

# Numerical modelling of wave and non-Newtonian mud interaction

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## Introduction

An extensive experiment and field measurements show surface waves attenuate significantly at muddy coasts and estuaries. Moreover, strong waves can force the mud particle movements resulting the mass transport of fluid mud layer. One noteworthy element in modelling these phenomena of wave-mud interaction is a description of the rheology of the mud. Owing to the geo-environmental conditions and diverse chemical/biological compositions, it is reasonable to expect that mud at different localities and in different periods of time may exhibit vastly different rheological behaviours. Different constitutive equations of viscoelastic (e.g. Kelvin-Voigt, Maxwell and Jeffreys), viscoplastic (e.g., Bingham, Casson and Herschel-Bulkley plastics) and visco-elastic-plastic models as well as site-specific empirical formulas have been widely used in the past to represent the rheological behaviour.

The objective of this study is to implement various viscoelastic and viscoplastic rheological models of the non-Newtonian fluid mud in numerical modelling of wave-mud interaction. The two-dimensional numerical model of Hejazi *et al.*, (2013), originally developed for two-layer of viscous mud-fluid system, is extended to both viscoelastic and viscoplastic non-Newtonian continua in order to simulate the wave and non-Newtonian mud interaction.

## Numerical solution

The numerical model uses a structured non-orthogonal curvilinear staggered mesh and is capable of simulating non-homogeneous, stratified flow fields. The linearized equations of motion for a compressible viscoelastic medium, including both viscous and elastic responses, and FVM non-hydrostatic RANS equations of viscoplastic fluids are solved using the projection method.

The stress tensor in momentum equations of viscoplastic mud is related to the rate-of-distortion,  $\dot{\gamma}$ , and usually is expressed as follows:

$$\tau = \mu \dot{\gamma} \quad (1)$$

For a Newtonian fluid,  $\mu$  presents a constant viscosity. However, a viscosity function for a general viscoplastic fluid is given by:

$$\mu = \mu(|\dot{\gamma}|) \quad (2)$$

where  $|\dot{\gamma}| = \sqrt{0.5 \text{tr}(\dot{\gamma}^2)}$  is the second invariant of the rate of the deformation tensor. Then the constitutive equation for viscoplastic fluids may be written as follows:

$$\tau = \mu(|\dot{\gamma}|)\dot{\gamma} \quad (3)$$

where  $\mu(|\dot{\gamma}|)$  is referred as the effective viscosity. Hence the viscosity varies in space and time according to the variation of the deformation-rate. Therefore in each time step, the second invariant of the deformation-rate tensor is calculated from the velocity field and then the corresponding viscosity of each cell is determined according to the rheological equation. The overall numerical solution for viscoplastic mud is obtained in two steps. In the first step the pressure gradient terms are omitted from the momentum equations, and the unsteady equations are advanced in time to obtain a provisional velocity field. The provisional velocity is corrected by accounting for the pressure gradient and the continuity equation in the second step. The water elevation is computed through the solution of the free surface equation obtained from the application of normal and tangential dynamic boundary conditions, and the integration of the continuity equation across the depth. The interface elevation is obtained by the application of the integration of continuity equation over mud depth and the kinematic boundary conditions at bed and the interface.

## Results

To test the capability of the modified numerical model for simulating the relatively strong interaction between non-linear wave and uneven bottom, periodic wave propagation over a submerged bar was modelled first. The comparisons of the free-surface elevation simulations with the measured values of the experiments carried by Beji and Battijes (1994), at seven wave gauge locations, show good agreements. As an example of the model applications on interaction of wave and non-Newtonian fluid mud, the simulated results have been compared with the laboratory

measurements reported by Hsu *et al.*, (2013). The rheological experiments indicate that mud behaves as a non-Newtonian fluid, where the yield stress and the shear thinning effect depend on both shear rate and mud concentration. The suggested rheological model exhibits hybrid properties of a Bingham and pseudo-plastic fluid. Numerical prediction of surface wave elevation at three different wave gauges in comparison with measured data show good agreements (Fig. 1). Fig. 2 shows the surface wave height variation across the mud layer. The linear model of Dalrymple and Liu (1978) with an exponential wave height decay has also been plotted. Comparisons of the velocity profile in the water and mud layers for the predicted results of the model against experimental data and linear model have been presented in Fig. 3.

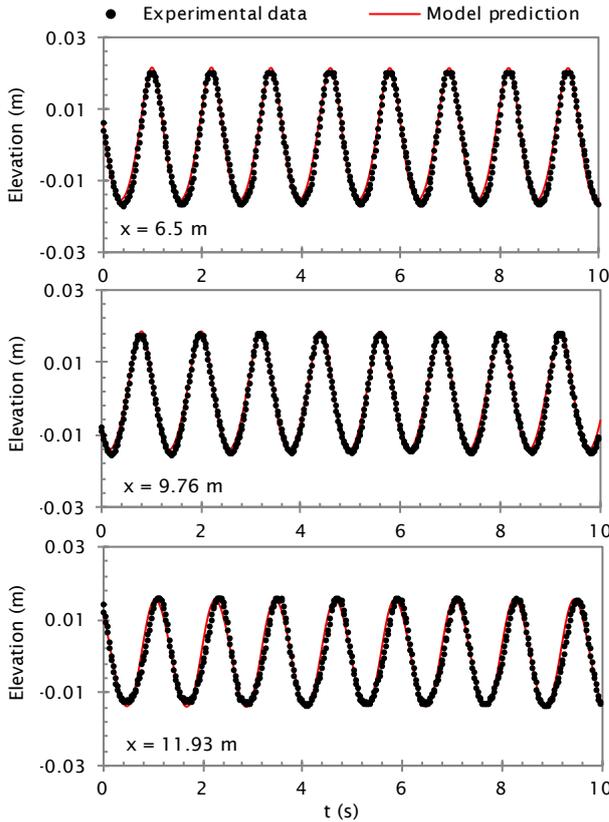


Fig. 1. Free-surface elevation at three wave gauge locations ( $H_0 = 0.04\text{m}$ ,  $T = 1.2\text{s}$ , water depth =  $0.24\text{m}$ , mud depth =  $0.06\text{m}$ , mud density =  $1420\text{kg/m}^3$ , bulk density =  $2500\text{kg/m}^3$ ).

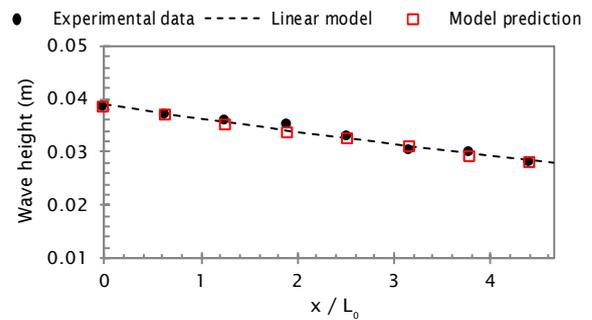


Fig. 2. Wave attenuation ( $H_0 = 0.04\text{m}$ ,  $T = 1.2\text{s}$ , water depth =  $0.24\text{m}$ , mud depth =  $0.06\text{m}$ ).

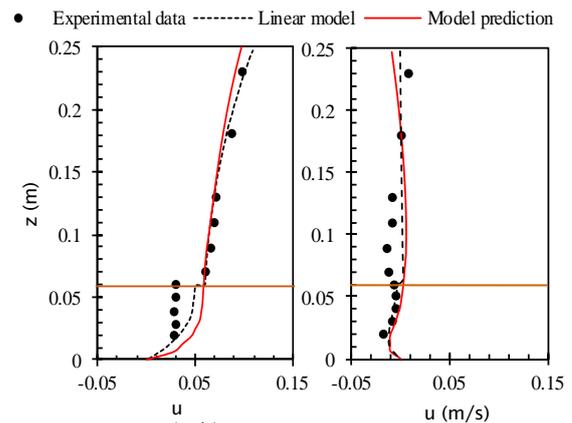


Fig. 3. Velocity profiles at wave crest and flow reversal ( $H_0 = 0.04\text{m}$ ,  $T = 0.9\text{s}$ , water depth =  $0.24\text{m}$ , mud depth =  $0.06\text{m}$ ).

## Conclusions

A 2DV numerical model has been proposed for the simulations of a two-layer mud-fluid system, where the mud behaviour is considered like a non-Newtonian fluid. The decay rate and wave shape predicted by the numerical model are in agreement with the measured data. Above the water-mud interface, where the flow is not affected by the rheological stress, the linear model proposed by other researchers and the present model agree well with the measured velocity profiles. The velocity decreases significantly approaching the bottom mud layer with a clear boundary at the water-mud interface. The results also show that although the present model affords for the simulation of non-linear wave propagation, the linear model also provides satisfactory results.

## References

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