Available online at www.sciencedirect.com
ScienceDirect
PALAEO
Palaeogeography, Palaeoclimatology, Palaeoecology 248 (2007) 473 – 476
www.elsevier.com/locate/palaeo

Discussion

Comment on: Paleoclimatic inference from stable isotope profiles of accretionary biogenic hardparts—a quantitative approach to the evaluation of incomplete data, by Wilkinson, B.H., Ivany, L.C., 2002. Palaeogeogr. Palaeocl. Palaeoecol. 185, 95–114

Fjo De Ridder [a,*], Anouk de Brauwere [b], Rik Pintelon [a], Johan Schoukens [a], Frank Dehairs [b], Willy Baeyens [b], Bruce H. Wilkinson [c]

[a] *Department of Fundamental Electricity and Instrumentation; Team B: System Identification and Parameter Estimation, Vrije Universiteit Brussel, Pleinlaan 2, B-1050 Brussels, Belgium*
[b] *Department of Environmental and Analytical Chemistry, Vrije Universiteit Brussel, Pleinlaan 2, B-1050 Brussels, Belgium*
[c] *Department of Geological Sciences, University of Michigan, 2534, CC Little Bldg, 1100 North University Ave, Ann Arbor, MI 48130, USA*

Received 14 December 2005; received in revised form 11 May 2006; accepted 17 August 2006

Abstract

The time base reconstruction method proposed by Wilkinson and Ivany [Paleoclimatic inference from stable isotope profiles of accretionary biogenic hardparts—a quantitative approach to the evaluation of incomplete data. Palaeogeogr. Palaeocl. Palaeoecol. 185 (2002), 95–114] can be further refined. They have shown that variations in accretion rate can be reconstructed from variations in the period of the measured signal. Here, it is shown that such variations influence not only the period, but also the phase of the signal. So, a refined estimator for the time base is constructed, which takes both period and phase variations into account.

Keywords: Accretionary; Hardparts; Isotopes; Palaeoclimate; Seasonality; Sinusoid

1. Introduction

Palaeo-environmental studies are often based on measurements of proxies or trace elements in corals, sponges or shells. One major problem with these records is the dating of the individual observations. Any proxy record is measured as function of a distance, while we would like to have the time series, which can be interpreted in the framework of palaeo-climate reconstruction and compared with other records or with models. Many of these records are so short in time (e.g. shells) that radio-isotopes can hardly be used for dating purposes. Even if it could be used, it will give an idea about time scales (is the record 100 or 150 years old?), not about the time base, which describes the variation in accretion rate from one observation to another. An elegant method developed to construct such time bases is proposed by Wilkinson and Ivany (2002; Ivany et al., 2003) and assumes that the signal model is sinusoidal, reflecting the large summer/winter variations. To reconstruct the time base, a window moves over the record by one observation per step. For each window the

DOI of original article: 10.1016/S0031-0182(02)00279-1.
* Corresponding author. Tel.: +32 2 629 29 79; fax: +32 2 629 28 50.
E-mail address: federid@vub.ac.be (F. De Ridder).

0031-0182/$ - see front matter © 2006 Elsevier B.V. All rights reserved.
doi:10.1016/j.palaeo.2006.08.004

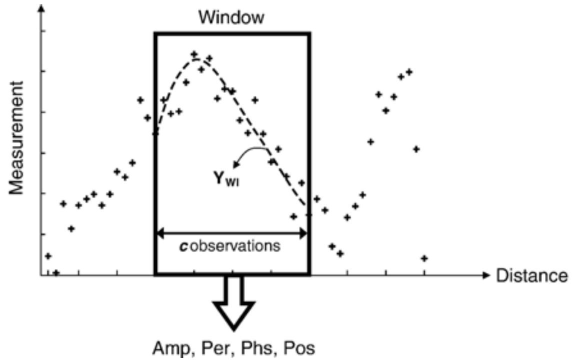

Fig. 1. Illustration of the parameters used to estimate the amplitude, period, phase and position of Wilkinson and Ivany's method (2002). The observations are shown by the '+' sign, the dotted line is the model and the rectangular is the window.

amplitude Amp (dimension of the measurement, e.g. per mil), period Per (distance), phase Phs (distance) and position or offset Pos (dimension of measurement), are estimated by matching

$$Y_{\mathrm{WI}} = \frac{\mathrm{Amp}}{2}\sin\left[(W_{\mathrm{mid}} - \mathrm{Phs})\frac{2\pi}{\mathrm{Per}}\right] + \mathrm{Pos} \tag{1}$$

on the samples in the window (see Fig. 1 for a visualization of the parameters). When the accretion rate is large this will be reflected in a large and vice versa. By moving the window over the observations, the varying period is finally estimated over the total length of the record and the varying accretion rate can be calculated from the variations in period.

2. First refinement

Note, firstly, that it is better to estimate the position and amplitude before windowing, because these parameters do not change from one window to another. If these parameters would be estimated in each window, they will slightly differ from one window to another, due to the stochastic noise. Moreover, the differences will cause additional variations in the estimated period and phase (Schoukens and Renneboog, 1984), which would make the method less stable. Therefore, we propose to estimate the position and amplitude on the total record, remove these parameters by subtracting the position and dividing by the amplitude. This has the advantage that the noise is averaged over much more observations. Besides this, in a next step the amplitude and position must be fixed anyway to estimate variations in accretion rate (see Eq. (3)). This means that this method cannot be applied if variations in amplitude or position are present.

3. Second refinement

The method proposed by Wilkinson and Ivany does not explicitly take into account that a part of the variations in accretion rate will appear in the phase (De Ridder et al., 2004). The result of this neglection is illustrated in Fig. 2: the artificial proxy record is a sine, shown by the dotted and the full line. The accretion rate decreases slowly from left to right. The last window of 10 samples is represented by the full line. On this window, a sinusoidal model, i.e. Eq. (1), is matched. This model is extrapolated over the total measurement window. At the origin, it can be seen that the phase is now $\pi/2$ (cosine). Indeed, a change in the time base does not only influence the period, but also the phase. So, if we want to reconstruct the time base more accurately, both the period and phase variations have to be taken into account.

To find a mathematical expression for this idea, we start by assuming that the true signal $Y_{\sin}$ (dimension of the measurement) is sinusoidal and is described by

$$Y_{\sin} = \frac{\mathrm{Amp}_0}{2}\sin\left\{\frac{2\pi}{\mathrm{Per}_0}\mathrm{Time} + \mathrm{Phs}_0\right\} + \mathrm{Pos}_0 \tag{2}$$

with Time the corresponding time instance (dimension: time) Amp_0, and Pos_0 respectively the amplitude and position of the signal (dimension of the measurement), Per_0 the period of the sine (dimension is now time) and Phs_0 the phase (scalar). The index 0 indicates that these parameters do not vary with time. If the time instances Time are larger then average, the time interval between sample n and the previous one is larger than average and the accretion rate was thus smaller than average (assuming a constant sample distance). For time instances smaller than average the opposite is true. The clue of this

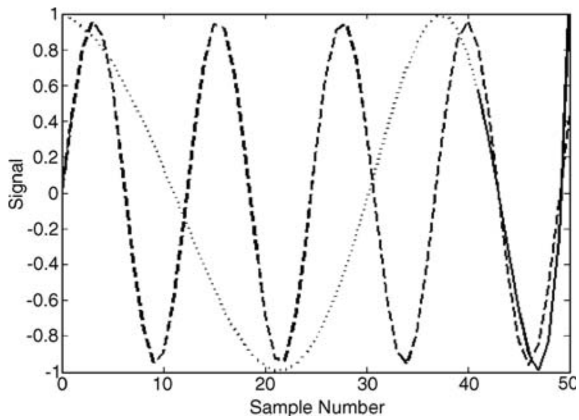

Fig. 2. The dotted and the full line represent a proxy-signal. On the full line a sinusoidal signal is matched. This sinusoidal is shown by the dashed line.

refinement is the identification of these time instances: we have followed the procedure proposed by Wilkinson and Ivany, i.e. a window of c samples is moved over the record. For each step Eq. (1) is matched on the windowed data.

In order to reconstruct the variation in accretion rate, Eq. (2) is set equal to Eq. (1) and the resulting expression is solved for Time. In order to derive an explicit expression for Time (pull it out of the sine's argument), we have had to assume that the amplitude and position do not vary with time. So,

$$\mathrm{Amp} = \mathrm{Amp}_0 \text{ and } \mathrm{Pos} = \mathrm{Pos}_0 \tag{3}$$

which leads to

$$\mathrm{Time} = \mathrm{Per}_0\left(\frac{W_{\mathrm{mid}} - \mathrm{Phs}}{\mathrm{Per}} - \frac{\mathrm{Phs}_0}{2\pi}\right) \tag{4}$$

See the appendix for a derivation. If we look in more details to this expression, one can notice that the Period Per_0 acts as a scaling factor and that the phase Phs_0 acts as an offset of Time. To put it more simply, the time instance between two subsequent samples is scaled by the period Per_0 Fortunately, we usually have a good idea about the real period of the measured signal. So, this period can be fixed in advance (e.g. for annually resolved archives this period is 1 year). As a result, we are able to date each observation w.r.t. the other observations, but the complete record can still be translated in time. In order to fix this, the phase Phs_0 has to be identified and this can sometimes be a problem: this method is not able to tell when the observations were formed; the latest period can correspond to this year or to any other year. In order to overcome this, the investigator has to use additional information, like radiocarbon dates or the sampling date, which can be linked to the latest observation. If this date is known, the phase Phs_0 can be fixed.

So to conclude, two parameters have to be chosen by the user, i.e. Per_0 and Phs_0. The first defines the time scale of the time base and the second positions the time base in real time. If this is done, all observations can be assigned an absolute date (Time).

4. Results

The refinement is illustrated on a simulated $\delta^{18}O$ record (see Fig. 3a), consisting of a sinusoidal signal (100 observations sampled over 5 years) with amplitude one per mil, disturbed by stochastic noise (with a standard deviation of 0.1 per mil), no systematic errors are present and the accretion rate increases exponentially. The change in accretion rate was chosen sufficiently

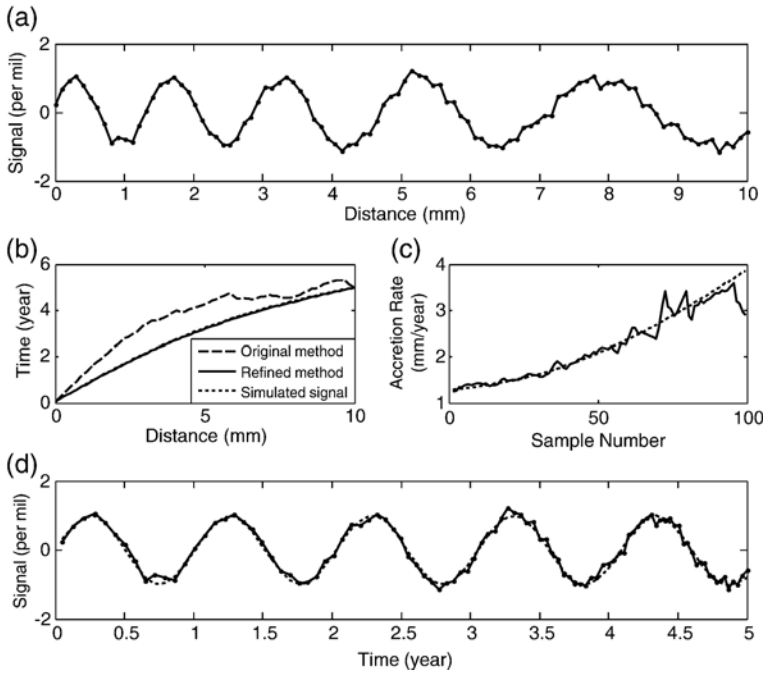

Fig. 3. (a) Simulated signal with noise: the amplitude of the simulated sine is 1 per mil, while the standard deviation of the noise is 0.1 per mil. (b) The estimated time bases: the divergence of the original signal is mainly due to the changing phase, which was not incorporated in the time model. Note that the refined method and the 'true' time base fall almost together. Therefore, the accretion rate of these is shown in (c). (d) shows the signal on the reconstructed time base. The dotted line represent the function Y_{sin}.

large, so that the importance of the phase is pronounced. The width of the window used is 20 samples. In Fig. 3b the time is shown as function of distance, i.e. the function relating the physical position of an observation with its date. Note that neglecting the phase can have a large influence on the estimated time, which even inverses near the end of the record. In Fig. 3c, the simulated accretion rate and the refined reconstruction are shown. Both are very similar, which illustrates that the refined method is very robust in the presence of stochastic noise. However, one should be careful if the sinusoidal model assumption is violated (i.e. a non-sinusoidal signal) or if the accretion rate varies rapidly compared to the width of the window.

Acknowledgements

Fjo De Ridder and Anouk de Brauwere are researchers of the Flemish Fund for Scientific Research (FWO-Vlaanderen) and are grateful for its support. This work is supported by the Belgian Government (IUAP V/22), the Flemish Government and the Vrije Universiteit Brussel (GOA22/DSWER4, GOA23-ILiNoS, and HOA9). This work was partially funded by EU contract No. EVK2-2001-00179-6C and the support from the European Science Foundation (ESF) under the EUROCORES Programme "EuroCLIMATE", through contract No. ERAS-CT-2003-980409 of the European Commission, DG Research, FP6. Finally, we would like to thank Guy Munhoven, Didier Paillard and an anonymous reviewer for their useful comments and critics on this work.

Appendix B. Supplementary data

Supplementary data associated with this article can be found, in the online version, at doi:10.1016/j.palaeo.2006.08.004.

List with symbols

Symbol	Description	Dimension
Amp	Amplitude	Dimension of measurement
c	Width of the window (number of samples)	Scalar
Per	Period	Time
Phs	Phase	Scalar
Pos	Estimated position or offset	Dimension of measurement
Time	Time instance	Time
W_{mid}	Number in the middle of the window	Scalar
$Y_{\sin}$	Signal model, proposed in this comment	Dimension of measurement
Y_{WI}	Signal model proposed by Wilkinson and Ivany	Dimension of measurement

Derivation of Eq. (4)

Eq. (1) is set equal to Eq. (2), so

$$\frac{\mathrm{Amp}}{2}\sin\left[(W_{\mathrm{mid}}-\mathrm{Phs})\frac{2\pi}{\mathrm{Per}}\right]+\mathrm{Pos} = \frac{\mathrm{Amp}_0}{2}\sin\left\{\frac{2\pi}{\mathrm{Per}_0}\mathrm{Time}+\mathrm{Phs}_0\right\}+\mathrm{Pos}_0 \tag{5}$$

We assumed that $\mathrm{Amp}=\mathrm{Amp}_0$ and $\mathrm{Pos}=\mathrm{Pos}_0$ (Eq. (3)), i.e. the amplitude and position are no function of time. This is actually necessary, because otherwise, as far as we know the arguments of the sinuses cannot be entered. Eq. (5) simplifies to

$$\sin\left[(W_{\mathrm{mid}}-\mathrm{Phs})\frac{2\pi}{\mathrm{Per}}\right]=\sin\left\{\frac{2\pi}{\mathrm{Per}_0}\mathrm{Time}+\mathrm{Phs}_0\right\} \tag{6}$$

Now, we apply an arcsin on both sides, which leads to

$$(W_{\mathrm{mid}}-\mathrm{Phs})\frac{2\pi}{\mathrm{Per}}=\frac{2\pi}{\mathrm{Per}_0}\mathrm{Time}+\mathrm{Phs}_0+2\pi k \tag{7}$$

with k an integer. From a mathematical point of view, the $2\pi k$ can be dropped because adding 2π to the phase would not change the signal. However, from a physical point of view, this corresponds to a shift of k periods in the time domain (for annually resolved archives this corresponds to a shift of exactly k years). So, now the time can be written as function of the other variables

$$\begin{aligned}&(W_{\mathrm{mid}}-\mathrm{Phs})\frac{2\pi}{\mathrm{Per}}=\frac{2\pi}{\mathrm{Per}_0}\mathrm{Time}+\mathrm{Phs}_0\\ &\Leftrightarrow \mathrm{Time}=\mathrm{Per}_0\left(\frac{W_{\mathrm{mid}}-\mathrm{Phs}}{\mathrm{Per}}-\frac{\mathrm{Phs}_0}{2\pi}\right)\end{aligned} \tag{8}$$

which is Eq. (4).

References

De Ridder, F., Pintelon, R., Schoukens, J., Gillikin, D.P., André, L., Baeyens, W., de Brauwere, A., Dehairs, F., 2004. Decoding nonlinear growth rates in biogenic environmental archives. Geochem. Geophys. Geosyst. 5 (12), Q12015. doi:10.1029/2004GC000771.

Ivany, L.C., Wilkinson, B.H., Jones, D.S., 2003. Using stable isotopic data to resolve rate and duration of growth throughout ontogeny: an example from the surf clam, *Spisula solidissima*. Palaios 18, 126–137.

Schoukens, J., Renneboog, J., 1984. Spectrum analysis of signals with very low frequency components. Conference on Precision Electromagnetic Measurements, Delft, Netherlands, pp. 1–4.

Wilkinson, B.H., Ivany, L.C., 2002. Paleoclimatic inference from stable isotope profiles of accretionary biogenic hardparts—a quantitative approach to the evaluation of incomplete data. Palaeogeogr. Palaeoclimatol. Palaeoecol. 185, 95–114.