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Large eddy simulations for quasi-2D turbulence in shallow flows: A comparison between different subgrid scale models

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ABSTRACT

In this study, the performance of the horizontal large eddy simulation module, developed at the University of Leuven (HLES-KULeuven module) is assessed. A comparison between different subgrid scale models has been carried out. The study is concerned with the non-rotating and unstratified flows. The results of the simulation for an oscillatory backward facing (BFS) flow are presented in case of an expanding flume based on a one-length scale approach and a two-length scale approach. Three subgrid scale (SGS) models have been tested: Smagorinsky SGS model (Smagorinsky, J., (1963). General circulation experiments with the primitive equations, I. the basic experiments. Monthly Weather Review, 91(3), 99–164), Uittenbogaard SGS model (Uittenbogaard, R.E., and van Vossen, B., (2004). Subgrid-scale model for quasi-2D turbulence in shallow water. Shallow Flows. Jirka and Uijttewaal (Eds.), Taylor & Francis Group, London, ISBN 90 5809 700 5) and a proposed two-length scale approach. The first two models are considered to be a one-length scale models. A simulation without a subgrid scale model for the horizontal mixing has also been conducted. In all simulations, a quadratic friction model parameterizes the dissipation produced by the 3D-subdepth scale turbulence. The two-length scale concept uses a newly mixing length formulation for the quasi-2D turbulence and doesn't depend on the filter width in contrast to the one-length scale approach, in which the mixing length is function of the filter width. The outputs of the HLES-KULeuven module have been compared with the experimental data taken from Stelling, G.S., and Wang, L.X., (1984). Experiments and computations on separating flow in an expanding flume. Dept. Civil Engineering, Delft University of Technology, Report 2–84.). The two-length scale approach has been validated with experimental data from SERC Flood Channel Facility at HR Wallingford. In general, there is a qualitative agreement with the experimental data. It has also been found that the two-length scale approach produces more elongated and less isotropic vortex than the one-length scale models.

1. Introduction

In three-dimensional numerical models, there are usually two types of eddy coefficients implemented in the eddy viscosity concept: the horizontal eddy coefficient for modelling the horizontal mixing process and the vertical eddy coefficient used for describing the vertical mixing process in the flow domain (Dunsbergen, 1994; Costa, 1995). The main aim of this study is to get better insight into the horizontal mixing process with more focus on the quasi-2D turbulence in shallow confined shear flows. The target study areas are the shallow flows such as the estuarine environments and coastal zones, which are characterized by a geometrical disparity. Two-distinct separated scales of turbulence can be found in the shallow flows: the three-dimensional subdepth scale turbulence generated by the wall bounded shear flow and the quasi-2D turbulence with vortex scales larger than the water depth (Fig. 1).

The horizontal large-scale eddies (quasi-2D turbulence) occur frequently in shallow free surface water flow (Ghidaoui and Kolyshkin, 1999). The quasi-2D turbulence might be

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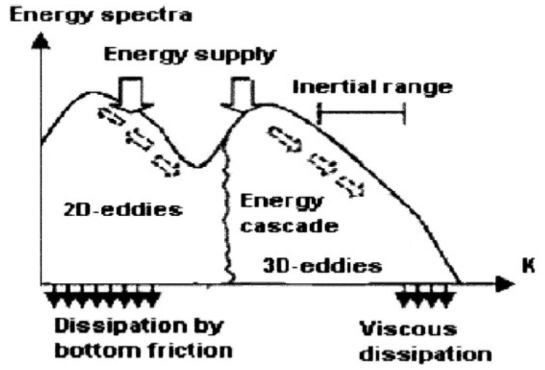


Fig. 1. The 2D turbulence and the 3D turbulence in the spectral domain of shallow-water turbulent flow, where k is the wave number (Nadaoka and Yagi, 1998).

created due to a topographic forcing (type A), transverse shear instabilities (type B) and secondary instabilities of the base flow (type C). Another important generation mechanism of 2D turbulence is the baroclinic instability. The highly resolved three-dimensional large eddy simulation (3D-LES) is not practical for large geophysical scale applications due to the limitation of the present computing capacity. Thus, the two-dimensional or horizontal large eddy simulation (Horizontal LES), as intermediate level of details, is a useful tool.

In this article, the performance of the horizontal large eddy simulation module developed at the University of Leuven (HLES-KULeuven module) is assessed. The HLES-KULeuven module is based on an implicit filter operator and has been implemented for oscillatory flows in an expanding flume, for which experimental data are available from Stelling and Wang (1984). In addition, the outputs of different subgrid scale models have been compared and analysed.

2. Governing equations

The governing equations of the horizontal large eddy simulations are expressed mathematically as follows (Uittenbogaard, 2003):

$$\frac{\partial \xi}{\partial t} + \nabla.\{(h+\eta)\underline{u}\} = 0 \tag{1}$$

η	free-surface elevation
h	bed level
$\underline{u}$	depth-averaged velocity vector

Based on the hydrostatic pressure assumption, the horizontal momentum equations read:

$$\frac{\partial \underline{u}}{\partial t} + \underline{u}.\nabla\underline{u} + g\nabla\eta = \upsilon\nabla^2\underline{u} + \underline{T} - \left\{\frac{c_f|\underline{u}|}{h+\eta}\right\}\underline{u} \tag{2}$$

Where;

g	the acceleration of gravity
υ	Kinematic viscosity of water

The velocity vector $\underline{u}$ is the depth-averaged horizontal velocity vector. The force $\underline{T}$ will appear due to the turbulent stresses and is defined as follows:

$$\underline{T} = \nabla.\left(2\upsilon_T^{3D}\underline{\underline{S}}\right) + \nabla.\left(2\upsilon^{SGS}\underline{\underline{S}}\right) \tag{3}$$

$\underline{\underline{S}}$ is the rate of deformation tensor;

$$\underline{\underline{S}} = \frac{1}{2}\left(\nabla\underline{u} + (\nabla\underline{u})^T\right) \tag{4}$$

υ_T^{3D} is the eddy-viscosity representing the momentum exchange by 3D turbulent motions. The last term on the right

Table 1
Eddy viscosity models for the shallow mixing region.

Author	Model	Notes
Prandtl, 1925	$l_m^2\left(\frac{\partial U}{\partial y}\right)$	l_m is proportional to the width of the mixing region ($=0.085\times\delta$)
Von Karman, 1930	$k^2\frac{(\partial U/\partial y)^2}{\left(\partial^2 U/\partial y^2\right)^2}\left\|\frac{dU}{dy}\right\|$	k is von Karman constant = 0.4
Taylor, 1932	$\frac{1}{2}l_w^2\left(\frac{\partial U}{\partial y}\right)$	$l_w = \sqrt{2}l_m$
Prandtl, 1942	$l_m^2\sqrt{\left(\frac{\partial U}{\partial y}\right)^2 + l'^2_m\left(\frac{\partial^2 U}{\partial y^2}\right)^2}$	Requires two mixing lengths l_m and l_m' In this study, $l_m = l_m' = l_m$
Prandtl, 1942	$C\delta(U_{max} - U_{min})$	$C = 0.014$ for plane jets
Hinze, 1975	$0.016\ U_eD^*$	This equation is deigned for the wake of a circular cylinder U_e is the local boundary layer edge velocity, D^* is the a characteristic thickness of the cylinder (e.g. cylinder diameter)
Ogink, 1985	$\upsilon_t = 0.5\ m^2/s$	
Alavian and Chu, 1985	$\varepsilon_o\delta\Delta$	ΔU is velocity difference across the shear layer, ε_o is the inverse of the turbulent Reynolds number In thus study, $\varepsilon_o\Delta = 0.08(U(y_{75\%}) - U(y_{25\%}))$ {see Section 4 for the definition of $U(y_{75\%})$ and $U(y_{25\%})$}
Lambert and Sellin, 1996	$(C_{ml}D)^2\ (dU/dy)$	$C_{ml} = 0.6$, and D is the water depth
van Prooijen et al., 2005	$\frac{D_m}{D}\beta^2\delta^2(dU/dy)$	D_m = average water depth of the main channel and the floodplain, D = local water depth, β = empirical coefficient (between 0.07 and 0.124). In this study, $\beta = 0.085$

Note: The empirical parameters in these models have been calibrated with turbulence experimental data from free shear flows and shallow compound channel flows.

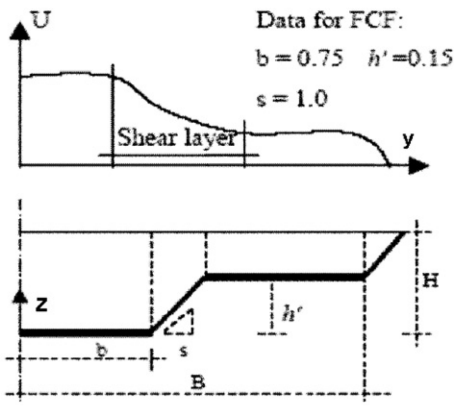


Fig. 2. Geometry of FCF (Shiono and Knight, 1991).

hand side of (Eq. (2)) represents the momentum sink due to the bottom shear stress (vertical shear) which is modelled by the quadratic friction law.

2.1. Two-dimensional Smagorinsky SGS model

In principle, the Smagorinsky SGS model implicitly assumes that turbulence is dissipated where it is produced, namely that there is no transport of turbulence (or of variables describing it). If the turbulence state at a certain point is affected significantly by the turbulence production in another position in the flow or by the history influence (the turbulence generation at previous times), such simple models are not valid. The Smagorinsky model is applicable to all turbulent flows, and arguably the simplest turbulence model (Pope, 2000). In the 2D Navier–Stokes equations, (Madsen et al., 1988) replaced the 3D-Smagorinsky subgrid scale (SGS) model (Smagorinsky, 1963) by the following relation:

$$\upsilon^{SGS} = l^2.\left[\left(\frac{\partial U}{\partial x}\right)^2 + \left(\frac{\partial V}{\partial y}\right)^2 + \frac{1}{2}\left(\frac{\partial U}{\partial y} + \frac{\partial V}{\partial x}\right)^2\right]^{1/2} \tag{5}$$

The instantaneous velocities have been replaced by the depth-averaged velocities U and V (the streamwise and the spanwise velocity components).

Such that;

l is the mixing length $= C_s\Delta$, (C_s is the Smagorinsky coefficient and Δ is the filter width).

It should be noticed that the Smagorinsky SGS model is based on the assumption that the grid scale of the LES model lies within a forward energy cascade. This SGS model is not reliable for the quasi-2D turbulence since 2D turbulence is characterized by an inverse energy cascade.

2.2. Uittenbogaard's model

A subgrid-scale model for quasi-2D turbulence has been defined based on a leaky energy cascade (energy loss by bed friction) in Uittenbogaard and van Vossen (2004). This model is similar to the Smagorinsky SGS model. The main difference between these two SGS models is that the Uittenbogaard's model takes into account the dissipative effect of the bottom shear stress on the turbulent kinetic energy of the quasi-2D turbulence.

A temporal high-pass filter on the resolved velocity field is used in order to omit the very large-scale velocity gradient from the strain rate tensor that enters the subgrid scale model. Uittenbogaard's model takes the following form:

$$\upsilon^{SGS} = \frac{1}{k_s^2}\left(\sqrt{(\gamma\sigma_T S^*)^2 + B^2} - B\right) \tag{6}$$

$$B = \frac{3}{4}c_f\frac{|\underline{U}|}{H} \text{ or } B = \frac{3}{4}\frac{g}{C^2}\frac{|\underline{U}|}{H} \tag{7}$$

k_s	truncation wave number depending on the local grid area $\Delta x.\Delta y$
γ	coefficient depends on the slope of the small-scale energy spectrum
σ_T	turbulent Prandtl/Schmidt number
$\underline{U}$	low-pass filtered velocity vector
c_f	Roughness coefficient $\left(=\frac{g}{C^2}\right)$
C	Chezy coefficient
H	water depth ($= h + \eta$)
B	accounts for the damping caused by bottom friction

$$\frac{1}{k_s^2} = \frac{\Delta x.\Delta y}{\left(\pi f_{lp}\right)^2} \tag{8}$$

f_{lp} is the spatial low-pass filter ($=1.0$).

$$\left(S^*\right)^2 = 2\left(\frac{\partial u^*}{\partial x}\right)^2 + 2\left(\frac{\partial v^*}{\partial y}\right)^2 + \left(\frac{\partial u^*}{\partial y}\right)^2 + \left(\frac{\partial v^*}{\partial x}\right)^2 + 2\frac{\partial u^*}{\partial y}\frac{\partial v^*}{\partial x} \tag{9}$$

$(\ )^*$ refers to Eulerian fluctuating flow variables obtained through the following recursive high-pass filter operator with time step number n:

$$\psi^* = \psi_{n+1} - \overline{\psi}^t_{n+1} \tag{10}$$

$$\overline{\psi}^t_{n+1} = (1-a)\psi_{n+1} + a\overline{\psi}^t_n \tag{11}$$

$$\overline{\psi}^t_0 = 0 \text{ and } a = exp(-\Delta t/\tau) \tag{12}$$

Δt is the time step, n is the number of time steps and τ is the relaxation time ($\approx$twice the period of the largest eddy) chosen such that deterministic (tidal) fluctuations do not contribute to S^*.

3. Two-length scale approach

Table 1 summaries some of the eddy viscosity models for plane free shear flows and the traditional models for shallow open channel flows (Tennekes and Lumley, 1972; Hinze, 1975; Rodi, 1980; Alavian and Chu, 1985; Lambert and Sellin, 1996; Tukker, 1997; van Prooijen et al., 2005). Since there is an uncertainty in the definition of the mixing length, some modifications have been suggested. The parabolic law is the simplest formulation (Hinze, 1975; Thomas and Williams,

Table 2
The shear layer thickness for different Dr of the FCF.

$Dr(-)$	0.11	0.157	0.196
H(m)	0.169	0.177	0.187
$\delta = 2(y_{75\%} - y_{25\%})$ (m)	0.5	0.375	0.46

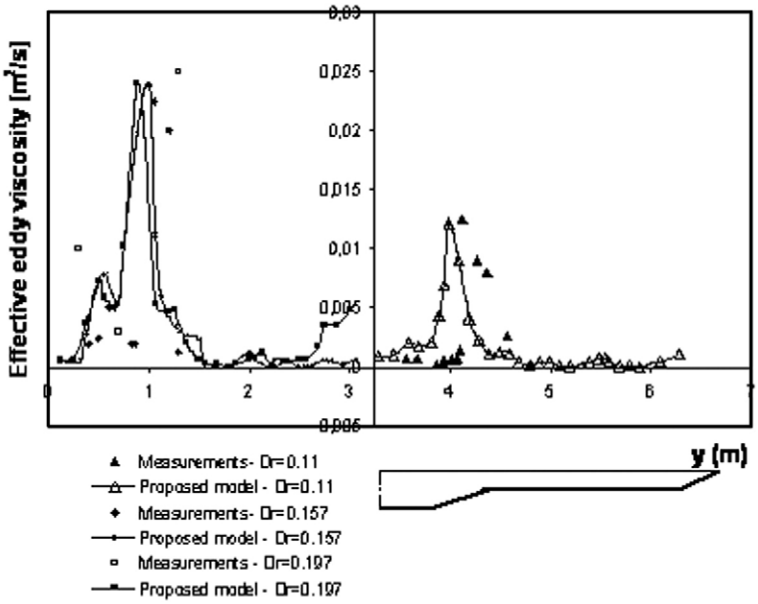


Fig. 3. Comparison between the proposed two-length scale model and the experimental data (where, y is the transverse (spanwise) direction of the flume). The total contribution of the 2D and 3D turbulence is defined as effective eddy viscosity.

1995b; Xing and Davies, 1996). In this law, two empirical mixing lengths are used as follows:

$$\frac{1}{l} = \frac{1}{l_1} + \frac{1}{l_2} \tag{13}$$

Where l_1 and l_2 are two mixing lengths.

The main idea of the proposed two-length scale model is to eliminate the weaknesses of the Smagorinsky SGS model. The Smagorinsky model has two weaknesses. Firstly, the model does not take into account the effect of the bottom friction on the turbulent kinetic energy of the quasi-2D turbulence. This problem generates unbalance between the dissipation produced by the bottom shear stress and the dissipation of the eddy viscosity model. The second weakness is that the Smagorinsky model does not represent the transport of turbulence. These are the reasons why the Smagorinsky model is not a proper model for the Horizontal LES. The proposed two-length scale model treats these

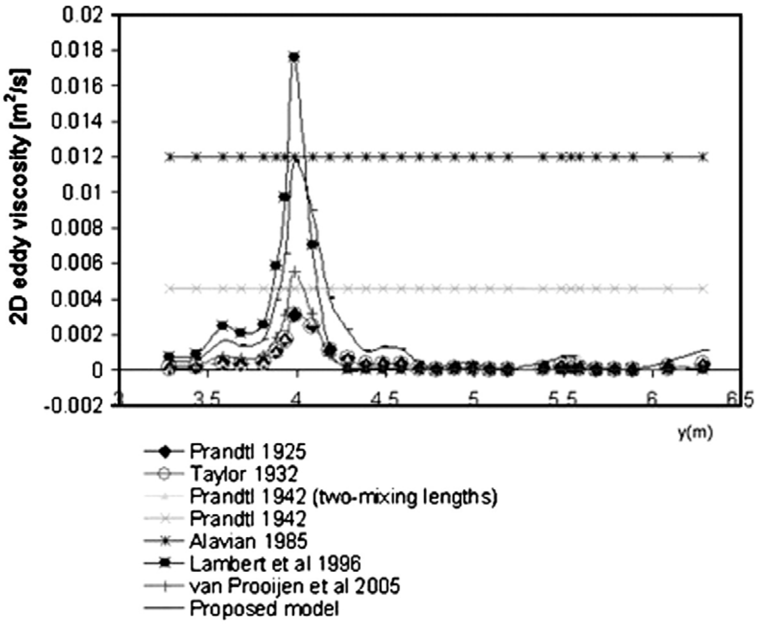


Fig. 4. Comparison between the proposed 2D eddy viscosity model (Eq. (15)) and selected eddy viscosity models from Table 1.

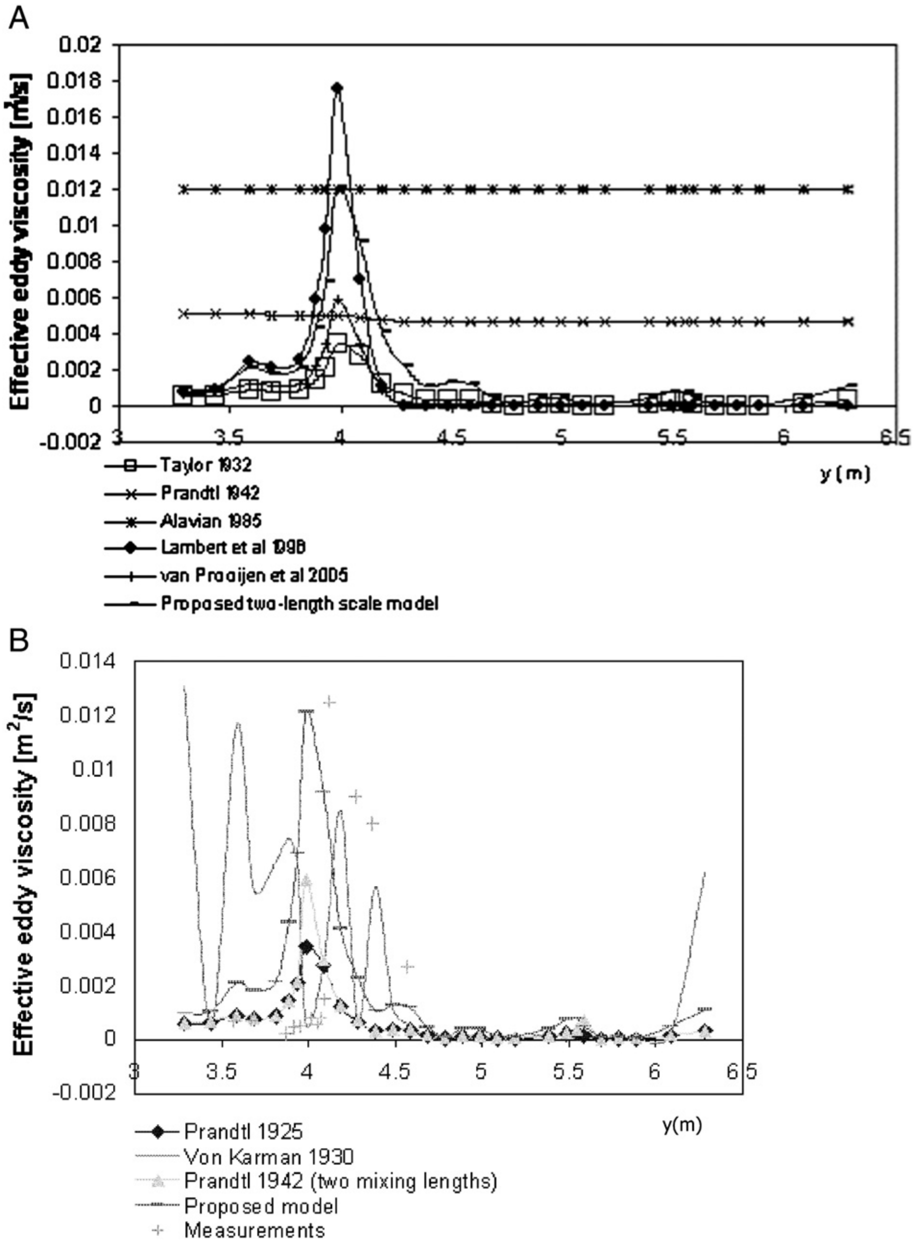


Fig. 5. Comparison between the proposed two-length scale model (Eq. (18)) and selected eddy viscosity models from Table 1. The total contribution of the 2D and 3D turbulence is defined as effective eddy viscosity.

problems empirically based on the analogy between the behaviour of the large scale vortices in plane free shear flow and the quasi-2D turbulence in shallow confined shear flows.

In the one-length scale models such as the Smagorinsky SGS model, it is assumed that there is a single length scale for the quasi-2D turbulence and the three-dimensional

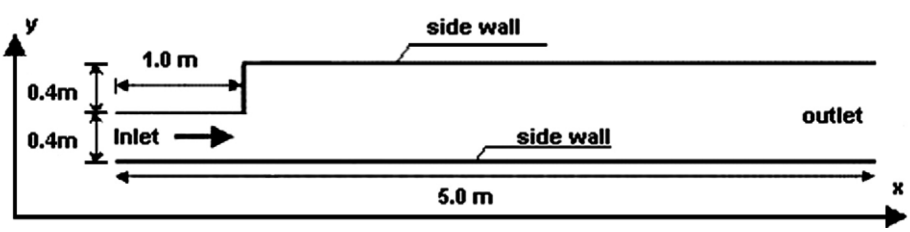


Fig. 6. Layout of the expanding flume (Stelling and Wang, 1984; Awad et al., 2007).

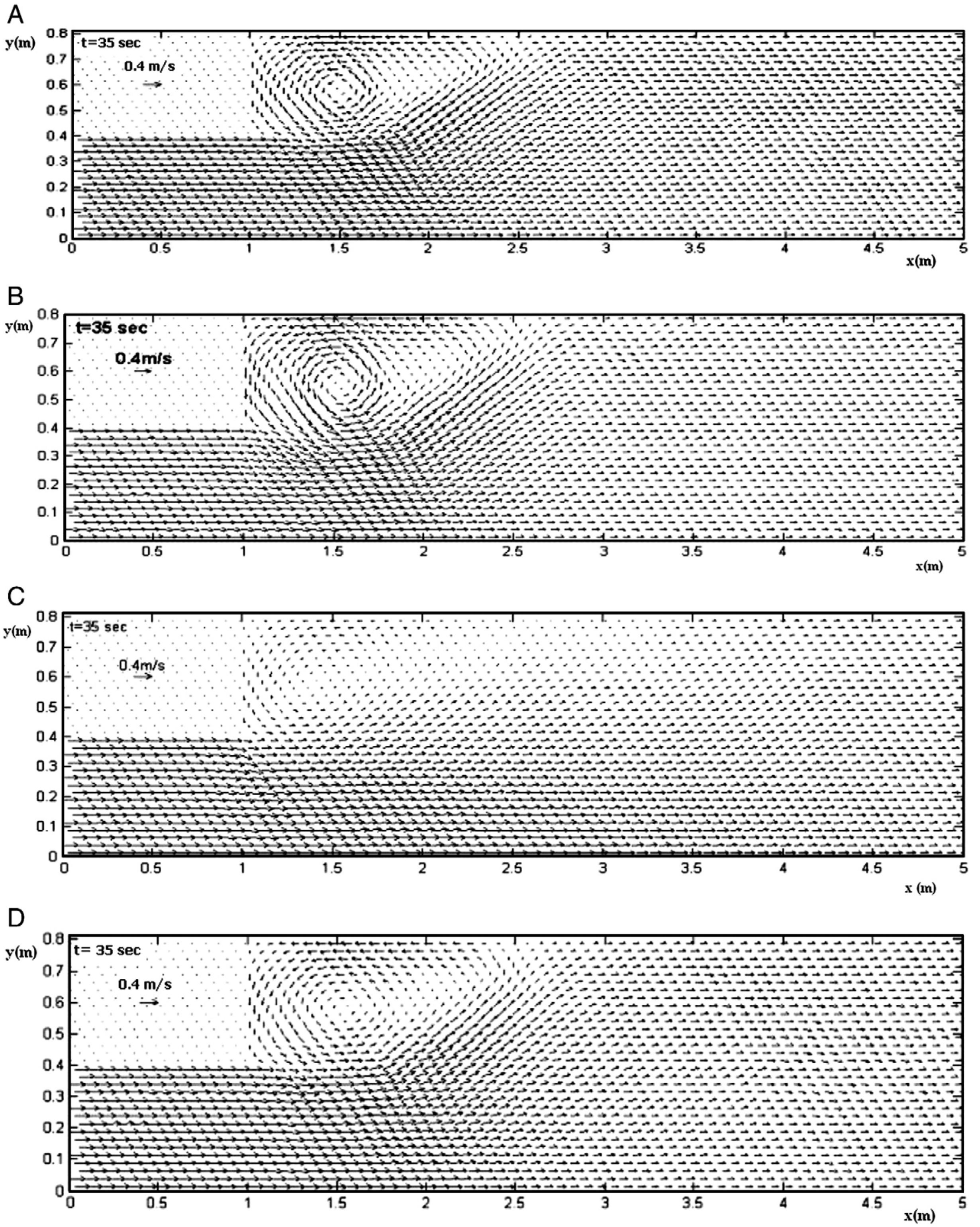


Fig. 7. Comparison between the results of the HLES-KULeuven module and measurements after 35 s. A: Simulation without SGS model. B: Simulation without SGS model and vertical shear. C: Two-length scale model. D: Smagorinsky SGS model. E: Uittenbogaard SGS model using high pass filter operator. F: Uittenbogaard SGS model using resolved velocities. G: Velocity measurements (Stelling and Wang, 1984).

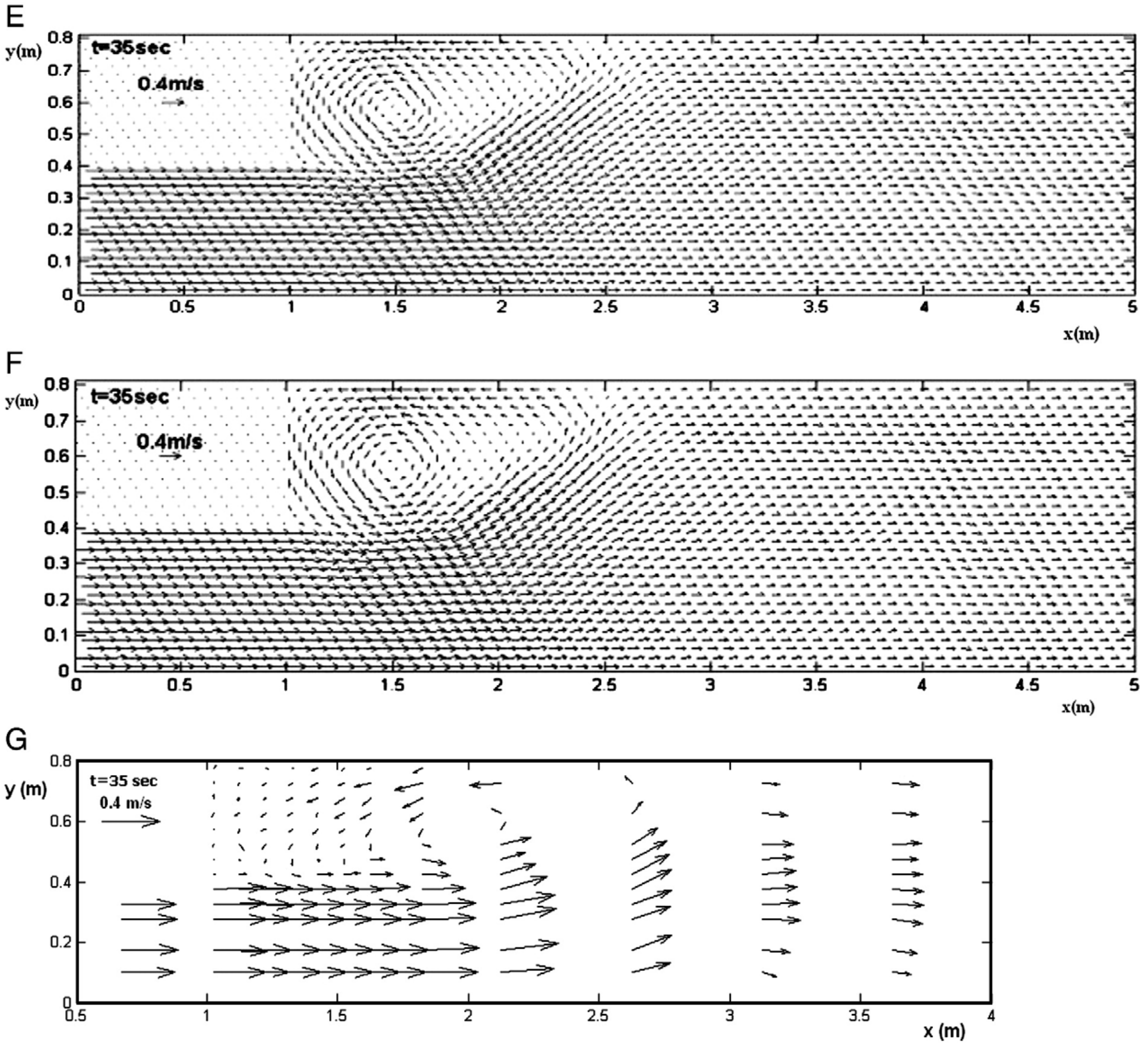


Fig. 7 (*continued*).

subdepth scale turbulence. The bottom friction has a stabilizing effect on the quasi-2D turbulence in shallow flows (Alavian and Chu, 1985; Babarutsi et al., 1989). This effect is not taken into consideration in the one-length scale models. According to Jirka and Uijttewaal (2004), a key element in the horizontal large eddy simulation (HLES) is the inclusion of the effect of the bottom friction on the quasi-2D turbulence.

For unidirectional flow conditions, the two-dimensional eddy viscosity υ^{2D} can be expressed using the Prandtl mixing length concept. Thus,

$$\upsilon^{2D} = \left(l^{2D}\right)^2 \frac{\partial U}{\partial y} \tag{14}$$

Such that;

l^{2D} is the mixing length of the quasi-2D turbulence.

U is the depth-integrated primary components of velocity.

The key issue of this concept is the mathematical definition of l^{2D}. According to Prandtl (1925), the mixing length is the undisturbed distance that a molecule travels. This implies that all the previous trials to describe the mixing length have empirical characters as can be noticed in Table 1. Despite this weakness, the mixing length models can be calibrated to give good engineering and trend predictions (Tennekes and Lumley, 1972). Therefore, the mathematical expression of the mixing length of the plane free shear flow may be reformulated so that it can be implemented for the quasi-2D turbulence.

The bed friction reduces the effectiveness of the transverse shear to produce turbulence (Chu and Babarutsi, 1988; Babarutsi et al., 1989; Chu et al., 1991). Thus, it can be deduced

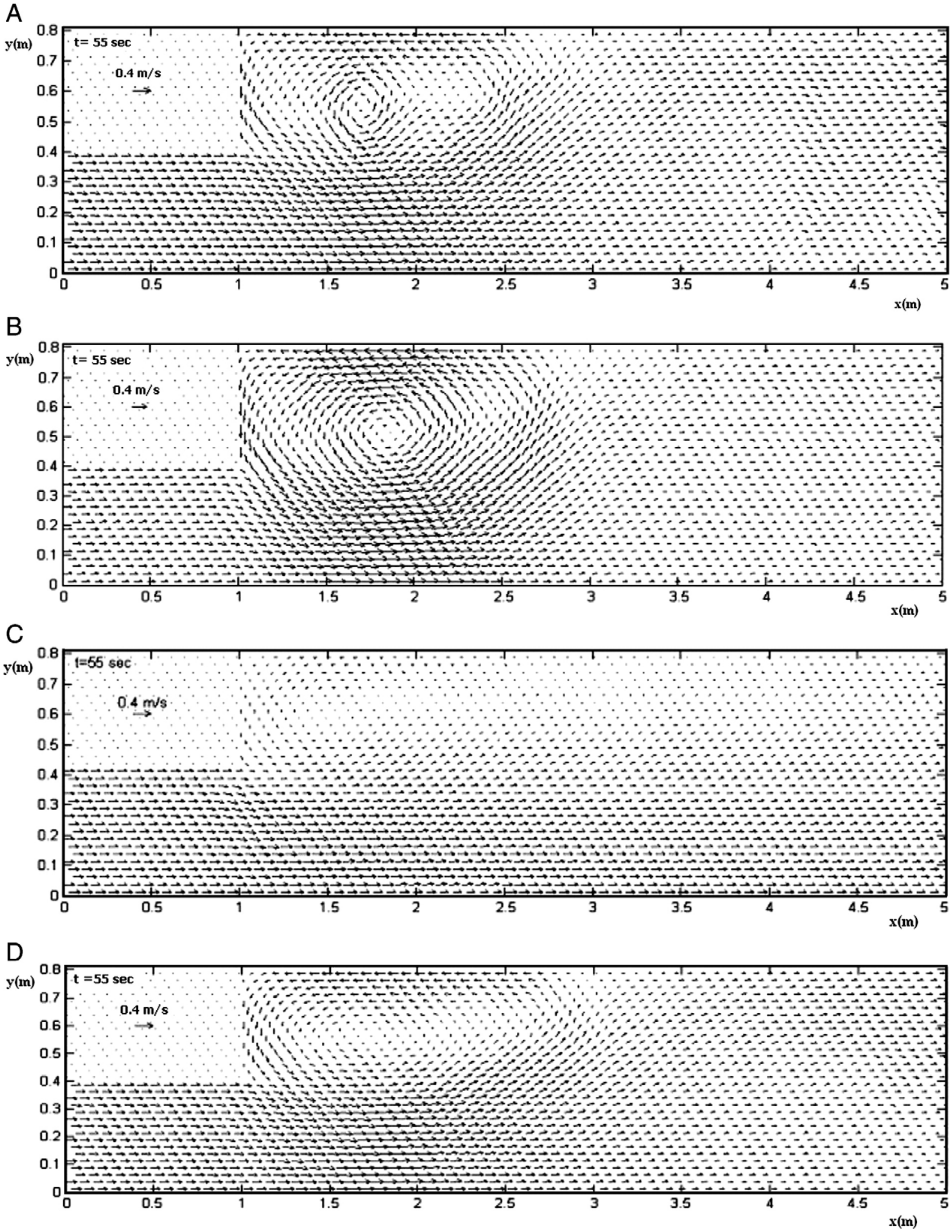


Fig. 8. Comparison between the results of the HLES-KULeuven module and measurements after 55 s. A: Simulation without SGS model. B: Simulation without SGS model and vertical shear. C: Two-length scale model. D: Smagorinsky SGS model. E: Uittenbogaard SGS model using high pass filter operator. F: Uittenbogaard SGS model using resolved velocities. G: Velocity measurements (Stelling and Wang, 1984).

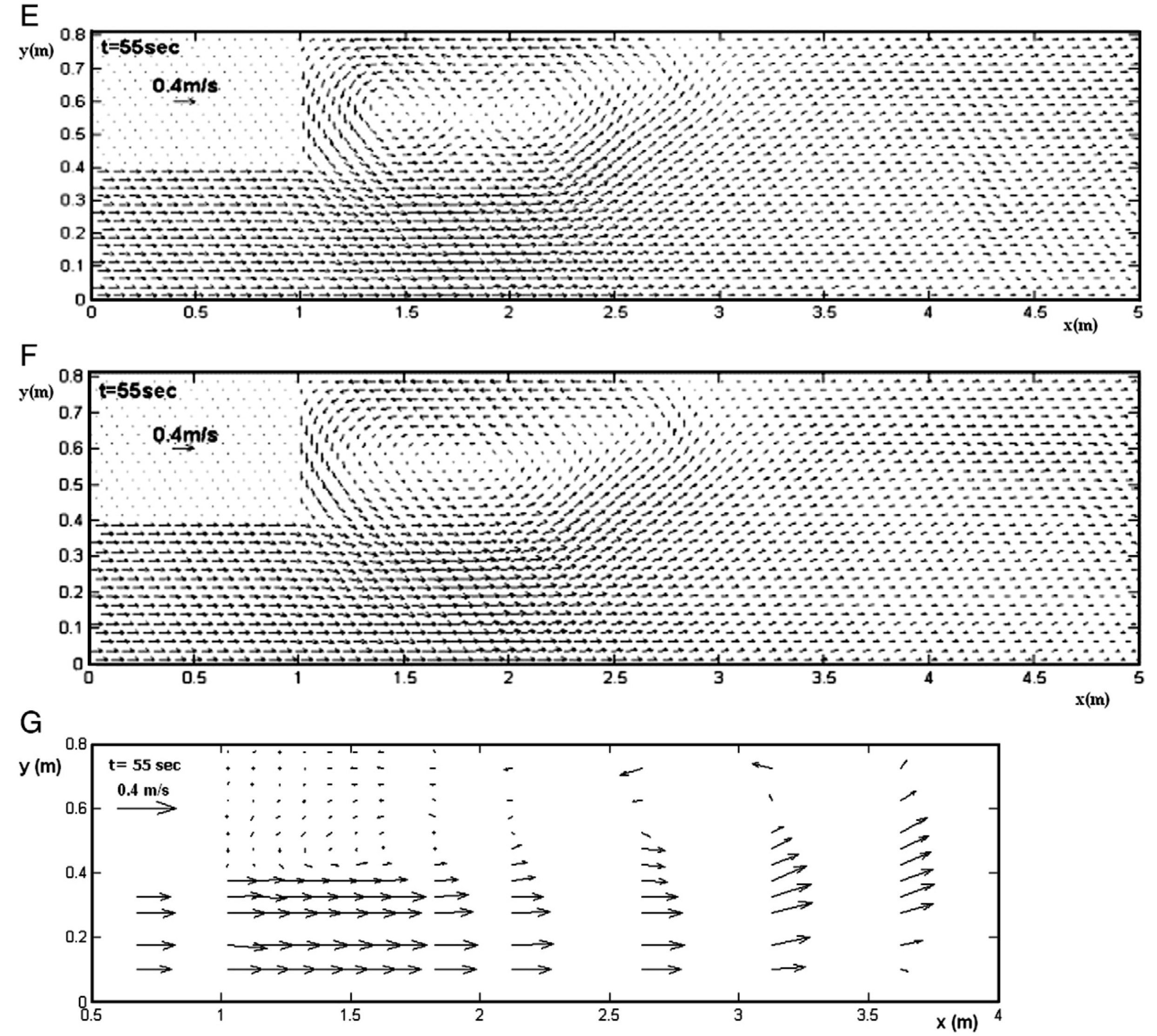


Fig. 8 (*continued*).

that the mixing length of the 2D-turbulence l^{2D} is proportional to $\left(\frac{1}{c_f}\right)$, where c_f is the bottom friction coefficient. In addition, l^{2D} is proportional to δ based on the analogy between the behaviour of the 2D-turbulence in confined geophysical flows and the large-scale eddy in the plane free shear flow, where δ is the mixing layer width. Thus, the proposed model for the 2D eddy viscosity can be written as follows:

$$\upsilon^{2D} = \left(\frac{\psi\delta}{c_f^{\phi}}\right)^2\left(\frac{\partial U}{\partial y}\right) \tag{15}$$

Such that;

- ψ proportionality empirical parameter (approximately equal to 0.085 based on the plane free shear flow applications)
- c_f the roughness coefficient
- ϕ exponential coefficient describes the relation between the mixing length of 2D-coherent turbulent structures and the bottom friction.

The turbulent viscosity of the confined flows is thought to be proportional to two components: the large-scale eddy viscosity and the small-scale eddy viscosity. Therefore, our proposed two length-scale model is formulated as follows:

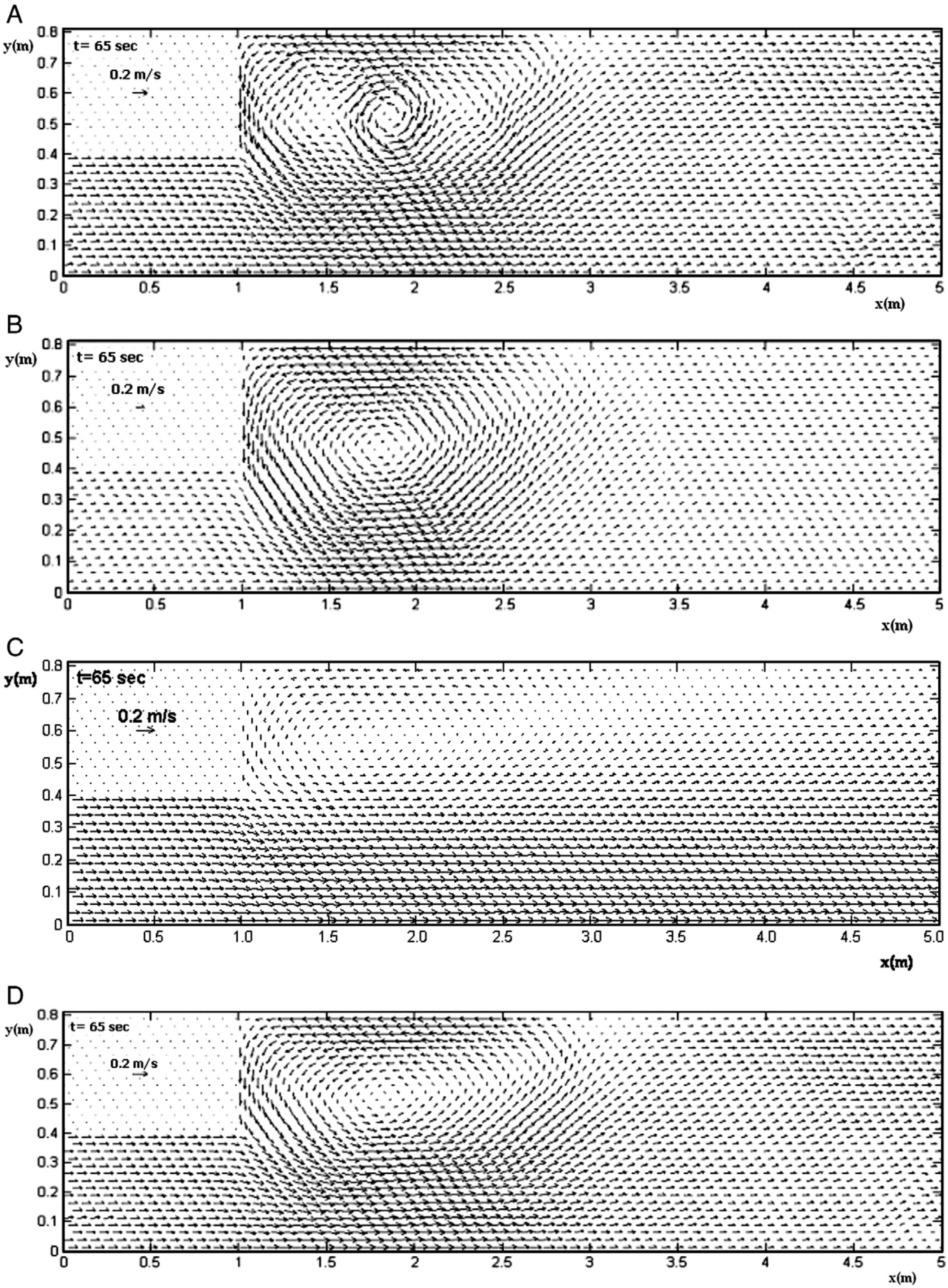
$$\upsilon_H = \left(\frac{\psi\delta}{c_f^{\phi}}\right)^2\left(\frac{\partial U}{\partial y}\right) + \frac{1}{H}\int_H \upsilon_T^{3D} dz \tag{16}$$

In principle, Eq. (16) can be generalized using the strain rate tensor. Thus,

$$\upsilon_H = \left(\frac{\psi\delta}{c_f^{\phi}}\right)^2\sqrt{2S_{ij}S_{ij}} + \frac{1}{H}\int_H \upsilon_T^{3D} dz \tag{17}$$

Such that,

- S_{ij} is the strain rate tensor $= \frac{1}{2}\left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i}\right)$
- i, j are indices indicating spatial dimensions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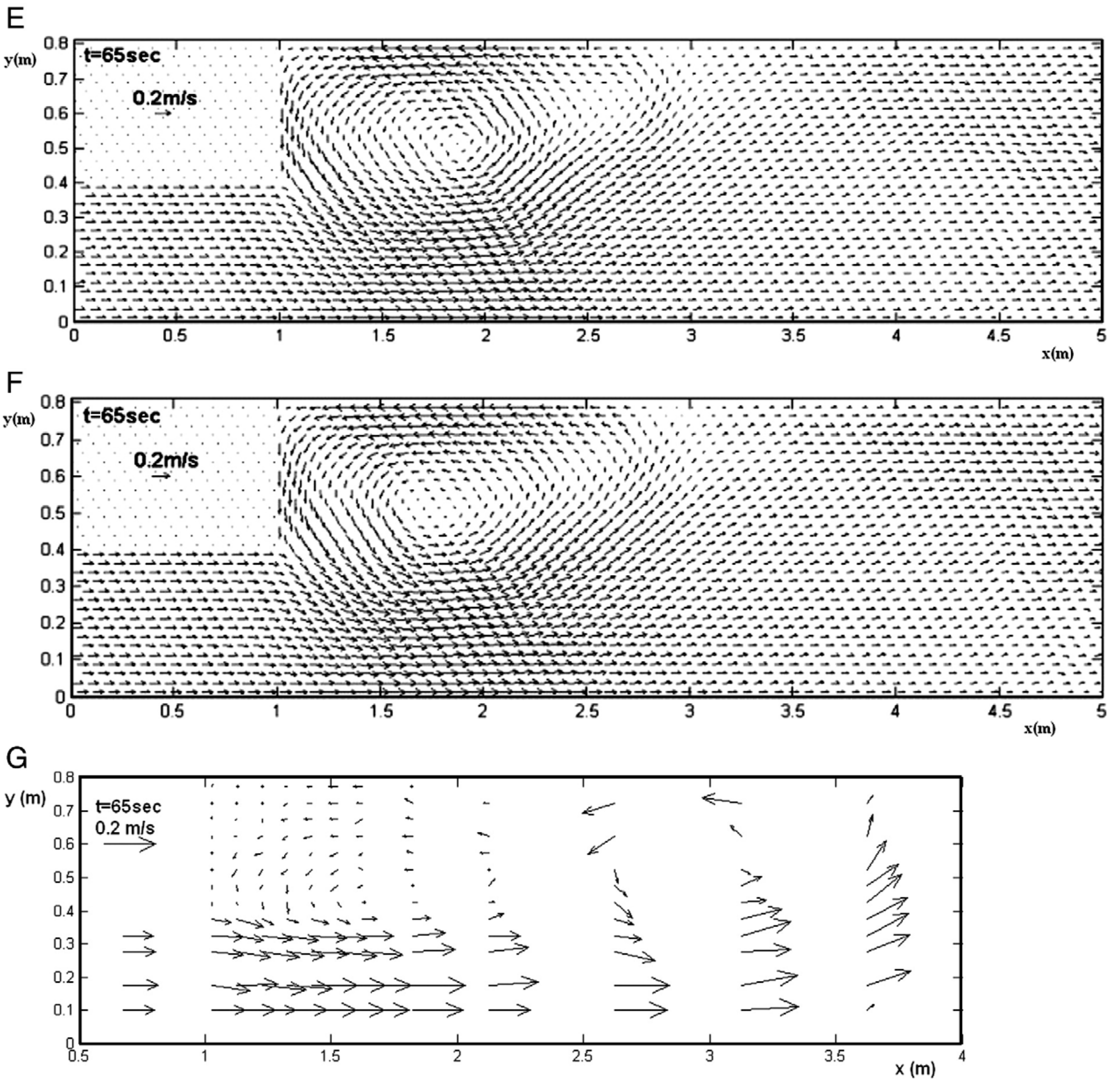

Fig. 9. Comparison between the results of the HLES-KULeuven module and measurements after 65 s. A: Simulation without SGS model. B: Simulation without SGS model and vertical shear. C: Two-length scale model. D: Smagorinsky SGS model. E: Uittenbogaard SGS model using high pass filter operator. F: Uittenbogaard SGS model using resolved velocities. G: Velocity measurements (Stelling and Wang, 1984).

x_i, x_j the coordinates axes. x_1, x_2, x_3 correspond to the Cartesian coordinates x, y, and z, respectively
u_i, u_j components of the instantaneous velocity

υ_H represented by Eq. (17) can be seen as an effective horizontal mixing coefficient (or effective eddy viscosity). The first term in this equation is equivalent to the eddy viscosity of the quasi-2D turbulence and the second term reflects the effect of the 3D turbulence on the horizontal mixing process. In this study, the contribution of the wall-bounded shear flow has been taken into account by the depth integration of the parabolic distribution of the 3D-eddy viscosity. In addition, the model will be calibrated with unidirectional flows. Thus, the following simple form of Eq. (17) will be used for the calculations of Section 4.

$$\upsilon_H = \left(\frac{\psi\delta}{c_f^{\phi}}\right)^2 \left(\frac{\partial U}{\partial y}\right) + \frac{k}{6} u^* H \quad (18)$$

Such that,

k Von Karman constant = 0.4

In principle, the proposed two-length scale approach is different from the parabolic mixing length formulations

Table 3
Separating length scales Ls of the horizontal mixing layer.

Case	Computation	Separating length Ls (m)					
		Time (s)					
		15	25	35	45	55	65
Measurement (Stelling and Wang 1984)		0.7	1.4	1.9	2.3	2.4	2.6
HLES-KULeuven module	Without SGS model	0.6	1.0	1.5	1.6	1.8	1.9
	Without SGS model and vertical shear	0.6	1.0	1.5	1.6	1.8	1.9
	Two-length scale model	0.65	0.80	1.75	2.0	2.4	2.6
	Smagorinsky SGS model	0.6	1.0	1.5	1.6	1.8	2.0
	Uittenbogaard SGS-high pass filter	0.6	1.0	1.5	1.6	1.8	2.0

mentioned in Eq. (13). This equation has been proposed to allow more degrees of freedom to tune the mixing length for a good agreement with the experimental data. This method is similar to the model of Prandtl (1942).

4. Results and discussion

4.1. Experimental calibration of the proposed model

A compound channel flow has been chosen as a test case for the calibration of the newly developed concept [Eq. (18)]. The geometry of the compound channel can be considered as a prototype for many estuarine environments. In the compound channel flow, the quasi-2D turbulence is generated at the interface between the floodplain and the main channel due to the internal transverse shear instabilities. Data are taken from the Flood Channel Facility (FCF) programme carried out in UK. Information about these measurements can be found in Thomas and Williams (1995a), Lambert and Sellin (1996), Ervine et al. (2000), Castanedo et al. (2005), Morvan (2005) and van Prooijen et al. (2005) (Fig. 2). The channel has a constant bed slope equals to $S_o = 1.027 \times 10^{-3}$. The friction coefficients used in the computations are 0.0023 for the main channel and 0.0030 for the floodplain (van Prooijen et al., 2005).

The shear layer width has been estimated using the procedures of van Prooijen (2004). The outputs are mentioned in Table 2.

Fig. 3 presents a comparison between the proposed two-length scale model (Eq. (18)) and the data of the eddy viscosity for the three different depth ratios $Dr = H - h'/H$ mentioned in Table 2. It is noted that the trend of the modelled and measured turbulent viscosity is identical. There is a shift between the modelled and the measured peaks. But in general there is a qualitative agreement between the results of the effective horizontal mixing model (Eq. (18)) and the experimental data. A value of ϕ between 0.11 and 0.18 gives a reasonable output. These values are equivalent to the depth ratio Dr in each case.

(Figs. 4 and 5) illustrates a comparison between selected models from Table 1 and the proposed model in case of $Dr = 0.11$. In Fig. 4, the proposed 2D eddy viscosity (Eq. (15)) is compared with the other approaches and in (Fig. 5A and B), the output of the proposed two-length scale model (Eq. (18)) is demonstrated.

It can be observed that the difference between the result of Eq. (15) in Fig. 4 and the result of (Eq. (18)) is negligible, which can be attributed to the domination of the 2D-turbulence in the shear layer region.

As indicated in (Fig. 5b), the concept of Von Karman (1930) does not provide acceptable results. This concept leads to infinite eddy viscosity at the position where the velocity gradient is zero (e.g. the centre line of the main channel).

It has been noticed that each model produces different outputs but the trend of all profiles is almost identical except

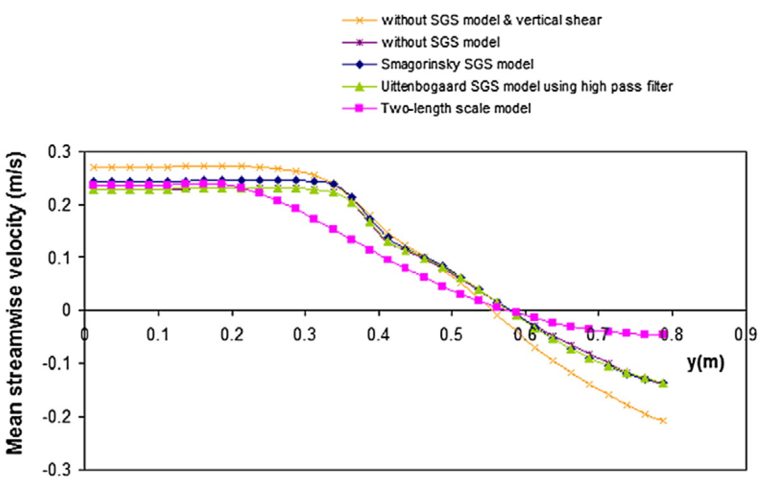

Fig. 10. Transverse distribution of the mean streamwise velocities at $x = 1.24$ m.

the Von Karman (1930) model, Alavian (Alavian and Chu, 1985) concept and Prandtl (1942) model. The result of Prandtl, 1942 based on two mixing lengths gives higher values than the Prandtl, 1925 model.

The modelled eddy viscosities are underestimated except the output of Lambert and Sellin (1996) and the proposed concept. The Lambert and Sellin (1996) model relies on a single length scale as an average between the 2D-turbulence

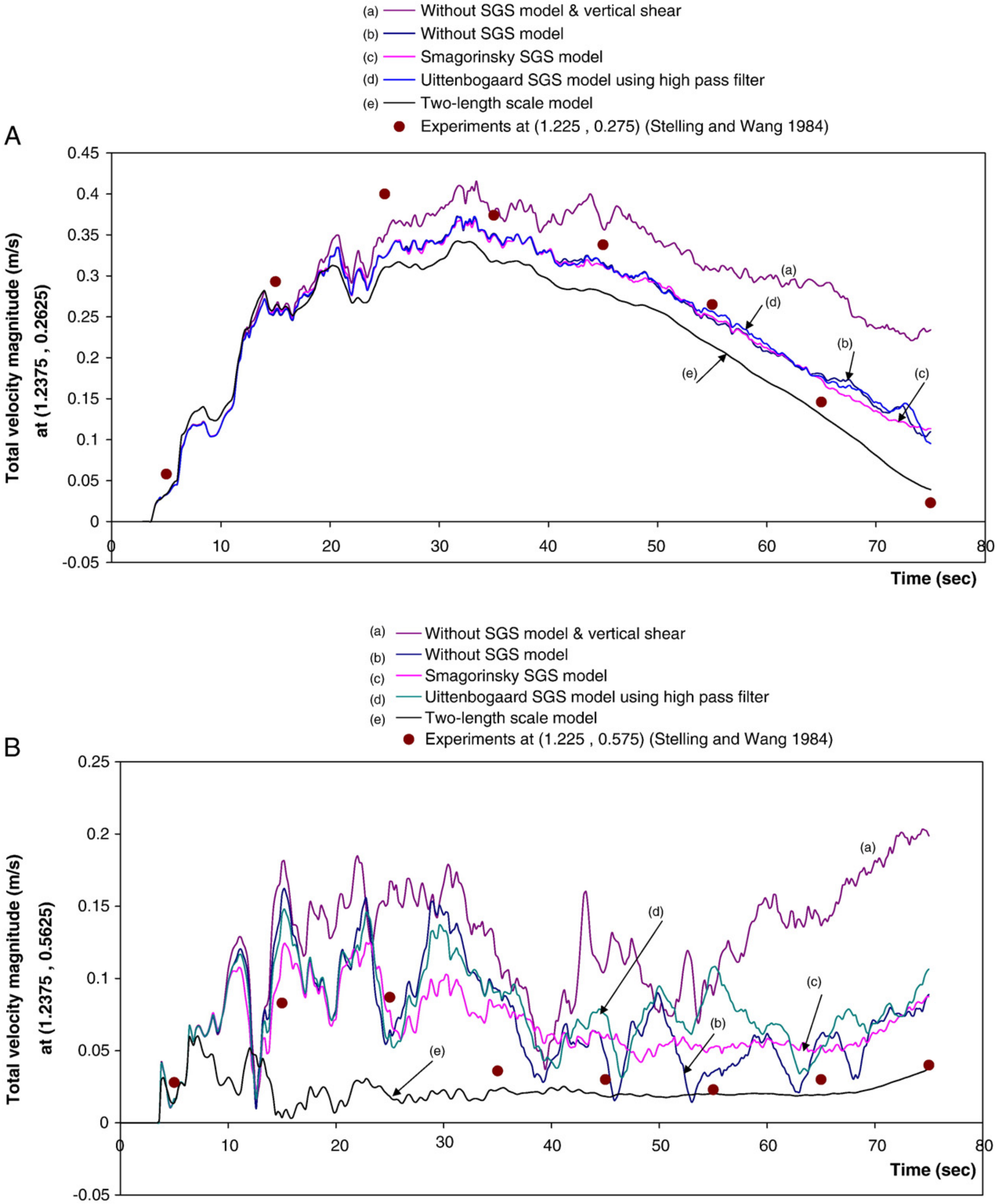

Fig. 11. Comparison between the measured velocities and the HLES-KULeuven outputs.

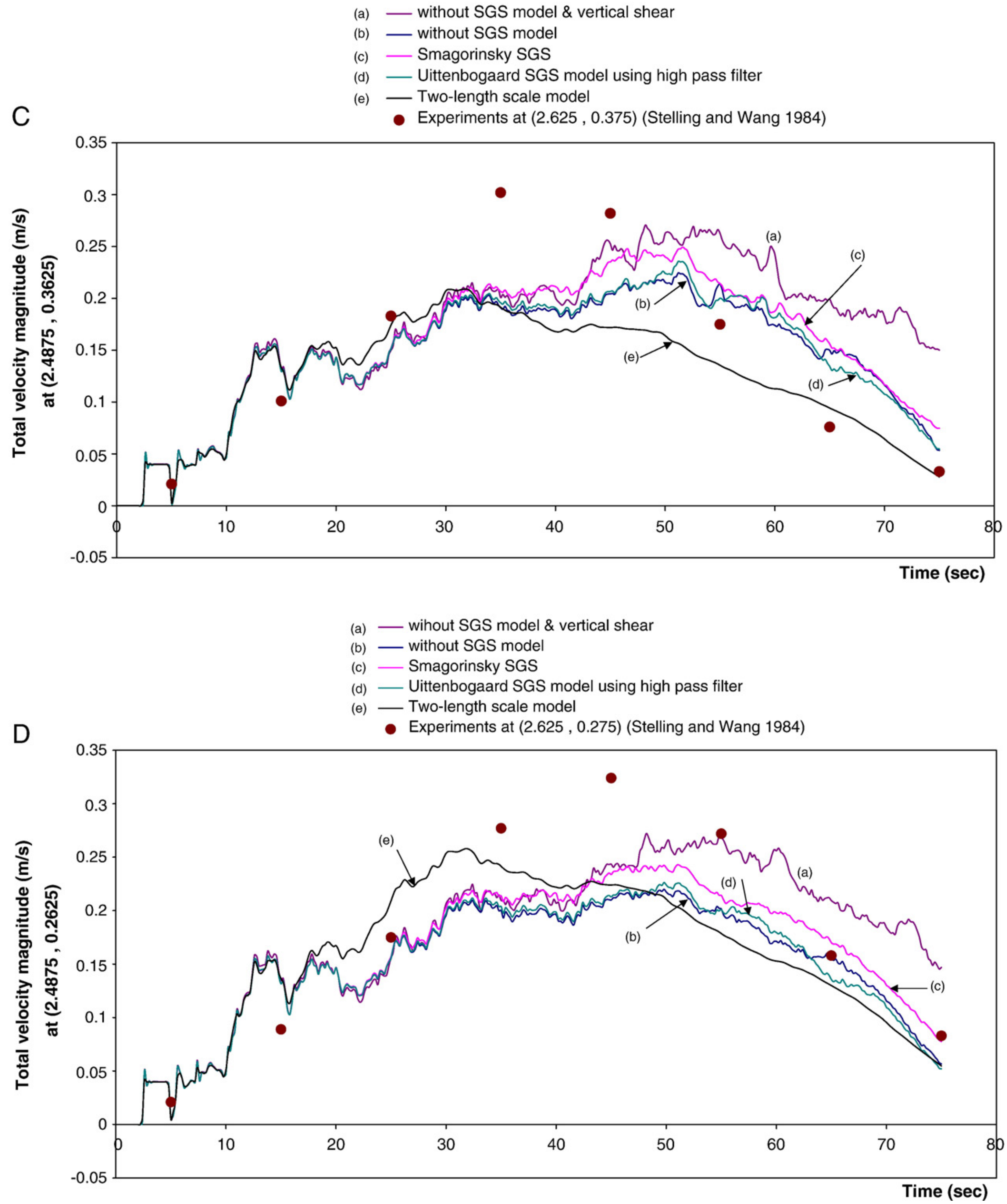

Fig. 11 (*continued*).

and the 3D-turbulence, which might be a reason for this discrepancy. Furthermore, all these models do not consider the influence of the bottom friction on the mixing length of the quasi-2D turbulence, which has been defined in the proposed model.

4.2. HLES of a backward facing step flow

In the following sections, the results of the horizontal large eddy simulations for an oscillatory flow in the expanding flume of Stelling and Wang (1984) are presented for three subgrid

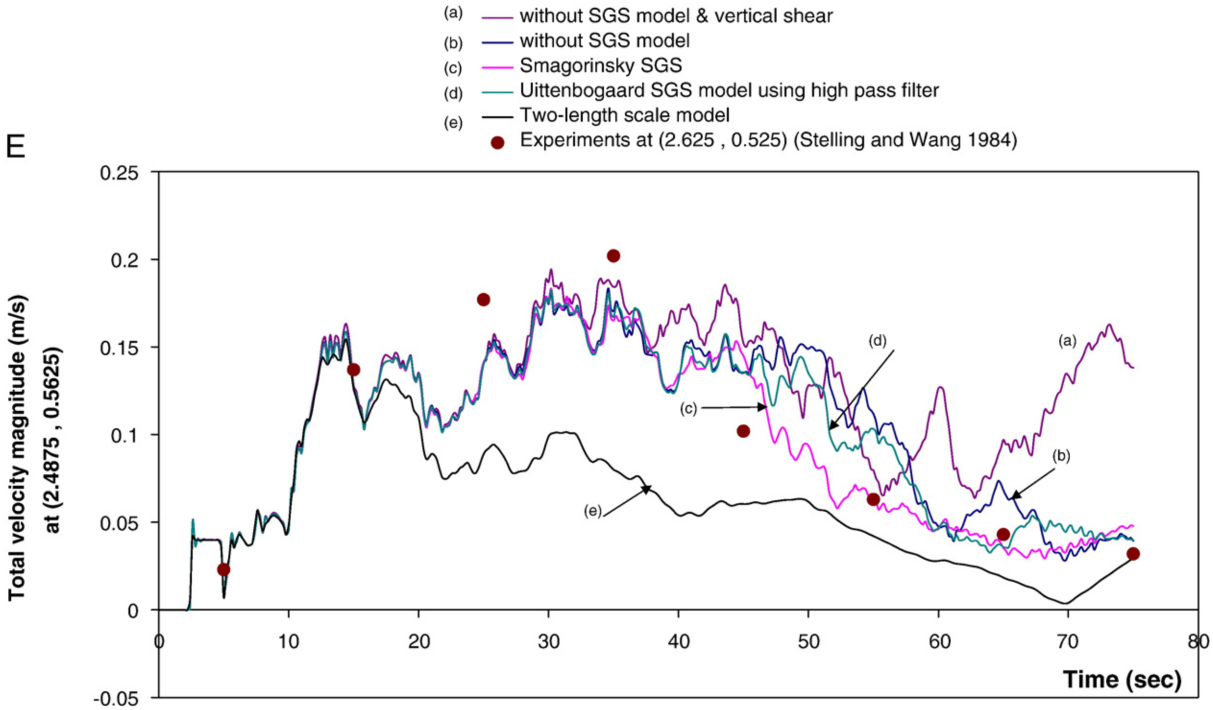

Fig. 11 (*continued*).

scale models: the Smagorinsky SGS model, the Uittenbogaard SGS model, and the proposed two-length scale model.

4.2.1. Numerical model setup

The dimensions of the expanding flume, in which the backward facing step flow measurements are carried out, are shown in Fig. 6 (Stelling and Wang, 1984; van Vossen, 2000). The total variation-diminishing scheme with a superbee limiter is used as a numerical solution for the advection terms of the HLES-KULeuven module. This module is currently integrated in COHERENS which is a three-dimensional community model developed by Luyten et al. (1999). The simulated time is 75 s, which is the total time of the experiments. The initial water depth in the flume was equal to 0.096 m and the flow velocity was zero. Two simulations have been carried out. The first simulation has been made with a uniform grid resolution equals to 25 mm ($\approx H/4$) in the streamwise and the spanwise directions (6400 cells). The second simulation has been conducted using a uniform grid resolution equals to 12.5 mm ($\approx H/8$) in the streamwise and the spanwise directions (25600 cells). This yields mesh sizes in terms of wall units of $\Delta x^+ = \Delta y^+ = 250$ in the first simulation and $\Delta x^+ = \Delta y^+ = 125$ in the second simulation based on the time-averaged shear velocity at the flume inlet. The filter width is equal to the grid resolution. The boundary conditions are defined based on inlet and outlet conditions in the form of harmonic expansion functions. A time-dependent velocity profile is imposed at the flume inlet $\{u(t)|_{\text{inlet}} = \sum_{k=1}^{3} \mathrm{u_k} \sin \mathrm{k}\varpi t$, u_k is velocity coefficients, h_k is water elevation coefficients, ϖ is the frequency $\left(= \frac{\pi}{T} = \frac{\pi}{75} sec^{-1}\right)\}$ and a time-dependent water level is imposed at the flume outlet. No-slip boundary conditions are imposed at the solid walls.

4.2.2. Results and discussion

4.2.2.1. Flow pattern. The following plots (Figs. 7–9) show the vectors of the simulated velocity between 35 s and 65 s from the beginning of the simulation. It can be noticed that the two-length scale model produces a weak horizontal large-scale eddy. In addition, it leads to more elongated vortex in the streamwise direction (less isotropic eddy). It has been found that the two-length scale approach produces better performance than the Smagorinsky SGS model and Uittenbogaard SGS model regarding the vortex size and stretching. This is attributed to the effect of the wall bounded shear on the two-dimensional flow. This effect has been taken into account by including the bottom friction and the three-dimensional eddy viscosity in the mathematical description of the horizontal mixing coefficient (see Eq. (17) and (18)). The incomplete representation of the wall bounded shear flow (the vertical shear) in case of Smagorinsky SGS model leads to less stretching than the two-length scale model. Furthermore, the Uittenbogaard SGS model does not show significant differences with respect to the results of the simulation without SGS model. Also, it can be seen that the results of the Uittenbogaard SGS model in case of using the resolved and the fluctuating velocity (obtained by a high pass filter operator using Eqs. (10)–(12)) are similar.

(Table 3) shows the separating length scale of the mixing layer (horizontal vortex) for different modelling approaches. All simulations give an underestimated separating length scale except the two-length scale model which results in approximately equivalent values to the measurements especially after 35 s. It should be noticed that the separating length scale, in case of the simulation without SGS model and the

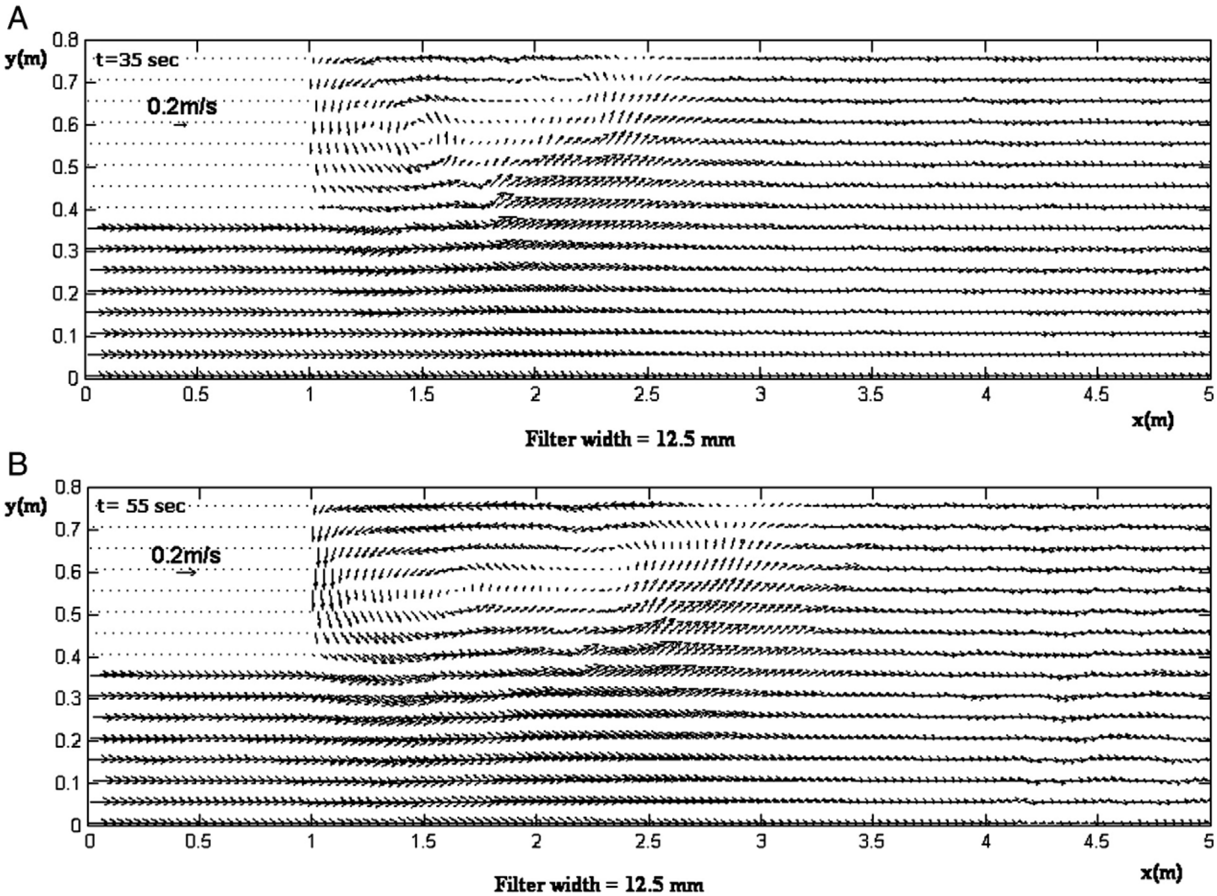

Fig. 12. Results of the HLES-KULeuven module (based on 12.5 mm using Smagorinsky SGS model).

vertical shear parameterization, is created mainly by the effect of the numerical dissipation of the advection scheme.

4.2.2.2. Quantitative comparisons. Fig. 10 presents the time-averaged streamwise velocities at a cross section located at $x \approx 1.24$ m after the flume expansion. It is clearly seen in (Fig. 10) that there is no significant difference between the results of the Smagorinsky SGS model and the simulation without SGS model. This indicates that the numerical dissipation is dominant. The two-length scale model produces lower velocities in the mixing layer (position of the horizontal vortex).

Fig. 11 shows a time-series for the measured and simulated velocity profiles at different positions inside the flow domain.

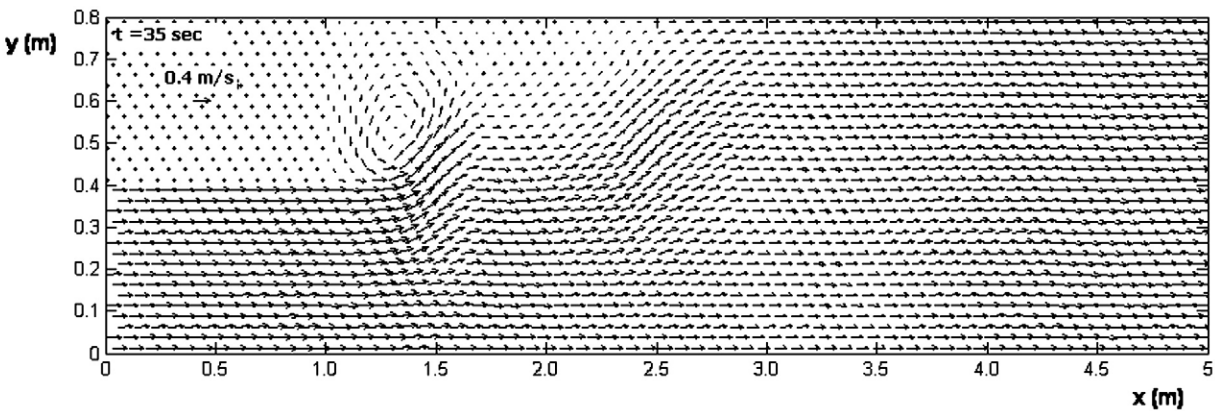

Fig. 13. Results of the HLES-KULeuven module (based on 25 mm + 10 layers in the vertical direction using Smagorinsky SGS model).

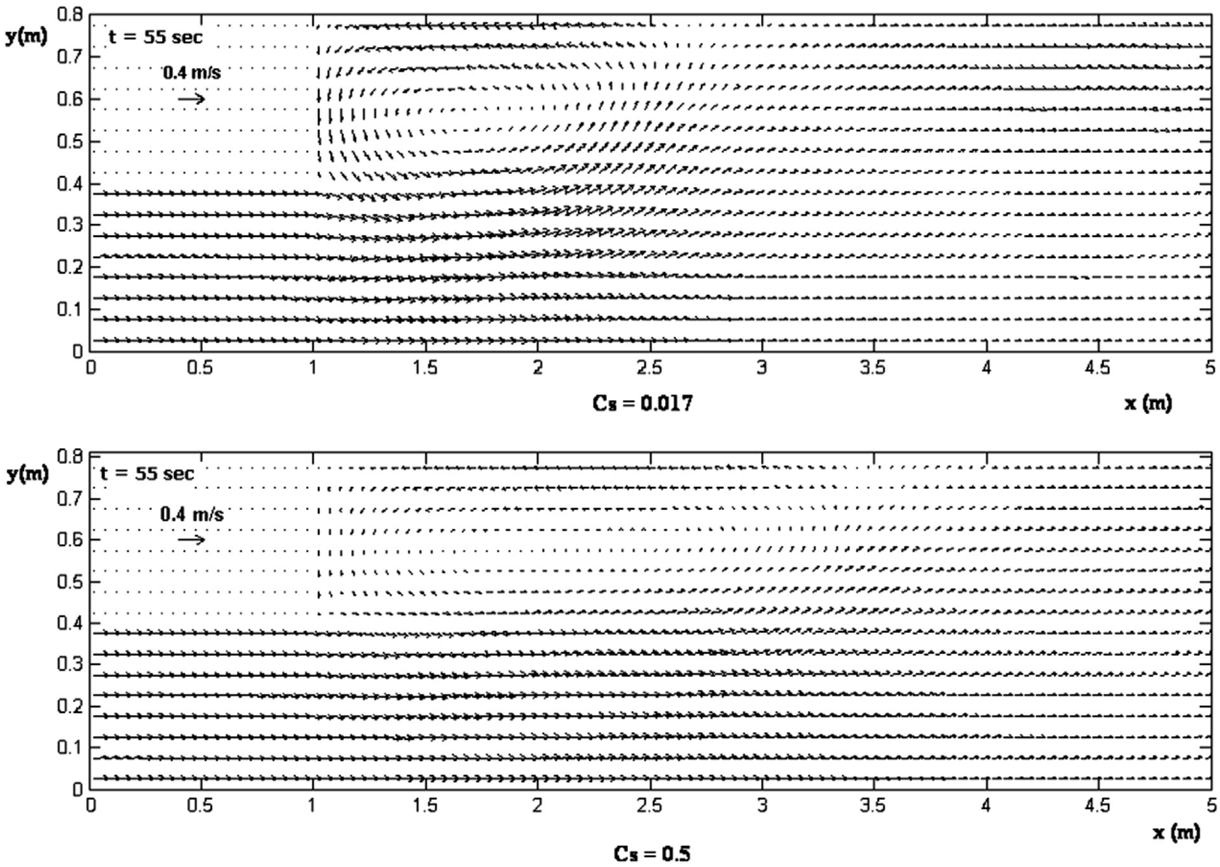

Fig. 14. Sensitivity analysis for Smagorinsky coefficient Cs.

The outputs of the simulations are in a fair agreement with the measurements. The dissipation of the two-length scale approach is found to be high leading to less resolved-turbulent fluctuations as indicated by the velocity signals. It produces lower velocities than the other SGS models at most of the positions. The possible reason of this discrepancy is the empirical inclusion of the effect of the bottom friction on the turbulent kinetic energy of the quasi-2D turbulence. As stated earlier, the calibration of the proposed two-length scale model has been made using turbulence experimental data from compound channel flows, in which type B quasi-2D turbulence is generated due to the internal transverse shear instabilities. This is in contrast to the backward facing step flow, in which type A quasi-2D turbulence is produced due to the effect of the flume expansion. In addition, the two-length scale approach is a non-grid size dependent model but the other models are grid-size dependent.

It has been found that increasing the resolution to 12.5 mm does not produce much difference compared to the 25 mm filter width (Fig. 12). In addition, the three dimensional simulation produces approximately similar results to the horizontal LES based on one layer in the vertical direction (Fig. 13).

It has also been found that increasing the Smagorinsky coefficient results in increasing the vortex stretching. This is clearly observed in the results of the simulation after 35 s in the following flow patterns (Fig. 14).

5. Conclusions

In the first part of this paper, several equations relying on the mixing length concept have been analysed and the weaknesses have been determined. A new concept has been proposed to estimate the mixing length of the horizontal large-scale turbulent motions in order to take into consideration two properties. Firstly, the analogy between the behaviour of the quasi-2D turbulence in geophysical shallow flows and the large-scale eddies in plane free shear flows (i.e. jet flows). Secondly, the suppression of the turbulent kinetic energy of the quasi-2D turbulence due to the effect of the bottom friction, which is the major mechanism that leads to the decay of the horizontal large-scale vortex. The experimental calibration of the proposed two-length scale approach with the new definition of the mixing length has been carried out based on turbulence experimental data from FCF programme [see (Shiono and Knight, 1991)]. The measured and modeled peaks of the turbulent viscosity have identical trend. As expected, the proposed two-length scale approach produces the peak values at the interface between the floodplain and the main channel. It has also been noted that there is a shift between the measured and the modeled peaks. But in general a fair agreement has been found.

In the second part of this article, the HLES-KULeuven module developed at the University of Leuven has been implemented for simulating an oscillatory backward facing step flow. This simulation has been made based on different approaches for subgrid scale modelling: the proposed two-length scale model, the Uittenbogaard SGS model (Uittenbogaard and van Vossen, 2004) and the Smagorinsky SGS model (Smagorinsky, 1963). The outputs of these models have also been compared with the simulations without SGS model and vertical shear parameterizations, and the experimental data of Stelling and Wang (1984). No significant difference has been found between the results of the Uittenbogaard and Smagorinsky SGS models. The two-length scale approach produces more elongated mixing layer in the streamwise direction. It results in underestimated velocities and a weaker vortex in the mixing layer (position of the horizontal vortex). The simulation without SGS model and a parameterization for the vertical shear produces more isotropic horizontal vortex. The splitting process of the horizontal vortex has not been noticed in all simulations, which is attributed to the effect of the numerical dissipation of the advection scheme in this study. In the accelerating phase of all simulations, the main stream drives the horizontal mixing layer. In the decelerating phase, the mixing layer is widening and the main stream is suppressed. It is recommended that a sensitivity analysis is conducted to test the effect of the empirical parameter ϕ of the proposed two-length scale approach on the outputs of the HLES-KULeuven module.

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References

Alavian, V., Chu, V.H., 1985. Turbulent exchange flow in shallow compound channel. Proc., 21st IAHR Congress, Melbourne.

Awad, E., Toorman, E., Lacor, C.h., 2007. Two-dimensional large eddy simulations for shallow-flows: a case of a backward facing step flow. Proceedings of 32nd IAHR Congress, Venice. 9 pp. (CD-ROM).

Babarutsi, S., Ganoulis, J., Chu, V.H., 1989. Experimental investigation of shallow recirculating flows. Journal of Hydraulic Engineering 115 (7), 906–924.

Castanedo, S., Medina, R., Mendez, F.J., 2005. Models for the turbulent diffusion terms of shallow water equations. Journal of Hydraulic Engineering 131 (3), 217–223.

Chu, V.H., Babarutsi, S., 1988. Confinement and bed–friction effects in shallow turbulent mixing layers. Journal of Hydraulic Engineering 114 (10), 1257–1275.

Chu, V.H., Wu, J.H., Khayat, R.E., 1991. Stability of the transverse shear flow in shallow mixing layers. Journal of Hydraulic Engineering 117, 1371–1388.

Costa, R.C.F.G., (1995). Three dimensional modelling of cohesive sediment transport in estuarine environments. PhD thesis, Department of Civil Engineering, University of Liverpool.

Dunsbergen, D., (1994). Particle models for transport in three dimensional shallow water flow. PhD thesis, Delft University, Netherlands.

Ervine, D.A., Babaeyan-koopaei, K., Sellin, R.H.J., 2000. Two-dimensional solution for straight and meandering overbank flows. Journal of Hydraulic Engineering 126 (9), 653–669.

Ghidaoui, M.S., Kolyshkin, A.A., 1999. Linear stability analysis of lateral motions in compound open channels. Journal of Hydraulic Engineering 125 (8), 871–880.

Hinze, J.O., 1975. Turbulence. McGraw-Hill.

Jirka, G.H., Uijttewaal, W.S.J., 2004. Shallow flows: a definition. In: Jirka, G.H., Uijttewaal, W.S.J. (Eds.), Proc. "Shallow Flows". Balkema publ., 2004. ISBN: 90 5809 7005.

Lambert, M.F., Sellin, K., 1996. Discharge prediction in straight compound channels using the mixing length concept. Journal of Hydraulic Research 34 (3), 381–394.

Luyten, P.J., Jones, J.E., Proctor, R., Tabor, A., Tett, P., Wild-Allen, K., 1999. A coupled hydrodynamical–ecological model for regional and shelf seas: user documentation. MUMM report. Management Unit of the Mathematical Models of the North Sea. 914 pp.

Madsen, P.A., Rugbjerg, M., Warren, I.R., 1988. Subgrid modelling in depth integrated flows. Coastal Engineering conference, vol. 1, pp. 505–511. Malaga, Spain.

Morvan, H.P., 2005. Channel shape and turbulence issues in flood flow hydraulics. Journal of Hydraulic Engineering 131 (10), 862–865.

Nadaoka, K., Yagi, H., 1998. Shallow-water turbulence modeling and horizontal large-eddy computation of river flow. Journal of Hydraulic Engineering 124 (5), 493–500.

Ogink, H.J.M., 1985. The effective viscosity coefficient in 2-D depth-averaged flow models. Proc., 21 st IAHR Congress, Melbourne.

Pope, S.B., 2000. Turbulent Flows. Cambridge University Press.

Prandtl, L., 1925. Uber die ausgebildete Turbulenz (On fully developed turbulence). ZAMM 5, 136–139.

Prandtl, L., 1942. Bemerkungen zur Theorie der freien Turbulenz. ZAMM 22, 241–243.

Rodi, W., 1980. Turbulence Models and Their Application in Hydraulics—A State of the Art Review. IAHR, Delft.

Shiono, K., Knight, D.W., 1991. Turbulent open channel flows with variable depth across the channel. Journal of Fluid Mechanics 222, 612–646.

Smagorinsky, J., 1963. General circulation experiments with the primitive equations, I. the basic experiments. Monthly Weather Review 91 (3), 99–164.

Stelling, G.S., Wang, L.X., 1984. Experiments and computations on separating flow in an expanding flume. Dept. Civil Engineering, Delft University of Technology, Report, pp. 2–84.

Taylor, G.I., 1932. The transport of vorticity and heat through fluids in turbulent motion. appendix by In: Fage, A., Faulkner, V.M. (Eds.), Proc. R. Soc. Lond., vol. A135, pp. 685–705.

Tennekes, H., Lumley, J.L., 1972. A First Course in Turbulence. The MIT press, Cambridge.

Thomas, T.G., Williams, J.J.R., 1995a. Large eddy simulation of a symmetric trapezoidal channel at a Reynolds number of 430,000. Journal of Hydraulic Research 33 (6), 825–841.

Thomas, T.G., Williams, J.J.R., 1995b. Large eddy simulation of turbulent flow in an asymmetric compound open channel. Journal of Hydraulic Research 33 (1), 27–41.

Tukker, J., (1997). Turbulence structures in shallow free-surface mixing layers., PhD thesis, TUDelft, The Netherlands.

Uittenbogaard, R.E., 2003. Points of view and perspectives of horizontal large-eddy simulation at Delft. Lecture notes (6–7 February 2003, Sapporo, Japan). WL/Delft Hydraulics, Delft.

Uittenbogaard, R.E., van Vossen, B., 2004. Subgrid-scale model for quasi-2D turbulence in shallow water. In: Jirka, Uijttewaal (Eds.), Shallow Flows. Taylor & Francis Group, London. ISBN: 90 5809 700 5.

van Prooijen, B., (2004). Shallow Mixing Layers. PhD thesis, Delft University of Technology, Delft.

van Prooijen, B., Battjies, J.A., Ujittewaal, W.S.J., 2005. Momentum exchange in straight uniform compound channel flow. Journal of Hydraulic Engineering 131 (3), 175–183.

van Vossen, B., 2000. Horizontal large eddy simulations; evaluation of computations with DELFT3D-FLOW. Report MEAH-197. Delft University of Technology, The Netherlands.

Von Karman, T.H., 1930. Mechanishe Ahnlichkeit und Turbulenz (Mechanical Similarity and Turbulence). Nachrichten von der Gesellschaft der Wissenschaften zu GoÉttingen, Mathematisch-Physikalische Klasse 58.

Xing, J., Davies, A.M., 1996. Application of turbulence energy models to the computation of tidal currents and mixing intensities in shelf edge regions. Journal of Physical Oceanography 26 (417), 447.