MATHEMATICAL MODELING OF FENDER FORCES AND MEMORY EFFECTS FOR SIMULATION OF SHIP MANOEUVRES IN CONFINED WATERS

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ABSTRACT

A reliable estimation of contact forces between a ship's hull and constructions protected by fenders requires the knowledge of the time history of hydrodynamic forces acting on the ship. As large decelerations may occur, memory effects become important and a quasisteady approach, which is commonly in use in manoeuvring simulation, cannot be applied.

The first part of the paper gives an alternative review of mathematical models suited for calculation of hydrodynamic forces on ships in impact or collision situations. Particular attention is paid to mathematical models making use of impulse response function techniques and the application of state vectors. The second part gives the outlines of the theoretical base for the implementation of memory effects in a manoeuvring simulation program.

INTRODUCTION

The forces acting on a ship which is navigating in confined waters can be influenced substantially when the distance to the boundaries of the navigational area decreases. The forces caused by the vicinity of these boundaries can be divided in three types.

A first group of forces, *bank suction*, is caused by the component of the ship's speed parallel to a closed boundary. Unless the ship's course follows the centerline of the waterway, lateral forces are induced by the asymmetric flow around the ship's hull. Forces of this kind will not be discussed in this paper.

Contact forces, are caused by contact between the ship's hull and the boundary of the navigational area (bank, quay wall, fender, bridge, bridgehead, jetty, ...). Following effects have to be taken into account:

- a restoring force which is a function of the deformation of the boundary by the ship's hull or v.v.;
- a damping force which is a function of the relative speed component perpendicular to the contact plane;
- an inertia term depending on the movable mass of the boundary;
- a frictional term caused by the component of the relative speed in the contact plane.

Finally, a third group of forces can be defined as *unsteady hydrodynamic forces*, acting on the ship as a consequence of relatively large accelerations or decelerations. The latter not only occur if the ship comes into contact with the boundaries of the navigational area, but can also be induced by other causes (tugs, anchors, rudder and machine manoeuvres). Unsteady hydrodynamic forces act on the ship in case of contact with open (e.g. quay wall) as well as closed (e.g. jetty) boundaries, but their characteristics are influenced substantially by the boundary's nature. If a ship is laterally approaching a closed boundary, the water level between hull and boundary increases, causing a transverse flow in the underkeel clearance and a longitudinal flow in the quay clearance.

A reliable mathematical model of these forces is required for simulation of manoeuvres in confined waters. In a preliminary stage of the implementation of contact forces and unsteady hydrodynamic forces into the manoeuvring simulator of the Hydraulics Research Laboratory of Antwerp-Borgerhout, the Office of Naval Architecture of Ghent University was charged with the selection and development of a suitable mathematical model. The choice of the mathematical model is subject to some restrictions, as simulations take place in real time :

- the integration time increment cannot be decreased without restriction; it should not be less than 0.2 s;
- the additional computing time required for force calculation should be as small as possible: this implies e.g. that the unsteady hydrodynamic force components should be presented as functions of a limited number of parameters expressing the relative position between ship and boundary.

REVIEW OF MATHEMATICAL MODELS

INTRODUCTION

A large majority of the publications handling contact forces and hydrodynamic memory effects have been developed from the viewpoint of the engineer responsible for the design of berthing and mooring facilities. As a consequence, most of the calculation methods described in literature are focused on an overall approach incorporating all force effects or on the estimation of extreme conditions. On the contrary, in a mathematical model suited for manoeuvring simulation an analysis of all effects acting on the ship is required, resulting into a reliable prediction of the time history of kinematic and dynamic characteristics.

This is the reason why the approach followed by the author in this review might seem rather unusual. Calculation methods based on considerations, based on kinematics or statistics, about the fraction of ship's kinetic energy which has to be absorbed by the mooring facility and its fenders will not be discussed in this paper, as they cannot be applied to real-time simulation. For a comprehensive review of these *energy methods* the reader is referred to [1],[2],[3].

These reviews mention two kinds of *mathematical models* as well:
- models based on the ship's equations of motion combined with *impulse response functions*;
- models based on a long wave approximation.

However, the latter can be considered as the simplest case of a model making use of *state vectors*, storing the ship's kinematical history; so far, these models have only been used for the simulation of forces acting on moored ships or ships in collision situations, but they can also be applied to berthing conditions.

More recent publications also describe *direct time approach* methods, based on simplified differential equations describing the dynamics and kinematics of the ship and the surrounding water, which are integrated together with the equations of motion.

IMPULSE RESPONSE FUNCTION TECHNIQUE.

General theory.

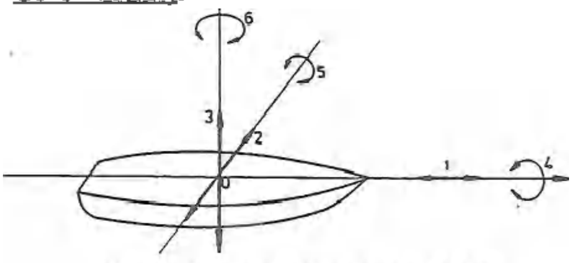

Fig. 1. Degrees of freedom of a ship.

Following general formulation of the equation of motion in mode j (see fig. 1 for definitions) is given by the classical theory of ship motions (see e.g. [4]) :

$$\sum_{j=1}^{6}\left\{(M_{kj}+a_{kj}(\omega))\ddot{x}_j+b_{kj}(\omega)\dot{x}_j+c_{kj}x_j\right\}=F_k(t) \tag{1}$$

in which $a_{kj}(\omega)$, $b_{kj}(\omega)$ and c_{kj} denote added mass, hydrodynamic damping and restoring hydrostatic coefficient, respectively; multiplied with acceleration, velocity and displacement, respectively, in mode j, they contribute to the force in mode k. M_{kj} stands for the factor with which the acceleration in mode j must be multiplied for the calculation of the inertia force in mode k (e.g. mass, moment of inertia).

Added mass and hydrodynamic damping are related with the radiation problem: due to the presence of a free water surface, ship motions generate a wave pattern. This causes a memory effect: even if the motion of the ship is stopped, the force action on the ship caused by the formerly generated wave system will continue, so that the ship dynamics is not only related to the instantaneous kinematics, but also to the kinematic past of the ship.

As the added mass and hydrodynamic damping coefficients are frequency-dependent, expression (1) is only valid if the external force F_k is a harmonic function of time. If this is not the case, a general expression for the motion of a floating body caused by the action of external forces is given by (see e.g. [5],[6]):

$$\sum_{j=1}^{6}\left\{M_{kj}\ddot{x}_j(t)+\int_{-\infty}^{\infty}h_{kj}(\tau)\,\dot{x}_j(t-\tau)\,d\tau+c_{kj}x_j(t)\right\} = F_k(t) \tag{2}$$

$h_{kj}(\tau)$ denotes the impulse response function for the hydrodynamic force in mode k at $t=\tau$ caused by a velocity pulse in mode j at $t=0$. The relation between frequency domain approach (1) and time domain formulation (2) yields an expression for this function:

$$\begin{aligned} h_{kj}(\tau) &= \mu_{kj}\frac{d\delta(\tau)}{d\tau}+\lambda_{kj}\delta(\tau)+K_{kj}(\tau), & \tau>0 \\ &= 0, & \tau<0 \end{aligned} \tag{3}$$

$\delta(t)$ being the Dirac function. μ_{kj} and λ_{kj} denote the high frequency limits for added mass and damping:

$$\begin{aligned} \mu_{kj} &= a_{kj}(\omega_0)+\frac{1}{\omega_0}\int_0^{\infty}K_{kj}(\tau)\sin\omega_0\tau\, d\tau \\ &= \lim_{\omega\to\infty} a_{kj}(\omega) \end{aligned} \tag{4}$$

$$\lambda_{kj} = \lim_{\omega\to\infty} b_{kj}(\omega) \tag{5}$$

while the *retardation function* $K_{kj}(t)$ is given by :

$$\begin{aligned} K_{kj}(t) &= \frac{2}{\pi}\int_0^{\infty}(b_{kj}(\omega)-b_{kj}(\infty))\cos\omega t\, d\omega \\ &= -\frac{2}{\pi}\int_0^{\infty}(a_{kj}(\omega)-a_{kj}(\infty))\,\omega\sin\omega t\, d\omega \end{aligned} \tag{6}$$

Introducing matrix notations, the equations of motion of the ship can be formulated as follows :

$$\begin{aligned} [F] &= \\ &([M]+[\mu])[x]+[\lambda]\,[x]+\int_{-\infty}^{t}[K(t-\tau)]\,[x(\tau)]\,d\tau \\ &+[c]\,[x] \end{aligned} \tag{7}$$

where [F],[x] denote 6x1-, and [M],[μ],[λ],[K],[a],[b], [c] 6x6-matrices. These dimensions decrease if only a limited number of motion modes is considered.

Models based on impulse response function techniques are applied rather currently for determining the time history of forces and displacements in cases of contact between a ship and a fixed construction (off shore construction, fender): Petersen & Pedersen [7]; Blok, Brozius & Dekker [8]; Fontijn [9],[10],[11]. Similar methods were used by Petersen [12] for collisions between ships, and by Van Oortmerssen [13],[14], Van Oortmerssen, Pinkster & van den Boom [15] and Remery [16] for moored ships.

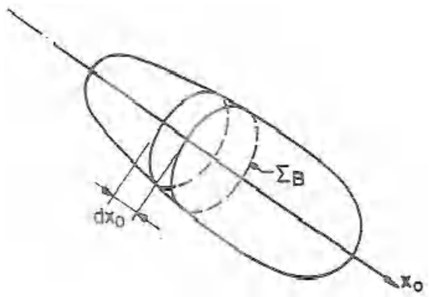

Fig. 2. Slender-body approximation : conventions.

__Strip theory formulation.__

The study of hydrodynamic forces acting on a body can often be simplified if it is assumed that the order of magnitude of one of the dimensions surpasses the other ones significantly, so that the body can be considered as *slender*. A ship can, for instance, be called slender in its longitudinal direction.

The *slender body approximation* implies that the longitudinal component of the fluid velocity is neglected. In this case, Gauss' theorem leads to following expression for the lateral force ([4], see fig. 2) :

$$Y(x) = -\rho \frac{d}{dt} \oint_{\Sigma_B(x)} \phi\, n_y\, dl \qquad (8)$$

Φ being the velocity potential. A general formulation of Φ for six degrees of freedom is given in [14] :

$$\phi(t) = \sum_{j=1}^{6} \left| x_j(t)\, \psi_j + \int_{-\infty}^{t} \chi_j(t-\tau)\, x_j(\tau)\, d\tau \right| \qquad (9)$$

where ψ_j and $\chi_j(t)$ denote the instantaneous and time-dependent fractions of the fluid reaction to a velocity impulse in mode j at t=0. Introduction of (9) into (8) leads to following expressions for the total lateral force Y and resulting yawing moment N (see Appendix A for definitions). From these expressions (10-11) it can be concluded that a consequent application of a strip theory not only requires an addition of memory terms to the hydrodynamic inertia forces, but also to the lifting forces (terms in ur and uv).

$$\begin{aligned} Y_h(t) = & +\mu_{22} v(t) + \int_0^{\infty} K_{22}(\tau)\, v(t-\tau)\, d\tau \\ & +\mu_{26} r(t) + \int_0^{\infty} K_{26}(\tau)\, r(t-\tau)\, d\tau \\ & -u(t)\, v(t)\, (\mu_F - \mu_A) \\ & -u(t) \int_0^{\infty} \left[\xi_F(\tau) - \xi_A(\tau)\right] v(t-\tau)\, d\tau \\ & -u(t)\, r(t)\, (x_F\mu_F - x_A\mu_A) \\ & -u(t) \int_0^{\infty} \left[x_F\xi_F(\tau) + x_A\xi_A(\tau)\right] r(t-\tau)\, d\tau \end{aligned} \qquad (10)$$

$$\begin{aligned} N_h(t) = & +\mu_{26} v(t) + \int_{-\infty}^{t} K_{26}(t-\tau)\, v(\tau)\, d\tau \\ & +\mu_{66} r(t) + \int_{-\infty}^{t} K_{66}(t-\tau)\, r(\tau)\, d\tau \\ & +u(t)\, v(t)\, (\mu_{22} - x_F\mu_F + x_A\mu_A) \\ & +u(t) \int_0^{\infty} \left[\xi_{22}(\tau) - x_F\xi_F(\tau) + x_A\xi_A(\tau)\right] \\ & \qquad v(t-\tau)\, d\tau \\ & +u(t)\, r(t)\, (\mu_{26} - x_F^2\mu_F + x_A^2\mu_A) \\ & +u(t) \int_0^{\infty} \left[\xi_{26}(\tau) - x_F^2\xi_F(\tau) + x_A^2\xi_A(\tau)\right] \\ & \qquad r(t-\tau)\, d\tau \end{aligned} \qquad (11)$$

__STATE VARIABLES.__

__General theoretical formulation.__

The theoretical background of the use of state variables for calculating unsteady hydrodynamic forces is developed by Schmiechen [17],[18]. The hydrodynamic forces (7) are considered as a functional :

$$\begin{aligned} [F_h] &= [\mu]\,[\ddot{x}] + [\lambda]\,[\dot{x}] + \int_{-\infty}^{t} [K(t-\tau)]\,[\dot{x}(\tau)]\, d\tau \\ &= [P([V(\tau)])] \quad , \quad -\infty \le \tau \le t \end{aligned} \qquad (12)$$

Such a formulation can be approximated by a number of recursive relationships in which a number of *state variables* $[s_0],[s_1],\ldots,[s_n]$ have been introduced :

$$\begin{aligned} [\dot{s}_{n-k}(t)] &= [Q_{n-k}([s_{n-k}], [v])] \\ &= [s_{n-k+1}(t)] - [A_k]\,[s_0(t)] - [B_k]\,[v(t)] \end{aligned} \qquad (13)$$

where k = 0,1,...n and $[s_{n+1}] = [0]$;

$$\begin{aligned} [F_h] &= \left[R([s_j], [v^{(i)}])\right] \\ &= [s_0] + \sum_{i=0}^{m} [C_i]\,[v^{(i)}] \end{aligned} \qquad (14)$$

In (13-14) the second equation is valid for linear systems. If m=1 in (14), while $[C_0]$ and $[C_1]$ are chosen to be $[\lambda]$ and $[\mu]$, respectively, $[s_0]$ expresses the hydrodynamic memory forces acting on the ship :

$$[s_0] = \int_{-\infty}^{t} [K(t-\tau)]\,[\dot{x}(\tau)]\,d\tau \tag{15}$$

The calculation of $[s_0]$ requires the knowledge of the other state variables $[s_1],\ldots,[s_n]$, so that the latter can be considered as parameters for the memory effects.

An estimation of the matrices $[A_k]$ and $[B_k]$ in (13) has to be based on the frequency response matrices $[a(\omega)]$ and $[b(\omega)]$. For this reason, system (13) of n+1 first order differential equations is rewritten as one *single differential equation with order n+1 :*

$$[s_0^{(n+1)}(t)] + \sum_{k=0}^{n} [A_k]\,[s_0^{(k)}(t)] = -\sum_{k=0}^{n} [B_k]\,[v^{(k)}(t)] \tag{16}$$

Fourier transformation of (16) yields :

$$\left\{(i\omega)^{n+1}[I] + \sum_{k=0}^{n} (i\omega)^k [A_k]\right\}[\bar{s}_0(\omega)] = -\left\{\sum_{k=0}^{n} (i\omega)^k [B_k]\right\}[\bar{v}(\omega)] \tag{17}$$

in which $[s_0]$ can be eliminated making use of following expressions for the Fourier transform of the hydrodynamic forces, derived from (14) and (1) :

$$\begin{aligned}[\bar{F}_h(\omega)] &= [\bar{s}_0] + (i\omega[\mu] + [\lambda])[\bar{v}(\omega)] \\ &= (i\omega[a(\omega)] + [b(\omega)])[\bar{v}(\omega)]\end{aligned} \tag{18}$$

Manipulation of (17-18) leads to a matrix equation:

$$\sum_{k=0}^{n} (i\omega)^{k-n-1}[B_k] - \left(\sum_{k=0}^{n} (i\omega)^{k-n-1}[A_k]\right)(i\omega[a(\omega)-\mu] + [b(\omega)-\lambda]) = i\omega[a(\omega)-\mu] + [b(\omega)-\lambda] \tag{19}$$

Making use of a least square method, the values of $[a(\omega)]$ and $[b(\omega)]$ for a number of (preferably equidistant) *values of ω lead to an optimal estimation for* the matrices $[A_k]$ and $[B_k]$.

This theory has been applied to the case of two colliding ships by Schmiechen [17],[18]. Jiang, Schellin & Sharma [19] have simulated the horizontal motion of a moored tanker, making use of an approximation with n=3, which increases the number of differential equations of motion from 3 to 15.

Marginal case n=0 - Long-wave approximation.

(19) takes following form if n=0 :

$$(i\omega[I] + [A_0])^{-1}[B_0] = i\omega[a(\omega)-\mu] + [b(\omega)-\lambda] \tag{20}$$

If only the lateral force caused by the sway motion is taken into account, (20) yields:

$$a_{22}(\omega) - \mu_{22} = \frac{-B_0}{A_0^2 + \omega^2} \tag{21}$$

$$b_{22}(\omega) - \lambda_{22} = \frac{A_0 B_0}{A_0^2 + \omega^2} \tag{22}$$

Differential equation (16) takes following form :

$$[\dot{s}_0] + [A_0]\,[s_0] = -[B_0]\,[v(t)] \tag{23}$$

which, taking account of (14), leads to :

$$[\dot{F}_h] + [A_0]\,[F_h] + [\mu]\,[\ddot{v}] + ([\lambda] + [A_0]\,[\mu])[\dot{v}] + ([A_0]\,[\lambda] + [B_0])[v] = 0 \tag{24}$$

leading to following expression for the lateral force caused by the uncoupled sway motion :

$$\dot{Y}_h + A_0 Y_h + \mu_{22}\ddot{v} + (\lambda_{22} + A_0\mu_{22})\dot{v} + (A_0\lambda_{22} + B_0)v = 0 \tag{25}$$

A remarkable similarity exists between this equation and the results of a *long-wave approximation*, applied by Middendorp [20] (see also [3],[11]). Following this approach, a two-dimensional situation is considered in which a simplified, rectangular ship's section performs a lateral motion, causing an elevation of the water level at one side and a sinkage at the other side of the ship. The velocity of these denivellations equals $c_w = (gh)^{1/2}$. Dynamics and kinematics of the ship and the surrounding water is described by equations expressing continuity, mass conservation, momentum conservation and the equation of motion. Application to a harmonic sway motion of the ship yields expressions for added mass and hydrodynamic damping coefficients as a function of frequency, which take following form if viscous friction in the underkeel region is neglected :

$$a_{22}(\omega) = \rho LBT\,\frac{h-T}{T}\,\frac{4gT^2}{\omega^2 B^2 h + 4g(h-T)^2} \tag{26}$$

$$b_{22}(\omega) = \rho LBT\,\frac{2c_w}{B}\,\frac{\omega^2 B^2 T}{\omega^2 B^2 h + 4g(h-T)^2} \tag{27}$$

Equations (21-22) and (26-27) are identical if :

$$A_0 = 2\sqrt{gh}\,\frac{h-T}{Bh} \tag{28}$$

$$B_0 = -\rho LBT\,\frac{h-T}{T}\,\frac{4gT^2}{B^2 h} \tag{29}$$

$$\mu_{22} = 0 \tag{30}$$

$$\lambda_{22} = -\frac{B_0}{A_0} = \rho LBT\,\frac{2\sqrt{gh}}{B}\,\frac{T}{h} \tag{31}$$

so that (25) is simplified to :

$$\dot{Y}_h + A_0 Y_h - \frac{B_0}{A_0} v_2 = 0 \tag{32}$$

which can be used for the implementation of nonstationary hydrodynamic forces into a manoeuvring simulation program.

This approximation is clearly useless for high frequencies, as the high frequency limits of (21-22) for added mass and damping are in contradiction with experimental and theoretical data (see fig. 3).

Case n=1.

If n=1, (19) takes following form :

$$\begin{aligned}([A_0] - \omega^2 [I] + i\omega [A_1])^{-1}([B_0] + i\omega [B_1]) \\ = i\omega [a(\omega) - \mu] + [b(\omega) - \lambda]\end{aligned} \tag{33}$$

If the lateral motion is uncoupled, (33) leads to following expressions for added mass and hydrodynamic damping if the latter is assumed to vanish for very small and very large frequencies (see fig. 4):

$$a_{22}(\omega) - \mu_{22} = \frac{B_1 (A_0 - \omega^2)}{(A_0 - \omega^2)^2 + \omega^2 A_1^2} \tag{34}$$

$$b_{22}(\omega) = \frac{\omega^2 A_1 B_1}{(A_0 - \omega^2)^2 + \omega^2 A_1^2} \tag{35}$$

(see also [18]). Following differential equation for the unsteady hydrodynamic force is obtained :

$$\begin{aligned}&\ddot{Y}_h + A_1 \dot{Y}_h + A_0 Y_h \\ &+ \mu_{22} \ddot{v}_2 + A_1 \mu_{22} \dot{v}_2 + (A_0 \mu_{22} + B_1) v_2 = 0\end{aligned} \tag{36}$$

which is equivalent with following system of first order differential equations:

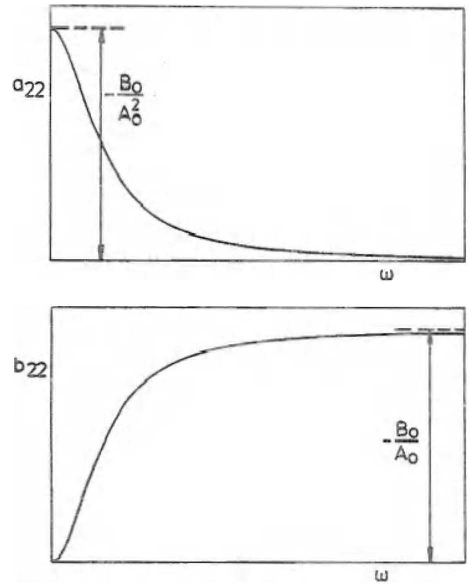

Fig. 3. Two-dimensional sway added mass and damping : state space model, n = 0.

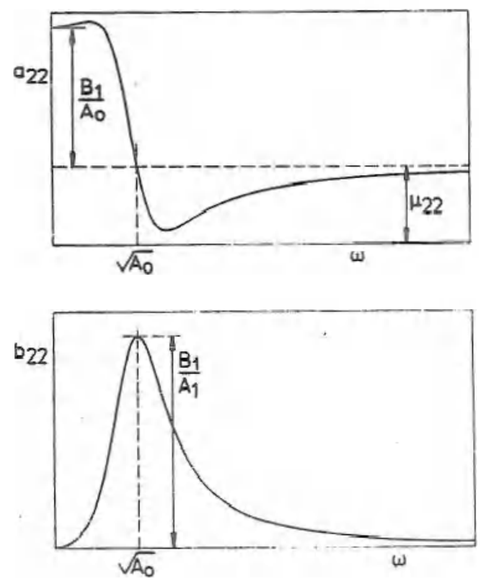

Fig. 4. Two-dimensional sway added mass and damping : state space model, n = 1.

$$\dot{s}_{1,2} = -A_0 s_{0,2} \tag{37}$$

$$\dot{s}_{0,2} = s_{1,2} - A_1 s_{0,2} - B_1 v_2 \tag{38}$$

$$Y_h = s_{0,2} - \mu_{22} v_2 \tag{39}$$

The correspondence between (34-35) and the real curves for added mass and hydrodynamic damping as functions of frequency has already improved compared with a long-wave approximation. Moreover, the typical shape of the curves due to the presence of a closed boundary (quay wall) can be obtained by a suitable choice of A_0, A_1, B_0, B_1 (see fig. 5).

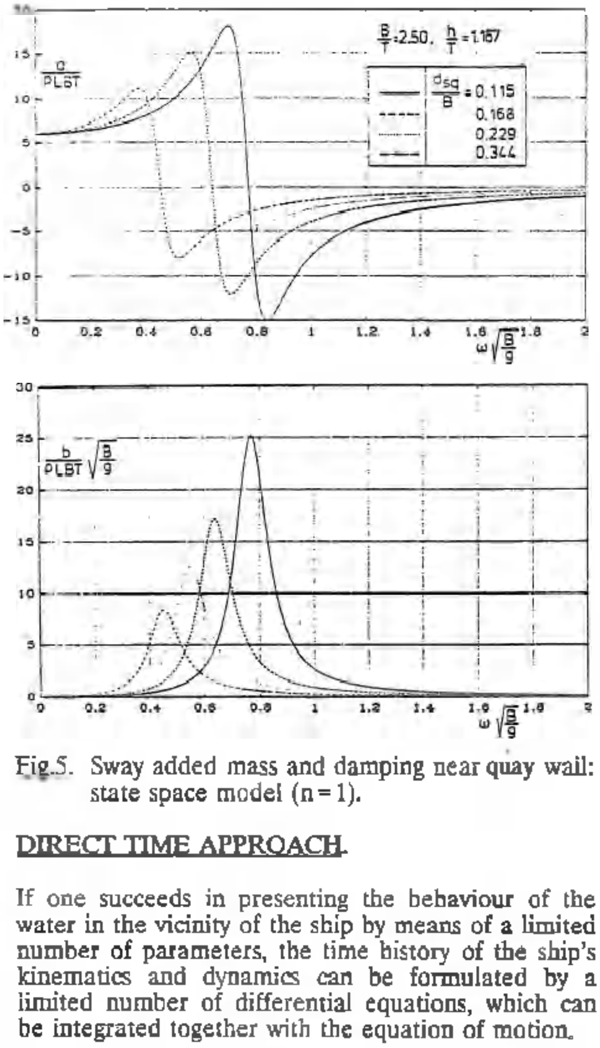

Fig.5. Sway added mass and damping near quay wall: state space model (n = 1).

DIRECT TIME APPROACH

If one succeeds in presenting the behaviour of the water in the vicinity of the ship by means of a limited number of parameters, the time history of the ship's kinematics and dynamics can be formulated by a limited number of differential equations, which can be integrated together with the equation of motion.

Such a method is based on a number of hypotheses; a typical example is the long-wave approximation, mentioned in a former paragraph, which assumes that the hydrodynamic forces acting on a (rectangular) ship in a pure sway motion depend on only five parameters (η_a, η_c, v_a, v_b and v_c, see fig. 6), so that the equation of motion in lateral direction is completed with five supplementary differential equations. Elimination of η_a, η_c, v_a and v_c leads to following system of two differential equations with two unknown time functions x_2 and v_b :

$$\dot{v}_b + 2\frac{g}{B}\frac{c_w}{c_w^2-\dot{x}_2^2}[v_b(h-T)+\dot{x}_2T] + 2\frac{\gamma}{\rho(h-T)}(v_b-\tfrac{1}{2}\dot{x}_2) = 0 \tag{40}$$

$$\ddot{x}_2 + 2\frac{g}{BT}\frac{c_w}{c_w^2-\dot{x}_2^2}\frac{c_wT-v_b(h-T)}{c_w+\dot{x}_2}[v_b(h-T)+\dot{x}_2T] + \frac{2g}{LBT}\frac{c_w^2}{(c_w^2-\dot{x}_2^2)^2}[v_b(h-T)+\dot{x}_2T]^2 - \frac{\gamma}{\rho LT}(v_b-\dot{x}_2) = \frac{F_2(t)}{\rho LBT} \tag{41}$$

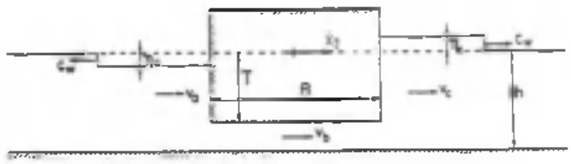

Fig. 6. Long-wave approximation [11].

With some additional simplifications, (40-41) lead to one single differential equation (32). However, direct integration of (40-41) makes it possible to account for several nonlinear effects, thus leading to an approach with a more solid physical base.

A direct time approach of problems concerning ship motions in confined waters is developed by Fontijn [11], who succeeded in formulating the lateral motion of a rectangular vessel parallel with a vertical wall by means of three simplified differential equations.

SELECTION OF A MATHEMATICAL MODEL

EVALUATION

The use of *impulse response techniques* can be considered as the most direct and logical method for the implementation of memory effects into a simulation program. The integration procedure of the equations of motion does not need to be modified, as the unsteady hydrodynamic forces are calculated separately and added to the other forces acting on the ship. On the other hand, the calculation of these forces requires numerical integration of the product of a number of retardation functions with the time history of the respective velocity components, which is a rather lengthy operation. If the interaction of surge motion with sway and yaw are not taken into consideration, four different retardation functions have to be handled; these functions depend on the water depth and the relative position between the ship and the closed boundaries of the navigation area.

Models based on a *state variables approach* offer the advantage that the information about unsteady hydrodynamic forces is stored in a rather limited number of parameters. For example, if $n=1$, the matrices $[A_0]$, $[A_1]$, $[B_0]$, $[B_1]$, $[\mu]$ and $[\lambda]$ contain 30 nonzero coefficients in case of an uncoupled surge motion, replacing the static added-mass coefficients $X'_{\dot{u}}$, $Y'_{\dot{v}}$, $Y'_{\dot{r}}$, $N'_{\dot{v}}$, $N'_{\dot{r}}$. These coefficients also depend on the water depth and ship's position. On the other hand, such models look rather artificial: neither the coefficient matrices [A] and [B] nor the state variables [s] theirselves are clearly related with the physical reality. The number of differential equations that have to be integrated is increased with $3(n+1)$; as an acceptable approximation for the frequency response characteristics requires $n \geq 1$, at least six differential equations must be added to the three equations of motion. Moreover, some variables are implicitely integrated several times, which can cause numerical complications if the time increment is too large. For this reason, application for real-time simulation does not seem feasible.

The application of a *direct time approach* also requires additional variables and differential equations, but there is a clear relationship with the physical reality. Unlike the other methods, nonlinear terms can be taken into account. On the other hand, Fontijn's publications [9]-[11] show that such an approach even for a rather simple case leads to complex formulations. Furthermore, quite drastic corrections are required for fitting theoretical and experimental results, so that it is rather doubtful that a direct time approach would lead to more reliable results than the other methods. The latter are mainly based on the frequency response characteristics of the ship; if the latter are determined in an experimental or a verified theoretical way, all influences which are considered in a direct time approach are implicitely taken into account.

As a conclusion, the use of *impulse response techniques* seems to be most appropriate for application in real-time simulation, although the theoretical developments concerning state variables will prove to be quite useful for the formulation of retardation functions. Moreover, the author would like to emphasize that state variable techniques have proved to be successful in studies handling unsteady hydrodynamic forces on ships (see [17]-[19]), although they have not been applied yet to problems related with interaction between ships and (fendered) harbour constructions.

Resuming, the mathematical presentation of unsteady hydrodynamic forces in the horizontal plane will be given by (10-11), together with an expression for the longitudinal force:

$$X_h(t) = \mu_{11}\dot{u}(t) + \int_0^\infty K_{11}(\tau)\, u(t-\tau)\, d\tau \qquad (42)$$

so that cross-coupling effects between the longitudinal mode and the other ones is neglected.

A strip-theory will be applied for the calculation of the retardation functions occurring in (10-11). For this purpose, general expressions of the retardation functions of each strip in open water and in the vicinity of a vertical wall will be determined. These functions will be integrated along the ship's length; subsequently the integrated functions will be approximated by means of a limited number of parameters determining the position of the ship relative to the boundaries of the navigation area.

RETARDATION FUNCTIONS OF A STRIP.

General expressions.

Expressions for the time functions $\xi(x,t)$ and $K(x,t)$ required for the calculation of the retardation functions occurring in (10-11) (see Appendix A) can be based on an approximation by means of a transfer function of order $n+1$, as suggested by (17-19):

$$T(x,\omega) \approx i\omega(a(x,\omega)-\mu(x)) + (b(x,\omega)-\lambda(x))$$
$$= \frac{\sum_{k=0}^{n} (i\omega)^k B_k}{(i\omega)^{n+1} + \sum_{k=0}^{n} (i\omega)^k A_k} \qquad (43)$$

where A_k and B_k are functions of x. (43) can be rewritten as follows :

$$T(x,\omega) = \sum_{j=1}^{n_1} \frac{B_0^{(j)'}}{i\omega + A_0^{(j)'}} + \sum_{j=1}^{n_2} \frac{i\omega B_1^{(j)''} + B_0^{(j)''}}{(i\omega)^2 + i\omega A_1^{(j)''} + A_0^{(j)''}} \qquad (44)$$

with $n=n_1+2n_2$. The retardation function $K(x,t)$ is obtained by inverse Fourier transformation:

$$K(x,t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} T(x,\omega)\, e^{i\omega t} d\omega$$
$$= H(t)\left\{\sum_{j=1}^{n_1} B_0^{(j)'} e^{-A_0^{(j)'} t} + \sum_{j=1}^{n_2}\left[B_1^{(j)''}\cos\omega_0^{(j)} t + \frac{B_0^{(j)''} - \alpha^{(j)} B_1^{(j)''}}{\omega_0^{(j)}}\sin\omega_0^{(j)} t\right] e^{-\alpha^{(j)} t}\right\} \qquad (45)$$

with

$$\alpha^{(j)} = \frac{1}{2} A_1^{(j)''} \qquad (46)$$

$$\omega_0^{(j)} = \sqrt{A_0^{(j)''} - \frac{1}{4}\left(A_1^{(j)''}\right)^2} \qquad (47)$$

$$H(t) = 0 \; , \; t<0 \quad ; \quad H(t) = 1 \; , \; t>0 \qquad (48)$$

$\xi(x,t)$ is obtained by integration of (45). As a consequence, $K(x,t)$ and $\xi(x,t)$ are composed of exponential and/or exponentially decaying harmonic functions.

Application to stationary situations shows that a consequent use of strip theory requires zero values for the hydrodynamic damping at zero and infinite frequency, so that the long-wave approximation is in contradiction with the general theoretical principles.

Sections in open water.

A second order transfer function is used:

$$T(x,\omega) = i\omega\,(a(x,\omega) - \mu(x)) + b(x,\omega) = \frac{i\omega B_1}{(i\omega)^2 + i\omega A_1 + A_0} \qquad (49)$$

leading to following expressions for added mass and hydrodynamic damping:

$$a(x,\omega) - \mu = \frac{B_1\,(A_0 - \omega^2)}{(A_0 - \omega^2)^2 + A_1^2\,\omega^2} \qquad (50)$$

$$b(x,\omega) = \frac{B_1 A_1 \omega^2}{(A_0 - \omega^2)^2 + A_1^2\,\omega^2} \qquad (51)$$

This implies that the hydrodynamic behaviour of a section is determined by means of four parameters (μ, A_0, A_1, B_1) which can be estimated if added mass and damping are known functions of frequency (fig. 4). In a first approximation, a rectangular section can be considered, for which results of a two-dimensional potential theory or even, for low frequencies, a long-wave approximation can be used (see Fontijn, [11]).

Sections in confined water (single wall).

Although a similar procedure applied to a half-open water configuration leads to a fair approximation of added mass and damping, the curves do not tend to the open-water curves with increasing quay clearance (QC). For this reason, the influence of the vicinity of a closed wall is approximated by:

$$T(x,\omega) = \frac{\epsilon\; i\omega B_1^{(O)}}{(i\omega)^2 + i\omega A_1^{(O)} + A_0^{(O)}} + \frac{(1-\epsilon)\; i\omega B_1^{(W)}}{(i\omega)^2 + i\omega A_1^{(W)} + A_0^{(W)}} = \epsilon T^{(O)}(x,\omega) + (1-\epsilon)\,T^{(W)}(x,\omega) \qquad (52)$$

The parameter ϵ takes a value between 0 (QC=∞) and 1 (QC=0); acceptable results are obtained as follows:

$$\epsilon = \frac{A_0^{(O)}}{A_0^{(O)} + A_0^{(W)}} = \frac{\omega_a^{(O)2}}{\omega_a^{(O)2} + \omega_a^{(W)2}} \qquad (53)$$

In a first approximation, Fontijn's formulae [11] for rectangular sections in the vicinity of a vertical wall can be used in order to obtain curves for added mass and hydrodynamic damping.

SIMPLIFIED HYDRODYNAMIC CHARACTERISTICS OF THE COMPLETE SHIP.

Integration over the ship's length.

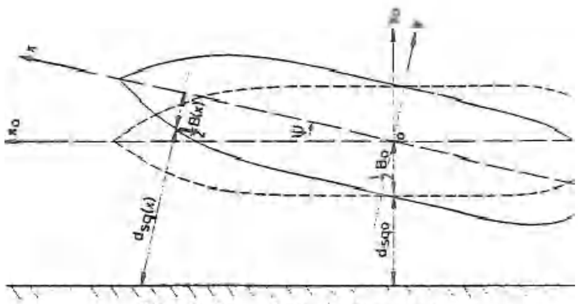

Fig. 7. Local quay clearance.

Added mass $a(x,\omega)$, hydrodynamic damping $b(x,\omega)$ and retardation function $K(x,t)$ are integrated over the ship's length; in the vicinity of a vertical wall, the local quay clearance is taken into account (fig. 7):

$$d_{sq}(x) = \frac{d_{sq0} + \frac{1}{2}B_0}{\cos\psi} + x\,tg\psi - \frac{1}{2}B(x) \qquad (54)$$

Simplification of integrated characteristics.

A fair approximation for open-water characteristics can be obtained by means of a second-order transfer function (49).

In the vicinity of a vertical wall, the frequency response characteristics display extrema at one or two frequencies. Following transfer function is suggested :

$$T(x,\omega) = \epsilon^{(0)} T^{(0)}(x,\omega) + \epsilon^{(1)} T^{(1)}(x,\omega) + \epsilon^{(2)} T^{(2)}(x,\omega) \qquad (55)$$

with (i = 0,1,2)

$$T^{(i)}(x,\omega) = \frac{i\omega B_1^{(i)}}{(i\omega)^2 + i\omega A_1^{(i)} + A_0^{(i)}} \qquad (56)$$

$$\epsilon^{(0)} + \epsilon^{(1)} + \epsilon^{(2)} = 1 \qquad (57)$$

$T^{(0)}$ representing the open-water transfer function.

A relationship between the coefficients determining the retardation functions and position parameters (quay clearance d_{sq} and heading angle ψ) is now examined. For each combination (d_{sq};ψ) two "equivalent quay clearance" values $d_{sq,eq}^{(1)}$ en $d_{sq\,eq}^{(2)}$ are defined:

$$d_{sq,eq}^{(i)} = d_{sq} + c^{(i)}\psi \qquad (58)$$

$c^{(1)}$ and $c^{(2)}$ being independent of the motion mode. $A_0^{(i)}$, $A_1^{(i)}$, $B_1^{(i)}$ and $\epsilon^{(i)}$ can be approximated by a function $[m\, d_{sq\,eq}^{(i)} + q]^{-1}$. Some of the coefficients m and q depend on the motion mode.

Retardation functions resulting from strip theory are compared with the approximation in fig. 8.

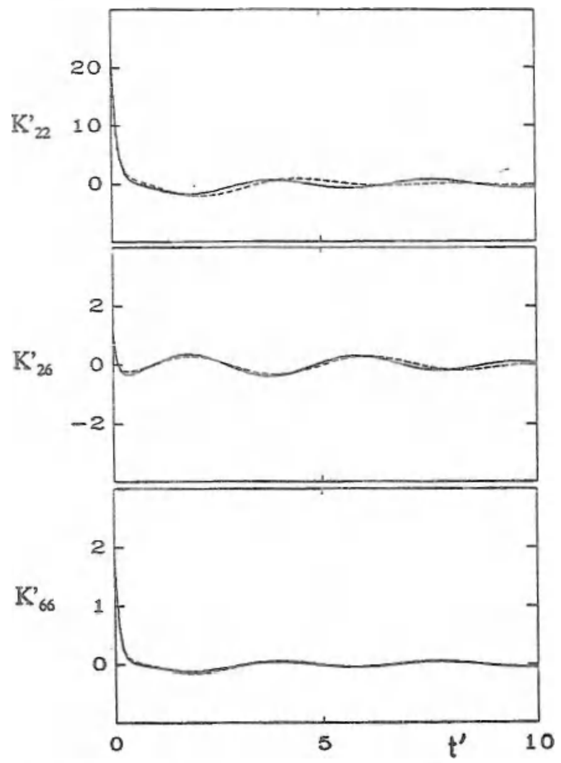

Fig.8. Retardation functions for a tanker near a quay (d_{so0}÷B=0.34; h÷T=1.2; ψ=5°): strip theory (---) and parameterized approximation (——).

CONCLUSIONS

The use of impulse response techniques can be considered as the most appropriate method for implementing hydrodynamic memory effects, which have to be taken into account in impact or collision situations, in a real-time manoeuvring simulation model.

The outlines of a practical procedure for this implementation are given, taking account of the relative position between the ship and the closed boundaries of the navigation area by means of a limited number of parameters, influencing the hydrodynamic coefficients and retardations functions. So far, the latter have been calculated by means of a simplified strip method, but the presented method for parameterisation can also be applied to results of more sophisticated computational methods (e.g. 2- or 3-dimensional boundary element methods) or experimentally determined frequency response characteristics.

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APPENDIX

A simple representation of results of integration of the velocity potential Φ in (9) over the contour length of each strip requires following definitions:

$$\mu(x) = \rho \int_{\Sigma_s(x)} \psi_2(x)\, n_y\, dl \qquad (A.1)$$

$$\xi(x,t) = \rho \int_{\Sigma_s(x)} \chi_2(x,t)\, n_y\, dl \qquad (A.2)$$

Integration of these functions over the ship's length yields:

$$\mu_{22} = \int_{x_A}^{x_F} \mu(x)\, dx \qquad (A.3)$$

$$\mu_{26} = \int_{x_A}^{x_F} \mu(x)\, x\, dx \qquad (A.4)$$

$$\mu_{66} = \int_{x_A}^{x_F} \mu(x)\, x^2\, dx \qquad (A.5)$$

$$\xi_{22}(t) = \int_{x_A}^{x_F} \xi(x,t)\, dx \qquad (A.6)$$

$$\xi_{26}(t) = \int_{x_A}^{x_F} \xi(x,t)\, x\, dx \qquad (A.7)$$

μ and ξ take following values at the aft and fore perpendicular:

$$\mu_A = \mu(x_A) \qquad (A.8)$$

$$\mu_F = \mu(x_F) \qquad (A.9)$$

$$\xi_A(t) = \xi(x_A, t) \qquad (A.10)$$

$$\xi_F(t) = \xi(x_F, t) \qquad (A.11)$$

Time derivation of $\xi(x,t)$ leads to an expression for the retardation function for lateral motion of a strip:

$$\frac{\partial \xi}{\partial t}(x,t) = K(x,t) \qquad (A.12)$$

which is integrated over the ship's length:

$$K_{22}(t) = \int_{x_A}^{x_F} K(x,t)\, dx \qquad (A.13)$$

$$K_{26}(t) = \int_{x_A}^{x_F} K(x,t)\, x\, dx \qquad (A.14)$$

$$K_{66}(t) = \int_{x_A}^{x_F} K(x,t)\, x^2\, dx \qquad (A.15)$$

u,v and r respectively denote the ship's longitudinal and transversal velocity components and rate of turn. The corresponding force and moment components are X,Y,N.