MINIMUM WEIGHT OF STRUCTURAL PARTS OF SHIPS.

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Summary.

An exact calculation of minimum weight of structural parts should be made in the designing office where the details of each case are available. Be means of a few examples the present paper gives some outline of the general principles for such calculation.

The example on hatch end beams and hatch side girders gives as a result that the ratio between the moments of inertia of the girders and the beams should be very nearly proportional to the ratio between length and breadth of hold when this ratio is greater than say 1,0 and that the ratio between reactional force between beams and girders and the total load then is very nearly independent of the L/B *ratio.*

The example on beam and frame portal with one row of pillars gives for minimum weight the ratio between the moments of inertia of beam and frame as the product of $n = B/D$ *and a second order function of the parameter* $\nu = chn^2 / (2\rho_1 h_1 \varepsilon \gamma)$.

The minimum weight of a deck panel with 'thwartship beams and a longitudinal centreline girder, subjected to an evenly distributed lateral load, is obtained with a beam spacing nearly a constant proportion of the ship breadth.

A deck panel subject to longitudinal compression has a minimum weight for 'thwartship beams with about 31 per cent of the weight in the beams and a beam spacing proportional to $b^{12/13} (p/E)^{1/13}$ *and a minimum weight for longitudinal girders with about 44 per cent of the weight in the girders and a girder spacing proportional to* $l^{6/7} (p/E)^{1/7}$.

Here b is the unsupported width of plating, l length of hold or distance between heavy transverse web beams and p load per unit width. The transversely stiffened deck will be considerably heavier than the longitudinally stiffened one unless l is more than six times b.

Introductory.

One of the objects of strength calculations is to save weight. For complex structures like ships, or even structural parts of ships, the question of minimum weight might deserve a separate treatment. The following is an endeavour to start such a treatment. So little has yet been done on these lines and so meagre is our knowledge of the more complex strength calculations that final and in every respect reliable results must not be expected immediately. But it is hoped that the line of thought may be followed up by necessary tests and the collection of practical experience so that the results may be corrected and improved upon and finally given in such a form thet they may be of use to the designers of ships.

A short paper like this can also only give a few examples.

Hatch End Beams and Hatch Side Girders.

As an example of a redundant system of beams and girders subjected to lateral loading has been chosen the symmetric arrangement of hatch end beams and hatch side girders shown in Fig. 1 for the deck of a hold of length L and breadth B. The hatch end beams are fitted at a distance αL from the bulkheads and the hatch side girders at a distance βB from the ships sides, where α and β are ratios less than 0.5.

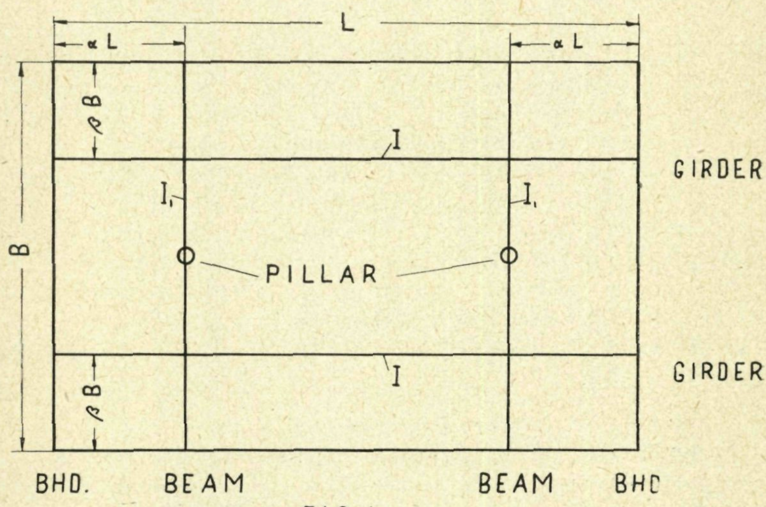

FIG. 1

The beams and girders shown may be considered as the main carrying members of the deck. The object of the ordinary beams and half-beams may be said to be distributing the load to the main members. Each of the two longitudinal girders may be assumed to carry an evenly distributed load q per unit length. The two hatch end beams will assist them by taking a reaction force R at each of the four hatch corners.

In the example a pillar is fitted at midlength of each of the hatch end beams. As shown on page 136 in reference [1] the double bottom under such pillars may deflect downwards or upwards, depending upon the amount of cargo in the hold in comparison with the draft. For simplicity it will here be assumed that the double bottom does not deflect, i.e. that the hatch end beams have zero deflection at the pillars. The deflection at the pillar caused by the pillar force P must, therefore, be equal in magnitude and opposite in direction of the deflection at the same point caused by the two reaction forces R. The deflections for any degree of fixity f of the beam ends may be expressed by means of the values given in Table III of reference [1]. This gives

$$P/R = 8\beta[3-4\beta^2-3f(1-\beta)]/(4-3f). \qquad (1)$$

Now the deflections of beams and girders at the hatch corner may also be written down by means of the expressions given in the same table III. These deflections are put equal in magnitude and direction, whereby we obtain

$$4R/qL = (4-3f)(1+\alpha-\alpha^2-f_0)(1-\alpha)\alpha n^3/\{(4-3f)[3-4\alpha-3f_0(1-\alpha)^2]\alpha^2 n^3+[3(1-f)+2\beta](1-2\beta)^3\ \beta^2\ m\}, \qquad (2)$$

where $n = L/B$ and $m = I/I_1$, I being the moment of inertia of a girder and I_1 the moment of inertia of a hatch end beam. The sections of beams and girders have been assumed to be constant over their lengths. f_0 is the degree of fixity of the girder ends. For hinged ends $f = f_0 = 0$, for encastre ends $f = f_0 = 1$.

The hatch end beams are subjected to the forces shown in the upper part of Fig. 2 and to the bending moments shown in the lower part of the same figure. The bending moment at the pillar force P, at midlength, is

$$M^P = -2\beta(1-2\beta)RB\ [1+2\beta-f(1+\beta)]/(4-3f). \qquad (3)$$

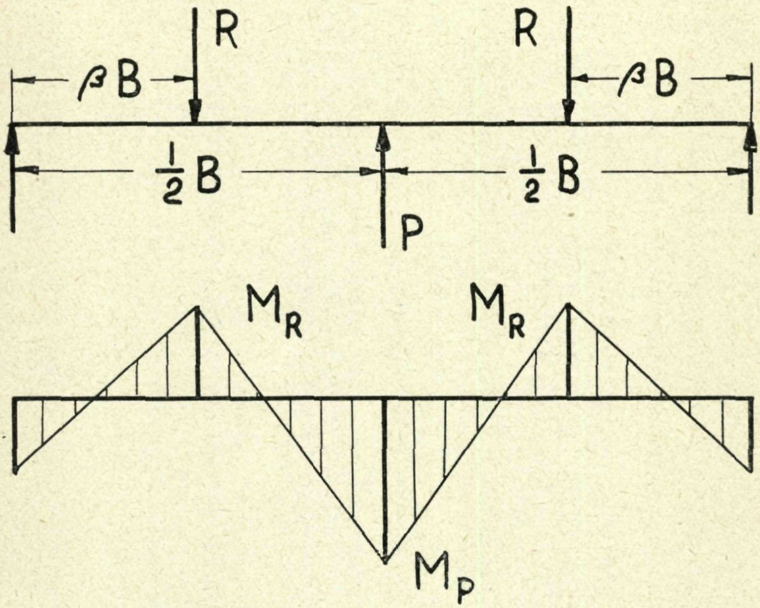

FIG. 2

The bending moment at the reaction force R, at hatch corners, is

$$M_R = 4\beta(1-2\beta)^2 RB\ (1+\beta-f)/(4-3f). \qquad (4)$$

The two moments are of equal magnitude when

$$\beta = -1/8(4-5f-\sqrt{32-56f+25f^2}), \qquad (5)$$

which for $f = 0$ equals 0,207,
» $f = 2/3$ » 0,217,
» $f = 1$ » 0,25.

When β is larger than this quantity M^P is the larger, when β is less M_R is the larger of the two.

The bending moment diagram of the girders has been

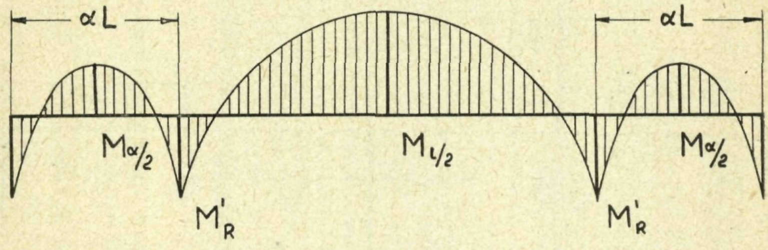

FIG. 3

sketched in Fig. 3. The bending moment at the hatch corners is

$$M_{R1} = L \left\{ \alpha[\tfrac{1}{2}(1-\alpha)qL - R] - f_0[1/12qL - \alpha(1-\alpha)R] \right\}, \quad (6)$$

and the bending moment at midlength

$$M_{L/2} = L \left\{ 1/8qL - \alpha R - f_0[1/12qL - \alpha(1-\alpha)R] \right\}. \quad (7)$$

The two moments are of equal magnitude when

$$R/qL = \frac{3 + 12\alpha - 12\alpha^3 - 4f_0}{48[1 - f_0(1-\alpha)]\alpha} \quad (8)$$

When R/qL is larger than this quantity M_R^1 is the larger, when R/qL is less $M_{L/2}$ is the larger of the two. If $f_0 =$ 2/3 equation (8) gives R/qL = 0,737 when $\alpha = 0{,}1$,
» = 0,431 » $\alpha = 0{,}25$,
» = 0,315 » $\alpha = 0{,}3575$.

For the sake of completeness the bending moment at midlength of the two sidespans will also be given (9)

$$M_{\alpha/2} = L\ \tfrac{1}{2}\alpha[\tfrac{1}{2}(1-\alpha/2)qL - R] - f_0[1/12qL - \alpha(1-\alpha)R]\}.$$

This moment equals $M_{L/2}$ when $R/qL = (1-\alpha)^2/(4\alpha)$, (10) which for $\alpha = 0{,}1$ equals 2,025,
$\alpha = 0{,}25$ » 0,5625,
$\alpha = 0{,}3575$ » 0,289.

When R/qL is larger than this quantity $M_{\alpha/2}$ is the larger, if R/qL is less $M_{L/2}$ is the larger of the two. Comparing (10) with (8) it will be seen that when $\alpha > 0{,}331$ for $f_0 = 2/3$, there may be a range of L/B-values for which $M_{\alpha/2}$ is larger than $M_{L/2}$ as well as M_R^1.

The maximum stress in the girders can now be obtained by dividing the maximum bending moment with the section modulus W. If the section had been symmetric, with equal flanges at top and bottom, one might have written $W = 2kI/L$, where k is the ratio between the span L and the height of the girders. For normal unsymmetric girders one may instead write $W = ckI/L$, where c is a coefficient less than 2. Similarly the maximum stress in the hatch end beams can be obtained by dividing their maximum bending moment by their section modulus $W_1 = c_1k_1I_1/B$. If beams and girders have the same height, as they usually have, $k/k_1 = L/B$.

It can be shown that the total weight of hatch end beams and girders will be a minimum when they are all subjected to their maximum allowable stress. With all parts made of steel this means that beams and girders must be subjected to the same maximum stress.

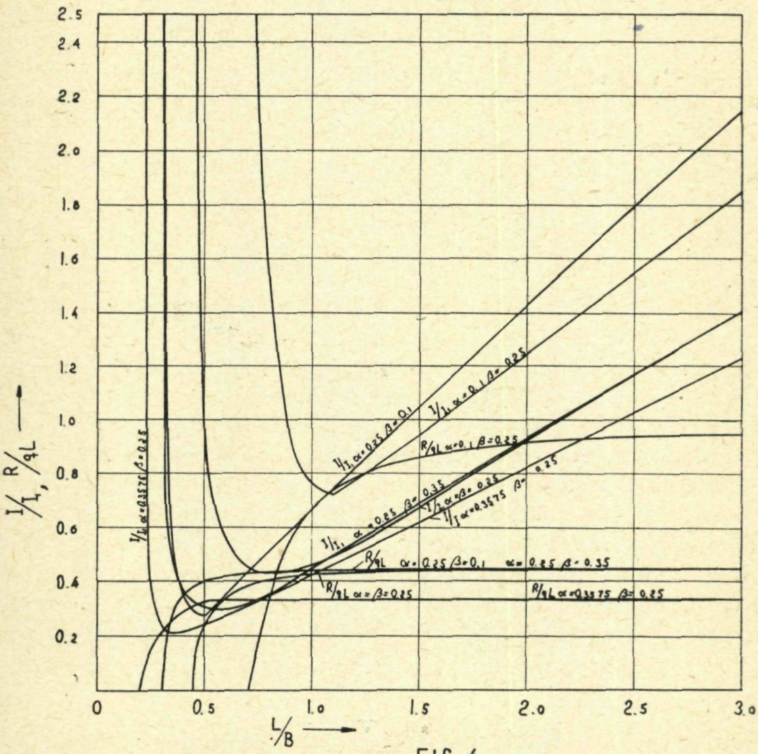

FIG. 4

In the diagram Fig. 4 curves have been plotted for ratios I/I_1 which must be chosen to obtain minimum weight, with L/B as abscissa and different values of α and β as parameters. The values have been obtained on the assumption that $c/c_1 = 1$, which means that beams and girders must not differ too much in size and design. A degree of fixity $f = f_0 = 2/3$ has been used for the curves.

Most of the curves for I/I_1 consist of two parts, one steeply rising part at low values of L/B, for which the maximum bending moment occurs at midlength of the girders, and one not quite so steeply rising, for higher values of L/B, for which the maximum bending moment of the girders occur at the hatch corners. The two parts meet at or near the minimum value of I/I_1 in accordance with equation (8). The last mentioned parts of the curves, which are roughly valid for L/B larger than unit and therefore will be of the greatest interest, are nearly straight lines. Their equation will be approximately

$$\frac{I}{I_1} = \frac{(3-14\alpha+12\alpha^2+27\alpha^3-30\alpha^4)\alpha}{(1-\alpha)(1+3\alpha-3\alpha^2)(1+4\beta)\ (1-2\beta)\beta}\ \frac{L}{B} \quad (11)$$

when $\beta > 0{,}217$, and

$$\frac{I}{I_1} = \frac{(3-14\alpha+12\alpha^2+27\alpha^3-30\alpha^4)\alpha}{2(1-\alpha)(1+3\alpha-3\alpha^2)(1+3\beta)\ (1-2\beta)^2\beta}\ \frac{L}{B} \quad (12)$$

when $\beta < 0{,}217$.

The ratio will be zero when $\alpha = 0{,}3575$. This is the reason why a curve for this value of α has been drawn. Already from $\alpha \geqslant 0{,}331$ we have, however, the case of equation (9), which means that the maximum bending moment of the girders will occur halfway between bulkhead and hatch corner and not at midlength of the hatch side for which equations (11) and (12) are valid, (11) when the maximum bending moment of the beams occurs at the pillar, (12) when it occurs at the hatch corners. For $\alpha \geqslant 0{,}331$ we have approximately

$$\frac{I}{I_1} = \frac{(-3+16\alpha-15\alpha^2-9\alpha^3+6\alpha^4)\alpha}{2(1-\alpha)(1+3\alpha-3\alpha^2)(1+4\beta)\ (1-2\beta)\beta}\ \frac{L}{B} \quad (13)$$

for large L/B.

Curves for R/qL have also been plotted according to equation (2) for the $m = I/I_1$ corresponding to minimum weight. R/qL is very nearly constant for values of L/B larger than unit. For $f_0 = 2/3$ we can write approximately

$$\frac{R}{qL} = \frac{(1-\alpha)(1+3\alpha-3\alpha^2)}{12\alpha(1-2\alpha^2)} \qquad (14)$$

for large L/B.

Frame and Beam Portal.

For the combination of an ordinary beam with adjacent frames it can again be shown that the minimum weight is obtained when beam as well as frames are designed for maximum allowable stress. The reason why such a combination is dealt with here is that the stiffness of each member of the framework influences the degree of fixity of the adjacent member. For designing purposes the framework must therefore be looked at as a whole.

The beam is subjected to a certain vertical load on the deck while the frames are subjected to horizontal water pressure from outside. If deck load and water pressure act simultaneously the deck load will reduce the stress in the frames and the water pressure the stress in the beam. To be able to stand the worst possible conditions the beam must therefore be designed to stand deck load only, without water pressure on frames (which may occur with a wave trough at these particular frames), and the frames must be designed to stand waterpressure only, with no load on deck.

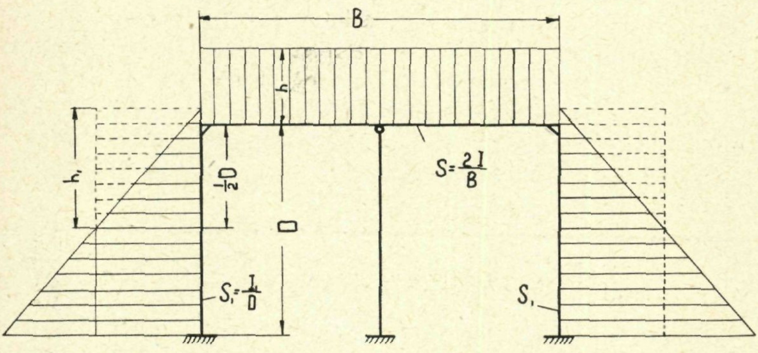

FIG. 5

Fig. 5 shows the framework under consideration. The numerical calculation here will be confined to one row of pillars. For the sake of simplicity we again assume the beam to have no deflection at the pillar. The deck load to consist of an evenly distributed cargo of height h and specific gravity ρ (say 0,72 for coal). The water pressure on the sides to reach some distance above deck. To get as simple expressions as possible the trapezoidal load on each frame is substituted by a rectangular load of the same total magnitude, i.e. by an evenly distributed load of height h_1 and specific gravity ρ_1 (say 1,025 for sea water). The midspan bending moment due to this substitution is slightly greater than the maximum positive bending moment due to the trapezoidal load. The frame spacing be a.

With these assumptions it is easy to show that the midspan bending moment on the frames will be

$$M_{D,2} = \frac{\rho_1 a h_1 D^2}{48} \cdot \frac{2S + 3S_1}{S + S_1}, \qquad (15)$$

and the maximum bending moment on the beam (at the pillar)

$$M_{B,2} = -\frac{\rho a h B^2}{96} \cdot \frac{3S + 2S_1}{S + S_1}, \qquad (16)$$

whe $S = 2\,I/B$ and $S_1 = I_1/D$ are the stiffnesses of beam and frame, respectively.

The maximum stresses are obtained by dividing these bending moments by the section moduli, which may again be written $W_1 = c_1 k_1 S_1$ and $W = ckS$, respectively. By putting $M_{D,2}/W_1 = M_{B,2}/W$ one finally obtains for the condition of minimum weight

$$m = I/I_1 = n/8[3(v-1) + \sqrt{9v^2 - 2v + 9}], \qquad (17)$$

where $v = \rho h n^2/(2\rho_1 h_1 \varepsilon\gamma)$, $n = B/D$, $\varepsilon = c/c_1$ and $\gamma = k/k_1$.

For convenience some numerical values of this equation are given in the table below :

$v =$ 0.25	0.5	0.75	1.0	2.0	3.0	4.0	5.0
$m/n =$ 0.0951	0.2127	0.3493	0.5	1.176	1.894	2.632	3.372

To start with one may use the value $\varepsilon\gamma = 1$ in the expression for v. The preliminary values of the section moduli may then be substituted in the correct expression $\varepsilon\gamma = nW/(2mW_1)$ and a revised value of m/n obtained.

A similar procedure as explained here may also be used for two and three rows of pillars, but the expressions will be more complicated.

Deck Panel with Lateral Load.

We consider a deck panel, say between two hatches and reaching from one ship's side to the other, with an evenly distributed vertical load $p = \rho h$ per square unit. The 'thwartship beams to have a length b, a moment of inertia I, a sectional modulus W, a cross-sectional area (without deck plating) F and a spacing $a = l/(n+1)$, where l is the longitudinal length of the panel and n the number of beams over this length. We again consider the case with one row of pillars, l actually being the longitudinal distance between two pillars or between a pillar and a bulkhead. A longitudinal girder is fitted in the line of the pillars, i.e. at midlength of the beams. It has a moment of inertia I_0 a sectional modulus W_0 and a cross-sectional area (without deck plating) F_0.

The maximum bending moment of the beams (at the pillar) is

$$M = \frac{6-5f}{4-3f} \cdot \frac{pab^2}{48} = j\ pab^2, \qquad (18)$$

where $j = (6-5f)/[48(4-3f)]$ and f is the degree of fixity of the beam ends. Alternatively one can use the equivalent equation (16) if some information is available about the ratio between the expected stiffnesses of beams and frames. From the maximum allowable stress $\sigma = M/W$ one gets W, picks a suitable beam and obtains I.

With a maximum allowable stress σ_0 in the girder one now computes the numerical value

$$u^2A_0 = c_0k_0\,\sigma_0 I(n+1)/(8qpb^4), \qquad (19)$$

(see reference 2), by means of which the value of u can be lifted from the curve of Fig. 6. Here $k_0 = lW_0/(c_0I_0)$ is the ratio between the span and height of the girder (say $k_0 =$ 15—30). $c_0 = 2$ for symmetrical sections, but here one may tentatively put say $c_0 = 1{,}15$ and afterwards adjust if

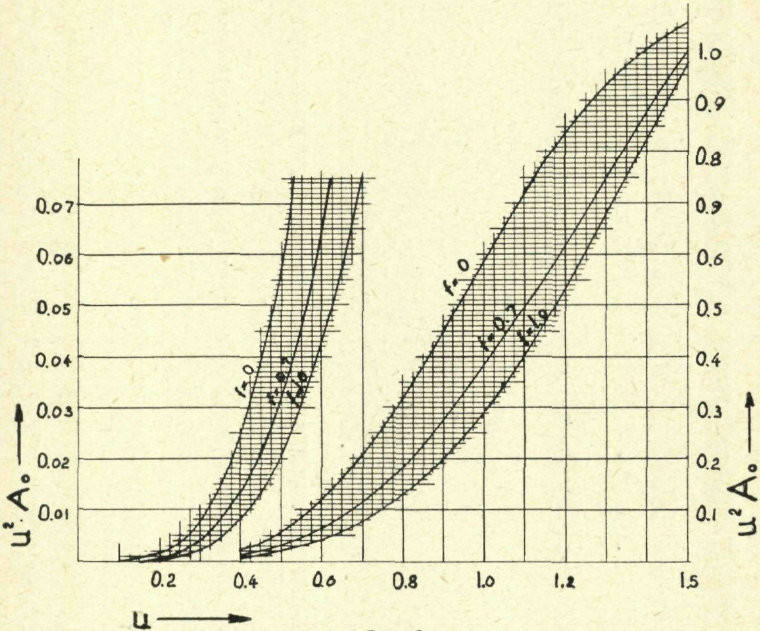

FIG. 6

no exact value is available. $q = (5-4f)/384$ is the factor in front of the expression for the deflection $d_0 = qplb^4/[(n+1)EI]$ which the beam would have had at midlength if there had been no pillar nor girder.

By means of u one finally finds

$$I_0 = (n+1)(l/b)^3 I/(gu^4), \tag{20}$$

where here with one girder $g = 4/3 - f$.

After this brief description of the method of calculation, explained more fully in reference [2], we shall consider the question of minimum weight. The total weight of the deck panel under consideration can be written

$$w = \rho(nbF + lF_0 + tbl), \tag{21}$$

where ρ is the specific gravity of the material (steel) and t is the thickness of the deck plating, which is assumed to be constant over the panel.

For the beams one may put $I = \lambda F^2$, where normally $\lambda = 2{,}5 - 3{,}5$. From $\sigma = M/W$ with $W = 2\,kc\,I/b$ one finally gets

$$F = \sqrt{\frac{jplb^3}{2kc\lambda\sigma(n+1)}}, \qquad (22)$$

where c equals about 1,9 and $k = 30$—40.

It can be shown that with sufficient approximation within the range of values of interest here $A/u^2 = (3-2f_0)/[0{,}33(6-5f_0)u^4+3]$. Hence $(A_0/u^2).u^4 = (3-2f_0)u^4/[0{,}33(6-5f_0)u^4+3]$.

By substituting $I(n+1)$ from $\sigma = jplb^3/[2kcI(n+1)]$ in expression (19) one obtains
$u^2A_0 = c_0k_0\sigma_0 jl/(16ck\sigma qb)$. Equating the two expressions for u^2A_0 gives
$u^4 = 3c_0k_0\sigma_0 jl/[16(3-2f_0)ck\sigma qb - 0{,}33(6-5f_0)c_0k_0\sigma_0 jl]$.

Dividing the last u^2A_0 by this u^4 one gets
$A_0/u^2 = 1-2f_0/3-0{,}11(6-5f_0)c_0k_0\sigma_0 jl/(16ck\sigma qb)$.

Substituting this and $I_0 = \lambda_0F^2_0$ (where normally $\lambda_0 = 7$–9)

$$\text{into } \sigma_0 = M_0/W_0 = \frac{epbl^2}{8W_0}\,A_0/u^2 = \frac{epbl^3}{8c_0k_0I_0}\,A_0/u^2,$$

where $e = (5-4f)/[2(4-3f)]$ varies between 5/8 and 1/2, one finally obtains (23)

$$F_0 = \sqrt{\frac{epbl^3}{8c_0k_0\lambda_0\sigma_0}\left[1-\frac{2f_0}{3}-\frac{0.11}{16}(6-5f_0)\frac{c_0k_0\sigma_0 jl}{ck\sigma qb}\right]}$$

f_0 is the degree of fixity of the girder ends.

For the maximum stress in the plating one can use the approximate expression $\sigma_1 = \frac{1}{2}p(a/t)^2$, which gives

$$t = a\sqrt{p/(2\sigma_1)} = l/(n+1)\sqrt{p/(2\sigma_1)}. \qquad (24)$$

Substituting (22), (23) and (24) in (21) we can write an expression for $w/(p^{1/2}bl^2)$ which is a function of n and l/b only, all other magnitudes being constants, at least within reasonable variations of u. By putting the derivative of this expression with respect to n equal to zero one finds that the weight expression, with sufficient approximation, will be a minimum when

$$n+1 = -2/3+(l/b)\sqrt[3]{\frac{4kc\lambda\sigma}{j\sigma_1}} \qquad (25)$$

If we choose as an example $k = 35$, $c = 1{,}9$, $\lambda = 3$, $j = 1/40$, $\sigma = \sigma_1$, this gives

$$n+1 = -2/3 + 31{,}7l/b, \tag{25a}$$

which is the straight line shown in the diagram Fig. 7. It may be of interest to note that the beam spacing $a = l/(n+1)$ equals $b/30$ for $l/b = \frac{1}{2}$ and $b/31$ for $l/b = 1$, i.e. nearly a constant proportion of the ship breadth.

The complete weight expression mentioned reads

$$\frac{w}{\rho p^{1/2} b l^2} = \frac{n}{\sqrt{n+1}} \sqrt{\frac{jb^3}{2ck\lambda\sigma l^3}} \tag{26}$$

$$+ \sqrt{\frac{el}{8c_0k_0\lambda_0\sigma_0 b}} \left[1 - \frac{2}{3} f_0 - \frac{0.11}{16} (6 - 5f_0) \frac{c_0k_0\sigma_0 jl}{ck\sigma qb} \right]$$

$$+ \frac{1}{(n+1)\sqrt{2\sigma_1}} .$$

The first term on the right hand side represents the total weight of the beams, the second term the weight of the girder and the third term the weight of the plating. Substituting (25a) one gets the minimum weight with the magnitude of the constants mentioned. $j = 1/40$ corresponds to $f = 6/7$. If in addition $f_0 = 0{,}5$, $k_0 = 20$, $c_0 = 1{,}15$, $\lambda_0 = 8$, $e = 9/16$, $q = 1/245$ and $\sigma = \sigma_0 = \sigma_1$ the percentages of the weights of beams, girder and plating for the minimum condition will be as given by the three curves drawn in Fig. 7 with the ratio l/b as abscissa. The curves represent points of minimum total weight for constant l/b and varying n.

The minimum of (26) with constant n and varying l/b will be obtained when approximately

$$(l/b)^2 = 59{,}2(-1 + \sqrt{1 + 3{,}17n}) \tag{27}$$

with the same numerical values of the constants as used above. This l/b is much greater than can be practically attained. If the problem of variation should occur in this way it may, however, be of value to know that the l/b should be taken as large as possible. This is of course due to the rôle played by the weight of the beams.

Returning to the more important problem of a panel of given dimensions bl and a possibility of varying the number of beams n it will be seen that equation (25a) gives a much shorter beam spacing and therefore according to equation

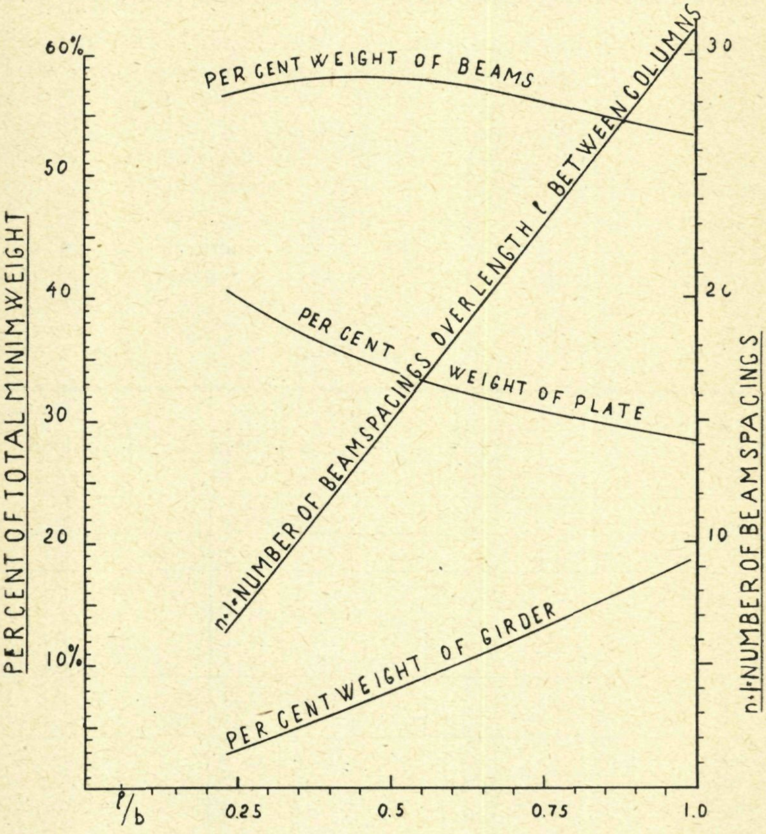

FIG. 7

(24) also a much thinner plating than usual according to the rules of the classification societies, especially for upper decks. This is due to our considering vertical load only. In reality the most important stress in an upper deck plating amidships is the horizontal normal stress in the longitudinal direction due to the longitudinal bending moment on the hull as a whole. This may be considered by using $\sigma_1 > \sigma$ in equation (25).

Deck Panel under Compression in the Longitudinal Direction.

H. L. Cox has dealt with this problem for decks stiffened by transverse beams only [3]. His investigation has been based on the assumption that the beams are hinged at both ends. The calculations are so complicated that it is difficult to draw general conclusions and may therefore not appeal to shipbuilders. Cox has, however, given a table with figures computed as an example for a ship breadth of 50 feet (= 15,2 meters). This table has been copied below with an extension to higher load and stresses more likely to be found in ships and figures given in metric units :

Table 1.

	Free width m.	Total weight kg/m2	o/o weight in beams	Plate thickness mm.	Beams Spacing cm.	Beams Sectional area cm2	Working stress kg/mm2
	Load 895 kg/cm. (= 5,000 lb./in.)						
Unstiffened plating	15.2	510	0	65.0	—	0	1.37
Min, weight with beams	»	310	29.6	27.8	198	232	3.22
Percentage beam	»	313	36.0	25.5	175	248	3.50
weight altered	»	320	42.1	23.6	152	261	3.79
from optimum	»	331	47.8	22.1	137	275	4.05
	»	313	23.1	30.7	229	210	2.92
Beam spacing reduced	»	334	34.8	27.8	107	158	3.22
from optimum	»	334	40.0	25.5	107	182	3.50
Min. weight with	7.6	202	28.6	18.4	107	78.8	4.87
reduced free	5.1	157	30.0	14.0	71	42.8	6.40
width	3.8	131	27.4	12.2	58	26.8	7.35
	2.5	102	28.7	9.3	38	14.6	9.63
	1.5	72	25	7.3	35	6.45	12.23
	Load 1790 kg/cm (= 10,000 lb/in)						
Min. weight	15.2	403	28.6	36.7	203	315	4.87
with different	7.6	263	27.4	24.3	117	107	7.35
free widths	5.1	205	28.7	18.6	76	57.3	9.63
	3.1	144	25	14.6	70	25.8	12.23
Unstiffened plating	15.2	642	0	81.7	—	0	2.20
	3.1	220	0	28.0	—	0	6.42

It will be noticed that minimum weight is obtained with about 30 per cent of the weight in the beams and that there will be very little additional weight with considerable alteration of this percentage if the area and spacing of the beams is altered in conformity with the table. Another remarkable thing, seen from the last column of the table, is that the high stresses occuring in ships cannot be obtained with minimum weight unless the free width of the plating is considerably reduced, say to about 2 meters with a load of 900 kg/cm and about 4 meters with a load of 1.800 kg/cm. This means that transverse stiffening of decks is not efficient unless it is combined with some longitudinal stiffening which reduces the free widths of plating.

The procedure for obtaining minimum weight can be considerably simplified in order to make it more suitable for general use. This is possible by making use of the principle that the most efficient designs are those in which failure occurs simultaneously in all possible buckling modes [4]. It is equivalent to the principle of maximum stress occurring simultaneously in all members of a structure subjected to bending by a lateral load, as used previously in this paper.

The buckling stress of a wide strip of plating between two beams is

$$\sigma = 0{,}905\, E\, (t/a)^2. \tag{28}$$

Simultanous buckling of transverse beams is obtained by giving them a moment of inertia

$$I = \lambda F^2 = b^4 t^3/(43{,}7\, a^3), \tag{29}$$

as explained in reference [5]. In a stability problem like this the structural index is not stress but p/a, as for wide columns [7], [8], where the load per unit width p is obtained by multiplying the stress of equation (28) with the plate thickness t. Relative weight only being of interest the factor of safety may, for the sake of brevity, be taken as equal to unit.

The weight is proportional to the mean sectional area per square unit, which is again equivalent to a mean thickness $t_m = t + F/a$. Substituting from (28) and (29) one finally gets

$$t_m = a \left(\frac{p}{0{,}905\, Ea}\right)^{1/3} \left[1 + (b/a)^2 \left(\frac{p}{0{,}905\, Ea}\right)^{1/6} (43{,}7\lambda)^{-1/2}\right] \tag{30}$$

where the first term in the bracket represents the plating and the second term the beams. Making the derivative with respect to a equal to zero it is found that (30) has a minimum value when the beam spacing

$$a = \left[\frac{9}{4} \; b^2 \left(\frac{p}{0.905\ E}\right)^{1/6} (43.7\lambda)^{-1/2} \right]^{6/13} \tag{31}$$
$$= 0.61278\ \lambda^{-3/13}\, b^{12/13}\, (p/E)^{1/13}.$$

Substituting in (30) we find the minimum mean thickness with 4/13 or 30,8 per cent. of the weight in the beams (*).

$$(t_m)_{min} = \frac{13}{9} \left[\frac{9}{4} \; b^2 \left(\frac{p}{0.905\ E}\right)^{5/4} (43.7\lambda)^{-1/2} \right]^{4/13} \tag{32}$$
$$= 1.4406\ \lambda^{-2/13}\, b^{8/13}\, (p/E)^{5/13},$$

With $\lambda = 5$ as used in the example by Cox the two equations read

$$a = 0{,}42267\ b^{12/13}\ (p/E)^{1/13} \tag{31a}$$
$$(t_m)_{min} = 0{,}84103\ b^{8/13}\ (p/E)^{5/13} \tag{32a}$$

The two load values used in the table from Cox correspond to p/E = 1/6.000 and 1/3.000 in., or 0,000423 and 0,000846 cm. respectively. The values obtained in the equation (31) and (32) correspond surprisingly accurately with the values in the table and can easily be used much more generally for any free width b and any load p.

A similar procedure can be used when we now consider a deck panel of length l stiffened by longitudinal girders only with a spacing r. The unstiffened plate between two girders will have a critical load per unit length

$$p = \sigma t = \pi^2 E t^3/(2{,}73\, r^2). \tag{33}$$

A simultaneous buckling of the girders will occur if they have a moment of inertia

$$I_1 = \lambda_1 F^2{}_1 = t^2 l^2\ (tr + F_1)/(2{,}73\, r^2), \tag{34}$$

(*) If in eq. (29) we write $I = \Psi \cdot F^n$ we find by a similar procedure that for minimum weight $2n/(5n+3)$ of the total weight should be in the beams. For $n = 3$ this means that the beam weight should be 1/3 or 33,3 % of the total. $n = 3$ undoubtedly is an extreme figure. The departure from the previous percentage (for $n = 2$) being small we can safely conclude that minimum weight is obtained with 31 or 32 per cent of the weight in the beams.

see reference [6]. The latter equation gives

$$F_1 = \frac{t^2\, l^2}{5.46\lambda_1\, r^2}\left(1 + \sqrt{1 + \frac{10.92\lambda_1\, r^3}{tl^2}}\right). \qquad (35)$$

t and F_1 from equations (33) and (35) are now substituted in the expression for the sectional area per square unit or mean thickness, giving

$$t^1_m = t + F_1/r = \left(\frac{2.73\,p}{\pi^2 E}\right)^{1/3} r^{2/3} + \qquad (36)$$

$$\frac{l^2}{5.46\lambda_1}\left(\frac{2.73\,p}{\pi^2 E}\right)^{2/3} r^{-5/3}\left[1 + \sqrt{1 + \frac{10.92\lambda_1}{l^2}\left(\frac{\pi^2 E}{2.73p}\right)^{1/3} r^{7/3}}\right]$$

From making the derivative with respect to r equal to zero we find that minimum mean thickness is obtained when the girder spacing equals (37)

$$r = \left[\frac{2.73p}{\pi^2 E}\left(\frac{15l^2}{14.56\lambda_1}\right)^3\right]^{1/7} = 0.84295\lambda_1^{-3/7}\, l^{6/7}\,(p/E)^{1/7}.$$

Substituting this in (49) we find the minimum mean thickness

$$(t^1_m)^{min} = 1.0465\left(\frac{l^2}{\lambda_1}\right)^{2/7}(p/E)^{3/7} \qquad (38)$$

with 44,4 per cent. of the weight in the girders.

With $\lambda_1 = 5$ the formula for the girder spacing with minimum weight will be

$r = 0{,}4229\, l^{6/7}\,(p/E)^{1/7}$ (37a)

and the minimum mean thickness will be

$$(t^1_m)_{min} = 0.66074\, l^{4/7}\,(p/E)^{3/7}. \qquad (38a)$$

Having obtained this result it is interesting to compare the minimum weight of a deck with 'thwartship beams with the minimum weight of a deck with longitudinal stiffeners (beams, girders or whatever they may be named). The ratio between the weights equals the ratio between the mean thicknesses given by equations (32) and (38), viz.

$$\frac{(t_m)_{min}}{(t'_m)_{min}} = 1.3766\,\frac{\lambda_1^{2/7}}{\lambda^{2/13}}\left(\frac{b}{l}\right)^{7}\left(\frac{bE}{p}\right)^{4/91} \qquad (39)$$

With $\lambda = \lambda_1 = 5$ we obtain the following values of this ratio :

Table 2.

Ratio between minimum weights of transversely and longitudinally stiffened decks.

b/l \ bE/p	5.10^5	10^6	2.10^6	3.10^6	4.10^6	5.10^6
0.5	2.04	2.10	2.17	2.21	2.23	2.26
1.0	3.03	3.12	3.22	3.28	3.32	3 35
1.5	3.82	3.94	4.06	4.13	4.18	4.26
2.0	4.50	4.64	4.78	4.87	4.93	4.98
2.5	5.11	5.27	5.43	5 53	5.60	5.65
3.0	5 68	5.85	6 03	6.14	6.22	6.28

It will be noticed that the minimum weight ratio mainly depends upon the ratio b/l between the width and the length of the hold. For b/l = 0,5 the transversely stiffened deck will be twice the weight of the longitudinally stiffened deck, for b/l = 1,0 it will be three times the weight of the latter, etc. For bE/p = 10^6 transversely and longitudinally stiffened decks wille have the same minimum weight when b/l = 0,136 or l/b = 7,34.

It should, however, be remembered that the deduction here, as mentioned in connection with reference [3], is based on the assumption of freely supported beams and girders. It can be shown that the right hand side of equation (29) must be multiplied by a factor

$$k = \frac{1 + 1{,}32\,m}{1 - 1{,}48\,m} = \frac{3 - 2{,}32\,f}{3 + 0{,}48\,f}$$

if the beam ends for a transversely stiffened deck have a degree of fixity f or a carry-over factor m as defined in reference [5]. This means that the right hand side of equation (31) for the beam-spacing must be multiplied by

$$k^{3/13}$$

and the right hand side of equation (32) for the minimum mean thickness, which is proportional to the minimum weight, must be multiplied by

$$k^{2/13}$$

As an example may be mentioned that if $k = 0,5$, which corresponds to $f = 0,586$,
$k^{3/13} = 0,8522$ and $k^{2/13} = 0,8989$.

The values in table 2 must then be multiplied by the latter figure. This means e.g. that for $bE/p = 10^6$ transversely and longitudinally stiffened decks will have the same minimum weight when $b/l = 0,164$ or $l/b = 6,09$.

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Intervention de M. DIEUDONNE.

Le Mémoire que nous a présenté M. le Professeur Vedeler est extrêmement intéressant puisqu'il nous montre, sur des exemples précis, les moyens de diminuer le poids de la charpente à égalité de résistance ou, ce qui revient au même, d'augmenter la résistance à égalité de poids.

La remarque que je désirerais présenter ne vise pas directement le texte du Mémoire, mais me paraît cependant s'y rattacher puisqu'elle pourrait être comprise sous le titre de ce mémoire.

Les calculs de résistance s'appliquent à des éléments satisfaisant aux conditions de la résistance des matériaux et, en particulier, toutes les poutres sont supposées continues. Dans un bâtiment réel il existe toujours des discontinuités dues en particulier aux attaches des éléments partiels et aux traversées d'éléments différents et ces discontinuités donnent lieu à des concentrations d'efforts, c'est-à-dire à des points faibles. Des expériences de laboratoire ont été faites à ce sujet dans divers pays et des résultats ont été publiés aux Etats-Unis et en France. Ils font ressortir que des concentrations d'efforts atteignant ou dépassant 2 sont courantes dans les charpentes réelles. Je crois qu'on pourrait obtenir des résultats très importants en ce qui concerne la résistance vraie d'une charpente de poids donné ou, ce qui revient au même, la réduction de poids d'une charpente de résistance donnée en s'attachant à l'étude des dispositions constructives de détail qui seraient susceptibles de diminuer l'importance de ces concentrations.

Dr. J. M. MURRAY.

This paper treats in a very convincing way the important subject of minimum weights of structure. Since, as has been stated recently (« Ships Structures — A Century of Progress » by R. B. Shepheard Esq., C.B.E., B.Sc., International Conference of Naval Architects and Marine Engineers 1951) compared with earlier years, there remains little scope for reduction in the structural weight of steel ships, except by the most effective use of welded design, it is evident that the savings within the limited scope now available will only be obtained by the application of such methods as are detailed in this paper.

Confirmation of this point of view is given in Fig. 4 of the paper dealing with the total weight of hatch end beams and girders. It will be remarked that on the basis of the assumptions made for a normal hold, a common minima exists for quite large differences in the positions of transverse and longitudinal members.

Perhaps one of the most interesting features of the paper is the exposition given in the section on welded deck panel under compression in the longitudinal direction. Here, it is shown very clearly that to obtain a reasonable efficiency of structure in regard to weight, it is necessary to adopt longitudinal framing. This is important, as from the point of view of main structural strength also, longitudinal framing has clearly many advantages over transverse framing.

It may be observed that in the course of the development of steel shipbuilding, solutions to problems have been made by the practical shipbuilder which have later been found to agree with theoretical considerations. It will be found in many cases that the arrongement of scantlings adopted in a ship does in fact give the minimum weight shown in this Table; that, of course, should not constitute a reason for avoiding the investigation of the problem.

Réponse du Prof. Ir. G. VEDELER.

If the weight expression is covered by a single curve which has a mathematical minimum, the fact that the curve has a horizontal tangent at the minimum point, implies that it is rather flat in this neighbourhood, wherefore comparatively large variations can be made to the structural parts without affecting the weight very much. But the example dealing with hatch end beams and girders is not such a case. Fig 4 gives the ratio between moments of inertia necessary to obtain the least possible weight, but the weight itself has not been given, simply because the weight expression was considered too involved to be quoted in such a short and simple paper. Actually there are two weight expressions, one with constant maximum stress (say equal to the maximum permissible stress) in the hatch end beams and varying stress in the girders, and another with constant stress in the hatch end beams. None of the expressions has a mathematical minimum, but the least possible weight is obtained at the point of intersection of the two curves, i.e. when the stress has reached its maximum permissible value in the hatch end beams as well as in the girders. But the weight rises rather quickly with any departure from this condition. The ratios α and β, giving the positions of beams and girders, have been considered constant in the treatment of this problem.

With regard to the deck panel in longitudinal compression it may be correct that common practice may give a weight not far from the minimum values of Table 1. But this table is concerned only with transverse stiffening, which is in itself not a very efficient design. When the practical man has changed over to longitudinal stiffening, he seems, however, to have used more or less the same ratio between the weights of plating and stiffeners as long experience with transverse stiffening had shown to be acceptable. My investigation shows that to get most efficient panel with longitudinal stiffening, a considerably larger part of the weight should be in the stiffeners than is the case with transverse stiffening. This has the advantage that with longitudinal stiffening the thickness of the plating can be reduced, which should be very welcome in large welded tankers.