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Key Points:

- Mesoscale quasi-Stokes flow and buoyancy (Gent-McWilliams) diffusivity are estimated observationally
- The estimates are rooted in the idea that mesoscale eddies diffuse PV, rather than buoyancy
- These provide real-world constraints for mesoscale eddy parameterizations and eddy-resolving models

Supporting Information:

Supporting Information may be found in the online version of this article.

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Global Observational Estimates of Mesoscale Eddy-Driven Quasi-Stokes Velocity and Buoyancy Diffusivity

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Abstract Mesoscale eddies are a dominant reservoir of kinetic energy in the ocean and play an important role in the transport and stirring of tracers such as heat and carbon. These effects need to be parameterized in most ocean models. A number of studies have observationally quantified eddy effects, but most of these have focused on the diffusivity of passive tracers (Solomon-Redi diffusivity). However, passive tracer diffusivity is not directly informative about the quasi-Stokes transport, since this requires estimates of the buoyancy diffusivity (also known as the Gent-McWilliams diffusivity). Here we obtain the first global observational estimates of the quasi-Stokes transport, assuming that mesoscale eddies diffuse potential vorticity (PV) rather than buoyancy. These are also used to estimate the corresponding buoyancy diffusivity, which is what is needed in coarse-resolution ocean models employing the Gent-McWilliams parameterization. These observational estimates may be used as constraints for future development and tuning of mesoscale parameterizations.

Plain Language Summary Large swirls in the ocean, called mesoscale eddies, play an important role in the distributing dissolved ocean properties such as heat and carbon and mass. The scales of these eddies are too small to be resolved for most computer simulations of ocean and climate. Consequently the effects of these eddies need to be prescribed by parameterizations, that is, simplifications depending on variables that can be calculated. A number of studies have calculated the impact of these eddies using observations, but mostly for their effect to redistribute dissolved material. However, the impact of these eddies on transporting mass is less well studied from observations. In this study we do estimate their impact on mass transport by means of a velocity and coefficients that represents their strength in parameterizations. These estimates can be used as observationally based constraints to improve parameterizations that describe the effects of mesoscale eddies in numerical simulations of the ocean and climate.

1. Introduction

Tracers in the sea, such as heat, oxygen, or carbon, are transported by advection and diffusion, which are a combined result of the large scale slowly varying flows and the smaller and faster scale turbulent flows. In the ocean, mesoscale eddies are the largest reservoir of the turbulent kinetic energy (Ferrari & Wunsch, 2009), and consequently play a central role in tracer transport (de Lavergne et al., 2022). Firstly, by their action on ocean density, in the process of release of available potential energy (APE) via baroclinic instability (Smith, 2007), mesoscale eddies contribute an advection at the large scales (Gent & McWilliams, 1990; Gent et al., 1995; McDougall & McIntosh, 2001). This contribution, known as the eddy-driven velocity or the *quasi-Stokes transport*, combined with the transport by the large scale *Eulerian mean flow* forms the *residual circulation* (Andrews et al., 1987; McDougall & McIntosh, 2001; Young, 2012). In addition to contributing to the residual circulation, mesoscale eddies also transport tracers by stirring along isopycnals (Abernathey et al., 2021; Redi, 1982).

The quasi-Stokes transport is a close cousin of along-isopycnal bolus transport, and arises due to adiabatic stirring of the density field (McDougall, 1998). It has been recognized for over three decades that a skillful parameterization of the quasi-Stokes transport is essential for producing a realistic ocean state and tracer transport in coarse resolution models (Danabasoglu & McWilliams, 1995; Kuhlbrodt et al., 2012; Viebahn & Eden, 2010). Numerous studies have since been dedicated to deciphering the theoretical nuances, and sensitivity to the implementation details and parameter choices (e.g., Ferrari & Ferreira, 2011; Ferrari et al., 2008; Gent, 2011; Treguier et al., 1997; Visbeck et al., 1997). However, observational quantification and constraints on quasi-Stokes transport is extremely sparse (Katsumata, 2016; Sévellec et al., 2019).

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Most observationally based quantifications of the effects of mesoscale eddies have focussed on estimating the *passive tracer diffusivity* due to stirring. These include estimates derived from Argo hydrographic profiles (Cole et al., 2015); drifter and float trajectories (Balwada et al., 2016; Roach et al., 2018); tracer concentrations (Ledwell et al., 1993; Polzin et al., 1997; Tulloch et al., 2014); and inverse methods (Groeskamp et al., 2017; Hautala, 2018; Kusters et al., 2024; Mackay et al., 2018; Zhang & Hogg, 1992; Zika et al., 2010). Recently, Groeskamp et al. (2020) combined elements from a few decades of theoretical and observational advances, and derived full-depth global estimates of mesoscale eddy diffusivity. However, passive tracer diffusivity estimates are not directly informative about the quasi-Stokes transport, since quasi-Stokes transport requires estimates of *buoyancy diffusivity*. While many ocean models assume that the two diffusivities are the same out of convenience (Griffies, 1998), direct diagnosis in eddy resolving simulation have shown that the two diffusivities can have distinct magnitudes and vertical structures in the adiabatic interior (Abernathey et al., 2013; Smith & Marshall, 2009). Thus, there remains a need to observationally quantify the buoyancy diffusivity and the quasi-Stokes velocity and stream function.

In this study we fill this gap by obtaining observationally based estimates for the quasi-Stokes velocities and streamfunction, as well as estimates of the buoyancy diffusivity. We achieve this by use of the Taylor-Bretherton relationship, which links fluxes of PV to the fluxes of buoyancy, and assuming that PV stirring is more closely linked to passive tracer stirring than buoyancy stirring. The obtained estimates have good agreement with prior idealized modeling studies and expectations based on our understanding of large scale ocean dynamics. They can be used as observationally based constraints for eddy parameterizations or studies that explore the effects of eddies in using hydrographic atlases.

2. Theory

2.1. Quasi-Stokes Streamfunction and Buoyancy Diffusivity

The quasi-Stokes streamfunction (eddy-induced transport, in $\text{m}^2 \text{s}^{-1}$) is generally formulated as 2D vector streamfunction $\mathbf{\Upsilon} = (\Upsilon^x, \Upsilon^y, 0)$ for use in depth coordinates. The corresponding 3D quasi-Stokes velocity ($\mathbf{v}^* = (u^*, v^*, w^*)$, in m s^{-1}) is non-divergent ($\nabla \cdot \mathbf{v}^* = 0$), whose horizontal components are defined as:

$$\mathbf{u}^* = (u^*, v^*) = \frac{\partial}{\partial z} \mathbf{\Upsilon} \quad (1)$$

and the vertical component is inferred using continuity to be: $w^* = -\nabla_H \cdot \mathbf{\Upsilon}$ (Ferrari et al., 2010). Here $\nabla = (\partial_x, \partial_y, \partial_z)$ is the 3D gradient, and $\nabla_H = (\partial_x, \partial_y)$ is the horizontal gradient operator.

$\mathbf{\Upsilon}$ is meant to represent the adiabatic impact of eddies as advection in a z-coordinate model, which arises naturally as an eddy thickness flux or bolus velocity in isopycnal coordinate model. In Z-coordinates, $\mathbf{\Upsilon}$ may be physically interpreted as the horizontal eddy-driven transport below an isopycnal that on average resides at a certain depth (McDougall, 1995, 1998; McDougall & McIntosh, 2001). This is closely related to the transformed-Eulerian mean (TEM) eddy streamfunction, which arises under zonal averaging adopted in the atmospheric literature (Vallis, 2017; Wolfe, 2014).

Using the Transformed Eulerian Mean (TEM) approach, under the assumptions of Quasi-Geostrophic (QG) scaling, particularly horizontal homogeneity of statistics, the streamfunction can be expressed in terms of the eddy buoyancy ($b = -g \frac{\rho}{\rho_0}$, m s^{-2}) flux as (Treguier et al., 1997):

$$\mathbf{\Upsilon} \approx \mathbf{\Upsilon}^{TEM} = -\frac{\overline{\mathbf{u}' b'}}{\partial_z \overline{b}}, \quad (2)$$

where $\mathbf{u}$ is the horizontal velocity (m s^{-1}), $\overline{(\cdot)}$ represents the mean, and $(\cdot)'$ represents the eddy fluctuation. The TEM and QG assumptions can be relaxed and a number of other additional factors can be considered to get progressively more precise formulae for $\mathbf{\Upsilon}$ (Wolfe, 2014), but given the limitations of observations they will not be considered here.

Most parameterizations of the horizontal eddy flux of buoyancy originate from Gent and McWilliams (1990), who assumed a downgradient diffusion of buoyancy in the horizontal direction:

$$\overline{\mathbf{u}'b'} = -\kappa_b \nabla_H \bar{b}, \quad (3)$$

which, using Equation 2, results in:

$$\mathbf{Y}^{\text{TEM,GM}} = \kappa_b \frac{\nabla_H \bar{b}}{\partial_z \bar{b}} = -\kappa_b \mathbf{S}. \quad (4)$$

Here κ_b ($\text{m}^2 \text{s}^{-1}$) is the buoyancy or Gent-McWilliams diffusivity, and $\mathbf{S} = -\frac{\nabla_H \bar{b}}{\partial_z \bar{b}} = (S^x, S^y)$ is the 2D isopycnal slope vector. While this 2D horizontal buoyancy flux is assumed downgradient, the parameterization is formulated as a skew flux, such that the 3D flux vector ($\mathbf{v}'b'$) is along isopycnals. This is insured by inferring a vertical flux component ($w'b'$), such that $\mathbf{v}'b' \cdot \nabla \bar{b} = 0$, rather than directly parametrizing the vertical flux (Vallis, 2017). Apart from being along isopycnals (skew-flux), this parameterization is also a potential energy sink everywhere—mimicking the bulk effect of baroclinic instability.

The final component of the quasi-Stokes stream function parameterizations are boundary conditions for the bottom and surface. These are prescribed such that the quasi-Stokes velocity follows a no-normal flow condition at the surface and along the bottom ($-H(x, y)$), which is insured by setting the streamfunction to zero at the boundaries:

$$\mathbf{Y}(\eta) = \mathbf{Y}(-H) = 0. \quad (5)$$

These boundary conditions also ensure that the eddy-driven quasi-Stokes transport does not have a barotropic component ($\int_{-H}^{\eta} \mathbf{Y} dz = 0$), which follows from the kinematic constraints on the flow approaching a boundary (Killworth, 2001; McDougall & McIntosh, 2001). This also imposes that eddies only redistribute but not create momentum (Marshall et al., 2012).

In practice, the direct formulae for the quasi-Stokes streamfunction (e.g., Equation 3) do not naturally obey these boundary conditions, and ad-hoc approaches like linear tapering or low-mode smoothing are employed to smoothly transition the streamfunction to zero at the boundaries (Ferrari et al., 2008, 2010). The physical justification for the tapering is that in boundary layers near the surface and the bottom, mesoscale buoyancy fluxes transition from being along isopycnals to along the boundary. This necessitates that the flux must have a diabatic component (across mean buoyancy surfaces), which is not being parameterized by the adiabatic quasi-Stokes streamfunction.

2.2. Taylor-Bretherton Relationship

The Taylor-Bretherton relationship relates the eddy flux of potential vorticity (PV) and eddy flux of buoyancy, for a quasi-geostrophic flow assuming that the length scales are large enough to ignore the relative vorticity contribution to PV (Vallis, 2017), the relationship becomes:

$$\overline{\mathbf{u}'q'} = f \frac{\partial}{\partial z} \left(\frac{\overline{\mathbf{u}'b'}}{\partial_z \bar{b}} \right), \quad (6)$$

where q is the PV and f is the Coriolis frequency (s^{-1}). Combining with Equations 1 and 2, we see that the PV flux is directly related to the quasi-Stokes velocity as,

$$\overline{\mathbf{u}'q'} = -f \frac{\partial}{\partial z} \mathbf{Y}^{\text{TEM}} = -f \mathbf{u}^{*,\text{TEM}}. \quad (7)$$

A number of studies have argued that the downgradient diffusive flux assumption may be more appropriate for PV (Treguer et al., 1997), such that:

$$\overline{\mathbf{u}'q'} = -\kappa_q \nabla_H \bar{q}, \quad (8)$$

where κ_q ($\text{m}^2 \text{s}^{-1}$) is the PV diffusivity.

Substituting this relationship and Equation 3 into the Taylor-Bretherton relationship and using the definition of interior QGPV gradient ($\nabla_H \bar{q} = \beta \hat{j} - f \frac{\partial \mathbf{S}}{\partial z}$), we can relate the PV and buoyancy diffusivity as (Smith & Marshall, 2009):

$$\kappa_q \left(\frac{\partial \mathbf{S}}{\partial z} - \frac{\beta \hat{j}}{f} \right) = \frac{\partial}{\partial z} (\kappa_b \mathbf{S}), \quad (9)$$

where β ($\text{m}^{-1} \text{s}^{-1}$) is the meridional gradient of f .

3. Data and Methods

In this study we use the relationship of Equation 9 to estimate the quasi-Stokes velocities, streamfunction and consequently the buoyancy diffusivity. To apply this relationship we need estimates of the isopycnal slope and the PV diffusivity.

3.1. Estimates of Isopycnal Slope

Here the observational estimates of the isopycnal slopes are obtained by applying the “Vertical Non-local Method (VENM)” of Groeskamp et al. (2019) to the annual means of in situ temperature and practical salinity data from World Ocean Atlas 2018 (WOA18) (Locarnini et al., 2019; Zweng et al., 2019). WOA18 is a globally gridded climatology of hydrographic data at 1° resolution and 102 vertical levels.

3.2. Estimates of Mesoscale PV Diffusivity

Observational estimates of PV diffusivity are not available, so we approximate it from observational estimates of tracer diffusivity. PV, an active tracer, evolves according to an equation analogous to that of a passive tracer (Meunier et al., 2023; Vallis, 2017). This similarity suggests that in the ocean interior, PV and passive tracers may be stirred similarly, and if the eddy flux of a tracer is also parameterized diffusively ($\overline{\mathbf{u}'C'} = -\kappa_C \nabla_H \bar{C}$) then $\kappa_q \approx \kappa_C$. Also, since the vertical velocity vanishes at the surface of the ocean, the governing equation for b , C , and q take the same form. This suggests that these active and passive tracers are stirred similarly, and $\kappa_b \approx \kappa_q \approx \kappa_C$ near the surface and in the mixed layer. Idealized eddy-resolving channel studies (e.g., Abernathey et al., 2013; Lee et al., 1997; Smith & Marshall, 2009; Treguier, 1999) support these hypotheses in the zonal average and away from boundaries. Although these assumptions remain untested in more realistic models, we adopt $\kappa_q \approx \kappa_C$ globally for isotropic diffusivities in the interior and $\kappa_b \approx \kappa_C$ in the mixed layer.

Here we use observational estimates of κ_C that were calculated by Groeskamp et al. (2020) using observations and a parameterization of eddy diffusivity. The functional form of the parameterization was based on mixing length theory (Prandtl, 1925), mean flow suppression theory (Ferrari & Nikurashin, 2010), and vertical surface modes (LaCasce & Groeskamp, 2020). Observations of sea surface height were used to estimate eddy kinetic energy, and hydrographic observations were used to estimate mean geostrophic shear and stratification.

3.3. Estimating the Quasi-Stokes Velocity ($\mathbf{u}^*$) and Streamfunction (Υ)

Using the isopycnal slope and diffusivity estimates, the horizontal quasi-Stokes velocity ($\mathbf{u}^* = (u^*, v^*)$) is computed as:

$$\mathbf{u}^{*,\text{TEM}} = -\frac{\overline{\mathbf{u}'q'}}{f} = \kappa_q \left(\frac{\beta \hat{j}}{f} - \frac{\partial \mathbf{S}}{\partial z} \right). \quad (10)$$

This expression is not appropriate for use near the equator, since its derivation requires us to assume that relative vorticity contributions are small relative to the buoyancy flux contributions in Equation 6 (Vallis, 2017). So in this study we only consider the regions outside of $\pm 10^\circ$.

Consequently, the quasi-Stokes streamfunction is estimated as (Equation 1),

$$\Upsilon(z) = \Upsilon(z_{ml}) + \int_{z_{ml}}^z \mathbf{u}^{*,\text{TEM}} dz, \quad (11)$$

where z_{ml} is the vertical coordinate corresponding to the base of the mixed layer. Estimates of $\mathbf{u}^{*,\text{TEM}}$, which are estimated using the PV flux, only determine the vertical gradient of the streamfunction; thus a PV diffusion based parameterization of the quasi-Stokes velocity is not a full prescription of the quasi-Stokes streamfunction (a longer discussion can be found in Section 11 of McDougall and McIntosh (2001)). Here this indeterminacy is represented as a boundary condition ($\Upsilon(z_{ml})$). Also, since the QG theory based expression in Equation 10 is not applicable in the boundary layer, it is not appropriate to integrate using the data up to the surface and using the boundary condition from Equation 5 (also see Section 8 of McDougall and McIntosh (2001)).

To complete the parameterization of a quasi-Stokes streamfunction, we make the choice to revert to the buoyancy based parameterization for quasi-Stokes streamfunction near the boundary (Equation 4). As discussed in Section 3.2, this is theoretically appropriate since near the boundary all tracers are advected similarly in QG theory (also see Section 3 in Treguer et al. (1997)). We set $\Upsilon(z_{ml}) = \frac{\int_{z_{ml}}^z \Upsilon^{\text{TEM,GM}}(z) dz}{\int_{z_{ml}}^z dz} = \frac{-\int_{z_{ml}}^z \kappa_b(z) S(z) dz}{\int_{z_{ml}}^z dz}$, using the average value over the mixed layer depth rather than a value at a specific location in the mixed layer. This reduces the sensitivity to the estimate of the mixed layer depth. We calculated the mixed layer depths based on a $\Delta\sigma = 0.03 \text{ kg m}^{-3}$ criterion applied to the WOA18 (de Boyer Montégut et al., 2004), where potential density (σ) was calculated using the TEOS10-GSW software toolbox (IOC SCOR & IAPSO, 2010; McDougall & Barker, 2011). Extreme values were limited to a maximum mixed layer depth of 400 m.

We could have made a similar choice as above to instead prescribe a boundary condition near the bottom (Equation 11). We avoid this choice because we trust our data more in the upper ocean, since estimates of slopes in the upper ocean are based on a larger amount of data and the diffusivities have been verified against a number of different studies (Groeskamp et al., 2020).

The above estimate of $\Upsilon(z)$ will not naturally satisfy the boundary condition (Equation 5), and a few more choices are needed. $\Upsilon(z)$ in the mixed layer is set by linearly tapering from $\Upsilon(z_{ml})$ to 0 at the surface, ensuring that the surface boundary condition is satisfied. Near the bottom $\Upsilon(z)$ was linearly tapered from its estimated value at 300 m above the bottom to 0 at the bottom.

The quasi-Stokes streamfunction may be integrated zonally to obtain the eddy-induced meridional overturning streamfunction $\Psi^{\text{MOC-eddy}}$,

$$\Psi^{\text{MOC-eddy}}(y, z) = -\oint_{-H}^z v^* dz dx = -\oint \Upsilon^y dx. \quad (12)$$

This sign convention is chosen such that the zonally integrated velocities can be related to the meridional overturning 2D streamfunction as $(\oint v^* dx, \oint w^* dx) = (-\partial \Psi^{\text{MOC-eddy}} / \partial z, \partial \Psi^{\text{MOC-eddy}} / \partial y)$. This sign convention for 2D streamfunctions ensures that when the corresponding vorticity ($\xi = \nabla \times \mathbf{v}$) component is computed, it is related to the 2D streamfunction through a positive Laplacian. For example, in this case the corresponding vorticity component is the zonal component, $\xi^x = \nabla^2 \Psi^{\text{MOC-eddy}}$ (Abernathy et al., 2011; Ferrari et al., 2010; Vallis, 2017). Thus, a region with a minima in $\Psi^{\text{MOC-eddy}}$ would correspond to an anticlockwise circulation around it.

3.4. Estimating the Buoyancy Diffusivity (κ_b) as an Inverse Problem

In principle, we can estimate the κ_b by using Equation 4 once the streamfunction is estimated (shown in Figures S7 and S8 in Supporting Information S1), or by solving Equation 9. However, these approaches results in a few

practical issues: (a) depths with small $\mathbf{S}$ may produce unrealistically large values of κ_b , due to division by ~ 0 ; (b) the boundary conditions on κ_b may not be easily available (see Section 3.3); (c) the estimated κ_b may be negative, which is physically unrealistic and there is no simple way to impose a positivity constraint; (d) there is no straightforward way to obey the isotropy assumption, as the equations from the x and y direction may result in distinct $\kappa_b \mathbf{S}$; and (e) any noise in the data (e.g., in estimates of S_z) may get compounded to other depths.

Given the above concerns, we approach the estimation of κ_b as an inverse problem, which is the appropriate approach to solving problems where both model uncertainty (in equations or boundary conditions) and/or data uncertainty are present (Wunsch, 1996). The equation being considered here is:

$$\frac{\partial}{\partial z}(\kappa_b \mathbf{S}) = -\mathbf{u}^*, \quad (13)$$

where we will estimate κ_b , given the $\mathbf{u}^{*,\text{TEM}}$ estimate and $\mathbf{S}$.

For y -direction, the discretization of Equation 13 from surface to the bottom provides the following N equations,

$$\begin{aligned} \kappa_b(z_0) S^y(z_0) - \kappa_b(z_1) S^y(z_1) &= -v^*(z_{1/2}) \Delta z_{1/2}, \\ \kappa_b(z_i) S^y(z_i) - \kappa_b(z_{i+1}) S^y(z_{i+1}) &= -v^*(z_{i+1/2}) \Delta z_{i+1/2}, \quad i \in (1, N-2) \\ \kappa_b(z_{N-1}) S^y(z_{N-1}) - \kappa_b(z_N) S^y(z_N) &= -v^*(z_{N-1/2}) \Delta z_{N-1/2}. \end{aligned} \quad (14)$$

Here the interface points are denoted as z_i with $i \in (0, N)$, and the center points are denoted as z_c , with $c \in (1/2, N - 1/2)$. Here $\mathbf{u}^*(z_c)$ is given at z_c and $\mathbf{S}(z_i)$ is given at z_i . A similar set of equations can be written for the x -direction. Assuming isotropy (see point d above), the equations for the x and y directions are combined into the same system of equations to obtain a single $\kappa_b(z)$ at every depth.

Regularization is needed where the isopycnal slopes become flat ($\mathbf{S} \approx 0$). Physically speaking, these are regions where the diffusive buoyancy flux should be small (Equation 3). Without regularization this may not hold for observational estimates, possibly leading to the estimated κ_b being unrealistically large, spiky, and noisy. We therefore apply regularization to regions with small slopes, when $|\mathbf{S}| < \delta$, by setting these slopes to $\text{sign}(\mathbf{S})\delta$ if this condition is met.

Additionally, we expect that the diffusivity should be positive. This positivity constraint comes from the principle that we expect mesoscale eddies to reduce available potential energy. We impose a non-negativity constraint by using a non-negative least squares approach (function `lsqnonneg` of Matlab 2022a) to the inverse problem.

Finally, following the reasoning that $\kappa_b \approx \kappa_q \approx \kappa_C$ in the mixed layer and the discussion on the boundary condition for $\mathbf{Y}$, we set the estimates for κ_b equal to the average κ_q in the mixed layer.

4. Results

4.1. Eddy-Driven or Quasi-Stokes Velocity

The magnitude of the observed quasi-Stokes velocities is $\mathcal{O}(10^{-3}-10^{-4})$ m s⁻¹ (Figure 1 and Figures S1 and S2 in Supporting Information S1), with complex spatial patterns that are set by the interplay between the patterns of diffusivity and isopycnal structure. Generally, u^* is a little weaker than v^* , and both components are large where the eddy kinetic energy is large, which is often also associated with regions of steep isopycnal slopes and related vertical gradients ($\partial \mathbf{S} / \partial z$). These regions are foremost the Antarctic Circumpolar Current (ACC) and the subpolar North Atlantic (SPNA). Isopycnal slopes are often steeper in meridional direction than zonal, allowing for larger vertical gradients of the slopes, which is why v^* is larger. Additionally, the impact of the planetary PV gradient (β/f -term) leads to a weak poleward flow, which can become dominant in regions where isopycnals are flat (e.g., below 2,000 m in the mid-latitudes).

In the ACC, large values of $\mathbf{u}^*$ are observed at mid-depth (1500–2500 dbar), with a directionality that flattens isopycnals and works against wind-driven isopycnal steepening. Dipole structures are found around local maxima or minima of the magnitude of isopycnal slopes (where $\partial S^y / \partial z = 0$ and changes sign), enhanced by mid-depth maxima in eddy diffusivity that results from mean-flow suppression (Figure S7 in Supporting Information S1). In

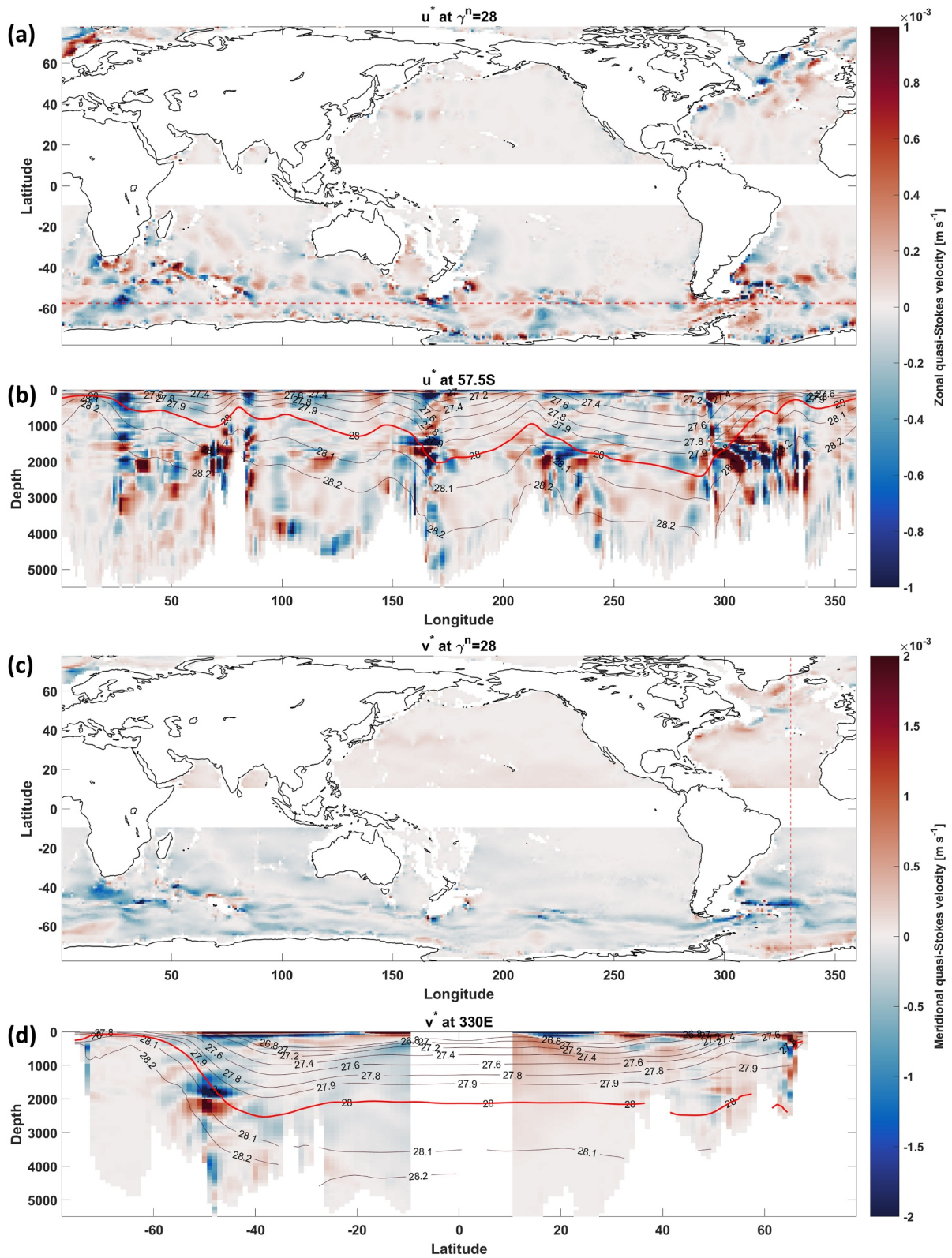


Figure 1. Maps of the zonal (a, b) and meridional (c, d) quasi-Stokes velocities. Black contours (b, d) indicate Neutral density as a proxy for the neutral slope. Red (dashed) lines mark the location of the corresponding cross-section (a, c) and thick red contour (b, d) indicate the density surface plotted in (a, c). Note the different colorscales.

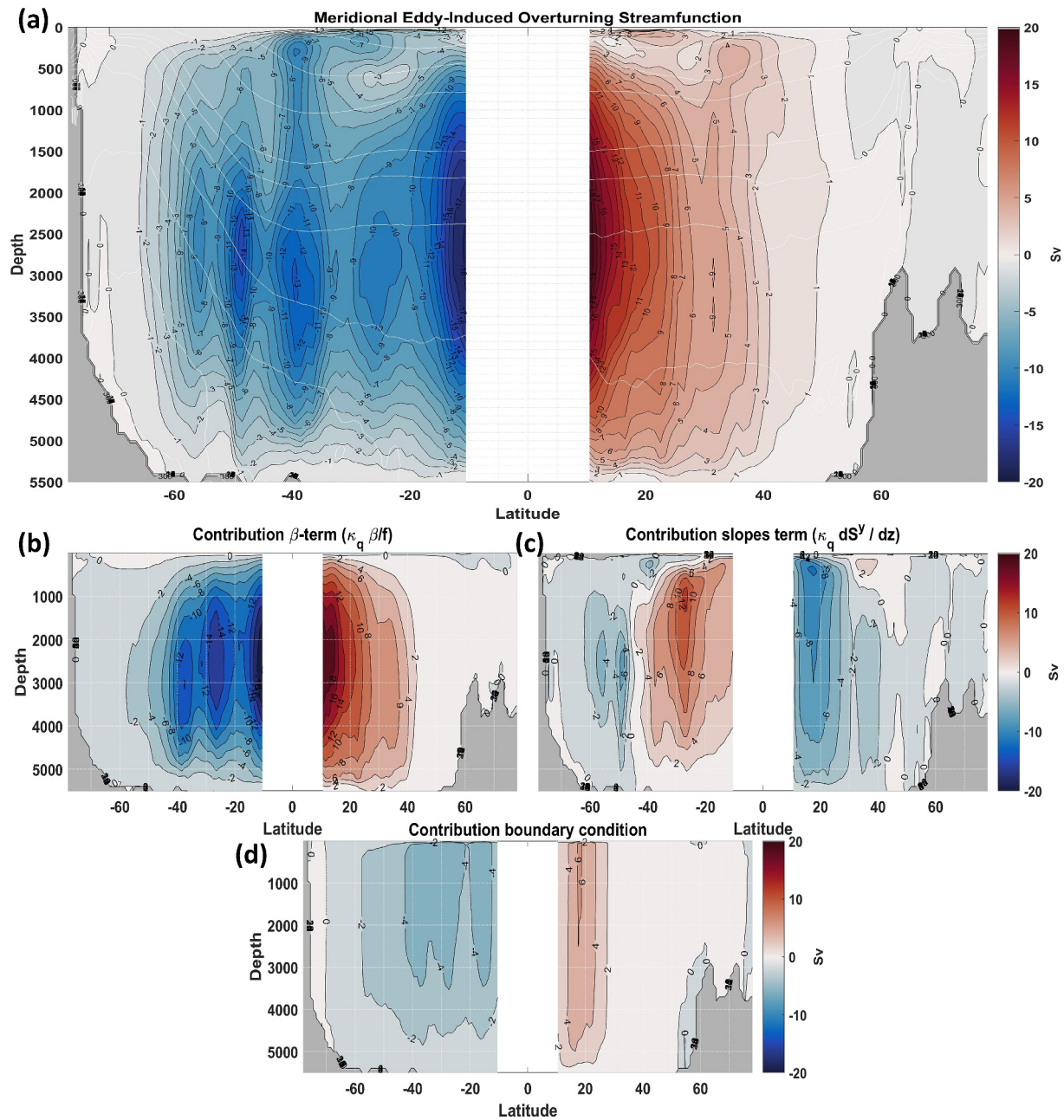


Figure 2. The meridional eddy-induced overturning streamfunction $\Psi^{\text{MOC-eddy}}$ (a), with the average $\kappa_q S^y$ in the ML as BC. White contours are zonally averaged Neutral Density contours for reference. $\Psi^{\text{MOC-eddy}}$ is further decomposed into (b) the contribution of the β/f -term, (c) the isopycnal slopes ($\kappa_q dS^y / dz$) and (d) the contribution from the boundary condition on Υ^y .

contrast the large eddy-driven velocities seen in the SPNA are more directly a result of steep isopycnals. In the tropics, surface intensified diffusivities clearly enhance the eddy-driven velocities in the upper ocean and act in a direction that oppose the wind-driven sub-tropical cells.

4.2. Eddy-Induced Meridional Overturning Streamfunction

The eddy-induced meridional overturning streamfunction $\Psi^{\text{MOC-eddy}}$ shows a hemispherically asymmetric pattern (Figure 2a, related patterns of the quasi-Stokes streamfunction Υ are not detailed here but displayed in Figures S3–S5 in Supporting Information S1). This asymmetry stems from the large zonal extent of the ocean in

the Southern Hemisphere (SH), large $\partial S^y/\partial z$ due to the ACC, and higher eddy diffusivity in the SH, including the large diffusivities seen even outside the western boundary currents and the ACC (e.g., Indian Ocean Figure S7 in Supporting Information S1). The largest eddy transport (outside of $\pm 20^\circ$, ~ 10 Sv) is found in the SH in the ACC, where a counter-clockwise rotating multiple core structure works to flatten the sloped isopycnals. The multiple cores result due to varying meridional position of the ACC in different basins (Figure S6 in Supporting Information S1).

The Northern Hemisphere (NH) eddy transport is relatively small, even at latitudes of the Gulf Stream and Kuroshio (notice the ~ 4 Sv cell associated with these boundary currents centered around 400 m and 35°N). This is partly due to the smaller region of enhanced eddy activity in the NH than in the ACC, and due to reduced basin width at higher latitudes. Locally the eddy-induced transport is significant as evidenced by the quasi-Stokes velocities (Figure 1), especially in the SPNA. Note that such zonal variations may lead to a net meridional tracer transport even if the corresponding meridional volume transport is weak. Next to the prominent full depth SH cells, sub-tropical eddy-driven cells (~ 2 Sv), that oppose the wind-driven cells, also appear on both sides of the equator in the upper ocean.

When decomposing the eddy-driven overturning streamfunction into contributions from the planetary PV gradients, interior PV gradients, and the boundary condition (Figures 2b–2d), we find that the contribution of the boundary condition is weak. This suggests that the choices made in Section 3.3 don't obfuscate the patterns being discussed here. Apart from the Southern Ocean, the contributions from the planetary PV gradient (β/f) and the interior PV gradient ($\partial S/\partial z$) oppose each other. This likely results from the tendency of the flow to homogenize large scale PV gradients. In the Southern Ocean both PV gradients have the same sign and together oppose the wind-driven isopycnal steepening.

The planetary PV gradient also leads to a very strong signal in $\Psi^{\text{MOC-eddy}}$ at lower latitudes that is associated with a small interior poleward quasi-Stokes velocity. This suggests an eddy-driven meridional flow and downwelling on either side of the equator, for example, notice the vertically oriented overturning streamfunction at latitudes 10° – 30° . The structure of these strong tropical eddy-driven meridional overturning cells tapers back to zero near the bottom as a result of the applied tapering (Figure S4 in Supporting Information S1). These structure require further investigation.

4.3. Buoyancy or Gent-McWilliams Diffusivity (κ_b)

The global maps of buoyancy diffusivity (κ_b) show a rich structure (Figure 3), which has similarities and differences to passive tracer or PV diffusivity (κ_q , Figure S7 in Supporting Information S1). In general, estimates of κ_b are smaller than κ_q , illustrated by their volume weighted global mean values of $47 \text{ m}^2 \text{ s}^{-1}$ and $165 \text{ m}^2 \text{ s}^{-1}$, respectively (Figure 3c and Figure S13 in Supporting Information S1). Large estimates for κ_b are observed in regions with strong mesoscale eddies, for example, the Agulhas current, the Gulf Stream, the ACC etc (Figure 3a). The cross-section of Figure 3b shows that in the ACC, the estimates of κ_b are about 100 – $500 \text{ m}^2 \text{ s}^{-1}$ and these values extend over the entire water column (e.g., loc. B and C in Figure 3c). There is relative little vertical variation compared to κ_q , which are larger in the upper part of the water column and has subsurface maxima, however κ_b does occasionally show subsurface maxima or minima. These differences between κ_b and κ_q , especially in the Southern Ocean, are in line with diffusivity structure elucidated using idealized channel models (e.g., Abernathey et al., 2013; Smith & Marshall, 2009). In contrast, at low latitudes, the estimates for κ_b are very small for large parts of the water column (Figure 3b), except near the surface.

When estimating κ_b from least squares fitting of Equation 14, we applied a slope threshold ($\delta = 2 \times 10^{-4}$). As δ increases κ_b becomes smoother and smaller. This threshold is based qualitatively on the observed distribution of isopycnal slopes (Groeskamp et al., 2019) and quantitatively on an L-curve criterion (Hansen, 2010) (see Figure S11 in Supporting Information S1). This compromise ensures that $\mathbf{u}^*$ estimates are well fit, without overfitting to unrealistically large values of κ_b . Regardless of the regularization constraints, the $\mathbf{u}^*$ are fit surprisingly well (Figure S11 in Supporting Information S1). By design, this slope regularization mainly impacts regions with small isopycnal slopes (e.g., at lower latitudes in the interior), which are also usually regions where $\mathbf{u}^*$ are relatively weak. Consequently, the parameterized flux due to using a buoyancy diffusion parameterization (Equation 4) would lead to a negligible eddy velocity in these regions. Areas with strong eddy activity (e.g., the ACC and

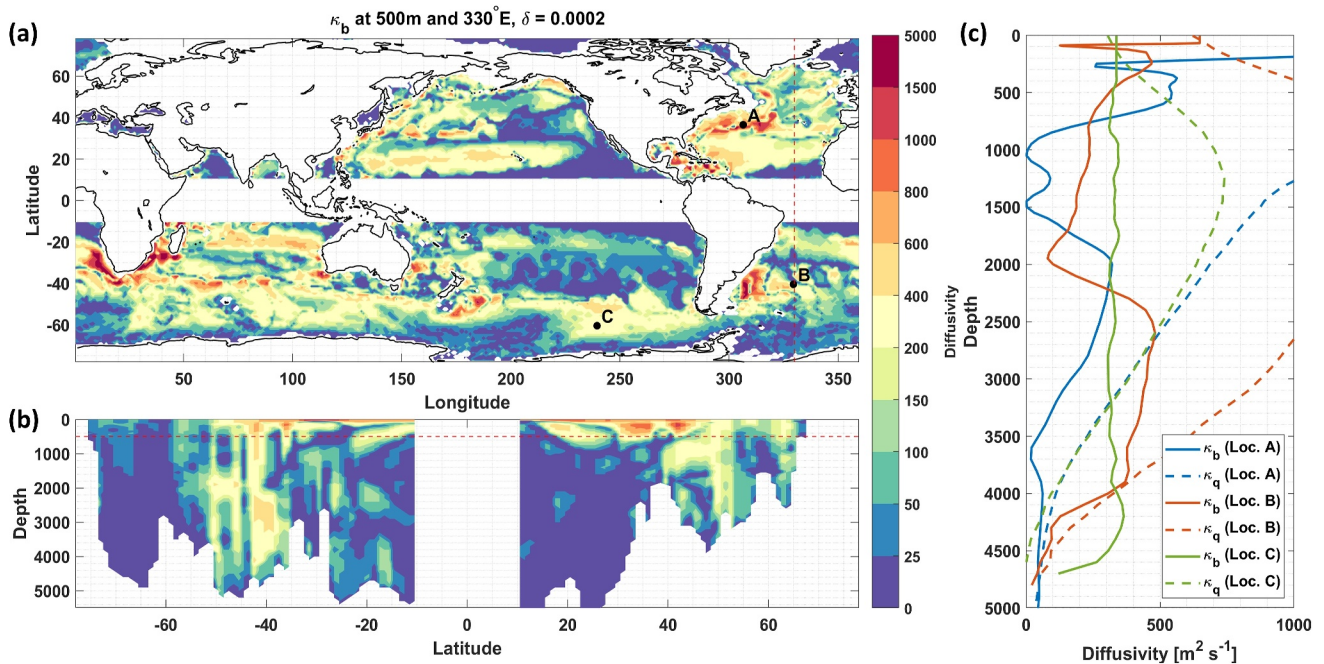


Figure 3. Inverse estimates of κ_b . With (a), a map at 500 m, (b) a cross-section at 330°E and (c) some example profiles of κ_b and the corresponding profiles of κ_q . The locations of the profiles are marked in panel (a) (e.g., the Gulfstream (loc A), southern Atlantic (loc B) and SO/Pacific (loc C)) by black dots. Dashed red lines (a, b) mark the location of the other panel. Note the nonlinear colorbar.

boundary currents), generally have large isopycnal slopes and thus are only minimally affected by the slope regularization.

5. Discussion and Conclusions

We provide the first global full-depth observational estimates of the mesoscale eddy-driven or quasi-Stokes velocities ($\mathbf{u}^*$). These estimates rely on the idea that oceanic flows are more likely to stir PV like a passive tracer in the interior rather than buoyancy or thickness, and thus that stirring of PV may be a better candidate than buoyancy to be modeled using a diffusive parameterization (Marshall et al., 1999; Treguier et al., 1997). The estimated eddy-driven velocity magnitudes, $\mathcal{O}(10^{-4} - 10^{-3} \text{ m s}^{-1})$, and patterns are roughly in agreement with the one direct observational estimate of this velocity from the Scotia Sea region, which also showed that $\mathbf{u}^*$ was smaller than 0.01 m s^{-1} and changed sign over vertical scales of $\mathcal{O}(300 - 400 \text{ m})$ (Sévellec et al., 2019).

We also used these quasi-Stokes velocity estimates to estimate the corresponding quasi-Stokes streamfunction, and the buoyancy (Gent-McWilliams) diffusivity that would be needed to represent this eddy transport using the Gent-McWilliams parameterization. The estimated streamfunction matches our dynamical understanding of the effects of mesoscale eddies. For example, we see an eddy-driven overturning cell in the Southern Ocean, which operates to remove the APE that is added due to the wind-driven circulation (Abernathey et al., 2011; Ferrari et al., 2010; Johnson & Bryden, 1989; Mazloff et al., 2013; Olbers et al., 2004). The estimated overturning streamfunction is in good agreement with previous observational-based estimates in the upper 2,000 m of the ACC (Chapman & Sallée, 2017) and that from Argo floats in the upper 1,000 m (Katsumata, 2016). Furthermore, empirically we find that our buoyancy diffusivity estimates are generally smaller than the corresponding PV or tracer diffusivity (Figure S13 in Supporting Information S1), which are in alignment with studies using idealized channel models to distinguish between the two diffusivities (Abernathey et al., 2013; Balwada et al., 2019; Smith & Marshall, 2009; Treguier, 1999).

We assumed that the PV diffusivity is similar to the passive tracer diffusivity, which is unlikely to hold everywhere. A consequence of this assumption is that the column integrated PV flux is non-zero (Marshall et al., 2012) and that the quasi-Stokes streamfunction does not naturally satisfy the boundary conditions; thus tapering is necessary. By imposing a linear tapering scheme any error that is made in the interior due to these assumptions, is

lumped into the boundary regions. More sophisticated tapering approaches could be used (e.g., Ferrari et al., 2010), but remain only loosely rooted in dynamics. More work needs to be done to understand how PV fluxes relate to passive tracer fluxes, specially in regions where flows are not akin to channel flows and eddy statistics may be spatially non-homogeneous and anisotropic.

A PV flux parameterization does not fully determine the quasi-Stokes streamfunction, as described in Section 3.3 and McDougall and McIntosh (2001). For applications where only the quasi-Stokes velocity is needed, this is of no consequence. However, for applications where the streamfunction is needed, additional assumptions are needed. Here we chose to address this by reverting to a Gent-McWilliams-style parameterization for buoyancy fluxes near the upper boundary, similar to past studies (Meunier et al., 2023; Sallée et al., 2010) and to what is done in ocean models using the Gent-McWilliams parameterization (Gent, 2011; Kuhlbrodt et al., 2012). In response to the conundrum raised by McDougall and McIntosh (2001), it seems to us that making such a choice gives a good balance between the benefits of mixing PV in the interior and buoyancy near the surface, both of which are dynamically reasonable choices with some support in the literature (Abernathey et al., 2013; Meunier et al., 2023). This choice also resonates with the work of Mazloff et al. (2013), who showed that density outcrops play an equally important role to interior PV structure in setting the eddy-driven overturning circulation.

Our work shows that even though the contribution of the planetary PV gradient is small locally, it can become quite significant in a zonally averaged sense (Figure 2) since it is single signed in each hemisphere. Past studies, (e.g., Abernathey et al., 2013; Smith & Marshall, 2009), have shown that this impact from the planetary PV gradient is small but present, even in layers where the thickness is relatively uniform (Lee et al., 1997). In regions close to the equator ($10^\circ - 30^\circ$), where β/f is large, this component becomes particularly large, and suggests a poleward and downwelling flow on both sides of the equator. This flow is in agreement with Sverdrup dynamics (Vallis, 2017), where instead of winds producing vertical divergence or convergence and a meridional flow, a meridional flow results due to PV stirring that results in a stretching of the water column. This process is a dynamically plausible scenario, but if it is present it seems to be in perfect balance with the mean flows in the region as almost no signature of any downwelling is apparent in the isopycnal (Figure 2) or tracer structure. An ocean model employing the Gent-McWilliams parameterization would produce no such overturning cells, since the isopycnals are flat in this region and the associated streamfunction would be zero (e.g., see Figure 6 of Gent et al. (1995)), and thus result in a very different zonal mean transport budget. We were unable to find any past studies discussing this structure, and its validity and impact would need to be scrutinized in future studies.

The estimated κ_b is contrasted against alternative approaches, such as dividing components of $\mathbf{V}$ by the corresponding isopycnal slope, or using a direct analytical expression also based on the Taylor-Bretherton relationship (Meunier et al., 2023) (see Text S3 and Figures S8–S10 in Supporting Information S1 for maps of these estimates). However, these methods did not provide adequate diffusivity estimates, as the estimated κ_b can be negative, become unrealistically large in regions with small isopycnal slopes, and not provide estimates that correspond to the assumption of isotropy. In contrast, the non-negative least squares optimization allowed us to resolve some of these issues using regularization, and the estimated κ_b are able to decently reconstruct the $\mathbf{u}^*$ (Figure S12 in Supporting Information S1). In future work, other regularization conditions, for example, requiring smoother structure in the vertical, could easily be included.

These observation-based global estimates of the quasi-Stokes velocity and streamfunction, eddy-induced overturning circulation, and buoyancy diffusivity provide observational constraints for eddy parameterizations, which could help improve coarse resolution ocean and climate models and better inform ocean state estimates. Furthermore, these estimates may also be useful for studies using hydrographical atlases, for example, in the context of water mass transformations and for studies considering the residual circulation, or for eddy-induced tracer transports of heat, carbon and other (biogeochemical) tracers (e.g., Palter et al., 2013; Thompson et al., 2014).

Data Availability Statement

World Ocean Atlas 2018 data (Locarnini et al., 2019; Zweng et al., 2019) can be downloaded from <https://www.ncei.noaa.gov/access/world-ocean-atlas-2018/>. The VENM Matlab code from Groeskamp et al. (2019) is available on <https://github.com/Sjoerdgr/VENM>. Estimates of the passive tracer diffusivity κ_C (Groeskamp et al., 2020) and related Matlab scripts can be obtained from Groeskamp (2020). The estimates of

$\mathbf{u}^*$, $\mathbf{r}^{\text{TEM}}$, $\Psi^{\text{MOC-eddy}}$ and the estimates of κ_b , as well as the Matlab scripts to reproduce the results are available as Kusters et al. (2025).

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