EXPERIMENTS ON A SPHEROID RELATED TO THE THEORY OF STEERING OF SHIPS.

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SUMMARY

Theory indicates that virtual mass coefficients may have in important effect on the turning path of ships. In the past assumptions have perforce been based on constant values of the coefficients, as evaluated for submerged bodies. Recent mathematical work has shewn that the coefficients are greatly influenced by the free surface and accordingly by the ship speed. Measurements have been made of the virtual mass of a semi-spheroid. Moment and force have been measured in azimuth on a yawed spheroid. The object is to guide mathematical investigations and including the appropriate boundary assumptions.

The model was 16 feet long and 1.6 feet beam to the shape of a semi-spheroid below water and wall sided above water.

The measured rotational virtual mass coefficient ranged from 0.8 to 2.3. The result can be compared with the theoretical value of 0.34 for a free surface without gravity and 0.88 for a fixed boundary. The transverse virtual mass coefficients ranged from 0.6 to 1.5 compared with theoretical values of 0.365 for a free surface with gravity and 0.86 for a fixed boundary. Resistance is neglected in the theory to date and deductions from the limiting velocities measured in the experiments shew that little or no error is involved in the theory in this respect.

The variation of moment coefficient with yaw was found to be fairly close to that predicted by theory. The measured value of the coefficient was 0.71 at low speed and 0.93 at high speed compared with the theoretical value of unity for the fixed boundary assumption and 0.39 for the free surface assumption. Increase of moment coefficient with speed suggests that wave making may be important. The assumption of a moment coefficient of unity used in theoretical work in the past does not appear valid for the semi-spheroid.

The change of normal force with yaw is of the same

character as the theoretical solution for a flat plate but the magnitude is not unexpectedly only about one half or less of that for flat plates depending on the speed. The position of the centre of pressure is almost independent of speed and is further aft at greater yaw but even at 12 degrees yaw the centre of pressure is just before the bow and is well before at 6 degrees yaw.

The resistance coefficient when yawed at 12 degrees is about 68 per cent greater at a Froude number of 0.3 than when fore and aft. Drift angle of ships with good turning qualities is about 12 degrees and the result affords one explanation of the loss of speed of a ship when turning.

Thre broad conclusion is that the application of coefficients for a fixed boundary is not valid. Mathematical investigations should take account of the variation of the coefficients with speed and be directed towards the determination of the influence of wave making in order that the results may be confidently used to improve the steering qualities of ships.

§ 1. — INTRODUCTION

The equations of motion of a ship when turning have been developed on a number of occasions a good example being in Reference 1 and in greater detail in Reference 2. The solution of the equations presents many complications which have to date severely restricted the extent to which mathematical theory can be usefully applied to elucidate the problem of ship steering and to play its part in improving the steering qualities of a ship. A necessary line of advance is to assign values to the various quantities entering into the equations.

One set of quantities is the virtual mass coefficient in linear and in rotational motion respectively. These have been assumed in Reference 2 to be the same as those of a totally submerged spheroid of the same length and of twice the displacement of a ship. The values of the coefficient are given in Reference 3 and their application involves the assumption that the free surface of water can be treated substantially as a fixed boundary. It is clear mathematically that the coefficients of rotational and transverse virtual mass have an important effect on the turning path of a ship. This effect cannot be assumed to be the same at all speeds as for the comparatively simple case of a body of similar dimensions when deeply submerged. The coefficients are greatly influenced by the free surface and therefore by the speed of the ship. In order to guide theoretical investigations including the appropriate boundary assumptions, experiments have been made on a spheroid to determine the change of virtual mass, if any, with speed. Measurements with a similar purpose have also been made of the turning moment and force in azimuth when moving in a straight line when yawed and at a range of uniform speeds.

§ 2. — MODEL

The model was shaped as a semi-spheroid below the waterline and wall sided above water. It was made of paraffin wax 16 feet long, 1.6 feet beam and 0.8 foot draught. The method of construction and machine shaping was as usual for ship models. It was ballasted to a displacement of 668 lbs. in fresh water and to a radius of gyration of 4.35 feet about a vertical axis through the centre of gravity. The radius of gyration was determinal by bifilar suspension.

§ 3. — DETERMINATION OF THE VIRTUAL MASS OF A SPHEROID

The object of the tests was to ascertain both the rotational and the transverse virtual mass. In order to determine the former the model was rotated in azimuth from rest to its limiting angular velocity by applying a constant moment. Similarly for the latter the model was displaced transversely by successive applications of a constant transverse force. Tests were made for each of a series of constant loads of progressively increasing magnitude. The movement of the model was photographed by cine camera and the very small time intervals involved were recorded by photographing a special clock with quick hand. The velocity and acceleration were deduced graphically and thence the virtual mass. The arrangements for the experiments are shewn in outline on Diagrams 1 and 2, and described in detail in Appendix 1.

A typical set of space time records for one applied load is shewn on Diagram 3, both for rotational and translational motion. The experiment spots were obtained by reading off the position of the model and the corresponding time of movement from each frame of the film when magnified

DIAGRAM 1

DIAGRAMMATIC ARRANGEMENT FOR EXPERIMENTS.

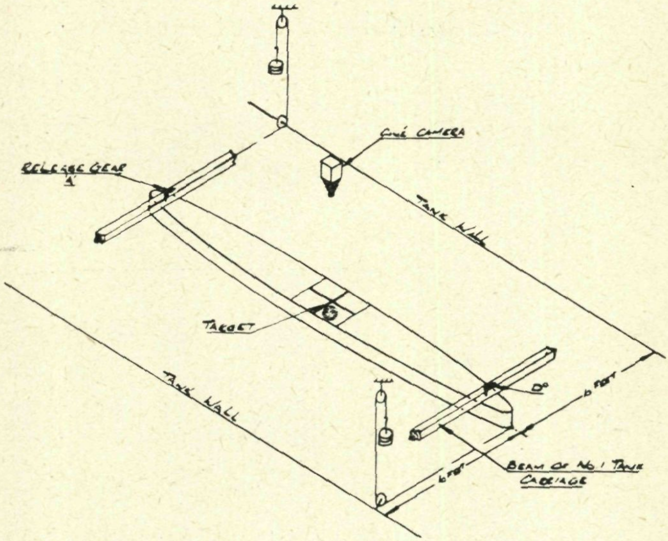


FIG 1 ROTATIONAL MOVEMENT

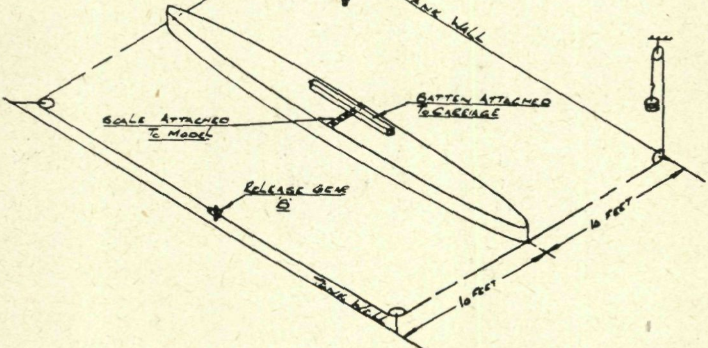


FIG 2 TRANSVERSE MOVEMENT

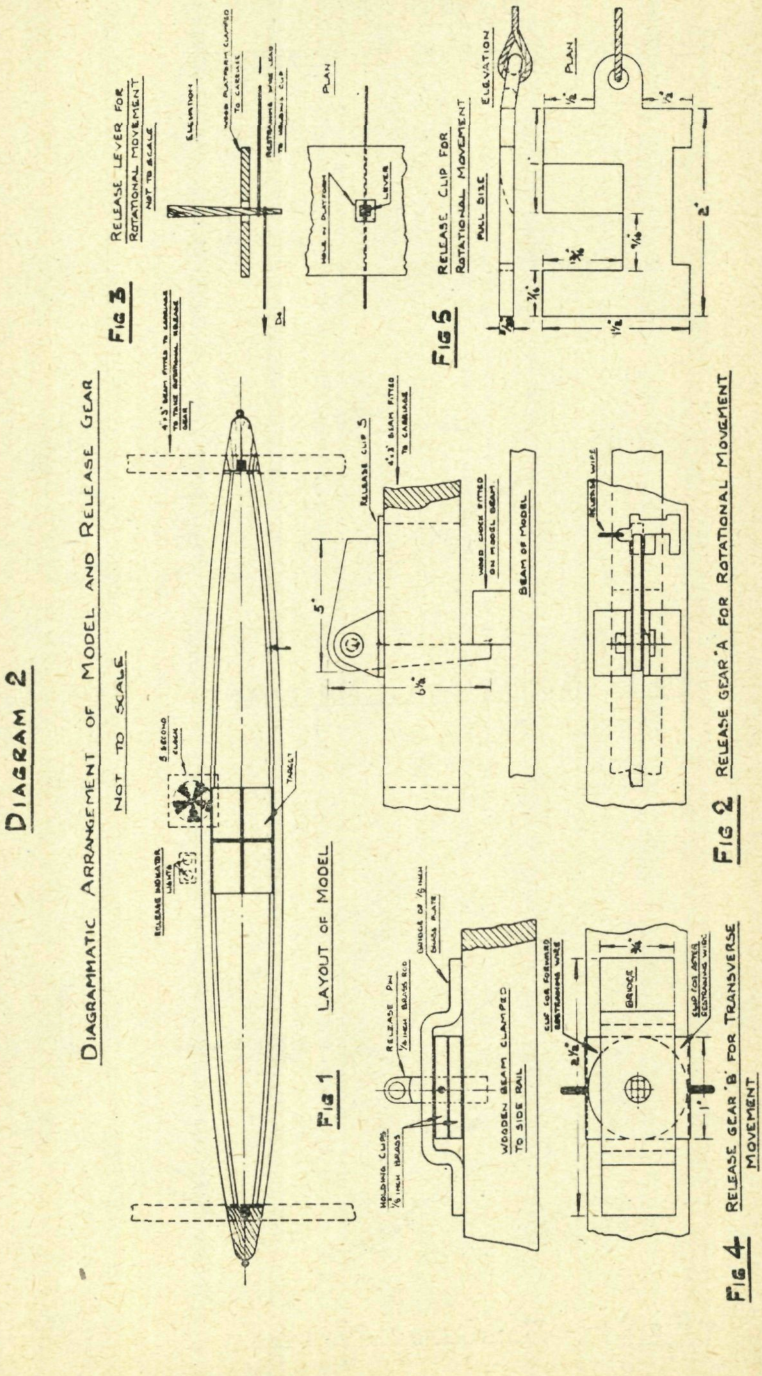


DIAGRAM 2
DIAGRAMMATIC ARRANGEMENT OF MODEL AND RELEASE GEAR
NOT TO SCALE
FIG 1
LAYOUT OF MODEL
FIG 2
RELEASE GEAR 'A' FOR ROTATIONAL MOVEMENT
FIG 3
RELEASE LEVER FOR ROTATIONAL MOVEMENT
NOT TO SCALE
ELEVATION
PLAN
LEVER
FIG 4
RELEASE GEAR 'B' FOR TRANSVERSE MOVEMENT
WOODEN BEAM CLAMPED TO SIDE RAIL
RELEASE PIN
BRIDGE
FIG 5
RELEASE CLIP FOR ROTATIONAL MOVEMENT
FULL SIZE
ELEVATION
PLAN
BEAM OF MODEL
RELEASE CLIP 5
5"
2"
1"

DIAGRAM 3

TYPICAL SPACE-TIME CURVES.

FIG I ROTATIONAL EXPERIMENT

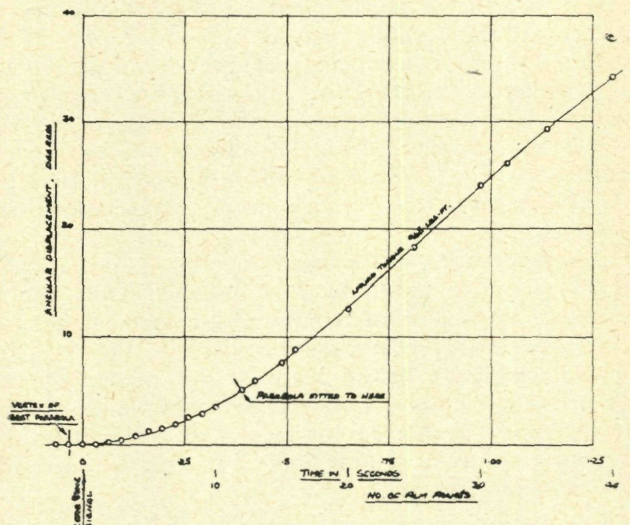
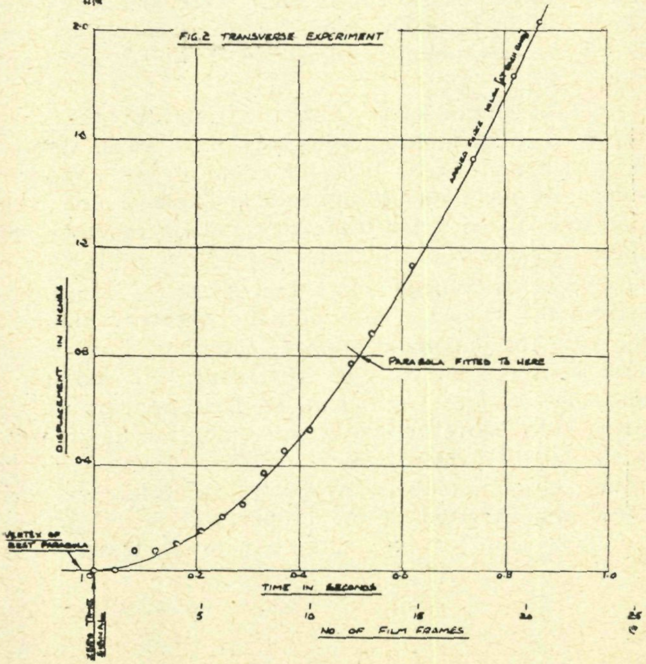


FIG. 2 TRANSVERSE EXPERIMENT



12 fold in a Watson viewer. Initial accelerations were then deduced first by double differentiation of the space time spots, but the results were inconsistent. It became clear that this was due to the impracticability of reading the small initial displacement to a sufficient degree of accuracy, e.g. in 1/10 second the rotation was of the order of 0.25 degrees and the transverse movement of the order of 0.06 inches.. The expedient was therefore adopted of fairing a parabola through the space time spots. The average initial acceleration was then deduced from the law of the parabola. Sets of curves similar to those on Sheet 3 were plotted for each applied load and the average initial accelerations similarly determined.

The accelerations are plotted to a base of initial load in Diagram 4. The initial load is obtained from the applied load reduced by an amount appropriate to the acceleration of the weights on the scale pans indicated by the results. This was appreciable at heavy loading and amounted to about 0.3 g as a miximum. Consideration was also given to an allowance for the moment of inertia of the pulleys, but this amounted to a total of only 0.0017 in feet lbs. second units, which is negligible.

Initial virtual mass coefficients were deduced for each applied loading from Diagram 4 and are shewn in appended Table 1 for rotational motion and Table 2 for transverse motion.

It will be appreciated from the trend of the curves in Diagram 3 that limiting velocities have been approached. This was found to be true for each of the applied loads. Results are recorded in appended Table 3 for rotational motion and Table 4 for transverse motion together with the deduced resistance coefficients.

The rotational virtual mass coefficient, as measured, ranged from 0.8 to 2.3. Although the greater value is associated with the maximum applied loading and the lower with a light load the increase in coefficient with loading is not progressive. The coefficient at first appears to decline, followed by a more or less steadly increase at greater load. The theoretical value of the coefficient for the fixed boundary assumption has been evaluated in Reference 3 and is 0.88 for the spheroid of the proportions tested. The theoretical value for the free surface without gravity has been

DIAGRAM 4

ACCELERATION - LOAD CURVES

FIG 1 ROTATIONAL EXPERIMENTS

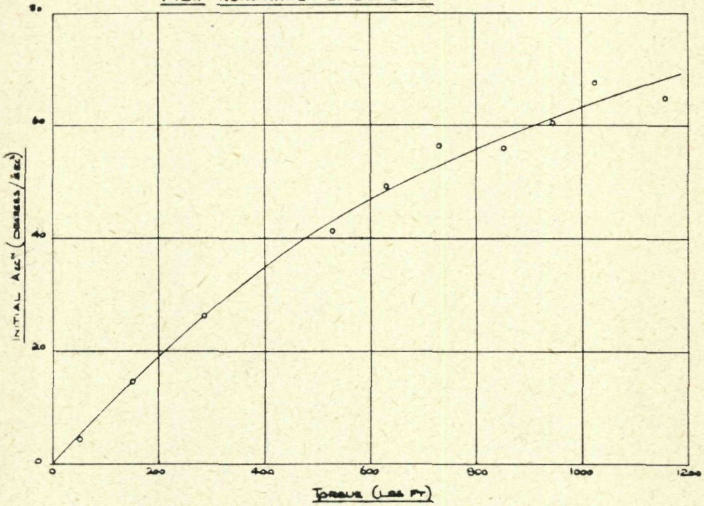
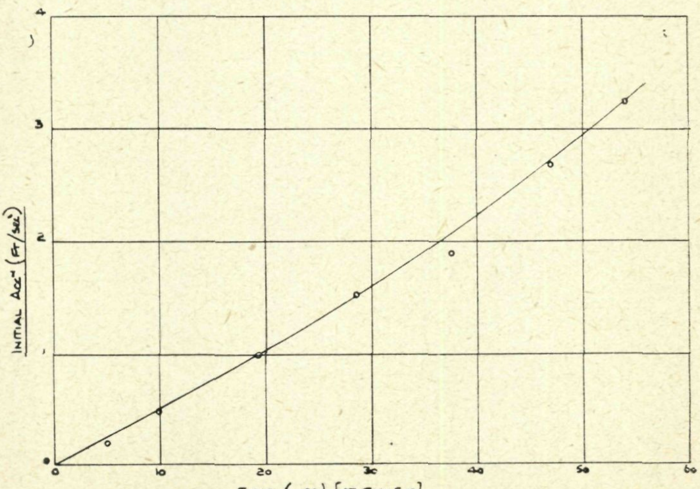


FIG 2 TRANSVERSE EXPERIMENTS



worked out by Mr. Wigley and found to be 0.34. The measured values are thus well above the latter and generally greater than the former. The transverse virtual mass coefficients declined from 1.5 at light load to 0.6 at heavy load. The theoretical value for the fixed boundary assumption as evaluated in Reference 3 is 0.895 while that deduced by Mr. Wigley for free surface without gravity is 0.371. The theory as at present developed is inexact, but it can be shewn that analogy with the solution for a cylinder at various submersions in Reference 4 insofar as it is valid suggests that the values of the virtual mass coefficients deduced from the mean initial accelerations vary consistently with theory, but further investigation is required to prove or disprove this. Since there is not as yet any mathematical solution for a body moving on the surface, it is not possible to give a conclusive comment based on theory though by analogy with the case of the transverse motion the measured values cannot be regarded as improbable. It has been deduced by Mr. Wigley that the effect of resistance as deduced from the limited velocity in the experiments would theoretically only increase the rotational virtual mass coefficient by about 2 or 3 per cent at all loadings so that the variation of virtual mass coefficient with loading cannot be ascribed to the effect of resistance. This applies with even more force to lateral motion.

Readers who desire to pursue the mathematical aspects of the subject may be interested to consult the recent work by Professor Sir Thomas H. Havelock in Reference 7 and that by F. Ursell in References 8 and 9.

§ 4. — DETERMINATION OF MOMENT AND FORCE ON A YAWED SPHEROID

The procedure for experiments was similar to that for ascertaining the turning moment on ship models and as described in Reference 5. The lateral force was measured at two points, one near each end of the model. The moment is azimuth and resultant transverse force were deduced from these measurements. The model was towed from the resistance dynamometer in No.1 Ship Tank at a series of uniform speeds from 150 to 370 feet per minute at angles of steady yaw of 12, 9 and 6 degrees to Port and Starboard respectively. Tests were also made at zero yaw. The general arrangements for the experiments are indicated on Diagram 5.

Improved lateral force « guiders » or balances were designed and made for the tests as shewn in the photograph on Diagram 6. A feature was the provision for null reading, weights being added until the lateral force was balanced. The guiders were connected to the lateral model through a thrust ball race shewn in the photograph so that the lateral force did not contribute to heel of the model. Special care was taken to eliminate backlash.

Results deduced from the experiments are on Diagram 7 to 11. The moments and forces are shewn as non-dimensional coefficients. The presentation is generally in accordance with the mathematical treatment by Mr. C. Wigley the relevant results of which are indicated on the diagrams.

Briefly the variation of moment with angle of yaw is not completely in accordance with the mathematical solution, but is fairly close as càn be seen from Diagram 7 and 8. Taking a mean of the experiment results the moment coefficient is 0.73 at low speed (Froude number 0.11) and 0.91 at high speed (Froude number 0.27). This result compares with the theoretical coefficient of unity for the fixed boundary assumption in the mathematical treatment and 0.39 for the free surface assumption. It is clear that the assumption made in some theoretical work on steering, such as in Reference 2, namely that the moment coefficient can be taken as unity is not really valid for a semi-spheroid and that allowance must be made for variation of the coefficient with speed.

The extent to which the increase of coefficient with speed may be due to wave making forces remains to be determined by development of the theory to take account of wave making. Meanwhile it is not without interest to note that the coefficient at the lowest speed which is least likely to be affected by wave making is about a mean of the values obtained by the alternative assumptions as to boundary condition at the free surface.

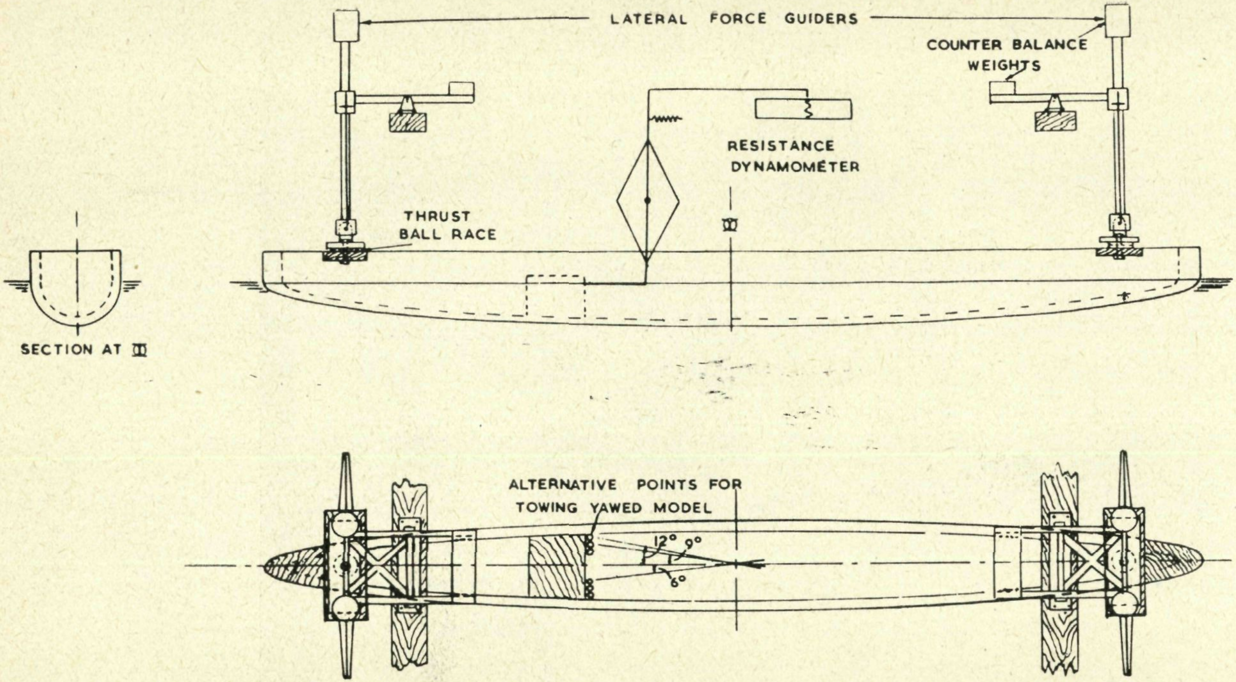
Although no theoretical solution is available for comparison it was considered useful to investigate the lateral force or « lift ». The results shewn on Diagram 9 indicate a slight increase with speed and a marked increase with yaw.

Curves of resistance coefficient are shewn on Diagram 10. Here again no comparison with theory is possible as the relevant mathematical work has not yet been developed. There is a fairly substantial increase in resistance with angle of yaw, and at 12 degrees yaw, for example, the resistance is 68 per cent greater than when the model is fore and aft. This percentage applies to a Froude number of 0.3 ($V/\sqrt{L}$ = unity in knots and feet units). The percentage increase varies to some small extent with speed. Since 12 degrees is approximately the drift angle for some classes of ship with good turning qualities, the increase of resistance is of interest in affording one explanation of the decline in speed of ship when turning. The analogy with a ship turning is of course not complete since the flow conditions are different but the result is of some relevance. The increase of resistance at 6 degrees yaw is small, namely about 11 per cent, but as far as steering is concerned, this would only be of interest for a very large turning circle. The resistance coefficient in fore and aft motion is rather large and at a Froude number of 0.3 is about 50 per cent greater than that of a ship model of good lines of the same length and displacement. This result is not surprising and may be attributed to large wave making consistent with the rather large prismatic coefficient, namely 0.665. Although the wave making may be large it does not follow that yawing couple due to wave making referred to above will also be large.

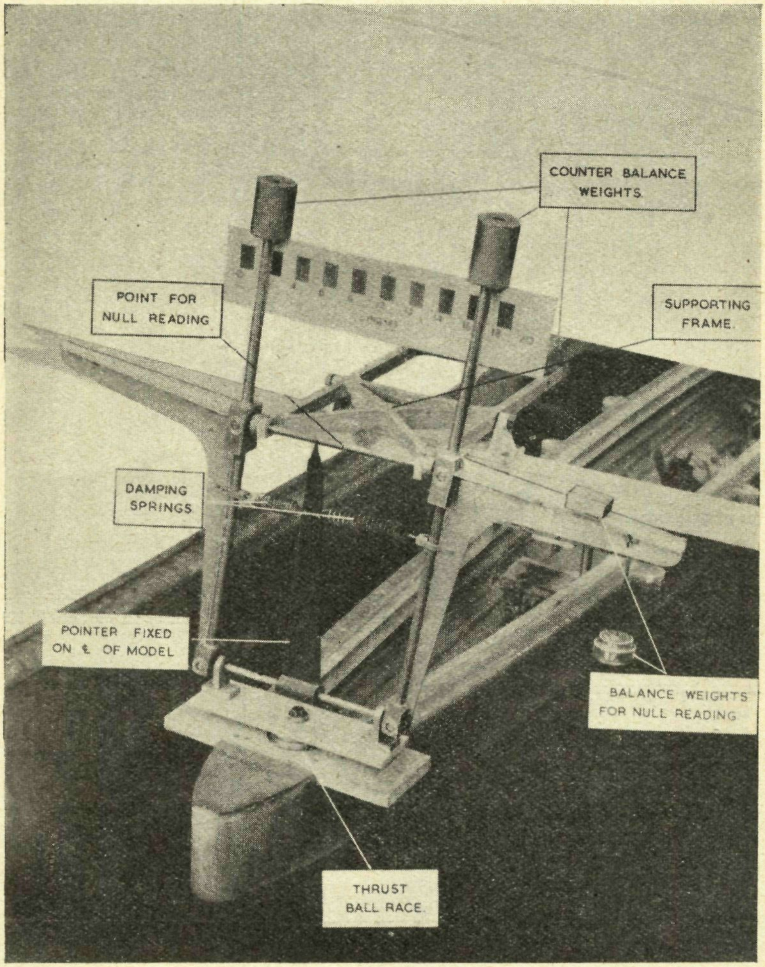
Diagram 11, for which the author is indebted to Mr. Wigley, shews that the character of the variation of normal force with angle of yaw is in accordance with a theoretical solution for flat plates in Reference 6, but the measured force is less than that theoretically obtained, the average ratio being 0.5 at the highest speed of test and 0.4 at the lowest. This divergence is probably attribuable to the difference between the flow over the spheroid and a flat plate.

The position of the centre of pressure is almost independent of speed at 9 degrees yaw and at 12 degrees. It is approximately 0.24 of the length before the bow at 9 degrees yaw and 0.05 of the length at 12 degrees yaw. At 6 degrees yaw the centre of pressure is 0.52 lengths before the bow at high speed and 0.77 at the lowest speed of experiment. The results at 6 degrees are therefore not so consistent over the speed range as at the larger angles of yaw. It should be remarked that both moment and force are comparatively small at 6 degrees yaw and any lack of precision of measu-

DIAGRAM 5



GENERAL ARRANGEMENT OF MODEL DURING EXPERIMENTS.


COUNTER BALANCE WEIGHTS
POINT FOR NULL READING
SUPPORTING FRAME
DAMPING SPRINGS
POINTER FIXED ON ℄ OF MODEL
BALANCE WEIGHTS FOR NULL READING
THRUST BALL RACE

VARIATION OF MOMENT COEFFICIENT WITH SPEED.

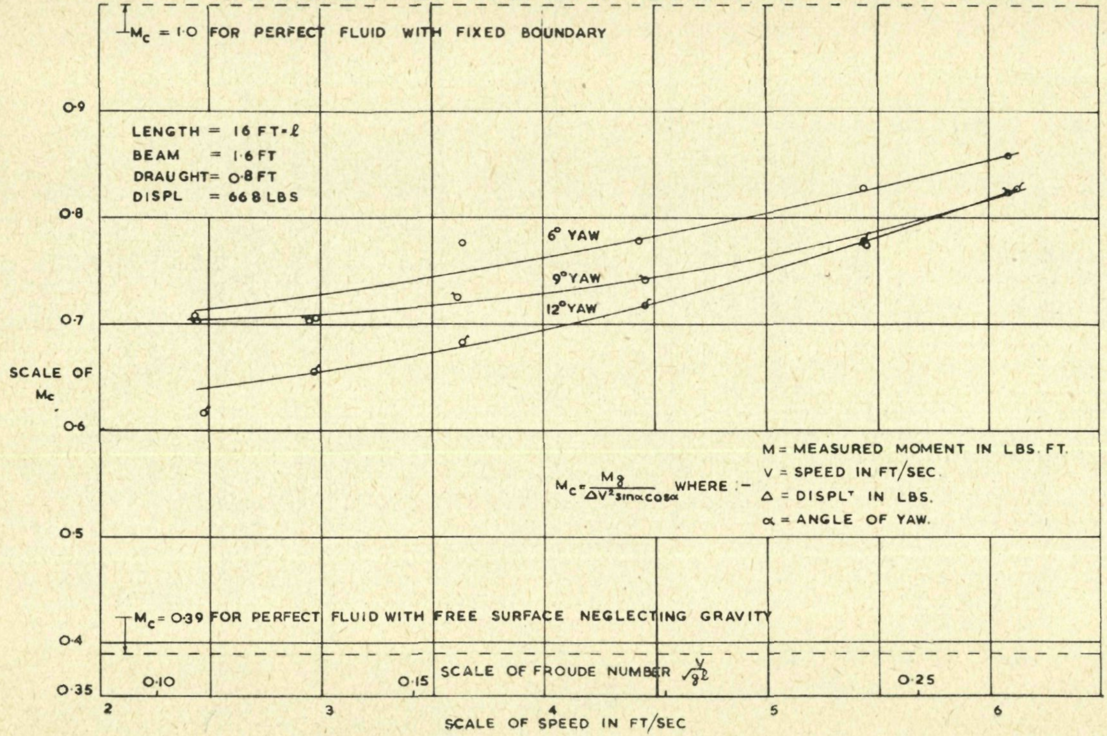


DIAGRAM 7

Diagram 8

EFFECT OF VARIATION OF ANGLE OF YAW ON MOMENT COEFFICIENT.

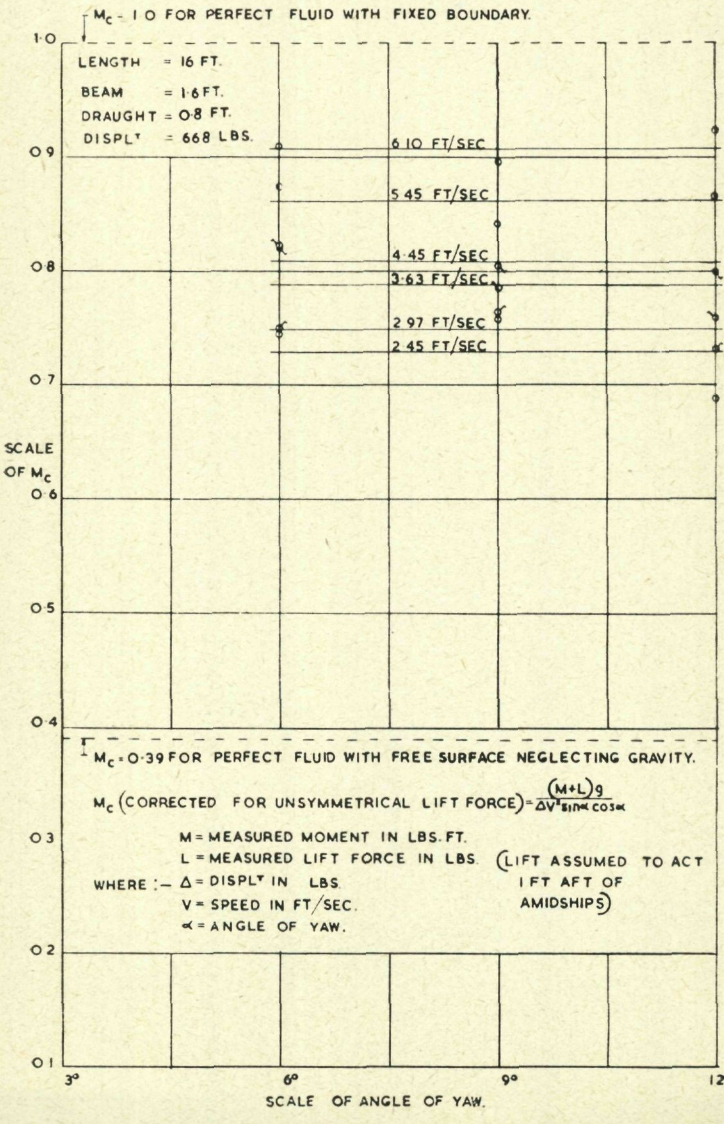


DIAGRAM 9

VARIATION OF LIFT COEFFICIENT WITH SPEED.

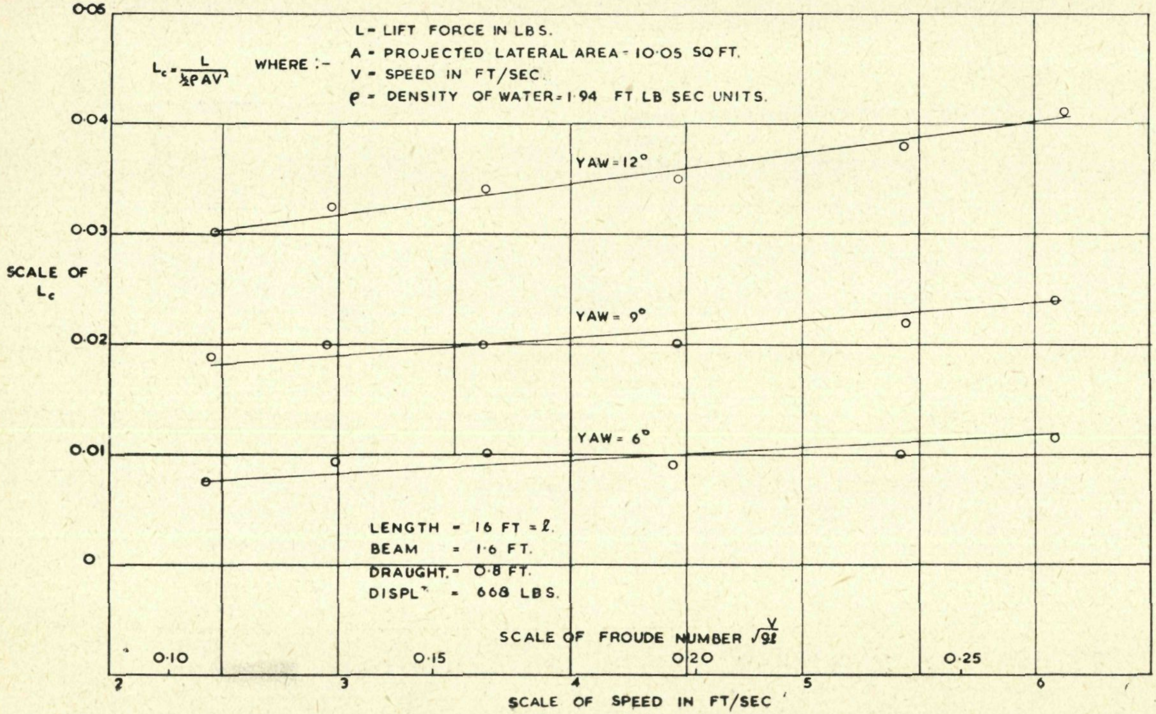


DIAGRAM 10

VARIATION OF DRAG COEFFICIENT WITH SPEED

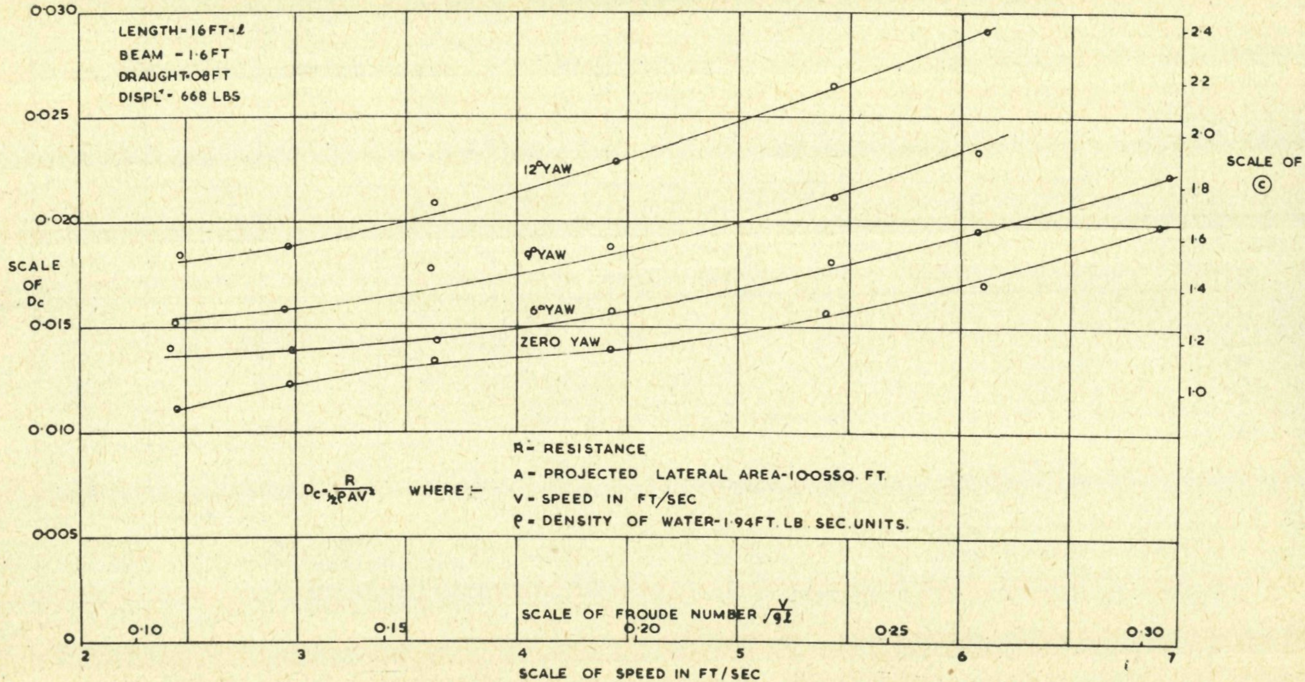
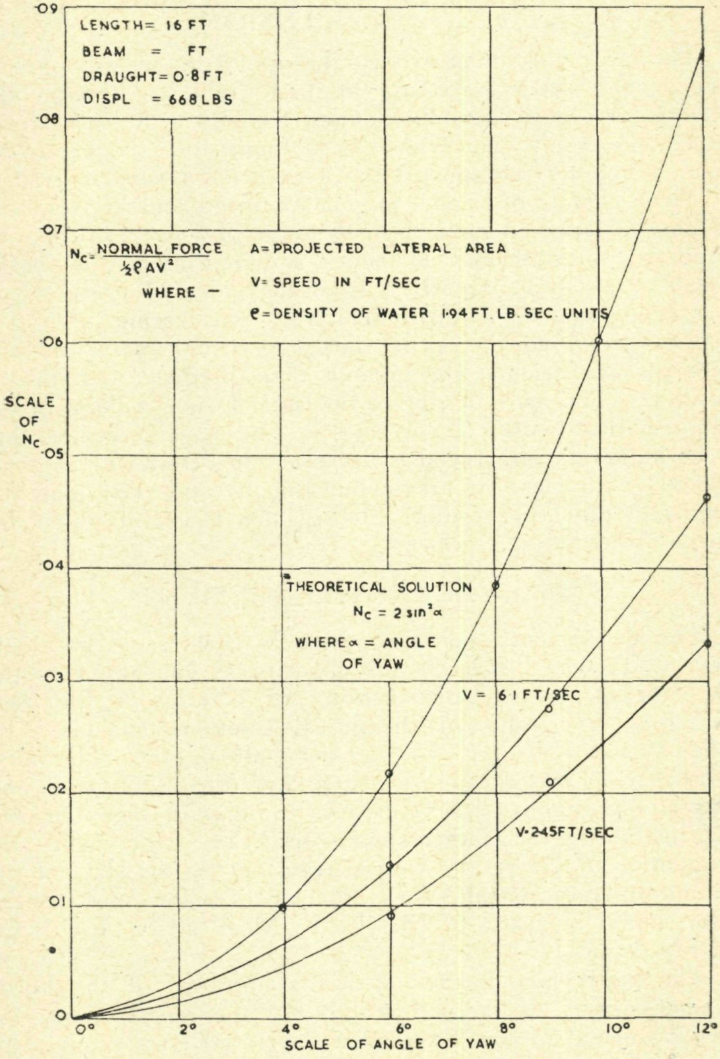


DIAGRAM 11

VARIATION OF NORMAL FORCE WITH SPEED COMPARED WITH THEORETICAL SOLUTION FOR FLAT PLATES.



* FROM BOLLAY'S RESULTS FOR FLAT PLATES WITH ZERO ASPECT RATIO

(PUBLICATION Nº III OF GUGGENHEIM AERONAUTICS LABORATORY CALIFORNIA INSTITUTE OF TECHNOLOGY 1939) rement can lead to additive error in the deduced position of centre pressure. The results bring out the rapid movement of centre of pressure aft with increase of angle of yaw consistent with the corresponding increase of lift.

§ 5. — CONCLUSION

The large variations of virtual mass with acceleration and speed may seem surprising at first sight. Although the theory for a body actually floating has still to be fully developed, the results which have been found for a body moving near a free surface support the experiment results in respect of large change of virtual mass with speed and acceleration.

The variation of moment with angle of yaw is fairly well predicted by mathematical theory, but here again the coefficients show great departure from the value of unity which has been assumed in theoretical work on steering.

The broad conclusion is that the virtual mass and the moment coefficients are greater than predicted by theory to-date. This may be due to neglect of wave making in the mathematical work. Experience in the tests indicates that even higher camera speed is necessary to determine the coefficients with close accuracy, but the investigation generally confirms that the results obtained are a useful guide for further theoretical work.

§ 6. — ACKNOWLEDGEMENT

The experiments were suggested by Professor Sir Goeffrey I. Taylor, F.R.S., in his capacity as consultant to the Director of Naval Construction, Sir Charles S. Lillicrap, K.C.B. They followed theoretical solutions evaluated by Mr. W.C.S. Wigley, M.A. on behalf of the Admiralty. Mr. Wigley was closely associated with the experiments and has kindly supplied the references to some of the published information on the mathematical work. The experiments and evaluation of the results were carried out by the staff of the Admiralty Experiment Works, Haslar, of whom special mention should be made of Mr. J.R.F. Moss, M.A., R.C.N.C., M.I.N.A.

The paper is published with the approval of the Lords Commissioners of the Admiralty, but the responsibility for any statement of fact or opinion expressed rests solely with the Author.

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APPENDIX 1

Detailed description of the virtual mass experiments.

The model was initially at rest with its centre on the middle line of No.1 Ship Tank under the experiment carriage. A flexible wire fixed to a small eye in the bow of the model about 1/4 inch above the waterline was led athwartships then vertically over a grooved pulley on one side of the tank and finally vertically downwards over another grooved pulley terminating in a scale pan to which weights could be added. A similar and parallel wire with scale pan and weights was secured to the stern of the model, but leading in the opposite direction. The wires were flexible of bowden type about 1/16 inch diameter. The pulleys, 4 in number, were 3 inches diameter and mounted on ball bearings to reduce friction to a minimum. The two lower pulleys were secured to the side of the waterway and the two upper ones to the roof girders of the waterway building.

In order to ensure free release of the model two triggers were fitted, one forward and one aft as shewn on Figure 1, Diagram 1, and in greater detail on Diagram 2. Each trigger was supported on a wooden beam secured to the carriage. The lower arm of the crank of each trigger locked the model by bearing against a small wooden chock secured to the beam of the model. The triggers were released by removal of the thin brass wedge « S » in Figure 2 of Diagram 2. The horizontal arm of the model was free to move under the influence of the accelerating moment. Following trial of preliminary devices, simultaneous release was arranged by a separate wire led to amidships from each trigger wedge and secured to a vertical lever about the centre of the model, as shewn in Figure 3 of Diagram 2.

The movement of the model was recorded by cine camera mounted on the experiment carriage and operating at 32 frames per second. Time was recorded by synchronous electric clock with a large hand making one revolution in 5 seconds which was secured in the model and photographed in each frame. The view of the cine camera contained two small electric light bulbs, the circuits of which were made through the two release triggers, one forward and one aft. The lights were thus automatically switched off when the triggers tripped, thus providing zero time marking and a check on the simultaneous trip of the two triggers. In order to assist identification of the path of the model and its orientation in azimuth, a small target in the form of a cross was secured across the gunwhale of the model amidships, and photographed by the camera. Illumination for photography was provided by 4 in number lamps, each of one kilowatt, arranged 7 feet vertically above the centre of the model.

At the instant of release, the bearing of the model was at 10 degrees yaw, in order that the lever of the couple, (i.e. distance between the tension wires) should be sensibly constant for 20 degrees of turn.

The applied torque in the experiments ranged from about 50 to 1600 lbs. feet. Tests were made with 10 in number initial torques within this range by successive additions of weights to the scale pans. Thus a wide range of load was covered, the limits being decided from preliminary trial which confirmed that the accuracy of the experiments would not be projudiced by significant stretch of the wire or by other strain at heavy load, nor by undue proportion of friction at light load.

The arrangements for the transverse experiments were as described for the rotational experiments except that the tensioned wires were each led to the same side of the tank. Release was effected by the removal of a pin in the tensioned wire, the arrangements being as in Figure 2 of Diagram 1 and Figure 3 of Diagram 2. At the instant of release the bearing of the model was along the centre line of the tank and the identification target was in the form of a linear scale as shewn in Figure 2 of Diagram 1.

It was intended to extend these tests to the same initial load as in the rotational tests, namely 100 lbs. of tension at each end of the model, but maximum load was restricted to 60 lbs. at each end because greater loads involved binding of the release pin « B ».

The experiment arrangements as finally evolved proved generally satisfactory. Free release of one end of the model in the rotation tests was in step with that at the other end within one cine film space. The quick drop in acceleration implies that records must be completed within a fraction of a second. In anticipation of this requirement a reasonably fast camera speed of 32 frames per second was arranged, guidance being obtained from some older tests on turning of ship models. The results may be accepted as a measure of the variation of virtual mass which may be expected, but the actual values are probably not of closest accuracy in view of the difficulties of experimental measurement. To determine this variation was the object with which the experiments were made, and it is thought that they have produced a picture of the type of variation which is probably general and will be helpful in the mathematical work. The results should be interpreted accordingly.

Table 1. *Rotational Virtual Mass Coefficients*

Nominal Torque lbs.-ft.	Initial Angular Acceleration ω Radians/sec ^{2}	Actual Torque (Ø) lbs.-ft.	Inertia Torque (Ø1) $= MK_M^2 \omega$ lbs.-ft.	(Ø) - (Ø1) lbs.-ft.	Inertia Torque for a uniform SPHEROID (ØU) lbs.-ft.	Virtual Mass Coefficient (Ø - Ø1) / (ØU)
56.3	0.076	55.2	29.9	25.3	20.5	1.2
160.8	0.252	150.7	99.2	51.5	67.8	0.8
321.7	0.454	285.2	178.8	106.4	122.1	0.9
643.3	0.718	528.0	282.7	245.3	193.2	1.3
804.2	0.856	632.0	337.0	295.0	230.3	1.3
965	0.981	728.6	386.3	342.3	263.9	1.3
1125	0.974	851.8	383.5	468.3	262.0	1.8
1286	1.053	948.2	414.6	533.6	283.3	1.9
1447	1.180	1021	464.6	556.4	317.5	1.8
1608	1.134	1153	446.5	706.5	305.0	2.3

Note :

Column N°.

EXPLANATION

1 Nominal applied torque from added weight and lever.

2 Mean acceleration for first half degree of motion deduced from acceleration load curves (cf. Diagram 4).

3 Effective torque allowing for acceleration of added weights.

4 Inertia torque of ellipsoid $= MK_M^2 \omega$
where M is the mass of model = 668 lbs.
K_M is radius of gyration of model = 4.35-ft.
and ω is angular acceleration in Col. 2.

5 Net Effective torque by subtracting Col. 4 from Col. 3.

6 Inertia torque for a uniform spheroid $= MK_U^2 \omega$
where $K_U^2 = 0.2\ (a^2 + b^2)$ i.e. $K_U = 3.59$ ft.
where *a* and *b* are semi axes of spheroid.

7 Virtual mass coefficient i.e. Col. 5 divided by Col. 6.

Table 2. *Transverse Virtual Mass Coefficients*

Nominal Force lbs.	Acceleration ($\ddot{y}$) ft. sec.2	Actual wire Tension (2T) lbs.	Inertia Force $M\ddot{y}$	$2T-M\ddot{y}$	Virtual Mass Coefficient $\frac{(2T-M\ddot{y})}{(M\ddot{y})}$
10	0.19	9.9	3.95	5.95	1.5
20	0.47	19.7	9.78	9.92	1.0
40	0.98	38.8	20.4	18.4	0.9
60	1.53	57.1	31.8	25.3	0.8
80	1.88	75.3	39.1	36.2	0.9
100	2.68	91.6	55.8	35.8	0.7
120	3.23	107.9	67.2	40.7	0.6

Note :

Column N°.

EXPLANATION

1 Nominal applied force from added weight.

2 Mean transverse acceleration for first half inch travel deduced from acceleration load curves (cf. Diagram 4).

3 Effective load allowing for acceleration of added weight.

4 Transverse inertia force of ellipsoid $= M\ddot{y}$ where M is mass of model = 668 lbs. and $\ddot{y}$ is acceleration in Col. 2.

5 Net effective load by subtracting Col. 4 from Col. 3.

6 Virtual mass coefficient by dividing Col. 5 by Col. 4.

Table 3. *Limiting Angular Velocity and Resistance in Rotational Motion*

Applied Torque 1bs.-ft.	Limiting Angular Velocity Radians/sec.	Resistance Coefficient
56.3	0.21	0.032
160.8	0.42	0.024
321.7	0.47	0.036
643.3	0.57	0.049
804.2	0.54	0.060
965	0.61	0.065
1125	0.59	0.080
1286	0.68	0.070
1447	0.63	0.092
1608	0.72	0.078

Note :

Resistance Coefficient $= \frac{T}{\frac{1}{2} e A \omega^2 l^2}$

where T = applied torque
e = density = 1.94 1bs. sec²/ft4
A = area of middle line plane $= \frac{\pi}{2}$ a b
a = semi length on waterline
b = draught
l = length on waterline = 2a
ω = limiting angular velocity in radians/sec.

Table 4. *Limiting Velocity and Resistance in Transverse Motion*

Applied Load 1bs.	Limiting Velocity ft./sec.	Resistance Coefficient
10	0.78	1.7
20	1.26	1.3
40	1.48	1.9
60	1.74	2.0
80	1.82	2.5
100	2.17	2.2
120	2.28	2.4

Note :

Resistance Coefficient $= \frac{R}{\frac{1}{2} e A V^2}$

where R = applied load
e = density = 1.94 1bs. sec²/ft⁴
A = area of middle line plane $= \frac{\pi}{2}$ a b
a = semi length on waterline
b = draught
V = limiting velocity

Comments by Dr. J. F. ALLAN.

As the author indicates in the paper, these experiments are largely of an exploratory nature and they show the importance of pursuing the subject.

Referring to the first part of the paper, it is noted that whereas the transverse virtual mass coefficient decreases with increase in load or initial acceleration, the rotational virtual mass coefficient decrases and then increases. In this connection it may be significant that the maximum transverse acceleration was just over 3 ft./sec.^{2}, whereas the maximum linear acceleration towards the ends of the model in the rotation experiments was 6 tot 8 ft./sec.^{2}.

It is probable that the increase in the rotational coefficient at higher acceleration is due to the wave formations near the ends of the model, causing a pile up of water on one side and a deficiency on the other side. If this explanation is true, then a similar result would have been obtained had the transverse experiments been pushed to higher accelerations.

The theoretical values for both types of motion, either for the fixed boundary assumption or for the free surface without gravity assumption are in close agreement, namely, 0,88 and 0,895, and 0,34 and 0,37.

Te disagreement between these values and the experimental results indicates clearly that the assumptions of the theory in both cases are incorrect or inadequate. It seems not unlikely that a suitable correction or allowance for the effect of gravity at the free surface will improve matters, but if so it will probably affect the comparison between theory and experiment in the second part of the paper.

However, the comparisons between the moment coefficients in yaw as measured and as calculated for the two limiting assumptions are wide enough to permit of considerable adjustment.

I am a little perturbed at the extreme and very forward positions of centre of pressure stated in this part of the paper, and perhaps the author would agree thet these should not be given too much emphasis.

It may be worth while drawing attention to the complete absence of forefoot or deadwood in a spheroid as tested, and remarking on the spectacular effect the addition of deadwood fore and aft would have on the results obtained.