# Evaluation of ESTMORF 

## Mass balance and additional simulation

## B. Karssen



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## 1 Introduction

The Directoraat-Generaal Rijkswaterstaat is interested in morphological models predicting the consequences of (human) interference (e.g. dredging, land reclamation) in the geometry of estuaries and tidal basins.

In view of previous studies, the RWS decided to build a model for the long-term morphological development of estuaries (estmorf). Phase I of the development of estmorf was completed in January 1994. Phase II of the development of the ESTMORF computer program for RWS/RIKZ (DYnastar) is presently under completion. The results of the calibration of the Westerschelde model (Karssen, 1994) have given rise to a further evaluation of the Estmorf program and its application to the Westerschelde case.

Rijkswaterstaat/Directie Zeeland commissioned DelfT hydraulics by 'Opdrachtbon' nr. 554040 dated 9 December 1994 to perform an additional evaluation and a simulation. In detail, the following activities were part of the scope of work:

- a simulation with the Westerschelde model over the period 1968-1993, using the dumping and dredging volumes such as provided by Rws during Phase II (Karssen, 1994) and a transfer of the dumping volumes from the Land van Saeftinghe to the area around Bath.
- a sediment mass balance check for each branch of the Westerschelde model for the simulation described above, by computing/determining
- the inflowing and outflowing sediment transports,
- the dredged and dumped sediment volumes, and
- the net deposition or erosion in the channel, low flats and high flats
for each of the branches.
For a correct mass balance, the following equation has to be satisfied for each time step and for the whole simulation period:

$$
\begin{equation*}
\left(S_{i n}-S_{o u t}\right) \Delta t=L \Delta A+V_{D} \tag{1}
\end{equation*}
$$

where:

| L | $=$ | length of the branch | $[\mathrm{m}]$ |
| :--- | :--- | :--- | ---: |
| $\mathrm{S}_{\text {in }}$ | $=$ | sediment transport rate into the branch/area | $\left[\mathrm{m}^{3} / \mathrm{s}\right]$ |
| $\mathrm{S}_{\text {out }}$ | $=$ | sediment transport rate out of the branch/area | $\left[\mathrm{m}^{3} / \mathrm{s}\right]$ |
| $\mathrm{V}_{\mathrm{D}}$ | $=$ | dredged/dumped volume | $\left[\mathrm{m}^{3}\right]$ |
| $\Delta \mathrm{A}$ | $=$ | cross-sectional area change in time step $\Delta \mathrm{t}$ | $\left[\mathrm{m}^{2}\right]$ |
| $\Delta \mathrm{t}$ | $=$ | morphological time step $=$ human interference time step | $[\mathrm{s}]$ |

Further, for the Dutch part of the Westerschelde model, the above mass balance has to be checked.

- the development of a computer program for the tabulation of the net water volumes and sediment transport through the branches.

This report contains a description of the results of the above activities.
The present study has been performed by B. Karssen. The useful discussions with and guidance of A. Langerak of Rijkswaterstaat/RIKZ are gratefully acknowledged.

## 2 Results

### 2.1 General study set-up

The main objective of the study is to validate the software program ESTMORF with respect to the solution of the concentration equation, the subsequent cross-sectional area change and the incorporation of human interference in the system. In order to do so, existing subroutines have been changed to produce the necessary output for the validation and new post-processing software has been developed.

The results can be considered as the outcome of an iterative process, consisting of the following steps:
a. programming of the output procedures for the validation
b. simulation
c. assessment of the results
d. adjustment of the software, if necessary

Steps $\mathrm{b}-\mathrm{d}$ have been repeated until the validation showed good results.
In the next sections, the results of the above activities are worked out in detail. In view of the fact that some (minor) abbreviations were necessary, Section 2.2 contains an overview of the adjustments made to the software, In the Sections 2.3-2.5, the validation results after adjustment are presented.

### 2.2 Software adjustements

## human interference

During the tests, it appeared that the dumping of sediment was not correctly implemented for a certain shape of the channel part of the cross-section. As a result, the impact of dumping on the relevant channels was not noticable. This was caused by a numerical effect: for large slopes of the channel banks, a multiplication factor became too small. This factor had to be multiplied in the next command by a large number. Analytically, this gives the correct results. Numerically, the multiplication became too small.
The problem was solved by a change of the order of the computations. The final human interference subroutine now fully satisfies the mass balance equation.

## residual flow

The residual flow was determined by a division of the residual discharge by the cross-sectional area. However, in the subroutine under consideration, the cross-sectional area parameter denotes half of the cross-sectional area.
The problem was solved by multiplying the cross-sectional area parameter in the relevant subroutine by 2 before computing the residual flow.

## accuracy

It appeared that the mass balance was not satisfied after taking into account deposition/erosion and/or human interference. Although the differences were small, it was considered important to find the cause for this. The origin of the deviation appeared to be the accuracy of the coordinates of the cross-sectional profile. After a cross-sectional area change due to human interference and/or deposition/erosion, the internal full accuracy of the coordinates (single precision) was reduced to the precision of the format in which the coordinates were written to file.

Example: a cross-sectional area change of $1000 \mathrm{~m}^{3}$ in a rectangular channel of width 1000 m and length 1000 m leads to a change of depth of 0.001 m . With an accuracy of centimeters, this change is not detected in the profile, and thus sediment is lost, With smaller area changes and larger channels, full accuracy in single precision will even be insufficient.

Originally, the ESTMORF cross-sectional profiles were saved on file with an accuracy of centimeters for both the horizontal (width) and the vertical coordinates (height/depth).
This was solved by writing the cross-sectional profiles free formatted, i.e. with the full precision.
Note: The mass balance is not yet satisfied completely. The deviations are, however, very small, see the next sections. These deviations are caused by the internal precision of the parameters (single precision). It is recommended to change this single precision to double precision in a later stage.

### 2.3 Simulation

A simulation has been performed for the period 1968-1993, with Runid 500. For this simulation, boundary conditions, input parameters, etc. have been kept equal to those of Run 300, as described in Karssen (1994). However, the human interference (dumping and dredging) has been changed relative to the previous simulation Run 300: the dumping in the Land van Saeftinghe has been moved to dumping in the area near Bath. In other words, the sum of the volumes dumped in the branches $93,94,95$ and 96 were divided equally over the branches $98,99,100$ and 185 (for branch numbers, see Karssen (1994)).

Similar to the previous simulation, channels silted up during the simulation. The years of total filling up have been listed in Table 2.3.1 below.

| Branch | 82 | 83 | 84 | 85 | 102 | 105 |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: |
| Year | 1972 | 1984 | 1984 | 1978 | 1991 | 1979 |

Table 2.3.1 Years of total filling up in Run 500

Further, the volumes of six areas in the model were compared with measurement data over the simulation period. The results are presented in the Figures 2.3.1 and 2.3.2. Comparison with the Figures 4.4.4-4.4.7 of Karssen (1994) learns that the results have improved much for the Areas 1 and 2, and to a lesser extent also for the other areas. It is concluded that this is a combined effect of the moved dumping sites, the improved accuracy and the removal of the computational error in the human intereference routine.

### 2.4 Sediment balance check

One of the main objectives of the study is to identify whether the mass balance is satisfied for both solution methods of the concentration field when the impact of human interference is computed, as described in Section 2.3. In order to do so, a sediment balance check has been performed on three levels:
i) branch level
a. the net sediment transport into the branch should equal the deposition/erosion plus the human interference (see Equation 1)
b. the net volume change of a branch (thus after a profile change!) should equal the volume of human interference plus the net input volume of sediment.
ii) nodal level
the net transport through each node should be zero.
iii) area or model level
the net volume change of an area should equal the human interference plus the net input of sediment.
The above three points should hold for the whole simulation such as described in Section 2.3.
i) branch level
a. For the whole simulation, the deviations from Equation (1) were written to a file. Afterwards, a computer program was used to determine some statistics on the error (deviation from mass balance). The results are summarized in Tables 2.4.1. It is noted that the volumes are computed per time step of one month.

| Maximum error in the computed transport $\left[\mathrm{m}^{3}\right]$ | 0.15 |
| :--- | :---: |
| Maximum relative error in the computed transport one month ${ }^{1}[-]$ | 0.12 |
| Transport magnitude during maximum relative error $\left[\mathrm{m}^{3}\right]$ | 0.21 |
| Average relative error $[-]$ | 0.00008 |

Table 2.4.1 Error statistics of the mass balance check based on the transport through the branches.

Tables 2.4.1 shows that the maximum error in the transports is small. However, this figure is only an absolute measure, and may be very large compared to the occurring transports. Therefore, the maximum relative error gives some indication on the magnitude of the error compared to the magnitude of the occurring transports. At first sight, the maximum relative error is quite high, but it appeared that this maximum relative error only occurs during very small transports (less than $1 \mathrm{~m}^{3}$ per month). By way of comparison, sediment transports of order 50000 $\mathrm{m}^{3}$ per month are very common during the simulation.
b. For the whole simulation, the net volume change, the net deposition/erosion and the human interference were written to file for all branches. A computer program was used to determine some statistical quantities, which are listed in Table 2.4.2.

[^0]| Average volume change per month $\left[\mathrm{m}^{3}\right]$ <br> (averaged over all branches and all time steps) | -1500 |
| :--- | :---: |
| Maximum absolute error in mass balance $\left[\mathrm{m}^{3}\right]$ | 10258 |
| Average absolute error in mass balance $\left[\mathrm{m}^{3}\right]$ | 5 |

Table 2.4.2 Error statistics of the mass balance check based on the volume changes in the branches.


#### Abstract

The average absolute error is small, especially compared to the average volume change (about $0.3 \%$ ). It also appeared that zero volume changes were found in combination with small erosion/deposition volumes. After some study, it was found that this is due to the accuracy problem, such as described in Section 2.2.

The maximum absolute error may seem large. A detailed study of the results showed that such errors do not occur frequently, and if so, only for a duration of one time step. These relatively large errors were found each time in combination with a change of Mean High Water. It appears that the subsequent profile change is not fully mass conserving for all cases. It should be noted that the error is still relatively small: $10258 \mathrm{~m}^{3}$ over a branch of 1000 m width and 1000 m length is about 1 cm vertical change on average over the profile. Although errors are small and do not occur frequently, it is recommended to study the profile change with respect to MHW-changes in more detail later.


It is concluded that the error in the mass balance in the branches is negligible.

## ii) nodal level

For each node, the net transport through that node was written to file. In order to satisfy the mass balance equation, this net transport should be zero. Again, a computer program was used to determine some statistical quantities, which are listed in Table 2.4.3.

| Maximum error in the computed net transport $\left[\mathrm{m}^{3}\right]$ | 0.03 |
| :--- | :---: |
| Maximum relative error in the net transport $[-]$ | 0.3 |
| Transport through the node during maximum relative error $\left[\mathrm{m}^{3}\right]$ | 0.08 |
| Average relative error $[-]$ | 0.00009 |

Table 2.4.3 Error statistics of the mass balance check based on the net transport through the nodes.

The conclusions that can be drawn for the nodes rae similar to those drawn for the branches. The maximum relative error is high, about $30 \%$, but again occurring during very small transports.
iii) area or model level

Basically, the mass balance error in an area equals the sum of the errors made (see item i)b.) in those branches included in the area under consideration. A computer program has been developed to sum the volume changes, human interference and net transport of the
individual branches in the model over the whole simulation period of 25 years. The statistics of these results are listed in Table 2.4.4.

| Volume error (whole simulation) $\left[\mathrm{m}^{3}\right]$ | $-112,862$ |
| :--- | :---: |
| Total volume change $\left[\mathrm{m}^{3}\right]$ | $-74,343,520$ |
| Relative error $[-]$ | 0.0015 |

Table 2.4.4 Volume errors in the model over the whole simulation period of 25 years

It is concluded that the error for the area/model is negligible.

### 2.5 Computer program for discharge/transport tabulation

An additional feature has been built in in the estmorf software: it is now possible to produce a table containing a list of the net volume of water flowing through a branch during a tidal period and a list of the sediment transport through the ends of each of the branches. The transports are subdivided in an advection and a diffusion part. This table enables post-processing with respect to residual transports and flows. A typical example of the layout of this table can be found in Appendix A-1.

Furthermore, a computer program has been developed which uses the aforementioned output table as input to produce another table contaning residual flows and transports of areas defined by the user. In this way, it is possible to identify the 'residual characteristics' of areas which are of special interest to the user. An example of this table is presented in Appendix A-2.

## 3 Conclusions and recommendations

Some (minor) errors were found in the software, with respect to the human interference routine and the computation of the residual flow. Improvements to the software are made to solve these problems.

The mass balance check also led to the conclusion that the accuracy of the computation depends on the accuracy of the storage of the cross-sectional profile coordinates. The accuracy has been improved by storage of the profiles with full accuracy (single precision). The mass balance is now satisfied with sufficient accuracy, but improvements can be made by storage in double precision. It is recommended to make such an adjustment in a later stage.

A similar accuracy problem gives a deviation for mass balance after a change of Mean High Water, but not always and only for certain profiles. The resulting error in the mass balance of the whole model is small and short-term action is thus not considered necessary. However, it is recommended to study this feature in more detail later.
estmorf satisfies the mass balance equation with sufficient accuracy. A summary of the errors in the mass balance are presented in Table 3.1 below.

| Branch transport, maximum absolute error $\left[\mathrm{m}^{3}\right]$ | 0.15 |
| :--- | :---: |
| Branch volume, average absolute error $\left[\mathrm{m}^{3}\right]$ | 5 |
| Nodal transport, maximum absolute error $\left[\mathrm{m}^{3}\right]$ | 0.03 |
| Model volume, relative error $[-]$ | 0.0015 |

Table 3.1 Errors in mass balance

A computer program has been developed for post-processing purposes with respect to residual flows and transports. It is recommended to develop a post-processing program to present these results graphically in two-dimensional maps.

A simulation has been performed with the Westerschelde model, including a shift of the dumping and dredging volumes from the Land van Saeftinghe to the area near Bath. The volumetric results now compare much better than for previous runs such as reported in Karssen (1994).

In order to be able to evaluate the performance of the Westerschelde model in more detail, it is recommended to make a comparison of the computed and observed cross-sectional areas below msL for the individual channels of the model. Further, the behaviour of the model with respect to the simulation of the area and heights of the tidal flats should be studied.

## References

Karssen, B. 1994: A dynamic/empirical model for the long-term morphological development of estuaries, Part II: Development of the model, Phase II. Report Z715. DELFT HYDRAULICS, Delf, November 1994 (3rd Draft).



Appendices

| val | Vin | Vuit | Tadvin | Tdisin | Tadvuit | Tdisuit |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | -. $131891 \mathrm{E}+09$ | -. 131889E+09 | -. 146546E-02 | .689556E-03 | -. 138644E-02 | . $644128 \mathrm{E}-0.3$ |
| 2 | -. $949780 E+08$ | -. $949790 \mathrm{E}+08$ | -. $970391 \mathrm{E}-03$ | . $278225 \mathrm{E}-03$ | -. $996429 \mathrm{E}-03$ | -. $714825 \mathrm{E}-03$ |
| 3 | -. $949790 \mathrm{E}+08$ | -. $949780 \mathrm{E}+08$ | -. $996429 \mathrm{E}-03$ | -. $714827 \mathrm{E}-03$ | -. $106151 \mathrm{E}-02$ | -. 191099E-03 |
| 4 | -. $364630 \mathrm{E}+08$ | -. $364620 \mathrm{E}+08$ | -. $407523 \mathrm{E}-03$ | -. 122079E-03 | -. 400164E-03 | . $645870 \mathrm{E}-03$ |
| 5 | --732810E+08 | -. $732590 \mathrm{E}+08$ | -. $804245 \mathrm{E}-03$ | . $415781 \mathrm{E}-03$ | -. $774295 \mathrm{E}-03$ | . $119651 \mathrm{E}-02$ |
| 6 | -. $468530 \mathrm{E}+08$ | -. $468300 \mathrm{E}+08$ | -. $495202 \mathrm{E}-03$ | . $194575 \mathrm{E}-02$ | -. 451714E-03 | . $120966 \mathrm{E}-02$ |
| 7 | - . $369060 \mathrm{E}+08$ | -. 368920E+08 | -. $377069 \mathrm{E}-03$ | -. 174515E-03 | -.389495E-03 | -. 945726E-04 |
| 8 | -. $368710 \mathrm{E}+08$ | -. $368430 \mathrm{E}+08$ | -. 389274E-03 | . $272952 \mathrm{E}-04$ | -. $391074 \mathrm{E}-03$ | -. 695806E-04 |
| 9 | -. $368430 \mathrm{E}+08$ | -. $368190 \mathrm{E}+08$ | -. $391074 \mathrm{E}-03$ | -. 695818E-04 | -. 404082E-03 | -. 230072E-03 |
| 10 | -. $206200 \mathrm{E}+05$ | . $122680 \mathrm{E}+01$ | -. $217700 \mathrm{E}-06$ | -. 121869E-03 | . $134633 \mathrm{E}-10$ | . $000000 \mathrm{E}+00$ |
| 11 | . $131372 \mathrm{E}+09$ | . $131495 \mathrm{E}+09$ | . $145969 \mathrm{E}-02$ | . $150281 \mathrm{E}-03$ | . $137893 \mathrm{E}-02$ | . 101990E-02 |
| 12 | . $585159 \mathrm{E}+08$ | . $585158 \mathrm{E}+08$ | .613631E-03 | . $513847 E-03$ | .604230E-03 | -. 421101E-03 |
| 13 | . $585158 \mathrm{E}+08$ | . $585164 \mathrm{E}+08$ | . $604230 \mathrm{E}-03$ | -. $421100 \mathrm{E}-03$ | .654000E-03 | . $690243 \mathrm{E}-04$ |
| 14 | .729780E+08 | . $731700 \mathrm{E}+08$ | . $765289 \mathrm{E}-03$ | . 506051E-03 | . $755229 \mathrm{E}-03$ | -. $383890 \mathrm{E}-03$ |
| 15 | . $264069 \mathrm{E}+08$ | . $264070 \mathrm{E}+08$ | . $272560 \mathrm{E}-03$ | -.623739E-03 | . $323130 \mathrm{E}-03$ | . $548549 \mathrm{E}-04$ |
| 16 | .458370E+08 | . $458490 \mathrm{E}+08$ | . $473110 \mathrm{E}-03$ | . $218683 \mathrm{E}-03$ | . $423141 \mathrm{E}-03$ | . 150831E-03 |
| 17 | . $926200 \mathrm{E}+06$ | . $119650 \mathrm{E}+07$ | . $955983 \mathrm{E}-05$ | . $211646 \mathrm{E}-04$ | .146181E-04 | -. 129272E-03 |
| 19 | -. $465200 \mathrm{E}+07$ | -. $453500 \mathrm{E}+07$ | -. 516889E-04 | . $535603 \mathrm{E}-04$ | -. $544357 \mathrm{E}-04$ | -. $285866 \mathrm{E}-03$ |
| 20 | -. $453500 \mathrm{E}+07$ | -. $449600 \mathrm{E}+07$ | . $544357 \mathrm{E}-04$ | -. $285867 \mathrm{E}-03$ | -. 541136E-04 | . $267148 \mathrm{E}-03$ |
| 21 | $-.449600 \mathrm{E}+07$ | -. $445200 \mathrm{E}+07$ | -. 541136E-04 | . $267148 \mathrm{E}-03$ | -. 517518E-04 | -. $103378 \mathrm{E}-03$ |
|  |  | . |  |  |  |  |
|  |  | - |  |  |  |  |
|  |  |  |  |  |  |  |
| 175 | . $416226 \mathrm{E}+06$ | . $419715 \mathrm{E}+06$ | .481433E-05 | . $272268 \mathrm{E}-06$ | .479972E-05 | . 547437E-06 |
| 176 | . $419714 \mathrm{E}+08$ | . $423461 \mathrm{E}+06$ | . $479970 \mathrm{E}-05$ | . $547428 \mathrm{E}-06$ | . $476575 \mathrm{E}-05$ | . $125284 \mathrm{E}-06$ |
| 177 | . $423461 \mathrm{E}+06$ | . $581100 \mathrm{E}+06$ | . $476575 \mathrm{E}-05$ | . $125282 \mathrm{E}-06$ | .645667E-05 | . $299728 \mathrm{E}-06$ |
| 178 | -. $293900 \mathrm{E}+05$ | -. $264990 \mathrm{E}+05$ | -. $339943 \mathrm{E}-06$ | -. $862843 \mathrm{E}-06$ | -. 307970E-06 | . $747443 \mathrm{E}-06$ |
| 179 | -. $264990 \mathrm{E}+05$ | -. $223210 \mathrm{E}+05$ | -. 307970E-06 | . $747443 \mathrm{E}-06$ | -. $253360 \mathrm{E}-06$ | -. $155413 \mathrm{E}-06$ |
| 180 | -. $223210 \mathrm{E}+05$ | -. 171970E+05 | -. $253360 \mathrm{E}-06$ | -. $155415 \mathrm{E}-06$ | -. $197103 \mathrm{E}-06$ | . 504517E-07 |
| 181 | -. $171970 \mathrm{E}+05$ | -. 117050E+05 | -. $197103 \mathrm{E}-06$ | . $504510 \mathrm{E}-07$ | -. 131767E-06 | . $204943 \mathrm{E}-07$ |
| 182 | -. $117050 \mathrm{E}+05$ | -. $544800 \mathrm{E}+04$ | -. 131767E-06 | . $204947 \mathrm{E}-07$ | -.610178E-07 | . 186749E-07 |
| 183 | $-.544800 \mathrm{E}+04$ | -. 140509E-01 | -. $610178 \mathrm{E}-07$ | . $186742 \mathrm{E}-07$ | $-.156025 \mathrm{E}-12$ | . $246325 \mathrm{E}-12$ |
| 184 | -. $131889 \mathrm{E}+09$ | -. $131883 \mathrm{E}+09$ | -. $138644 \mathrm{E}-02$ | . $644127 \mathrm{E}-03$ | -. $134745 \mathrm{E}-02$ | . $103708 \mathrm{E}-03$ |
| 185 | -. $397550 \mathrm{E}+08$ | -. $396923 \mathrm{E}+08$ | -. 683601E-03 | . $524286 \mathrm{E}-02$ | -. $294477 \mathrm{E}-03$ | . $121738 \mathrm{E}-02$ |
| 187 | -.664701E+07 | -. $664801 \mathrm{E}+07$ | -. $641159 \mathrm{E}-04$ | . $259623 \mathrm{E}-02$ | -. 613545E-04 | -. $173291 \mathrm{E}-02$ |
| 190 | .264070E+08 | . $264068 \mathrm{E}+08$ | . $323130 \mathrm{E}-03$ | . $548534 \mathrm{E}-04$ | .279101E-03 | . $749263 \mathrm{E}-03$ |

Appendix A-2: Residual transport and flow table for user-defined areas (example)

| groep | vak | Vin | Vuit | Tadvin | Tdisin | Tadvuit | Tdisuit | Tin | Tuit |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 6 |  | $-.131891 \mathrm{E}+09$ |  | -.659457E+02 | $.310300 \mathrm{E}+02$ |  |  | -. 349157E+02 |  |
| 6 | 2 |  | $-.949790 \mathrm{E}+08$ |  |  | $-.448393 E+02$ | $-.321671 \mathrm{E}+02$ |  | -. $770064 \mathrm{E}+02$ |
| 5 |  | -. $949790 \mathrm{E}+08$ |  | $-.448393 \mathrm{E}+02$ | $-.321672 \mathrm{E}+02$ |  |  | $-.770065 \mathrm{E}+02$ |  |
| 6 | 7 |  | -. $368920 \mathrm{E}+08$ |  |  | -. $175273 \mathrm{E}+02$ | $-.425577 \mathrm{E}+01$ |  | -. $217830 \mathrm{E}+02$ |
| 5 |  | $-.368710 \mathrm{E}+08$ |  | -. $175173 \mathrm{E}+02$ | . $122828 \mathrm{E}+01$ |  |  | -. $162890 \mathrm{E}+02$ |  |
| 6 | 11 | $.131372 \mathrm{E}+09$ |  | .656861E+02 | . $676265 \mathrm{E}+01$ |  |  | . $724487 \mathrm{E}+02$ |  |
| 6 | 12 |  | . $585158 \mathrm{E}+08$ |  |  | . $271903 \mathrm{E}+02$ | $-.189495 \mathrm{E}+02$ |  | .824080E+01 |
| 5 | 13 | . $585158 \mathrm{E}+08$ |  | . $271903 \mathrm{E}+02$ | -. $189495 \mathrm{E}+02$ |  |  | . $824085 \mathrm{E}+01$ |  |
| 6 | 14 |  | . $731700 \mathrm{E}+08$ |  |  | $.339853 \mathrm{E}+02$ | -. $172750 \mathrm{E}+02$ |  | . $167103 \mathrm{E}+02$ |
| 5 | 15 | . $264069 \mathrm{E}+08$ |  | . $122652 \mathrm{E}+02$ | $-.280683 E+02$ |  |  | $-.158031 \mathrm{E}+02$ |  |
| 5 | 16 | . $458370 \mathrm{E}+08$ |  | . $212899 \mathrm{E}+02$ | . $984073 \mathrm{E}+01$ |  |  | . $311307 \mathrm{E}+02$ |  |
| 5 | 17 | . $926200 \mathrm{E}+06$ |  | . $430192 \mathrm{E}+00$ | . $952407 \mathrm{E}+00$ |  |  | . $138260 \mathrm{E}+01$ |  |
| 6 | 19 | -. $465200 \mathrm{E}+07$ |  | $-.232600 \mathrm{E}+01$ | . $241021 \mathrm{E}+01$ |  |  | $.842129 \mathrm{E}-01$ |  |
| 6 | 20 |  | $-.449600 \mathrm{E}+07$ |  |  | $-.243511 E+01$ | $.120217 \mathrm{E}+02$ |  | $.958655 \mathrm{E}+01$ |
| 5 | 21 | -. $449600 \mathrm{E}+07$ |  | $-.243511 \mathrm{E}+01$ | . $120217 \mathrm{E}+02$ |  |  | $.958655 \mathrm{E}+01$ |  |
| 5 | 34 |  | . $356790 \mathrm{~F}+08$ |  |  | . $195252 \mathrm{E}+02$ | $-.391284 E+00$ |  | . $191339 \mathrm{E}+02$ |
| 4 | 35 | $.356790 \mathrm{E}+08$ |  | $.195252 \mathrm{E}+02$ | -. $391277 \mathrm{E}+00$ |  |  | . $191340 \mathrm{E}+02$ |  |
| 4 | 39 |  | . $360560 \mathrm{E}+08$ |  |  | . $208456 \mathrm{E}+02$ | -. $419207 \mathrm{E}+02$ |  | -. $210751 \mathrm{E}+02$ |
| 5 | 43 |  | -. 392850玉+08 |  |  | -. $183666 \mathrm{E}+02$ | . $268652 \mathrm{E}+01$ |  | -. $156801 \mathrm{E}+02$ |
| 4 | 44 | $-.392850 \mathrm{E}+08$ |  | $-.183666 \mathrm{E}+02$ | $.268650 \mathrm{E}+01$ |  |  | -. $156801 \mathrm{E}+02$ |  |
| 4 | 48 |  | -. 391200E+08 |  |  | $-.200512 \mathrm{E}+02$ | -. $697243 \mathrm{E}+02$ |  | -. $897756 \mathrm{E}+02$ |
| 3 | 54 |  | -. $182110 \mathrm{E}+08$ |  |  | -.933417E+01 | . $382541 \mathrm{E}+01$ |  | -. 550876E+01 |
| 3 | 54 | $-.182150 \mathrm{E}+08$ |  | -. $105309 \mathrm{E}+02$ | . $418205 \mathrm{E}+02$ |  |  | . $312895 \mathrm{E}+02$ |  |
| 3 | 55 | -. 573320E+08 |  | -. $293859 \mathrm{E}+02$ | -. $658989 \mathrm{E}+02$ |  |  | -. $952848 \mathrm{E}+02$ |  |
| 3 | 61 | . $542710 \mathrm{E}+08$ |  | . $313765 \mathrm{E}+02$ | $-.837414 \mathrm{E}+02$ |  |  | -. $523649 \mathrm{E}+02$ |  |
| 3 | 68 |  | . $465299 \mathrm{E}+07$ |  |  | . $199426 \mathrm{E}+01$ | $.106531 \mathrm{E}+03$ |  | . $108525 \mathrm{E}+03$ |
| 2 | 69 | . $465299 E+07$ |  | . $199426 E+01$ | $.106529 E+03$ |  |  | $.108524 E+03$ |  |
| 3 | 72 |  | -. $685720 \mathrm{E}+07$ |  |  | -. $302682 \mathrm{E}+01$ | $-.227718 \mathrm{E}+02$ |  | -. $257987 \mathrm{E}+02$ |
| 2 | 73 |  | -. $195930 \mathrm{E}+08$ |  |  | -. $864851 \mathrm{E}+01$ | -. $445868 \mathrm{E}+02$ |  | -. $532353 \mathrm{E}+02$ |
| 2 | 78 |  | . $239320 \mathrm{E}+07$ |  |  | . $745466 \mathrm{E}+00$ | -. $287821 \mathrm{E}+01$ |  | -. $213275 \mathrm{E}+01$ |
| 2 | 81 |  | . $375470 \mathrm{E}+07$ |  |  | . $116956 \mathrm{E}+01$ | . $403127 \mathrm{E}+01$ |  | . $520083 \mathrm{E}+01$ |
| 2 | 88 |  | . $185670 \mathrm{E}+08$ |  |  | $.113661 \mathrm{E}+02$ | $-.325494 \mathrm{E}+02$ |  | -. $211833 \mathrm{E}+02$ |
| 1 | 89 | . $578160 \mathrm{E}+07$ |  | . $180092 \mathrm{E}+01$ | . $154287 \mathrm{E}+01$ |  |  | . $334380 \mathrm{E}+01$ |  |
| 1 | 90 | . $366200 \mathrm{E}+06$ |  | . $114069 \mathrm{E}+00$ | $-.389814 \mathrm{E}+00$ |  |  | -. $275745 \mathrm{E}+00$ |  |
| 2 | 91 | -. $264510 \mathrm{E}+08$ |  | -. 116757E+02 | $-.673587 \mathrm{E}+02$ |  |  | $-.790344 \mathrm{E}+02$ |  |
| 2 | 96 |  | -. $256810 \mathrm{E}+08$ |  |  | -. $157211 \mathrm{E}+02$ | $.599364 \mathrm{E}+02$ |  | . $442153 \mathrm{E}+02$ |
| 1 | 98 | $-.664000 \mathrm{E}+07$ |  | $-.406479 \mathrm{E}+01$ | . $268763 \mathrm{E}+02$ |  |  | $.228115 \mathrm{E}+02$ |  |
| 1 | 119 |  | . $400000 E+06$ |  |  | +206122E+00 | $-.436630 \mathrm{E}+01$ |  | $-.416018 E+01$ |
| Groep |  |  |  |  |  |  |  |  |  |
| 1 |  | -. $979880 \mathrm{E}+06$ | . $797008 \mathrm{E}+06$ | -. $432499 \mathrm{E}+01$ | . $531631 \mathrm{E}+02$ | . $409942 \mathrm{E}+00$ | -. $859716 \mathrm{E}+01$ | . $258795 \mathrm{E}+02$ | -. $416018 \mathrm{E}+01$ |
| 2 |  | -. $436660 \mathrm{E}+08$ | -. $411953 \mathrm{E}+08$ | -. 194312E+02 | . $761706 \mathrm{E}+02$ | -. $222656 \mathrm{E}+02$ | -. $358621 \mathrm{E}+02$ | . $294893 \mathrm{E}+02$ | -. $271352 \mathrm{E}+02$ |
| 3 |  | -. 425790E+08 | $-.408533 \mathrm{E}+08$ | -. $170580 \mathrm{E}+02$ | $-.215857 \mathrm{E}+03$ | -. 207230E+02 | . $173847 \mathrm{E}+03$ | -. 116360E+03 | . $772176 \mathrm{E}+02$ |
| 4 |  | -. $721400 \mathrm{E}+07$ | -. 612500E+07 | . $227849 \mathrm{E}+01$ | . $454124 \mathrm{E}+01$ | . $159188 \mathrm{E}+01$ | $-.223323 E+03$ | . $345388 \mathrm{E}+01$ | -. 110851E+03 |
| 5 |  | -. $932549 \mathrm{E}+07$ | -. $721400 \mathrm{E}+07$ | -. $720635 \mathrm{E}+01$ | -. $109965 \mathrm{E}+03$ | . $227849 \mathrm{E}+01$ | . $454129 \mathrm{E}+01$ | -. 587579E+02 | . $345389 \mathrm{E}+01$ |
| 6 |  | -. 103510E+08 | -. 936700E+07 | -. $517541 \mathrm{E}+01$ | $.829332 \mathrm{E}+02$ | -. $722594 \mathrm{E}+01$ | -. 121115E+03 | . $376172 \mathrm{E}+02$ | -. $642519 \mathrm{E}+02$ |


[^0]:    1 relative error $=$ error in the computed transport divided by the computed transport

