

## Numerical comparison of sediment transport formulae

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### Abstract

Near coast sediment transport is a challenge due to the complexity of wave-current interaction, but also because of the variety of acting phenomena (breaking waves, undertow ...). Most transport formulae have been calibrated with experiments, but have a restricted range of application. Nonetheless, these formulae are often used beyond these ranges. The aim of this paper is to point out the limits of four of these formulae : the Engelund-Hansen, Bijker, Bailard and Dibajnia-Watanabe formulations. The sensitivity of the formulae to wave orbital velocity, wave period, sediment grain size and water flux has been studied. It appears that they behave in very different ways if one of these parameters is slightly modified.

### Introduction

Many different formulae are found to estimate sediment transport on the beach. These formulae, based on theoretical or experimental study have mostly an energetic approach (cf. Bagnold, 1966) or a probabilistic approach (cf. Einstein, 1950). According to the authors, their formulae give the best result for the studied case. The problem is that there are as many experiments or case studies as formulae ! Thus, it is interesting to compare the dependence of these formulae with the main parameters of a classical test case.

#### 1- The different sediment transport formulae

We chose four formulae interesting for their different approach of the problem. The first one is the Engelund-Hansen formula (1972) modified by Cholley and Cunge (1980) to take into account different types of transport according to the flow regime. Designed for a river environment, this formula is not adapted for a coastal environment but give a good reference as a comparison with the others formulae for steady currents.

$$Q_s = \frac{0.05}{1-n} \sqrt{\frac{(s-1)d^3}{g}} K^2 h^{1/3} \tau_*^{5/2}$$

where  $d$  is the sand grain diameter,  $K$  the Strickler coefficient,  $n$  the porosity,  $s$  the relative density of the sediment ( $=2.65$ ),  $g$  the acceleration due to gravity,  $h$  the water height and  $\tau_*$  is non-dimensional bed shear stress :

$$\tau_* = 0 \quad \text{si } \Psi < 0.06 \quad \text{no transport,}$$

$$\tau_* = \sqrt{25(\Psi - 0.06)} \quad \text{si } 0.06 < \Psi < 0.384 \quad \text{dune regime,}$$

$$\tau_* = 1.065\Psi^{0.176} \quad \text{si } 0.384 < \Psi < 1.08 \quad \text{transition regime,}$$

$\tau_* = \Psi$  si  $1.08 < \Psi$  sheet flow regime,

where  $\Psi = \frac{0.5C_f \bar{u}^2}{(s-1)gd}$  is the Shields parameter,  $C_f = 2 \left( \frac{0.4}{1 + \ln(z_0/h)} \right)^2$  the friction coefficient,  $z_0$  the bed roughness length ( $z_0 = d_{50}/12$  for hydrodynamically rough flow) and  $\bar{u}^2$  the average velocity over the depth.

The second one is the Bijker formula (1968). It is the first transport formula for combined waves and currents conditions, but is still widely used. It is just a modified formula for current alone by Frijlink using a new bed shear stress through a wave-current interaction model.

$$Q_s = \frac{5d}{1-n} \sqrt{\frac{\mu\tau_c}{\rho}} \exp\left(-0.27 \frac{(\rho_s - \rho)gd}{\mu\tau_{cw}}\right) \left( 1 + 1.83 \left[ I_1 \ln\left(\frac{\sqrt{33}h}{\delta}\right) + I_2 \right] \right)$$

where  $\mu$  is the ripple factor,  $\tau_c$  the bed shear stress due to current alone,  $\tau_{cw}$  the bed shear stress due to the combined wave and current,  $\rho_s, \rho$  the density of sand and water,  $I_1, I_2$  the integrals of suspended sediment from bottom to water level and  $\delta$  the bedload thickness. The second part of the formula correspond to suspended load effect based on Einstein's works (1950).

The third one is the Bailard formula (1981), one of the most used methods. It was developed from the energetic approach proposed by Bagnold (1966). It takes into account wave effects, averaging over several wave periods. The formula can be simplified as follows :

$$\bar{Q}_s = \frac{C_f}{g(s-1)(1-n)} \left( \frac{\varepsilon_c}{tg\phi} \langle |\bar{u}|^2 \bar{u} \rangle + \frac{\varepsilon_s}{W_s} \langle |\bar{u}|^3 \bar{u} \rangle \right)$$

where  $\bar{u}$  is the combined wave plus current bed velocity,  $W_s$  the settling velocity,  $\varepsilon_c, \varepsilon_s$  the bedload and suspended load efficiency ( $\varepsilon_c = 0.2, \varepsilon_s = 0.025$  according to Soulsby, 1995). The first term corresponds to bedload transport and the second term to suspended load transport.

The last one is the Dibajnia-Watanabe formula (1992). It has an interesting approach, decomposing the sediment transport into two half cycles due to waves (see figure 1). In the first cycle, sediments move in the wave direction ; and in the second one, they move in the opposite direction. Therefore, it can take into account wave asymmetry effects on sediment transport.

*Fig 1 : Near bottom velocity profile in the direction of propagation of the waves (Dibajnia and Watanabe, 1992).*

*The volumetric load is then :*

$$\bar{Q}_s = 0.001 \frac{W_s d \bar{\Gamma}}{1-n} \Gamma^{-0.55} \quad \text{with} \quad \bar{\Gamma} = \frac{T_c(\Omega_c^3 + \Omega_t'^3)\bar{u}_c + T_t(\Omega_t^3 + \Omega_c'^3)\bar{u}_t}{(u_c + u_t)T}$$

where  $T_c$  and  $T_t$  are the half periods of wave taking into account the influence of a current (see figure 1),  $\Omega_c$  and  $\Omega_t'$  the amount of sediment in movement during the half periods  $T_c$  and  $T_t$ ,  $\Omega_c'$  and  $\Omega_t$  the amount of sediment in movement after the half periods  $T_c$  and  $T_t$ ,  $\bar{u}_c$  and  $\bar{u}_t$  the

quadratic velocities (wave + current) over each half period,  $w_j = \frac{u_j^2}{2(s-1)gW_s T_j}$  the phase-lag parameter with  $j$  is  $c$  or  $t$ ,  $w_{cr}$  the critical parameter ( $0.03 < w_{cr} < 1$  in accordance to Shields parameter due to wave-current interaction).

$$\text{if } w_j \leq w_{cr} \text{ then } \Omega_j = 2w_j W_s T_j / d$$

$$\Omega_j' = 0$$

$$\text{if } w_j > w_{cr} \text{ then } \Omega_j = 2w_r W_s T_j / d$$

$$\Omega_j' = 2(w_j - w_r) W_s T_j / d$$

## 2. Numerical comparisons

We have compare these four formulae for different cases. The hydrodynamic and sedimentary characteristics are chosen to correspond to a flume experiment. Then, we have :

$$\text{for all cases : } d = 310^{-4}m, n = 0.4, K = 50m^{1/3}s^{-1}, h = 0.6m,$$

$$\text{for cases without waves : } Q = 0.25m^2s^{-1},$$

$$\text{for cases with waves : } Q = 0.1 m^2s^{-1}, T_w = 1.5s, u_w = 1ms^{-1}$$

The wave parameters  $H_w$  and  $T_w$  control more or less directly the wave velocity  $u_w$  at the bottom and the friction coefficient due to the waves  $f_w$ . We compute the new Shields parameter :

$$\Psi_w = \frac{0.5f_w u_w^2}{(s-1)gd}$$

$$\text{with } f_w = \min \left( \exp \left[ -6 + 5.2 \left( \frac{A_w}{2.5d} \right)^{-0.19} \right]; 0.3 \right) \quad (\text{Nielsen, 1992}) \text{ and } u_w = \frac{\pi H_w}{T_w sh(kh)},$$

where  $k$  is the wave number and  $A_w = \frac{H_w}{2sk(kh)} = \frac{T_w u_w}{\pi}$  the wave excursion.

From the procedure given by Soulsby (1997), we find the Shields parameter under the coexistence of waves and currents. At last, for the computation of quadratic velocity of the Dibajnia-Watanabe formula and the terms  $\langle |\vec{u}|^2 \vec{u} \rangle$  and  $\langle |\vec{u}|^3 \vec{u} \rangle$  of the Bailard formula, we suppose an interaction of a steady current and a linear and sinusoidal wave. Moreover, we suppose that the directions of the current and the wave are the same.

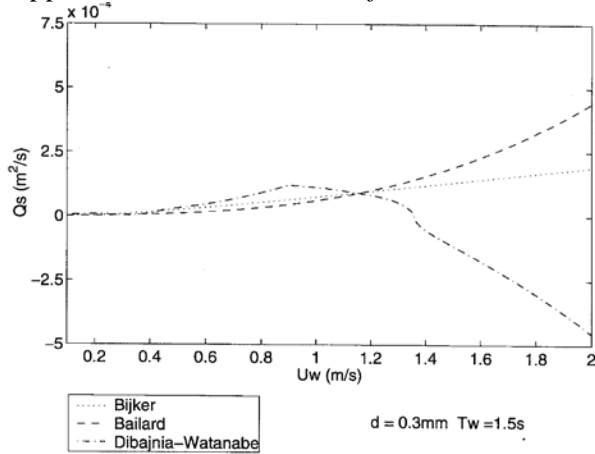
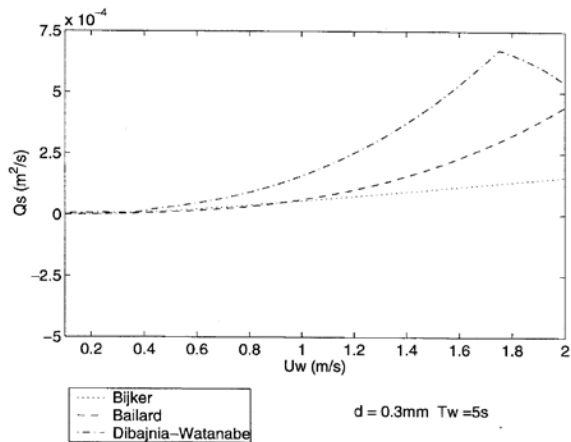
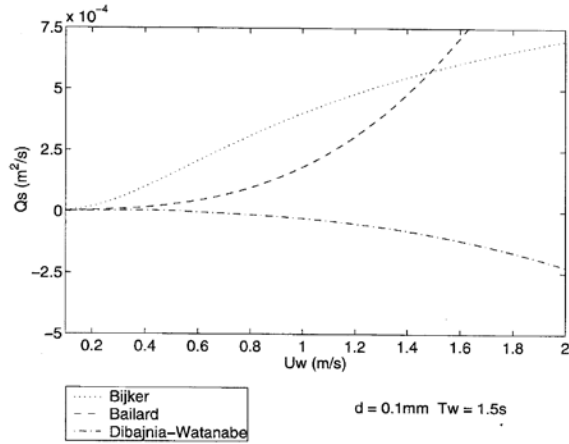


Fig 2 : Effects due to wave orbital velocity.

We can observe on figure 2 that the Bailard and Bijker formulations show an increasing sediment transport with an increasing wave velocity which seems to be logical as energy of wave is directly proportional to the square orbital velocity. But for high sheet flow ( $u_w > 1.5\text{m/s}$ ), the Bailard formula gives a transport rate twice bigger than the Bijker formula when it predicts smaller values for orbital velocity down to 1m/s. On the other hand, the Dibajnia-Watanabe formulation predicts a decrease for values of velocities above 1m/s (sheet flow) and even a negative sediment transport (i.e. against the direction of the current). This behavior corresponds to the phase-leg effect predict by the formula when the phase-lag parameter is greater than 0.1. This has been observed in situ and it is therefore one of the great interests of this formula. For little values of  $u_w$  ( $0.2 < u_w < 0.4$ ), the Dibajnia-Watanabe formula gives yet an other slight decrease which corresponds to the moment where the orbital velocity becomes larger than the flow velocity and then, a negative part of the sediment flux appears.





Figures 3a and 3b show the influence of a smaller grain sand diameter or a larger wave period on the previous graphs (i.e. sediment rate function of wave orbital velocity). The first one increases the effects observed previously : a negative transport rate for smaller value of  $u_w$  for the Dibajnia-Watanabe formula, a general increase of the sediment transport rate for the Bailard and Bijker formulae. The wave period has only an influence on the Dibajnia-Watanabe formula (opposite effects than grain size effects). This can easily be explained by the fact that the influence of wave period on these formula only

appear through the wave friction factor which is a quite small effect.

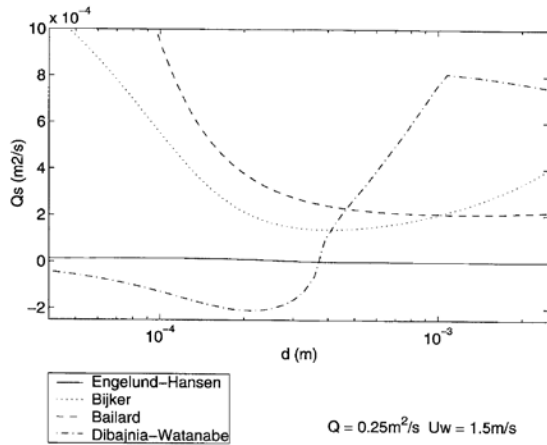


Fig 3 : Influence of grain size (a) and wave period (b) on effects due to wave orbital velocity.

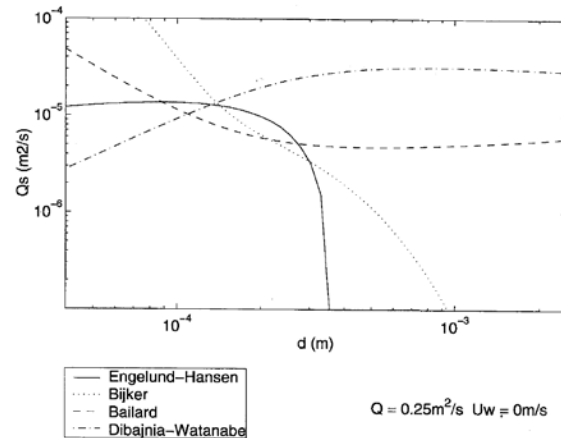


Fig 4 : Influence of grain sand parameter without (a) and with (b) wave effects.

The influence of grain sand diameter give also interesting results (see figure 4). It shows the part of empiricism or at least, the limitation of all these formulae due to the lack of experimental data. In that way, the formulae give quite the same value for classical grain size ( $0.3 < d < 0.4\text{mm}$ ) without waves and have great differences of behavior for other values. Thus, if the Bailard and Bijker formulae predict an increasing sediment rate for finer sediment, the

Engelund-Hansen and Dibajnia-Watanabe have an inverse behavior. Supposing that fine non-cohesive sediment is more easily subject to suspension, the first behavior seems to be more realistic. Moreover, for great value of grain size, sediment rate must logically stop. The Bailard and Dibajnia-Watanabe predict an increase or a maximal rate for these values. On the other

hand, the Bijker and Engelund-Hansen estimate the limit of no-transport for very small values of grain size ( $d < 1\text{mm}$ ), that does not seem much realistic!

Following the range of value given by Cholley and Cunge for the Shields parameter, we can in that way see the influence of water flux on these formulae from the no transport regime to the sheet flow regime. The first observation we can make from the figure 5 is that all these sediment transport formulae follow the proportionality which seems to be accepted nowadays :  $Q_s \propto \bar{u}^3$ , where  $u$  is the velocity of the current. Nevertheless, they do not give the same results. We note a factor 10 between the Dibajnia-Watanabe and the Bijker or the Bailard formulae. The discontinuity of the Engelund-Hansen formula curve is simply due to the four different expressions of the non-dimensional bed shear stress  $\tau^*$  following the different type of flow regime.

We can also see in figure 5b that a presence of waves does affect this proportionality. For the low values of current velocity, wave velocity is no longer negligible and creates an increase of the sediment rates.

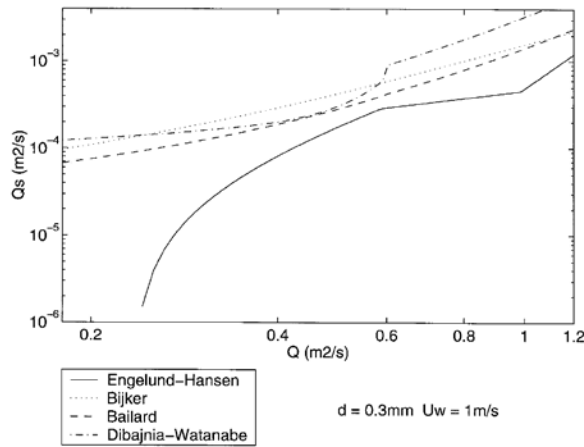
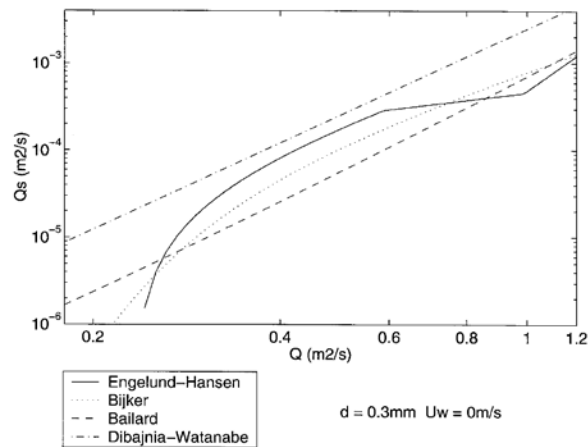


Fig 5 : Influence of water flux without (a) and with (b) wave effects.



### Conclusion

As a conclusion, we can resume results obtained with these simple numerical tests with this table :

| formula                   | Engelund-Hansen          | Bijker                   | Bailard            | Dibajnia-Watanabe                                 |
|---------------------------|--------------------------|--------------------------|--------------------|---|
| experimental range of $d$ | $0.2 < d < 0.4\text{mm}$ | $0.2 < d < 0.9\text{mm}$ | -                  | $0.2\text{mm}$                                    |
| exp. kind of regime       | currents only            | little waves             | littoral processes | sheet flow (waves)                                |
| $Q_s \propto f(u_w)$      | -                        | $f$ increasing           | $f$ increasing     | $f$ increasing / decreasing with phase-lag effect |

|                                |               |                 |              |                                 |
|--------------------------------|---------------|-----------------|--------------|---------------------------------|
| $Q_s \propto f(T_w)$           | -             | no              | no           | f decrease the phase-lag effect |
| $Q_s \propto f(d)$             | f decreasing  | f decreasing    | f increasing | f increasing                    |
| $Q_s \propto u^n$ (total load) | $1 < n < 4.5$ | $3.0 < n < 3.5$ | $n = 3.5$    | $n = 3.2$                       |

If for academic cases, all these formulae estimate similar results, they differ a lot rapidly if some parameters are slightly changed. Grain is a typical example : the four formulae shows four different behavior! The Bailard, Bijker and Dibajnia-Watanabe formulae are very sensitive for fine diameters, the two first ones giving easily a high transport rate and the last one a negative transport rate. If all these formulae have same behaviors face to current velocity (near to the power three), the order of magnitude can differ from one to ten from the Bijker formula to the Dibajnia-Watanabe formula. At last, big differences can be observe for the prediction of transport rate for sheet flow due to waves. If the Dibajnia-Watanabe formula predicts high negative values (that seems to be the reality), the Bailard formulation predicts high positive values.

These few tests show that sediment transport formulae are often adapted to a little part of sediment processes. Then, it is important to use them very cautiously, avoiding using a formula for a domain it is not prepared for. At last, it could be interesting to make an experiment to compare these formulae for less academic cases.

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