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Frequency of occurrence of storm surges in an estuary: a stochastic approach

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Abstract

This paper presents the results of a study on the frequency of occurrence of storm surge levels in the river Scheldt at Antwerp, related to the design of a storm surge barrier and the evaluation of the dike safety in the Scheldt basin.

The basic principles of the extreme value distribution methods, the joint probability methods and simulation models are examined.

A new technique, based upon the simulation of storms by variables characterizing the resulting surge is proposed.

Results are compared with those of other methods.

l. Introduction

This study concerns a statistical evaluation of extreme waterlevels on the river Scheldt near Antwerp, for which the highest registrated level is NKD+8m with an "estimated frequency of 1/100 years, based upon extrapolation of data along the Dutch North Sea coast.

Due to hydrodynamic and hydraulic actions in the estuary, the accuracy of such frequencies is poor, and since enough observations are available for Antwerp (1902 - present), the frequency of occurrence of storm levels was investigated for this city itself.

At this moment all levels exceeding NKD+6.5m are called "storm surges" (till 1954 that datum was NKD+6.0m).

This study doesn't analyse tidal characteristics either in a statistical way, nor in a fundamental, physical and hydrodynamic way.

2. Analysis of storm surge levels

The different components of a surge level on the river Scheldt are represented in Fig. 1.

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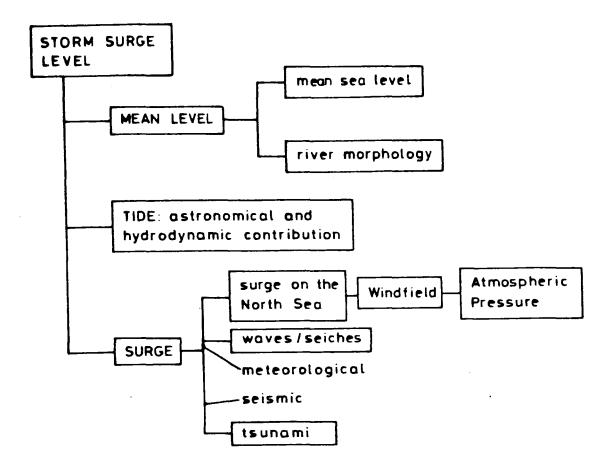


Fig.1: Components of storm surge level

2.1. Mean level

The variation of mean level, dealt with in this paragraph consists of periodic and secular changes, in which periodic means 'with a period of at least one year', thus excluding diurnal and shorter-period oscillations, classified as tidal effects. The variation in mean level, often substituted for by 'mean tidal level', is caused by mean sea level variations and by morphological changes of the river, due to the dynamic physical processes in the river and due to human interventions (35, 40). The most important causes of mean sea level variation are : changes in the total water balance, movements in the land reference level (bench mark), time variations in the atmospheric forces (wind stress and air pressure), time variations in the oceanographic forces (temperature, salinity, currents) and finally long period astronomical tides. For a description of these phenomena reference is made to specific geophysical literature (24, 35, 37, 39). With a view to obtain an homogeneous set of water level data along an estuary, one has to investigate not only mean sea level variations, but (depending upon the calculationprocedure to determine frequencies of occurrence) also trends in annual maxima and in tidal amplitude must be considered (see § 3.1.).

2.2. Tidal movements

In the river Scheldt tidal activity is a cooscillating tide in an estuary, propagated in the basin and caused by tidal movements on the North Sea. The principal constituents are the M2 lunar semidiurnal tide, with a period of 12.42 hours and the S2 solar semidiurnal tide (period = 12h). Furthermore it must be mentioned that both observed and predicted tidal levels consist of pure "astronomical tides" and hydrodynamic influences.

2.3. Surge elevation

Surges on the river Scheldt are mainly caused by meteorological activities, acting on the water mass in the North Sea. For a description of the sources, physical phenomena and mathematical calculation schemes we refer to literature (32, 42). Waves and oscillations have neglectable influence on resulting water levels, while seismic surges didn't occur till now.

2.4. Interaction of surge and tide

The interaction effects between surge and tide are caused by frictional resistance and variations in the celerity of waves (due to differences in waterdepth), modifying surges in tidal regions (2, 15, 21, 28, 36, 38, 48).

The main conclusions of several investigators are : Keers deduced that the interaction increases linearly with the surge height and that the degree of interaction is quite correlated with the tidal amplitude and with the ratio of the amplitude and mean waterdepth (21). Banks stated that the most important interaction is between the M2 semidiurnal tide and the surge (2). Although Pugh and Vassie (30) concluded that any interactions were of second order, so that surge and tide can be treated statistically independent, it seems important to mention the qualitative investigations of the dutch 'Deltacommissie' (33) : the increasing levels due to surge cause an advancement with regard to the predicted tide (in absence of surge). they cause a enfeeblement of the tide since low levels remain higher and finally they cause a deformation because geometrical elements, such as the topography of banks and gullies) which are boundaries for the tidal movement are oriented in a different way. Walden, Prescott and Webber (48) compared two methods to determine the degree of interaction, based upon hourly surge residuals: a first approach is to examine the distribution of these residuals at various phases of the tidal cycle (see also (28)), whilst the second method determines the distribution of residuals at different tidal levels (see also (29)). They concluded that the more a port is remote from the open sea, the more the interaction is prominent. This implies that the use of the joint probability method (see § 3.2.) is justified if the degree of interaction is negligible.

3. Methods to determine the frequency of occurrence of high surge levels

Depending upon the set of available tidal records, two principal methods can be used to determine the frequency of occurrence of high

water levels : extreme value distributions and methods based upon the joint surge-tide probability.

3.1. Extreme value distributions

For a series of M annual maximum water levels an extreme value distribution is assumed, characterized by a set of parameters, which are estimated in procedures such as the method of moments, maximum likelihood, etc. (4). Some of these distributions, such as Fisher-Tippetdistribution (10), Barricelli (3), Gumbel (14) and Jenkinson (19, 20), have been applied to estimate the frequency of occurrence of extreme sea-levels. Suthons (44) and Lennon (22) published studies with respect to the south-east and the west coast of England. Akers e.a. (1), Webber e.a. (9, 50), Blackman e.a. (6, 12) reviewed these reports to add more recent information. For the Scheldt estuary frequencies were calculated by SVKS (43), Janssens e.a. (19), Sas e.a. (40) and Berlamont e.a. (5). The results are listed in Table 1.

As conclusion, we believe that

- 1. Although no exact information is available, frequencies depending upon the extreme value distributions are reliable for design periods up to 4 times the length of the series of observed data (12).
- 2. Using the method of extreme value distributions includes the disadvantage of neglecting important information about exceptional but not maximal events : such as the increase of the frequency of occurrence of high water levels from 1960 on (see Table 2, and (40)).
- 3. If a persistent trend is apparent, its effect must be estimated and data must be adjusted accordingly in order to get a homogeneous set of data (6, 44, 47), using for example Suthon's adjustment method, based upon 10-year forward or backward accumulation (13, 44) or 19-yearly mean values to eliminate nodal periodic variations of mean sea level (34, 40).
- 4. To achieve any reliability, the observed data must cover a period of a least 30 years (11).

3.2. Joint surge tide probability methods

If the set of observed tidal data is not long enough, extreme value distributions can't provide reliable information concerning the frequency of occurrence. In this case several methods based upon the principal of joint probability can be applied. A recorded level (H) can be split up into three constituents, after filtering for wave influences:

$$H(t) = Z_{O}(t) + G(t) + M(t)$$
 [1]

in which Z_O(t): mean level (i.e. mean sea level), G(t): tidal component, M(t): meteorological component (i.e. surge).

3.2.1. The convolution method

From a series of observations H the hourly surge component M can be derived by calculating the mean level $Z_0(t)$, and the tidal component G(t) from its harmonic constituents. If surge and tide can be treated as statistically independent (see § 2.4.); the probability density function p(h) can be written as the convolution of the individually

estimated density functions of tide p_C and surge p_S .

$$p(h) = \int_{-\infty}^{\infty} p_G(h-y) p_S(y) dy$$
 [2]

with h = H(t) - Z, y = M(t) and x = h - y = G(t). Eq. [2] is valid only if h, y and x are deduced from a stationary stochastic process, which condition is nearly never completely satisfied. The probability of exceedance of a level h can be evaluated from the cumulative distribution function $P_h(h_0)$:

$$P_h(h_o) = 1 - \int_0^\infty p(h) dh = \int_0^\infty P_G(h_o - y) p_S(y) dy$$
 [3]

"o and the return period of height h_0 :

$$T(h_0) = 1/[(1 - P_h(h_0))]$$
. (average number of hourly values in a year) [4]

Pugh e.a. (30, 31) applied this method to the English south coast port of Portsmouth over a period of 12 years. For return periods of 50, 100 an 250 yr, they estimated sea levels of 2.86, 2.91 and 2.98 whereas application of the extreme value theory using the 12 annual maxima resulted in 2.69, 2.71 and 2.74: which is an underestimate of 6, 7 and 8 % respectively.

3.2.2. Duration methods

In the aforementioned methods the duration of the storm is not considered; the surge is characterized by its magnitude only. Other methods however account for the transient nature of the surge event by considering the effective duration of the process.

Tayfun (45) developed an analytical procedure to determine the probability distribution function of extreme levels, caused by coincidence of a rare event with the astronomic tide. If we consider tide and surge to be statistically independent and if we characterize a storm surge by an equivalent magnetidue X and an effective duration τ (Fig. 2) and if

$$Z = \max_{\substack{t_0 \le t \le t_0 + \tau}} [G(t) + M(t)]$$

in which M(t) is a positive surge, starting at any time toof the tidal cycle, and if we assume that the water level increase caused by M(t) is gradual enough that an approximation by a constant height surge X is allowed over the period (t_0 , t_0 + τ) then

$$Z \approx \max_{\substack{t \\ 0 \le t \le t}} [G(t) + X]$$

in which τ and X are random variables. Considering the conditional probability function of Z for given value of τ :

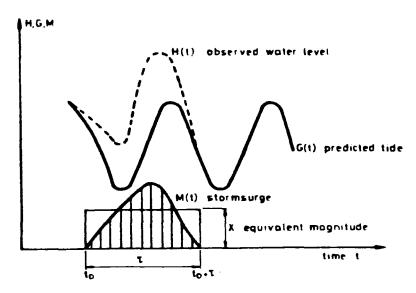


Fig. 2: Tayfun method

$$P_{Z/\tau}(z/\tau) = \text{Prob} \left[G(t) + X < z \text{ for } t \le t \le t_0 + \tau/\tau \right]$$

$$= \int_{-\infty}^{z} P\left[X < z - g/(G = g \text{ for } t_0 \le t \le t_0 + \tau/\tau) \right]$$

$$P_{G/\tau}(g/\tau) dg \qquad [5]$$

and assuming that the non-negative X is statitically independent of τ and G:

$$P[X < z - g/(G = g \text{ for } t_0 \le t \le t_0 + \tau/\tau)] = P_X(z - g)$$
 [6]

and

$$p_{G/\tau}(g/\tau)$$
 dg = Prob (g \leq G \leq g + dg for t_o \leq t \leq t_o + τ/τ) [7]

in which tide is assumed to be a Gaussian process. For τ varying over all possible values:

$$P_{Z}(z) = \int_{0}^{\infty} \int_{-\infty}^{z} P_{X}(z - g) p_{G/\tau}(g/\tau) p_{\tau}(\tau) dg d\tau$$
 [8]

so that the return period for a water leverl z = h caused by positive surges, occuring at a mean rate of σ per year:

$$T(h_0) = 1/[\sigma (1 - P_Z(h_0))]$$
 [9]

Walden, Prescott and Webber (49) developed an adapted Tayfun method for use in European waters since tide can not be represented by a Gaussian process because frequency distributions tend to be bimodal (30). Furthermore, surge duration is longer than one tidal cycle (~ 13 hours) and the magnitude is rather moderate compared with the tidal

amplitude. For surges with duration τ greater than 13 hours (= T_0), only the largest equivalent magnitude X over a 13-hour interval must be considered. This X value results from moving the T_0 -interval through the τ -interval (= surge) in steps of one hour, and at each step T_1 equating the area above (T_1 , T_1 + T_0) to T_0 X (in general $\tau < 50$ hours (18, 49)). The conditional density function $P_{C/T}$ (g/ τ) can be obtained from the conditional probability distribution function

$$P_{G/\tau}(g/\tau) = \text{Prob} \{G(t) \leq g \text{ for } t_0 \leq t \leq t_0 + \tau/\tau\}$$
 [10]

by numerical differentation.

Such probability can be estimated for different values of τ for a series of g values by moving the interval of duration τ through the hourly predicted G(t) series and dividing the number of hours for which the condition of eq. 10 is satisfied by the total number of G(t) values. The conditional distribution function of the maximum sea level Z for given τ is derived in the following way. If g_{\min} and g_{\max} are the lowest and highest hourly predicted tides and if Z is chosen as a sea level which can occur only by adding a positive surge to g_{\max} , i.e. $z \Rightarrow g_{\max}$, then Prob ($X \le z - g/\tau$) = 1 for a range of g values between g_{\min} and g_{\max} . Let us denote this boundary by g_b , hence

$$P_{Z/\tau}(z/\tau) = \int_{g_{min}}^{g_{max}} P(X \le z - g/\tau) p_{A/\tau}(g/\tau) dg$$

Further derivation leads to

$$P_{Z/\tau}(z/\tau) = 1 - \int_{g_h}^{g_{max}} [1 - P(X \le z - g/\tau)] p_{G/\tau}(g/\tau) dg$$
 [11]

The adapted Tayfun method is completed by using eq. [8] and [9]. For return periods of 50, 100 and 250 yr, Walden, Prescott and Webber (49) estimated water levels of 2.74, 2.78 and 2.83 differing from the "convolutionestimates" by 4.3, 4.7 and 5.3 % respectively (see § 3.2.1.).

Beside these analytical methods to calculate the frequency of occurrence of high water levels, several simulation procedures were developed in which the transient event is reconstructed.

- 1. Myers (25) pointed out three important aspects in developing a simulation procedure
 - assess the behaviour of surges from past records, analyze all factors causing and determining the event, such as atmospheric pressure, forward speed and direction of the storm
 - develop a hydrodynamic mathematical model to simulate storms (hurricanes) from a random input of the stochastic parameters, determined in the first step, and calculate the resultant surge
 - calculate density distributions and probability functions for all parameters and deduce the frequency of occurrence of the rare event, simulated in step 2.

This method has been applied several times in the U.S.A. (ref. 16, 17, 25, 26, 27). The major disadvantage is the development of the hydrodynamic model and the required calculation time for each simulation.

2. For this reason, the author proposes a simulation technique, based upon the parameters, characterizing the resulting surge (and not

based upon the event-causing parameters), such as duration and magnitude of surge and tidal amplitude and phase angle. Application of the Tayfun method is rejected since hourly water level data had to be digitized over a period of several years. As a list of storm events, exceeding the storm level of NKD+6.5m was available from 1900 on, this information was used to determine surge characteristics.

4. Determination of surge

4.1. Predicted tide during storm

For all storm periods predicted levels of high and low tide were available, as well as the time of occurrence. In order to avoid harmonic calculation of tidal levels, tide is reconstructed using the typical (mean) tidal cycle of the period 1961-1970 (8). This curve is transformed both in time and height (so that in the extremes coincide with the predicted values) by linear interpolation. In this way the tidal component during the storm is reconstructed similar with the typical tidal curve.

4.2. Surge

To determine the surge from the observed levels and from the tidal elevations, several methods were examined. The Dutch Deltacommission (32) suggests to subtract predicted tide from observations which leads to inadequate surge levels, since the interaction between storm and tide causes transformations in the tidal phenomenon. Theuns (46) draws a smooth curve S through the registered storm water levels R, so that the area above and below S and limited by R are equal one to the other. Subtraction of mean water level from curve S gives the surge event. This is a practical method, producing reliable information in regions where mean water level doesn't vary with time. Due to the secular and periodic variations (Fig. 3) on the river Scheldt this method was rejected. Schalkwijk (42) proposes a method based upon harmonic analysis and recalculation of tidal levels. Subtraction of these levels from the observations and application of forward and backward means over 3 hours each to eliminate the M4 lunar component, produces a reliable approximation of the surge in the "linear" parts of the tidal cycle. To improve the surge residue in the neighbourhood of the extremes a flattening procedure is developed, based upon a trial and error method to draw a smooth curve through the surge residue. method was also rejected for it requires a harmonic tidal calculation and because of the 'trial and error'-nature of the graphical flattening procedure. Janssens & Sas (18) propose a numerical technique. Subtraction of predicted tide, calculated in the aformentioned way (§ 4.1.), from the observed levels doesn't account for tide-surge interaction. Since no quantative data are available on this interaction mechanism, the predicted tide is moved through the observations until no periodically oscillating surge residue is obtained. In this way the tidal 'advancement' due to storm is accounted for. The flattening procedure is realized by an approximation of the surge residue by a third order polynomial (higher order approximations cause inflexions and successive maxima), determinded by the least square method and applying a Lagrangion multiplier so that the area below the resultant surge equals the one below the first non-oscillating surge approximation

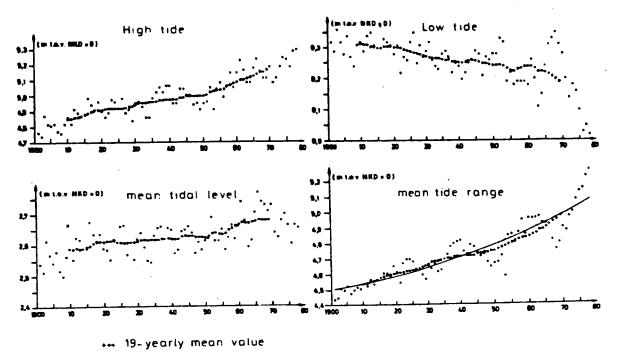


Fig. 3: Variation of tidal variables

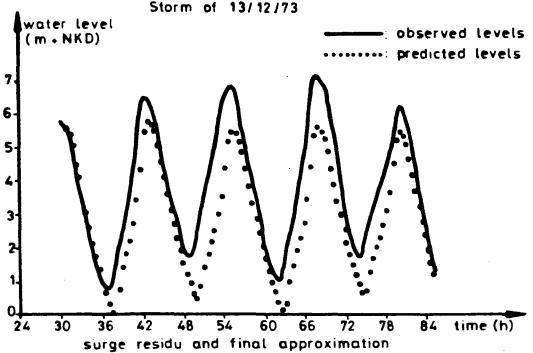
(Fig. 4). For 33 storm periods the surge elevation was calculated in this way. It was discovered that there was no tendency at all for the maximum surge to occur at high tide, nor at any other stage of the tidal cycle, which is in strong agreement with the statements of Heaps (15) and Wemelsfelder (32). The resultant duration and magnitude for each surge was derived: 'Duration' is determined (with an accurracy of 0.5 hour) by comparing the time between the zero-points of the surge, the time between the minima of the surge and the time between the points of coıncidence of predicted and observed water level. The 'magnitude' is assumed to be the maximum of the polynomial. After checking the dependency of both characteristics, it seems that there is no linear correlation (coefficient = 0.013). Hence magnitude and duration are treated as statistically independent stochastic variables. The probability distribution functions are a normal distribution (μ = 42.5 h, σ = 11.6 h) for the duration and a Gumbel distribution

$$p(x) = \alpha \exp \{-\alpha(x - u) - \exp [-\alpha(x - u)]\}, u = 1.32, \alpha = 3.04$$
 for the magnitude.

4.3. Frequency of occurrence of storms

On the basis of the information concerning the penetration of storms in the Scheldt estuary (50 arrivals in 77 years) a uniform distribution of these storms was assumed. The time interval Δt of this Poisson process has a distribution function

$$p(\Delta t) = a \exp(-a\Delta t)$$



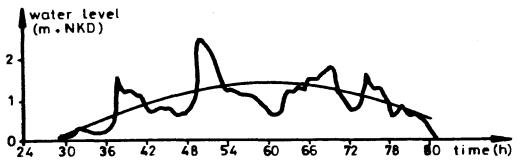


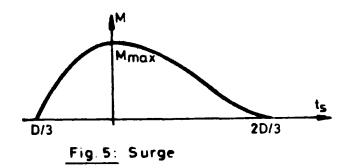
Fig. 4: Determination of a storm surge

with a = $1/E(\Delta t)$. Since more storms occur during winter (October till March): $E_{\text{winter}}(\Delta t) = 10.74$ winter months and $E_{\text{summer}}(\Delta t) = 66$ summer months (or 1 storm occurs in summer during 11 years).

4.4. Simulation of surge

From the set of surges, determined as explained in § 4.2., the most suitable shape was found to be a polynmial of order 3. For a surge of duration D and magnitude M_{max} , the component reads (Fig. 5)

$$M(t) = 6.75 M_{max} \left[\left(\frac{ts}{D} \right)^3 - \left(\frac{ts}{D} \right)^2 \right] + M_{max}.$$



5. Determination of tidal variables

As described in § 3.1. and illustrated in Fig. 3 tidal variables vary with time. Since the simulated tide is characterized by mean water level and tidal amplitude, the appropriate values of those variables must be determined. To eliminate secular, i.e. nodal, variations 19-yearly (forward and backward) mean values are calculated and are used to estimate actual values of both mean tidal level (= NKD+2.70m) and tidal amplitude (= 2.54 m). To account for variations of amplitude between neap and spring tide (= luni-solar fortnightly M_{Sf} constituent) with a period of 14.76 days, the tidal amplitude is assumed to vary in a sinusoidal way with amplitude = 0.42 m. Variations in tidal amplitude of periodic nature are determined from annual mean amplitudes so that relative magnitude of all effects can be analysed:

amplitude (t) = A cos
$$(\frac{2\pi}{T} t)$$
 + B sin $(\frac{2\pi}{T} t)$ + C + DT [12]

(where T = period of variation, t = time in years, C + DT is the trend due to secular variation). Using the method of least squares, the most important long term period can be determined. For the river Scheldt the nodal tide (T = 18.67 years, amplitude = 0.06 m) was found to be the most important). If tide is simulated by a simple sinusoidal variation; the tidal constituent during the surge event can be represented by

$$G(t) = A(t) \sin \left(\frac{2\pi}{12.42} t_{s} + \varphi_{o}\right)$$

$$= (\overline{A} + A_{nodal}^{*} + A_{Msf}^{*}) \sin \left(\frac{2\pi}{12.42} t_{s} + \varphi_{o}\right)$$

$$G(t) = [\overline{A} + A_{nodal} \sin \left(\frac{2\pi}{18.67} t\right) + A_{Msf} \sin \left(\frac{2\pi}{14.76 \times 24} t_{h} + \varphi_{Msf}\right)]$$

$$\cdot [\sin \left(\frac{2\pi}{12.42} t_{s} + \varphi_{o}\right)]$$
[13]

(with t = time in years since the beginning of the simulation, t_h = hours since the surge in started, φ_M = phase angle of the M contituent, sf

A = annual mean amplitude, A nodal = nodal amplitude, A = luni-solar

fortnightly amplitude). For Antwerp $\overline{A} = 2.54 \text{ m}$, $A_{\text{nodal}} = 0.06 \text{ m}$, $A_{\text{m}} = 0.42 \text{ m}$.

6. The simulation model

In this model tide during the surge is described as observed during recent years, including periodic variations over a nodal period. Surge and interaction are treated together and their magnitude is based upon observations of the last century. From the probability distribution functions and the frequency distribution, the model parameters to reconstruct all components are generated by random numbers. These parameters, treated as stochastic variables are: Δt , D, $M_{\rm Max}$ and $\varphi_{\rm O}$. Superposition of $Z_{\rm O}$, G(t) and M(t) eacht 10 minutes (according to eq[1]) during a storm (Fig. 6) leads to a series of n maximal storm levels $H_{\rm max}$ over a range of j simulated years. The return period T of a level $h_{\rm o}$ is calculated as j/m, with m the number of surges exceeding level $h_{\rm o}$:

$$T(h_0) = \frac{\text{simulated years}}{\text{number of storms with } H_{\text{max}} > h_0}$$
 [14]

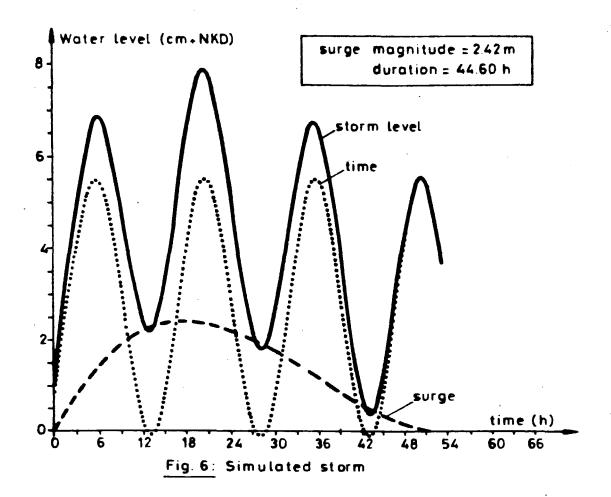
To obtain suitable results:

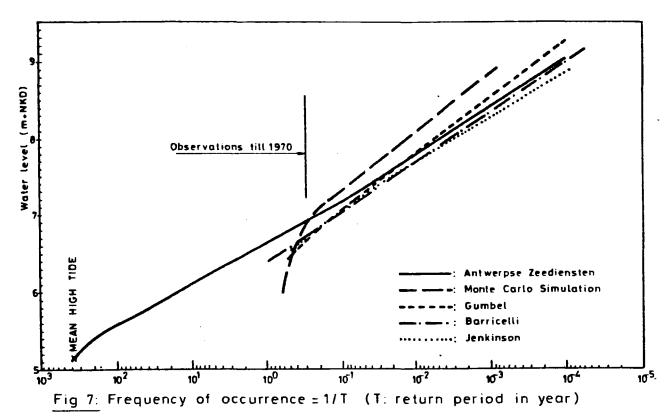
- the simulation model must use enough random numbers so that the results are not influenced by the initial random number
- the simulation must cover a span of years of at least 10 times the largest requested return period
- the simulation results must be compared with historical data and if possible with other calculation methods.

Furthermore it seems obvious that the model can not give more information or more accurate data than those included in the set of observations. Hence one should try to collect as much data as possible to increase the reliability of the model parameters and their probability distributions. For the river Scheldt near Antwerp 10000 years were simulated, in which 5952 storms exceeded the lower limit level of NKD+6m. The results are given in Table 1 and Fig. 7.

7. Comparison of results - conclusions

The results in Table 1 indicate that the simulation model gives a higher water level than the extreme value distributions, for the same return period. Since the frequency of occurrence of high levels increases with time (Table 2), one should account for trends in determining the frequency of occurrence of extreme water levels. Classical extreme value distributions assume the series of annual maximal water levels as homogeneous, and thus underestimate the frequencies. To avoid inhomogeneity, trends must be corrected for, but one must be aware of the different possible estimates of these trends (Table 3) and the influence on the water levels corresponding to several return periods. The proposed simulation technique avoids these difficulties by using tidal variables based upon recent years observations and by simulating surge events as they happened over the last century. Examination of Table I shows that the extreme value distributions give estimates of water levels which become higher the more recent years are considered or the more a homogeneous set of annual maxima is used, whilst the estimates derived from the simulation model are greater than the pure annual maxima results but smaller than the estimates obtained after correction for trends.





The simulation technique overcomes the difficulties of applying extreme value distributions. Compared with the joint probability methods it avoids the problem of correlated residuals of the convolution method by considering surge duration and magnitude. It has been illustrated that in comparison with the Tayfun method, detailed observations are required in storm periods and not hourly values over 10 or 15 years. In this way laborous work of data collection is avoided. Compared with the simulation model the proposed technique has the big advantage of not using a hydrodynamic model, since surges are simulated depending on surge-characterizing parameters and not on surge-causing parameters. Although it remains difficult to draw any firm conclusions this simulation technique seems quite promising, espcially in estuaries where shallow water components influence surge propogation so that hydrodynamic models become complex, and where trends in annual maxima are apparent. The technique serves to calculate a best estimate for extreme water level frequencies, which is a problem for which no definitely correct answer exists.

Table 1: Water Levels h_0 (m + NKD) and corresponding return periods xextrapolation of results calculated till T = 1000 yr.

		Return period (years)					
	1901 - 1978	10	50	100	1000	10000	
	Simulation	7.29	7.82	8.06	8.85	9.58 ^X	
	Barricelli	ļ	7.03	7.48	7.68	8.33	8.99
	Jenkinson		7.01	7.47	7.56	7. 87	7.94
1	Trend	Case					
- 1	(m/year)						
ĺ	0.000	(0) no adaptation	7.11	7.66	7.88	8.64	9.40
- 1	0.0029	(1)	7.20	7.72	7.94	8.66	9.38
I	0.0032	(2)	7.21	7.73	7.94	8.66	9.38
Ì	0.00304	(3)	7.21	7.72	7.94	8.66	9.38
1978	0.0025	(4)	7.18	7.71	7.93	8.65	9.38
2	0.0040	(5)	7.24	7.75	7.97	8.68	9.39
th reference	0.0020	(6)	7.17	7.70	7.92	B.65	9.37
	0.0050	(7)	7.27	7.78	7.99	8.71	9.42
	0.0014	(8)	7.15	7.68	7.91	8.64	9.38
	0.0046	(9)	7.26	7.77	7.98	8.70	9.41
	0.009/0.0067/0.0020	(10)	7.18	7.70	7.92	8.65	9.39
with	0.0037/0.009	(11)	7.33	7.83	8.04	8.7 5	9.45
	0.0070	(12)	7.35	7.86	8.07	8.78	9.49
E T	0.0096	(13)	7.46	7.98	8.20	8.93	9.65
maxima,	0.0084	(14)	7.41	7.92	8.14	8.86	9.57
1,	0.0071	(15)	7.35	7.86	8.08	8.79	9.50
שתו	0.0118	(16)	7.57	8.11	8.33	9.09	.9.84
homogenéous annual	1941 - 1978						
neor	0.0000	(17)no adaptation	7.29	7.87	8.12	8.93	9.74
noge	0.0046	(18)	7.35	7.92	8.15	8.94	9.72
+	1951-1978						
	0.0000	(18) no adaptation	7.36	7.94	8.19	9.01	9.83
CUMBEL	0.0046	(20)	7.41	7. 9 9	8.23	9.04	9.84
ಕ	0.0092	(21)	7.47	8.04	8.28	9.08	9.88

Table 2: Frequency of occurrence (number of tides exceeding level h)

Waterlevel	1901	1911	1921	1931	1941	1951	1961
h _o	-	-	-	-	-	-	_
m + N.K.D.	1910	1920	1930	1940	1950	1960	1970
5	211	297	319	359	383	399	471
5,5	29	40,7	40	41	75	86	131
6	3,2	3	4,2	4,2	5,1	B,4	14,7
6,5	0,5	0,5	0,3	0,6	0,6	0.7	1,4

Table 3: Trends in tidal observations

E	Period	Mean tidal level	Trend (cm/c Mean see level	-	e Annual maximum	Reference
vlissingen	1900-1962 (x) 1890-1962 1880-1970	. +29(1) +25(4)	+30.4(3)	+32(2)		20 26 61
	1888-1970 1910-1969(m) 1910-1954(m) 1955-1961(m) 1961-1969(m) 1955-1969(m) 1955-1978	+20(6) +14(8) +9 +67 (10) +20		+50(7) +46(9) +37 (11	70(12)	61 20 20 20 20 20 20
Antwerpen	mean minimum	Mean values till 1925 t 5 year +96(13) +70	18 18			
Ĺ	maximum	+118(16)	+105	+108	+73	18

⁽x) 19-yearly mean values; data cover a period of 9 years forward and backward to the indicated period.

^() case number of adaptation to obtain homogeneous data (see also Table 1).

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