A semi-analytical morphodynamic model of the Western-Scheldt

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delft hydraulics

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1 Introduction

The Directoraat-Generaal Rijkwaterstaat/Dienst Getijdewateren of the Ministry of Public Works and Transport (RWS/DGW) is interested in morphological models predicting the consequences of (human) interference, such as dredging and land reclamation, on the geometry of estuaries.

RWS has commissioned DELFT HYDRAULICS to perform various studies on fundamental problems in 1D morphological modelling. A long-term model called ESTMORF, which predicts morphological developments over a period of 50 to 100 years, is studied in the DYNASTAR project. The implementation of this model is currently carried out. The middle long-term model EENDMORF, which predicts morphological development over a period of 20 to 30 years, is also studied in the DYNASTAR project. So far, this study has been focused on stability problems of 1D morphological network models.

By letter NWL 6570 dated July 5th, 1993, RWS/ZL commissioned DELFT HYDRAULICS to carry out a study on a semi-analytical morphodynamic model of the Western-Scheldt as a part of the DYNASTAR and MAST G8 project. The study is partly funded by the Commity of the European Community. The results of this study are described in this report.

1.1 Morphological models of estuaries

An estuary is the transition area between a river and a sea. It usually has a very complicated geometry: networks of ebb channels and flood channels; shoals and tidal flats; meanders, etc. This means that estuaries are, morphologically, very active. It is not easy, therefore, to build accurate morphological models of estuaries.

Three different types of morphological models of estuaries can be distinguished (Karssen and Wang, 1992):

- Empirical models, based on empirical relations for morphological quantities.
- Dynamic-empirical models, based on empirical relations for morphological quantities, combined with hydrodynamic models derived from physical laws.
- Morphodynamic models, based on hydrodynamic models combined with equations of sediment transport.

The difference between these three types is to what extent they consider the dynamics of the estuary. Empirical models only consider equilibrium equations, i.e., dynamical processes are not incorporated in these models. Dynamic-empirical models compute water flow and derive morphological quantities from it, i.e., hydrodynamic processes are incorporated but sediment transport is not. Finally in morphodynamic models, both water flow and sediment transport are computed.

Morphodynamic models take processes with the smallest time-scales into account, therefore these models seem to be preferable. However, they require the largest computational effort, which is why morphodynamic models are still in their infancy. More specifically, this is due to the following problems:

- The tidal motion requires relatively short time-steps, much shorter than the time-steps for sediment transport because morphological changes occur on a long time-scale.
- There is insufficient knowledge about the dominant conditions in estuaries, so that it is hard to decide what should be put in the model and what can be left out.

This report describes a morphodynamic model of an important estuary: the Western-Scheldt. The model is based on a semi-analytical method, proposed by Krol (1990), which requires little computational effort. The method is designed to solve the first problem and to enhance the knowledge of morphodynamic models. The aim of the model is to study morphological scales: the time-scale of morphological changes due to (human) interference and the spatial scale on which the development occurs. A second aim is to test Krol's method for an actual estuary. Up to now the method has only been tested for schematic cases (Krol, 1990; Fokkink, 1992 and 1993).

The aims have been reached partially. The model does give insight into the time-scale of morphological development in the Western-Scheldt. It also gives insight into the consequences of dredging and depositing. However, Krol's method does not succeed in modelling the water flow in the Western-Scheldt correctly. The main problem is to calibrate the M_4 harmonic of the tide.

1.2 Acknowledgements

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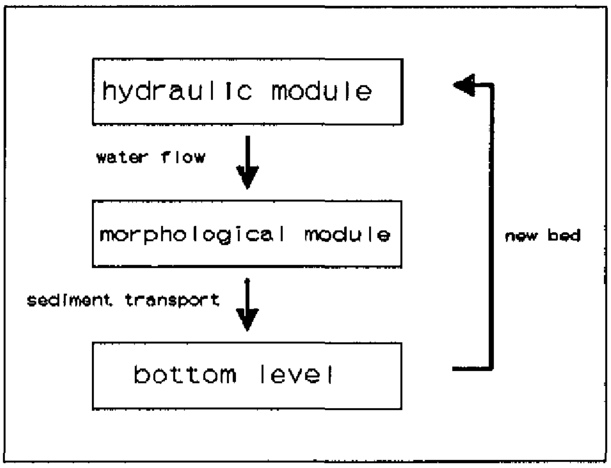
2 The morphodynamic model

Morphodynamic models consists of two parts: a hydraulic module and a morphological module. The hydraulic module describes water flow, whereas the morphological module describes sediment transport. Schematically, the model works as follows:



The modelling problem is mainly in the morphological module, because the equations for the morphological evolution of estuaries are not yet well described. In contrast, the computational problem is in the hydraulic module. The equations of water flow, the shallow-water equations, are well known, but in the standard hydraulic models the solution requires time-steps in the order of a minutes. For morphological computations, which involve time-spans of decades or centuries, this time-step is very short. Krol (1991) has proposed an efficient method to solve a simplified version of the equations of water flow. His method reduces the computational effort drastically.

2.1 The morphological module

Morphological changes are slow. The net transport into the Western-Scheldt, for instance, is in the order of a million m^3 of sediment every year. The Western-Scheldt is about 80 km long and, on average, about 2.5 km wide. The area of the bed is in the order of 200 km^2. The average bottom change, therefore, is in the order of 1.000.000/200.000.000 m = 0.5 cm a year. Some regions are morphologically more active than others, so, this is a very crude estimate. It does show, however, that the time-scale for morphological changes is large.

One of the most common sediment transport formula's in river engineering is Engelund en Hansen's power law

$$S = B_s M u^5 \tag{1}$$

S =	sediment transport	[m^3/s]
B_s=	flow width	[m]
M =	transport coefficient	[s^4/m^3]
u =	flow velocity	[m/s]

This formula is simple and accurate for total-load transport in a non-tidal river. For oscillating flow in tidal rivers, sediment transport is not so easy to describe. We use a slightly modified Engelund-Hansen formula

$$S = B_s M u^5 (1 + \alpha a_x) \tag{2}$$

α = down-slope coefficient, its sign depends on the direction of the water flow.
a = depth of the water

The extra term αa_x expresses that sediment is more easily transported downward than upward. This is called the down-slope effect. It has a stabilizing effect because the down-slope effect tends to fill the holes in the bed.

The equation of continuity of sediment is

$$S_x - a_t = 0 \tag{3}$$

Equations (2) and (3) form the core of the morphological module. Two boundary values are required, both at the upstream and downstream boundary, because sediment is transported into the estuary at both sides. We choose a fixed depth of the bed at both sides of the estuary.

The change of bed level over one period of the tide is:

$$\Delta a = \int_0^T S_x \, dt \tag{4}$$

T = tidal period = 44700 s

Substitution of the power law (2) yields

$$\begin{aligned} \Delta a &= \int_0^T 5 M u^4 u_x (1 + \alpha h_x) + M u^5 \alpha h_{xx} \, dt \\ &= 5 M (1 + \alpha h_x) \int_0^T u^4 u_x \, dt + M \alpha h_{xx} \int_0^T u^5 \, dt \end{aligned} \tag{5}$$

The down-slope effect induces a diffusion term in the equation. The diffusion is negligible if the bed is smooth, but it can be considerable at sudden jumps in the level of the bed.

2.2 The hydraulic module

The morphological time-step may be large, but the hydraulic equations require much smaller time-steps. The flow in the estuary is oscillating with a tidal period of about 12 hours and 25 minutes. To compute the shallow-water equations accurately requires a time-step in the order of a few minutes. One tidal period requires in the order of a hundred time-steps. Moreover, in most hydraulic models it is necessary to compute a few tidal periods before the influence of the initial condition disappears. All in all, the hydraulic module requires much more computational effort than the morphological module. It is necessary to reduce the computational effort in some way.

In 1918, the Dutch government called upon a committee to plan the construction of a dam across the Zuiderzee, an inland sea in the Netherlands, to disconnect it from the Wadden Sea. The main task of the committee, presided by the well-known physicist H.A.Lorentz, was to predict the water level in the Wadden Sea, and thus calculate the required level of the dam, after the closure of the Zuiderzee. In those days, before the invention of the computer, this was not an easy task and Lorentz himself simplified the equations of water flow, so that the calculations could be done by hand. Once the dam was built, the calculations turned out to be very precise, predicting the actual water level up to a few centimetres.

Lorentz' method to compute water flow, also known as the harmonic method, is now out of date in large-scale hydraulic models, as computers can handle more intricate equations. However, for morphological computations, which extend over a span of many years and require many calculations of water flow, it still is attractive. Krol (1990) has proposed a refinement of Lorentz' method, which takes the generation of the M_4 tide into account, as well as the variability of the bed level. This is the method which is used here.

2.3 The Lorentz method

The tidal wave, which propagates into the Western-Scheldt, is a composition of infinitely many harmonic constituents. The idea of the harmonic method is to decompose the tidal wave and describe the propagation of each harmonic constituent separately. It is impossible to take all constituents into account, as this would require too much computer time. In our model of the Western-Scheldt, only the semi-diurnal constituent M_2 and the higher harmonic M_4 are implemented. These are the two most important constituents for the morphological development.

The following simplified equations describe the water flow in one dimension. The equation of continuity:

$$\frac{\partial Q}{\partial x} + B\frac{\partial h}{\partial t} = 0 \tag{6}$$

and the equation of motion

$$\frac{\partial Q}{\partial t} + \frac{\partial}{\partial x}\left(\frac{Q^2}{B_s(a+h)}\right) + g\,B_s\,(a+h)\,\frac{\partial h}{\partial x} + \beta Q = 0 \tag{7}$$

Q = water flux [m^3/s]
h = water level [m]
a = depth of the water [m]
B = total width [m]
B_s = flow width [m]
β = linearized friction coefficient [1/s]
g = gravitational acceleration [m^2/s]

The tidal wave is periodic, so Q and h can be described by a Fourier series

$$Q = Q_1 e^{i\omega_1 t} + Q_2 e^{i\omega_2 t} + Q_3 e^{i\omega_3 t} + \dots \tag{8}$$

$$h = h_1 e^{i\omega_1 t} + h_2 e^{i\omega_2 t} + h_3 e^{i\omega_3 t} + \dots \tag{9}$$

The complex functions Q_1, Q_2, Q_3, h_1, h_2, h_3 depend on place but not on time. Only the real part of the functions has physical meaning. The idea is to solve equations (6) and (7) for each constituent $e^{i\omega t}$ separately.

We consider only the harmonics M_2 and M_4, which are most important for the morphological development in the Dutch tidal waters. The period of M_2 is twice the period of M_4.

$$Q = Q_1 e^{i\omega t} + Q_2 e^{2i\omega t} \tag{10}$$

$$h = h_1 e^{i\omega t} + h_2 e^{2i\omega t} \tag{11}$$

ω = angular velocity = $2\pi/T$
i = complex unity

M_2 is the main component of the lunar tide, M_4 is its first higher harmonic, generated by inertia. In equation (7), inertia is represented by the second term, containing Q^2 of angular velocity 2ω.

Substitute (10) and (11) into the equations of water flow (6) and (7). This gives two sets of equations: one for Q_1, h_1 and one for Q_2, h_2.

$$\frac{\partial Q_1}{\partial x} + B\, i\, \omega\, h_1 = 0 \tag{12}$$

$$(\beta + i\omega)\, Q_1 + g\, B_s\, a\, \frac{\partial h}{\partial x} = 0 \tag{13}$$

Equations (12) and (13) are linear, homogeneous equations. Numerically they are easy to handle.

For Q_2, h_2, the equations are almost the same. There are two extra terms at the right hand side of (15), representing inertia and a correction on the pressure.

$$\frac{\partial Q_2}{\partial x} + B\, i\, \omega\, h_2 = 0 \tag{14}$$

$$(\beta + i\omega)\, Q_2 + g\, B_s\, a \frac{\partial h_2}{\partial x} = -2 \frac{Q_1}{B_s a} \frac{\partial Q_1}{\partial x} - g\, B_s\, h_1 \frac{\partial h_1}{\partial x} \tag{15}$$

There are two physical boundaries to the estuary: downstream the water level is ruled by the tide; upstream the influence of the tide is negligible. This is translated into the following boundary conditions:

$$\begin{aligned} h_1(0) &= A_2 e^{i\omega t - \theta_1},\ Q_1(L) = 0, \\ h_2(0) &= A_4 e^{2i\omega t - \theta_2},\ Q_2(L) = 0 \end{aligned} \tag{16}$$

L =	length estuary	[m]
A_2 =	amplitude M_2 at sea-side	[m]
A_4 =	amplitude M_4 at sea-side	[m]
Θ_1 =	phase angle M_2 at sea-side	
Θ_2 =	phase angle M_4 at sea-side	

The equations (12), (13), (14), (15) with these boundary values constitute the hydraulic module.

2.4 Numerical implementation of the model

The grid is linear, it contains fifty points. The distance between the points is 3 km. Three equations need to be solved numerically: equation (5) and the two pairs of equations (12), (13) and (14), (15). They are discretized by finite differences. The equation of continuity of sediment, Equation (5), integrates the gradient S_x. This gradient is determined numerically by a second order upwind scheme. The direction of the upwind scheme is the direction of the flow.

By cross differentiation, equations (12) and (13) reduce to the one-dimensional wave equation

$$g B_s a \frac{\partial^2 Q_1}{\partial x^2} - \frac{g B_s a}{B} \frac{\partial B}{\partial x} \frac{\partial Q_1}{\partial x} - (\beta + i\omega) B i \omega Q_1 = 0 \tag{17}$$

which is solved by central differences. The central difference matrix is tridiagonal. Equations (14) and (15) are treated the same way.

3 Results of the simulations

This section contains the results of a few simulations. The first simulation does not concern the Western-Scheldt, but demonstrates two morphological extremes: a tidal basin, for which the upstream boundary is closed, and a tidal river with a river discharge which dominates the tide completely.

3.1 Simulation 1: two schematic cases

The first simulation shows how the model works. Two cases are considered:
a tidal basin and a tidal river. For the tidal basin the boundary values are as follows:

Length of the basin	=	10 km
Initial bed level	=	5 m below sea-level
Amplitude M_2	=	1 m
Amplitude M_4	=	0.1 m
Widths : B	=	1000 m
B_s = 500 m		
Friction coefficient β	=	0.001 s^{-1}
Transport coefficient M	=	3 10^{-4} s^4/m^3

There is only one important parameter for the tidal basin: the phase lag between M_2 and M_4, which determines the direction of transport. If the tide is flood dominated, sediment is transported into the tidal basin because the velocity of the water flow is higher during flood; if the tide is ebb-dominated, sediment is transported out of the basin. In this simulation the tide is flood dominated. The amplitudes of the M_2 and M_4 components determine the magnitude of the transport and thereby the length of the morphological time-scale.

The result of the simulation is demonstrated in Figure 1.1. A wave of sediment slowly penetrates in the estuary. After 80 years, the front of the wave has gone 4 km beyond the mouth of the basin. It takes much more time before it finally reaches the end. The equilibrium position of the bed is linear, of depth 5 m at the mouth and 0 m at the end of the basin.

The second example is a tidal river with a large river discharge. Upstream the discharge is constant and the length of the tidal river should be chosen sufficiently large so that the tide is damped out.

Length of the river	=	150 km
Initial bed level	=	5 m below sea-level
Amplitude M_2	=	1 m
Amplitude M_4	=	0.1 m
Widths: B	=	500 m
B_s	=	500 m
Friction coefficient β	=	0.001 s^{-1}
Transport coefficient M	=	3 10^{-4} s^4/m^3
River discharge	=	1000 m^3/s

Again the tide is chosen to be flood dominated. However, this does not cause a net transport into the estuary because the river flow dominates the tidal flow in this example. The important parameters are the velocity of the flow upstream and the amplitude of the tidal velocity downstream. So, only the river discharge and the M_2 component are important. Furthermore, the friction coefficient has to be sufficiently large, otherwise resonance might occur.

The result of the simulation is demonstrated in Figure 1.2. The bed grows towards an equilibrium position, more or less exponentially shaped. So, in both examples, there is an equilibrium position of the estuary. For the tidal basin, it is a linear bed, for the tidal river it is an exponential bed. This is the general result for these schematic cases, e.g. (Fokkink, 1992), (de Jong, 1992) and many others.

Both the tidal basin and the tidal river are very stable. There are one or two parameters which determine the evolution, M_2 and M_4 in the tidal basin, M_2 and river flow in the tidal river, and all other parameters are of minor importance. For the Western-Scheldt, the *important morphological factors are much harder to determine.*

3.2 Simulation 2: the Western-Scheldt

The Western-Scheldt is not a tidal basin, because there is a small river discharge. Neither is it a tidal river, because the river discharge is of no importance at the mouth of the estuary. Moreover, unlike the previous two examples, sediment is transported into the estuary at both boundaries.

Boundary values

The tidal components at the mouth near Vlissingen are:

$$\begin{aligned} M_2: &\quad 1.8\cos(\omega t - 1.03) \quad [m] \\ M_4: &\quad 0.14\cos(2\omega t - 2.04) \quad [m] \end{aligned} \tag{18}$$

The tide at Vlissingen contains many significant harmonics. The most important generating harmonic is M_2 with an amplitude of 1.74 m. The other semi-diurnal harmonics which generate the tide are N_2, of amplitude 0.29 m, and S_2, of amplitude 0.48 m. The most important higher harmonics are M_4, of amplitude 0.13 m, and MS_4, of amplitude 0.09 m. For morphological computations, the interaction between the semi-diurnal harmonic and the first higher harmonic are most important.

The tide at Vlissingen is flood dominated: the average flood period takes about 6 hours, and the average ebb period takes about 6,5 hours (Allersma, 1992). The M_2-M_4 tide, however, is not flood dominated. As these are the only two harmonics in the model, the harmonics M_2 and M_4 have to be adjusted. The harmonics are chosen such that the average tide at Vlissingen is simulated. Compared to (18), the amplitude of M_2 is put at 2.0 m instead of 1.8 and the phase angle of M_4 is 0.47 instead of 2.04, a phase shift of 90°.

The upstream river discharge of the river Scheldt is about 105 m^3/s. Together with the water level at Vlissingen, this constitutes the boundary values for the model.

The geometry of the model is as follows. The flow width is equal to the storage width. The estuary is funnel-shaped, which means that the width decays exponentially. At the mouth of the estuary it is 5 km wide. It decays to 100 m at the upstream boundary, which is chosen at 150 km. This strong funnel shape is an important factor in the model.

$$\begin{gathered} B(x) = B_0 \exp(-2.61\ 10^{-5}\ x),\ B_0 = 5000\ m \\ B_s = B,\ L = 150\ km \end{gathered} \tag{19}$$

The initial bed level of the model is exponential as well. It runs from -12 m at the mouth to -5 m at the upstream boundary. The x-coordinate at the mouth is 0, at the upstream boundary it is 150 km.

$$a(x) = -12 \exp(-5.84\ 10^{-6}\ x) \tag{20}$$

The geometry of the model is depicted in Fig. 2.1.

Calibration of the model

The only free variable to calibrate the water flow is the friction coefficient β. In the Lorentz model it is calculated by an iterative method. In our model, it is a parameter to calibrate water flow.

The model is calibrated mainly on the amplitude of the semidiurnal tide and the amplitude of the first overtide. The propagation of M_2, S_2 and N_2 is of the same character: the amplitude increases up to Bath, and then decreases until it is almost damped out at the upstream boundary. In contrast, the amplitude of the first overtide, notably M_4 and MS_4, stays more or less the same. So, the damping of the harmonics is different and in the model this is met by a different β for each harmonic. Beyond Antwerp, the magnitude of the harmonics decays. So, the damping increases and β should be chosen dependent on place.

The coefficient β is chosen as in Fig. 2.2: almost no bottom friction at the mouth, slightly increasing bottom friction up to Antwerp and strong bottom friction beyond Antwerp. Up to Antwerp, the friction parameter β/ω is less than 1, which means that there is almost no damping.
Beyond Antwerp, the parameter β reaches an exceptionally high value. This is the only way in which the tide can be damped out in the model, as it does in reality.

The amplitudes correspond well with reality, see Fig 2.3.a, which means that the magnitude of the sediment transport should correspond well with reality. The phase angles, however, do not correspond well with reality, see Fig. 2.3.b. In the model, the phase angle of M_2 approximates reality, but the phase angle of M_4 does not. As a consequence, the tide in Antwerp is ebb dominated in the model, whereas in reality it is flood dominated. A possible explanation for this deviation of the model is given in the Appendix.

Since the amplitudes of the harmonics is calibrated well, sediment transport has the right magnitude. However, since the phase angle is not calibrated well, the residual sediment transport over one period of the tide does not have the right direction. This means that the time-scales for morphological development in the model should coincide with reality, see Simulation 3 and 4, but that erosion and sedimentation in the model occurs at places which may not be realistic.

Results of the simulation

The morphological development in the model turns out to be different from the morphological development observed in nature. This is demonstrated in Fig. 2.5. About 15 km beyond Vlissingen, an ever growing bar appears. After thirty years, the simulation has to be stopped because the morphological time-step is too large: wiggles appear in the bar. Because the M_4 component is shorter in the model, the wave of sediment penetrates less far in the estuary and instead of building up a bar closer to Antwerp, it builds up a bar close to the mouth of the estuary.

The model behaves like a combination of the tidal basin and the tidal river from Simulation 1. At the mouth it behaves like a basin and beyond the bar it behaves like a tidal river. There is no morphological activity beyond 100 km. Due to the large friction coefficient, M_2 and M_4 are small, and the storage width is small compared to the cross section.

The residual sediment transport is shown in Fig. 2.6. The transport into the estuary at the mouth is about 3.5 thousand m^3 per tidal period, which amounts to 2.5 million m^3 per year. This is of the same order as observed in nature. The bar builds up at the place where sediment transport changes its direction, i.e., where it changes sign. The change of sign gets ever more sharp during the simulation, and it moves to the left. Apparently, the model tries to push the bar out of the estuary, but it is stopped by the boundary condition at the mouth: the flood dominant tide causes a perpetual transport into the estuary.

The choice of the initial bed hardly makes any difference. No matter what bed, the bar invariably builds up at about 15 km. The choice of the width of the estuary, however, is of more importance. A different geometry of the estuary is shown in Fig. 2.7. and the development of the bed in this estuary is shown in Fig. 2.8. The bar now is not pushed to the mouth, but stays in the same place, slightly further into the estuary. Because the storage area is larger at the mouth, the residual transport into the estuary is larger. Therefore, it reaches farther into the estuary and that is why the bar builds up at a greater distance from the mouth. The qualitative behaviour is the same: the estuary is split in a tidal basin and a tidal river.

3.3 Simulation 3: changing storage width and flow width

In this simulation, the flow width and the storage width is changed between Hansweert and the border, that is, between 30 and 60 km upstream from Vlissingen.

In Fig. 3.1 the result is shown if the flow width is decreased by 30%. The velocity of the water increases accordingly by about 30%. Sediment transport is proportional to the fifth power of the velocity, and therefore it increases by about 370%. As a result, the bed erodes very fast as can be seen in Fig. 3.1.
A bar builds up at the downstream end, 30 km from the mouth. This bar grows and finally reaches the mouth of the Western-Scheldt. It is clear from Fig. 3.2 that the model tries to push the bar out of the estuary, just like in Simulation 2. At the upstream boundary, 60 km from Vlissingen, hardly anything happens.

In Fig. 3.3 the storage area of the estuary has been increased by 1000 Ha near the border, between 54 and 63 km from Vlissingen. The result is about similar: the velocity goes up and therefore the bed erodes fast, around 60 km from Vlissingen. Again a bar appears at the downstream border, about 50 km from Vlissingen, and the bar is pushed towards the mouth of the estuary.

3.4 Simulation 4: dredging and deposition

In the Western-Scheldt, bars build up at the end of the channels. To keep the channels navigable, the bars are dredged continually. It is important to investigate the consequences of dredging.
In our model, there is only one bar, at about 15 km from the mouth. Therefore it does not make sense to simulate the actual dredging of the Western-Scheldt, which takes place at about 30 to 50 km from the mouth.

It appears that it requires very intense dredging to deepen this bar, in the order of 100 million m^3 of sediment per year. This is much more than the actual dredging in the Western-Scheldt, which is in the order of 10 million m^3 and does not take place at 15 km from Vlissingen, but at about 50 km upstream.

In Fig. 4.1 it is demonstrated what happens if the dredging is 3 million m^3: nothing happens. In Fig. 4.2 dredging is 6 million m^3 between 0 and 20 km from the mouth. In Fig. 4.3 the dredged sediment is deposited 30 to 50 km from the mouth.

3.5 Conclusions from the simulations

The main purpose of this report is to get more insight into morphological models of estuaries. As Simulation 2 shows, the water flow in this model, and hence the sediment transport, does not correspond to the actual flow in the Western-Scheldt. The results of Simulation 3 and 4, however, do give insight into the time-scales of the morphological processes. This section reviews the results of the simulations.

As Fig. 3.1 demonstrates, decreased flow width leads to a deepening of the channel. The deepening is proportional to the narrowing. The bed in the narrow channel is of constant slope. At the upstream end, a bar builds up. The morphological time-scale of the initial response is 10 years.

The response to increased storage width is about the same, Fig. 3.3. The channel deepens and a bar builds up at the end of the narrowing. The morphological time-scale of initial response is 10 years. The bed is not of constant slope, but it deepens towards the end. The response to increased storage area is stronger at the upstream end.

Fig. 4.1 shows that the bar can be controlled by dredging 3.000.000 m^3 of sediment a year. If the dredged sediment is deposited, a bar has to build up somewhere, as no sediment is removed from the system. Fig. 4.3 shows that if 6.000.000 m^3 is dredged and deposited upstream, the bar builds up and moves downstream. The time-scale is 10 years, dependent of course on the distance between the place of dredging and deposition.

The time-scales are proportional to the magnitude of the sediment transport. In the model, the net sediment transports are of magnitude equal to sediment transports observed in the Western-Scheldt (Allersma, 1993). A substantial part of the sediment transport in the Western-Scheldt, however, is circulating. No such transport is possible in the 1-dimensional model. This means that the net transport in the model is an upper bound for the actual net transport in the Western-Scheldt. The morphological time-scales in the model, therefore, are a lower bound for the actual time-scales in the Western-Scheldt.

The downstream boundary condition for morphological development is a fixed bed. As Simulation 2 shows, at some point the bar wants to extend across the boundary, but is stopped by this condition. In Fig. 5.1 and 5.2 it is shown what happens if the boundary condition is replaced by a constant net transport into the estuary. The outcome is more or less the same, regardless the amount of the net transport. A bar builds up close to the mouth and the height of the bar does not depend on the boundary condition. This shows that the morphological development does not depend on the morphological boundary conditions, but on the hydrodynamic conditions.

On the whole it can be said that, although the model does not simulate the morphological processes in the Western-Scheldt, it does give estimates on time-scales in the processes. Moreover, it indicates that modelling water flow can be difficult especially for the overtides.

4 Analysis of the present problem

The main feature of the model is that a persistent bar builds up close to the mouth of the estuary. During the simulation, the bar is pushed closer to the mouth. It moves from about 30 km to about 15 km from the mouth. The bar more or less divides the estuary into two parts: in front of the bar, at the mouth, the estuary behaves like a tidal basin; behind the bar, the estuary behaves like a tidal river. The position of the bar depends on the direction of the residual transport. To be more precise, the bar marks the place where the residual transport changes direction: from both sides sediment is transported to the bar.

The residual transport depends on the interaction of the components of the tide. If the flood period is shorter than the ebb period, then the residual transport is directed generally into the estuary, if it lasts longer it is directed generally out of the estuary. The bar builds up at the place where the ebb period and flood period are equal. In the model only two components of the tide, M_2 and M_4, are present and they determine the position of the bar.

To explain the phenomenon, consider a rectangular channel of constant width, constant depth 10 m and no bottom friction. The wavelength of M_2 is about 450 km, and the wavelength of M_4 is 225 km. Suppose that the waves are in phase at the mouth of the channel, which means that the tide is flood dominated there. At 225 km, the waves are in opposite phase, which means that the tide is ebb dominated. So, the ebb period and flood period are equal at 112,5 km and this is where the bar builds up in this idealized situation. This is the maximal distance. For various phase shifts, the bar builds up in a range from 0 to 112,5 km from the mouth.

In the model M_2 and M_4 are in phase at the mouth. Nevertheless, the bar builds up at about 15 km from the mouth. This seems to imply that, in our model, the tidal wave is propagated in a way very different from the propagation in the idealized rectangular channel. There are two factors which deform the propagation of the tidal signal: bottom friction and the channel width. Both factors dissipate of energy of the wave.

The relative importance of bottom friction is expressed by the friction parameter:

$$\beta / \omega \tag{21}$$

Notice that ω is different for M_2 and M_4.
If the parameter is larger than one, friction deforms the wave. In the model, the friction parameter is small up to Antwerp, less than 0.5, and it increases rapidly beyond Antwerp.

The other dimensionless parameter indicates the importance of the width of the estuary. The parameter is equal to

$$\left(\frac{\partial B}{\partial x}\right)\frac{\sqrt{g.a}}{2B\omega} \tag{22}$$

as is shown in the Appendix. It is called the width parameter. If the width parameter is larger than one, the width of the channel deforms the tidal wave: it decreases water discharge and increases the water level. In our model of the Western-Scheldt the width parameter is equal to 0.92 for the M_2 component and to 0.46 for M_4. The funnel shape affects the M_2 wave more than M_4, but since M_2 generates M_4, it indirectly influences M_4 as well.

Both the influence of friction and funnel shape may be too large in the model. The friction parameter is small upstream, but it is large upstream in the estuary. Since the water flow is solved from a boundary value problem, the high upstream friction influences the tidal wave in the entire estuary. The width parameter may be too large as well, because of the strong schematization by the exponential function. This may explain why the bar invariably builds up at 15 km: due to the funnel shape, the bar is being pushed to the mouth of the estuary.

5 Conclusions and recommendations

5.1 Conclusions

It turns out that it is very hard to model the water flow correctly. In the model, there is essentially only one parameter to calibrate the water flow: the coefficient of bottom friction. This coefficient has to be tuned in such a way that the tidal wave runs through the model like it runs through the Western-Scheldt.

Each component of the tidal wave is represented by two numbers: water level and phase lag. The coefficient of bottom friction has to be adjusted in such a way that both numbers agree with the tide in the Western-Scheldt. It turns out that one parameter is too little to calibrate both water level and phase lag correctly. In the model, the water level is modelled well, but the phase lag is not.

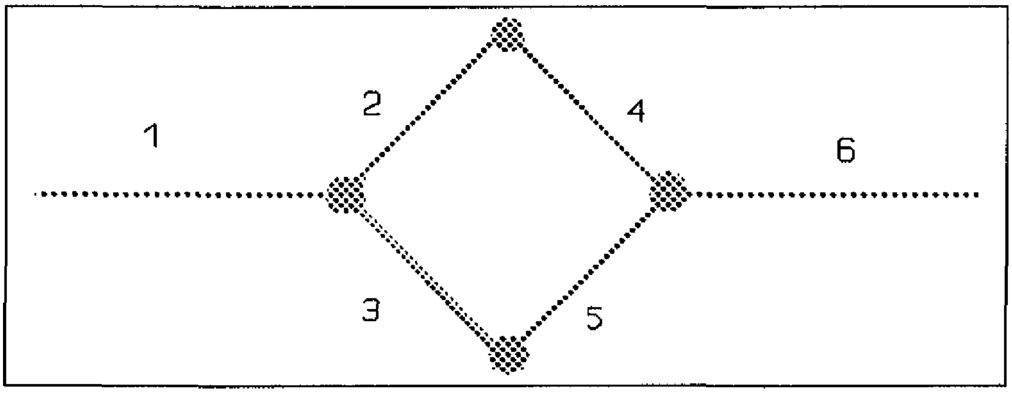
There is a residual transport of sediment in the estuary both upstream and downstream. At the mouth of the estuary, the incoming transport is about 2.5 million m^3 per year. This the right order of magnitude: measurements give about 1.5 million m^3 (Allersma, 1992), and very accurate measurements can hardly be made.

Since sediment is transported into the model from both sides, a bar must build up somewhere in the estuary. In the model, it builds up very close to the mouth, at about 15 km. In fact, it starts building up at about 30 km but during the process it is pushed to the mouth. Behind the bar, the bed erodes. This is different from the actual behaviour of the Western-Scheldt. In fact, there is intense dredging in the Western-Scheldt at the place where there is erosion in the model. The sediment transport is of the right magnitude, but not of the right direction. This means that the model gives an indication of the morphological time-scales, but not of the actual place where a bar builds up.

This model is not really a morphological model of the Western-Scheldt. It does show one of the characteristics of the Western-Scheldt, the building up of a bar, but the place of the bar in the model is different from the actual place in the Western-Scheldt. The data of the Western-Scheldt, therefore, could not be tested against the results of the model.

5.2 Recommendations

- Instead of a one channel model, it is advisory to build a network model of the Western-Scheldt. In this way it is possible to model ebb channels and flood channels, which is very important in the Western-Scheldt. The network should be a sequence elementary blocks, each block consists of six channels, as illustrated below. These blocks represent the braiding ebb channel - flood channel network.



Although this is a very natural schematization, so far channel networks have been applied in hydraulic models only. They are not used in operational morphological models.

- The actual place of the bar is determined by the M_2-M_4 interaction. In the model, the M_4 wave is not calibrated well. This wave is the first overtide, generated by M_2 due to advective acceleration and bottom friction. In the model, the M_4 wave is generated only through advective acceleration since friction is linearized. It is advisory, therefore, that the linearized friction is replaced by a quadratic approximation of friction.

- The model has only one parameter to calibrate the water flow: bottom friction. A second parameter is needed: the geometry of the estuary. It turns out that the funnel shape of the Western-Scheldt deforms the tidal signal significantly (see Appendix). The funnel shape can be a second parameter to calibrate the model.

- To test whether the problem to calibrate water flow indeed is due to the funnel shape, other 1D test computations should be made, for instance with the WENDY model.

- The direction of the sediment transport depends on the interaction of the tidal components. In the model there are two constituents: M_2 and M_4. It follows that the direction of the sediment transport is more or less fixed during the process. In reality, the bar in the Western-Scheldt does not build up in one place, but it moves up and down through the estuary. This is because there are more constituents to the tide, so, the direction of the sediment transport varies. It is advisory, therefore, to take more constituents into account.

Appendix: the influence of the funnel shape

As the results of the model show, the tidal propagation is distorted: both M_2 and M_4 have wavelength which is too short. Since the model is semi-analytical, an estimate of the distortion can be made. It is derived in this appendix.

The propagation of the M_2 wave is described by equation (17), the one-dimensional wave equation. The widths B_s and B are equal and exponentially decreasing as in equation (19). Substitution in (17) yields the equation

$$\frac{\partial^2 Q}{\partial x^2} + k\frac{\partial Q}{\partial x} - \frac{(\beta + i\omega)}{ga} i\omega\, Q = 0 \tag{23}$$

where k, the constant which expresses the funnel shape, is 2.61 10^{-5} for the Western-Scheldt.

This is the standard differential equation for the forced-damped pendulum. The solutions are linear combinations of the exponential functions

$$\exp(c_{1,2}\,x),\;\; c^2 + k\,c - \frac{\beta + i\omega}{ga} i\omega = 0 \tag{24}$$

If friction is neglected, the equation can be approximated by

$$(c + \frac{k}{2})^2 = -\frac{\omega^2}{ga} + \frac{k^2}{4} \tag{25}$$

This equation shows that the character of the differential equation depends on the constant

$$\frac{k\sqrt{g\,a}}{2\omega} \tag{26}$$

which is the width parameter, mentioned in section 4. If it is smaller than 1, the solution is wave like; if it is larger than 1, the solution is exponentially damped.
Note that if k=0, the equation reduces to

$$c = \pm\frac{i\omega}{\sqrt{ga}} \tag{27}$$

The standard equation of wavelength.

In the model of the Western-Scheldt, the values of the parameters are

$$k = 2.61\;10^{-5}\,m^{-1},\; \omega = 1.15\;10^{-4}\,s^{-1},\; a \approx 10\,m. \tag{28}$$

So, the width parameter is about 0.92 for the M_2 tide and 0.46 for the M_4 tide. It follows that the harmonics M_2 and M_4 are significantly deformed by the funnel shape. Remind that the distorted M_2 generates a distorted M_4.

If the width parameter is close to 1, the right hand part of (25) almost disappears. In this case the friction term in (24) gives the most important contribution. The wave like character depends on β. Under this assumption, equation (24) can be approximated by

$$(c + \frac{k}{2})^2 = \frac{\beta\,\omega}{ga}\,i \tag{29}$$

The imaginary part of c determines the length of the wave. It is equal to

$$\pm\sqrt{\frac{\beta\omega}{2ga}}\ i \tag{30}$$

In the model, the friction coefficient β is small in the western part of the estuary, but it is extremely large in the eastern part to damp out the tidal wave. As equation (30) shows, a large friction coefficient increases the imaginary part of c, so, it reduces the length of the wave. The extremely large value of β at the upstream boundary is the reason why the wavelength of the harmonics is too short. On the other hand, the tidal influence is damped out in reality, so, a large value of β is needed in the model.

In the model, β is the only free parameter to calibrate the water flow. As this computation shows, it is not a very convenient parameter to calibrate the water flow in the Western-Scheldt.

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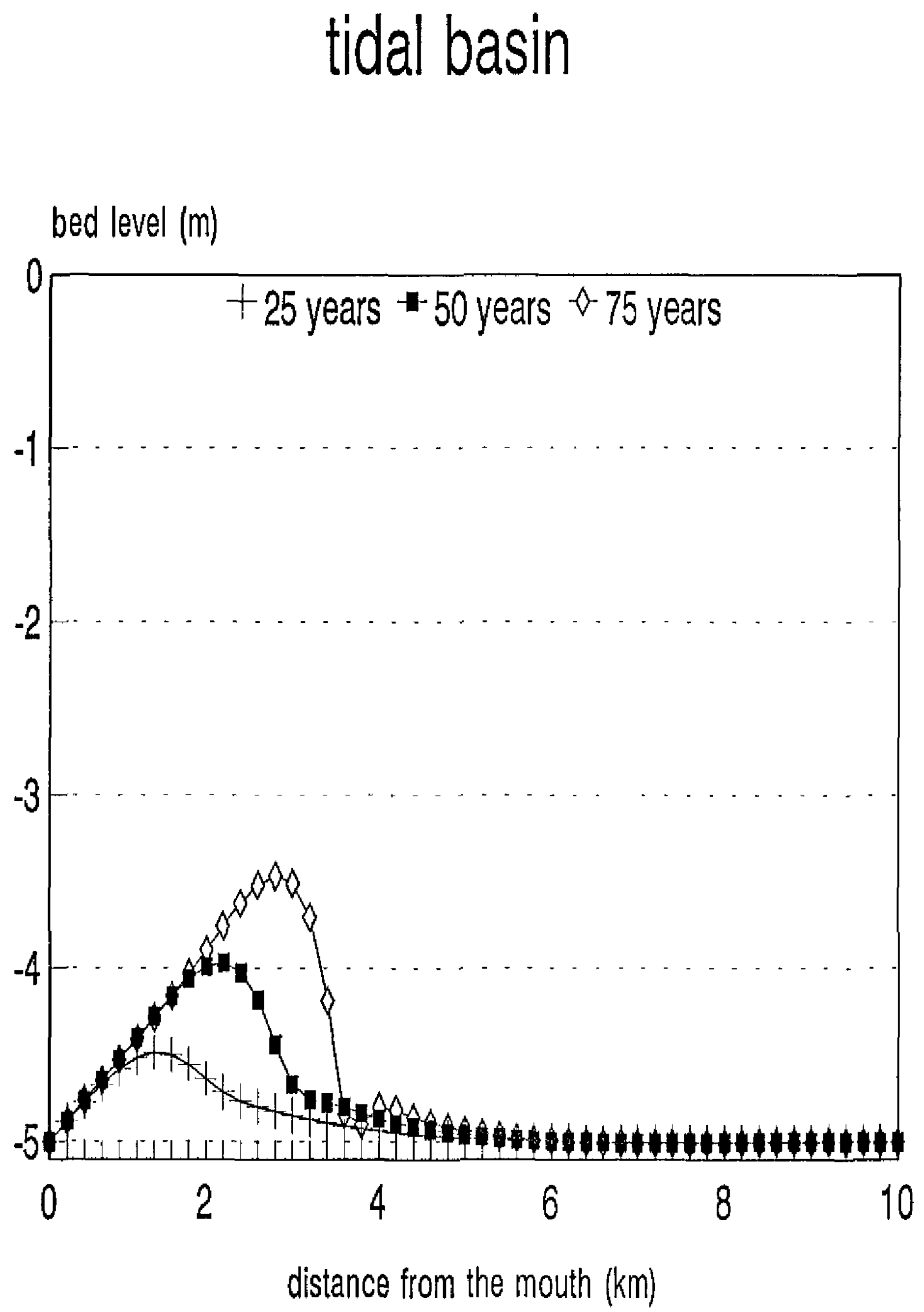
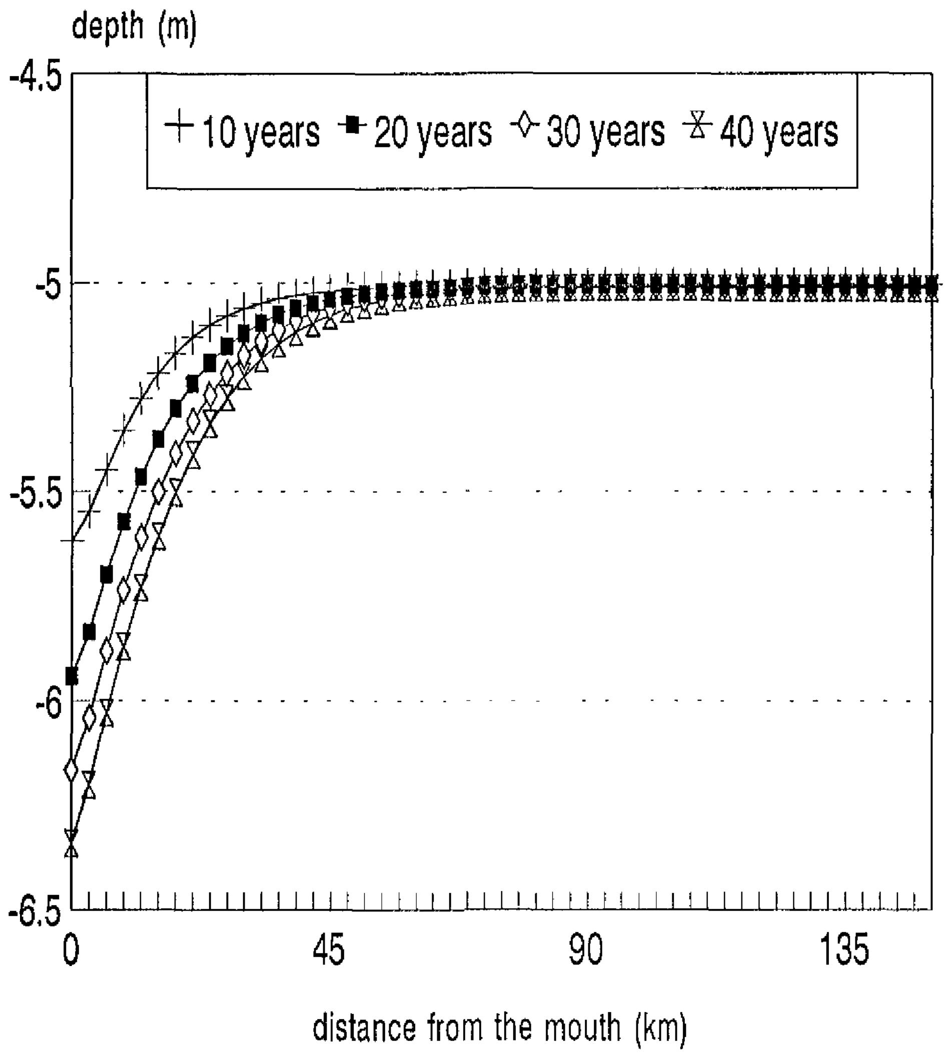


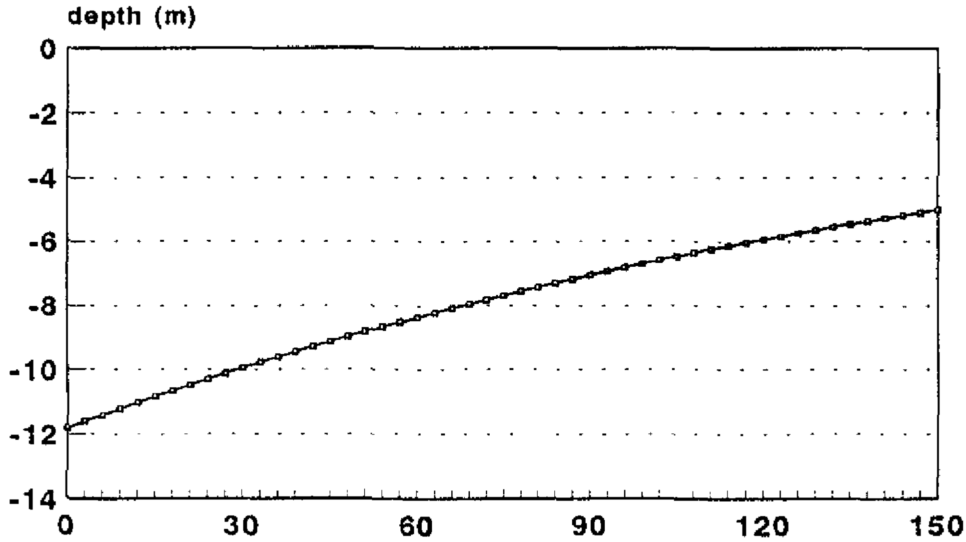
Evolution of the tidal basin		Nov 93
	Western-Scheldt	
DELFT HYDRAULICS	Proj: Z-695	Fig.1.1

tidal river

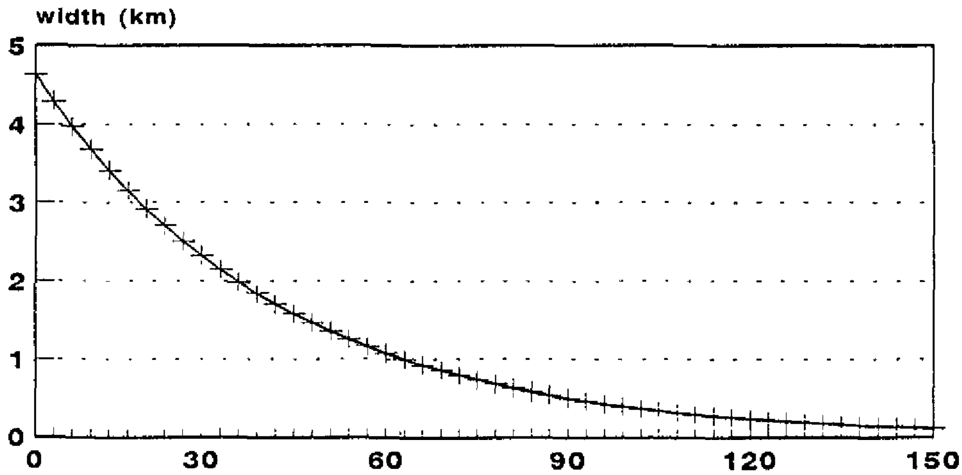


Evolution of the tidal river		Nov 93	
	Western-Scheldt		
DELFT HYDRAULICS	Proj: Z-695	Fig.1.2	

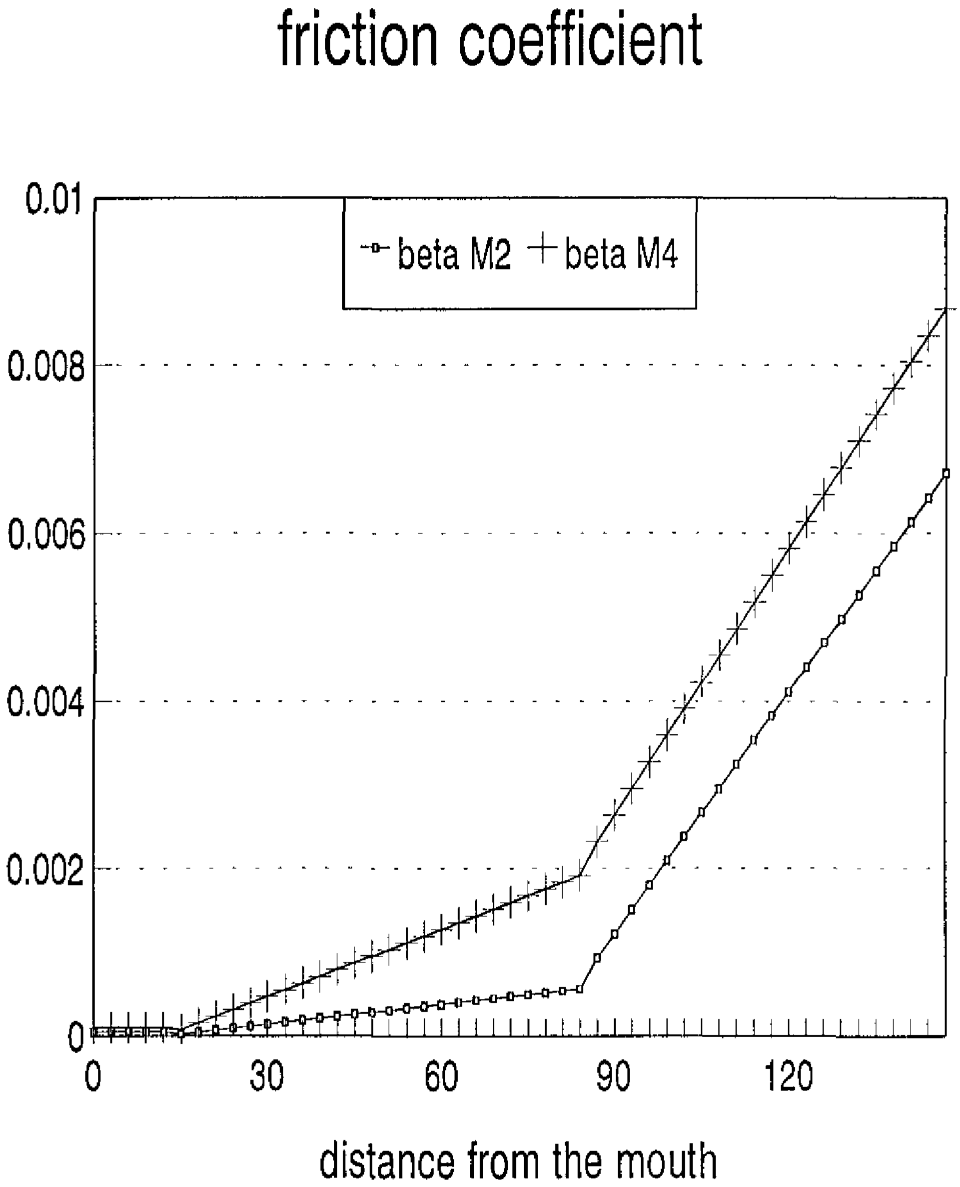
depth of the estuary



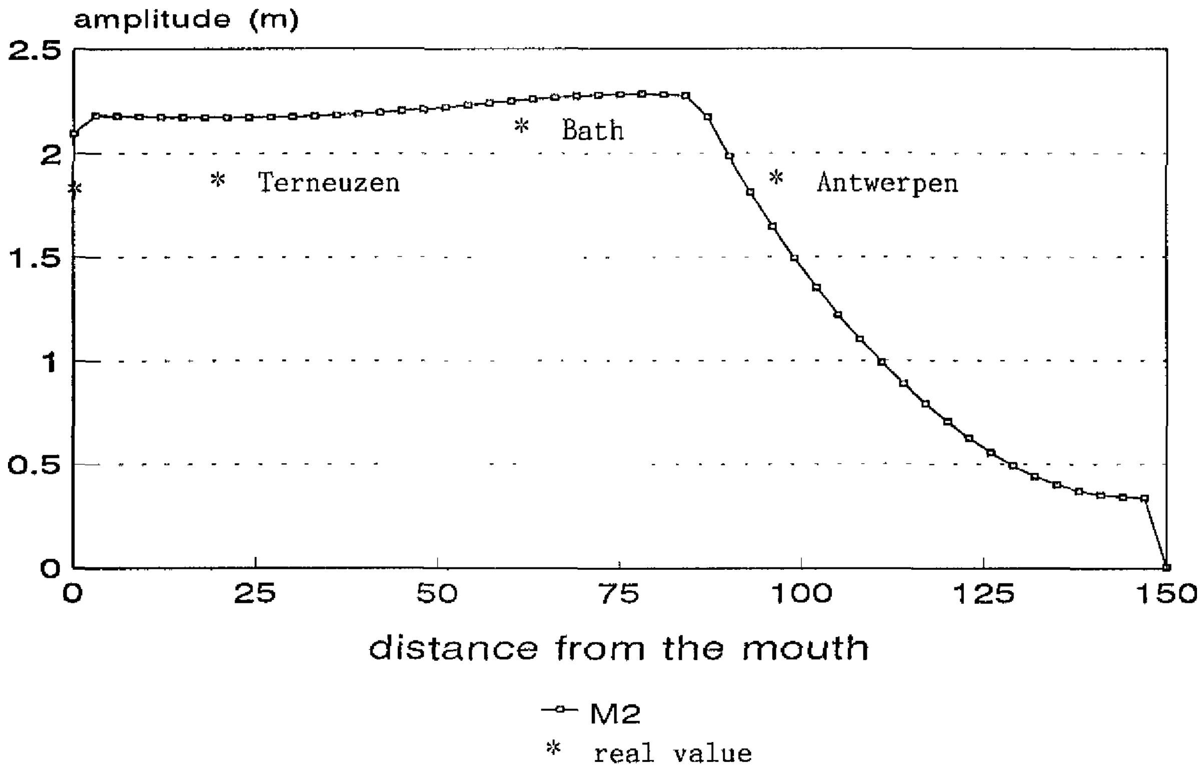
width of the estuary

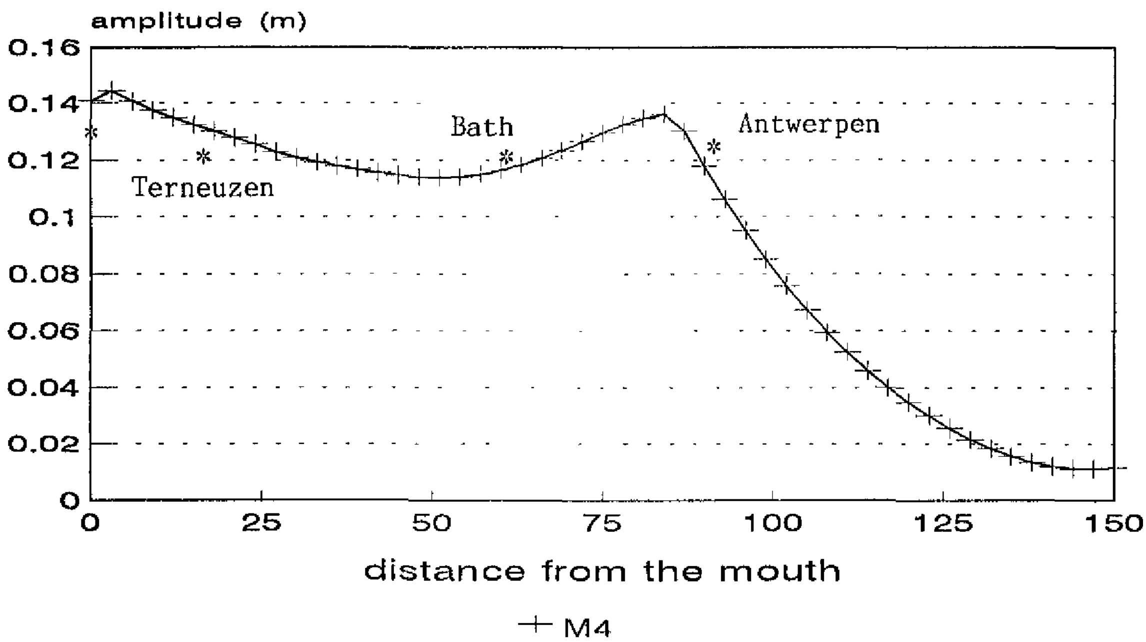


Schematization of the Western-Scheld		
D E L F T H Y D R A U L I C S	Proj: Z-695	Fig.2.1

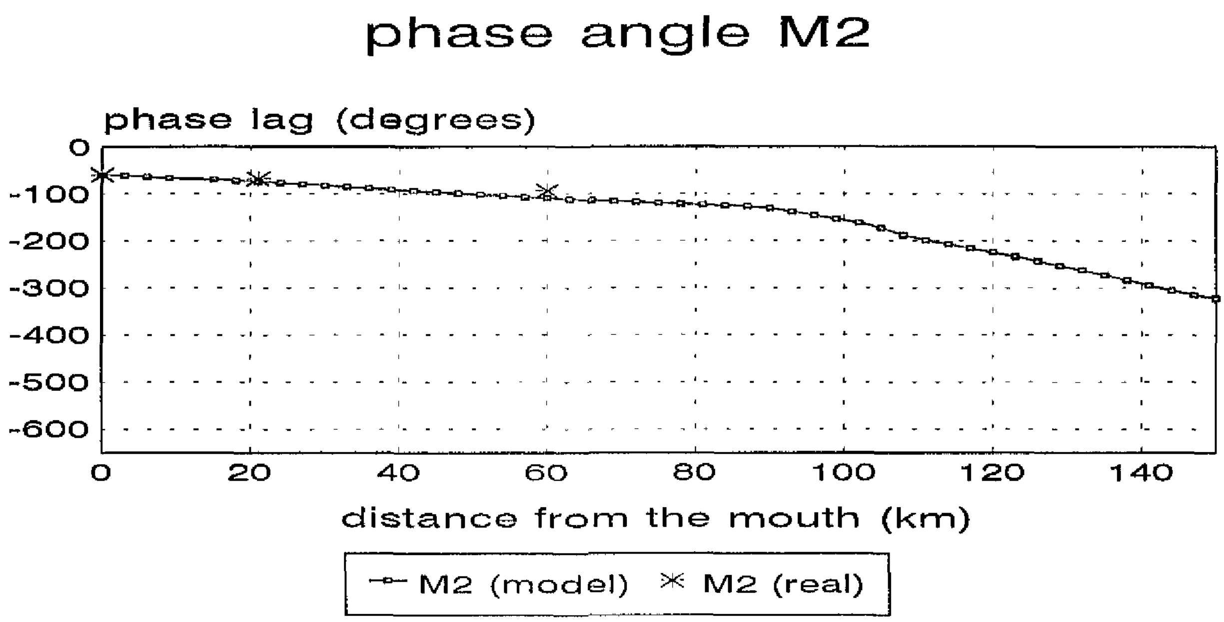


The friction coefficient		Nov 93
	Western-Scheldt	
DELFT HYDRAULICS	Proj: Z-695	Fig.2.2

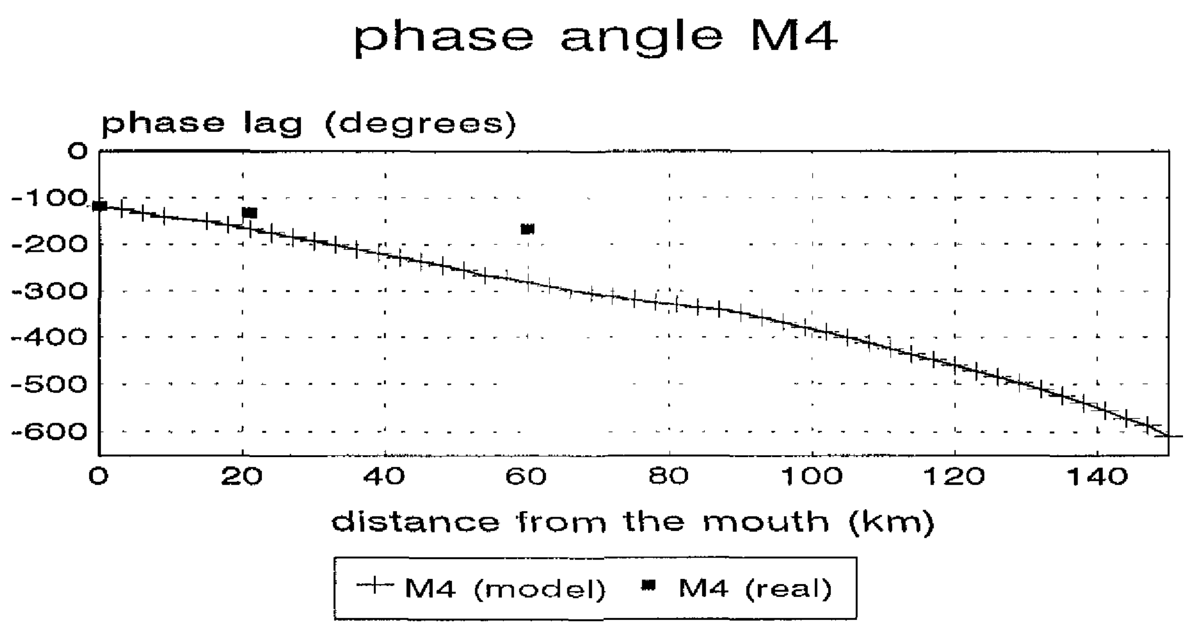




Amplitude of the components throught the estuary		Nov 93
	Western-Scheldt	
DELFT HYDRAULICS	Proj: Z-695	Fig.2.3



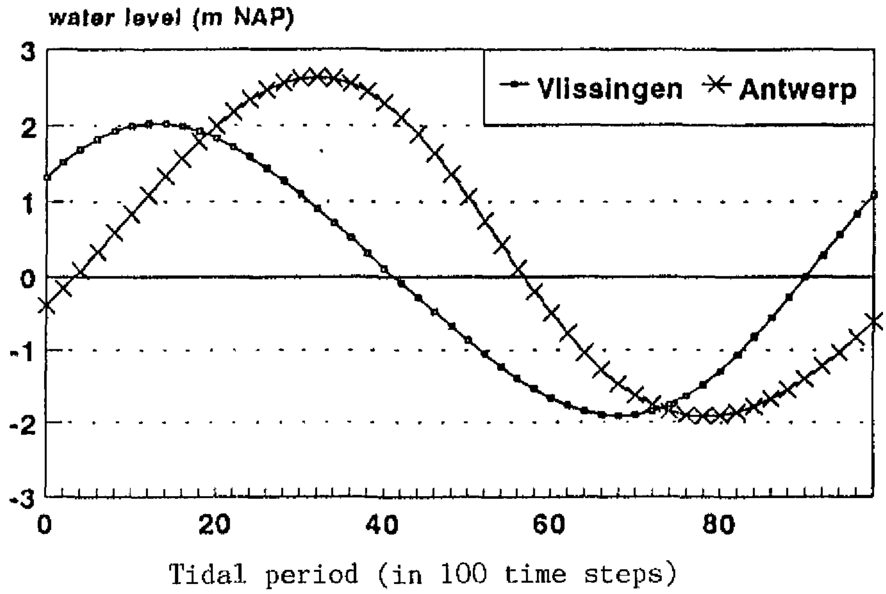
points represent the phase angle at
Vlissingen (0 km), Terneuzen (20 km) and Bath (60 km).



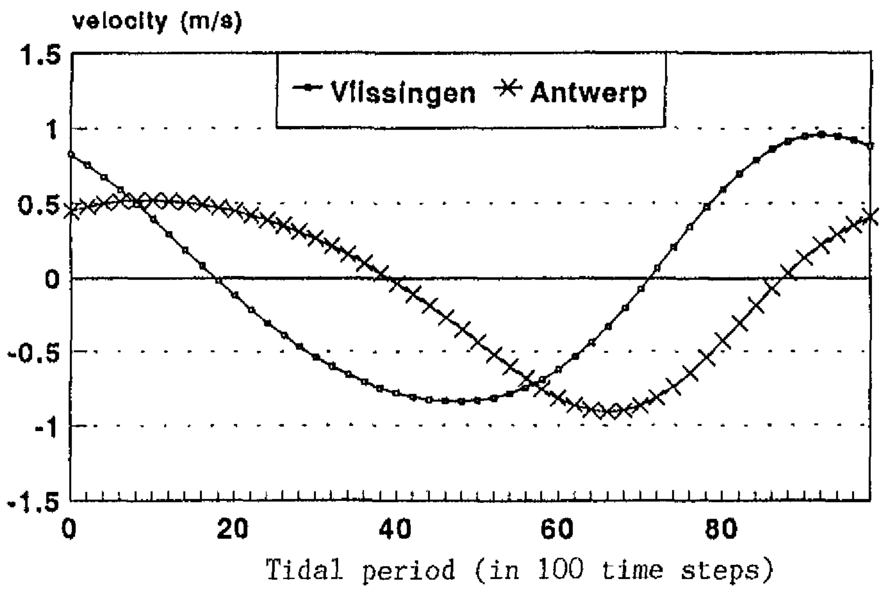
points represent the phase angle at
Vlissingen (0 km), Terneuzen (20 km) and Bath (60 km).

Phase angle of the components throughout the estuary		Nov 93
	Western-Scheldt	
DELFT HYDRAULICS	Proj: Z-695	2.3.b

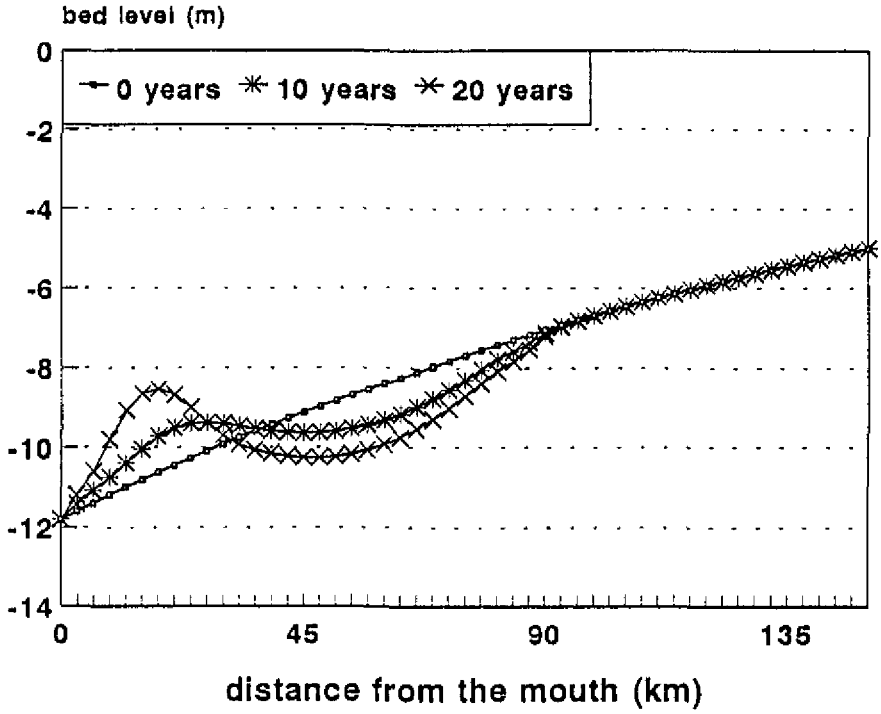
water level

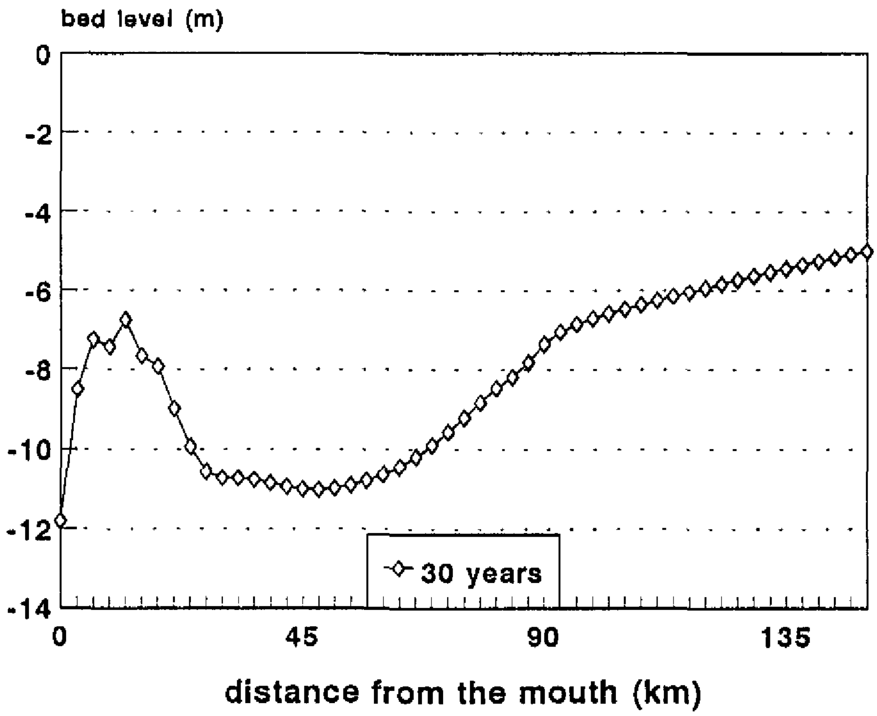


velocity

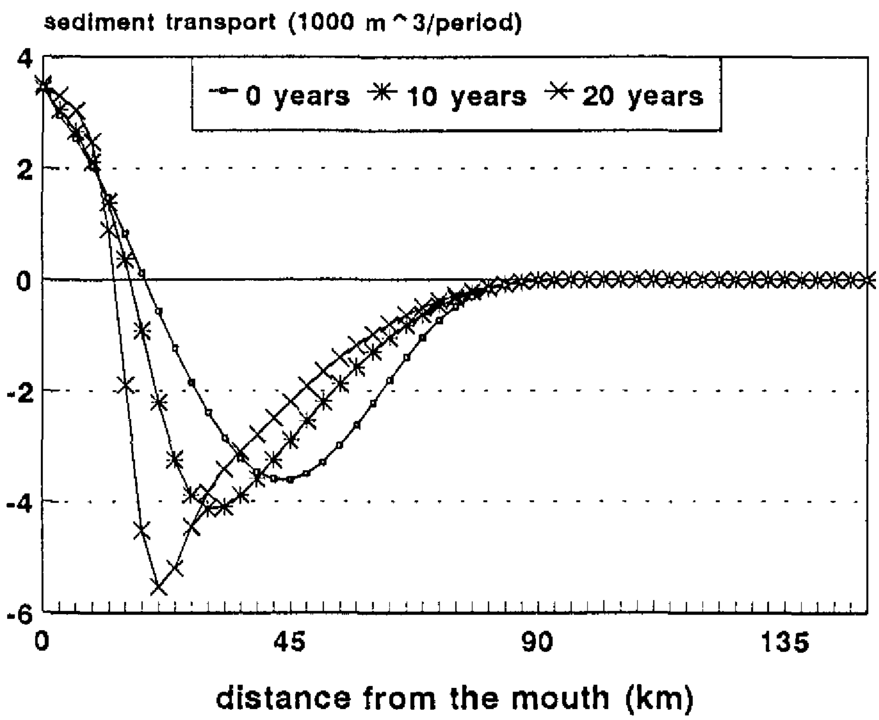


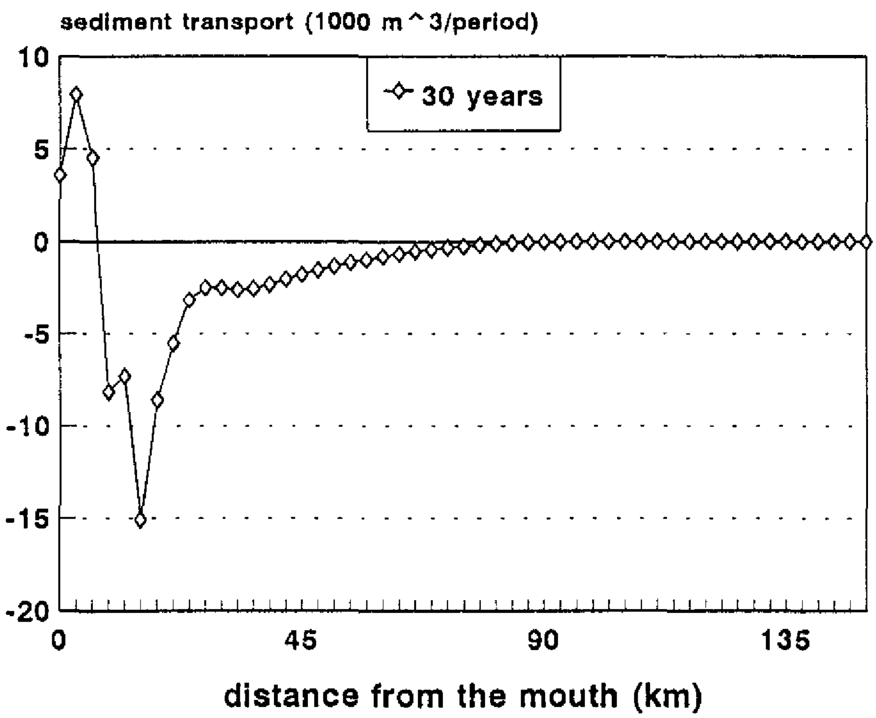
The water flow at Vlissingen and Antwerp		
DELFT HYDRAULICS	Proj: Z-695	Fig.2.4





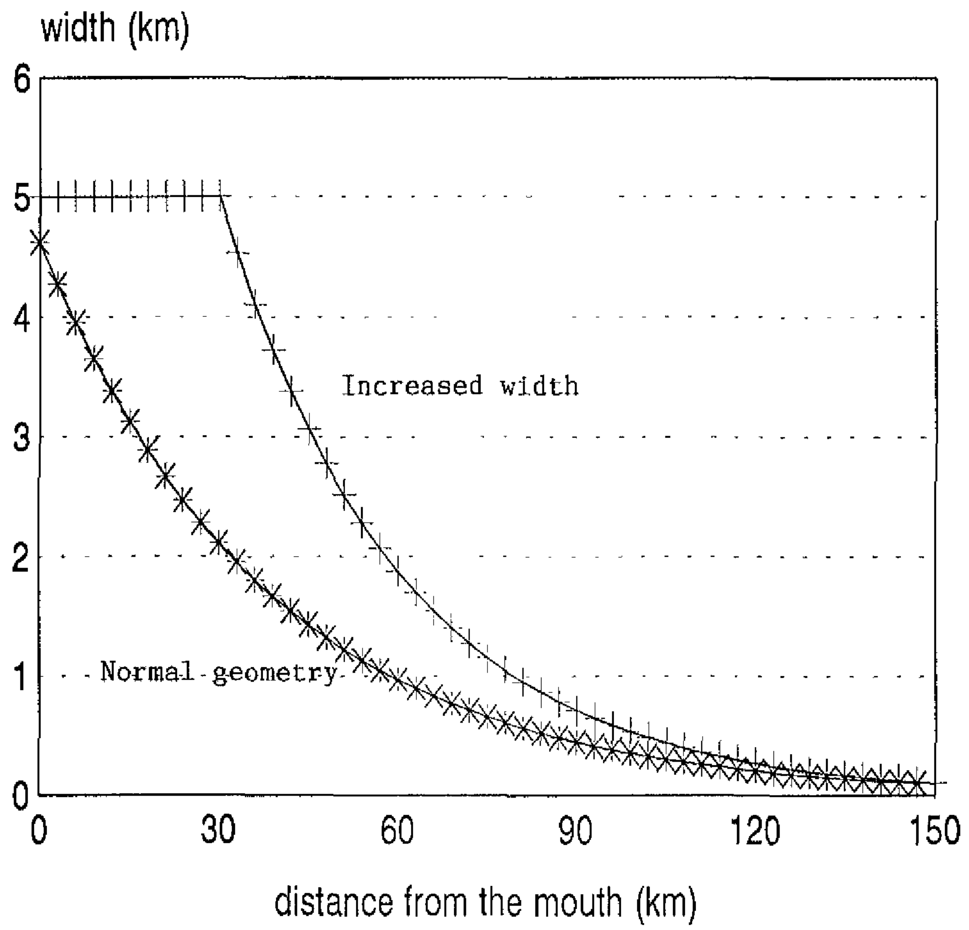
The development of the bed		
DELFT HYDRAULICS	Proj: Z-695	Fig.2.5



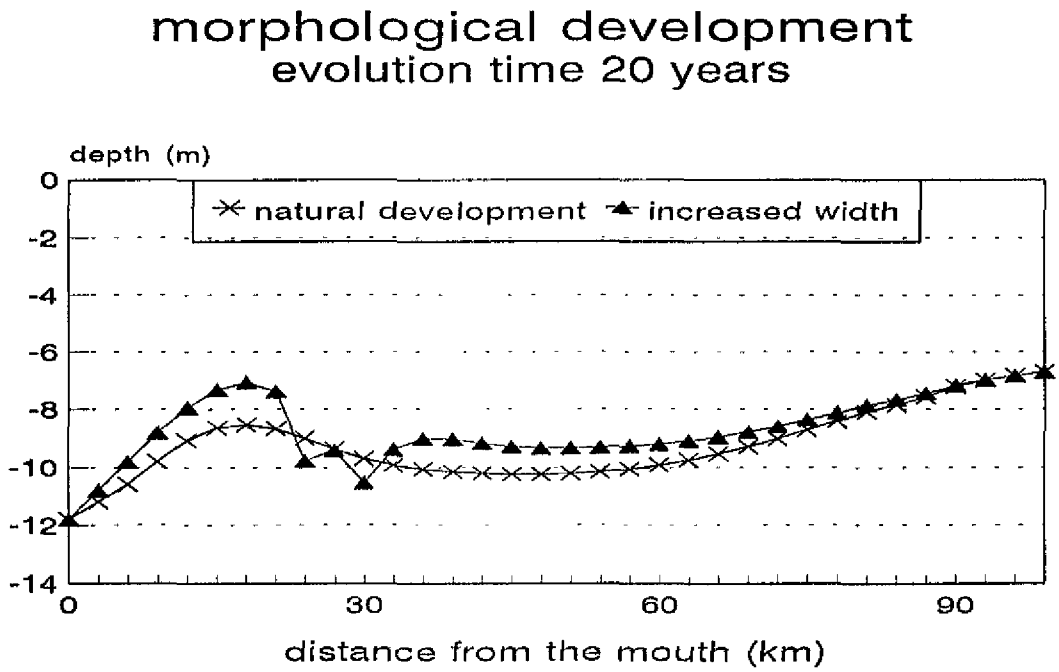


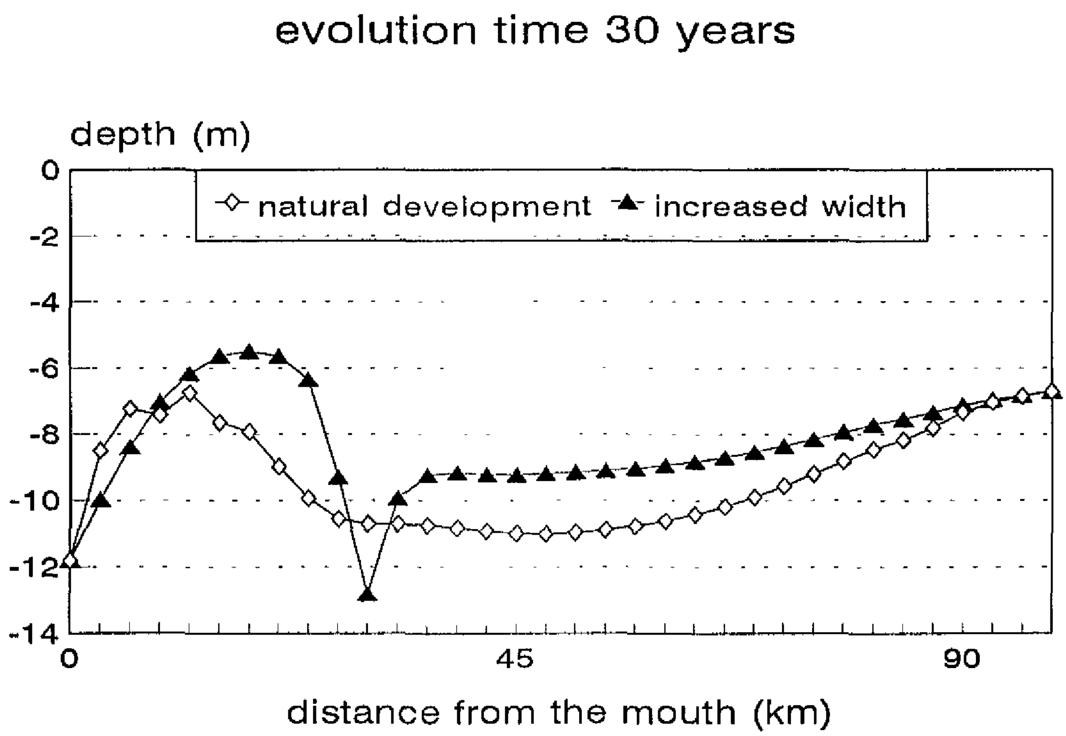
The residual sediment transport		
DELFT HYDRAULICS	Proj: Z-695	Fig.2.6

width of the estuary

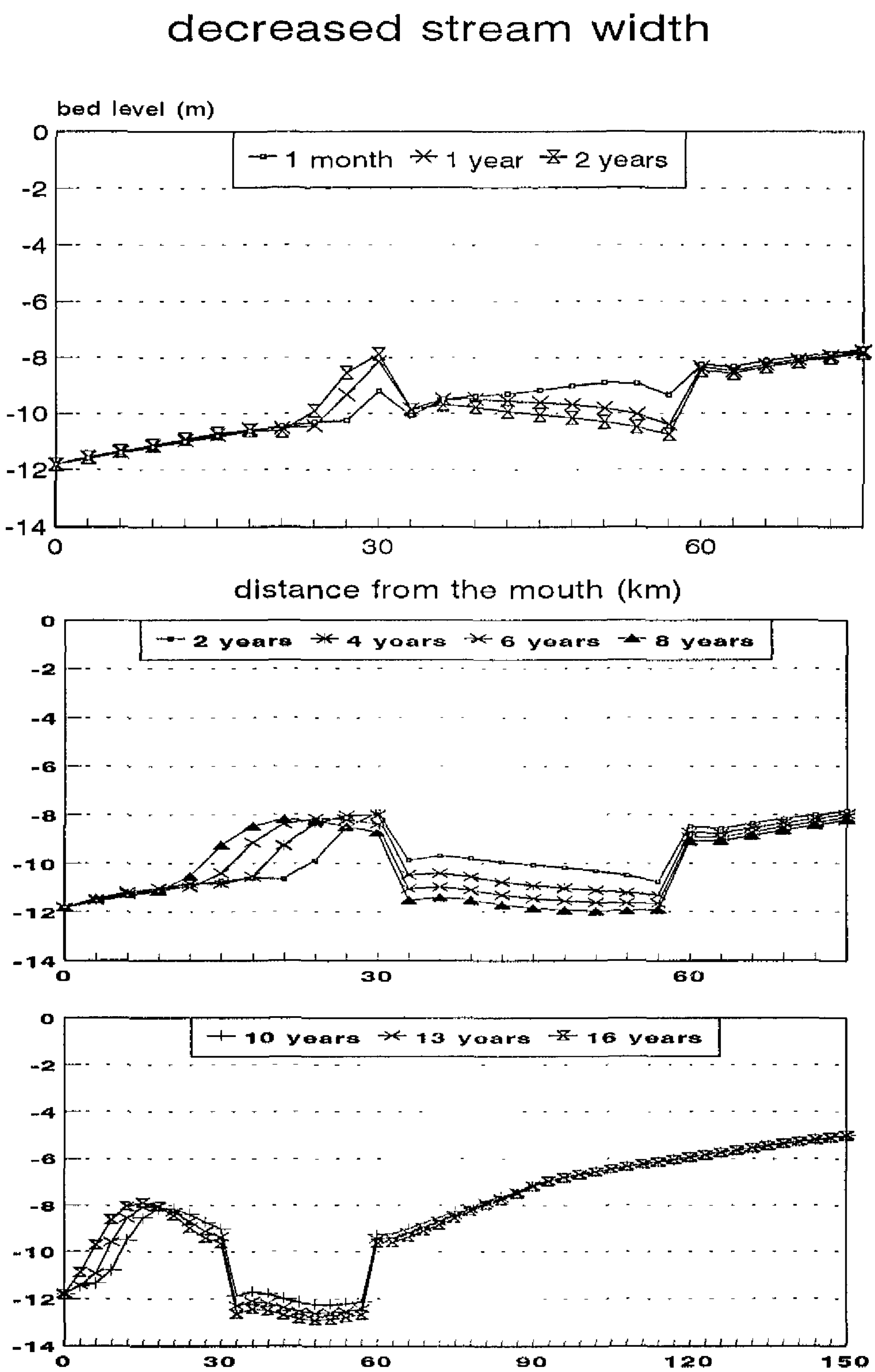


The storage width at the mouth isincreased.	Western-Scheldt	
DELFT HYDRAULICS	Proj: Z-695	Fig.2.7





The morphological development if the storage width at the mouth is increased. A comparison.		Nov 93
	Western-Scheldt	
DELFT HYDRAULICS	Proj: Z-695	Fig.2.8



The stream width is decreased by 30% between Hansweert and the border (30-60 km)..

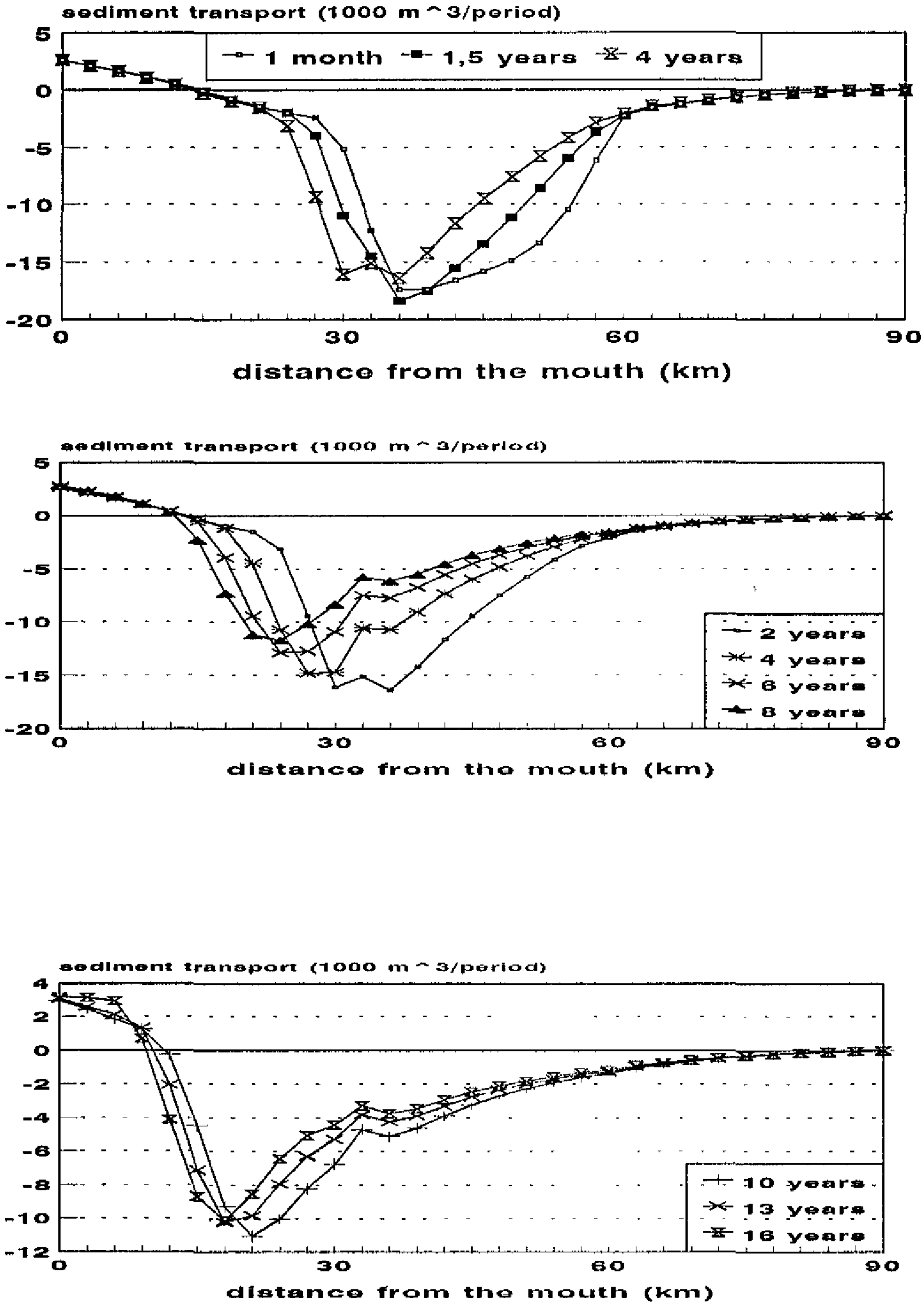
Western-Scheldt

DELFT HYDRAULICS

Proj: Z-695

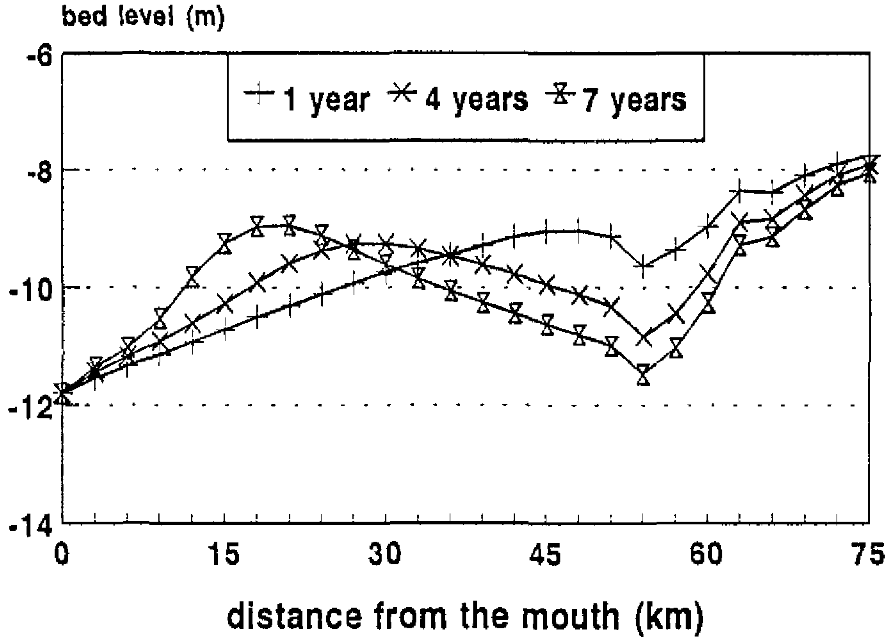
Fig.3.1

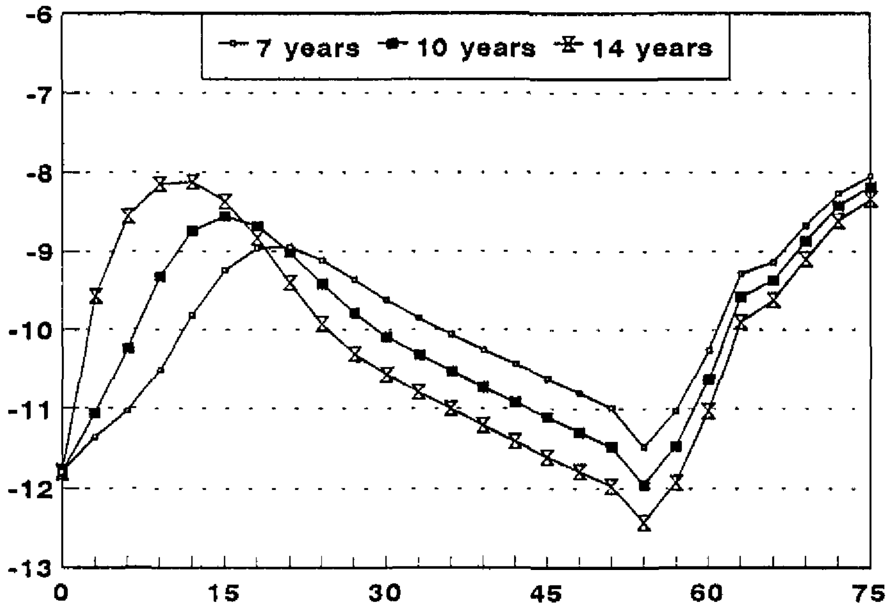
Decreased stream width



Sediment transport if the stream width is decreased by 30%.		Nov 93
	Western-Scheldt	
DELFT HYDRAULICS	Proj: Z-695	Fig.3.2

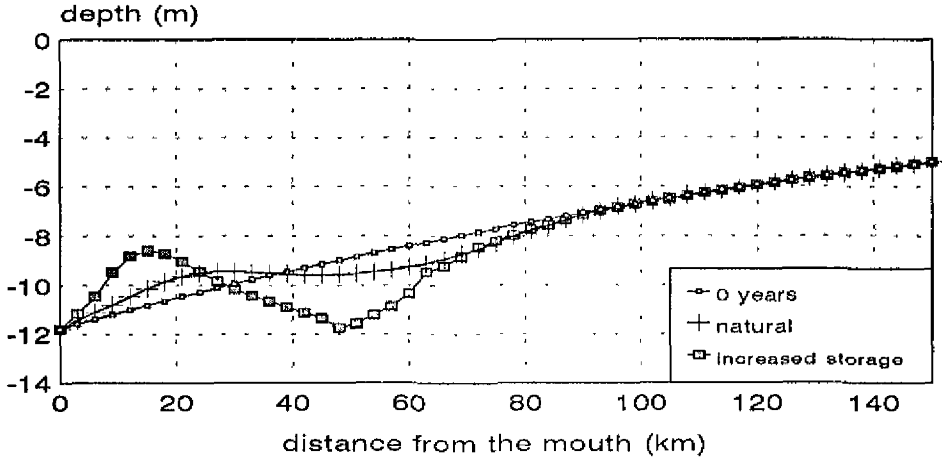
Storage width at the border is increased by 1000 Ha.



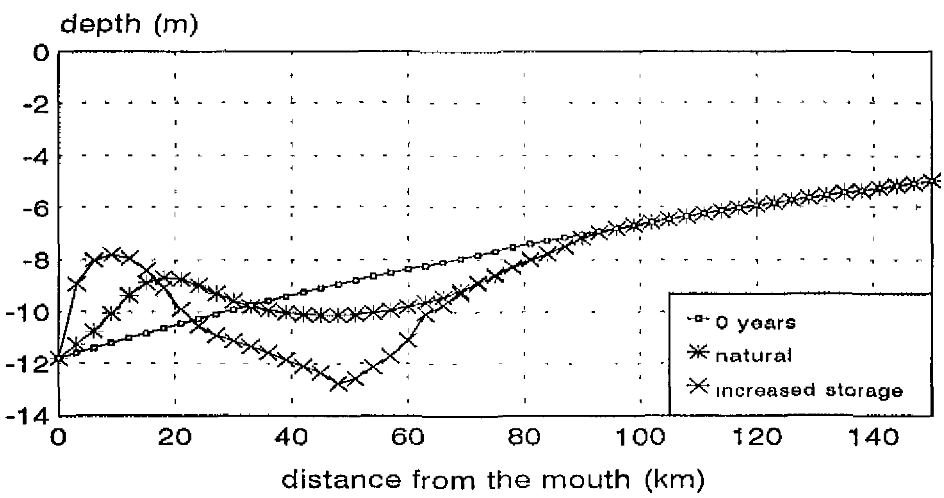


The storage width is increased between Hansweert and the border.		
DELFT HYDRAULICS	Proj: Z-695	Fig.3.3

evolution time 10 years
comparison between natural development
and increased storage area.

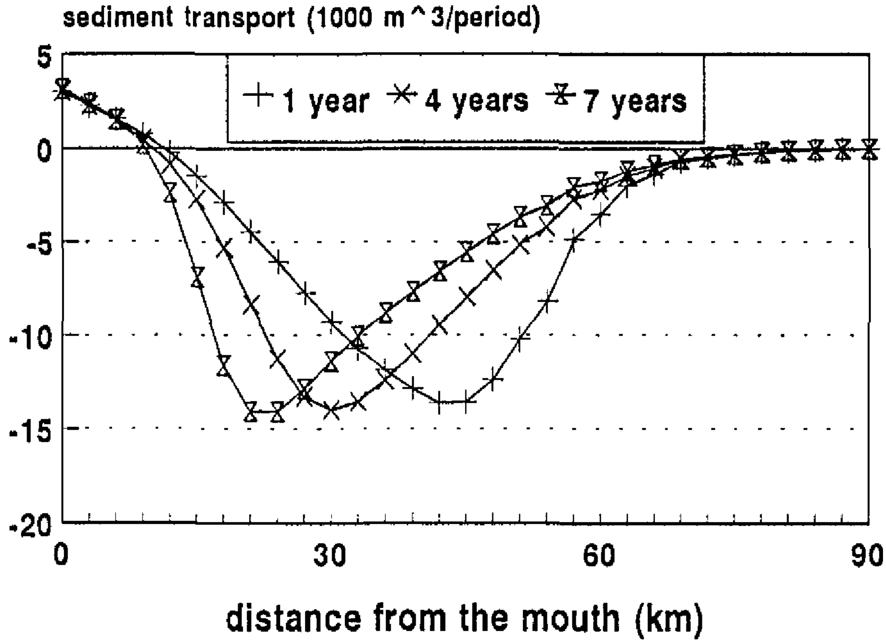


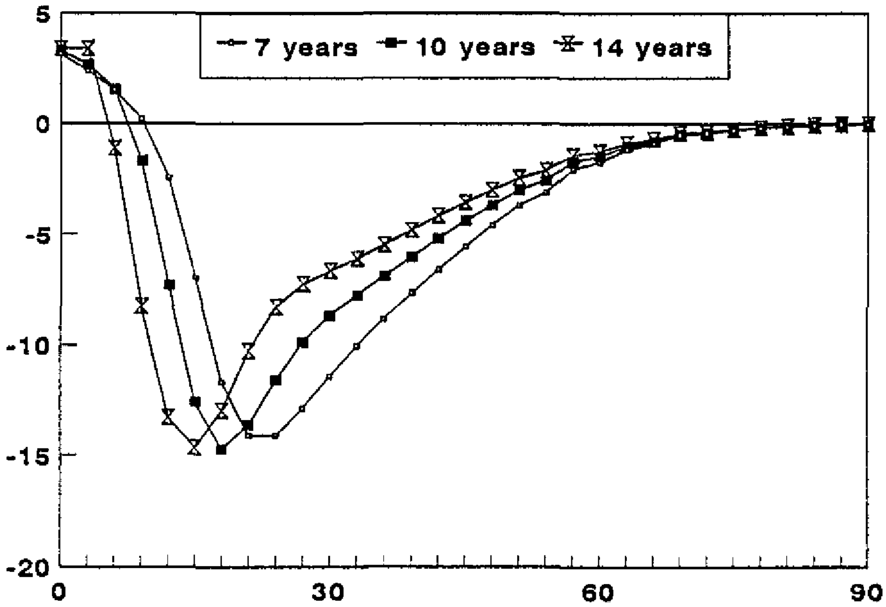
evolution time 20 years
comparison between natural development
and increased storage area.



A comparison between increased storage width and natural development..		Nov 93
	Western-Scheldt	
DELFT HYDRAULICS	Proj: Z-695	Fig.3.3.b

Storage area at the border is increased by 1000 Ha

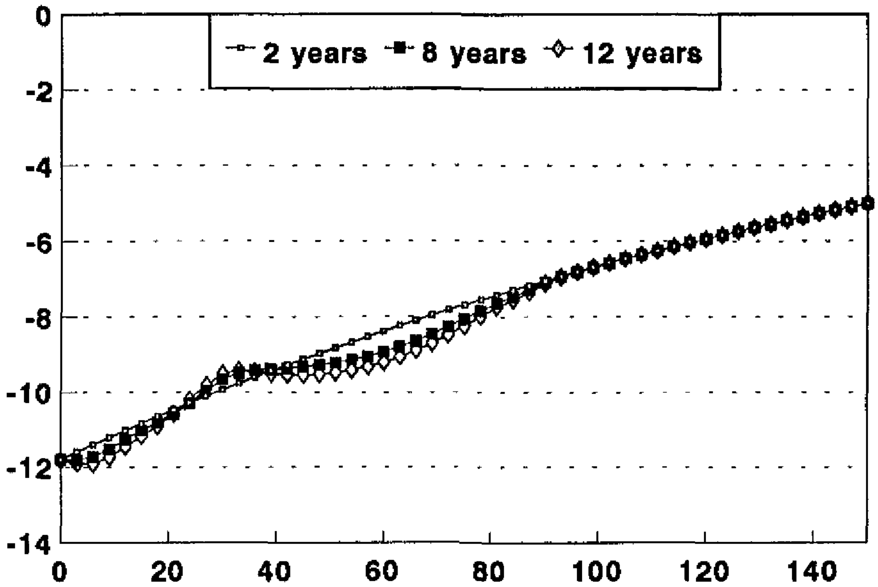




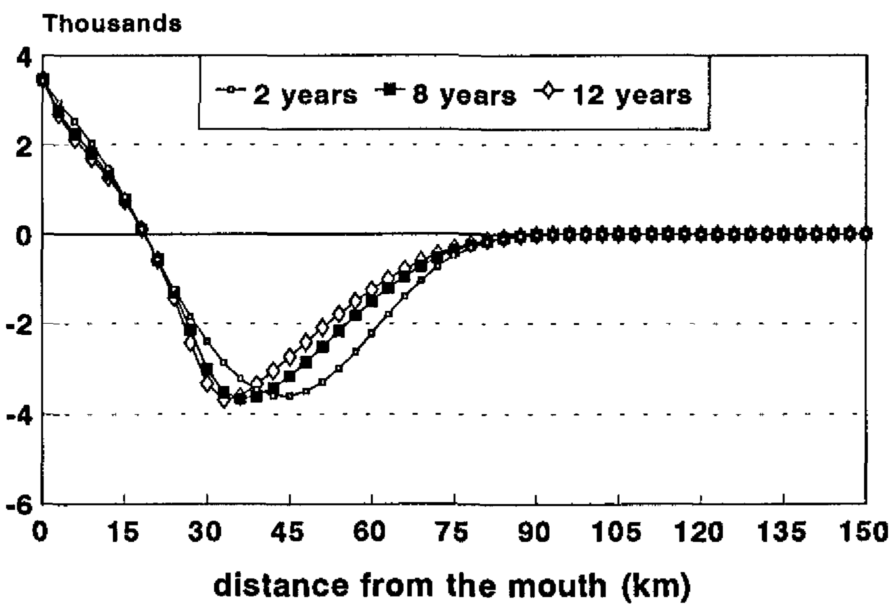
The storage width is increased between Hansweert and the border.

DELFT HYDRAULICS	Proj: Z-695	Fig.3.4

bed level

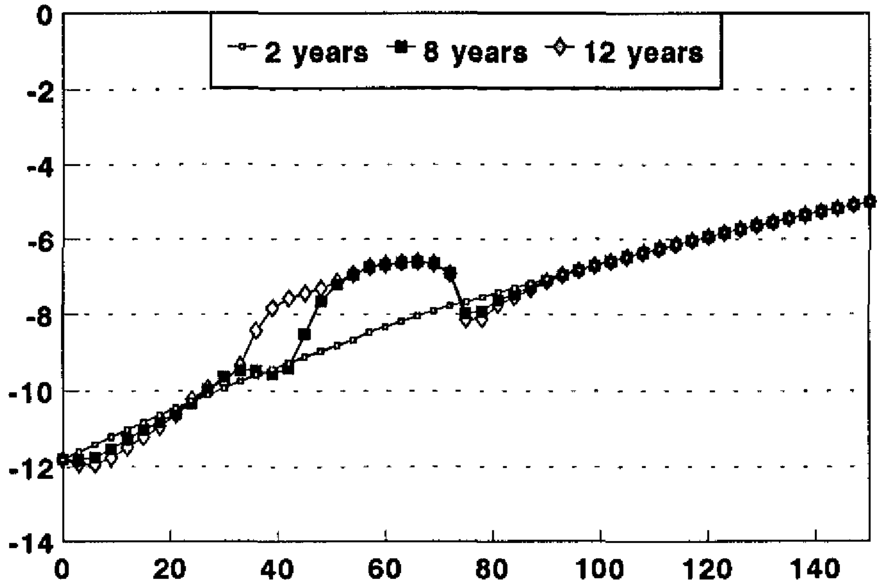


sediment transport

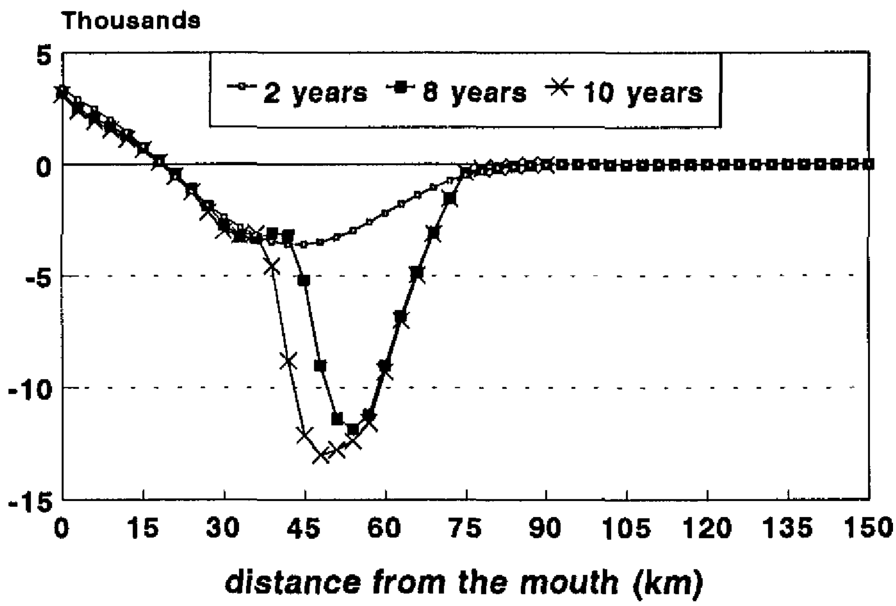


Evolution of the tidal basin when annual dredging is 3 million m^3 at the mouth: 0 - 30 km		
D E L F T H Y D R A U L I C S	Proj: Z-695	Fig.4.1

bed level

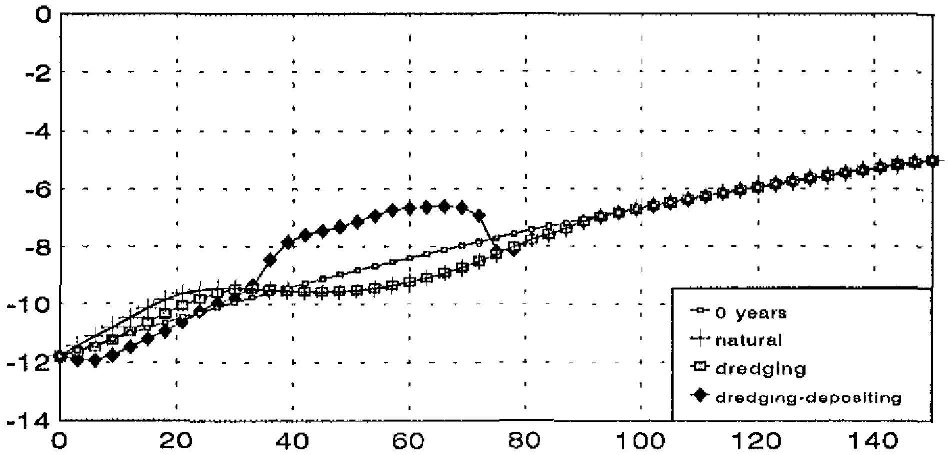


sediment transport

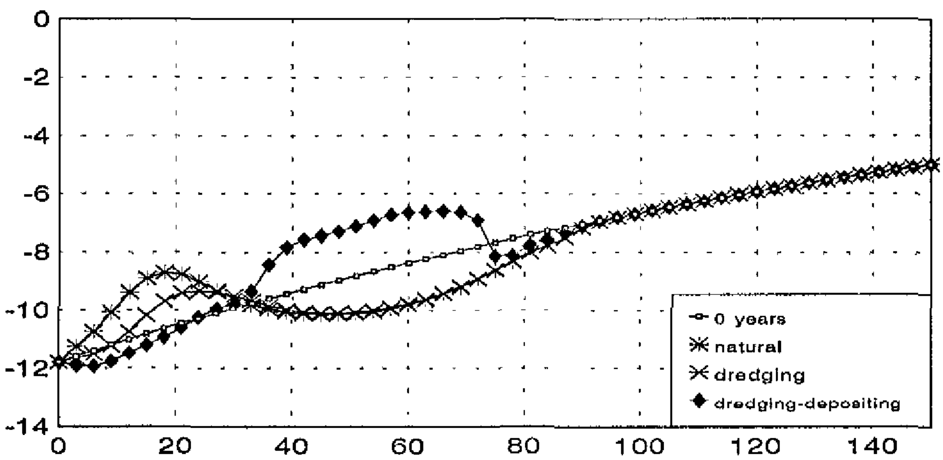


The annual dredging is 6 million m^3 at the mouth, 0 - 30 km, which is deposited upstream, 60 - 75 km.		
DELFT HYDRAULICS	Proj: Z-695	Fig.4.2

evolution time 10 years
comparison between natural development, dredging, and dredging and depositing.

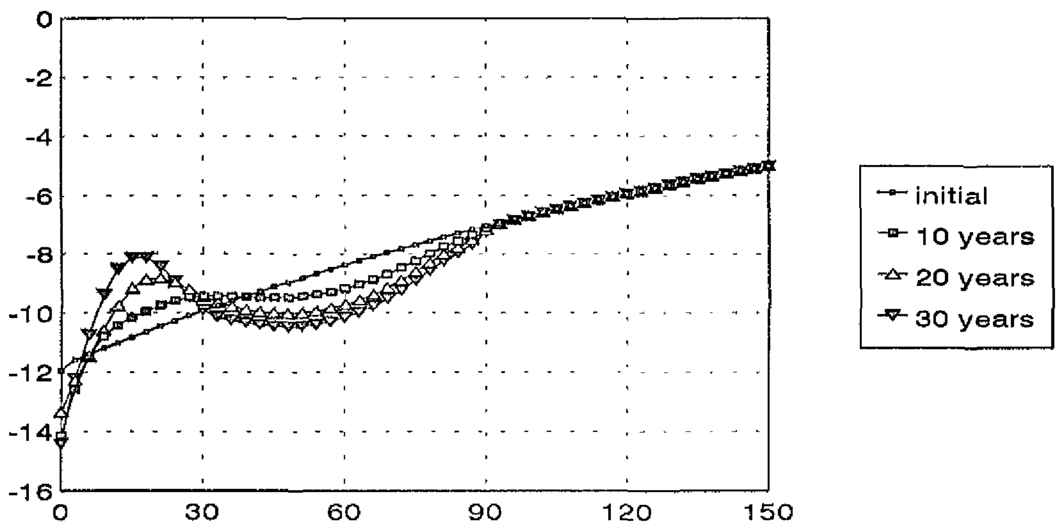


evolution time 20 years
comparison between natural development, dredging, and dredging and depositing.

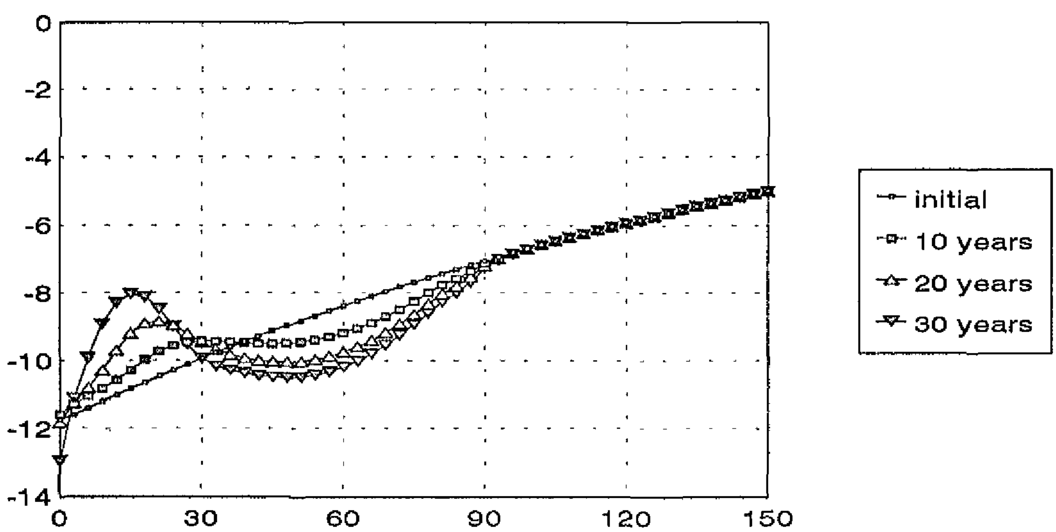


Comparison between the scenarios		Nov 93
	Western-Scheldt	
DELFT HYDRAULICS	Proj: Z-695	Fig.4.3

transport at the mouth 2000 m ^ 3/period

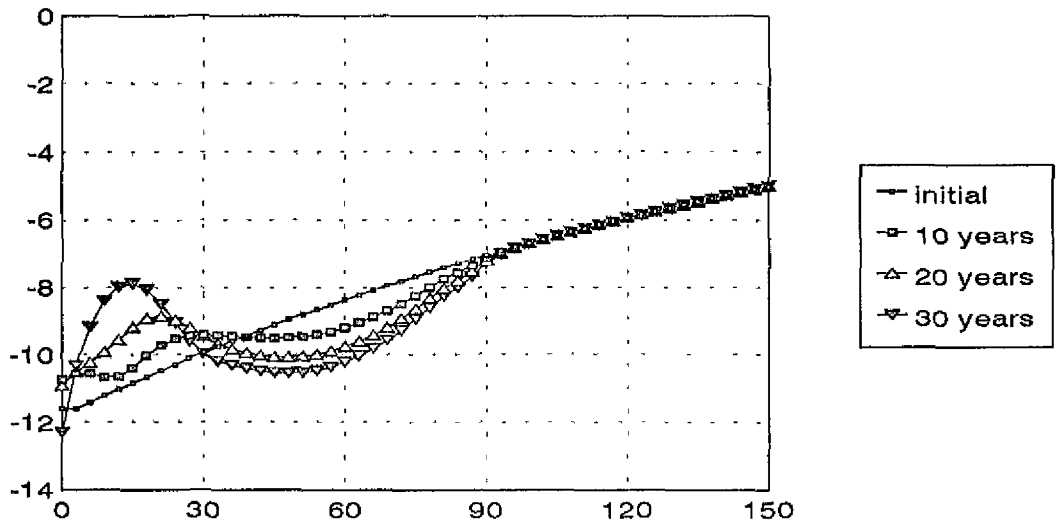


transport at the mouth 3000 m ^ 3/period

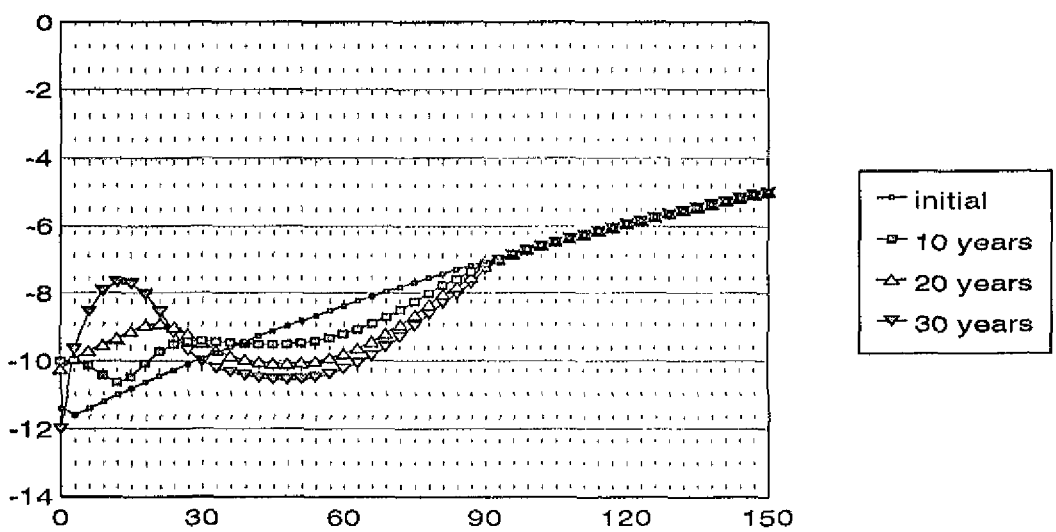


Boundary condition at the mouth: fixed sediment transport instead of fixed bottom level.		Nov 93
	Western-Scheldt	
DELFT HYDRAULICS	Proj: Z-695	Fig.5.1

transport at the mouth 4000 m ^ 3/period



transport at the mouth 5000 m ^ 3/period



Boundary condition at the mouth: fixed sediment transport instead of fixed bottom level.		Nov 93
	Western-Scheldt	
DELFT HYDRAULICS	Proj: Z-695	Fig.5.2