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Toward a general theory of the age in ocean modelling

Eric J.M. Delhez [a,*], Jean-Michel Campin [b], Anthony C. Hirst [c], Eric Deleersnijder [b]

[a] *GeoHydrodynamics and Environment Research, Université de Liège, Sart Tilman B5, B-4000 Liège, Belgium*
[b] *Institut d'Astronomie et de Géophysique G. Lemaître, Université Catholique de Louvain, 2 Chemin du Cyclotron, B-1348 Louvain-la-Neuve, Belgium*
[c] *CRC for Southern Hemisphere Meteorology, CSIRO Atmospheric Research, PMB 1, Aspendale Vic 3195, Australia*

Abstract

Seawater is a mixture of several constituents. The age of a parcel of a constituent is defined to be the time elapsed since the parcel under consideration left the region where its age is prescribed to be zero. Estimating the age is an invaluable tool for understanding complex flows and the functioning of the numerical models used for representing them. A general theory of the age is developed, according to which the age of a constituent of seawater is a time-dependent, pointwise quantity that may be obtained from the solution of two partial differential equations governing the evolution of the concentration of the constituent under study and an auxiliary variable called the "age concentration". It is seen that previous applications of the notion of age are in agreement with the present theory, or may be viewed as approximations deriving from this theory. A few particular problems are touched upon, including the estimation of the age of the water and the determination of the age with the help of radioactive tracers.

Keywords: Age; Age of seawater; Radioisotope; Tracer

1. Introduction

Ocean flows depend on a number of external forcings and phenomena which can be characterised by appropriate timescales, such as the inverse of the Coriolis parameter or the periods of tidal components. On the other hand, there are timescales which, instead of constraining the flow, are inherent properties of the flow which can be evaluated a posteriori, as diagnostic variables, to

* Corresponding author. Tel.: +32-4-366-33-55; fax: +32-4-366-23-55.
E-mail address: e.delhez@ulg.ac.be (E.J.M. Delhez)

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help describe and understand the processes under consideration. The age of the water is one such timescale. Many definitions of the age have been given, according to the needs of the study in which it was used (e.g. Bolin and Rodhe, 1973; Zimmerman, 1976; Stuiver et al., 1983; Prandle, 1984; Takeoka, 1984; Broecker et al., 1991; Hirst, 1994; England, 1995; Salomon et al., 1995; Anon., 1997; Karstensen and Tomczak, 1998; Campin et al., 1999). It is, however, possible to work out a theory of the age sufficiently general that most of the applications of the notion of age to ocean phenomena that were carried out over recent years can now be seen to be consistent with this theory. This is outlined below.

A review of previous applications of the concept of age suggests that a general theory must offer the opportunity to distinguish between the age of a particular constituent of seawater, that of an aggregate of constituents and that of seawater itself. In addition, it must be suited to problems in which there is either a pointwise constituent source, such as an outfall pipe, or a – sometimes loosely defined – origin region of the constituent or the water of which the age is to be evaluated. Finally, it is appropriate to conceive the age as a pointwise, time-dependent variable, an option more general than that adopted in many previous studies in which the age was defined to be a steady-state quantity, frequently integrated over the whole domain of interest or, at least, a significant fraction of this domain. In accordance with these requirements and with the definitions of authors such as Zimmerman (1976) or Takeoka (1984), it is suggested that the age of a tracer or water parcel be defined as the time elapsed since the parcel under consideration left the region in which its age is prescribed to be zero, which may be zero- to three-dimensional, i.e. a point, a curve, a surface or a volume. The nature of the region where the age is prescribed to be zero depends obviously on the flow considered and the purpose for which the age is introduced.

The present approach to the concept of age is intended for numerical models, i.e. systems for solving partial differential equations. However, some of its aspects are believed to be useful to studies that rely exclusively on field data to estimate the age.

Age, as defined above, is a concept intimately associated with that of fluid parcels. Hence, evaluating the age in a Lagrangian model is presumably trivial, for the variables of such a model are "tied" to fluid parcels, rather than being functions of the position in the domain of interest as must be the case in the Eulerian description of fluid flows. On the other hand, most ocean models are formulated in the Eulerian way. Therefore, for a given Eulerian model, it is tempting to recommend that a Lagrangian module be included at least for the purpose of computing the age. Nonetheless, this may not be the best option, for several reasons. Firstly, Lagrangian solvers, such as that described in Hunter et al. (1993), demand generally more computer time than their Eulerian counterparts. Secondly, the age, as a diagnostic quantity, should be evaluated by means of an algorithm as similar as possible to those used in the prognostic model, which is here assumed to be Eulerian. In particular, water masses in a model are moved and modified by diffusive processes as well as by resolved advection, and it is often desirable to include these effects properly in the calculation of the age. This is particularly important since the age is sometimes computed for investigating the functioning of the model, rather than the physics of the flow. Thirdly, an age field is best interpreted through the examination of synoptic maps which are easier to obtain from Eulerian results than from Lagrangian ones. Thus, it is desirable that the equation governing the age be derived in an Eulerian framework.

2. Age equation

2.1. Age distribution function

Strictly speaking, seawater must be regarded as a mixture of $I+1$ constituents – i.e. pure water, dissolved salts, pollutants, plankton, etc. – that can be identified by the index i $(0 \leqslant i \leqslant I)$. The latter is defined here in such a way that pure water corresponds to $i=0$. Let x, y and z denote Cartesian coordinates such that the position vector of any point in the domain of interest reads $\mathbf{x}=(x,y,z)$. Let the age distribution function of the i-constituent $(0 \leqslant i \leqslant I), c_i(t,\mathbf{x},\tau)$, be defined as follows: at time t, the mass of the i-constituent contained in the volume

$$(x-\Delta x/2, y-\Delta y/2, z-\Delta z/2) \leqslant (x,y,z) \leqslant (x+\Delta x/2, y+\Delta y/2, z+\Delta z/2), \tag{1}$$

with an age τ lying in the interval

$$\tau-\Delta\tau/2 \leqslant \tau \leqslant \tau+\Delta\tau/2, \tag{2}$$

tends to $\rho c_i(t,\mathbf{x},\tau,)\Delta x\Delta y\Delta z\Delta\tau$ as $\Delta x, \Delta y, \Delta z$ and $\Delta\tau$ tend to zero, where seawater density ρ is assumed to be a constant in accordance with the Boussinesq approximation.

Relations (1) and (2) delineate a four-dimensional volume, Ω, in the four-dimensional space comprising the physical – three-dimensional – space, associated with coordinates x, y and z, and the age dimension, associated with coordinate τ. The mass of the i-constituent contained in Ω varies as a result of local production/destruction, due to processes such as chemical reactions, and transport through the boundaries of Ω. Therefore, the mass budget of the i-constituent in Ω reads

$$\begin{aligned}\rho\Delta x\Delta y\Delta z\Delta\tau\frac{\partial}{\partial t}c_i(t,\mathbf{x},\tau) &= \rho\Delta x\Delta y\Delta z\Delta\tau(p_i-d_i)\\ &\quad-\rho\Delta y\Delta z\Delta\tau[q_{i,x}(t,x+\Delta x/2,y,z,\tau)-q_{i,x}(t,x-\Delta x/2,y,z,\tau)]\\ &\quad-\rho\Delta x\Delta z\Delta\tau[q_{i,y}(t,x,y+\Delta y/2,z,\tau)-q_{i,y}(t,x,y-\Delta y/2,z,\tau)]\\ &\quad-\rho\Delta x\Delta y\Delta\tau[q_{i,z}(t,x,y,z+\Delta z/2,\tau)-q_{i,z}(t,x,y,z-\Delta z/2,\tau)]\\ &\quad-\rho\Delta x\Delta y\Delta z[q_{i,\tau}(t,x,y,z,\tau+\Delta\tau/2)-q_{i,\tau}(t,x,y,z,\tau-\Delta\tau/2)],\end{aligned} \tag{3}$$

where $p_i(\geqslant 0)$ and $d_i(\geqslant 0)$ are the rates of production and destruction, respectively, i.e. the source and sink terms; $q_{i,x}, q_{i,y}, q_{i,z}$ and $q_{i,\tau}$ denote the fluxes in the x, y, z and τ directions. In the limit $\Delta x, \Delta y, \Delta z, \Delta\tau \to 0$, (3) yields

$$\frac{\partial c_i}{\partial t} = p_i - d_i - \nabla\cdot\mathbf{q}_i - \frac{\partial q_{i,\tau}}{\partial\tau}, \tag{4}$$

where $\mathbf{q}_i=(q_{i,x},q_{i,y},q_{i,z})$ and ∇ is the nabla or del vector operator in the physical space.

The flux $\mathbf{q}_i$ accounts for the transport of mass in the physical space. It consists of an advective part, due to the fluid velocity $\mathbf{u}(t, \mathbf{x})$ resolved by the model, and a contribution from the phenomena that are not resolved by the model. Parameterising the latter in a Fourier–Fick manner with the help of the eddy diffusivity tensor $\mathbf{K}$, $\mathbf{q}_i$ reads

$$\mathbf{q}_i = \mathbf{u}c_i - \mathbf{K}\cdot\nabla c_i. \tag{5}$$

The expression of the subgrid-scale flux, $-\mathbf{K}\cdot\nabla c_i$, is sufficiently general that it is capable of taking into account the impact of a wide variety of phenomena, ranging from the three-dimensional turbulent motions (e.g. Mellor and Yamada, 1982; Rodi, 1993), the typical scale of which does not exceed a few metres, to the meso-scale eddies that most ocean general circulation models cannot – yet – resolve (e.g. Redi, 1982; Cox, 1987). Other kinds of parameterisations of the subgrid-scale processes can also be used in Eq. (5) without any other change to the theory developed here than the formulation of the corresponding subgrid-scale fluxes – and the influence of these fluxes on the age in Eq. (17) – provided that the parameterisation remains linear with respect to c_i.

The flux $q_{i,\tau}$ is related to ageing, i.e. the process by which the age of every fluid parcel tends to increase by a certain amount of time as time progresses by the same amount of time. This may be viewed as advection with a unit velocity in the age direction. Hence,

$$q_{i,\tau} = c_i. \tag{6}$$

Substituting Eqs. (5) and (6) into Eq. (4) leads to the equation governing the evolution of the age distribution function, i.e.

$$\frac{\partial c_i}{\partial t} = p_i - d_i - \nabla \cdot (\mathbf{u} c_i - \mathbf{K} \cdot \nabla c_i) - \frac{\partial c_i}{\partial \tau}. \tag{7}$$

2.2. Mean age

Assuming that the age is positive definite, the concentration at time t and location $\mathbf{x}$ of the i-constituent is

$$C_i(t, \mathbf{x}) = \int_0^\infty c_i(t, \mathbf{x}, \tau)\, \mathrm{d}\tau. \tag{8}$$

Then, by integrating Eq. (7) over τ, taking into account Eq. (8) and the common-sense boundary conditions

$$\lim_{\tau \to \infty} c_i(t, \mathbf{x}, \tau) = 0, \tag{9}$$

the equation obeyed by the i-constituent concentration is obtained, i.e.

$$\frac{\partial C_i}{\partial t} = P_i - D_i - \nabla \cdot (\mathbf{u} C_i - \mathbf{K} \cdot \nabla C_i), \tag{10}$$

with

$$P_i(t, \mathbf{x}) = c_i(t, \mathbf{x}, \tau = 0) + \int_0^\infty p_i(t, \mathbf{x}, \tau)\, \mathrm{d}\tau \quad \text{and} \quad D_i(t, \mathbf{x}) = \int_0^\infty d_i(t, \mathbf{x}, \tau)\, \mathrm{d}\tau. \tag{11}$$

Obviously, $c_i(t, \mathbf{x}, \tau = 0)$ is zero, unless there is a source producing i-constituent with the age prescribed to be zero at time t and location $\mathbf{x}$. In the latter case, however, $c_i(t, \mathbf{x}, \tau = 0)$ is directly related to the production rate, implying that it does not appear as an additional unknown in Eq. (10). As an alternative, $c_i(t, \mathbf{x}, \tau = 0^-)$ can be assumed to be zero in Eq. (11) and a source at $\tau = 0$ may be accounted for by including an appropriate Dirac function in p_i, as shown further.

That the widely accepted Eq. (10) may be derived from Eq. (7), which is the core of the theory built herein, is obviously reassuring as to the well-foundedness of this theory. Since physical-space transport is independent of the age, the physical-space transport terms have similar expressions in the equations governing the age distribution function, (7), and the concentration, Eq. (10).

In accordance with the definition of the age distribution function, at time t and location $\mathbf{x}$, the mean age of the i-constituent is given by

$$a_i(t,\mathbf{x}) = \frac{1}{C_i(t,\mathbf{x})}\int_0^\infty \tau c_i(t,\mathbf{x},\tau)\,\mathrm{d}\tau. \tag{12}$$

Let the age concentration be defined as

$$\alpha_i(t,\mathbf{x}) = C_i(t,\mathbf{x})a_i(t,\mathbf{x}). \tag{13}$$

It seems reasonable to assume that the age distribution function verifies

$$\lim_{\tau\to 0}\tau c_i(t,\mathbf{x},\tau) = 0 = \lim_{\tau\to\infty}\tau c_i(t,\mathbf{x},\tau). \tag{14}$$

Then, multiplying Eq. (7) by τ, integrating over τ, it is readily seen that the age concentration obeys

$$\frac{\partial\alpha_i}{\partial t} = C_i + \pi_i - \delta_i - \nabla\cdot(\mathbf{u}\alpha_i - \mathbf{K}\cdot\nabla\alpha_i), \tag{15}$$

with

$$\pi_i(t,\mathbf{x}) = \int_0^\infty \tau p_i(t,\mathbf{x},\tau)\,\mathrm{d}\tau \quad \text{and} \quad \delta_i(t,\mathbf{x}) = \int_0^\infty \tau d_i(t,\mathbf{x},\tau)\,\mathrm{d}\tau. \tag{16}$$

So, the age concentration α_i satisfies an equation similar to that governing the evolution of the – mass – concentration of every constituent, hence its name. As the independent variable τ, the age, will rarely be used from here on, it is decided, for simplicity, to call the variable a_i the "age" rather than the "mean age".

The equation governing every constituent concentration and its age concentration are derived from the same equation, i.e. the equation obeyed by the age distribution function. Therefore, the subgrid-scale parameterisations in the equations governing the constituent concentration and the age concentration, respectively, are consistent with each other. This is a key aspect of the present theory, since it was previously underscored that the impact of eddy mixing on the age must be taken into account properly (Hirst, 1994; Karstensen and Tomczak, 1998).

It is now clear how the age is to be calculated in an Eulerian model. First, the concentration of every constituent is computed from Eq. (10). Then, using a numerical scheme similar to that needed to obtain the constituent concentration, the age concentration is calculated from Eq. (15) – provided appropriate boundary conditions for the age concentration are set. Finally, according to Eq. (13), the age of every constituent is evaluated as the ratio of the age concentration to the constituent concentration.

A direct evaluation of the age could be performed without any detour through the evaluation of the age concentration. Indeed, by manipulating Eqs. (10) and (15), the equation governing the age is obtained

$$D_t a_i = 1 + \frac{\pi_i - a_i P_i}{C_i} - \frac{\delta_i - a_i D_i}{C_i} + \nabla\cdot(\mathbf{K}\cdot\nabla a_i) + \frac{\nabla C_i\cdot\mathbf{K}\cdot\nabla a_i + \nabla a_i\cdot\mathbf{K}\cdot\nabla C_i}{C_i}, \tag{17}$$

where $D_t = \partial/\partial t + \mathbf{u}\cdot\nabla$ denotes the material derivative. It is however recommended that the age concentration equation (15) be used rather than the age equation (17), because the form of the latter is so different from the constituent concentration equation (10) that it should be solved by means of a specific numerical scheme, which would demand unnecessary model development efforts and diminish the relevance of the computed age as a tool for diagnosing the functioning of the associated ocean model. By contrast, the transport terms of the age concentration equation (15) can be discretised by the same method as that used in the concentration equation. However, as will be seen below, Eq. (17) is useful to understand the impact of production/destruction terms on the age.

The production/destruction terms in Eqs. (11) and (16), being formulated as integrals over the independent age variable τ, may appear as unnecessarily complex. It is believed, however, that in most cases these integrals may be easily computed, leading to simple expressions. Consider, for instance, a radioisotope, the concentration of which is denoted C_r . Assume that its decay proceeds with characteristic time λ. Since radioactive decay is a process independent of the age, the sink term to be included in Eq. (7) is $d_r = c_r/\lambda$. Hence, using Eqs. (11) and (16), the destruction terms to be included in the equations governing the tracer concentration and its age concentration are simply $D_r = C_r/\lambda$ and $\delta_r = C_r a_r/\lambda = \alpha_r/\lambda$. Note that the corresponding destruction term of the age equation (17) is identically zero for this radioisotope, indicating that radioactive decay does not tend to modify the age of a radioisotope parcel. This is not surprising, since the radioactive disintegration probability for a given atom is independent of the age of this atom and since every radioisotope atom is "destroyed together with its age".

Another illustrative example is that of a constituent which is produced with a prescribed age. This is, for instance, the case when new material is introduced into the system with an initial age set to zero in order to diagnose the advection/dispersion of this material. This also applies to intermediate products of a series of chemical reactions that can be considered as produced with the age of the reagents if the age is used to study the kinetics of the whole reaction. Let 'pa' be the subscript identifying this constituent and, $\mu(t, \mathbf{x})$ the prescribed age just mentioned. Then, $p_{\rm pa}(t, \mathbf{x}, \tau) = P_{\rm pa}(t, x)\delta(\mu - \tau)$, where $P_{\rm pa}$ denotes an appropriate rate of production while δ is the Dirac function. Hence, the source terms to be included in Eqs. (10) and (15) read $P_{\rm pa}$ and $\pi_{\rm pa} = P_{\rm pa}\mu$, respectively. In this case, the second term in the age equation (17) is $(\pi_{\rm pa} - a_{\rm pa}P_{\rm pa})/C_{\rm pa} = (P_{\rm pa}/C_{\rm pa})(\mu - a_{\rm pa})$, implying that this type of production tends to nudge the age of every parcel of the constituent considered toward μ, the age at which the production occurs.

As the production/destruction rates are likely to be relatively simple expressions, deriving the mean age of to active constituents is not just a mere speculation that would hardly ever become reality because of insuperable difficulties in the formulation of the production/destruction rates. This does not imply, however, that parameterisations which explicitly retain the age dimension, Eq. (7), should never be utilised. It might be appropriate, for example, to set up ecological models in which the behaviour of living organisms would depend explicitly on their ages, demanding use of the four-dimensional space comprising the physical space and the age dimension. One of the improvements that such an approach could bring about would be offering the opportunity of formulating the death rate as a function of the age. Nonetheless, investigating this is far beyond the scope of the present note.

3. Discussion

It is now worth further explaining some aspects of the age theory developed above and touching upon some of its consequences.

3.1. Integral approach

Integrating Eqs. (7) and (10) over the domain of interest leads to expressions that are the starting point of the theory developed by Bolin and Rodhe (1973). This approach is particularly useful if the mixing within the domain of interest is sufficiently strong that the space variations of the concentration of every constituent may be neglected (e.g. Deleersnijder et al., 1997, 1998).

This domain integrated approach was an inspiration to Prandle (1984), who was concerned with the fate of caesium-137 released into the Irish Sea and the English Channel by the nuclear fuel reprocessing plants of Windscale and Cap de la Hague, respectively. The transport of the radioisotope considered was modelled over the north-western European continental shelf. Assuming constant rates of release at each of the sources and zero initial concentration of the tracer in the domain of interest, the tracer concentration was computed until a steady state was obtained. The corresponding time-independent age was evaluated in every grid box by means of a formula involving the temporal evolution of the caesium concentration during the phase of adjustment toward a steady state. This formula is in agreement with the integral approach but might not be entirely valid for the evaluation of the age as a pointwise variable.

3.2. Aggregates and age of the water

The number of constituents of seawater is so large that it is unreasonable to develop a strategy that would demand that the concentration and, possibly, the age of each of them be computed. It is more appropriate to deal with suitably defined groups of constituents. Herein such a group is called an "aggregate". So, seawater may be viewed as made up of a certain number of aggregates, the concentration C_k^{ag} and age concentration α_k^{ag} of which must obviously be defined as

$$(C_k^{\mathrm{ag}}, \alpha_k^{\mathrm{ag}}) = \sum_{j=0}^{l} \omega_{k,j}(C_j, \alpha_j), \tag{18}$$

where $\omega_{k,j}$ is equal to 1 or 0 according to whether the j-constituent is included or excluded from the k-aggregate. Applying to Eqs. (10) and (15) a weighted sum similar to that used in Eq. (18), it is readily seen that the equations obeyed by the concentration and the age concentration of an aggregate are similar to those governing the evolution of the concentration and age concentration of any individual constituent. Finally, the age of the k-aggregate is evaluated as

$$a_k^{\mathrm{ag}} = \frac{\alpha_k^{\mathrm{ag}}}{C_k^{\mathrm{ag}}} = \frac{\sum_{j=0}^{l} \omega_{k,j}\alpha_j}{\sum_{j=0}^{l} \omega_{k,j}C_j}. \tag{19}$$

Thus, as the mathematical relations pertaining to aggregates are equivalent to those derived for the individual constituents, there is no need to keep distinguishing aggregates from individual constituents. Accordingly, for simplicity, only the term "constituent" will be used from here on, even if the constituent under consideration is, in fact, an aggregate.

Seawater is the aggregate that comprises all of the constituents. Its concentration is equal to 1, since concentration is considered in the present note as a mass fraction such that $\sum C_j = 1$ – where the sum is taken over all constituents. There should be no net local production/destruction of mass, implying that the production/destruction rates must verify $\sum(P_j - D_j) = 0$. Then, the sum over all the constituents of Eq. (10) yields the continuity equation $\nabla \cdot \mathbf{u} = 0$, indicating once again that the present theory is in agreement with the widely accepted basic principles of ocean modelling.

The concentration of pure water, C_0, is close to 1 at any time and location, implying that the concentrations of the other constituents of seawater are all much smaller than 1. Hence, the age of seawater, which is defined to be $\sum C_j a_j / \sum C_j = \sum C_j a_j$ in accordance with Eqs. (18) and (19), is approximately equal to the age of pure water, unless the age of some constituents other than pure water are overwhelmingly larger than the age of pure water.

Pure water is a passive constituent, i.e. a constituent which is neither produced ($P_0 = 0$) nor destroyed ($D_0 = 0$). That C_0 is close to 1 implies that the last two terms of the age Eq. (17), are such that

$$|\nabla \cdot (\mathbf{K} \cdot \nabla a_0)| \gg C_0^{-1} |\nabla C_0 \cdot \mathbf{K} \cdot \nabla a_0 + \nabla a_0 \cdot \mathbf{K} \cdot \nabla C_0|. \tag{20}$$

Then, taking into account the continuity equation $\nabla \cdot \mathbf{u} = 0$, the age of pure water may be seen to obey approximately

$$\frac{\partial a_0}{\partial t} = 1 - \nabla \cdot (\mathbf{u} a_0 - \mathbf{K} \cdot \nabla a_0). \tag{21}$$

It is worth noting that this equation is equivalent to that used by Haidvogel and Bryan (1992), and England (1995) to evaluate the age of the water in their studies of the ventilation rate of the ocean.

3.3. Age determination from tracer concentration

This section briefly reviews a few methods for determining the age which are based on the hypothesis that subgrid-scale transport may be neglected. Thus, if $\mathbf{K}$ is assumed to be zero in Eqs. (10), (15) and (17), then any arbitrarily small seawater volume, i.e. a seawater parcel, comprises parcels of every constituent which all follow the same trajectory, defined by the temporal evolution of the position vector $\mathbf{r}(t)$. The latter is the solution of $d\mathbf{r}(t)/dt = \mathbf{u}[t, \mathbf{r}(t)]$. Along a trajectory, the concentration of every constituent and its age obey differential equations

$$\frac{\mathrm{d}}{\mathrm{d}t} C_i[t, \mathbf{r}(t)] = P_i - D_i \tag{22}$$

and

$$\frac{\mathrm{d}}{\mathrm{d}t} a_i[t, \mathbf{r}(t)] = 1 + \frac{\pi_i - a_i P_i}{C_i} - \frac{\delta_i - a_i D_i}{C_i}. \tag{23}$$

According to Eqs. (22) and (23), the concentration of a passive constituent – i.e. a constituent for which P_i, D_i, π_i and δ_i are all zero – in a seawater parcel remains constant, while its age increases as time. Taking advantage of these properties, Salomon et al. (1995) made an attempt to estimate the age – which they called "transit time" – of a (quasi-)passive tracer released into the English Channel. They defined the transit time as the time lag between the time series of the concentration computed at the release point and that computed at the observation point. Such a method relies on the hypothesis that subgrid-scale transport may be neglected and requires that the tracer concentration at the release point varies in time. With diffusion in the system, however, the time series of the concentration at the observation point is not only delayed but is also distorted with respect to the concentration time series at the release point, so that the transit time can only be estimated by looking at the maximum of the cross-correlation between the two time series. Another drawback of this approach is that it only provides a time-independent estimate of the transit time.

Another method for estimating the age requires radioisotope concentration measurements. Along the trajectory passing at time t^1 and t^2 through points $\mathbf{r}^1 = \mathbf{r}(t^1)$ and $\mathbf{r}^2 = \mathbf{r}(t^2)$, respectively, the concentration and the age verify $C_r(t^2, \mathbf{r}^2) = C_r(t^1, \mathbf{r}^1)\exp[-(t^2 - t^1)/\lambda]$ and $a_r(t^2, \mathbf{r}^2) = a_r(t^1, \mathbf{r}^1) + (t^2 - t^1)$. Hence, if the concentration field and its age are in a steady state, the age verifies

$$a_r(t^2, \mathbf{r}^2) = a_r(t^1, \mathbf{r}^1) + \lambda \log \frac{C_r(t^1, \mathbf{r}^1)}{C_r(t^2, \mathbf{r}^2)}. \tag{24}$$

A special form of this relation has been used to estimate ventilation rates in the World Ocean from carbon-14 data (e.g. Stuiver et al., 1983; Broecker et al., 1991). To do so, the age was prescribed to be zero at the ocean surface, so that the age in the interior of the ocean was to be interpreted as the time elapsed since the radioisotope particle considered was last exposed to the atmosphere. Thus, assuming that the flow is at a steady state and that the radiocarbon concentration $C_{r,s}$ is known – and constant – in the ocean surface layer, the age at any point $\mathbf{x}$ in the interior of the ocean is

$$a_r(\mathbf{x}, t) = \lambda \log \frac{C_{r,s}}{C_r(\mathbf{x}, t)}. \tag{25}$$

The timescale derived from Eq. (25) is no more than an approximation of the age, because it was necessary to neglect the impact of subgrid-scale transport to obtain it. In addition, problems with boundary conditions were also seen to be detrimental to this approach (Campin et al., 1999). Finally, it must be kept in mind that the age of the radioisotope is different from that of the water, a distinction that is not allowed by this method.

To account for the impact of subgrid-scale transport, it was suggested (e.g. Anon., 1997) that a model be made of the fate of a passive tracer which is subjected to boundary conditions equivalent to those used for the radioactive tracer. The age at time t and location $\mathbf{x}$ is then estimated by the expression $\lambda \log[C_p(t, \mathbf{x})/C_r(t, \mathbf{x})]$ where λ, C_p and C_r denote the timescale of radioactive decay, the concentration of the passive tracer and the concentration of the radioactive tracer. Though this approach is better than that relying on one single – radioactive – tracer as it makes it possible to separate the effects of the radioactive decay and of the subgrid scale processes, it is readily seen that this method is not fully compatible with the more general theory advocated in this note since the complex effect of the subgrid-scale transport on the age dynamics described in Eq. (17) is still not taken into account.

Hirst (1994) suggested simulating the evolution of two variables p_1 and p_2 defined mathematically as the solutions of

$$\frac{\partial p_1}{\partial t} = -\nabla \cdot (\mathbf{u}p_1 - \mathbf{K} \cdot \nabla p_1), \tag{26}$$

and

$$\frac{\partial p_2}{\partial t} = \gamma p_1 - \nabla \cdot (\mathbf{u}p_2 - \mathbf{K} \cdot \nabla p_2), \tag{27}$$

where $1/\gamma$ is an appropriate timescale. Hirst stated that the ratio $p_2/(\gamma p_1)$ was to be viewed as the age of a passive tracer, which was, in his study, North Atlantic Deep Water. If p_2/γ is interpreted as the age concentration of the passive tracer of which the concentration is p_1, it is readily seen that Hirst's method is equivalent to that developed herein.

4. Conclusion

The concept of age developed above may be applied to all of the constituents of seawater, be they active or passive. It was shown that most of the previous applications of the age may be regarded as particular cases of the present approach or might be improved by taking advantage of the developments above. Nonetheless, the general theory of the age in ocean modelling has only been outlined in this note. This is why a detailed article is being prepared (Deleersnijder et al., 1999), which will include:

- a detailed derivation of the age concentration equation;a discussion of the equivalence of the Lagrangian and Eulerian ages;
- a comprehensive discussion of the boundary conditions to be applied to the age concentration equation, a subject that has been ignored in this note;
- a discussion of the mathematical properties of the age, as obtained from the age concentration equation, which may be useful for model validation;
- numerical results obtained in the world ocean and the North Sea which illustrate various aspects of the theoretical developments;
- case studies demonstrating the importance of being able to distinguish between the ages of different constituents.

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