ABSTRACT

This paper focuses on the methodology used to determine the design wave impact loading on a pedestrian walkway on top of a breakwater. The wave loading on the walkway is an impact loading with high forces acting for short times. The actual impact loading is depending on the characteristics of the overtopping waves and on the dynamic characteristics of the pedestrian walkway.

In order to assess the impact effect of the overtopping waves on the structure tests are run on a physical 1/20 scale model is built and run in the large wave flume of Flanders Hydraulics Research Laboratory (Antwerp, Belgium). Each run generates a series of waves in the flume shoaling up the model breakwater with the model pedestrian walkway situated on top of it.

The physical model cannot represent all the relevant walkway material properties like mass and elasticity, such that the forces in the (flexible) connections between walkway and breakwater could be derived accurately for the prototype design. Therefore the model is solely used to assess the integral wave impact loading on the walkway, whose boundary conditions in the model have been adapted for measuring time series of horizontal and vertical reaction forces with load cells. These time series are converted to actual wave loading on the model by using a numerical model for the dynamic response of the scale model, thus taking into account the dynamic characteristics of the scale model. Doing so, a numerical filter is derived to eliminate possible eigenmodes due to the characteristics of the model set up.

Time series of the reaction forces were used as input for the numerical filter. Eventually the design values for the wave impact loading in prototype are derived using Froude scaling (1/8000 = 1/20^3).

INTRODUCTION

At the port of Ostend, Belgium, a pedestrian walkway is being designed on top of a new low-crested breakwater. The crest of the breakwater is situated only ca. 2 m above design storm water level (see Figure 1). Design storm conditions at the toe of the structure are a significant wave height Hs of 3.6 m and a peak period Tp of 11 s.
Important wave loading can be expected on the pedestrian walkway. The pedestrian walkway is conceived as a structure in stainless steel elements: a deck with a width of 8 m situated 2 m above the breakwater crest, two rows of supporting piles and a balustrade of ca. 1 m high.

**HYBRID MODELLING DETERMINATION OF WAVE IMPACT DESIGN LOADING**

Today there are no systematic design standards for wave impact loading on marine constructions situated on top of a breakwater. Because of the complex hydraulic phenomena involved, a physical scale model is built in a laboratory. During simulation runs in the laboratory test set up, scaled design waves are generated and recordings are made of their impact (components) on a scale model of the actual designed prototype construction structure to be built is recorded.

To enable these measurements, the connections of the scale model with the outer world necessarily differ from those in the prototype. For practical constructive reasons relevant material properties like mass and elasticity cannot be in consequent agreement (to scale) with the prototype either.

Nevertheless reliable estimates of the loading to be considered for the prototype design are to be derived from the measurements in the laboratory scale model.

Therefore a numerical model is drawn up of the scale model’s dynamic behavior, which may cause disturbances in the recordings due to possible eigenoscillations at the (possibly inadequately scaled) natural frequencies of the scale model. Applying the filter operator derived from this numerical model to the measured wave impact recordings eventually yields back scaled time series for reliable extreme (design) value estimation of actual wave impact loading on the prototype.

**PHYSICAL SCALE MODEL**

The physical model is built and run in the large wave flume of Flanders Hydraulics Research Laboratory (Antwerp, Belgium). Each run generates a series of waves in the flume shoaling up the model breakwater with the model pedestrian walkway situated on top of it (see Photo 1). The length scale is 1/20 and the time scale is \((1/20)^{0.5} = 1/(4.472..)\). Forces are scaled down with a factor \((1/20)^3 = 1/8000\).

Experiments include runs with different heights of the walkway and different roughness conditions of the breakwater. In general tests are run for different wave heights (below and above the design value) to allow for interpolation of the measured forces at wave impact for the design wave height.
For each test run are analyzed:

- Measured values of the incident wave height at the toe of the breakwater
- Time series of the force measurements (at three vertical and two horizontal connections)
- All recorded peak values of measured forces above a certain threshold value
- Correlated peak values of the total horizontal action, the upward vertical action, the downward vertical action and the horizontal action at the moment of maximum downward action, which are caused by the same wave impact but with a certain mutual time lag.

The number of force peaks, the largest force peaks and the variance on the measurements are computed. Subsequently statistical analysis of the force peaks is performed.

**NUMERICAL MODEL**

**Introduction – methodology**

The modeling performed is based on the assumption that the scale model behaves as a rigid body towards its (flexible) fixations, such that the loading-displacement behavior may be represented by considering only a limited (finite) number of degrees of freedom (d.o.f.) of the scale model motions due to wave impact. This implies only a restricted number of possible eigenoscillations of the scale model.

The research presented aims at identifying the manifestation of possible eigenmodes in the laboratory force recordings under wave impact and assessing the need to eliminate such disturbing signal components for reliable design value estimation from the laboratory recordings.

The following subjects will be briefly addressed in some more detail next.

- Linear elastic loading-displacement behavior model
- Kinematic relations and force equivalents
- Stiffness matrix
• Mass matrix
• Dynamic model equations and filter operator
• Filter application to measured force recordings – discussion of results

Linear elastic loading-displacement behavior model

A linear elastic modeling is performed of the laboratory scale model with a common static structural analysis software package (ESA-PrimaWin), including the geometry, boundary conditions and material properties (specific mass, elasticity, Poisson ratio) of the actual components of the test set up.

Subsequently a unit displacement \( u_i = 1 \text{ mm} \) is imposed at each of the 5 fixations \( (i=1,...,5) \) of the scale model to the outer world. In each case the reaction forces \( R_j \) are computed along all of the 5 linear spring (pendulum) elements \( (j=1,...,5) \) connecting the scale model to the outer world, yielding a 5x5-matrix \( L \) (with elements \( L_{ij} \) at the \( i^{th} \) row and the \( j^{th} \) column = \( R_j \) for each case \( i \)) representing an approximative linear elastic model of the scale model loading-displacement behavior.

The following laboratory model properties are required to justify the simple model approximations mentioned above:

- The elasticity of the connecting (pendulum) spring elements is relatively large compared to the relatively stiff ('rigid') scale model.
- The mass of the connecting pendulum elements is relatively small compared to the mass of the actual scale model.

In such cases the reaction forces perpendicular to the connecting spring elements may be neglected. Conversely displacements perpendicular to the connecting elements will not generate significant reaction force contributions in the direction of the connecting spring elements.

Kinematic relations and force equivalents

Assuming a rigid body, the displacements \( u_i \) at each of the 5 fixations \( (i=1,...,5) \) of the scale model to the outer world are linearly geometrically related to the central displacement components \( v_j \) \( (j=1,...,5) \) (non-hindered translations and rotations of the mass center) of the scale model by a non-singular 5x5-matrix \( S_u \):

\[
 v = S_u \cdot u
\]

wherein \( u \) and \( v \) are the 5x1-column vectors containing the displacements \( u_i \) \( (i=1,...,5) \) and \( v_j \) \( (j=1,...,5) \) respectively.

A similar force equivalence analysis relates the central force components \( F_i \) \( (i=1,...,5) \) (forces and moments in the mass center) of the scale model to the forces \( R_j \) along the 5 (pendulum) elements \( (j=1,...,5) \) connecting the scale model to the outer world, by a non-singular 5x5-matrix \( S_R \):

\[
 F = S_R \cdot R
\]

wherein \( F \) and \( R \) are the 5x1-column vectors containing the force components \( F_i \) \( (i=1,...,5) \) and \( R_j \) \( (j=1,...,5) \) respectively.

Note that the non-singularity of the matrices \( S_u \) and \( S_R \) requires a well-chosen (mutually independent) set of boundary conditions (fixations) with corresponding displacement degrees of freedom for the laboratory scale model.
Under these conditions following mutual inversion relations hold:

\[ S_u = S_R^{-1} \quad \text{and} \quad S_R = S_u^{-1} \]  
(3)

**Stiffness matrix**

With the notations introduced above, the earlier mentioned loading-displacement model (matrix \( L \)) yields the following relation between corresponding (reaction) forces \( R \) and displacements \( u \) at the flexible fixations of the scale model:

\[ R = L \cdot u \]  
(4)

With relations (1), (2) and (3) and some matrix algebra, (4) transforms into:

\[ F_{el} = k \cdot v \quad \text{with:} \quad k = S_R \cdot L \cdot S_u^{-1} \]  
(5)

The 5x5-matrix \( k \) is the stiffness matrix relating the elastic contributions \( F_{el} \) in the force components acting on the mass center of the scale model, to the corresponding mass center displacement components \( v \).

**Mass matrix**

From the mass, the dimensions and the positions of the different components of the laboratory scale model, and using Steiner's theorem, its total mass and central dynamic moments of (mass) inertia (around the free rotation axes in the mass center) are computed. These quantities are grouped in a 5x5-diagonal matrix \( m \) in the order of the corresponding displacement degrees of freedom of the scale model: \( m = \text{diag} [m, m, I_{xx}, I_{yy}, I_{zz}] \).

The mass of the applied scale model fixations (spring elements) is negligible.

**Dynamic model equations and filter operator**

With the notations introduced above, and neglecting possible visco-elastic damping phenomena, the dynamic system equations read:

\[ f(t) = m \cdot v''(t) + k \cdot v(t) \]  
(6)

which relates \( f(t) \), the 5x1-column vector containing the instantaneous central external driving force components \( f_i \) (i=1,..,5) (forces and moments in the mass center) of the scale model, to the corresponding (time dependent) inertia and elastic force components.

Denote \( P(t) \) the 5x1-column vector containing the instantaneous external driving force components \( P_j \) (j=1,..,5) along the 5 (pendulum) elements (j=1,..,5) connecting the scale model to the outer world, for which holds (see (2)):

\[ f(t) = S_R \cdot P(t) \]  
(7)

then using the formulae above, matrix equation (6) can be transformed into:

\[ P(t) = R(t) + A \cdot R''(t) \quad \text{with:} \quad A = (S_R^{-1} \cdot m \cdot S_u) \cdot L^{-1} \]  
(8)
defining the dynamic filter operator, which transforms instantaneous (reaction) forces $R(t)$ measured at the flexible fixations of the scale model, into the corresponding instantaneous “pure” external driving force (loading) components $P(t)$ which actually caused them. Thus accounting for the mass inertia and elastic properties (“dynamic behavior”) of the scale model, the measured reaction force recordings can be “filtered” with (8) to yield the original driving forces (loading), independent of the geometrical and material properties.

It may be verified that if eigenoscillations of the scale model would occur and ‘disturb’ the signals $R(t)$, the operator (8) yields signals $P(t)$ from which such contributions are removed.

The angular frequencies (pulsations) of the eigenoscillations are found as the square root of the eigenvalues of the 5x5-matrix $m^{-1} \ast k$ which is equivalent to the inverse filter matrix $A^{-1}$. As all eigenvalues are positive, 5 corresponding eigenmodes of oscillation exist.

Table1: Eigenfrequencies and –periods of the discrete numerical approximation of the laboratory scale model, derived from force-displacement relations (stiffness matrix) along the connections with the outer world (3 vertical and 2 horizontal)

<table>
<thead>
<tr>
<th>Vertical oscillations</th>
<th>Horizontal oscillations</th>
</tr>
</thead>
<tbody>
<tr>
<td>Period (sec)</td>
<td>Frequency (Hz)</td>
</tr>
<tr>
<td>0.135</td>
<td>7.40</td>
</tr>
<tr>
<td>0.064</td>
<td>15.62</td>
</tr>
<tr>
<td>0.041</td>
<td>24.25</td>
</tr>
</tbody>
</table>

Doing so, the force-displacement model approximation (= ‘discretization’ as a rigid body of an in fact ‘continuous’ structure with an infinite number of displacement degrees of freedom) by 5x5-stiffness and mass-matrices yields the eigenfrequencies listed in the table above, more or less independent of the fixations used for the scale model.

Note that the (necessarily mutually independent) 5 displacement degrees of freedom correspond to the (necessarily ‘well’-)chosen 5 flexible connections equipped with load cells for reaction force recordings. Thus the latter choice determines the adopted scale model ‘discretization’ and subsequently the (time invariant) filter operator (8), including the eigenmodes (frequencies and oscillation patterns).

Also in the time domain a discretization is necessary in order to evaluate the ‘correction’ (inertia) term with the second time derivative of $R(t)$ in the right hand side of (8), based on recordings of $R(t)$ at discrete time points $t$. The least complex discrete approximation of the second time derivative is the second order central difference operator. However its accuracy rather sensitively depends on the time step compared to the frequencies (periods) present in the signal to be processed, calling for careful use.

Eventually the ‘filtered’ signals $P(t)$ may be combined to yield the signals of the actual integral (horizontal and vertical) wave impact loading on the prototype, using Froude scaling. From these, reliable design values can be estimated.

Application to measured force recordings – discussion of results

For the purpose of our research we need to investigate the quantitative effect of the dynamic filter (8) in the vicinity of (peak) values of $R(t)$ which correspond to peak values of $P(t)$.
In general a single harmonic component of \( R(t) \) will be more amplified in the signal \( P(t) \) by the operator (8), if it has a relatively high frequency in combination with a considerable amplitude. Indeed, the ‘dynamic correction’ (inertia) term (with second time derivative) rises proportional to the amplitude and the square of the frequency of the component. Harmonic components with frequencies near one of the eigenfrequencies will vanish.

Following findings were obtained after performing filtering computations on several peak value containing intervals of signals actually recorded in the laboratory test set up described in this paper.

- The dynamic correction on the vertical force components remains relatively limited and merely has a reducing effect on peak amplitudes. Conversely this indicates that relatively less ‘extreme’ (filtered) external driving forces (loading) cause (recorded) internal (reaction) forces which are affected by a certain resonance at near eigenfrequency(ies). Indeed in the measured laboratory model recordings of the internal reaction forces oscillations appear with a period close to the smallest analyzed ‘discrete’ eigenperiod of ca. 0,04 sec (see table above), corresponding to a vertical oscillation mode. As expected, these oscillations are suppressed by applying the dynamic filter (8).

- For the horizontal force components a similar suppression was observed of a relatively persistent oscillation with a period of ca. 0,1 sec, which also was “recognized” by the dynamic filter as an eigenperiod (corresponding to a horizontal eigenoscillation mode).

These findings constitute an important indication of the validity of the adopted discrete numerical model approximation (rigid body with a limited number of displacement degrees of freedom).

**CONCLUSIONS**

It appears that for modeling its loading-displacement behavior, the physical scale model may be approximated sufficiently accurate as a rigid body, involving only a limited (finite) number of well-chosen degrees of freedom (d.o.f.) of the scale model motions due to wave impact. Some simple static structural analysis of the laboratory model components is sufficient to predict the dynamic behavior (eigenoscillation modes) of the scale model.

Applying the proposed numerical filtering procedure to the laboratory reaction force recordings during wave impacts, it is verified that no significant peak-enhancing contributions of spurious eigenmodes occur. Care has to be taken though with the numerical approximation of the second time derivative of the recordings, to avoid artificial peaks and high-frequent noise introduction in the filtered signals. Contrarily in some impact cases significant peak reductions occur, in a non-systematic way however. Therefore it is concluded that reliable design value estimations directly be obtained from (unfiltered) time series of reaction forces measured in the physical scale model considered. A fortiori this conclusion holds for a comparative scenario analysis of two different walkway heights levels above the crest of the breakwater’s crest on which it is fixed.

Further research to validate these conclusions is recommended. The rigid body approximation may be tested by FE Modeling of the laboratory scale model, yielding an increased number of eigenmodes. Harmonic analysis of the measured series of physical scale model reaction forces may confirm the (in)sensitivity for eigenoscillations of a particular laboratory scale model. Dynamic structural analysis of the prototype may reveal the robustness of the proposed filtering procedure against non-accurate numerical approximation of the second time derivative.