APPLICATIONS OF SEQUENTIAL SAMPLING TO MARINE RESOURCE SURVEYS

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Abstract

Computational procedures for sequential marine resource surveys are outlined and the results of some applications are presented in graphic form. The negative binomial distribution was found to fit hard clam Mercenaria mercenaria, survey data when samples were obtained with a construction bucket sampler. The normal distribution fitted the standard towed dredge data. Sequential sampling plans based on both distributions for classification of areas into three abundance classes are demonstrated. The advantages of sequential techniques for certain classes of marine sampling problems are briefly considered. Suggestions for additional applications and extensions in marine sampling are made.

INTRODUCTION

The objective of this report is to introduce marine resource survey techniques which permit rapid classification of environments in terms of relative abundance of organisms. The specific approach to the problem involves sequential analysis of sample data.

Briefly, sequential analysis is a technique of statistical inference whose characteristic feature is that the number of observations required by the procedure is not predetermined. In classical fixed-sample tests of linear hypotheses a decision is reached on the basis of a single sample of size n. These classical tests utilize two regions (acceptance and rejection) which are functions of a predetermined number of samples. In sequential tests a third region is introduced (uncertainty) where no final decision is made because the observations taken up to this point do not satisfy acceptance or rejection of the test hypothesis. Observations are added until the observations fall outside the third (uncertainty) region. The number of observations is a sequence of values of a random variable.

Techniques of sequential analysis were developed during World War II and have since been applied to numerous problems of sampling inspection and experimentation. Wald (1945, 1947) describes general formulae for most classes of problems, and these are illustrated in several standard statistics texts such as Davies (1954) and Goulden (1952). Biological applications of sequential analysis techniques include: medicine (Anscombe, 1963; Bross, 1952), forest insects (Cole, 1960; Morris, 1954; Stark 1952; Waters, 1955), bacteria (Morgan et al., 1951) and white fish parasites (Oakland, 1950). Lander (1956) has also indicated the utility of sequential sampling for biological problems and provided a simple hypothetical example. However, no mention of sequential sampling was made in a recent review article on sampling marine benthos by Holme (1964). It seems timely to introduce this approach to marine sampling problems, especially for benthic organisms.

Sequential sampling seems particularly suited for reconnaissance type surveys, which are often the case in marine environments. These surveys provide a means for classification rather than a means for estimating population parameters directly. Sequential plans involve no fixed sample size and are therefore efficient. These plans usually result in sampling economies even when it is necessary to set an upper limit on sample size (Bliss and

1. Northeast Shellfish Sanitation Research Center, Public Health Service, U. S. Dept. of Health, Education, and Welfare, Narragansett, R. I. Contribution No. 3 from this center.

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Calhoun, 1954). Finally, classification of populations is possible within arbitrary specified limits of accuracy by means of these plans.

MECHANICS OF SAMPLING

It seems highly desirable to have rapid sampling techniques for estimating gross pollution effects, suitability of areas for transplanting, and for the general management of shellfish growing areas. It is for these and similar purposes that sequential sampling is proposed in this report. The empirical data used pertain exclusively to the hard clam or quahog (Mercenaria mercenaria), but the approach can be applied to other organisms for which adequate background data are available.

The techniques applied by field parties in sampling shellfish (primarily bivalve mollusks) have commonly involved one of two types of gear--the clamshell construction bucket and the towed commercial dredge. These will be considered as Method 1 and Method 2, respectively.

Method 1. A one-half cubic yard capacity construction bucket used in this and previous studies provided a sample of approximately five square feet in area. It has been found that this device permitted satisfactory sampling to depths at which bivalve mollusks were found under all substrate conditions except extremely rocky areas. The sampling plan involved a random or systematic sample over a defined area. Each sample was taken aboard the survey vessel and washed in a screened box. The screening material determines the size selectivity of the sample. Contours of abundance are frequently plotted from these data, and population estimates have been attempted from random samples after establishing the nature of the sampling distribution. Method 1 is relatively slow and expensive with a fixed sample size especially when the sampling interval is small. Furthermore, fitting contours of abundance by inspection involves considerable judgment.

Method 2. A standard scheme of unit tows with a 17 tooth Fall River (rocking-chair) dredge has been established. The duration of the tow was three minutes at a constant boat speed providing samples of approximately 1000 square feet (1.45 x 750 feet). Tows were made from randomly chosen points in an experimental area and at randomly chosen directions of tow. It was demonstrated that the towed dredge, when operated at uniform speeds and in relatively uniform substrate conditions provided a stable sampling unit. For example, five replicate tows over the same course gave a coefficient of variation (C) of less than 11 percent ($\overline{x} = 62.6$, $s = 6.84$). It is not known at this time whether the gear effectively takes all individuals of sufficient size to be retained by the mesh. However, it does appear that the gear is fairly consistent in taking approximately similar numbers from the same area. The mesh of the dredge was equipped with 2 inch diameter rings and the abundance classification for the towed dredge refers only to size classes retained by this mesh.

THE SEQUENTIAL PLAN

All sequential sampling plans are characterized graphically by one or more pairs of parallel lines. These lines, or values obtained from them, are criteria for making decisions (in our case, for distinguishing abundance classes).

The general formula for any pair of decision lines is:

$$d_{o,1} = b\,n \pm h_{o,1}$$

where d_o refers to the maximum value for the lower classification in terms of cumulative numbers of organisms and d_1 to the minimum value for the higher class, n refers to the number of sampling units examined, h_o and h_1 are Y intercepts and b is the slope of the lines.

For solving line equations the assigned risks, α and β, as well as the class limits, m_1 and m_2, are involved in various ways depending on the nature of the sampling distribution.

A prerequisite to the application of any sequential plan is to determine the nature of the frequency distribution for each type of gear and organism because this determines the formulas to be used in subsequent steps.

<u>Method 1</u>. The frequency distribution of hard clams from 745 one-half yard capacity construction bucket samples is shown in Figure 1 (solid circles).[4] It is apparent that this frequency distribution exhibits a high degree of contagion. The presence of one hard clam in the sampling unit increases the chances of others occurring in it. The wide applicability of the negative binomial to describe frequency distributions of many kinds of organisms suggested that these data might also conform. The negative binomial was therefore fitted to the observed frequencies. This distribution is described by $(q-p)^{-k}$, where $p = m/k$ and $q = 1 + p$. Expansion of this expression gives the two-parameter (m and k) frequency function:

$$f(x) = \frac{(k+x-1)!}{x!(k-1)!} \; \frac{m}{k+m}^{x} q^{-k} \qquad x = 1,2,3...$$

where $m/(k+m) = p/q$. The parameter k of the negative binomial was estimated by the efficient method of Haldane (1941) and its value was found to be 0.369 for 745 samples containing all size-classes of hard clams retained by a one-half inch mesh washing box. This result was obtained with a FORTRAN II computer program written for the IBM 1620 data processing system. Figure 1 also suggests (by the smooth curve which was fitted to the data points) that the observed sampling distribution conforms well to the negative binomial. For a given sampling method the parameter k should remain fairly constant within reasonable limits of m, the mean. This relation is shown for three size groups of hard clams which provided the following values:

	m	k
1	.103	.189
2	.315	.225
3	.969	.352

Some increase in k is apparent from increasing values of m in this study.

In addition to the above, a chi-square test for the fit of the negative binomial distribution to the observed frequencies was made. A value of 5.46 (d.f. = 9) was obtained. If an α risk of .05 is accepted then chi-square = 16.92 (d.f. = 9). Clearly since:

$$\chi^2 \leq \chi^2_{1-\alpha}$$

there is no reason to believe that the negative binomial does not adequately represent the data.

<u>Method 2</u>. The sampling distribution of hard clams as obtained from the towed dredge data was markedly different. A series of 89 unit tows from an experimental area of one square mile were utilized for the frequency distribution. Figure 2 illustrates graphically a test for the goodness-of-fit of the observed distribution to the normal distribution. In this procedure a

4. From "A Report on the Economically Important Shellfish Resources of Raritan Bay", by R. Campbell, USPHS, Northeast Shellfish Research Center, July 1964, (Processed).

straight line indicates a normal or Gaussian distribution. A test for linearity of the probit line gave a chi-square value of 2.732 (d.f. = 16). For an α risk of .05 the chi-square $_{1-\alpha}$ = 26.30 (d.f. = 16), indicating no significant deviation from linearity.

In this study tentative class limits have been established for determining low, medium and high population densities. These limits were set from a consideration of previous sample surveys for hard clams as well as from correspondence with shellfish management groups. It is recognized that these class limits may not be applicable in all areas or for other species. However, they can be readily changed without affecting the sequential sampling approach to classification. The class limits which have been established for hard clam sampling with the two types of gear described previously are as follows:

To distinguish between Class I (low) and Class II (medium) areas alternative hypotheses follow:

Bucket sampling--(5 square feet)

H_o--that the number of hard clams is 0.2 or less per square foot (1 per sample).
H_1--that the number of hard clams is 1.0 or more per square foot (5 per sample).

Towed dredge sampling--(3 min. tows)

H_o--that the number of hard clams is 20 per tow or less.
H_1--that the number of hard clams is 40 or more per tow.

To distinguish between Class II (medium) and Class III (high) areas:

Bucket sampling--

H_o--that the number of hard clams is 2.0 or less per square foot (10 per sample).
H_1--that the number of hard clams is 3.0 or more per square foot (15 per sample).

Towed dredge sampling--

H_o--that the number of hard clams is 60 or less per tow.
H_1--that the number of hard clams is 100 or more per tow.

The values of the distribution constants for both methods with alternate hypotheses are shown in Table 1.

Each pair of hypotheses (H_o and H_1) has two types of error.

α = probability of accepting H_1 when H_o is true.
β = probability of accepting H_o when H_1 is true.

For the purposes at hand both and risks have been established at 0.05. Acceptance of this probability of error seems justified inasmuch as the number of samples required by either method did not appear to be prohibitively large in preliminary studies.

It should be recognized that the numbers of hard clams per unit dredge tow as described above cannot be directly converted to numbers per unit area. Therefore, the results obtained by the two methods are not directly comparable. The reasons are: (1) the towed dredge is size selective according to the ring size of the bag, and (2) the efficiency of the towed gear is not completely known at this time in various substrates and at various densities of organisms. It is also suggested that the towed gear effectively obscured the contagious distribution of hard clams, probably by passing through several clusters (groups) in each unit tow.

Table 1. Values of the distribution constants under alternate hypotheses Method 1, based on the negative binomial; and Method 2, based on the normal curve.

Bucket Sampling (Method 1) k = 0.369				
	Relative Abundance			
Constant				
Mean = kp	kp_o = 0.200	kp_1 = 1.000	kp_o = 2.000	kp_1 = 3.000
p = kp/k	p_o = 0.542	p_1 = 2.712	p_o = 5.424	p_1 = 8.130
q = p+1	q_o = 1.542	q_1 = 3.712	q_o = 6.424	q_1 = 9.136
var = kpq	var = 0.308	var = 3.712	var = 12.849	var = 27.409

Towed dredge Sampling (Method 2)		
	Low vs. Medium	Medium vs. High
m_1 = smaller aver.	20	60
m_2 = larger aver.	40	100
s = std. dev.	26.10	26.10
$A' = \ln \frac{1-\beta}{\alpha}$	2.94	2.94
$B' = \ln \frac{1-\alpha}{\beta}$	2.94	2.94

ACCEPTANCE AND REJECTION LINES

Method 1. The following formulas and procedures refer to the calculations for Class I (low) versus Class II (medium) as well as for the sets of comparisons. These formulas apply for sampling distributions reasonably approximated by the negative binomial.

The formulas for the acceptance and rejection lines are:

$d = b\,n + h_o$ (lower line),

$d = b\,n + h_1$ (upper line).

The slope of the lines is:

$$b = k \frac{\log (q_1/q_o)}{\log (p_1 q_o / p_o q_1)} .$$

The intercepts are:

$$h_o = \frac{\log B}{\log (p_1 q_o / p_o q_1)} \quad \text{where } B = \frac{\beta}{1-\alpha} ,$$

$$h_1 = \frac{\log A}{\log (p_1 q_o / p_o q_1)} \qquad \text{where } A = \frac{1 - \beta}{\alpha}.$$

The subscripts for p and q refer to the columns under the alternate hypotheses, H_o and H_1, of Table 1. For bucket sampling substitution in the above provides, for the low versus medium abundance, the following:

accept H_o if $d \leq 0.443\ n - 4.023$,

accept H_1 if $d \geq 0.443\ n + 4.023$,

where d is the cumulative number of hard clams per sampling unit and n is the number of samples taken.

These acceptance and rejection lines as well as the lines for medium versus high abundance are plotted in Fig. 3. This figure may be used by field parties to determine how many samples should be taken in each area in order to define the abundance class within the accepted limits of α and β. The procedure for using the figure merely involves accumulating the number of hard clams per square foot and plotting the accumulated values for successive samples until the plotted points depart from the area of the figure enclosed by the regions of uncertainty.

Method 2. The following formulae apply to both sets of comparisons for data which can be approximated by the normal distribution.

The decision lines are:

$d = b\ n - h_o$ (lower line)

$d = b\ n + h_1$ (upper line)

where d is the accumulative number of hard clams, n is the number of tows made, b is the slope and h_o and h_1 are the intercepts.

The formulae for the slope and intercepts are:

$$b = \frac{m_1 + m_2}{2} = \text{slope},$$

$$h_o = \frac{B' \ s^2}{m_2 - m_1} \quad \text{and} \quad h_1 = \frac{A' \ s^2}{m_2 - m_1} = \text{intercepts}.$$

The constants for the equations are defined according to Table 1.

The average sample numbers, i.e. the average sample size to be drawn after the risks have been set are:

$$\bar{n}_{m_1} = \frac{(1 - \alpha)\ B' - \alpha A'}{\frac{(m_2 - m_1)^2}{2 s^2}}$$

$$\bar{n}_{m_2} = \frac{(1 - \beta)\ A' - \beta B'}{\frac{(m_2 - m_1)^2}{2 s^2}}$$

Sample size may be reduced by increasing the α and β risks as is apparent from the above equations.

For the towed dredge sampling, when Class I (low) versus Class II (medium) abundance is contrasted, the following alternatives arise:

accept H_o if $d \leq 30.0\ n - 100.5$,

accept H_1 if $d \geq 30.0\ n + 100.5$.

Figure 4 illustrates the acceptance and rejection lines separating the three abundance classes. This figure may be utilized in a manner exactly analogous to Figure 3 except that the sampling gear is a towed dredge in this case.

OPERATING CHARACTERISTIC CURVES

For any sampling plan it is sometimes desirable to know in advance what the chances are of correctly classifying an area, or for making other decisions at various population levels which are expected.

The operating characteristic curves for Method 1 (bucket sampling) and for Method 2 (towed dredge sampling) are shown in Figure 5 and Figure 6 respectively. These curves show the probability L(m) of accepting H_o for any level of the population mean. For example, when the mean (m_1) is 20 in the left curve of Figure 6, the probability of accepting H_o (low) is .95 and the probability of accepting H_1 (medium) is .05. When $m_2 = 40$ on the same curve, the probability of accepting H_o is .05 and of accepting H_1 is .95. These correspond, of course, to the probabilities set for the α and β risks. Upon careful examination there seems to be some overlapping between the curves, (especially Fig. 5), but this occurs only at the extreme probability levels. Examination of these curves also provides some insight into why gaps were left between classes of population abundance as defined previously.

The operating characteristic curves are calculated as follows for the two sampling methods:

<u>Method 1</u>.

$$L(m) = \frac{A^x - 1}{A^x - B^x}$$

where A and B are defined as previously for the negative binomial acceptance and rejection lines, and

$$m = \frac{1 - (q_o/q_1)^x}{(p_1\ q_o/p_oq_1)^x - 1}$$

In both expressions x is a "dummy" variable which is assigned a range of values convenient for calculation.

<u>Method 2</u>.

The formulae for the towed dredge sampling operating characteristic curves are:

$$K = \frac{2\ (b - m)}{s^2}\ ,$$

$$t_o = h_oK$$

$$t_1 = (h_o + h_1)K, \quad \text{and}$$

$$L(m) = \frac{e^{t_o} - 1}{e^{t_1} - 1}\ .$$

The symbols used herein are defined or have been defined previously. Values of L(m), m are obtained by assigning various values of m in the first equation above and computing corresponding values of L(m) from the last equation listed above. Values of e^{t_0} and e^{t_1} are found in tables of exponential functions.

AVERAGE SAMPLE NUMBER CURVES

The average sample number curves for both sampling methods are illustrated in Figure 7 and Figure 8. Use of these curves provides some indication of the mean number of samples E(n) which must be taken by field parties at any population level. Peaks in all sets of curves (both figures) occur at population levels which are intermediate between the classification limits. It is obvious from examination of these figures that bucket sampling requires more sampling units than towed dredge sampling. Furthermore, for the bucket sampling system a larger number of samples is required to distinguish between medium versus high abundance than for low versus medium. The reverse is true in this case for dredge sampling. It should be appreciated that the E(n) values (the average sample number) indicated by the figures may be considerably exceeded at some sampling points. Figure 7 and Figure 8 merely represent guides to the average expected sample number.

The average sample number curves for both sampling methods may be calculated as follows:

Method 1.

$$E(n) = \frac{h_1 + (h_o - h_1)L(m)}{km - b}$$

where all symbols are as defined previously. The symbols h_o, h_1, b and k have been defined for the acceptance and rejection lines, and L(m) and m have been defined for the operating characteristic curves.

Method 2.

$$E(n) = \frac{L(m)\ (h_o + h_1) - h_o}{m - b}$$

where the symbols have also been defined previously. The values of E(n) for plotting in Figure 8 were obtained by substitution in the formulae for the operating characteristic curve for Method 2 and then the corresponding L(m) values were substituted in the above equation.

APPLICATION AND DISCUSSION

In applying the sequential sampling plans outlined in this report, field parties may find it more satisfactory to utilize tabulations rather than figures demonstrating acceptance and rejection lines. Such tables are easily constructed directly from the figures or by substitution in the appropriate equations for the decision lines.

It is emphasized that strict adherance to proper sampling methods requiring random location of bucket samples and tows is necessary. That is, sampling points and samples at these points must be selected without personal bias.

A major problem in marine resource surveys is that the areas for which information is required are large and heterogenous with respect to environmental variables. Further, the occurrence of hard clams, even in relatively uniform areas, appears to demonstrate a clustering of organisms. It is necessary to subdivide a large area into as many uniform sub-areas (sampling points) as required in order to effectively utilize this sampling plan. The sequential plan is then applied to each of these uniform sub-areas for classification purposes. The data may then be combined or averaged if necessary.

The saving of time is probably the greatest advantage of the sequential plan, although sometimes increased sampling is required in borderline classification.

It is believed that the elements of the plan presented herein can be refined and empirically evaluated in future work, since sequential sampling has been effectively applied to a marine shellfisheries problem. The plan outlined in this report should have utility in diverse marine sampling problems.

ACKNOWLEDGEMENTS

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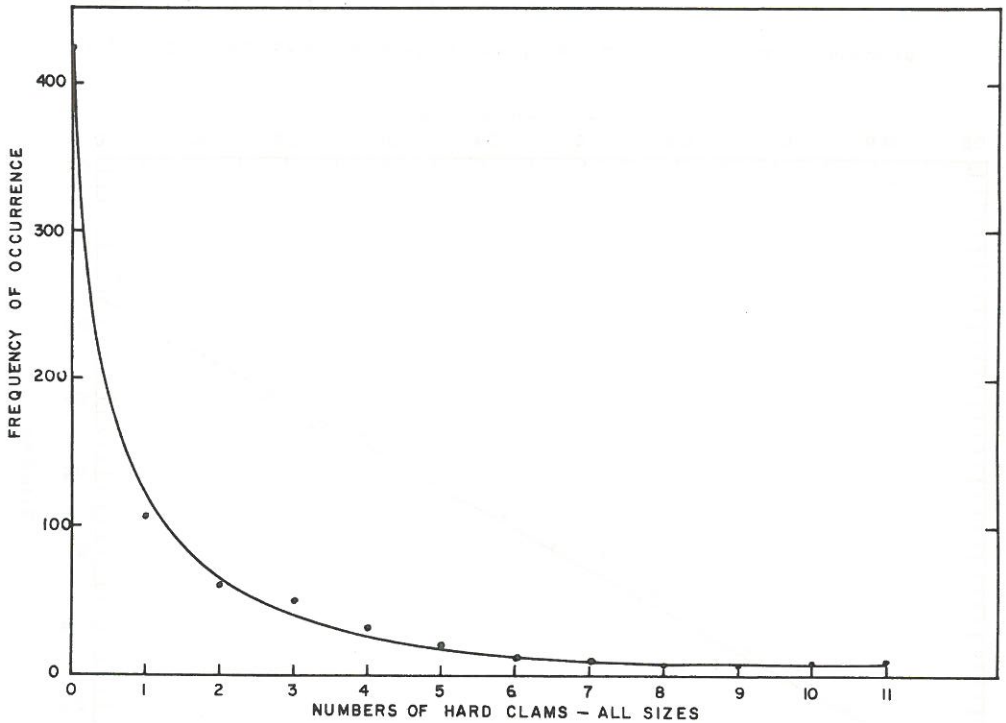

Figure 1. Frequency distribution of hard clams (all sizes) from bucket sampling with fitted negative binomial distribution.

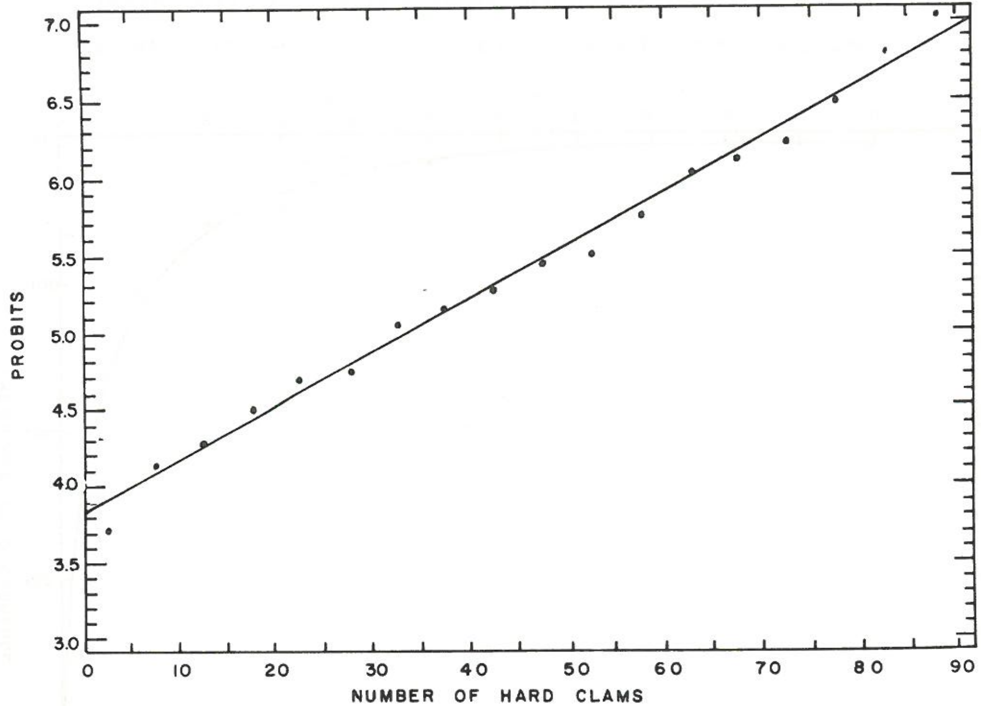

Figure 2. Test for goodness-of-fit on the number of hard clams per standard dredge tow. Test for normal distribution.

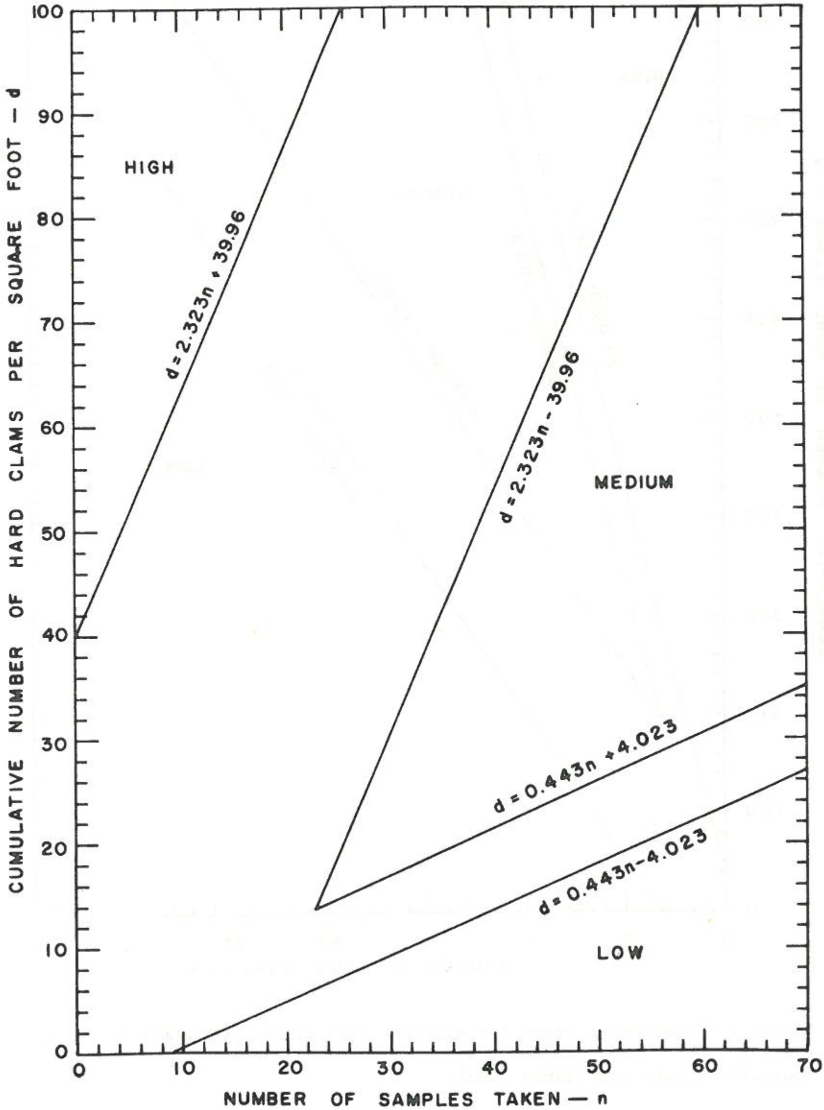

Figure 3. Sequential graph for sampling hard clams with construction bucket (5 square feet)--95 percent confidence level.

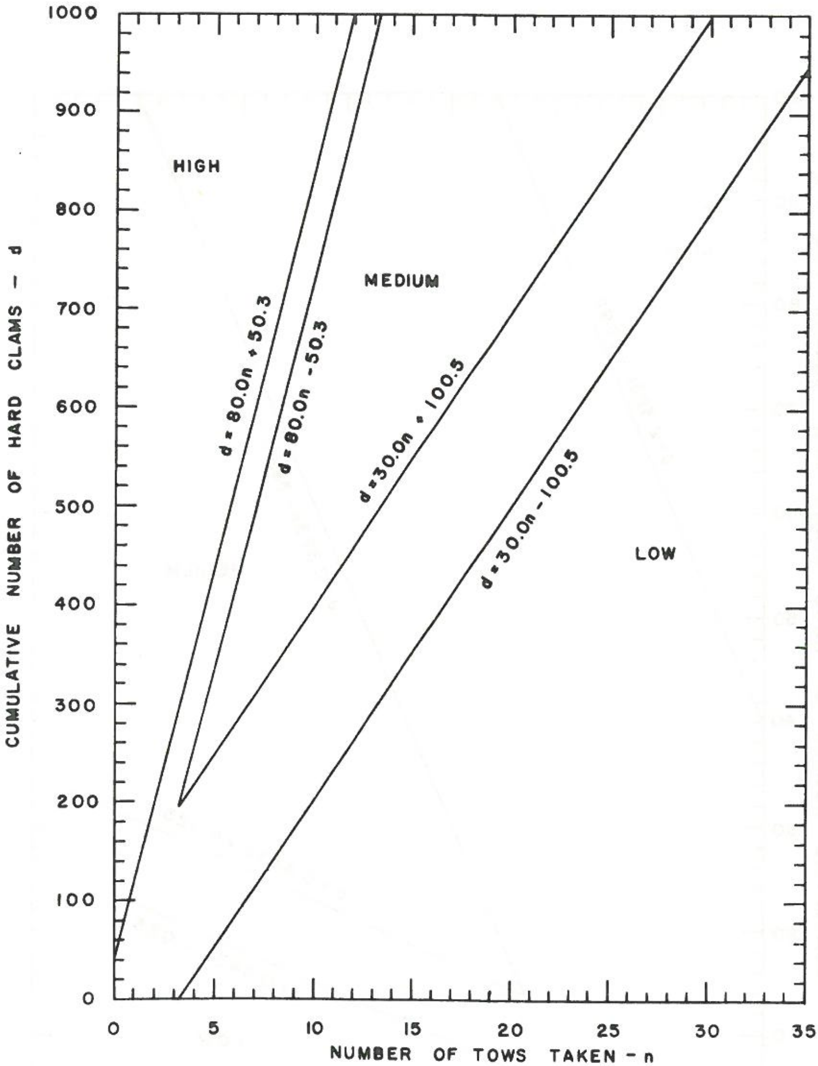

Figure 4. Sequential graph for sampling hard clams with standard dredge tows--95 percent confidence level.

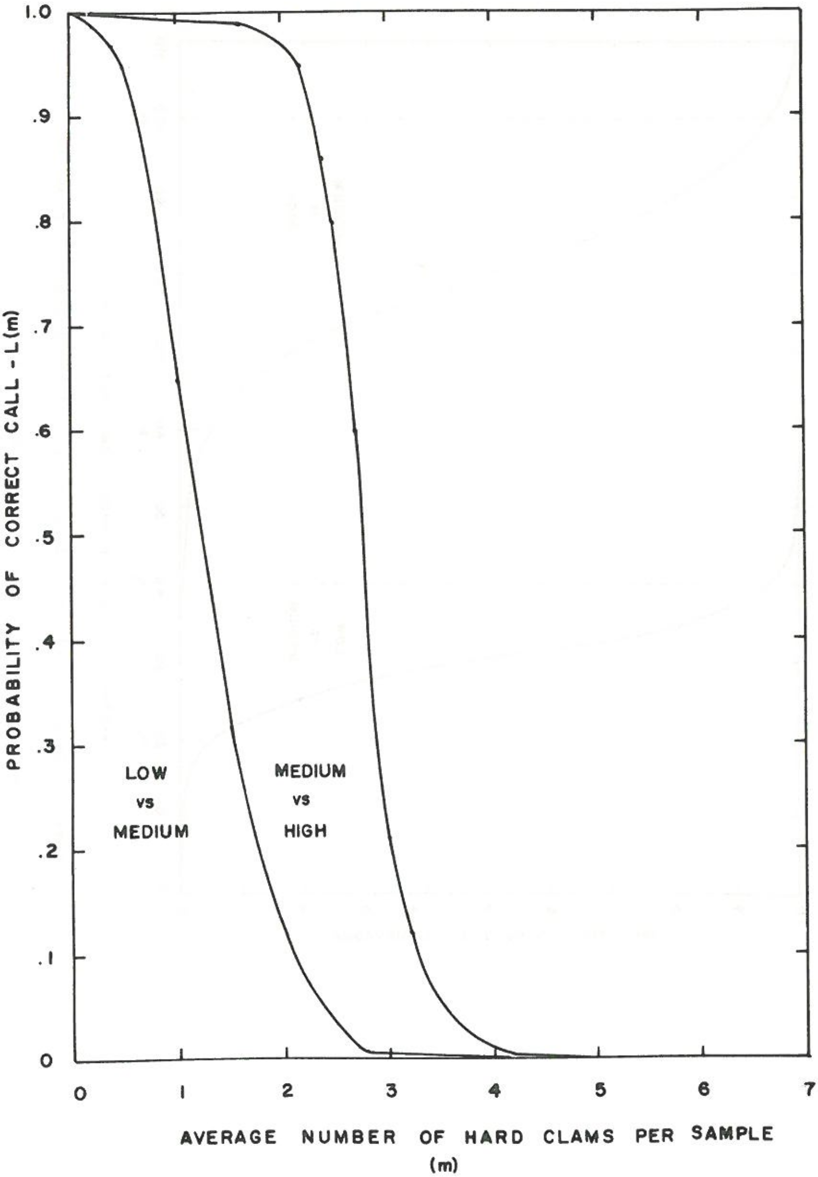

Figure 5. Operating characteristic curves of the sequential plan for bucket sampling of hard clams--95 percent confidence level.

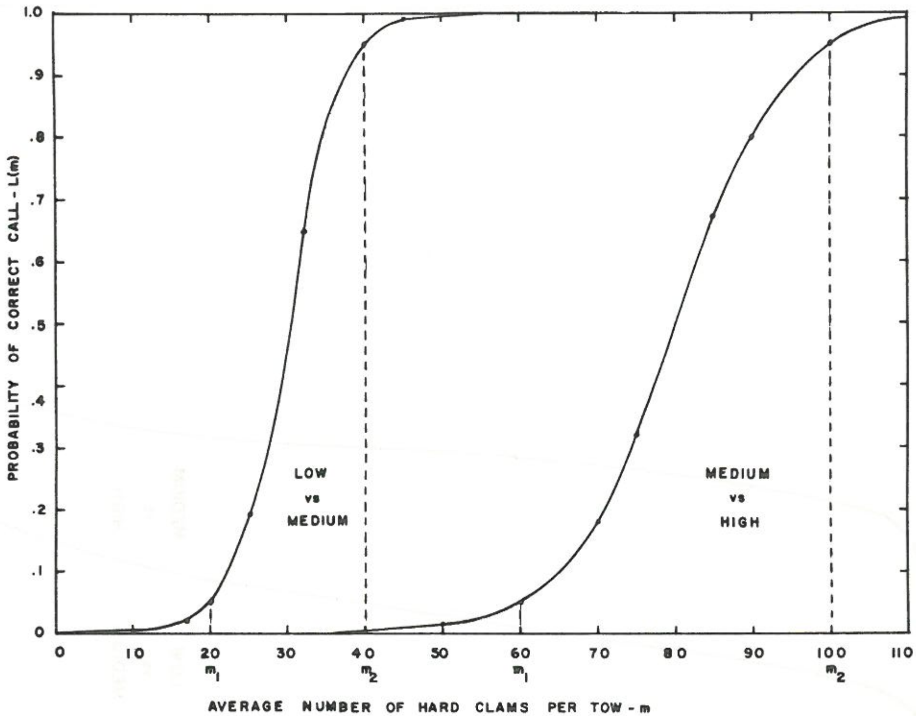

Figure 6. Operating characteristic curves of the sequential plan for dredge sampling of hard clams--95 percent confidence level.

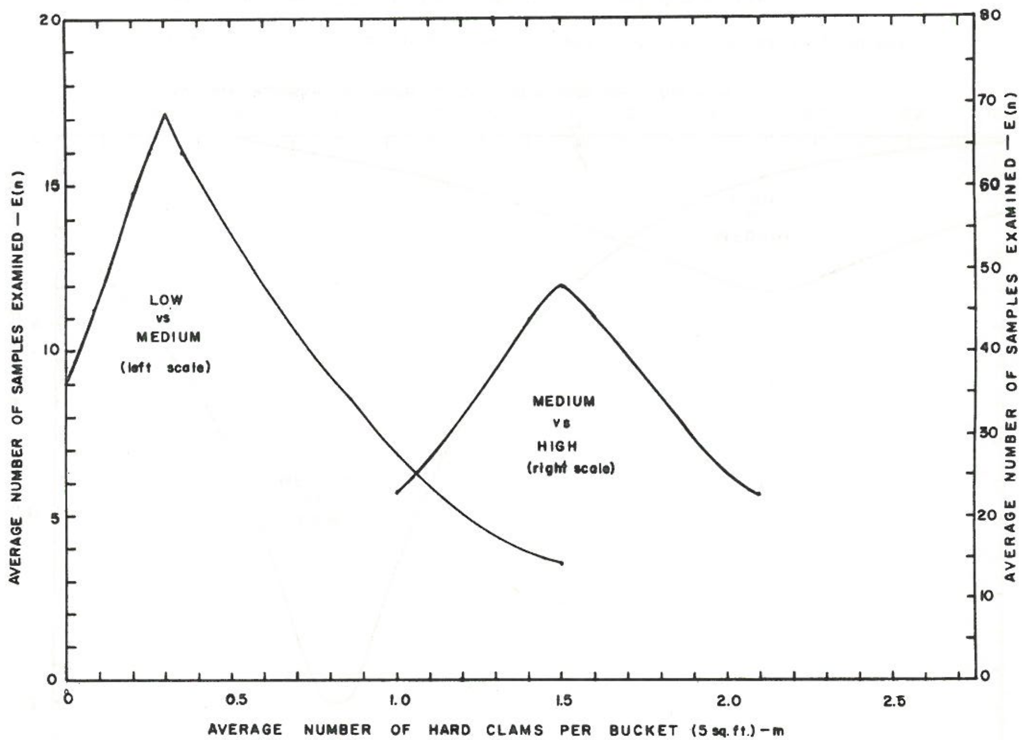

Figure 7. The average sample number curves of the sequential plan for bucket sampling of hard clams--95 percent confidence level.

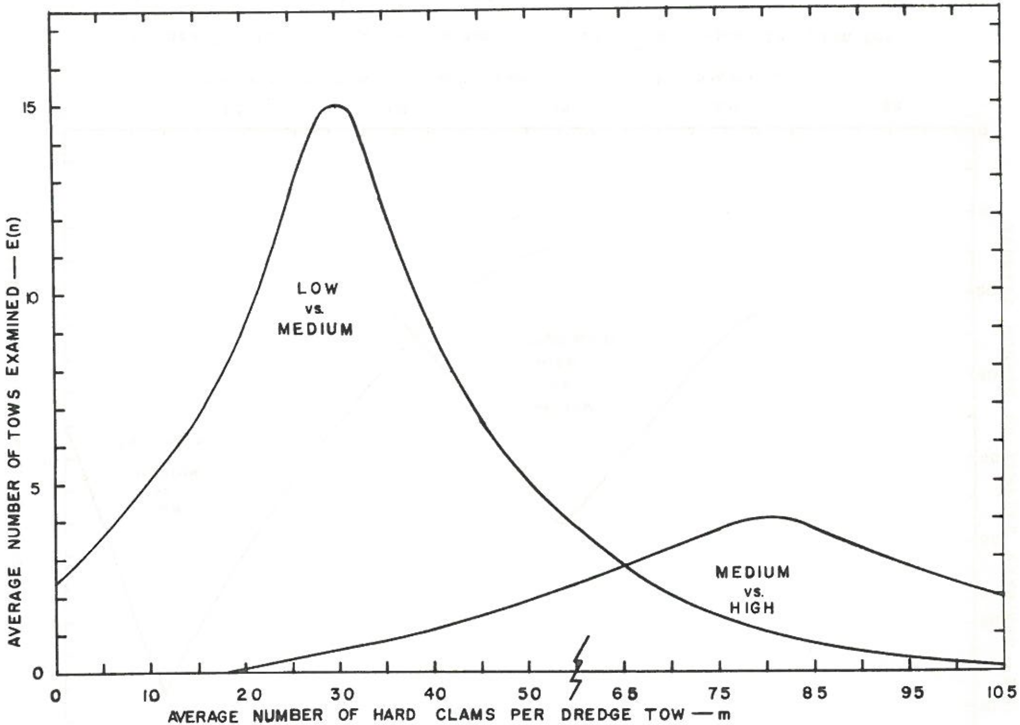

Figure 8. The average sample number curves of the sequential plan for dredge sampling of hard clams--95 percent confidence level.

APPENDIX I

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C  C         NEGATIVE BINOMIAL DISTRIBUTION
      DIMENSION GP(500),X(500),F(500)
   57 FORMAT(1H ,6HK EFF.)
   81 FORMAT(26H         X          O      E)
   82 FORMAT(I10,F10.0,F15.5)
   84 FORMAT(49H         X FREQ.OBS.      FREQ.EXP.       (O-E)2/E)
   85 FORMAT (49H  SUM OBS.     SUM EXP.     CHI SQUARE        DF )
   86 FORMAT(F10.0,F15.5,F15.5,F10.0)
   91 FORMAT(1H ,12HNEG.BINOMIAL)
   92 FORMAT(3H K=F15.8)
   93 FORMAT(3H P=F15.8)
  800 FORMAT(I10)
  801 FORMAT(2F15.5)
  802 FORMAT(1H1)
  902 FORMAT(F15.5,F15.5,F15.5)
  903 FORMAT (10H EXP.VAR.=F15.5)
  908 FORMAT(I10,I10,F15.5,F15.5)
  917 FORMAT(45H         MEAN          VARIANCE          STD.DEV. )
      WRITE(3,802)
  800 SP=0.
      SUMF=0.
      CHISQ=0.
      SO=0.
      SE=0.
      SUMFX=0.
      SUFX2=0.
      TE1=0.
      TE2=0.
      PI=0.
      DF=0.
      READ(1,800)N
      DO 31 I=1,N
      READ(1,801)X(I),F(I)
      IF(I-1) 94,94,95
   94 F1=F(I)
   95 SUMF=SUMF+F(I)
      SUMFX=SUMFX+F(I)*X(I)
   31 SUFX2=SUFX2+F(I)*X(I)*X(I)
      Z=SUMFX/SUMF
      VARX=(SUFX2-(SUMFX**2/SUMF))/(SUMF-1.)
      SIGMA=VARX**.5
      WRITE(3,91)
      WRITE(3,917)
      WRITE(3,902)Z,VARX,SIGMA
      I=1
      PO=F1/SUMF
      PI=1./(1.+Z)
      IF(PI-PO) 10,15,20
   10 FK=.5
      FKL=0.
      FKU=1.
   11 PI=1./(1.+Z/FK)**FK
      Q=FK
      IF(PI-PO) 25,30,35
   25 FKU=FK
      GO TO 40
   35 FKL=FK
   40 FK=(FKL+FKU)*.5
      IF(ABS(FK-Q)-1.E-5) 30,11,11
   15 FK=1.
```

```
      GO TO 30
   20 FK=ABS(Z**2/(VARX-Z))
   30 EK=FK
      A=SUMF
      DO 52 I=1,N
      A=A-F(I)
   52 GP(I)=A
      M=N-1
      L=1
    1 ELHS=SUMF*(ALOG(EK+Z)-ALOG(EK))
      ERHS=0.
      DO 51 I=1,M
      C=I-1
   51 ERHS=ERHS+(GP(I)/(EK+C))
      Q=EK
      GO TO (5,6,9,13), L
    5 IF (ERHS-ELHS) 3,2,4
    3 EKU=EK
      EK=EK-(.1*EK)
      L=2
      GO TO 1
    4 EKL=EK
      EK=EK+(.1*EK)
      L=4
      GO TO 1
   13 IF (ERHS-ELHS) 12,2,4
    6 IF (ERHS-ELHS) 3,2,7
    9 IF (ERHS-ELHS) 12,2,7
   12 EKU=EK
      GO TO 8
    7 EKL=EK
    8 L=3
      EK = (EKL+EKU)*.5
      IF (ABS(EK-Q)-1.E-6) 2,1,1
    2 WRITE(3,57)
      WRITE(3,92)EK
   27 P=Z/EK
      WRITE(3,93)P
      EVAR=Z+(Z**2/EK)
      WRITE(3,903)EVAR
      WRITE(3,81)
      DO 16 I=1,N
      IF(I-2) 17,19,28
   17 PX=1./(1.+Z/EK)**EK
      GO TO 18
   19 W=1.E-40
      T1=EK*1.E-40
      S=EK+1.
      PX=EK*P/(1.+P)**S
      GO TO 18
   28 IF(N-I) 62,62,63
   62 PX=1.-SP
      GO TO 18
   63 R=I-1
      RR=I-2
      T1=T1*(EK+RR)
      W=W*R
      IF(T1-1.) 66,69,69
```

```
 66 IF(W-1.) 67,69,69
 69 T1=T1*1.E-40
    W=W*1.E-40
 67 S=EK+R
    S3=S/40.
    R3=R/40.
    PX=(T1/W)*(P**R3/(1.+P)**S3)**40.
 18 FX=PX*SUMF
    SP=SP+PX
    J=I-1
    X(I)=FX
 16 WRITE(3,82)J,F(I),X(I)
 79 IF(X(N)-5.)89,99,99
 89 X(N-1)=X(N-1)+X(N)
    F(N-1)=F(N-1)+F(N)
    N=N-1
    GO TO 79
 99 WRITE(3,84)
    NN=N-1
    K=0
    DO 29 I=1,N
 47 TE1=TE1+F(I)
    TE2=TE2+X(I)
    IF(I-NN) 44,44,45
 44 IF(TE2-5.) 29,45,45
 45 M=TE1
    SO=SO+TE1
    SE=SE+TE2
    RA=ABS(TE1-TE2)**2/TE2
    WRITE(3,908)K,M,TE2,RA
    K=K+1
    CHISQ=CHISQ+RA
    TE1=0.
    TE2=0.
    DF=DF+1.
 29 CONTINUE
    DF=DF-3.
    WRITE(3,85)
    WRITE(3,86)SO,SE,CHISQ,DF
    GO TO 800
 68 STOP
    END
```