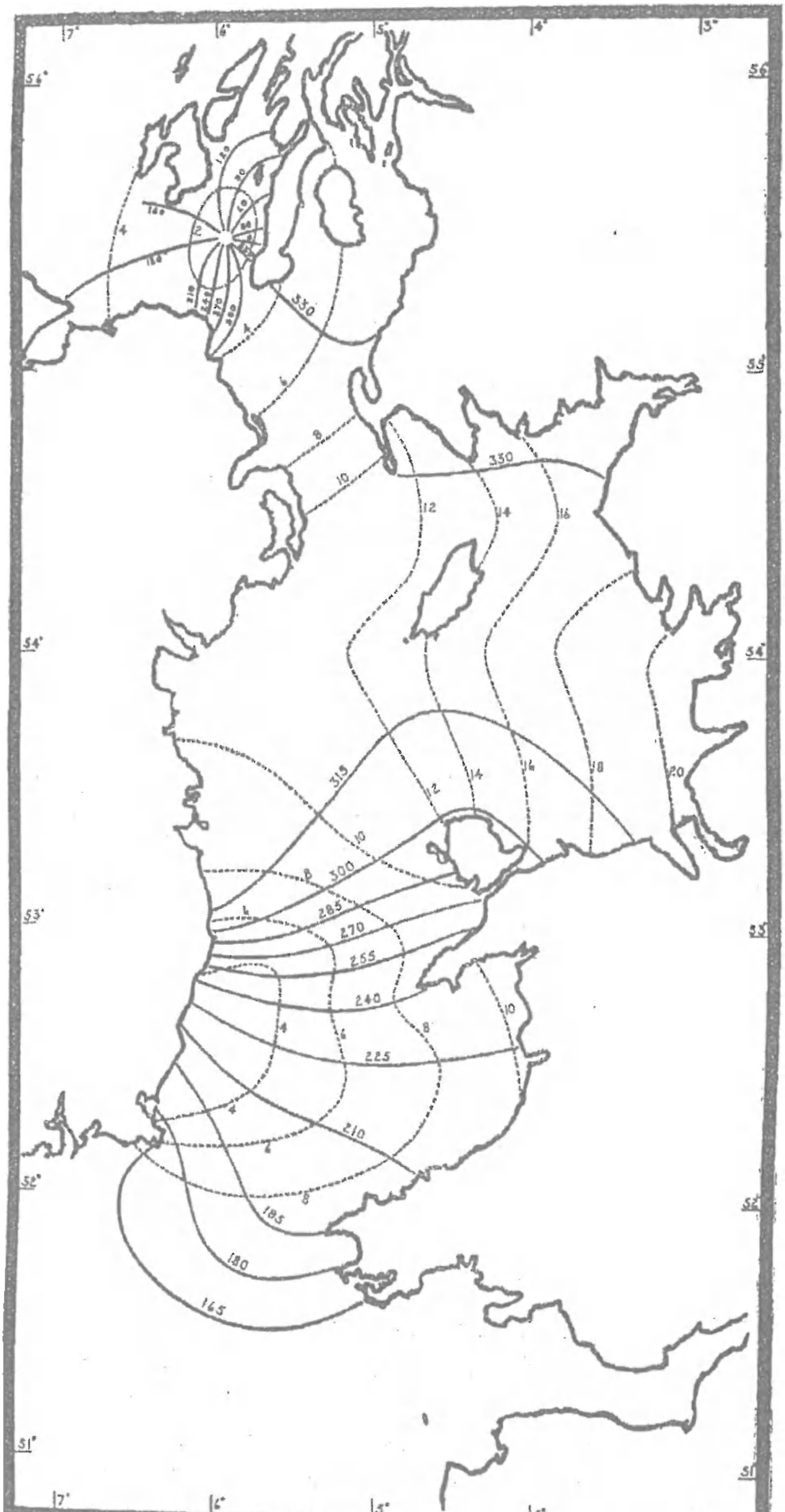

Fig. 4.—Co-tidal and co-range lines in the Irish Channel. (Reproduced from Admiralty Chart No. 301, by permission of the Controller of H.M. Stationery Office and the Hydrographer of the Navy.)

Summary.

Details are given of the method of construction of two new tidal charts for the English and Irish Channels, prepared for the Hydrographic Department of the Admiralty, and supplementing a similar chart for the North Sea.

The use of tidal currents for the calculation of gradients of the surface at a number of stations along a sectional line, together with the known coastal data, by graphical and numerical integration, enables the surface elevation to be computed from coast to coast. By a network of such lines the errors of observation are smoothed out and the resulting charts conform to the observational data and to the dynamical equations of motion. Previous charts have not satisfied the dynamical conditions. The new charts show that popular conceptions of the propagation of tides are largely erroneous, as the co-tidal lines do not bend inwards to the channel; that is, the time of high water is not accelerated in the centre of the channel, relatively to the times on the coasts. At certain places the new charts indicate errors of as much as 3 hours in the times of high water as shown by older charts.