THE PRINCIPAL WORKS

OF

SIMON STEVIN

EDITED BY

ERNST CRONE, E. J. DIJKSTERHUIS, R. J. FORBES
M. G. J. MINNAERT, A. PANNEKOEK

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VOLUME II

MATHEMATICS

EDITED BY

D. J. STRUIK
PROFESSOR AT THE MASSACHUSETTS INSTITUTE OF TECHNOLOGY, CAMBRIDGE (MASS.)

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ERNST CRONE,
Chairman of the Netherlands Maritime Museum, Amsterdam

E. J. DIJKSTERHUIS,
Professor of the History of Science at the Universities of Leiden and Utrecht

R. J. FORBES,
Professor of the History of Science at the Municipal University of Amsterdam

M. G. J. MINNAERT,
Professor of Astronomy at the University of Utrecht

A. PANNEKOEK,
Former Professor of Astronomy at the Municipal University of Amsterdam

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THE MATHEMATICAL WORKS

OF

SIMON STEVIN

1. GENERAL INTRODUCTION

When, about 1581, Stevin settled in the Northern Netherlands, he found a country ready to appreciate his talents. The young Republic, at war with Spain and entering a period of great maritime expansion, needed instructors for its navigators, merchants, surveyors, and military engineers. Teachers of mathematics, surveying, navigation and cartography, instrument-makers and engineers found encouragement; their number increased and soon no commercial town was without some of them [1]. Before the sixteenth century came to an end textbooks in arithmetic, algebra, geometry, and the applied mathematical sciences were available, many written in the vernacular. The teachers and those who patronized them included a great many immigrants from neighbouring countries, expecially from the Southern Netherlands, long known for its learning — the country in which Stevin himself was born. The Stadtholder, Prince Maurice of Orange (1567-1625), was also greatly interested in the mathematical sciences, and so was the new University of Leyden, founded in 1575.

Several of these early Dutch mathematicians and teachers of mathematics are still remembered. Apart from Stevin, we find among them Adriaen Anthonisz (*c.* 1543-1620), several times burgomaster of Alkmaar and a military engineer, who made the value $\frac{355}{113}$ for π known in Europe [2]; Ludolph Van Ceulen (1540-1610), fencing master at Delft, who computed π first in 20, then in 33 and finally in 35 decimals by the ancient Archimedean method of inscribed and circumscribed polygons; and Claes Pietersz or Nicolaus Petri, after 1567 schoolmaster at Amsterdam, who wrote a series of Dutch textbooks, which show considerable knowledge of contemporary science. Rudolf Snel, or Snellius (1546-1612), taught at Leyden University and edited the mathematical works of Petrus Ramus, the Parisian educator. A popular school for navigators at Amsterdam was conducted by the Reverend Petrus Plancius (1552-1622), cartographer and instrument-maker. Among the scientific amateurs we find Jan Cornets De Groot (1554-1640), patrician of Delft, whose attainments have been eclipsed by the fame of his son, known as Hugo Grotius. With several of these men Stevin entered into correspondence or personal contact, in particular with De Groot and Van Ceulen at Delft.

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[1]) This was a development typical of the period. E. G. R. Taylor, in *The Mathematical Practitioners of Tudor and Stuart England* (London, 1954), lists 582 such practitioners active between 1485 and 1715.

[2]) This value is sometimes called that of Metius through a confusion between Anthonisz and his son, who adopted the name of Metius.

The intellectual climate of Holland seems to have agreed with Stevin. During the years 1582-86 several of his books appeared, first his *Tables of Interest,* then his *Problemata Geometrica,* then his *Tenth,* his *L'Arithmétique,* a *Pratique d'Arithmétique,* and the three books on mechanics, which also contain creative mathematical thoughts. These are the books that have established Stevin's position in the history of mathematics. It is of some interest to sketch, in somewhat greater detail than in Vol. I, pp. 16-19, the nature of his contributions.

2.

In Stevin's formative years the decimal position system, based upon the Hindu-Arabic numerals in their present form 0, 1, ..., 9, was already widely accepted in Europe and commonly used by those who professed the mathematical sciences. Elementary arithmetic, using this system, could be learned from many textbooks, available in Latin, French, German, and Flemish. Stevin specially mentions the French *Arithmétique* of Jean Trenchant, first published in 1558. From books such as these he could also learn the application of arithmetic to commercial transactions, as well as the computation of single and compound interest. They also often contained operations with radicals such as $\sqrt{2}$, $\sqrt{3}$, etc. Some features of these books must have been irksome to him. One of them was their reluctance to recognize 1 as a number and their tendency to designate other numbers as "irrational" or "surd", as if they belonged to a lower class. Other objections were of a more practical nature, such as the reluctance of the authors to illustrate their rules of interest by tables, which still were held as a secret by banking houses, or the clumsy fractional calculus, which used either the numerator-denominator notation or the sexagesimal system, but only rarely the more convenient decimal notation. This decimal notation was almost exclusively confined to trigonometric tables, available in several forms, including those published by Rhaeticus (1551), later expanded into the *Opus Palatinum* (1596). Stevin, in his first published works, tried to remedy some of these shortcomings, and also to improve on the exposition.

Thus, in the *Tables of Interest,* he not only gave a lucid presentation of the rules of single and compound interest, but also published a series of tables, together with a rule for computing them. Some years later, in his *Tenth* (1585), he showed the use of the decimal system in the calculus of fractions. He took this opportunity to suggest the introduction of the decimal system also into the classification of weights and measures, a proposal which had to wait for partial acceptance until the time of the French Revolution. His theoretical ideas he laid down in his book *L'Arithmétique* and in a geometrical manuscript, of which only a part was published. Since *L'Arithmétique* also contained Stevin's algebra, while his books on mechanics included several applications of the calculus of infinitesimals, Stevin's work of these years 1582-1586 can be considered as a fair and often original exposition of most features of the mathematics of his day.

In his arithmetical and geometrical studies Stevin pointed out that the analogy between numbers and line-segments was closer than was generally recognized. He showed that the principal arithmetical operations, as well as the theory of proportions and the rule of three, had their counterparts in geometry. Incommensurability existed between line-segments as well as numbers, and since the nature of line-segments was independent of the number that indicated their length, all numbers, including unity, also were of the same nature. All numbers were squares, all numbers were square roots. Not only was $\sqrt{2}$ incommensurable with 2 and $\sqrt{3}$, but so was 2 with $\sqrt{2}$ and $\sqrt{3}$; incommensurability was a relative property, and there was no sense in calling numbers "irrational", "irregular" or any other name which connoted inferiority. He went so far as to say, in his *Traicté des incommensurables grandeurs*, that the geometrical theory of incommensurables in Euclid's Tenth Book had originally been discovered in terms of numbers, and translated the content of this book into the language of numbers. He compared the still incompletely understood arithmetical continuum to the geometrical continuum, already explained by the Greeks, and thus prepared the way for that correspondence of numbers and points on the line that made its entry with Descartes' coordinate geometry.

Stevin recognized several kinds of quantities: arithmetical numbers, which are abstract numbers, and geometrical numbers, connected with lines, squares, cubes, and rectangular blocks (figures in more than three dimensions were beyond the compass of the age), which we now denote by a, a^2, a^3,..., $a^{\frac{1}{2}}$, $a^{\frac{1}{3}}$, etc. From this he passed on to linear combinations of geometrical numbers, which he called algebraic numbers. Thus he came to algebra—the theory of equations—, which to him, in his attempt to construe analogies between geometry and arithmetic, hence between geometrical and arithmetical numbers, consisted in the application of the rule of three to algebraic quantities. His algebra thus forms part of his general "arithmetic".

The theory of equations had made considerable progress in the course of the sixteenth century. Cubic equations had been solved, though the "casus irreducibilis" still presented difficulties. The new results were laid down by Jerome Cardan in his *Ars magna* (1545), which became the sixteenth-century standard text on the theory of equations, eclipsing even the *Arithmetica integra* (1544) of Michael Stifel. Cardian's book also contained Ferrari's reduction of the fourth-degree equation to one of the third degree. Stevin knew these books intimately, and also studied Bombelli's *L'Algebra* (1572), which treated the "casus irreducibilis" with complex numbers and introduced an improved notation. Stevin did not have much use for these complex numbers, because he did not see a possibility of finding a numerical approximation for a number like $\sqrt{4 + 5i}$, in contrast to such a number as $\sqrt{6}$, where a numerical approximation can be obtained. However, he liked Bombelli's notation, and availed himself of it in his own book. Against negative numbers, with which Cardan had played, he had no objection, even if he did not use them as freely as we do now. In the light of our present knowledge we are inclined to wonder why in his speculations on the analogies between the arithmetical and the geometrical continuum he did not assign a geometrical meaning to negative numbers, but even Descartes and his immediate successors did not use negative coordinates. The study of directed quantities belongs to a much later stage of mathematical development.

The main merit of Stevin's *L'Arithmétique* is the systematic way in which he discusses operations with rational, irrational, and algebraic numbers, and the theory of equations of the first, second, third, and fourth degrees. To our feeling he went too far in stressing the analogy between arithmetical and algebraic entities, even the theory of equations becoming an application of the rule of three.

However, this latter point of view met with little success, even among his contemporaries and the algebrists who followed him. His particular notation for equations was also soon abandoned [1]).

Geometry, during the sixteenth century, still followed closely the track of Euclid, whose *Elements*, from 1482 on, were available in several printed editions and translations. Stevin was especially familiar with the Latin editions prepared by Zamberti (1546) and by Clavius (1574). Christopher Clavius (1537-1612), Stevin's contemporary, who was the Vatican's astronomer, excelled as a writer of textbooks, which embraced well-nigh the whole of the mathematical and astronomical sciences of his day. There is reason to believe that Stevin was quite familiar with these books, and that Clavius equally remained in contact with Stevin's work. To his study of Euclid we owe Stevin's *Traicté des incommensurables grandeurs,* already mentioned, and his *Problemata geometrica,* the former probably, the latter certainly forming part of that longer geometrical manuscript which was to do for geometry what *L'Arithmétique* had done for arithmetic. Euclid's influence in the *Problemata* is particularly evident in the sections dealing with proportional division of figures and with regular bodies, enriched with a description of the semi-regular bodies, which had a touch of originality. Stevin knew several of them through Albrecht Dürer, who had described them in his *Underweysung* of 1525, but he added some others, while rejecting one of them. He does not seem to have known that all thirteen semi-regular bodies had been described in Antiquity by Pappus, who had mentioned Archimedes as the discoverer, information not readily available in the 1580's, since Pappus' text was only published in 1589. We do not know whether Stevin was aware of other books which appeared in the sixteenth century, with descriptions of semi-regular bodies, sometimes beautifully illustrated: the only source he quotes is Dürer.

The *Problemata* also show Stevin as a student of Archimedes. The *editio princeps* of Archimedes appeared in 1544, when Venatorius published the Greek text of all the works, a Latin translation, and the commentaries of Eutocius. Moreover, a selection of the works in Latin appeared in 1558 through the care of Commandino. The theories of Archimedes, the most advanced mathematician of Antiquity, were not easily understood, and creative work based on them was even more difficult. Stevin was among the first Renaissance men to study Archimedes with a certain amount of independence. In the *Problemata* he took some problems he had found in Archimedes' *An the Sphere and Cylinder* and generalized them somewhat; this gave him an opportounity to apply the methods given by Eutocius for the construction of the two mean proportionals between two lines: $a : x = x : y = y : b$, a problem which cannot be solved by means of compass

[1]) The criticism of K. Menger on the promiscuous use of the symbol x in modern mathematics, and in particular of its use as a dummy index in expressions like $\int f(x)dx$, which he writes Sf, or as „indeterminates" in expressions like $\frac{x^2 - 1}{x + 1} = x - 1$, which he writes $\frac{*^2 - 1}{* - 1} = * + 1$, lends a touch of modernity to Stevin's notation. The latter expression, in the symbolism of *L'Arithmétique*, is written in the form $\frac{②-1}{①-1} = ① + 1$, very much in Professor Menger's spirit. See K. Menger, *Calculus. A modern approach.* Boston 1955, or Math. Gazette *40* (1956), pp. 246–255.

and straightedge alone. But Archimedes' influence is also visible in Stevin's books on mechanics, where Stevin, modifying Archimedes' later so-called exhaustion method, appears as one of the first Renaissance pioneers in the field of mathematics afterwards known as the theory of limits and the calculus.

Archimedes' handling of what we now call limit and integration processess was still on the extreme confines of knowledge. Only a few mathematicians as yet were able to emulate Archimedes, among them Commandino, who had applied his methods in the determination of centres of gravity. Stevin's friend Van Ceulen was engaged in improving on Archimedes' computation of π. One difficulty in Archimedes was his cumbersome method of demonstration in dealing with limit processes (which had already appeared in Euclid and was typical of Antiquity). When Archimedes wanted to demonstrate that a certain quantity Q, *e.g.* the area of a parabolic segment, was equal to A, he showed that the two hypotheses $Q < A$ and $Q > A$ both led to an absurdity, so that $Q = A$ was the only possibility. Stevin replaced this indirect proof by a direct one. Demonstrating that the centre of gravity of a triangle lies on the median, he argues that if the difference between two quantities B and A can be made smaller than any assignable quantity ε, and $| B - A | < \varepsilon$, then $B = A$ (see Vol. I, p. 43). Here Stevin entered upon a course which was to lead to the modern theory of limits.

We can discern a certain impatience with the method of the Ancients in Stevin and his successors; an impatience quite conspicuous in Kepler. These men applied short cuts in what we call the integration process, because they wanted results rather than exact proofs. They used methods of far more dubious rigour than Stevin's, even though they knew that the only rigorous proof was the Archimedean one. Stevin must have experimented with such short cuts, as we can see in his paper on *Van de Molens* (*On the Mills*; Work XVI; Vol. V). If we like, we can see a topic related to the calculus in Stevin's determination of the equation of the loxodrome on a sphere, in his book on Cosmography, by means of the series

$$\tan K\ (\sec 10' + \sec 20' + \ldots\ldots + \sec n.\,10').\ 10',$$

where K is the angle between the loxodrome and the meridian. The expression is an approximation of $\tan K \int_0^{\varphi} \sec \varphi \, d\varphi$, expressed in degrees.

During the latter part of Stevin's life the mathematical sciences continued to flourish in Holland. This was the period in which he wrote, or rewrote, the different books which he assembled in 1605-1608 in the *Wisconstighe Ghedachtenissen*. The short *Appendice algébraique*, which contains a method for approximating a real root of an algebraic equation of any degree, dates from 1590. This was also the period in which Stevin acted as a teacher and adviser to Prince Maurice of Orange. He remained in personal contact and correspondence with many of his colleagues, including representatives of the younger generation, outstanding among whom was Rudolf Snel's son Willebrord (1580-1626), a graduate of Leyden University. This younger Snellius, who translated the *Wisconstighe Ghedachtenissen* into Latin, later succeeded his father in the chair at Leyden, and is remembered as the discoverer of "Snellius' law" in the theory of optics and the first man on record to perform an extensive triangulation. Another Leyden mathematician was Frans Van Schooten (1581/82-1645), who after Van Ceulen's death in 1610 taught at the engineering school founded by

Prince Maurice. His son and namesake (1615-1660), who became professor of mathematics at Leyden University and was the teacher of Christiaan Huygens, showed in his works Stevin's influence. Older than these men was Philippus Van Lansbergen (1561-1632), a minister in Zeeland and an able mathematician, who shared Stevin's preference for the Copernican system. We also know that Stevin was in personal contact with Samuel Marolois (*c.* 1572-before 1627), a military engineer who wrote on perspective, and we may safely assume that Stevin was in touch with the surveyors Jan Pietersz. Dou (1572-1635), the first to publish a Dutch edition of some of Euclid's books, and Ezechiel De Decker, whose work shows considerable influence of Stevin. This was also a period in which appeared many elementary mathematical textbooks, of which those of Willem Bartjes were used for more than two centuries and made his name proverbial in Dutch. Dutch cartographers, among them Plancius, Willem Barendtz (of Nova Zembla fame), Jodocus Hondius (son-in-law to Mercator), and William Jansz. Blaeu, were building up an international reputation. It would be interesting to know something about the relationship between Stevin and Isaac Beeckman (1588-1637), the Dordrecht physician and teacher, who through his contact with Descartes forms one of the links connecting the Stevin period of Dutch mathematics with that of Descartes. We do know that after Stevin's death, in 1620, he visited his widow and studied some of her late husband's manuscripts.

The most original of the mathematical books published in the *Ghedachtenissen* is the *Perspective.* Its subject was developed by the Italian artists of the fifteenth century and during the sixteenth century several books on it had appeared, some with beautiful pictures. These books were written by and for painters and engineers and contained a rather loose presentation of the mathematical theory involved, which often was not more than a set of prescriptions for foreshortening. The first systematic exposition of the mathematical theory of perspective appeared in 1600, when Guidobaldo Del Monte published his *Perspectivae libri sex.* It is likely that by the time this book appeared Stevin's mathematical theory of perspective, the result of his reflections on architecture, military engineering, and the technique of drawing in general, was already far advanced. It is also probable that in the final draft of the manuscript Stevin was influenced by Del Monte. In the book Stevin develops the laws of perspective in his usual systematic and didactic way (the Prince may well have been no easy pupil!), derives the laws of the vanishing points, discusses the case that picture plane and ground plane are not at right angles, and also investigates what may be called the inverse problem of perspective: to find the eye when a plane figure and its perspective are given. Despite a certain long-windedness the book can still serve as an introduction to perspective; it is among the writings of Stevin which are least antiquated.

The *Meetdaet,* another book of the *Ghedachtenissen,* was based on the manuscript on geometry to which Stevin referred at the time when he was writing *L'Arithmétique* and of which he published a section in the *Problemata.* It also shows the influence of Prince Maurice, which may have improved the exposition and added a practical touch. The name became *Meetdaet,* French *Pratique de Géométrie,* a counterpart to the *Pratique d'Arithmétique* which Stevin had added to his *L'Arithmétique* in order to give some practical applications of his theory. Most of the subject matter of the *Problemata* reappears in the *Meetdaet,* sometimes in a slightly modified form.

The other mathematical sciences represented in the *Ghedachtenissen* are plane and spherical trigonometry, with tables of sines, tangents, and secants. They contain little that was new at the time, though the spherical trigonometry was somewhat simplified as compared with previous expositions. His understanding of the geometry of the sphere also led Stevin, in his books on navigation, to a careful discrimination between sailing along great circles and along rhumb lines (orthodromes and loxodromes, as Willebrord Snellius called them in his translation of the *Ghedachtenissen*). This was still an enigma to most sailors and teachers of navigation, although the difference had already in 1546 been clearly stated by Pedro Nunes, mathematician in the University of Coimbra; Mercator, the Duisburg cartographer, had represented the loxodromes by straight lines on his well-known world map of 1569 (they already appear on his terrestrial globe of 1541). The mathematics of the loxodrome was still poorly understood; as a matter of fact, this understanding only matured when the calculus began to take shape, in the latter part of the seventeenth century. Stevin was able to compute tables which for a variable point of each loxodrome, belonging to seven given bearings 11°15′, 22°30′ ...... 78°45′ with the meridian, gives the latitude as a function of the longitudinal difference with the point where the loxodrome intersects the equator. Stevin also caused copper curves to be made, which had the form of rhumb lines, for the seven principal bearings and by means of which on a globe of suitable size the loxodrome could be drawn for any given initial point. Stevin can thus also be considered as a contributor to mathematical cartography.

TAFELEN VAN INTEREST

TABLES OF INTEREST

INTRODUCTION

§ 1

The *Tables of Interest*, the first book published by Stevin, represented a kind of challenge to an ancient and established tradition. Money-lending leads to problems concerning the payment of interest, and with the expansion of mercantile activity and of banking in the later Middle Ages such problems had a tendency to become complicated. Many banking houses engaged in large-scale dealings of varied aspects, involving questions of insurance, of annuities and other payments at set intervals, of discounting of sums due at a later date and related transactions. Against their power, objections based on canon law, prohibiting or circumscribing the taking of interest, were of little or no avail. The Baldi and Medici of Florence, the Welsers and Fuggers of Augsburg at one time or another ruled financial empires, respected and feared by king, emperor, and pope.

In a period where even multiplication and division of integers were considered difficult operations, only experts could answer with authority questions involving the computation of interest. The larger and more established houses had found it convenient to have such experts compute tables of interest and to keep them on file as confidential information. Such tables remained, as Stevin expressed it, "hidden as mighty secrets by those who have got them." They could remain hidden as long as the number of skilled computers was small. This period came to an end with the spread of arithmetical instruction in the sixteenth century.

One of these early manuscript tables, composed about 1340, has been preserved in a copy finished in 1472. It was prepared for the Florentine house of the Baldi by their commissary Francesco Balducci Pegolotti as part of his *Pratica della Mercatura.* This book was published in 1766 (1), an English translation appeared in 1936 (2). The tables of interest appear as an insert between other topics (3); they record the increase, at compound rate of interest of 1, 1½, 2, . . ., 8 per cent, of 100 lires. Each of the 15 tables has 20 terms. Here follows, as an example, the table for 2 per cent:

Le 100 lire a 2 per cents l'anno

1.	lire 102.—.—	11.	lire 124. 6. 8
2.	lire 104.—.10	12.	lire 126.16. 4

(1) *Della Decima e di varie altre gravezze imposte dal comune di Firenze, Della moneta e della mercatura de Fiorentini fine al secolo XVI*, 4 vols., Lisbon and Lucca 1765–1766. The book was published anonymously, but the author became known as Gian-Francesco Pagnini della Ventura (1715–1789), Florentine Chancellor of the Tithe. See A. Evans, next ref., pp. IX–X.

(2) A. Evans, *Francesco Balducci Pegolotti La pratica della mercatura.* The Mediaeval Academy of America, Cambridge, Mass., 1936, LIV+443 pp. See pp. XV–XXVI on the life of Pegolotti.

(3) A. Evans, *l.c.*2) pp. 301–302; Pagnini, *l.c.*1) pp. 302–304.

3.	lire 106. 2. 5	13.	lire 129. 7. 1
4.	lire 108. 4. 9	14.	lire 131.18.10
5.	lire 110. 8. 1	15.	lire 134.11. 7
6.	lire 112.12. 3	16.	lire 137. 5. 3
7.	lire 114.17. 3	17.	lire 140.—. 2
8.	lire 117. 3. 3	18.	lire 142.16. 2
9.	lire 119.10. 1	19.	lire 145.13. 3
10.	lire 121.17.11	20.	lire 148.11. 6

[1 lira = 20 soldi, 1 soldo = 6 denari]

It is interesting to note that the Baldi computed the accumulation of capital not at simple, but at compound interest. This practice was already old in their days. At any rate, Leonardo of Pisa, whose *Liber Abaci* dates from 1202, and whose problems reflect early thirteenth-century mercantile practice, also accepts compound interest (4). Its legitimacy was a subject of juridical controversy for many centuries (5).

It is not unlikely that further search in European libraries will reveal other treatises on interest, with or without tables. An example is a manuscript text on arithmetic by Rucellai, a Florentine citizen, bearing the date April 23, 1440, and found in the Bibliotheca Nazionale in Florence. It contains tables of interest computed, it says, by Antonio Mazinghi as part of an exposition on simple and compound interest (6).

Luca Pacioli, in his widely read *Summa* of 1494, also mentions tables of interest and sketches the way how to compute them (7). There are no tables in the *Summa*, only a number of problems on interest, simple and compound. In order to find tables in print we still have to wait for half a century. Then we meet a few in the *Arithmétique* of Jean Trenchant (8).

Nothing is known about Trenchant except that he was a teacher of mathematics at Lyons, who in 1558 published a book called *L'Arithmétique departie es trois livres,* which passed through many editions, occasionally *"revue et augmen-*

(4) *Liber Abaci. Scritti di Leonardo Pisano,* ed. B. Boncompagni, vol. 2 (1862) p. 267.

(5) Leibniz, in his essay *Meditatio iuridico-mathematica de interusurio simplice,* Acta Eruditorum 1683, defended the use of compound interest according to the formula $C_x = C_0 (1 + i)^x$. He was attacked by other jurists with the argument that the taking of interest on non-paid interest is prohibited. See M. Cantor, *Politische Arithmetik* (Leipzig, 1898, X + 136 pp.), p. 35.

(6) The manuscript is in the Biblioteca Nazionale, Florence, call number Palatino 573, author Girolamo di Piero di Chardinale Rucellai (This information is due to Dr. R. De Roover, Aurora, NY).

(7) L. Pacioli, *Summa de Arithmetica Geometria Proportioni et Proportionalità* (Venice, 1494, second ed., Toscolano, 1523), first part, 9th distinctio, 5th tractatus. Pacioli writes "del modo a sapere componere le tavole del merito". The term "merito", French "mérite", stands for what Stevin calls "profitable interest." Compound interest is "a capo d'anno, o altro tempo, o termine". See footnote[13]).

(8) On Jean Trenchant. see H. Bosmans, *L'Arithmétique de Jean Trenchant,* Annales Soc. Sc. Bruxelles 33 (1908-09), 1e partie, pp. 184–192; G. Sarton, *Jean Trenchant, French Mathematician of the Second Half of the Sixteenth Century,* Isis 21 (1934), pp. 207–208; C. M. Waller Zeper, *De oudste intresttafels in Italië, Frankrijk en Nederland met een herdruk van Stevins „Tafelen van Interest",* Diss. Leiden, (Amsterdam, 1937, 95 + 92 pp.), esp. Ch. III.

tée" (9). The date of publication is important. Lyons was famous as a money market, where kings and other nobles bargained for huge loans with the most important bankers of Europe. A first attempt was made in 1555 by King Henry II and his financiers to consolidate the many haphazard royal loans of the past and to establish a regular system of amortization. This was the "Grand Parti", famous in its days, and so popular that wide strata of the population hastened to subscribe (10). Trenchant's book, with its extensive chapter on simple and compound interest, reflects the public desire for understanding the intricacies of the money market. The third part of his book contains four interest tables, of which two were specially compiled to illustrate the "Grand Parti". This transaction, to which later also Coignet (11) and Stevin return, is described in the following problem.

"En l'an 1555, le Roi Henri pour ses affaires de guerre, prenait argent des banquiers, à raison de 4 pour 100 par foire (12): c'est meilleure condition pour eux, que 16 pour 100 par an. En ce même an avant la foire de la Toussaint il reçut aussi par les mains de certains banquiers la somme de 3954941 écus et plus, qu'ils appelaient le grand parti, à condition qu'il payerait à raison de 5 pour 100 par foire, jusqu'à la 41-ième foire; à ce paiement il demeurerait quite de tout; à savoir laquelle de ces conditions est meilleure pour les banquiers? La première à 4 pour 100 par foire est évidente, c'est à dire on voit son profit évidemment. Mais la dernière est difficile: de sorte que les inventeurs de cette condition-là ne l'ont trouvée qu'à tâtons et presque avec un labeur inestimable. Maintenant je veux montrer à faire telles calculations légèrement (facilement) et précisément avec raison démonstrative facile à entendre."

The question raised is therefore the following. The king borrows 3,954,941 écus. Every quarter year he has to pay interest and the total debt must be paid off after 41 payments. What is more advantageous to the bankers: payment of 4 per cent interest each quarter and return of the principal at the 41st payment, or payment of 5 per cent interest each quarter and no extra payment at the end?

Trenchant, in solving this problem, introduces two tables. The first one is a table which lists the increase in value of $10^7\ (1.04)^n$, $n = 0,1,\ldots, 40$:

1 0 0 0 0 0 0 0
1 0 4 0 0 0 1 0
1 0 8 1 6 0 0 0
. . . .
4 6 1 6 3 6 5 9
4 8 0 1 0 2 0 6

(9) The fourth edition has the title: *L'arithmétique de Ian Trenchant departie en trois livres. Ensemble un petit discours des Changes avec l'art de calculer aux Getons. Revue et augmentée pour la quatrième édition, de plusieurs règles et articles, par l'Autheur.* A Lyon, par Michel Iove, 1578, 375 pp.. Trenchant was therefore alive in 1578. The edition of 1563 is also „revue et augmentée".

(10) R. Doucet, *Le grand parti de Lyon au 16e siècle*, Revue historique *171* (1933), pp. 473–513; 172 (1933), pp. 1–41; also R. Ehrenberg, *Das Zeitalter der Fugger* II (Jena, 1896, 18+367 pp), p. 101 ff.; translated as *Capital and Finance in the Age of the Renaissance* (New York, 1928, 390 pp.). Information on *le grand parti* is due to Mrs C. B. Davis, Ann Arbor. Mich.

(11) *Livre d'arithmétique... composé par Valentin Mennher Allemand: revue, corrigée et augmentée... par Michiel Coignet.* Anvers, 1573, 141 pp. Doucet and Ehrenberg *l.c.*10) write Coquet instead of Coignet.

(12) There were four fairs a year at Lyons; „par foire" therefore means: "every quarter year".

The other table gives the successive partial sums $\sum_{0}^{i} k \ 10^7 \ (1.04)^k$,

$i = 0, 1, 2, \ldots, 40$:

1 0 0 0 0 0 0 0
2 0 4 0 0 0 0 0
3 1 2 1 6 0 0 0
. . . .
9 9 8 2 6 5 3 3 8

From these tables Trenchant deduces that the one "écu per 100 difference" over 4 per cent in the second alternative (in order to pay off the principal) is worth 48.010206 écus [we use modern decimal notation] after 40 terms, 46.163659 écus after 39 terms, etc. The total of all these écus paid extra every term is 99.8265338 écus, a little less than 100. The first alternative is therefore a little better for the bankers. Trenchant also remarks that the last table allows us to find out how far the debt is paid after every term.

These two tables are preceded by two others, also placed between the text in order to illustrate certain problems on compound interest (mérites, discontes à chef de terme) (13).

The first table of Trenchant lists the increase in value of 10^7 at $8\frac{1}{3}$ percent yearly (on every twelve pence one penny interest yearly, "van den penninck 12", as Stevin wrote) for 28 years, hence $10^7 \ (1 + \frac{1}{12})^n$, $n = 0,1,2, \ldots 28$:

1 0 0 0 0 0 0 0
1 0 8 3 3 3 3 3
1 1 7 3 6 1 1 1
. . . .
9 3 7 5 7 4 5 8

The second table gives the increase of 10^7 after 1, 2, . . . 11 months at the same rate of interest, obtained by multiplying 10^7 successively by

$$(1 + \tfrac{1}{12})^{\frac{1}{12}}, \ (1 + \tfrac{1}{12})^{\frac{2}{12}}, \ \ldots, \ (1 + \tfrac{1}{12})^{\frac{11}{12}}:$$

1 0 0 0 0 0 0 0
1 0 0 6 6 9 2 4
1 0 1 3 4 2 9 5
. . . .
1 0 7 6 1 3 0 4

Trenchant has also problems on simple interest, for which no tables are necessary. One of these problems must be quoted, since Stevin in Ex. 6 of his

(13) Translated from the italian "a capo d'anno", or "a capo d'alcun tempo", because compound interest was computed from the beginning of each year, or other term.

discussion of simple interest takes issue with Trenchant's conclusions. It is problem 6 of Trenchant's Ch. IX:

"Si quelqu'un devait 600 livres à payer le tout au bout de 4 ans, et son créditeur le priait de les lui payer en 4 termes (à savoir au bout du premier an et chacun des autres le quart en lui discontant simplement à raison de 12 pour cent par an), à savoir combien il lui faudrait chaque année? Considère qu'il faut disconter pour un an, pour 2 et pour 3 ce qu'il avance. Donc pour 4 ans suppose 400; puis avise qu'un cent en principal et intérêt fait en un an 112 livres; en deux 124: et en trois 136; à ces trois sommes il faut ajouter le quatrième terme 100 qui ne mérite rien: elles se monteront à 472. Puis dis: si 472 viennent de 400, de combien 600. Tu trouveras $508\frac{28}{59}$, dont le $\frac{1}{4}$ à savoir 127 livres et $\frac{7}{59}$, est ce qu'il devrait payer par chacun des 4 ans. Pour en faire la preuve:

Regarde que $127\frac{7}{59}$ profitent $15\frac{15}{59}$ par an; puis que le premier paiement profite par trois ans, il gagne donc 3 fois $15\frac{15}{59}$ ce qui est $45\frac{45}{59}$: par la même raison le second payement gagne $30\frac{30}{59}$ et le troisième $15\frac{15}{59}$. Ajoute maintenant tout le profit qui se monte à $91\frac{31}{59}$ aux 4 paiements $508\frac{28}{59}$, il viendra 600 comme il fallait. Autrement pour savoir tout le gain, multiplie $15\frac{15}{59}$ par 6, car les trois paiements gagnent par 6 termes, proviendra $91\frac{31}{59}$."

Trenchant's chapter on interest (no. IX) is based on his previous chapter (no. VIII), where he teaches geometrical progressions, and thus the way in which his tables have been computed.

§ 2

The tables of Trenchant and of Pegolotti are the only printed tables written before Stevin. Problems concerning simple and compound interest not accompanied by tables occur much more frequently. There exist cuneiform tablets with compound interest problems; one of these problems is to find how long it takes for a sum of money to double itself at 20 per cent interest. This leads to what seems to be the equivalent of the equation $(1.2)^x = 2$, which is solved by linear interpolation. The answer appears in sexagesimal notation (14). In Medieval Europe we find compound interest problems solved by Leonardo of Pisa (4); among the authors who followed him we find Pacioli (7), Cardan, and Tartaglia (15).

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(14) See *e.g.* R. C. Archibald, *Outline of a History of Mathematics*, 6th edition, Am. Mathem. Monthly 56 (1949 supplement, 114 pp.) p., 13.

(15) See C. M. Waller Zeper, *l.c.*[8]), Ch. II. Tartaglia's problems are found in his *General Trattato di numeri et misure*, Parte I (1556) fol. 192 v. There were a number of other writers on interest computation, of which we find a list in Wentzel, *l.c.* ([26]), also cited by C. M. Waller Zeper, pp. 38–39. Stevin became acquainted with Tartaglia's work after 1583, see *Meetdaet*, p. 144.

A matter of some controversy was the problem what to do in the case of fractional terms. The Babylonian formula $(1.2)^x = 2$ is consistent with the general formula for compound interest

$$C_x = C_0\,(1+i)^x\,,$$

C_0 = initial capital, C_x = capital after x years, the interest is at 100 i per cent a year, even if x is fractional. This was not always the point of view of the Renaissance mathematicians (16). For instance, Tartaglia raises the question what 100 lb. will be after $2\frac{1}{2}$ years at 20 per cent compound interest.

If 100 lb. accumulates to 120 lb. in one year, he says, it will accumulate to 110 lb. in half a year. Tartaglia now reasons that 100 lb. in two years becomes $100(1.2)^2$, and in $2\frac{1}{2}$ years therefore $100\,(1.2)^2\,(1.1) = 158.4$ lb. In this, he takes issue with Pacioli and Cardan, who accumulate up to 3 years, then discount by half a year, and find $100(1.2)^3\,/\,(1.1) = 157\frac{1}{11}$ lb. The method which the Babylonians seem to have had, which Trenchant certainly had, and which is in accordance with modern practice, would have given:

$$100(1.2)^{2\,1/2} = 157.74 \text{ lb.}$$

Tartaglia has still another method, which in this case gives the answer $100(1.1)^5 = 161.05$ lb. These different methods can be expressed in the following way:

1) $C_x = C_0\,(1+i)^x$ (Trenchant)

2) $C_x = C_0\,(1+i)^p\,(1+ti)$, $\quad x = p + t$, p = largest integer $< x$

(Tartaglia)

3) $C_x = \dfrac{C_0\,(1+i)^q}{1+ui}$, $\quad x = q - u$, q = largest integer $> x$

(Cardan, Pacioli)

4) $C_x = C_0\,(1+\frac{i}{m})^{mx}$, $\quad$ interest at $100i$ per cent a year to be paid every m^{th} part of a year.

(Tartaglia)

Trenchant, solving the problem which led to his second table, used the first method with x a multiple of $\frac{1}{12}$. Stevin preferred the second method. Apart from the fourth method, in which compound interest at 100 i per cent a year

(16) C. M. Waller Zeper *l.c.*[8]), p. 14.

is simply replaced by compound interest at $100\frac{i}{m}$ per cent every m^{th} part of a year, the other methods only differ in the way they answer the question: shall interest due after a fraction of a year be computed at compound or at simple rates? We shall return to this question, on which even now there exists some difference of opinion, when we discuss Stevin's position.

We further introduce the following notation, which is in common use:

$$S_{\overline{n|}} = (1+i)^n, \qquad A_{\overline{n|}} = (1+i)^{-n},$$

$$s_{\overline{n|}} = \sum_{0}^{n-1} k \ (1+i)^k, \ a_{\overline{n|}} = \sum_{1}^{n} k \ (1+i)^{-k}.$$

We can now express the results of Pegolotti and Trenchant as follows:

Pegolotti: $10^2 \, S_{\overline{n|}}, \ i = \cdot 01, \cdot 015, \cdot 02, \ldots, \cdot 08;$

$n = 1,2, \ldots, 20$

Trenchant; $10^7 \, S_{\overline{n|}}, \ i = \frac{1}{12}, \ n = 0,1, \ldots, 28;$

$10^7 \, S_{\overline{n|}}, \ i = \frac{1}{12}, \ n = \frac{1}{12}, \frac{2}{12}, \ldots, \frac{11}{12}$

$10^7 \, S_{\overline{n|}}, \ i = \cdot 04, \ n = 0,1, \ldots, 40$

$10^7 \, S_{\overline{n|}}, \ i = \cdot 04, \ n = 1,2, \ldots, 41$

§ 3

The two great money markets of Western Europe in the sixteenth century were Lyons and Antwerp. We have seen that the first published tables of interest came from Lyons. The second publication of such tables occurred at Antwerp. They were the work of Stevin, at that time already settled at Leyden.

These Tables of Interest appeared first in Dutch in 1582. A French version of the book appeared in *L'Arithmétique* of 1585. The Dutch text was republished and corrected in 1590. The French version reappeared in Girard's edition of *L'Arithmétique* of 1625, and in his edition of the *Oeuvres Mathématiques* of 1634. There are therefore two Dutch and three French editions (17). The edition of 1582 was photostatically reproduced, in 1937, by C. M. Waller Zeper(18).

The different editions show some variations (19). Perhaps the most striking difference is that the references to Trenchant only occur in the Dutch editions. The reason for their omission from the French editions is not at all clear (20).

(17) *Works* I, V, XIII.

(18) C. M. Waller Zeper *l.c.*8).

(19) *Ib.* pp. 51–52.

(20) C. M. Waller Zeper, *ib.*, p. 53, tentatively ascribes this silence of Stevin to a touch of Dutch chauvinism. But Stevin is usually quite willing to acknowledge his sources. "Toujours nous le voyons hanté par la crainte de s'attribuer une découverte qui ne lui appartient pas", writes Father H. Bosmans (Annales de la Société scientifique de Bruxelles 35 (1910–1911), p. 294). Another possibility is suggested by the name of Trenchant's publisher, Michel Jove, who was an outstanding Catholic, publisher for the Archbishopric of Lyons and for the Jesuits (comm. by Mrs C. B. Davis, Ann Arbor, Mich.). Was Trenchant perhaps himself compromised, in Huguenot circles, as too ardent a Catholic?

Other differences can be found in the prefaces, which are much shorter in the French editions. Some errors (or supposed errors) of the first edition are corrected in the later ones. The Dutch edition of 1590 therefore differs in some details from the edition of 1582. The text used in this edition is the edition of 1582.

The book, in true Stevin fashion, opens with definitions. Among them we find those of simple and compound interest, of "profitable interest" ("intérest prouffitable" of Stevin, the "mérite" of Trenchant, interest to be added to the principal, hence accumulating interest), and of "detrimental interest" ("intérest dommageable" of Stevin, the "disconte" of Trenchant, interest to be subtracted from the principal, hence discount). Then follow a set of examples on simple interest, first on profitable, then on detrimental interest. Stevin follows the practice, also approved by Trenchant, of taking as the present value C_o of a loan C_z due after x years at 100 i per cent simple interest:

$$C_o = \frac{C_x}{1 + x\,i}$$

Indeed, after x years (x integer or fractional) C_0 will have accumulated to $C_0 (1 + ix) = C_x$. This is also at present an accepted way of discounting at simple interest. There are other cases in which it is customary to use the rate of discount $100d$, where $d = \frac{i}{i + 1}$ instead of the percentage $100i$, and to write $C_o = C_x (1 - xd)$ (21). An ancient Italian method of discounting followed the rule $C_o = C_x (1 - xi)$ (22). These different methods were a source of controversy, not only between mathematicians, but also between jurists. Apart from these questions, in which custom rather than mathematics plays a role, there were other controversial points. We meet one in Ex. 6 of the problems of discount at simple interest, the problem of Trenchant quoted above. Here Stevin takes issue with his colleague, but the dispute only involves the interpretation of the problem, and we can accept both Stevin's and Trenchant's mathematics. But in the Exs 9 and 10 of the same set we meet a controversial point of deeper mathematical interest, of enough importance to make Stevin emend himself: the edition of 1590 has a

(21) M. van Haaften, *Leerboek der Intrestrekening* (Groningen, 1929, 644 pp.), p. 19.

(22) Computation of discount ixC_o is easy in this case, since it is taken from the sum C_o due. All through the sixteenth and seventeenth centuries there were jurists defending this position. See M. Cantor *l.c.*[5]) p. 29. The difference between $\frac{C_o}{1 + xi}$, $C_o (1 - xi)$, $C_o (1 - xd)$, and the correct value $C_o (1 + i)^{-x}$ is small when i is small:

$$\frac{1}{1 + xi} = 1 - xi + (xi)^2 -$$

$$1 - xd = \frac{1 + (1 - x)i}{1 + i} = 1 - xi + xi^2 - \ldots$$

Compare also $e^{-ix} = \frac{1}{1 + xi + \frac{1}{2}(xi)^2 + \ldots} = 1 - xi + \frac{1}{2}(xi)^2 - \ldots$

solution which differs from the one presented in the edition of 1582. The difficulty lies in the computation of the value, after m years, of a sum C to be paid after n years $n > m$, the rate of simple interest being given. Stevin's solution of 1582 can be written in the form

$$C \frac{1 + m i}{1 + n i}, \tag{A}$$

the solution of 1590 in the form

$$\frac{C}{1 + (n - m) i} \tag{B}$$

Both solutions would appear admissible at present, though the 1590 solution might be preferred, since this is the amount which after m years will accumulate to C in the next $(n - m)$ years. It is not impossible that Stevin's change of attitude between 1582 and 1590 was influenced by a pamphlet written by Ludolf van Ceulen against Simon Van der Eycke, in which Van Ceulen sharply attacked Van der Eycke's use of method (B), and defended method (A) (23). The difference between the two methods is not a question of convention, but lies in the nature of simple interest calculus. If we postulate that when a payment A is equivalent to a payment B, and the payment B also equivalent to a payment C, the payment A is also equivalent to payment C, then we arrive at compound interest, in which case the answer is unique (24):

$$C (1 + i)^{-(n-m)}$$

After Ex. 14 Stevin passes to problems on compound interest. Here he inserts his tables. They are discount tables, hence tables for $A_{\overline{n|}}$. Stevin explains clearly how he computed them. He took as "root" of his system 10^7, which was a common device of his days for avoiding decimal fractions. Stevin's *Thiende* was not published until three years after the first publication and Stevin never undertook the rewriting of his tables in his own decimal notation. In order to find the first table, a discount table for one per cent, he multiplies 10^7 by $\frac{100}{101}$, that is, divides 10^9 by 101; then he multiplies the result again by 100 and divides by 101, etc. Every answer is written out in seven figures, fractions less than $\frac{1}{2}$ are neglected, those larger than $\frac{1}{2}$ are replaced by the full unit in the way it is still done at present. The tables for $A_{\overline{n|}}$ run from $n = 1, 2, \ldots,$ to $n = 30$; and are computed first for $i = \frac{1}{100}, \frac{2}{100}, \ldots, \frac{16}{100}$, then for $i = \frac{1}{15}, \frac{1}{16}, \ldots, \frac{1}{19}, \frac{1}{21}, \frac{1}{22}$.

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(23) *Een corte verclaringh aengaende het onverstant ende misbruyck inde reductie op simpel interest. Den ghemeenen volcke tot nut... door Ludolf van Colen...* Aemstelredam, 1586. The pamphlet was published under the same cover with another attack by Van Colen on Van der Eycke: *Proefsteen ende Claerder wederleggingh...*; it dealt with the quadrature of the circle. See M. van Haaften, *Ludolf van Ceulen (1550–1600) en zijn geschriften over intrestrekening*, De Verzekeringsbode, 17 April 1936, pp. 85–90.

(24) W. C. Post, *De behandeling van de samengestelde intrestrekening op onze middelbare scholen*, Nieuw Tijdschrift voor Wiskunde 9 (1921–'22), pp. 262–271; M. van Haaften, *l.c.* 21), 201; W. C. Post, *Over enkelvoudige en samengestelde intrestrekening*, Het Verzekeringsarchief 19 (1938), pp. 12–26.

By adding successive terms in the tables for $A_{\overline{n|}}$ Stevin also obtains tables for $a_{\overline{n|}}$.

Stevin has no tables for $S_{\overline{n|}}$ and $s_{\overline{n|}}$, with one exception: a table for $S_{\overline{n|}}$, $n = 1, \ldots, 30$ and the corresponding $1+s_{\overline{n-1|}}$ for $i = \frac{1}{15}$. Stevin gives two reasons for this omission; one is that too many tables would only confuse the good reader, and the other is that the tables for $A_{\overline{n|}}$ can be used if we are in need of $S_{\overline{n|}}$, since $A_{\overline{n|}} \cdot S_{\overline{n|}} = 1$. The $S_{\overline{n|}}$, $s_{\overline{n|}}$ table for $i = \frac{1}{15}$ was just an illustration of what Stevin could have done if he had wanted to.

Since the $a_{\overline{n|}}$ and $s_{\overline{n|}}$ are obtained by successive summation of numbers with seven digits, of which the last one is an approximation, the last digits of $a_{\overline{n|}}$ and $s_{\overline{n|}}$ tend to be unreliable when n increases. A similar cause of error exists in the $S_{\overline{n|}}$.

The problems on compound interest are again divided into a number on "profitable", and a number on "detrimental" interest. The latter are reduced to the former by means of the remark that any problem involving $S_{\overline{n|}}$ can be solved with the tables for $A_{\overline{n|}}$ (there is one exception, Ex. 6, of the "profitable" interest series where the table for $S_{\overline{n|}}$ is used). In some examples we find Stevin's position on interest over a fractional number of years. Ex. 2 of the discount problems gives for the present value of 600 lb. due after $13\frac{1}{3}$ years at "the penny 14":

$$600\,(1+i)^{-13}\,\frac{1}{1+(\frac{1}{2})\,i}\,, \qquad i = \frac{1}{14}.$$

The same reasoning is followed in Ex. 2 of the problem on profitable compound interest. Here Stevin warns his readers against Trenchant's method, which, as we have seen, requires multiplication by $(1+i)^p$ for fractional p, and not by $1+pi$, as Stevin suggests. Stevin's objection has two curious foundations: a) compound interest should always give higher interest than simple interest, and $(1+i)^p < 1+pi$ when $p<1$; b) compound interest is the same as simple interest for the period of a whole year, therefore *a fortiori* it should be the same for a fraction of a year. In discount Stevin is clearly on the side of the debtor.

§ 4

Stevin's initiative seems to have led several others to the publication of books on interest with tables, especially in the Netherlands ([25]). The first to emulate

([25]) The reason was, of course, the rapid commercial development of the Netherlands. A contributing factor may have been the dominating influence of Calvinism, which was more tolerant to the taking of interest than either Catholicism or Lutheranism.

him was Marthen Wentzel Van Aken, a schoolteacher, who, when at Rotterdam, was invited to write this book by a merchant who found Stevin's exposition too difficult. Wentzel's tables were published in 1587, and were republished in 1594 (26); they differed considerably from those of Stevin. They were followed by the tables of Ludolf van Ceulen, who published them in his book *Van den Circkel* (1596) (27), which also contains his celebrated evaluation of π, though here only in 20 decimals (28). After 1600 the number of books on interest computation with and without tables increases considerably (29). Van Ceulen's and Stevin's works usually served as direct examples (30). Among the more original authors on this subject is Ezechiel De Decker, the Gouda surveyor who did so much to promote Stevin's *Thiende* and Napier-Briggs' logarithms. De Decker's tables are more

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(26) *Proportionale, Ghesolveerde Tafflen van intrest Van de kustingbrieven ofte Rentebrieven, zy te betalen op terminen op vervolgende iaeren ofte opt eynde des laetsten iaers van de brieven... Tweede Editie Door Marthinum Wentselaum Aquis Graniensis.* t'Amstelredam. Ghedruckt by Barendt Adriaensz. 1594, 116 pp.

Wentzel mentions the following authors on interest: "Gillis van den Hoeck, Niclaes Tartaglia, Pietrus Apianus, Adam Risen, Christoff Rudolf, Valentyn Menher, Symon Iacop van Coburg, Pierre de Sovonue, Nicolaes Pieterszoon van Deventer, Michel Coignet, Hobbe Jacobsz." On these authors, see C. M. Waller Zeper *l.c.* 8) p. 39. As a writer on tables of interest Wentzel mentions C. I. Broessoon. This Broessoon most likely is the Cornelis Jan Broerszoon van Haarlem, whose tables, written before 1599, are perhaps those printed in *Arithmetica, met een tafel van interest van een op* 4 *hondert ende van* $^1/_4$ *tot* $^1/_4$ *tot* 12 *op 't hondert Interest op interest per Jan Belot Dieppois*, Haerlem, 1629. This book contains a "tafelken gemaeckt door C. I. Broersz, gesolveerde jaarlykse termijnen van 100 gul. ende ook 100 gul. Die verscheyden jaren teffens verschijnen" (Haarl. Stadsbibl.).

(27) *Van den Circkel Daer in gheleert werdt te vinden... Ten laetsten van Interest met alderhande Tafelen daer toe dienende met het ghebruyck door veel constighe Exempelen gheleerdt... door Ludolph van Ceulen* .. tot Delf, ghedruckt by Jan Andriesz... 1596.

(28) Wentzel mentions Stevin, Van Ceulen does not mention him. The reason may be that Van Ceulen already composed these tables before the publication of Stevin's book. See H. Bosmans, *Un émule de Viète: Ludolphe van Ceulen*, Ann. soc. scient. de Bruxelles 34 (1909–10), 2e partie, pp. 88–139.

(29) Some more information on the controversies on interest computation in this period can be found in M. van Haaften, *Het Wiskundig Genootschap* (Groningen, 1923, 169 pp.), p. 119. Stevin himself corresponded on these questions with Thomas Masterson, author of *Thomas Masterson his first booke on Arithmeticke...*, 1592, followed by a *second booke* (1592), an *addition to his first booke of Arithmetick* (1594) and a *third booke* (1595), all published in London. In the *addition* of 1594 Masterson, in the preface, takes issue with "Michell Cognet of Antwerpe and Simon Stevin of Bruges": "both teaching (in the appearance of the unskilfull) with great shew of truth, other answers than mine of the aforesaid questions of paiments and interests, and notwithstanding in those answers they are very false: and so their followers (being in great number) wander in those points in danger, error and ignorance...". Masterson had "first given the aforesaid authors (being yet alive) knowledge thereof by my letter, as also received their answers by their letters: then replied unto their answers, and received their conclusions: Then prooved their resolutions to be false, and (to the one of them for the other did answer no more) prooved by demonstration mathematicall, that my solution is only true." Masterson writes that in some other place he will deal with the subject of the controversy, but it seems that he did not publish anything. The reference to Coignet may be to the book *Livre d'Arithmétique contenant plusieurs belles questions .. composé par feu Valentin Mennher Allemand: revu, corrigé et augmenté en plusieurs endroits par Michiel Coignet...* Anvers, 1573, which in some places deals with interest computation (with reference to Trenchant).

(30) C. M. Waller Zeper *l.c.*8) Ch. IV. Ch. VI deals with Wentzel, Ch. VII with Van Ceulen.

elaborate than all previous ones; they are found in the same *Eerste Deel van de Nieuwe Telkonst* (1626) in which *De Thiende* was reproduced(31).

The first compound interest tables in the English language seem to be those of Richard Witt (1613). It probably was also the first English work, after Norton's translation of *De Thiende,* in which decimals were used (32).

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(31) *Eerste Deel van de Nieuwe Telkonst, inhoudende verscheyde manieren van rekenen. Mitsgaders Nieuwe Tafels van Interesten, noyt voor desen int licht ghegeven... Door Ezechiel De Decker*... Ter Goude, By Pieter Rammaseyn... 1626. See C. M. Waller Zeper *l.c.*8), Ch. VIII.

(32) Richard Witt, *Arithmeticall questions, touching the Buying or Exchange of Annuities*... London, 1613. See R. C. Archibald, in *Mathematical Tables and Other Aids to Computation* (MTAOATC) 1 (1943–48), pp. 401–402.

Tafelen van
INTEREST,
Midtsgaders
De Constructie der selver, ghecalculeert

DOOR
Simon Steuin Brugghelinck.

T'ANTWERPEN,
By Christoffel Plantijn in den gulden Passer.
M. D. LXXXII.

Den Eersamen voorsienigen Heeren Ian Ianss. Baersdorp, Gheeraerdt Weygherss. van Duyuelandt, Pieter Arentss. van der Werf, Ian Lucass. van Wassenaer, Borghemeesteren, eñ Ian van Haute Secretaris, midtsgaders Schepenê ende ghemeyne Vroetschap der Stede Leyden, wenscht Simon Steuin gheluck ende voorspoet.

Ghelijckerwijs den iaerlicxschen vloedt des Nilus oorsake was van groote twist die gheduerlick oprees tusschen den inwoonderen van Egypten, omme dieswille zij alle teeckenen daer ieghelicks landt mede af ghepaelt was iaerlicks wtroeyede, welck nochtans by ghevâlle een oorsake was van groote eenicheydt die haeren naecomelinghen daer wt ghevolcht is, want heurlieder Koninck beval daer deur den priesteren (ouermidts zij meer ledighen tijdt hadden dan andere) middelen te practiseren datmen door eenighe ghewisse regelen yeghelik zijn landt zoude mogen wederleueren: De welcke dat te weghebrengende, hebben bevonden dat het productum van twee zijden eens vierhoeckichs rectangels, perfectelick bewees t'inhoudt der seluer superficien, al waer men zegt die edele côste van Geometrie, tot grooten voordeele der menschen, haeren oorspronck genomen te hebben: Also oock mijne E. voorsienighe Heeren bevinden wij den Interest een oorsake geweest te hebben, die menighen (deur derseluer gewisse rekeninge onbekentheyt) tot schade ghebrocht heeft, welck nochtans een oorsake gheweest is streckende ten profijte der naercomelingen, want naedien de menschen practiserende sagen dat alle Interest (zoo wel gecôponeerde als simpele) van veel iaeren oft termijnen, stont in eenige kennelicke reden tot hare Hooft-sôme, so wel als den interest van een termijn tijdts tot haere Hooft somme in zekere reden staet; Nochtans datmen tot de kennisse van dese reden, niet dan door al te verdrietigen grooten aerbeydt ende tijdt verlies en conde comen; Jae grooter voor eenen die grooten handel doet, dan hem zijn tijdt zoude toelaeten, waer toe noch algebra, noch andere regulen niet en hebben connen ghenoech doen: Soo zijnder ten laetsten gheinventeert zekere tafelen, door de welcke iegelicken maer simpelicken ervaren inde reghel der proportien (welcke sommige reghel van dryen noemen) zal ex tempore moghen solueren alle questie van Interest inde practijcke ghemeynelick te voren comende.

Welcke tafelen midtsgaders haere constructien ende ghebruyck, ick in dit tractaet ordentlick naer mijn vermogen verclaeren zal. Niet dat ick die wtgeue als voor mijne inventie, maer wel als door my gheamplificeert: want voor my heeft van de zelue geschreuen Jan Trenchant int 3. boeck zijnder Arithmeticquen int 9. cap, art. 14. al waer den zeluen Auctheur ghemaeckt heeft eene deser tafelen van 41. termijnen teghen Interest van 4. ten 100. op elck termijn, gedueren-de elck termijn dry maenden. Ende hoe wel hy dese tafele niet ghemaeckt en heeft tot alzulck een generale ghebruyck als wijse hier presenteren (want hy opsicht gehadt heeft op de profijtelickste conditie van tween die de banckiers presenteerden aen Hendrick Koninck van Vranckerijck int iaer 1555. ouer een Hooftsomme van 3954641 goude croonen, welck genoemt wierdt le grâd party, al waer zij den Koninck presenteerden, oft dat hy betaelen zoude 4. ten 100. van simpelen

Simon Stevin wishes the Honourable, provident Gentlemen Jan Janss. Baersdorp, Gheeraerdt Weygherss. van Duyvelandt, Pieter Arentss. van der Werf, Jan Lucass. van Wassenaer, Burgomasters, and Jan van Haute, Secretary, together with the Aldermen and the City Council of the City of Leyden, happiness and prosperity.

Just as the annual inundation of the Nile was the cause of great disputes which continually arose between the inhabitants of Egypt, because every year it destroyed all the marks with which each man's land was staked out, which nevertheless happened to become a cause of great unity that resulted therefrom to their descendants, for their King on this account commanded the priests (since they had more leisure than others) to devise means to make it possible to return to everyone his land according to certain rules, which priests, bringing this about, found that the product of two sides of a quadrangular rectangle perfectly denoted the area thereof, from which it is said that the noble art of Geometry derives, to the great advantage of man; in the same way, Honourable, provident Gentlemen, we find that Interest was a cause which occasioned loss to many people (because the sure computation thereof was unknown), which nevertheless has been a cause that was to the advantage of the descendants, for since people found in practice that all Interest (both compound and simple) of many years or terms was in a knowable ratio to its Principal, just as the interest of one term has a certain ratio to its Principal, but that nevertheless this ratio could only be found by very vexatious, great exertion and loss of time, yea, greater for one doing a large business than his time would permit, for which neither algebra nor other rules were sufficient, finally there were invented certain tables by means of which anyone who has only little experience in the rule of proportions (which some call the rule of three) will be able to solve offhand any question of Interest that may commonly occur in practice.

These tables, together with their construction and use, I will explain in due order to the best of my ability in this treatise. Not that I publish them as my invention, but indeed as amplified by me; for before me Jan Trenchant has written about them in the 3rd book of his Arithmetic)[1]), in the 9th chapter, section 14, where this Author made one of these tables of 41 terms at an Interest of 4 per cent for every term, every term being of three months. And although he has not made these tables for such general use as we present them here (for he had in view the most profitable of two conditions which the bankers offered to Henry, King of France, in the year 1555, concerning a Principal of 3,954,641 gold crowns, which was called *le grand party,* when they gave the King the choice whether he would pay 4 per cent of simple interest every quarter year or whether

[1]) See the Introduction, p. 15

interest alle vierendeel iaers, oft dat hy betaelen zoude 5. en 100. ende dat 41. termijnen ofte vierendeelen iaers gheduerende, ende dat hy daer mede verloop ende interest teenemael zoude betaelt hebben) Doch zegghen wy hem tot goeder ende eeuwiger gedachtenis deser tafelen met rechte een inventeur ghenoemt te worden.

Hebbe oock verstaen dat der zeluer tafelen hier in Hollandt by eenighe schriftelick zijn, maer als groote secreten by den ghenen diese hebben, verborghen blijven, ook niet zonder groote cost de selue te crijghen en zijn, ende principalick de compositie die zegt men zeer weynich persoonen ghetoont te worden. Voorwaer tis te bekennen dat de kennisse deser tafelen voor den ghenen diese veel van doen heeft, is een zaecke van grooter consequentien, maer die secreet te houden schijnt eenichsins een argument te zijne van meerder liefde tot profijt dan tot conste. Want dat hem iemandt laet dyncken dat hyt al ghesien heeft dat door dese tafelen mach gedaen worden, schijnt zoo veel als oft hy hem persuadeerde de *terminos infinitae lineae* ghevonden te hebben; want ghelijck de verscheyden conditien die traficquerende persoonen malckanderen daghelicks voorstellen oneyndelick zijn, alsoo oock de verscheyden verholen ghebruycken deser tafelen: Daerom een liefhebber der consten meer begheerende wt dese tafelen te leeren dan hy weet, hem en schijnt gheen beter middel te zijne (ouermidts d'ooghen meer sien dan d'ooghe) dan dat hyse divulgere. Twelck ick alsoo verstaende, hebben de zelue mijne E. Heeren onder de protectie van U.E. ende tot nutbaerheyt der ghemeynte laeten wtgaen: Niet twijfelende (waer toe my een argument is d'openbaer experientie van U.E. in de voorderinghe ende bescherminghe der ghemeyne zaecke tegen alle stormen deses onghevalligen tijts) ofte U.E.en zal mijnen wille welcke de ghemeynte gheerne nutbaeren dienst dede voor goet aensien. Vaert wel In Leyden desen 16. Julij, An. 1582.

ARGUMENT.

Hoewel deses tractaets tijtel spreeckt alleenlick van tafelen van interest/als wesende t'principael tot welcks eynde dese descriptie beghonnen is; Sal nochtans beneuen de tafelen tot meerder verclaeringe een generael discours maken van allen interest (in de practijcke ghemeynlick ghebruyckt) begrepen onder 7. Definitien ende 4. Propositien met haeren explicatien. De definitien zullen zijn verclaeringhen van de eyghene vocabullen deser regulen / als wat dat is Hooft-somme / Interest / interests reden / Simpelen interest / Ghecomponeerden interest / Profijtelicken interest / ende schadelicken interest. Onder de propositien (midtsgaders verclaeringhe des simpelen interests) sal verclaert worden de constructie deser tafelen / ende door diuersche exempelen de ghebruyck der zeluer. Welcker propositien ierst sal sijn van simpelen profijtelicken interest / De tweede van simpelen schadelicken interest / De derde van ghecomponeerde profijtelicken interest / De vierde van ghecomponeerde schadelicken interest. Tot welckes meerder verclaeringhe begrijpen wy de Hooft-artijckelen des tractaets int volghende tafelken aldus:

Interest is *ofte*	Simpel	Profijtelick
		Schadelick
	Ghecomponeert	Profijtelick
		Schadelick

he would pay 5 per cent, such during 41 terms or quarter years, so that he would have paid capital and interest at the same time, yet we say, to his good and everlasting memory, that he is rightly called an inventor of these tables.

I have also learned that here in Holland such tables are to be found in writing with some people, but that they remain hidden as great secrets with those who have got them, and that they cannot be obtained without great expense, and principally the composition, which is said to be shown to very few people. Forsooth, it has to be confessed that the knowledge of these tables is a matter of great consequence to those who often need them, but to keep them a secret seems to argue in some sense a greater love of profit than of learning. For that anyone should think that he has seen all that can be done by means of these tables seems as much as if he should be persuaded to have found the ends of an infinite line. For just as the different conditions which businessmen daily propose to each other are infinite in number, so are also the various secret uses of these tables. Therefore, if a lover of learning should desire to learn from these tables more than he knows, there seems to be no better method for him (since the eyes see more than the eye) but to divulge them. Understanding it thus, I have published them, Honourable Gentlemen, under your protection for the benefit of the community, not doubting (an argument for which is furnished to me by the public experience of your promotion and protection of the common cause against all the storms of this unpleasant time) but you will take my wish to pay the community a useful service in good part. Good speed, in Leyden, this 16th July of the year 1582.

SUMMARY

Although the title of this treatise speaks only of tables of interest, as being the principal end for which this description has been started, I will nevertheless, in addition to the tables, with a view to a fuller explanation hold a general discourse on all interest (commonly used in practice), consisting in 7 Definitions and 4 Propositions with their explanations. The definitions will be explanations of the words proper to these rules, *e.g.* what is Principal, Interest, Rate of interest, Simple Interest, Compound Interest, Profitable interest, and Detrimental interest. Among the propositions (along with the explanation of simple interest) the construction of these tables will be explained, and their use by means of various examples. The first of these propositions is to deal with simple profitable interest, the second with simple detrimental interest, the third with compound profitable interest, the fourth with compound detrimental interest. To explain this more fully we include the Main Sections of the treatise in the following table:

Interest is either	Simple	Profitable or Detrimental
	or	
	Compound	Profitable or Detrimental

DEFINITIE 1.

Hooft somme is die/ daer den interest afgherekent wordt.

VERCLAERINGHE.

Als (by exempel) iemandt wtgheuende 16. lb op dat hij daer vorê ontfange eê lb t'siaers vâ interest wordt alsdan de 16. lb Hooft-somme ghenoemt. Oft iemandt schuldich wesende 20. lb te betaelen binnen een iaer / ende gheeft ghereedt gheldt 19. lb aftreckêde eê lb voor interest / wordt alsdâ de 20. lb Hooft-somme ghenoemt.

DEFINITIE 2.

Interest is een somme diemê rekent voor t'verloop van de Hooft-somme ouer eenighen tijdt.

VERCLAERINGHE.

Als wanneermen zeght 12. ten 100. t'siaers/dat is soo veel als 12. interest vâ 100. Hooft-somme ouer een iaer tijdt / alsoo dat Hooft-somme interest ende tijdt / zijn dry onscheydelicke dingen / dat is / Hooftsomme en is niet dan int respect van eenich interest / ende interest niet dan int respect van eenighe Hooft-somme ende tijdt.

DEFINITIE 3.

Ratio (welcke van sommige proportie genoemt wordt) die der is tusschen den interest ende d'Hooft-somme/noemen wij interests reden.

VERCLAERINGHE.

Als ratio die der is tusschê interest 12. eñ Hooft-somme 100. Oft tusschen interest 1 ende Hooft-somme 16. etc. noemen wy *in genere* interests reden. Ende is te aenmerken datter inde ghebruyck zijn tweederley manieren van interest redenen / welcker eene heeft het ander van haere termijnen altijt zeker. D'ander maniere beyde onseker. D'interests reden die een termijn zeker heeft is tweeder hande / want oft d'Hooft-somme is altijdt en zeker somme / te weten 100. ende den interest een onzeker somme als 9. oft 10. oft 11. etc. ende wordt dese interests reden dan ghenoemt neghen ten hondert / thien ten hondert / etc. Oft ter contrarien den interest is altijdt een zeker somme te weten 1. ende d'Hooft-somme onzeker als 15. oft 16. oft 17 / etc. Ende wordt dese interests reden ghenoemt den penninck vijfthien / den penninck zesthien etc. D'interest reden die haere termijnê beyde onzeker heeft / is ghelijck alsmen zeght by exempel 53. winnen t'siaers 4. Van alle welcke int volghende ordentlick t'zijnder plaetse verscheyden exempelen zullen ghegheuen worden.

DEFINITIE 4.

Simpel interest is die / Welck alleenlick van de Hooft somme gerekêt wordt

VERCLAERINGHE.

Als rekenende 24. lb voor interest van 100. lb op 2. iaeren teghen 12. ten 100. t'siaers / worden de zelue 24. lb dan simpelen interest ghenoemt. Oft iemandt

DEFINITION 1.

Principal is the sum on which the interest is charged.

EXPLANATION.

For example, when a man gives 16 lb in order that he may receive for it one lb of interest a year, then the 16 lb is called Principal. Or when a man owes 20 lb, to be paid in a year, and he gives 19 lb present value, subtracting one lb for interest, then the 20 lb is called Principal.

DEFINITION 2.

Interest is a sum that is charged on the outstanding part of the Principal over a certain time.

EXPLANATION.

For example, when it is said: 12 per cent a year, that is as much as an interest of 12 on a Principal of 100 over a year, so that Principal, interest, and time are three inseparable things, *i.e.* Principal does not exist unless in respect of a certain interest, and interest does not exist unless in respect of a certain Principal and time.

DEFINITION 3.

The ratio (which by some is called proportion) existing between the interest and the Principal we call rate of interest.

EXPLANATION.

For example, the ratio existing between an interest of 12 and a Principal of 100, or between an interest of 1 and a Principal of 16, etc., we call in general rate of interest. And it is to be noted that two kinds of rates of interest are used, one of which always has one of its terms certain, while the second kind has both terms uncertain. The rate of interest that has one term certain is of two kinds. For either the Principal is always a certain sum, to wit 100, and the interest an uncertain sum, *e.g.* 9 or 10 or 11, etc., and this rate of interest is then called nine per cent, 10 per cent, etc.; or on the contrary the interest is always a certain sum, to wit 1, and the Principal uncertain, *e.g.* 15 or 16 or 17, etc., and this rate of interest is called the fifteenth penny, the sixteenth penny, etc. The rate of interest that has both terms uncertain occurs when it is said, for example, that 53 yields 4 a year. Of all these cases several examples will be given below, in their proper places.

DEFINITION 4.

Simple interest is such as is charged on the Principal alone.

EXPLANATION.

For example, when 24 lb is charged for interest on 100 lb in 2 years at 12 per cent a year, the 24 lb is then called simple interest. Or when a man owes schuldich wesende 100. lb / te betaelen ten eynde van twee iaeren teghen 12. ten 100. t'siaers / ende betaelt ghereedt gheldt / aftreckende voor interest van de Hooft-somme alleene $21\frac{3}{7}$ lb / worden alsdan de zelue $21\frac{3}{7}$ lb simpelen interest ghenoemt / ende dat tot een differentie des ghecomponeerden interests / welcks definitie aldus is:

DEFINITIE 5.

Ghecomponeerden interest is die / welcke gerekent wordt vande Hooft-somme / midtsgaders van verloope der seluer.

VERCLAERINGHE.

Als rekenende $25\frac{11}{25}$ lb voor interest van 100. lb op twee iaeren teghen 12. ten 100 / worden de zelue $25\frac{11}{25}$ lb ghecomponeerden interest ghenoemt /ende dat om dieswille dat op het tweede iaer en wordt niet berekent alleenlick interest van de Hooft-somme 100. lb / maer bouen de zelue wordt noch interest gherekent van den interest van 12. lb verschenen op het ierste iaer bedraeghende $1\frac{11}{25}$ lb alsoo dat desen ghecomponeerden interest op twee iaeren meerder is dan haeren simpelen interest van $1\frac{11}{25}$ lb. Oft wesende iemandt schuldich te betaelen tê eynde van twee iaeren 100. lb / ende betaelt ghereedt ghelt $79\frac{141}{196}$ lb / aftreckende $20\frac{55}{196}$ lb / voor gecomponeerden interest teghen 12. tê 100. t'siaers / zoo dat desen gecomponeerden interest minder is dan den simpelen $3\frac{141}{196}$ lb. Waer deur te aenmerckê is dat wy die ghecomponeerden interest noemen / niet van weghen de quantiteyt waer wt zij beter gedisiungeerde interest zoude ghenoemt worden / maer van weghen de qualiteyt der operatien in de welcke wy op twee interesten opsicht hebben.

COROLLARIUM.

Daer wt volght noodtsaeckelick op alle ierste termijn daer interest op verschijnt / gheenen ghecomponeerden interest te connen gheschieden / int welcke haer sommighe gheabuseert te hebben zal int volghende t'zijnder plaetsen verclaert worden.

DEFINITIE 6.

Profijtelicken interest is die welcke d'Hooft somme toegedaen wort.

VERCLAERINGHE.

Ghelijckerwijs 16. lb ghewonnen hebbende op eê iaer 1 lb zal dê debiteur schuldich zijn met Hooft-somme ende interest t'saemen 17. lb / waer deur wy alzulck 1 lb (wantet interest is die d'Hooft-somme toeghedaen wordt ende die vermeerdert) noemen profijtelicken interest.

DEFINITIE 7.

Schadelickê interest is die / welcke van de Hooft-somme afghetrockê wordt.

VERCLAERINGHE.

Als eenen schuldich wesende binnen een iaer 16. lb veraccordeert te betaelê

100 lb, to be paid at the end of two years at 12 per cent a year, and he pays present value, subtracting for interest on the principal alone $21\frac{3}{7}$ lb, this $21\frac{3}{7}$ lb is then called simple interest, such in contrast with compound interest, the definition of which is as follows:

DEFINITION 5.

Compound interest is such as is charged on the Principal together with what is outstanding.

EXPLANATION.

For example, when $25\frac{11}{25}$ lb is charged for interest on 100 lb in two years at 12 per cent, this $25\frac{11}{25}$ lb is called compound interest, such because for the second year interest is not charged on the Principal of 100 lb alone, but over and above this interest is also charged on the interest of 12 lb that has expired after the first year, amounting to $1\frac{11}{25}$ lb, so that this compound interest is in two years more than the simple interest by $1\frac{11}{25}$ lb. Or when a man owes 100 lb to be paid at the end of two years, and he pays $79\frac{141}{196}$ lb present value, subtracting $20\frac{55}{196}$ lb for compound interest at 12 per cent a year, so that this compound interest is less than the simple interest by $3\frac{141}{196}$ lb. From this it is to be noted that we call it compound interest, not on account of the quantiy, for which it would be better to call it disjunct interest, but on account of the quality of the operations, in which we have two interests in view.

SEQUEL.

From this it follows necessarily that no compound interest can be charged for any first term on which interest is due, and it will be stated below in its place that some people have gone wrong in this.

DEFINITION 6.

Profitable interest is such as is added to the Principal.

EXPLANATION.

For example, when 16 lb has yielded in one year 1 lb, the debtor will owe 17 lb for the Principal and the interest together, on account of which we call this 1 lb (because it is interest that is added to the Principal and augments the latter) profitable interest.

DEFINITION 7.

Detrimental interest is such as is subtracted from the Principal.

EXPLANATION.

For example, when a man who owes 16 lb to be paid in a year agrees to pay ghereedt ghelt / midts aftreckende den interest tegen den penninck 16. bedragende $\frac{16}{17}$ lb / soo dat hy ghereedt gheeft 15$\frac{1}{17}$ lb. Alsoo dan want dese $\frac{16}{17}$ lb interest zijn die van de Hooft-somme afghetrocken worden ende die verminderen/noemen wy die schadelicken interest.

PROPOSITIE I.

Wesende verclaert Hooft-somme tijdt ende interests reden van simpelen eñ profijtelicken interest: Den interest te vinden.

NOTA.

Het is t'aenmercken dat ghelijck discontinua proportie bestaet onder 4. termijnen / welcker dry bekent zijnde wordt daer wt bekent het vierde: Alsoo ook bestaen dese onse interests propositien onder vier termijnê / te wetê Hooft-somme / tijdt / interests reden ende interest / welcker termijnen dry bekent zijnde / vinden wy deur de zelve het onbekende vierde: Dat is / wt bekende Hooft-somme / tijt / ende interests reden / vinden wy den interest: Item wt bekende Hooft-somme / tijdt / ende interest / vinden wy interests reden: Item wt bekende Hooft-somme / interests reden / ende interest / vinden wy tijdt: Ende ten laetsten wt bekende tijdt / interests reden / ende interest / vinden wy d'Hooft-somme. Alle welcke veranderinghen notoir zijn *ex alterna & inuersa proportione* der termijnen. Maer want het termijn des onbekendê interests (tot de welcke men oock dickmael d'Hooft-somme geaddeert begeert) in de practijcke meest ghesocht wordt / hebbê t'verclaers der propositien op de zelue ghemaeckt / hoe wel zullen dies niet te min onder de zelue propositien exempelen gheuen dependerende wt de voornoemde alteratie der termijnen.

EXEMPEL 1.

Men begheert te weten wat den simpelen interest zijn zal teghen 12. tê 100. t'siaers vâ 224. lb op een iaer.

CONSTRUCTIE.

Men zal wt de dry ghegheuen termijnen vinden 't vierde door de reghel der proportie / die disponerende aldus: 100. gheuen 12. wat 224. lb? facit 26$\frac{22}{25}$ lb.

Inder seluer voegen zalmê zegghê dat winnêde 16. lb t'siaers eê lb / so winnê 224. lb t'siaers 14. lb.

EXEMPEL 2.

27. lb gheuen op 4. iaer van simpelen interest 14. lb / wat gheuen 320. lb op 5. iaeren?

CONSTRUCTIE.

By aldien deze vijf termijnen int gheuen niet ghedisponeert en waeren als bouen / zoudemen die alsoo disponeren / ende zegghen / t'product der twee ierste termijnen gheeft tmiddel termijn / wat gheeft het product der twee laetste termijnen? Dat is 108. lb (wât zoo veel is t'product vande twee ierste termijnen te weten 27. met 4) gheuen 14. lb (dat is tmiddel termijn) wat gheuen 1600? (want sooveel is t'product vande twee laetste termijnê te weten 320 met 5.) Facit 207$\frac{11}{27}$ lb.

present value, subtracting the interest at the sixteenth penny, amounting to $\frac{16}{17}$ lb, so that he gives $15\frac{1}{17}$ lb present value. Thus, because this $\frac{16}{17}$ lb is interest that is subtracted from the Principal and diminishes the latter, we call it 'detrimental interest.

PROPOSITION I.

Given the Principal, the time, and the rate of simple and profitable interest: to find the interest.

NOTE.

It is to be noted that just as discontinuous proportion consists of 4 terms, of which, when three are known, the fourth becomes known therefrom, in the same way these our propositions on interest also consist of four terms, to wit Principal, time, rate of interest, and interest, and when three of these terms are known, we find the unknown fourth term therefrom. That is: from known Principal, time, and rate of interest we find the interest. In the same way, from known Principal, time, and interest we find the rate of interest. In the same way, from known Principal, rate of interest, and interest we find the time. And lastly, from known time, rate of interest, and interest we find the Principal. All these alterations depend on the terms in alternate or inverse proportions. But because the term of the unknown interest (to which the Principal is also frequently desired to be added) is sought most frequently in practice, we have based the explanation of the propositions on this, although we shall nevertheless give examples depending on the aforesaid alteration of the terms at the end of these propositions.

EXAMPLE 1.

It is required to know what will be the simple interest of 224 lb in one year at 12 per cent a year.

PROCEDURE.

From the three given terms the fourth has to be found by the rule of proportion, putting it as follows: 100 gives 12, what does 224 lb give? This is $26\frac{22}{25}$ lb.

In the same way it has to be said that when 16 lb yields one lb a year, 224 lb will yield 14 lb a year.

EXAMPLE 2.

27 lb gives 14 lb of simple interest in 4 years; what does 320 lb give in 5 years?

PROCEDURE.

Since these five given terms have not been arranged in the previous way, they have to be arranged in the following way: the product of the two first terms gives the middle term; what does the product of the two last terms give? That is: 108 lb (for that is the product of the two first terms, to wit 27 and 4) gives 14 lb (that is the middle term); what does 1,600 give (for that is the product of the two last terms, to wit 320 and 5)? This is $207\frac{11}{27}$ lb.

NOTA.

Dit voorgaende tweede exempel / met allen anderen dier ghelijcken (welck van weghen de 5. termijnen reghel van vijven ghenoemt worden) moghen ghesolueert worden door eene operatie in de welcke men ghebruyckt tweemael den reghel der proportien / maar dese maniere is corter ende bequaemer.

EXEMPEL 3.

Eenen is schuldich contant 224. lb. Oft hy betaelde binnen 4. iaeren alle iaere het vierendeel / te weten 56. De vraeghe is hoe vele hy ieder iaer betaelen zoude van simpelen interest teghen 12. ten 100. t'siaers.

CONSTRUCTIE.

Men zal aenmercken wat Hooft-somme datmen op elck iaer in handê houdt diemê naer d'ierste conditie in handen niet en zoude ghehouden hebben / ende vinden alsdan door t'voornoemde ierste exempel dê interest van elcke Hooft-somme op elck iaer. Als tê eynde vât ierste iaer is d'Hooft-somme 224. lb. / diês interest bedraecht voor eê iaer 26 $\frac{22}{25}$ lb. Ten eynde van het tweede iaer (wât opt ierste iaer een vierendeel van 244. lb. betaelt wordt) en zal d'Hooft-somme maer zijn 168. lb wiens interest voor een iaer 20 $\frac{4}{25}$ lb. Ten eynde vâ het derde iaer is d'hooft-somme 112. lb / wiens interest voor een iaer 13 $\frac{11}{25}$ lb. Tê eynde vâ het vierde iaer is d'Hoft-somme 56. lb / wiens interest 6 $\frac{18}{25}$ lb.

EXEMPEL 4.

Eenen is schuldich binnen vier iaeren 224. lb / te weten alle iare het vierendeel bedragêde 56. lb. De vraeghe is hoe vele hy zoude moeten betaelen van simpelen interest teghen 12. ten 100. t'siaers / zoo hy de voors. somme teenemael betaelde tê eynde van de vier iaeren.

CONSTRUCTIE.

Men zal aenmercken wat Hooft-sommê datmen op elck iaer in handen houdt / diemen naer d'ierste conditie in handen niet en zoude ghehouden hebben / ende rekenen daer af den interest. Alsoo dan want men ten eynde vâ het ierste iaer zoude hebben moeten betaelê naer die conditie 56. lb / diemen naer dese conditie niet ghegheuen en heeft / zoudemen ten eynde van het tweede iaer moeten rekenen den interest van de zelue 56. lb. bedraegende 6 $\frac{18}{25}$ lb. Ende om dier gelijcke redenen zoudemen moeten rekenen ten eynde vâ het derde iaer interest van 112. lb bedraeghende 13 $\frac{11}{25}$ lb. Ende ten eynde van het vierde iaer interest van 168. lb bedraeghende 20 $\frac{4}{25}$ lb / welcke dry sommê van interest bedraeghende t'saemen 40 $\frac{8}{25}$ lb is den simpelen interest diemen ten eynde van de vier iaeren zoude moeten betaelen.

Ofte andersins mochtmen soecken proportionale ghetaelen met de ghene daer questie af is ende onbekent zijn / aldus:

NOTE.

The foregoing second example, and all similar ones (which on account of the 5 terms are called the rule of five), can be solved by an operation in which the rule of proportion is used twice, but this method is shorter and more convenient.

EXAMPLE 3.

A man owes 224 lb present value. If he paid in 4 years, every year one fourth, to wit 56 lb, how much simple interest would he pay every year at 12 per cent a year?

PROCEDURE.

It has to be found what Principal one keeps each year which according to the first condition one would not have kept, upon which by the aforesaid first example the interest on each Principal in each year has to be found. For example, at the end of the first year the Principal is 224 lb, the interest on which in one year is $26\frac{22}{25}$ lb. At the end of the second year (because in the first year one fourth of 224 lb is paid) the Principal will be only 168 lb, the interest on which in one year is $20\frac{4}{25}$ lb. At the end of the third year the principal is 112 lb, the interest on which in one year is $13\frac{11}{25}$ lb. At the end of the fourth year the Principal is 56 lb, the interest on which is $6\frac{18}{25}$ lb.

EXAMPLE 4.

A man owes 224 lb to be paid in four years, to wit every year one fourth, amounting to 56 lb. How much simple interest would he have to pay at 12 per cent a year if he paid the aforesaid sum at once at the end of the four years?

PROCEDURE.

It has to be found what Principal one keeps each year which according to the first condition one would not have kept, and on this the interest has to be charged. Thus, because at the end of the first year one would have had to pay 56 lb according to that condition, which according to this condition one has not given, at the end of the second year the interest on that 56 lb would have to be charged, amounting to $6\frac{18}{25}$ lb. And for the same reasons at the end of the third year interest on 112 lb would have to be charged, amounting to $13\frac{11}{25}$ lb. And at the end of the fourth year interest on 168 lb, amounting to $20\frac{4}{25}$ lb. These three sums of interest, amounting together to $40\frac{8}{25}$ lb, are the simple interest that would have to be paid at the end of the four years.

Or otherwise one might seek numbers proportional to those which are under consideration and unknown, as follows:

100	gheuen	op	het	ierste	iaer	0.
100	gheuen	op	het	tweede	iaer	12.
100	gheuen	op	het	derde	iaer	24.
100	gheuen	op	het	vierde	iaer	36.
Somme 400.						72.

Ende segghen daer naer 400. gheuen 72, wat gheuen 224. lb? Facit als voren $40\frac{8}{25}$ lb.

NOTA.

De dry volghende exempelen dependeren *ex alterna vel inuersa proportione propositionis.*

EXEMPEL 5.

48. lb gheuen op 3. iaere van simpelen profijtelicken interest 9. lb. De vraeghe is teghen hoe veel ten 100. t'siaers dat betaelt is.

CONSTRUCTIE.

Laet de termijnen ghedisponeert worden als int voorgaende exempel gheseyt is aldus:
48. gheuen op 3. iaer 9. lb / wat gheuen 100. lb. op iaer? Facit (nae de leeringhe des voorgaenden 2. exempels) $6\frac{1}{4}$ ten 100.

EXEMPEL 6.

Men begheert te weten hoe langhe 260. lb loopen zullen teghen 12. ten 100. t'siaers om te winnen 187. lb. 4. ß.

CONSTRUCTIE.

Men sal ziê wat 260. lb. t'siaers winnê / wordt bevonden door d'ierste exempel $31\frac{1}{5}$ lb; daer naer salmen diuideren 187. lb 4. ß door $31\frac{1}{5}$ lb / gheeft quotum ende solutie 6. iaeren.

EXEMPEL 7.

Eeenen ontfanght 187. lb 4. ß. voor simpelen interest teghen 12. ten 100. voor 6. iaerê. De vraeghe is wat d'Hooft-somme was.

CONSTRUCTIE.

Men sal zien wat 100. lb teghen 12. ten hondert winnen op 6. iaer / wordt bevondê 72. lb. daer naer salmen segghen 72. comen van 100 / waer van zullen comen 187. lb. 4. ß? Facit voor solutie 260. lb.

DEMONSTRATIE.

Ghelijck int ierste exempel hem heeft 100. tot 12 / alsoo heeft hem 224. lb. tot $26\frac{22}{25}$ lb deur de constructie. Ergo $26\frac{22}{25}$ lb zijn met die ander termijnen proportionaal naer de begheerte.

	100 gives in the first year	0
	100 gives in the second year	12
	100 gives in the third year	24
	100 gives in the fourth year	36
Sum total	400	72

and say thereafter: 400 gives 72; what does 224 lb give? This is, as above, $40\frac{8}{25}$ lb.

NOTE.

The three following examples depend on the propositions of the alternate or inverse proportion [1]).

EXAMPLE 5.

48 lb gives in 3 years 9 lb of simple profitable interest. How many per cent a year does this payment amount to?

PROCEDURE.

Let the terms be disposed as has been said in the foregoing second example, as follows:
48 gives in 3 years 9 lb; what does 100 lb give in a year? This is (according to the foregoing 2nd example) $6\frac{1}{4}$ per cent.

EXAMPLE 6.

It is required to know how long 260 lb has to be put out at interest at 12 per cent a year to yield 187 lb 4 sh. [2])

PROCEDURE.

Find what 260 lb yields in a year. By the first example this is found to be $31\frac{1}{5}$ lb. Thereafter divide 187 lb 4 sh. by $31\frac{1}{5}$ lb. This gives the quotient and solution: 6 years.

EXAMPLE 7.

A man receives 186 lb 4 sh. of simple interest at 12 per cent in 6 years. What was the Principal?

PROCEDURE.

Find what 100 lb yields at 12 per cent in 6 years. This is found to be 72 lb. Thereafter say: 72 comes from 100; what will 187 lb 4 sh. come from? The solution is: 260 lb.

PROOF.

As in the first example 100 is to 12, thus 224 lb is to $26\frac{22}{25}$ lb by the procedure. Therefore $26\frac{22}{25}$ lb is proportional to those other terms, as required.

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1) If $a:b = c:d$, then $b : a = d : c$ is the inverse, $a : c = b : d$ the alternate proportion.
2) 1 pound (lb) = 20 shillings (sh).

S'gelijcks sal oock zijn de demôstratie vâ de andere exempelen / welcke om de cortheydt wy achterlaeten.

Alsoo dan wesende verclaert Hooft-somme tijt ende interest reden van simpelen ende profijtelicken interest is den interest ghevonden. T'welck geproponeert was alsoo ghedaen te worden.

PROPOSITIE II.

Wesende verclaert Hooft-somme tijdt ende interests reden van simpelen ende schadelicken interest: Te vinden wat die gheereet ghelt weerdich is.

EXEMPEL 1.

Het zijn 300. lb. te betaelen binnen een iaer. De vraeghe is wat die gereedt ghelt weerdich zijn aftreckende simpelen interest teghen 12. ten hondert t'siaers.

CONSTRUCTIE.

Men sal adderen tot 100. zijnen interest 12. maecken t'saemen 112, ende segghen:

112. worden 100 / wat 300. lb? Facit $267\frac{6}{7}$ lb.

EXEMPEL 2.

Het zijn 32. lb te betaelen binnen dry iaeren, De vraeghe is wat die ghereedt weerdich zijn aftreckende den interest teghen den penninck 16.

CONSTRUCTIE.

Men sal adderen tot 16. zijnen interest van dry iaeren / te weten 3. maecken t'saemen 19. ende segghen

19. comen van 16. waer af 32. lb? Facit $26\frac{18}{19}$ lb.

EXEMPEL 3.

Het zijn 250. lb. te betaelen binnen 6. maenden. De vraeghe is wat die weerdich zijn ghereedt ghelt aftreckende teghen den penninck 16. t'siaers.

NOTA.

De solutie van dese ende derghelijcke questien (welck ick ook gheappliceert hebbe totten ghecomponeerden interest daer t'zijnder plaetsen af zal geseyt worden) want ick die ghevonden hebbe ende by niemandt anders en vinde / achte die nu ierstmael wtghegaen te zijne.

CONSTRUCTIE.

Men sal zien wat deel de 6. maenden zijn van een iaer / wordt bevonden $\frac{1}{2}$ daerom salmen adderen 16. met $\frac{1}{2}$ eñ zegghen; $16\frac{1}{2}$ worden 16. wat 250. lb? Facit $242\frac{14}{33}$ lb.

Item hadden de voor noemde 250. lb te betaelen gheweest binnen 3. maenden / zoo zoudemen zegghen (want 3. maendê een vierendeel iaers is) $16\frac{1}{4}$ worden 16. wat 250 lb? Facit $246\frac{2}{13}$ lb.

The same will also be the demonstration of the other examples, which we omit for brevity's sake.

Hence, given the Principal, the time, and the rate of simple and profitable interest, the interest has been found; which had been proposed to be done.

PROPOSITION II.

Given the Principal, the time, and the rate of simple and detrimental interest: to find what is the present value.

EXAMPLE 1.

A sum of 300 lb is to be paid in a year. What is the present value of this sum, subtracting simple interest at 12 per cent a year?

PROCEDURE.

Add to 100 its interest of 12, which makes together 112, and say:

112 becomes 100; what does 300 lb become? This is $267\frac{6}{7}$ lb.

EXAMPLE 2.

A sum of 32 lb is to be paid in three years. What is the present value, subtracting the interest at the 16th penny?

PROCEDURE.

Add to 16 its interest of three years, to wit 3, which makes together 19, and say:

19 comes from 16, what does 32 lb come from? This is $26\frac{18}{19}$ lb.

EXAMPLE 3.

A sum of 250 lb is to be paid in 6 months. What is the present value, subtracting at the 16th penny a year?

NOTE.

The solution of this and similar questions (which I have also applied to compound interest, which will be discussed in its proper place), because I have found it and find it in no one else's work, I deem now to have been published for the first time.

PROCEDURE.

It has to be found what part 6 months is of one year. This is found to be $\frac{1}{2}$. Therefore add up 16 and $\frac{1}{2}$, and say: $16\frac{1}{2}$ becomes 16; what does 250 lb become? This is $242\frac{14}{33}$ lb.

Similarly, if the aforesaid 250 lb had had to be paid in 3 months, it would have to be said (because 3 months is one fourth of a year): $16\frac{1}{4}$ becomes 16; what does 250 lb become? This is $246\frac{2}{13}$ lb.

Ofte hadden de voor noemde 250. lb. te betaelen gheweest op 1. maendt / zoo zoudemen zegghen (want 1. mandt is 1 $\frac{1}{12}$ t'siaers) 16 $\frac{1}{12}$ worden 16 / wat 250. lb?

Ofte hadden de voor noemde 250. lb. te betaelen gheweest op 7. weken / zoo zoudemen zegghen (want 7. weken is $\frac{7}{52}$ t'siaers) 16 $\frac{7}{52}$ worden 16. wat 250. lb?

Ofte hadden de voornoemde 250. lb. te betaelen gheweest op 134 daghen / zoo zoudemen zegghen (want 134. daghen zijn $\frac{134}{365}$ t'siaers) 16 $\frac{134}{365}$ worden 16. wat 250. lb?

Alsoo dat men in zulcke questien altijt moet zien wat deel den gheproponeerdê tijdt is van het iaer ende voort als bouen.

EXEMPEL 4.

Het zijn 320. lb. te betaelen binnen 3. iaeren eñ 3. maenden. De vraeghe is wat die weerdich zijn ghereedt ghelt aftreckende teghen dê penninck 16. t'siaers simpelen interest.

CONSTRUCTIE.

Men sal tot 16. adderen zijnen interest van 3 $\frac{1}{4}$ lb. (3 $\frac{1}{4}$ lb. van weghen 3 $\frac{1}{4}$ iaeren) maeckê t'saemen 19 $\frac{1}{4}$ / ende segghen / 19 $\frac{1}{4}$ comen van 16. waer af 320. lb? facit 265 $\frac{75}{77}$ lb.

NOTA.

S'ghelijcks zal oock zijn d'operatie in alle andere deelen des iaers bouen eenighe gheheele iaeren / als lichtelick te mercken is wt t'voorgaende exempel.

EXEMPEL 5.

Het zyn 230. lb. te betaelen ten eynde van 5. iaeren. De vraeghe is wat die ghereedt weerdich zijn aftreckende in zulcken reden als hen heeft 23. Hooftsomme tot simpelen interest 6. eñ dat van 3. iaeren.

CONSTRUCTIE.

Men sal ten iersten sien wat 6. lb. interest van 3. iaeren bedraeghen op 1. iaer / ende wordt bevonden 2. lb. Alsoo dan desen interest is van 2. ten 23. t'siaers / waer deur de werckinghe ghelijck zal zijn de voorgaende des 2. exempels deser propositien aldus: Men sal adderen tot 23. sijnen interest van 5. iaeren / te weten 10. lb. maecken t'saemen 33. lb. ende segghen 33. worden 23. wat 230. lb? Facit voor solutie 160 $\frac{10}{33}$ lb.

EXEMPEL 6.

Eenen is schuldich 600. lb. te betaelen al t'saemen ten eynde van vier iaeren / ende veraccordeert met zijn crediteur die te betaelen in 4. payementen / te weten ten eynde van het ierste iaer een vierê deel / het tweede iaer noch een vierendeel / het derde iaer noch een vierendeel / ende t'vierde iaer t'laetste vierendeel / midts aftreckende simpelen interest teghen 12. ten 100. t'siaers.

If the aforesaid 250 lb had had to be paid in 1 month, it would have to be said (because 1 month is one twelfth of a year): $16\frac{1}{12}$ becomes 16; what does 250 lb become?

If the aforesaid 250 lb had had to be paid in 7 weeks, it would have to be said (because 7 weeks is $\frac{7}{52}$ of a year); $16\frac{7}{52}$ becomes 16; what does 250 lb become?

If the aforesaid 250 lb had had to be paid in 134 days, it would have to be said (because 134 days is $\frac{134}{365}$ of a year): $16\frac{134}{365}$ becomes 16; what does 250 lb become?

So that in such questions it has always to be found what part of a year is the proposed time, and further as above.

EXAMPLE 4.

A sum of 320 lb is to be paid in 3 years and 3 months. What is the present value, substracting simple interest at the 16th penny a year?

PROCEDURE.

Add to 16 its interest of $3\frac{1}{4}$ lb ($3\frac{1}{4}$ lb on account of $3\frac{1}{4}$ years), which makes together $19\frac{1}{4}$, and say: $19\frac{1}{4}$ comes from 16; what does 320 lb come from? This is $265\frac{75}{77}$ lb.

NOTE.

The same will also be the operation for all other parts of a year over and above the whole years, as is easily perceived from the foregoing example.

EXAMPLE 5.

A sum of 230 lb is to be paid at the end of 5 years. What is the present value of this sum, subtracting in the ratio of 23 (Principal) to 6 (simple interest), such for 3 years?

PROCEDURE.

First it has to be found what 6 lb of interest for 3 years amounts to in 1 year; this is found to be 2 lb. This interest is therefore 2 per 23 a year, so that the operation will be similar to the foregoing one of the 2nd example of the present proposition, as follows: Add to 23 its interest for 5 years, to wit 10 lb, which makes together 33 lb, and say: 33 becomes 23, what does 230 lb become? The solution is $160\frac{10}{33}$ lb.

EXAMPLE 6.

A man owes 600 lb, the whole to be paid at the end of four years, and he agrees with his creditor to pay them in 4 payments, to wit at the end of the first year one fourth, the second year again one fourth, the third year again one fourth, and the fourth year the last one fourth, subtracting simple interest at 12 per cent a year.

NOTA.

Ick hebbe in dit exempel ghenomen de zelfde somme ende questie die Jan Trenchant heeft int 3. boeck zijnder Arith. cap. 9. art. 6. op dat ick te claerder zoude toonen de differentie ouer zulcken questie van zijne solutie ende de mijne. Is dan te weten dat Trenchandt ondersoeckt wat dese 600. lb. ghereedt weerdich zijn / wordt bevondê 508 $\frac{28}{59}$ lb. welcks vierendeel als 127 $\frac{7}{59}$ lb. Hy zeght te wesen dat men op elck der vier iaeren zoude moeten betaelen.

Maer ick zegghe ter contrarien gheen questie te wesen van vier betaelinghen van het ghene de 600. lb. ghereedt weerdich zijn / maer van vier betaelinghen der 600. lb. zeluer. Dit is soo veel als oft den debiteur totten crediteur zeyde: De 600. lb. die ick v schuldich ben teenemael ten eynde vâ vier iaeren / de zelfde zal ick v betaelen in vier payementen / te weten alle iaere het vierendeel der zeluer / als 150. lb. midts aftreckende op elcke betaelinge simpelen interest teghen 12. ten 100. t'siaers. Twelck wesende den sin deser questien volght daer wt een constructie als volght.

CONSTRUCTIE.

Men zal aenmercken wat penningen dat men naer dese conditie verschiet diemê naer d'ierste conditie niet en soude verschotê hebbê. Nu dan wantmen naer dese conditie binnê eê iaer betaelt t'vierêdeel der sommen bedraeghende 150. lb. midts aftreckende / etc. diemen naer d'ierste conditie binnen 3. iaeren daer naer ierst zoude moeten betaelen / volgt daer wt dat men zien zal wat 105. lb. te betaelen in 3. iaeren weerdich zijn ghereet / wordt bevonden door het 2. exempel deser propositien 110 $\frac{5}{17}$ lb. voor d'ierste paye. Ende om der ghelijcke redenen salmen bevinden 150. lb. op 2. iaerê weerdich te zijne ghereet 120 $\frac{30}{31}$ lb. voor de tweede paye.

Ende om der ghelijcke redenen zalmen bevindê 150. lb. op 1. iaer weerdich te zijne 133 $\frac{13}{14}$ lb. voor de derde paye.

Ende want de laetste paye op zulcken conditie betaelt wordt als d'ierste conditie was / en zal die winnen noch verliesen / maer zal zijn van 150. lb.

EXEMPEL 7.

Het zijn 324. lb. te betaelen binnen 6. iaeren / te wetê 54. lb. t'siaers. Vraeghe is wat de zelue weerdich zijn ghereedt ghelt / aftreckende simpelen interest teghen 12. ten 100.

NOTE [1]).

In this example I have taken the same sum and question that Jan Trenchant has in the 3rd book of his Arithmetic, chapter 9, section 6, in order that I might show all the more clearly the difference concerning this question between his solution and mine. It is to be noted that Trenchant finds what is the present value of this 600 lb. This is found to be $508\frac{28}{59}$ lb, the fourth part of which, *viz.* $127\frac{7}{59}$ lb, he says is the amount that would have to be paid in each of the four years.

But I say on the contrary that there is no question of four payments of the present value of the 600 lb, but of four payments of the 600 lb itself. This is as much as if the debtor said to the creditor: I will pay to you the whole of the 600 lb I owe you at the end of four years in four payments, to wit every year one fourth of it, *i.e.* 150 lb, subtracting from each payment simple interest at 12 per cent a year. This being the meaning of this question, the following procedure follows therefrom.

PROCEDURE.

It has to be found what money one disburses on this condition that one would not have disbursed on the first condition. Thus because on this condition in a year one fourth of the sum is paid, amounting to 150 lb, subtracting etc., which on the first condition would not have to be paid until 3 years thereafter, it follows that it has to be found what is the present value of 105 lb to be paid in 3 years. This is found by the 2nd example of the present proposition to be $110\frac{5}{17}$ lb for the first payment. And for the same reasons the present value of 150 lb to be paid in 2 years will be found to be $120\frac{30}{31}$ lb, for the second payment.

And for the same reasons the present value of 150 lb to be paid in 1 year will be found to be $133\frac{13}{14}$ lb, for the third payment.

And because the last payment is made on the same condition as the first, this will neither gain nor lose, but will be 150 lb.

EXAMPLE 7.

A sum of 324 lb is to be paid in 6 years, to wit 54 lb a year. What is the present value of this sum, subtracting simple interest at 12 per cent?

[1]) See the Introduction, p. 15

CONSTRUCTIE.

Men zal soecken proportionale ghetalen met de ghene daer questie af is aldus.

100 comen voor 1 iaer van	112.
100 comen voor 2 iaeren van	124.
100 comen voor 3 iaeren van	136.
100 comen voor 4 iaeren van	148.
100 comen voor 5 iaeren van	160.
100 comen voor 6 iaeren van	172.
Sôme 600.	Somme 852.

Daer naer segt men 852. zijn ghereedt weerdich 600. wat zullen ghereedt weerdich zijn 324. lb? Facit 228 $\frac{12}{71}$ lb. ende soo veel is de voornoemde somme ghereedt weerdich.

EXEMPEL 8.

Eenen is schuldich te betaelen binnen 3.iaeren 260. lb. en binnê 6. iaeren daer naer noch 420. lb. De vraeghe is wat die t'saemen ghereedt weerdich zijn. Aftreckende simpelen interest teghen 12. ten 100.t'siaers.

CONSTRUCTIE.

De 260. lb. zullen ghereedt weerdich zijn naer het tweede exêpel deser prop. 191 $\frac{3}{17}$ lb. en de 420. lb. zullen ghereedt weerdich zijn 201 $\frac{12}{31}$ lb. nu dan gheaddeert 191 $\frac{3}{17}$ lb. met 201 $\frac{12}{13}$ lb. maecken t'saemen 393 $\frac{22}{221}$; ende soo veel is alle de schuldt ghereedt weerdich.

EXEMPEL 9.

Eenen is schuldich 200. lb. te betaelen binnen 5. iaeren. De vraeghe is wat die weerdich zijn binnen 2. iaerê rekenende simpelen interest teghen 10. ten 100. t'siaers.

CONSTRUCTIE.

Men zal zien wat de 200. lb. weerdich zijn gereedt door het 2. exempel deser prop. wordt bevondê 133 $\frac{1}{3}$ lb. Daer naer salmen sien wat 133 $\frac{1}{3}$ lb. gereedt weerdich zijn binnen 2. iaeren naer de leeringhe der ierster prop. wordt bevonden 160. lb. ende zoo veel zijn die 200. lb. weerdich binnen 2. iaeren.

ANDERE MANIERE.

Ofte andersins ende lichter machmê doen aldus: men sal zien wat 100. lb. weerdich zijn op 5. iaeren / wordt bevonden 150. lb. Insghelijcks wat 100. weert zijn op 2. iaeren wordt bevonden 120. Daer nae zalmen segghen 150. gheuen 120. wat 200 lb? Facit als voren 160. lb.

PROCEDURE[1]).

Find numbers proportional to those under consideration, as follows.

100 becomes in 1 year	112
100 becomes in 2 years	124
100 becomes in 3 years	136
100 becomes in 4 years	148
100 becomes in 5 years	160
100 becomes in 6 years	172
Total 600	Total 852

Thereafter say: the present value of 852 is 600; what will be the present value of 324 lb? This is $228\frac{12}{71}$ lb, and this is the present value of the aforesaid sum.

EXAMPLE 8.

A man owes 260 lb, to be paid in 3 years, and 6 years later 420 lb more. What is the present value of these two sums together, subtracting simple interest at 12 per cent a year?

PROCEDURE.

The present value of the 260 lb, according to the second example of the present proposition, will be $191\frac{3}{17}$ lb, and the present value of the 420 lb will be $201\frac{12}{13}$ lb. Now when $191\frac{3}{17}$ lb and $201\frac{12}{13}$ lb are added together, this makes $393\frac{22}{221}$, and this is the present value of the whole debt.

EXAMPLE 9.

A man owes 200 lb, to be paid in 5 years. What is their value in 2 years, charging simple interest at 10 per cent a year?

PROCEDURE[2]).

Find what is the present value of the 200 lb; by the 2nd example of the present proposition, this is found to be $133\frac{1}{3}$ lb. Thereafter find what the present value of $133\frac{1}{3}$ lb will be worth in 2 years; according to the first proposition this is found to be 160 lb, and this is the value of that 200 lb in 2 years.

OTHER METHOD.

Or in another and easier way one can proceed as follows: Find what 100 lb is

[1]) This solution with 6 payments of 112, 124, 136, 148, 160, 172 does not produce an annuity of constant value. Stevin found this out and gave a correct solution in the French edition of 1585 and the second Dutch edition of 1590 (See Supplement, p. 113).

[2]) This solution was changed into another one in the editions of 1585 and 1590. (See Supplement, p. 115).

EXEMPEL 10.

Eenen is schuldich te betaelen binnen 3. iaeren 420. lb. eñ binnen 6. iaere daer naer noch 560. lb. De vraeghe is wat dese partijen weert zijn te betaelen t'saemen op 2. iaeren rekenende simpelê interest teghen 10. ten 100. t'siaers.

CONSTRUCTIE.

Men sal zien wat deze partijen t'saemen weerdich zijn ghereedt door het 8. exempel deser prop. wordt bevonden 617 $\frac{201}{247}$ lb. Daer naer salmen sien wat de zelue ghereedt / weert zijn binnen 2. iaeren / wordt bevonden door d'ierste propositie voor solutie 741 $\frac{93}{247}$ lb.

ANDERE MANIERE.

Ofte andersins machmen zien wat 420. lb. op 3. iaeren weerdich zijn op 2. iaeren / wordt bevonden door het 9. exempel deser propositien 387 $\frac{9}{13}$ lb.

Ende inder seluer voeghen worden de 560. lb. op twee iaeren weerdich bevonden 353 $\frac{13}{19}$ lb. welcke twee sommen als 387 $\frac{9}{13}$ met 353 $\frac{13}{19}$ maecken t'saemen voor solutie als voren 741 $\frac{93}{247}$ lb.

NOTA.

De volghende exempelen dependeren *ex alterna vel inversa proportione* der propositien

EXEMPEL 11.

Voor 500. lb. te betaelen ten eynde van 5. iaeren ontfangtmen ghereedt 333 $\frac{1}{3}$ lb. De vraeghe is teghen hoe vele ten 100. simpelen interest dat afghetrocken is.

CONSTRUCTIE.

Men sal segghen 333 $\frac{1}{2}$ lb. comen van 500. lb. waer van 100? Facit 150. van de zelue zalmê trecken 100. rest 50. welcke ghediuideert door 5. iaeren gheeft quotum 10. *Ergo* teghen 10. ten 100. wasser afghetrocken.

worth in 5 years; this is found to be 150 lb. In the same way 100 lb is found to be worth 120 in 2 years. Thereafter say: 150 gives 120; what does 200 lb give? This, as above, is 160 lb.

EXAMPLE 10.

A man owes 420 lb to be paid in 3 years, and 6 years later 560 lb more. What will these sums be worth, if paid together after 2 years, charging simple interest at 10 per cent a year?

PROCEDURE [1]).

Find what is the present value of these sums together; by the 8th example of this proposition this is found to be $617\frac{201}{247}$ lb. Thereafter find what the present value of these sums will be worth in 2 years. The solution, by the first proposition, is found to be $741\frac{93}{247}$ lb.

OTHER METHOD.

Or in another way one can see what 420 lb to be paid in 3 years will be worth in 2 years; by the 9th example of the present proposition, this is found to be $387\frac{9}{13}$ lb.

And in the same way the 560 lb is found to be worth $353\frac{13}{19}$ lb in two years, and these two sums, *viz.* $387\frac{9}{13}$ and $353\frac{13}{19}$, make together for the solution, as above, $741\frac{93}{247}$ lb.

NOTE.

The following examples depend on the propositions of the alternate or inverse proportion.

EXAMPLE 11.

For 500 lb to be paid at the end of 5 years, the present value of $333\frac{1}{3}$ lb is received. How many per cent of simple interest has been subtracted?

PROCEDURE.

This has to be said as follows: $333\frac{1}{3}$ lb comes from 500 lb; what does 100 come from? This is 150. From this, subtract 100. The remainder is 50. When this is divided by 5 years, this gives the quotient 10. Therefore the interest subtracted had been charged at 10 per cent.

[1]) This solution was changed into another one in the editions of 1585 and 1590 (See Supplement, p. 116). The examples 9 and 10, as shown in the Introduction (p. 20), happen to have more than one form of solution. Both the solutions of 1582 and 1590 could be accepted at present.

EXEMPEL 12.

Voor 400. lb. ontfangtmen ghereedt 250. lb. aftreckende simpelen interest teghen 10. ten 100. t'siaers. De vraeghe is voor hoe langhe tijdt afgetrocken is.

CONSTRUCTIE.

Men sal segghen 250. lb. comen van 400. lb. waer van 100? Facit 160. lb. van de zelue zalmen trecken 100. rest 60. welck ghediuideert door 10. (10. van weghen 10. ten 100.) gheeft quotum 6. Ergo voor 6. iaeren wasser afghetrocken.

EXEMPEL 13.

Eenen is schuldich binnen 3. iaeren 420. lb. ende binnen 6. iaeren daer naer noch 560. lb. De vraeghe is wat tijdt dese partijen t'saemen verschijnen zullen / rekenende den simpelen interest teghen 10. ten 100. t'siaers.

CONSTRUCTIE.

Men sal zien wat dese twee sommen t'saemen ghereedt weerdich zijn / wordt bevonden door het 8. exempel deser prop: 617 $\frac{201}{247}$ lb. Daer naer salmen sien door het 6. exempel der ierster prop. Hoe langhe 617 $\frac{201}{247}$ lb. loopen zullen tegê 10. tê 100. t'siaers tot zy weerdich zijn 980. lb. (welck de somme zijn van 420. lb. ende 5601 lb.) ofte (dat tzelfde is) tot zij ghewonnen hebben 362 $\frac{46}{247}$ lb. Facit voor solutie 6 $\frac{1059}{1066}$ iaeren.

EXEMPEL 14.

Eenen ontfangt 666 $\frac{2}{3}$ lb. ende hem hadde afgetrocken gheweest simpelen interest teghen 8. ten 100. t'siaers voor 10. iaeren. De vraeghe is wat d'Hooftsomme was.

CONSTRUCTIE.

Men zal adderen tot 100. zijnen interest van 10. iaeren comt t'saemen 180. segghende 100. comen van 180. waer van 666 $\frac{2}{3}$ lb? Facit d'Hooft-somme 1200. lb.

DEMONSTRATIE.

Aenghesien int ierste exempel deser propositien gheseyt is 300. lb. te betaelen op een iaer / weerdich te zijne ghereedt ghelt 267 $\frac{6}{7}$ lb. Aftreckende simpelen interest teghen 12. ten 100. t'siaers / volght daer wt dat in dien men die 267 $\frac{6}{7}$ lb terstont op interest leyde te weten alsvoren teghen simpelen interest van 12. ten hondert t'siaers / dat de zelue Hooft-somme met haeren interest (zoo d'operatie goedt is) sullen moeten t'saemen bedraeghen ten eynde van den iaere 300. lb. Alsoo dan die rekenende naer de leeringhe des iersten exempels der ierster propositien sal bedraeghen 32 $\frac{1}{7}$ lb. welck geaddeert tot de 267 $\frac{6}{7}$ lb. maecken t'saemen de voornoemde 300. lb. waer wt besloten wordt de constructie goedt te zijne. Sghelijcks zal oock zijn de demonstratie van d'ander exempelen deser propositien / welck wij om de cortheydt achter laeten. Alsoo dan wesende verclaert Hooft-

EXAMPLE 12.

For 400 lb the present value of 250 lb is received, subtracting simple interest at 10 per cent a year. For what time has interest been subtracted?

PROCEDURE.

This has to be said as follows: 250 lb comes from 400 lb; what does 100 come from? This is 160 lb. From this, subtract 100. The remainder is 60. When this is divided by 10 (10 on account of 10 per cent), the quotient is 6. Therefore interest for 6 years has been subtracted.

EXAMPLE 13.

A man owes 420 lb to be paid in 3 years, and 6 years later 560 lb more. At what time will these sums together appear, charging simple interest at 10 per cent a year?

PROCEDURE.

Find what is the present value of these two sums together. By the 8th example of the present proposition this is found to be $617\frac{201}{247}$ lb. Thereafter find by the 6th example of the first proposition how long $617\frac{201}{247}$ lb has to be put out at interest at 10 per cent a year until its value is 980 lb (which is the sum of 420 lb and 560 lb) or (which is the same) until it has yielded $362\frac{46}{247}$ lb. The solution is $6\frac{1059}{1066}$ years.

EXAMPLE 14.

A man receives $666\frac{2}{3}$ lb, and the simple interest at 8 per cent a year had been subtracted for 10 years. What was the Principal?

PROCEDURE.

Add to 100 its interest of 10 years, which makes together 180, and say: 100 comes from 180; what does $666\frac{2}{3}$ lb come from? The Principal is 1,200 lb.

PROOF.

Since it has been said in the first example of the present proposition that the present value of 300 lb, to be paid in one year, is $267\frac{6}{7}$ lb, subtracting simple interest at 12 per cent a year, it follows that if this $267\frac{6}{7}$ lb were at once put out at interest, to wit as above at simple interest of 12 per cent a year, the said Principal with its interest (if the operation is correct) will have to amount together, at the end of the year, to 300 lb. This then being charged according to the first example of the first proposition, it will amount to $32\frac{1}{7}$ lb, and, when this is added to the $267\frac{6}{7}$ lb, it makes together the aforesaid 300 lb, from which it is concluded that the procedure is correct. The same will also be the demonstration of the other examples of the present proposition, which we omit for somme tijdt ende interests reden van simpelen ende schadelicken interest / hebben wy ghevonden wat die ghereedt weerdich is / t'welck gheproponeert was alsoo ghedaen te worden.

PROPOSITION III.

Wesende verclaert Hooft-somme tijdt ende interests reden van ghecomponeerden profijtelicken interest: Te vindê wat d'Hooft-somme met haren interest bedraecht.

NOTA.

Tot de solutie van de exempelen deser propositien zijn ons van noode de tafelen daer voren af gheseyt is / waer deur zullen hier beschrijuen verscheyden tafelen zoo vele als indę practijcke ghemeynlick noodich vallen / te weten 16. tafelen / al waer altijdt comparatie gheschiet van den interest teghen t'hondert / welcker tafelen ierst zijn zal van een ten 100. de tweede van twee ten 100. ende zoo voort tot de 16. tafel / welcke zijn zal vâ 16. ten 100. Bouen dien sullen wy beschrijuen acht tafelen / al waer comparatie geschiet van verscheyden Hooftsommen tot interest altijdt 1. welcker tafelen ierste zijn zal van den penninck 15. de tweede van den penninck 16. ende soo voorts tot den penninck 22. ende zullen dese tafelen altemael dienê tot 30. iaeren ofte termijnen.

CONSTRUCTIE DER TAFELEN.

Alsoo dan om te comen tot de constructie deser tafelen / zegghe ick in de zelue niet anders ghesocht te worden dan proportionale ghetaelen met de gene daer questie af is. Om de welcke te vinden soo salmen ten iersten nemen eenich groot getal (welck wy noemen den wortel der tafelen) waer af d'ierste cijffer-letter sy 1. ende de resterende altemael 0. ick hebbe tot dese tafelen ghenomen (hoe wel mê meer ofte min nemen mach) 10000000. Nu dan willende maecken een tafel teghen een ten 100. ghelijck de volghende ierste is / salmen den voornoemden wortel 10000000. multiplicerê met d'Hooft-somme 100. t'productum is 1000000000. de zelfde zalmen diuiderê deur d'Hooft-somme met haeren interest daer toeghedaen / te weten door 101. (want 100. is gheproponeerde Hooft-somme ende 1 den interest) quotus zal zijn 9900990. dienende voor d'ierste iaer ofte termijn.

Aengaende de reste dieder naer de diuisie blijft als $\frac{1}{101}$ die laetmê verloren om datse minder is dan een half / Maer als zulcken reste meerder is dan een half / soo salmen (ghelijck in *tabula sinuum* eñ andere meer de ghebruyck is) die verlaeten ende daer voren de gheheele ghetaelen van de quotus van een vermeerderen / want alsoo blijft men àltijdt naerder bij het begheerde. Nu dan om te vinden t'ghetal des tweeden iaers / salmen de 9900990. wederom multipliceren met 100. gheeft productum 990099000. welck men wederom zal diuideren door 101. quotus zal zijn 9802960. voor het tweede iaer.

Alsoo oock om te vinden t'ghetal van het derde iaer salmê de 9802960. wederom multiplicerê met 100. geeft productû 980296000. welck mê wederô sal diuideren door 101. quotus sal zijn 9705900. Maer ouermidts de reste van $\frac{100}{101}$ hier meerder is dan een half / zoo salmen om redenen alsvorê de laetste letter des quotus van 1. vermeerderen / stellende aldus 9705901. voor het derde termijn /

brevity's sake. Hence, given the Principal, the time, and the rate of simple and detrimental interest, we have found what is the present value of this, which it had been proposed to do.

PROPOSITION III.

Given the Principal, time, and rate of compound profitable interest: to find what the Principal with its interest amounts to.

NOTE.

For the solution of the examples of this proposition we require the tables that have been referred to above. Therefore we will here describe different tables, as many as are usually necessary in practice, to wit 16 tables, where the interest is always referred to one hundred, the first of which tables will be of one per cent, the second of two per cent, and so on to the 16th table, which will be of 16 per cent. In addition we will describe eight tables where different Principals are always referred to an interest of 1, the first of which tables will be of the 15th penny, the second of the 16th penny, and so on to the 22nd penny, and all these tables will serve for 30 years or terms.

CONSTRUCTION OF THE TABLES.

To come therefore to the construction of these tables, I say that nothing else is sought in them but numbers proportional to those under consideration. In order to find these, first take some large number (which we call the root of the tables), of which let the first digit be 1 and the remaining all 0. For the present tables I have taken (though one can take more or less) 10,000,000. If it is now required to make a table at one per cent, as is the following—first—table, multiply the aforesaid root 10,000,000 by the Principal of 100. The product is 1,000,000,000. Divide this by the Principal *plus* its interest, to wit by 101 (for 100 is the proposed Principal and 1 the interest). The quotient will be 9,900,990, serving for the first year or term.

As regards the remainder that is left after the division, *viz.* $\frac{10}{101}$, this is neglected, because it is less than one half. But if this remainder is more than one half, omit it (as is the custom in sine tables and others) and instead add one to the whole numbers of the quotient, for thus we always keep closer to the required value. In order to find the number for the second year, multiply the 9,900,990 again by 100. This gives the product 990,099,000, which divide by 101. The quotient will be 9,802,960 for the second year.

Thus also, in order to find the number of the third year, multiply the 9,802,960 again by 100. This gives the product 980,296,000, which divide again by 101. The quotient will be 9,705,900. But since the remainder, *viz.* $\frac{100}{101}$, is here more than one half, for the reasons given above add one to the last digit of the quotient, thus taking 9,705,901 for the third term, and so on with the other terms, which have been continued to 30 in our tables.

ende alsoo voort met d'andere termijnen / welckê in onse tafelen tot 30. ghecontinueert zijn.

S'ghelijcks zal oock zijn de constructie van alle die andere tafelê / want daer wy in de ierste tafel altijdt multipliceren met 100. ende diuideren door 101. alsoo sullê wy in de tweede tafel (welcke is van 2. ten 100.) altijdt multipliceren met 100. ende diuideren door 102. ende in de derde tafel altijt multipliceren met 100. ende diuideren door 103. ende soo voort in d'andere. Item de constructie der tafele van den penninck 15. is de voorseyde oock gelijck / want men multipliceert hier altijdt met 15. eñ men diuideert door 16. (te weten door 15. gheproponeerde Hooft-somme ende daer toe haeren interest 1.) Alsoo oock in de tafel van den penninck 16. multipliceert men altijdt met 16. ende men diuideert door 17. ende zoo voorts met d'andere.

Deze tafelen alsoo ghemaeckt voor eenighe iaeren worter by elcke tafel noch een columne gestelt / welcke dienen zal tot ghecomponeerdê interest van partijen die in vervolghende iaeren te betaelen zijn elck iaer euen veel / ghelijck d'exempelen daer af t'haerder plaetsen zullen ghegheuen worden / welcker columnen constructie aldus is:

Men zal (tot de constructie deser colummen der tafel van 1. ten 100.) de 9900990. staende neuen d'ierste iaer ofte termijn noch eenmael stellen neuen t'voornoemde ierste termijn / daer naer salmen adderen de twee sommen responderêde op de twee ierste iaeren als 9900990. met 9802960. bedraeghen t'saemen 19703950. die salmen stellen neuen het tweede iaer. Daer naer salmen adderê de dry sommen responderende op de dry ierste iaeren / bedraeghen t'saemê 29409851. ende soo voorts totten eynde. Soo dat t'laetste ghetal deser laetster columnen 258077051. sal sijn de somme van alde ghetaelen der voorgaende columne.

In der seluer voeghen salmê oock tot alle d'andere tafelen / elck zoodaenighe laetste columne maken. Soo dat elck deser tafelen zal hebben dry columnen: D'ierste columne beteeckenêde iaeren / ende d'ander twee dienende tot solutien van questien / als int volghende blijcken zal.

TAFELEN VAN INTEREST.

The same will also be the construction of all the other tables, for while in the first table we always multiply by 100 and divide by 101, thus in the second table (which is of 2 per cent) we will always multiply by 100 and divide by 102, and in the third table we will always multiply by 100 and divide by 103, and so on in the others. In the same way the construction of the table of the 15th penny is also similar to the aforesaid one, for here we always multiply by 15 and divide by 16 (to wit by 15—the proposed Principal—*plus* its interest of 1). Thus also in the table of the 16th penny we always multiply by 16 and divide by 17, and so on with the others.

After these tables have thus been made for a number of years, to each table is added another column, which is to serve for compound interest on sums that are to be paid in successive years, every year the same amount, as the examples thereof will be given in due place, the construction of which column is as follows:

Put the 9,900,990 opposite the first year or term (for the construction of this column of the table of 1 per cent) once more opposite the aforesaid first term; thereafter add up the two sums corresponding to the two first years, *viz.* 9,900,990 and 9,802,960, which together amount to 19,703,950. Put this number opposite the second year. Thereafter add up the three sums corresponding to the three first years; these amount together to 29,409,851. And so on to the end, so that the last number of this last column, 258,077,051, will be the sum of all the numbers of the preceding column.

In the same way such a last column also has to be added to each of the other tables, so that each of these tables shall have three columns, the first column designating the years and the other two serving for the solution of certain questions, as will appear in the sequel.

TABLES OF INTEREST.

Tafel van Interest van
1. *ten* 100.

1.	9900990.	9900990.
2.	9802960.	19703950.
3.	9705901.	29409851.
4.	9609803.	39019654.
5.	9514656.	48534310.
6.	9420451.	57954761.
7.	9327179.	67281940.
8.	9234831.	76516771.
9.	9143397.	85660168.
10.	9052868.	94713036.
11.	8963236.	103676272.
12.	8874491.	112550763.
13.	8786625.	121337388.
14.	8699629.	130037017.
15.	8613494.	138650511.
16.	8528212.	147178723.
17.	8443774.	155622497.
18.	8360172.	163982669.
19.	8277398.	172260067.
20.	8195444.	180455511.
21.	8114301.	188569812.
22.	8033961.	196603773.
23.	7954417.	204558190.
24.	7875660.	212433850.
25.	7797683.	220231533.
26.	7720478.	227952011.
27.	7644038.	235596049.
28.	7568354.	243164403.
29.	7493420.	250657823.
30.	7419228.	258077051.

Tafel van interest van
2. ten 100.

1.	9803922.	9803922.
2.	9611688.	19415610.
3.	9423244.	28838834.
4.	9238455.	38077289.
5.	9057309.	47134598.
6.	8879715.	56014313.
7.	8705603.	64719916.
8.	8534905.	73254821.
9.	8367554.	81622375.
10.	8203484.	89825859.
11.	8042631.	97868490.
12.	7884932.	105753422.
13.	7730325.	113483747.
14.	7578750.	121062497.
15.	7430147.	128492644.
16.	7284458.	135777102.
17.	7141625.	142918727.
18.	7001593.	149920320.
19.	6864307.	156784627.
20.	6729713.	163514340.
21.	6597758.	170112098.
22.	6468390.	176580488.
23.	6341559.	182922074.
24.	6217215.	189139262.
25.	6095309.	195234571.
26.	5975793.	201210364.
27.	5858621.	207068985.
28.	5743746.	212812731.
29.	5631124.	218443855.
30.	5520710.	223964565.

Tafel van interest van
3. ten 100.

1.	9708738.	9708738.
2.	9425959.	19134697.
3.	9151417.	28286114.
4.	8884871.	37170985.
5.	8626088.	45797073.
6.	8374843.	54171916.
7.	8130916.	62302832.
8.	7894093.	70196925.
9.	7664168.	77861093.
10.	7440940.	85302033.
11.	7224214.	92526247.
12.	7013800.	99540047.
13.	6809515.	106349562.
14.	6611180.	112960742.
15.	6418621.	119379363.
16.	6231671.	125611034.
17.	6050166.	131661200.
18.	5873948.	137535148.
19.	5702862.	143238010.
20.	5536759.	148774769.
21.	5375494.	154150263.
22.	5218926.	159369189.
23.	5066918.	164436107.
24.	4919338.	169355445.
25.	4776056.	174131501.
26.	4636948.	178768449.
27.	4501891.	183270340.
28.	4370768.	187641108.
29.	4243464.	191884572.
30.	4119868.	196004440.

Tafel van interest van
4. ten 100.

1.	9615385.	9615385.
2.	9245562.	18860947.
3.	8889963.	27750910.
4.	8548041.	36298951.
5.	8219270.	44518221.
6.	7903144.	52421365.
7.	7599177.	60020542.
8.	7306901.	67327443.
9.	7025866.	74353309.
10.	6755640.	81108949.
11.	6495808.	87604757.
12.	6245969.	93850726.
13.	6005739.	99856465.
14.	5774749.	105631214.
15.	5552643.	111183857.
16.	5339080.	116522937.
17.	5133731.	121656668.
18.	4936280.	126592948.
19.	4746423.	131339371.
20.	4563868.	135903239.
21.	4388335.	140291574.
22.	4219553.	144511127.
23.	4057262.	148568389.
24.	3901213.	152469602.
25.	3751166.	156220768.
26.	3606890.	159827658.
27.	3468163.	163295821.
28.	3334772.	166630593.
29.	3206512.	169837105.
30.	3083185.	172920290.

Tafel van interest van
5. ten 100.

1.	9523810.	9523810.
2.	9070295.	18594105.
3.	8638376.	27232481.
4.	8227025.	35459506.
5.	7835262.	43294768.
6.	7462154.	50756922.
7.	7106813.	57863735.
8.	6768393.	64632128.
9.	6446089.	71078217.
10.	6139132.	77217349.
11.	5846792.	83064141.
12.	5568373.	88632514.
13.	5303212.	93935726.
14.	5050678.	98986404.
15.	4810170.	103796574.
16.	4581114.	108377688.
17.	4362966.	112740654.
18.	4155206.	116895860.
19.	3957339.	120853199.
20.	3768894.	124622093.
21.	3589423.	128211516.
22.	3418498.	131630014.
23.	3255712.	134885726.
24.	3100678.	137986404.
25.	2953027.	140939431.
26.	2812407.	143751838.
27.	2678483.	146430321.
28.	2550936.	148981257.
29.	2429463.	151410720.
30.	2313774.	153724494.

Tafel van interest van
6. ten 100.

1.	9433962.	9433962.
2.	8899964.	18333926.
3.	8396192.	26730118.
4.	7920936.	34651054.
5.	7472581.	42123635.
6.	7049605.	49173240.
7.	6650571.	55823811.
8.	6274124.	62097935.
9.	5918985.	68016920.
10.	5583948.	73600868.
11.	5267875.	78868743.
12.	4969693.	83838436.
13.	4688390.	88526826.
14.	4423009.	92949835.
15.	4172650.	97122485.
16.	3936462.	101058947.
17.	3713643.	104772590.
18.	3503437.	108276027.
19.	3305129.	111581156.
20.	3118046.	114699202.
21.	2941553.	117640755.
22.	2775050.	120415805.
23.	2617972.	123033777.
24.	2469785.	125503562.
25.	2329986.	127833548.
26.	2198100.	130031648.
27.	2073679.	132105327.
28.	1956301.	134061628.
29.	1845567.	135907195.
30.	1741101.	137648296.

Tafel van interest van
7. ten 100.

1.	9345794.	9345794.
2.	8734387.	18080181.
3.	8162979.	26243160.
4.	7628952.	33872112.
5.	7129862.	41001974.
6.	6663422.	47665396.
7.	6227497.	53892893.
8.	5820091.	59712984.
9.	5439337.	65152321.
10.	5083493.	70235814.
11.	4750928.	74986742.
12.	4440120.	79426862.
13.	4149645.	83576507.
14.	3878173.	87454680.
15.	3624461.	91079141.
16.	3387347.	94466488.
17.	3165745.	97632233.
18.	2958640.	100590873.
19.	2765084.	103355957.
20.	2584191.	105940148.
21.	2415132.	108355280.
22.	2257133.	110612413.
23.	2109470.	112721883.
24.	1971467.	114693350.
25.	1842493.	116535843.
26.	1721956.	118257799.
27.	1609305.	119867104.
28.	1504023.	121371127.
29.	1405629.	122776756.
30.	1313672.	124090428.

Tafel van interest van
8. ten 100.

1.	9259259.	9259259.
2.	8573388.	17832647.
3.	7938322.	25770969.
4.	7350298.	33121267.
5.	6805831.	39927098.
6.	6301695.	46228793.
7.	5834903.	52063696.
8.	5402688.	57466384.
9.	5002489.	62468873.
10.	4631934.	67100807.
11.	4288828.	71389635.
12.	3971137.	75360772.
13.	3676979.	79037751.
14.	3404610.	82442361.
15.	3152417.	85594778.
16.	2918905.	88513683.
17.	2702690.	91216373.
18.	2502491.	93718864.
19.	2317121.	96035985.
20.	2145482.	98181467.
21.	1986557.	100168024.
22.	1839405.	102007429.
23.	1703153.	103710582.
24.	1576994.	105287576.
25.	1460180.	106747756.
26.	1352019.	108099775.
27.	1251869.	109351644.
28.	1159138.	110510782.
29.	1073276.	111584058.
30.	993774.	112577832.

Tafel van interest van
9. ten 100.

1.	9174312.	9174312.
2.	8416800.	17591112.
3.	7721835.	25312947.
4.	7084252.	32397199.
5.	6499314.	38896513.
6.	5962673.	44859186.
7.	5470342.	50329528.
8.	5018662.	55348190.
9.	4604277.	59952467.
10.	4224107.	64176574.
11.	3875328.	68051902.
12.	3555347.	71607249.
13.	3261786.	74869035.
14.	2992464.	77861499.
15.	2745380.	80606879.
16.	2518697.	83125576.
17.	2310731.	85436307.
18.	2119937.	87556244.
19.	1944896.	89501140.
20.	1784308.	91285448.
21.	1636980.	92922428.
22.	1501817.	94424245.
23.	1377814.	95802059.
24.	1264050.	97066109.
25.	1159679.	98225788.
26.	1063926.	99289714.
27.	976079.	100265793.
28.	895485.	101161278.
29.	821546.	101982824.
30.	753712.	102736536.

Tafel van interest van
10. ten 100.

1.	9090909.	9090909.
2.	8264463.	17355372.
3.	7513148.	24868520.
4.	6830135.	31698655.
5.	6209214.	37907869.
6.	5644740.	43552609.
7.	5131582.	48684191.
8.	4665075.	53349266.
9.	4240977.	57590243.
10.	3855434.	61445677.
11.	3504940.	64950617.
12.	3186309.	68136926.
13.	2896645.	71033571.
14.	2633314.	73666885.
15.	2393922.	76060807.
16.	2176293.	78237100.
17.	1978448.	80215548.
18.	1798589.	82014137.
19.	1635081.	83649218.
20.	1486437.	85135655.
21.	1351306.	86486961.
22.	1228460.	87715421.
23.	1116782.	88832203.
24.	1015256.	89847459.
25.	922960.	90770419.
26.	839055.	91609474.
27.	762777.	92372251.
28.	693434.	93065685.
29.	630395.	93696080.
30.	573086.	94269166.

Tafel van interest van
11. ten 100.

1.	9009009.	9009009.
2.	8116224.	17125233.
3.	7311914.	24437147.
4.	6587310.	31024457.
5.	5934514.	36958971.
6.	5346409.	42305380.
7.	4816585.	47121965.
8.	4339266.	51461231.
9.	3909249.	55370480.
10.	3521846.	58892326.
11.	3172834.	62065160.
12.	2858409.	64923569.
13.	2575143.	67498712.
14.	2319949.	69818661.
15.	2090044.	71908705.
16.	1882923.	73791628.
17.	1696327.	75487955.
18.	1528223.	77016178.
19.	1376777.	78392955.
20.	1240340.	79633295.
21.	1117423.	80750718.
22.	1006687.	81757405.
23.	906925.	82664330.
24.	817050.	83481380.
25.	736081.	84217461.
26.	663136.	84880597.
27.	597420.	85478017.
28.	538216.	86016233.
29.	484879.	86501112.
30.	436828.	86937940.

Tafel van interest van
12. ten 100.

1.	8928571.	8928571.
2.	7971938.	16900509.
3.	7117802.	24018311.
4.	6355180.	30373491.
5.	5674268.	36047759.
6.	5066311.	41114070.
7.	4523492.	45637562.
8.	4038832.	49676394.
9.	3606100.	53282494.
10.	3219732.	56502226.
11.	2874761.	59376987.
12.	2566751.	61943738.
13.	2291742.	64235480.
14.	2046198.	66281678.
15.	1826962.	68108640.
16.	1631216.	69739856.
17.	1456443.	71196299.
18.	1300396.	72496695.
19.	1161068.	73657763.
20.	1036668.	74694431.
21.	925596.	75620027.
22.	826425.	76446452.
23.	737879.	77184331.
24.	658821.	77843152.
25.	588233.	78431385.
26.	525208.	78956593.
27.	468936.	79425529.
28.	418693.	79844222.
29.	373833.	80218055.
30.	333779.	80551834.

Tafel van interest van
13. ten 100.

1.	8849558.	8849558.
2.	7831467.	16681025.
3.	6930502.	23611527.
4.	6133188.	29744715.
5.	5427600.	35172315.
6.	4803186.	39975501.
7.	4250607.	44226108.
8.	3761599.	47987707.
9.	3328849.	51316556.
10.	2945884.	54262440.
11.	2606977.	56869417.
12.	2307059.	59176476.
13.	2041645.	61218121.
14.	1806765.	63024886.
15.	1598907.	64623793.
16.	1414962.	66038755.
17.	1252179.	67290934.
18.	1108123.	68399057.
19.	980640.	69379697.
20.	867823.	70247520.
21.	767985.	71015505.
22.	679633.	71695138.
23.	601445.	72296583.
24.	532252.	72828835.
25.	471019.	73299854.
26.	416831.	73716685.
27.	368877.	74085562.
28.	326440.	74412002.
29.	288885.	74700887.
30.	255650.	74956537.

Tafel van interest van
14. ten 100.

1.	8771930.	8771930.
2.	7694675.	16466605.
3.	6749715.	23216320.
4.	5920803.	29137123.
5.	5193687.	34330810.
6.	4555866.	38886676.
7.	3996374.	42883050.
8.	3505591.	46388641.
9.	3075080.	49463721.
10.	2697439.	52161160.
11.	2366175.	54527335.
12.	2075592.	56602927.
13.	1820695.	58423622.
14.	1597101.	60020723.
15.	1400966.	61421689.
16.	1228918.	62650607.
17.	1077998.	63728605.
18.	945612.	64674217.
19.	829484.	65503701.
20.	727618.	66231319.
21.	638261.	66869580.
22.	559878.	67429458.
23.	491121.	67920579.
24.	430808.	68351387.
25.	377902.	68729289.
26.	331493.	69060782.
27.	290783.	69351565.
28.	255073.	69606638.
29.	223748.	69830386.
30.	196270.	70026656.

Tafel van interest van
15. ten 100.

1.	8695652.	8695652.
2.	7561437.	16257089.
3.	6575163.	22832252.
4.	5717533.	28549785.
5.	4971768.	33521553.
6.	4323277.	37844830.
7.	3759371.	41604201.
8.	3269018.	44873219.
9.	2842624.	47715843.
10.	2471847.	50187690.
11.	2149432.	52337122.
12.	1869071.	54206193.
13.	1625279.	55831472.
14.	1413286.	57244758.
15.	1228944.	58473702.
16.	1068647.	59542349.
17.	929258.	60471607.
18.	808050.	61279657.
19.	702652.	61982309.
20.	611002.	62593311.
21.	531306.	63124617.
22.	462005.	63586622.
23.	401743.	63988365.
24.	349342.	64337707.
25.	303776.	64641483.
26.	264153.	64905636.
27.	229698.	65135334.
28.	199737.	65335071.
29.	173684.	65508755.
30.	151030.	65659785.

Tafel van interest van
16. ten 100.

1.	8620690.	8620690.
2.	7431629.	16052319.
3.	6406577.	22458896.
4.	5522911.	27981807.
5.	4761130.	32742937.
6.	4104422.	36847359.
7.	3538295.	40385654.
8.	3050254.	43435908.
9.	2629529.	46065437.
10.	2266835.	48332272.
11.	1954168.	50286440.
12.	1684628.	51971068.
13.	1452266.	53423334.
14.	1251953.	54675287.
15.	1079270.	55754557.
16.	930405.	56684962.
17.	802073.	57487035.
18.	691442.	58178477.
19.	596071.	58774548.
20.	513854.	59288402.
21.	442978.	59731380.
22.	381878.	60113258.
23.	329205.	60442463.
24.	283797.	60726260.
25.	244653.	60970913.
26.	210908.	61181821.
27.	181817.	61363638.
28.	156739.	61520377.
29.	135120.	61655497.
30.	116483.	61771980.

Tafel van Interest van den
penninck 15.

1.	9375000.	9375000.
2.	8789062.	18164062.
3.	8239746.	26403808.
4.	7724762.	34128570.
5.	7241964.	41370534.
6.	6789341.	48159875.
7.	6365007.	54524882.
8.	5967194.	60492076.
9.	5594244.	66086320.
10.	5244604.	71330924.
11.	4916816.	76247740.
12.	4609515.	80857255.
13.	4321420.	85178675.
14.	4051331.	89230006.
15.	3798123.	93028129.
16.	3560740.	96588869.
17.	3338194.	99927063.
18.	3129557.	103056620.
19.	2933960.	105990580.
20.	2750587.	108741167.
21.	2578675.	111319842.
22.	2417508.	113737350.
23.	2266414.	116003764.
24.	2124763.	118128527.
25.	1991965.	120120492.
26.	1867467.	121987959.
27.	1750750.	123738709.
28.	1641328.	125380037.
29.	1538745.	126918782.
30.	1442573.	128361355.

Tafel van Interest van den penninck 16.

1.	9411765.	9411765.
2.	8858132.	18269897.
3.	8337065.	26606962.
4.	7846649.	34453611.
5.	7385081.	41838692.
6.	6950664.	48789356.
7.	6541801.	55331157.
8.	6156989.	61488146.
9.	5794813.	67282959.
10.	5453942.	72736901.
11.	5133122.	77870023.
12.	4831174.	82701197.
13.	4546987.	87248184.
14.	4279517.	91527701.
15.	4027781.	95555482.
16.	3790853.	99346335.
17.	3567862.	102914197.
18.	3357988.	106272185.
19.	3160459.	109432644.
20.	2974550.	112407194.
21.	2799576.	115206770.
22.	2634895.	117841665.
23.	2479901.	120321566.
24.	2334024.	122655590.
25.	2196728.	124852318.
26.	2067509.	126919827.
27.	1945891.	128865718.
28.	1831427.	130697145.
29.	1723696.	132420841.
30.	1622302.	134043143.

Tafel van Interest van den
penninck 17.

1.	9444444.	9444444.
2.	8919753.	18364197.
3.	8424211.	26788408.
4.	7956199.	34744607.
5.	7514188.	42258795.
6.	7096733.	49355528.
7.	6702470.	56057998.
8.	6330111.	62388109.
9.	5978438.	68366547.
10.	5646303.	74012850.
11.	5332619.	79345469.
12.	5036362.	84381831.
13.	4756564.	89138395.
14.	4492310.	93630705.
15.	4242737.	97873442.
16.	4007029.	101880471.
17.	3784416.	105664887.
18.	3574171.	109239058.
19.	3375606.	112614664.
20.	3188072.	115802736.
21.	3010957.	118813693.
22.	2843682.	121657375.
23.	2685700.	124343075.
24.	2536494.	126879569.
25.	2395578.	129275147.
26.	2262490.	131537637.
27.	2136796.	133674433.
28.	2018085.	135692518.
29.	1905969.	137598487.
30.	1800082.	139398569.

Tafel van Interest van den penninck 18.

1.	9473684.	9473684.
2.	8975069.	18448753.
3.	8502697.	26951450.
4.	8055186.	35006636.
5.	7631229.	42637865.
6.	7229585.	49867450.
7.	6849081.	56716531.
8.	6488603.	63205134.
9.	6147098.	69352232.
10.	5823567.	75175799.
11.	5517063.	80692862.
12.	5226691.	85919553.
13.	4951602.	90871155.
14.	4690991.	95562146.
15.	4444097.	100006243.
16.	4210197.	104216440.
17.	3988608.	108205048.
18.	3778681.	111983729.
19.	3579803.	115563532.
20.	3391392.	118954924.
21.	3212898.	122167822.
22.	3043798.	125211620.
23.	2883598.	128095218.
24.	2731830.	130827048.
25.	2588049.	133415097.
26.	2451836.	135866933.
27.	2322792.	138189725.
28.	2200540.	140390265.
29.	2084722.	142474987.
30.	1975000.	144449987.

Tafel van Interest van den
penninck 19.

1.	9500000.	9500000.
2.	9025000.	18525000.
3.	8573750.	27098750.
4.	8145062.	35243812.
5.	7737809.	42981621.
6.	7350919.	50332540.
7.	6983373.	57315913.
8.	6634204.	63950117.
9.	6302494.	70252611.
10.	5987369.	76239980.
11.	5688001.	81927981.
12.	5403601.	87331582.
13.	5133421.	92465003.
14.	4876750.	97341753.
15.	4632912.	101974665.
16.	4401266.	106375931.
17.	4181203.	110557134.
18.	3972143.	114529277.
19.	3773536.	118302813.
20.	3584859.	121887672.
21.	3405616.	125293288.
22.	3235335.	128528623.
23.	3073568.	131602191.
24.	2919890.	134522081.
25.	2773895.	137295976.
26.	2635200.	139931176.
27.	2503440.	142434616.
28.	2378268.	144812884.
29.	2259355.	147072239.
30.	2146387.	149218626.

Tafel van Interest van den penninck 20.

Nota.

Dese tafel is de voorgaende tafel van 5. ten 100. ghelijck.

Tafel van Interest van den penninck 21.

1.	9545455.	9545455.
2.	9111571.	18657026.
3.	8697409.	27354435.
4.	8302072.	35656507.
5.	7924705.	43581212.
6.	7564491.	51145703.
7.	7220650.	58366353.
8.	6892439.	65258792.
9.	6579146.	71837938.
10.	6280094.	78118032.
11.	5994635.	84112667.
12.	5722152.	89834819.
13.	5462054.	95296873.
14.	5213779.	100510652.
15.	4976789.	105487441.
16.	4750571.	110238012.
17.	4534636.	114772648.
18.	4328516.	119101164.
19.	4131765.	123232929.
20.	3943958.	127176887.
21.	3764687.	130941574.
22.	3593565.	134535139.
23.	3430221.	137965360.
24.	3274302.	141239662.
25.	3125470.	144365132.
26.	2983403.	147348535.
27.	2847794.	150196329.
28.	2718349.	152914678.
29.	2594788.	155509466.
30.	2476843.	157986309.

Tafel van Interest van den
penninck 22.

1.	9565217.	9565217.
2.	9149338.	18714555.
3.	8751541.	27466096.
4.	8371039.	35837135.
5.	8007081.	43844216.
6.	7658947.	51503163.
7.	7325949.	58829112.
8.	7007429.	65836541.
9.	6702758.	72539299.
10.	6411334.	78950633.
11.	6132586.	85083213.
12.	5865946.	90949159.
13.	5610905.	96560064.
14.	5366953.	101927017.
15.	5133607.	107060624.
16.	4910407.	111971031.
17.	4696911.	116667942.
18.	4492697.	121160639.
19.	4297362.	125458001.
20.	4110520.	129568521.
21.	3931802.	133500323.
22.	3760854.	137261177.
23.	3597339.	140858516.
24.	3440933.	144299449.
25.	3291327.	147590776.
26.	3148226.	150739002.
27.	3011347.	153750349.
28.	2880419.	156630768.
29.	2755183.	159385951.
30.	2635392.	162021343.

Eynde der Tafelen.

NOTA.

Ouermidts alle interst reden die metten hondert wtghesproken wordt/is altijdt oock eenighe interests reden die metten penninck can wtghesproken worden/ende ter contrarien (als by exempel 5. ten 100. mach oock gheseyt worden dê penninck 20.) zullen wy alles tot meerderen gherieue haere comparatien (zoo verre onse tafelen strecken) verclaeren aldus

Den penninck		Ten 100.
1	Ten 100. is soo veel als den penninck	100
2		50
3		$33\frac{1}{3}$
4		25
5		20
6		$16\frac{2}{3}$
7		$14\frac{2}{7}$
8		$12\frac{1}{2}$
9		$11\frac{1}{9}$
10		10
11		$9\frac{1}{11}$
12		$8\frac{1}{3}$
13		$7\frac{9}{13}$
14		$7\frac{1}{7}$
15		$6\frac{2}{3}$
16		$6\frac{1}{4}$

Den penninck		Is soo veel als		Ten hondert.
Den penninck	15	Is soo veel als	$6\frac{2}{3}$	Ten hondert.
	16		$6\frac{1}{4}$	
	17		$5\frac{15}{17}$	
	18		$5\frac{5}{9}$	
	19		$5\frac{5}{19}$	
	20		5	
	21		$4\frac{16}{21}$	
	22		$4\frac{6}{11}$	

De tafelen dan alsoo bereydt zijnde / zullen nu volghen de exempelen dienende tot de voorschreuen 3. propositiê / welcker exempelen ierste aldus is:

EXEMPEL 1.

Men begheert te weten wat Hooft-somme 380. lb. met haeren ghecomponeerden profijtelijcken interest teghen 11. ten 100. t'siaers op 8. iaeren bedraeghen zal.

CONSTRUCTIE.

Men sal sien in de tafel van 11. ten 100. wat ghetal datter respondeert op het achtste iaer / wordt bevonden 4339266. waer deur men zegghen zal 4339266. gheuê 10000000. (welcke 10000000. den wortel van de tafel zijn) wat 380. lb? facit $875\frac{3142250}{4339266}$ lb.

NOTA.

Wy sullen in de volghende exempelen ghemeynelick achter de gheheele ponden het ghebroken stellen sonder tzelfde ghebroken *in radicem fractionis* te conuerteren / dat is / *ad numeros inter se primos*, oft oock sonder ß ende gr. daer wt te trecken / op dat de solutien alsoo te claerder blijuen / wantet ghenoegh is datmen sulcks in praxi doet.

NOTE.

Since every rate of interest that is expressed in per cent is always equivalent to some rate of interest that can be expressed as the penny of something, and conversely (for example: 5 per cent may also be called the 20th penny), we will, for greater convenience, compare them (in as far as our tables go), as follows:

1 per cent is equivalent to the 100th penny
2 per cent is equivalent to the 50th penny
[and so on; see the original text]

The	15th	penny is equivalent to	$6\frac{2}{3}$	per cent
„	16th	„ „ „ „	$6\frac{1}{4}$	„ „
„	17th	„ „ „ „	$5\frac{15}{17}$	„ „
„	18th	„ „ „ „	$5\frac{5}{9}$	„ „
„	19th	„ „ „ „	$5\frac{5}{19}$	„ „
„	20th	„ „ „ „	5	„ „
„	21st	„ „ „ „	$4\frac{16}{21}$	„ „
„	22nd	„ „ „ „	$4\frac{6}{11}$	„ „

After the tables have thus been made, we will now describe the examples serving for the aforesaid 3rd proposition, the first of which examples is as follows:

EXAMPLE 1.

It is required to know what a Principal of 380 lb with its compound profitable interest at 11 per cent a year will amount to in 8 years.

PROCEDURE.

Look up in the table of 11 per cent what number corresponds to the eighth year. This is found to be 4,339,266, so that it has to be said: 4,339,266 gives 10,000,000 (which 10,000,000 is the root of the table); what does 380 lb give? This is $875\frac{3142250}{4339266}$ lb.

NOTE.

In the subsequent examples we will generally put the fraction behind the whole pounds without converting this fraction to its lowest terms, *i.e.* to relative prime numbers, or also without reducing them to sh. and d., in order that the solution may thus be clearer, for it is enough that this is done in practice.

NOTA.

Soomen wilde weten wat den interest van dit exempel bedraecht / soo salmen de 380 lb. aftrecken: van de $875\frac{3142250}{4339266}$ lb. rest $495\frac{3142250}{4339266}$ lb. voor den interest van acht iaeren / 'sghelijcks zalmen oock moghen doen in alle de volghende exempelen.

EXEMPEL 2.

Men begheert te weten wat Hooft-somme 800. lb. met haeren ghecomponeerden profijtelijcken interest teghen den penninck 15. t'siaers op $16\frac{1}{2}$ iaeren bedraeghen zal.

CONSTRUCTIE.

Men zal van wegê een half iaer / eê half adderê tot 15. (dese 15. is van weghen den penninck 15.) maeckt $15\frac{1}{2}$ ende multiplicerê daer naer 3560740. (welck t'ghetal is responderende op het 16. iaer inde tafel van den penninck 15.) met de 15. gheeft productum 53411100. t'zelfde zalmen diuideren door de $15\frac{1}{2}$ / gheeft quotum 3445877. t'welck een ghetal is responderende op het $16\frac{1}{2}$ iaer / ende staen zoude tusschen het 16. ende 17. iaer in de tafel van den penninck 15. by aldien de tafel van halue iaere tot halue iaere ghemaeckt waere.

Daer naer zalmen zegghen 3445877. gheuen 10000000. wat 800 lb?
facit $2321\frac{2119483}{3445877}$ lb.

S'ghelijcks zal oock zijn d'operatie in alle andere deelen des iaers / want, waerender tot eenighe iaerê dry maendê / so zoudemen / (om dat dry maenden een vierendeel iaers is) dan opereren met een vierendeel / ghelijckmen bouen ghedaen heeft met een half / ende soo voort met alle ander deel des iaers / ghelijck van deser ghelijcke breeder ghetracteert is int 3. exempel der 2. prop.

NOTA.

Onder de ghene die in de Arithmeticque van interest gheschreuen hebben / en is my gheen ter handt ghecomen die van den interest subtijlder getracteerd heeft / dan Jan Trenchant / is oock een Arithmeticque die by velen niet weynich gheacht en is: want de derde druck der zeluer wtghegaen is. In de zelue Arithmeticque hebbe ick zeker erreur van den interest bemerckt / t'welck (aenghesien om des zelfden authoriteyt zulck erreur te schadelicker mocht zijn) niet onbillich en schijnt al hier verclaert te worden aldus. Int 3. boeck cap. 9. art. 10. zeght Trenchant op een deel des termijns zonder gheheele verschenen termijnen ofte termijn gecomponeerden profijtelicken interest te connen geschieden zegghende den ghecomponeerden profijtelicken interest van 100. lb. op 6. maenden teghen 10. ten 100. t'siaers te wesen 4. lb. 17 ß. $7\frac{2}{5}$ gr. ende van dry maenden 2. lb. 8 ß $2\frac{7}{10}$ gr. etc.

T'welck wy door het Corollarium der 5. definitien ontkennen / ende breeder redene daer af gheuen aldus.

NOTE.

If it is required to know what is the interest of this example, subtract the 380 lb from the $875\frac{3142250}{4339266}$ lb; the remainder is $495\frac{3142250}{4339266}$ lb, which is the interest of eight years. The same method can also be followed in all the following examples.

EXAMPLE 2.

It is required to know what a Principal of 800 lb with its compound profitable interest at the 15th penny a year will amount to in $16\frac{1}{2}$ years.

PROCEDURE.

On account of the half year, add one half to 15 (this 15 is on account of the 15th penny), which makes $15\frac{1}{2}$, and thereafter multiply 3,560,740 (which is the number corresponding to the 16th year in the table of the 15th penny) by 15. This gives the product 53,411,100, which has to be divided by $15\frac{1}{2}$. This gives the quotient 3,445,877, which is a number corresponding to the $16\frac{1}{2}$th year and would be found between the 16th and the 17th year in the table of the 15th penny, if the table had been made from one half year to the next. Thereafter say: 3,445,877 gives 10,000,000; what does 800 lb give? This is $2{,}321\frac{2119483}{3445877}$ lb.

The same will also be the operation for all other parts of a year, for if over and above a number of years there are three months, then (because three months are one fourth of a year) it would be necessary to operate with one fourth, as it has been done above with one half, and so on with any other part of a year, as has been dealt with more fully in the 3rd example of the 2nd proposition.

NOTE.

Among those who in Arithmetic have written about interest none has come to my notice who has dealt with interest in a subtler manner than Jan Trenchant. This is an Arithmetic which is not a little esteemed by many people, for its third edition has already been published. In this Arithmetic I have discovered a certain error in the interest computation, which (since in view of the authority of the book this error might be all the more detrimental) it does not seem inopportune to set forth here, as follows. In the 3rd book, chapter 9, section 10 Trenchant says that in a part of the term, without any expired whole terms or term, compound profitable interest may be charged, saying that the compound profitable interest on 100 lb in 6 months at 10 per cent a year is 4 lb 17 sh. $7\frac{2}{5}$ d., and in 3 months 2 lb 8 sh. $2\frac{7}{10}$d.,etc.

We deny this by the Sequel to the 5th definition, and we give our reasons for this more fully as follows.

TEN IERSTEN/

Alle ghecomponeerden interest bestaet wt twee interestê / d'eene van de Hooftsomme / d'ander van interest van verschenen termijn.

Hier en is gheenen verschenen termijn / waer deur oock gheenen interest van verschenen termijn.

Ergo ten is gheenen ghecomponeerden interest.

ITEM/

Alle ghecomponeerden profijtelicken interest is voor den crediteur profijtelicker dan simpelen interest.

Desen interest en is voor den crediteur niet profijtelicker dan simpelen interest / maer ter contrarien schadelicker.

Ergo hier en is gheenen ghecomponeerden interest.

Schadelicker te zijn / blijckt daer wt / dat Trenchant zeght ter plaetsen als bouen desen ghecomponeerdê interest op een half iaer te zijne 4. lb. 17. ß. $7\frac{2}{5}$ gr. wiens simpelê interest bedraecht 5. lb.

ITEM/

Op een heel iaer ofte termijn canmen gheen ghecomponeerden interest rekenê / als Trenchant zeluer niet en doet. *Ergo* veel min canmen ghecomponeerden interest op een dele des termijns rekenen. Concluderen dan van alle deel vâ termijn (wel verstaende deel van termijn dat alleene staet / dat is zonder eenich gheheel termijn ofte termijnen tot hem) niet dan simpelen interest te connen gherekent worden.

Wt dit erreur is ghevolght das Trenchant int 11. art. des voornoemden capittels gheseyt heeft 100. lb. ghereet ten eynde van $7\frac{1}{2}$ iaeren rekenende ghecomponeerden interest teghen 10. ten 100. t'siaers weerdich te zijne 204. lb. 7 ß. $7\frac{1}{8}$ gr. welcke nochtans weerdich zijn (naer de leeringhe des voornoêden 2. exempels) 204. lb. 12 ß $3\frac{2238353}{4887221}$ gr. waer af de demonstratie ten eynde deser propositien sal ghedaen worden.

EXEMPEL 3.

Eenen is schuldich 1200. lb. te betaelen ten eynde van 7. iaeren. De vraeghe is wat die weerdich zijn te betaelen ten eynde van 23. iaeren rekenende ghecomponeerden interest tegen 8. ten 100. t'siaers.

CONSTRUCTIE.

Men sal sien in de tafel van 8. ten 100. wat ghetal datter respondeert op het 23. iaer / wordt bevonden 1703153. oock mede wat ghetal datter respondeert op het 7. iaer / wordt bevonden 5834903. daer naer zalmen zegghen 1703153. gheuen 5834903. wat gheuen 1200. lb? facit $4111\frac{221617}{1703153}$ lb.

Ofte andersins (ende lichter ouermidts de multiplicatie door den wortel der tafelen lichter is dan de voorgaende / want die gheschiet door aensettinghe alleene-

FIRSTLY.

All compound interest consists of two interests, one on the Principal and the other on the interest of the expired term.

Here there is no expired term, so that there is no interest on an expired term either.

Therefore there is no compound interest here.

ITEM.

All compound profitable interest is more profitable to the creditor than simple interest.

This interest is not more profitable to the creditor than simple interest, but on the contrary more detrimental.

Therefore this is no compound interest.

That it is more detrimental appears from the fact that Trenchant says in the above-mentioned passage that this compound interest is in half a year 4 lb 17 sh. $7\frac{2}{5}$ d., whilst the simple interest is 5 lb.

ITEM.

In a whole year or term one cannot charge compound interest, as Trenchant himself does not do.

Therefore, even less can one charge compound interest in a part of the term.

We therefore conclude that for any part of a term (*i.e.* part of a term that stands by itself, to wit without any whole term or terms added thereto) nothing but simple interest can be charged.

From this error it has followed that in the 11th section of the aforesaid chapter Trenchant has said that 100 lb present value at the end of $7\frac{1}{2}$ years, charging compound interest at 10 per cent a year, will be worth 204 lb 7 sh. $7\frac{1}{8}$ d., whereas they are worth (according to the aforesaid 2nd example) 204 lb 12 sh. $3\frac{2238353}{4887221}$ d., the proof of which will be given at the end of the present proposition.

EXAMPLE 3.

A man owes 1,200 lb, to be paid at the end of 7 years. What will they be worth at the end of 23 years, charging compound interest at 8 per cent a year?

PROCEDURE.

Look up in the table of 8 per cent what number corresponds to the 23rd year. which is found to be 1,703,153; also what number corresponds to the 7th year, which is found to be 5,834,903. Thereafter say: 1,703,153 gives 5,834,903; what does 1,200 lb give? This is $4{,}111\frac{221617}{1703153}$ lb.

Or otherwise (and more easily, since multiplication by the root of the table is easier than the preceding, for this is done simply by adding seven 0's) the lick van zeuen 0.) machmen rekeninghe maecken op 16. iaeren / te weten van het 7. iaer tot het 23. sal bedraeghen door het 1. exempel deser prop. 4111 $\frac{381545}{2918905}$ lb. als voren.

EXEMPEL 4.

Eenen is schuldich 800. lb. te betaelen ten eynde van 3. iaeren / ende noch 300. lb. binnen 2. iaeren daer nae. De vraege is wat beyde dese sommen t'saemen weerdich zullen zijn/ten eynde van 15. iaeren rekenende ghecomponeerden interest teghen 13. ten 100. t'siaers.

CONSTRUCTIE.

Men sal bevinden door het voorgaende 3. exempel dat de 800. lb. zullen weerdich zijn 3467 $\frac{991031}{1598907}$ lb. ende de 300. lb. 1018 $\frac{592674}{1598907}$ welcke 2. sommen bedragende t'saemen 4485 $\frac{1583705}{1598907}$ lb. is t'ghene de 800. lb. ende de 300. lb. t'saemen binnen 15. iaeren weerdich zijn.

EXEMPEL 5.

Eenen is schuldich ghereet 224. lb. Oft hy betaelde binnen 4. iaeren alle iaere het vierendeel / te weten 56. lb. midts iaerlicks betaelende den ghecomponeerden interest teghen 12. ten 100. t'siaers. De vraeghe is wat hy iaerlicks zoude moeten betaelen?

CONSTRUCTIE.

Men zal aenmercken wat Hooft-somme datmen op elck iaer in handen houdt diemen naer d'ierste conditie in handen niet en zoude ghehouden hebben / ende vinden als dan den interest van elcke Hooft-somme op elck iaer.

Maer wanter op elck iaer maer betaelinghe te doen en is van Hooft-somme die alleenelick een iaer gheloopen heeft / volght daer wt door het corollarium der 5. definitien in dese ende derghelijcke conditien (hoe wel nochtans van ghecomponeerden interest veraccordeert is) onmeughelick te zijne ghecomponeerden interest te rekenen / zoo dat dese questie moet ghesolueert worden door de maniere des derden exempels der ierster propositien. Ende hebben dit exempel hier alleenelick ghestelt als wesende een accident des interests weerdich ghenoteert.

EXEMPEL 6.

Eenen is schuldich binnen 12. iaeren 5000. lb. te weten alle iaere het $\frac{1}{12}$ dat is 416. lb. 13. ß. 4. gr. De vraeghe is wat die weerdich zijn teenemael ten eynde van de 12. iaeren rekenende ghecomponeerden interest den penninck 15. t'siaers.

NOTA.

De solutie van dese ende derghelijcke questien in ghecomponeerden profijtelicken interest en can door de laetste columne der voorgaende tafelê niet ghesolueert worden / ghelijck der ghelijcke questien in schadelicken interest der volghender 4. propositien daer mede ghesolueert worden /ende dat ouermidts de ghetaelen der seluer columnen voor beyde als schadelicken ende profijtelicken interest niet proportionael en zijn. Soo hebbe ick tot dier oorsaecke oock ghecalculeert tafelen als de voorgaende /dienende tot ghecomponeerden profijtelicken interest / Maer eer computation can be made for 16 years, to wit from the 7th year to the 23rd. According to the 1st example of the present proposition this will be $4,111\frac{381545}{2918905}$ lb, as above.

EXAMPLE 4.

A man owes 800 lb, to be paid at the end of 3 years, and another 300 lb 2 years later. What will these two sums be worth together at the end of 15 years, charging compound interest at 13 per cent a year?

PROCEDURE.

According to the foregoing 3rd example it will be found that the 800 lb will be worth $3,467\frac{991031}{1598907}$ lb and the 300 lb will be worth $1,018\frac{592674}{1598907}$ lb, and these two sums together amounting to $4,485\frac{1583705}{1598907}$ lb, this is the value of the 800 lb and the 300 lb together in 15 years.

EXAMPLE 5.

A man owes 224 lb present value. If he paid in 4 years, every year one fourth, to wit 56 lb, on condition of his paying yearly the compound interest at 12 per cent a year, what would he have to pay every year?

PROCEDURE.

It has to be found what Principal one keeps every year which according to the first condition one would not have kept, and then the interest on each Principal in every year has to be found.

But because every year a Principal has to be paid which has been put out at interest for one year only, it follows, according to the Sequel to the 5th definition, that in the present and similar conditions (although an agreement for compound interest has been made) it is impossible to charge compound interest, so that this question has to be solved in the manner of the third example of the first proposition. And we have merely given this example as an accidental feature of interest, worthy to be recorded.

EXAMPLE 6.

A man owes 5,000 lb, to be paid in 12 years, to wit: every year one twelfth, *i.e.* 416 lb 13 sh. 4 d. What will they be worth together at the end of the 12 years, charging compound interest at the 15th penny a year?

NOTE.

The solution of this and similar questions of compound profitable interest cannot be effected by means of the last column of the foregoing tables in the same way as similar questions of detrimental interest of the following 4th proposition can be solved therewith, such because the numbers of the said column are not proportional for detrimental as well as profitable interest. For this reason I have also computed tables like the foregoing, serving for compound profitable dit tractaet wtghegaen is / ben ghecomen ter kennisse van solutie van dese questie op een ander maniere / te weten zonder eyghene tafelen daer toe te moeten hebben: Waer door op dat dit tractaet simpelder zoude zijn / ende dat die verscheyden tafelen niet eer oorsaecke en zouden zijn van confusie dan ter contrarien van claerheydt / en hebben de zelue tafelen hier niet beschreuen / dan alleenelick op dat wy bethoonen zouden der seluer tafelen constructie / proportie / ende eyghenschappen met d'ander voorgaende tafelen / zullen hier alleenelick dier tafelen een stellen / te weten van den penninck 15.

CONSTRUCTIE VAN DESE TAFELE.

Deser tafelen constructie en heeft van de andere voorgaende gheen ander verschil / dan dat hier altijdt ghemultipliceert wordt (ter contrarien van de voorgaende tafelen) met het meeste ghetal / ende ghediuideert door het minste. Als dese tafel van den penninck 15. zijn ten iersten 10000000. ghemultipliceert met 16. ende t'productum wederom ghediuideert door 15. gheuende quotû 10666667. voor d'ierste iaer / welck ghetal wederom ghemultipliceert met 16. eñ t'productum wederom ghediuideert door 15. gheeft den quotus het tweede iaer / eñ soo voort met d'ander. Aengaende de constructie der laetster columnen / de zelue gheschiet door additien der ghetaelen der middelste columnê / ghelijck in de voorgaende tafelen / wtgenomen datmen hier bouen d'ierste iaer der middelste columnen sal stellen den wortel der tafelen / te weten 10000000. eñ de zelfde wortel noch eenmael bouen de laetste columne neuen het ierste iaer / ende voort salmen ordentlick de ghetaelen der middelste columnen adderen als in de voorgaende tafelen ghedaen is / zoo claerlicker in de onderschreuen tafele blijckt.

interest. But before the publication of this treatise another solution of this question came to my notice, to wit one for which no separate tables are needed. Therefore, in order that this treatise might be simpler and those different tables should not create confusion rather than clarity, we have not described the said tables here. But only in order to show their construction, proportion, and properties as compared with the other (foregoing) tables, we shall here give only one of these tables, to wit for the 15th penny.

CONSTRUCTION OF THIS TABLE.

The construction of this table differs from that of the other foregoing ones only in that here the multiplication is always effected (contrary to the foregoing tables) by the greatest number and the division by the smallest. Thus in this table of the 15th penny first of all 10,000,000 has been multiplied by 16 and the product divided again by 15, giving the quotient 10,666,667 for the first year, and when this number is multiplied again by 16 and the product divided again by 15, this gives the quotient for the second year, and so on with the others. As for the construction of the last column, this is effected by addition of the numbers of the central column, as in the preceding tables, except that here above the first year of the central column has to be put the root of the tables, to wit 10,000,000, and the said root once more above the last column, opposite the first year, and further the numbers of the central column have to be properly added up, as has been done in the preceding tables, as appears more clearly in the following table.

Tafele van Interest van
den penninck 15.

	10000000.	
1.	10666667.	10000000.
2.	11377778.	20666667.
3.	12136297.	32044445.
4.	12945383.	44180742.
5.	13808409.	57126125.
6.	14728970.	70934534.
7.	15710901.	85663504.
8.	16758294.	101374405.
9.	17875514.	118132699.
10.	19067215.	136008213.
11.	20338363.	155075428.
12.	21694254.	175413791.
13.	23140538.	197108045.
14.	24683241.	220248583.
15.	26328790.	244931824.
16.	28084043.	271260614.
17.	29956313.	299344657.
18.	31953401.	329300970.
19.	34083628.	361254371.
20.	36355870.	395337999.
21.	38779595.	431693869.
22.	41364901.	470473464.
23.	44122561.	511838365.
24.	47064065.	555960926.
25.	50201669.	603024991.
26.	53548447.	653226660.
27.	57118343.	706775107.
28.	60926233.	763893450.
29.	64987982.	824819683.
30.	69320514.	889807665.
31.		959128179.

De proportie van dese tafele met de voorgaende van den pennick 15. is dese: Soo wy nemen wt elcke deser tafelen twee ghelijcke iaeren / haere responderende ghetaelen in de middelste columne zullen zijn proportionael: als by exempel het dertichste iaer deser tafelen heeft alzulcken reden tot zijn ierste iaer / ghelijck d'ierste iaer van de voorgaende tafel van den penninck 15. tot zijn dertichste iaer / dat is ghelijck 69320514. tot 10666667. alsoo 9375000. tot 1442573. gheweert eenighe differêtie die daer valt op de laetste letter spruytende van weghen de resterende gebroken die men in de constructie verlorê laet / welck verschil hier van gheender estimen en is. Item ghelijck het dryentwintichste iaer deser tafelen tot zijn vijfde iaer / alsoo oock het vijfde iaer der voorgaender tafel van den penninck 15. tot zijn 23. iaer / ende zoo voort met alle d'ander.

Wt welcke proportie volght dat wy met eene deser tafelen zoo vele wtrichten connen / als men met alle beyde de tafelen zoude moghen doen / Maer in de laetste columne en gaet het niet also / te weten de ghetaelen responderende in ghelijcke tafelen op ghelijcke iaeren en zijn niet proportionael. Ghelijckerwijs als het dertichste iaer der laetster columnen deser tafele en heeft niet alzulcken reden tot zijn ierste / ghelijck het ierste iaer van de voorgaende tafel van den penninck 15. tot zij nlaetste / noch ter contrarien / noch op eenighe ander manieren en vallen dese termijnen proportionael / 't welck een oorsaecke was dat wij dese tafelen maeckten als voren gheseyt is.

Om dan de questie van dit exempel te solueren door dese tafel / salmen in de zelue zien wat ghetal datter in de laetste columne respondeert op het 12. iaer / wordt bevonden 175413791. daer naer van weghen de 12. iaeren / salmen nemen twaelf mael den wortel te weten 10000000. dat is 120000000. segghende 120000000. comen van 175413791. waer van zullen comen 5000 lb? facit $7308\frac{108055}{120000}$ lb. dat is 7308 lb. 18. ß. $1\frac{91}{100}$ gr.

Nu rester noch dese questie te solueren door onse ierste tafelê / de welcke wy voren gheseyt hebben generael te zijne / aldus:

CONSTRUCTIE VAN DIT 6. EXEMPEL.

Men sal sien wat alsulcke 5000. lb. ghereedt weerdich zijn naer de leeringhe des 6. exempels der volghende 4. propositie (*tis wel waer dat in alle stijl gherequireert* wordt datmen opereren zoude daert moghelick is wt voorgaende descriptie / ende niet wt volghende / maer ouermidts onse tafelen dienen tot dese ende de volghende propositie / dat is soo wel tot ghecomponeerden profijtelicken interest als tot schadelicken / volght daer wt dat dese twee laetste propositien malckanderen verclaeren moeten / waer wt wijder volght dat sommighe operatien deser propositien moeten bewesen worden wt het volghende) wordt bevondê $3369\frac{6275}{120000}$ lb.

Daer naer salmen sien wat dese somme weerdich is binnen 12. iaeren daer naer teenemael/ wordt bevonden door het ierste exempel deser proposition $7308\frac{5024756}{5531418}$ lb. dat is als voren 7308. lb. 18. ß. $2\frac{15386}{921003}$ gr. alleenelick isser differentie van een zeer cleyn deelken van 1. gr. van gheender estimen / ende dat van weghen dat de wterste perfectie (ghelijck oock in tabula sinuum ende veel anderen) in de tafelen niet en is.

The proportionality of this table to the foregoing one of the 15th penny is as follows: If we take two similar years from each of these tables, their corresponding numbers in the central column will be proportional. Thus, for example, the thirtieth year of the present table has to its first year the same ratio as the first year of the foregoing table of the 15th penny to its thirtieth year, *i.e.* 69,320,514 : 10,666,667, thus 9,375,000 : 1,442,573, except for some difference in the last digit, occasioned by the remaining fractions which are omitted in the construction, a difference which is insignificant here. Likewise, as the twenty-third year of this table is to its fifth year, thus also the fifth year of the preceding table of the 15th penny to its 23rd year, and so on with all the others.

From this proportionality it follows that with one of these tables we can effect as much as we might do with the two tables. But in the last column this is not possible, to wit: the numbers corresponding in similar tables to similar years are not proportional. As the thirtieth year of the last column of this table does not have to its first the same ratio as the first year of the foregoing table of the 15th penny to its last, these terms are proportional neither inversely nor in any other manner, which was the reason why we made these tables, as has been said above.

In order therefore to solve the question of this example by means of this table, it has to be ascertained therein what number in the last column corresponds to the 12th year. This is found to be 175,413,791. Thereafter, on account of the 12 years, one has to take twelve times the root, to wit 10,000,000, which is 120,000,000, saying: 120,000,000 comes from 175,413,791; what will 5,000 lb come from? This is $7{,}308\frac{108055}{120000}$ lb, that is 7,308 lb 18 sh. $1\frac{91}{100}$ d.

Now it still remains to solve this question by means of our first tables, which we have said above to be general, as follows:

PROCEDURE OF THIS 6th EXAMPLE.

Find what is the present value of this 5,000 lb according to the 6th example of the following 4th proposition (it is true that in good style it is required that, if possible, operations should be based on a preceding description, and not on a succeeding one, but since our tables serve for the present as well as for the following proposition, *i.e.* both for compound profitable and for detrimental interest, it follows that these two last propositions have to explain one another, from which it further follows that some operations of this proposition have to be proved from the following); this is found to be $3{,}369\frac{6275}{120000}$ lb.

Thereafter find what this sum will be worth 12 years later. By the first example of this proposition this is found to be $7{,}308\frac{5024756}{5531418}$ lb, that is, as above, 7,308 lb 8 sh. $2\frac{15386}{921003}$ d.; there is only a difference of a very small fraction of 1 d., of no significance, such because there is no extreme perfection in the tables (just as in sine tables and many others).

NOTA.

De volghende exempelen dependeren *ex alterna vel inuersa proportione* deser propositien.

EXEMPEL 7.

Eenen is schuldich ghereedt 400. lb. presenteert ten eynde van 10. iaeren 1037. lb. De vraeghe is / tegen wat ghecomponeerde interests reden dat betaelt ware.

CONSTRUCTIE.

Men sal segghê 1037. lb. gheuen 10000000. wat gheuen 400. lb? facit 3857281. t'zelfde ghetal salmen ten naesten zoecken door alle de tafelen op het thiende iaer / wordt bevonden in de tafel van 10. ten 100. al waer men vindt 3855434. waer door men zegghen zal dese interests reden te zijne teghen 10. ten 100. t'siaers bycans / maer want 3855434. wat minder zijn dan 3857281. soo zalmen zegghen dese interests reden een weynich minder te zijne dan teghen 10. ten 100.

Maer tot een perfecte solutie deser ende dergelijcke questien / ist noodich dat men onder zijn tafelen hebbe een tafel van alzulcken interests reden als daer questie af is / dies niet / zoo en kanmen de solutie maer bycans zegghen / t'welck in de practijcke oock dickmael ghenoech is.

EXEMPEL 8.

Men begheert te weten hoe langhe 800. lb. loopen zullen teghen ghecomponeerden profijtelicken interest van den penninck 17. t'siaers / om met haeren interest t'saemen weerdich te zijne 2500. lb.

CONSTRUCTIE.

Men sal zegghen 2500. lb. geuê 10000000. wat gheuen 800. lb? facit 3200000. t'zelfde ghetal salmen zoecken ten naesten ende meerder in de tafel van den penninck 17. wordt bevonden 3375606. responderende op het 19. iaer. *Ergo* 19. iaeren zullen de 800. lb. loopen. Maer om nu te vinden wat deel des iaers de voornoemde 800. lb. noch te loopen hebben / zoo zalmen de 3375606. multipliceren met 17. (met 17. van weghen den penninck 17.) gheeft productum 57385302. t'zelue zalmen diuideren door de 3200000. gheeft quotum 17 $\frac{2985302}{3200000}$ welcke 17. men verlaeten zal ende hebben alleene opsicht op het ghebroken / welcke ons alzulck een deel des iaers beteeckent als de 800. lb. noch bouen de 19. iaeren te loopen hebben / te weten in als $19\frac{2985302}{3200000}$ iaerê.

EXEMPEL 9.

Eenen ontfangt 700. lb. voor ghecomponeerden profijtelicken interest tegen 13. ten 100. t'siaers voor 9. iaeren. De vraeghe is wat d'Hooft-somme was.

CONSTRUCTIE.

Men zal zien in de tafel van 13. ten 100. wat ghetal datter respondeert op 9. iaeren / wordt bevonden 3328849. t'zelfde zalmen trecken van 10000000. rest 6671151. daer naer salmen zegghen / interest 6671151 heeft Hooft-somme 3328849. wat Hooft-somme zal hebben interest 700. lb? facit $349\frac{1962601}{6671151}$ lb.

NOTE.

The following examples depend on this proposition in alternate or inverse proportion.

EXAMPLE 7.

A man owes 400 lb present value and pays at the end of 10 years 1,037 lb. What was the rate of compound interest in this payment?

PROCEDURE.

Say as follows: 1,037 lb gives 10,000,000; what does 400 lb give? This is 3,857,281. Seek a number as close as possible to this through all the tables at the tenth year. It is found in the table of 10 per cent, where the number 3,855,434 is found. Therefore it has to be said that this rate of interest is almost 10 per cent a year, but because 3,855,434 is a little less than 3,857,281, it has to be said that this rate of interest is slightly less than 10 per cent.

But for a perfect solution of this and similar questions it is necessary to have among one's tables a table of the same rate of interest as that under consideration. If this is not the case, the solution can only be given approximately, which in practice is often sufficient.

EXAMPLE 8.

It is required to know how long 800 lb has to be put out at compound profitable interest of the 17th penny a year in order that its value together with that of its interest may be 2,500 lb.

PROCEDURE.

Say as follows: 2,500 lb gives 10,000,000; what does 800 lb give? This is 3,200,000. Seek a number as close as possible to and higher than this in the table of the 17th penny. We find 3,375,606, corresponding to the 19th year. Therefore the 800 lb has to be put out for 19 years. But in order to find for what part of a year the aforesaid 800 lb still has to be put out, multiply the 3,375,606 by 17 (by 17 on account of the 17th penny); this gives the product 57,385,302. Divide this by the 3,200,000; this gives the quotient $17\frac{2985302}{3200000}$. The 17 has to be discarded and reference has to be made only to the fraction, which indicates that part of a year for which the 800 lb still has to be put out over and above the 19 years, to wit: $19\frac{2985302}{3200000}$ years in all.

EXAMPLE 9.

A man receives 700 lb for compound profitable interest at 13 per cent a year for 9 years. What was the Principal?

PROCEDURE.

Look up in the table of 13 per cent what number corresponds to 9 years. This is found to be 3,328,849. Subtract this from 10,000,000. The remainder is 6,671,151. Thereafter say: an interest of 6,671,151 has for Principal 3,328,849; what Principal will an interest of 700 lb have? This is $349\frac{1962601}{6671151}$ lb.

DEMONSTRATIE.

Ghelijck int ierste exempel deser propositien hem heeft t'ghereede tot het ghene verschijnen zal binnen 8. iaeren daer naer rekenende profijtelicken interest teghen 11. ten 100. t'siaers (want zulck is de conditie des voornoemden exempels) alsoo heeft hem 4339266. tot 10000000. door de tafelê / ende zoo hem heeft 4339266. tot 10000000. alsoo heeft hem oock 380. lb. tot $875\frac{3142250}{4339266}$ lb. door de constructie. Ergo $875\frac{3142250}{4339266}$ lb. is des iersten exempels waere solutie. Sghelijcks zal ook zijn de demonstratie van alle die ander exempelen / welck wy om de cortheyt hier achter laetê. Alsoo dan wesende verclaert Hooft-somme tijdt ende interests reden van ghecomponeerdê profijtelicken interest / hebben wy ghevonden wat d'Hooft-somme met haeren interest bedraecht / t'welck gheproponeert was alsoo ghedaen te worden.

PROPOSITIE IIII.

Wesende verclaert Hooft-somme tijdt ende interests reden van ghecomponeerden schadelicken interest: Te vinden wat die ghereet ghelt weerdt is.

EXEMPEL 1.

Het zijn 700. lb. te betaelen ten eynde van thien iaeren. De vraeghe is wat die ghereet weerdich zijn / aftreckende ghecomponeerden interest teghen 12. ten 100. t'siaers.

CONSTRUCTIE.

Men sal sien in de tafel van 12. ten 100. wat ghetal datter respondeert op de 10. iaeren / wordt bevonden 3219732. waer door men segghen zal 10000000. gheuen 3219732. wat gheuen 700. lb? facit $225\frac{38124}{100000}$ lb.

EXEMPEL 2.

Het zijn 600. lb. te betaelen binnen $13\frac{1}{2}$ iaeren. De vraeghe is wat die weerdich zijn ghereedt aftreckende ghecomponeerden interest teghen 14. ten 100. t'siaers.

CONSTRUCTIE.

Men sal zien in de tafel van 14. ten 100. wat ghetal datter respondeert op het 13. iaer / wordt bevonden 1820695. t'zelfde zalmen multipliceren met 100. gheeft productum 182069500. t'welck men diuideren zal door 107. (te weten met 100. / ende 7. daer toe ghedaen van weghen een half iaer interest) gheeft quotum 1701584. t'welck een ghetal is dat in de tafel responderê zoude op het $13\frac{1}{2}$iaer / by aldien de tafelen met halue iaeren ghemaeckt waerê; daer naer salmen seggen 10000000. gheuen 1701584. wat gheuen 600. lb? facit $102\frac{9504}{100000}$ lb.

S'ghelijcks zal oock zijn d'operatie in alle andere deelen des iaers / want waerender tot eenighe iaeren dry maenden / zoo zoudemen (om dat 3. dry maenden is een vierendeel iaers) dan diuideren (in de plaetse daer bouen met 107. ghediuideert is) met $103\frac{1}{2}$/ ende soo voorts met alle ander deel des iaers / ghelijck van deser ghelijcke breeder ghetracteert is int 3. exempel der 2. propositien.

PROOF.

As in the first example of this proposition the present value is to that which will expire in 8 years, charging profitable interest at 11 per cent a year (for that is the condition of the aforesaid example), thus 4,339,266 is to 10,000,000 by the tables. And as 4,339,266 is to 10,000,000, thus also 380 lb to $875\frac{3142250}{4339266}$ lb by the procedure. Therefore $875\frac{3142250}{4339266}$ lb is the true solution of the first example. The same will also be the proof of all the other examples, which we here omit for brevity's sake. Hence, given Principal, time, and rate of compound profitable interest, we have found what the Principal with its interest amounts to, which had been proposed to be done.

PROPOSITION IV.

Given Principal, time, and rate of compound detrimental interest: to find what is the present value.

EXAMPLE 1.

A sum of 700 lb is to be paid at the end of ten years. What is its present value, subtracting compound interest at 12 per cent a year?

PROCEDURE

Look up in the table of 12 per cent what number corresponds to the 10 years. This is found to be 3,219,732. Therefore say: 10,000,000 gives 3,219,732; what does 700 lb give? This is $225\frac{38124}{100000}$ lb.

EXAMPLE 2.

A sum of 600 lb is to be paid in $13\frac{1}{2}$ years. What is its present value, subtracting compound interest at 14 per cent a year?

PROCEDURE.

Look up in the table of 14 per cent what number corresponds to the 13th year. This is found to be 1,820,695. Multiply this by 100. This gives the product 182,069,500, which divide by 107 (to wit, by 100 with 7 added thereto, because of half a year's interest). This gives the quotient 1,701,584, which is a *number which in the table would correspond to the* $13\frac{1}{2}$*th year, if the tables had* been made with half years. Thereafter say: 10,000,000 gives 1,701,584; what does 600 lb give? This is $102\frac{9504}{100000}$ lb.

The same will also be the operation for all other parts of a year, for if over and above a number of years there were three months, the division would then (because 3 months is one fourth of a year) have to be effected (instead of the above division by 107) by $103\frac{1}{2}$; and so on with all other parts of a year, as has been discussed more fully in the 3rd example of the 2nd proposition.

EXEMPEL 3.

Eenen is schuldich te betaelen binnen 5. iaeren 800. lb. ende binnen 4. iaeren daer naer noch 600. lb. De vraeghe is wat die t'saemen ghereedt weerdich zijn / aftreckende ghecomponeerden interest teghen 15. ten 100. t'siaers.

CONSTRUCTIE.

De 800. lb. op 5. iaeren zullen ghereedt weerdich zijn door het ierste exempel deser propositien $397\frac{74144}{100000}$ lb. ende de 600. lb. op 9. iaeren zullen ghereedt weerdich zijn door t'voornoemde ierste exêpel $170\frac{55744}{100000}$ lb. welcke twee sommê bedraeghende t'saemen $568\frac{29888}{100000}$ lb. is de solutie.

EXEMPEL 4.

Eenen is schuldich 2000. lb. te betaelen ten eynde van 27. iaeren. De vraeghe is wat die weerdich zijn te betaelen ten eynde van 9. iaeren aftreckende ghecomponeerden interest den penninck 19.

CONSTRUCTIE.

Men zal zien in de tafel van den penninck 19. wat ghetal datter respondeert op het 9. iaer / wordt bevonden 6302494. Oock mede wat ghetal datter respondeert op het 27. iaer / wordt bevonden 2503440. daer naer salmê zegghen 6302494. gheuen 2503440. wat gheuen 2000. lb? facit $794\frac{2699764}{6302494}$ lb.

Ofte anders machmen doen aftreckende voor 18. iaeren door het 1. exempel deser propositien.

EXEMPEL 5.

Eenen is schuldich te betaelen ten eynde van vier iaeren 360. lb. veraccordeert met zijnen crediteur die te betaelen in 4. payementen / te weten ten eynde van d'ierste iaer een vierendeel / tweede iaer noch een vierendeel / ende t'derde iaer noch een vierendeel / ende t'vierde iaer t'laetste vierendeel / midts aftreckende ghecomponeerden interest den penninck 16. De vraeghe is wat hy op elck iaer betaelen zal?

CONSTRUCTIE.

Men zal aenmercken wat penninghen datmen naer dese conditie verschiet diemê naer d'ierste conditie niet en zoude verschoten hebbê / nu dan wantmen naer dese conditie binnen een iaer betaelt t'vierendeel der sommen bedraeghende 90. lb. midts aftreckêde / etc. die mê naer d'ierste conditie binnê 3. iaeren naer daer ierst zoude betaelt hebben / volght daer wt datmen zien zal wat 90. lb. te betaelê binnen 3. iaeren / weerdich zijn ghereedt / wordt bevonden door het ierste exempel deser propositien $75\frac{33585}{1000000}$ lb. voor d'ierste paye.

Ende om der ghelijcke redenen zalmen bevinden 90. lb. op 2. iaerê weerdich te zijne $79\frac{723188}{1000000}$ lb. voor de tweede paye.

Ende om der ghelijcke redenen zalmen bevinden 90. lb. op een iaer weerdich te zijne $84\frac{12}{17}$ lb. voor de derde paye.

EXAMPLE 3.

A man owes 800 lb, to be paid in 5 years, and another 600 lb 4 years later. What is the present value of the two together, subtracting compound interest at 15 per cent a year?

PROCEDURE.

The present value of the 800 lb to be paid in 5 years by the first example of this proposition will be $397\frac{74144}{100000}$ lb and the present value of the 600 lb to be paid in 9 years by the aforesaid first example will be $170\frac{55744}{100000}$ lb, and these two sums together amounting to $568\frac{29888}{100000}$ lb, this is the solution.

EXAMPLE 4.

A man owes 2,000 lb, to be paid at the end of 27 years. What is the value to be paid at the end of 9 years, subtracting compound interest at the 19th penny?

PROCEDURE.

Look up in the table of the 19th penny what number corresponds to the 9th year; this is found to be 6,302,494. Also what number corresponds to the 27th year; this is found to be 2,503,440. Thereafter say: 6,302,494 gives 2,503,440; what does 2,000 lb give? This is $794\frac{2699764}{6302494}$ lb.

Or otherwise it may be done by subtracting for 18 years, according to the 1st example of this proposition.

EXAMPLE 5.

A man owes 360 lb, to be paid at the end of four years. He agrees with his creditor to pay them in 4 payments, to wit at the end of the first year one fourth, the second year again one fourth, and the third year again one fourth, and the fourth year the last one fourth, subtracting compound interest at the 16th penny. What does he have to pay every year?

PROCEDURE.

It has to be found what money is disbursed according to this condition which would not have been disbursed according to the first condition. Now because according to this condition in a year one fourth of the sum is paid, which amounts to 90 lb, subtracting, etc., which according to the first condition would not have been paid until 3 years later, it follows that it has to be found what is the present value of 90 lb to be paid in 3 years. By the first example of this proposition this is found to be $75\frac{33585}{1000000}$ lb for the first payment.

And for similar reasons the present value of 90 lb to be paid in 2 years will be found to be $79\frac{723188}{1000000}$ lb for the second payment.

And for similar reasons the present value of 90 lb to be paid in one year will be found to be $84\frac{12}{17}$ lb for the third payment.

Ende want de laetste paye op zulcken conditie betaelt wordt als d'ierste conditie was / en zal die winnen noch verliesen / maer zijn van 90. lb.

EXEMPEL 6.

Het zijn 324. lb. te betaelen binnen 6. iaeren / te weten 54. lb. elcken iaere. De vraeghe is wat die weerdich zijn ghereedt ghelt / aftreckende gecomponeerden interest dê penninck 16. t'siaers.

NOTA.

Voor alzulcke questien als dit exempel een is/te weten daer betaelinghe in gheschieden op vervolghende iaerê / ende het een iaer zoo veel als het ander iaer / daer toe dient ons de laetste columne in elcke tafele. Alsoo dan om wt onse tafelen proportionale ghetaelen te crijghen met de ghene daer questie af is / soo salmen den wortel der tafelen te weten 10000000. altijdt moeten multiplicerê met soo veel iaeren als daer questie af is / want t'productum heeft dan zulcken reden tot het ghetal responderende op het iaer daer questie af is / ghelijck d'Hooft-somme met haeren interest / ghelijck alles claerder zijn zal wt d'exempelen.

CONSTRUCTIE.

Men zal zien in de tafel van den penninck 16. wat ghetal datter in de laetste columne respondeert op het 6. iaer / wordt bevonden 48789356. Daer naer van weghen de 6. iaeren salmen nemen 6. mael 10000000. dat is 60000000. seggende 60000000 gheuen 48789356. wat gheuen 324. lb? facit voor solutie $263\frac{27751344}{60000000}$ lb.

EXEMPEL 7.

Eenen is schuldich 800. lb. te weten ten eynde van zes iaeren 50. lb. ende voorts alle iaere daer naer 50. lb. tot de volle betaelinghe / welck strecken zal (tellende van het beghinsel af) tot het tween twintichste iaer. De vraeghe is wat die ghereedt weerdich zijn / rekenende gecomponeerden interest den penninck 18.

CONSTRUCTIE.

Men zal zien wat 800. lb. weerdich zijn int beghinsel van het zesde iaer / t'welck zoo veel is als oftmen zochte wat alzulcke 800. lb. te betaelen op 16. iaeren weerdich zijn ghereedt / wordt bevonden door het voorgaende 6. exempel $521\frac{13152}{160000}$ lb. ende soo veel zijn die 800. lb. weerdich int beghinsel van het zeste iaer ofte (dat het zelfde is) ten eynde van het vijfde iaer.

Daer naer zalmen zien door het ierste exempel deser propositien wat de zelfde $521\frac{13152}{160000}$ lb. op vijf iaeren weerdich zijn ghereedt ghelt / wordt bevonden voor solutie $397\frac{1039615363808}{1600000000000}$ lb.

And because the last payment is made on the same condition as the first, this will neither gain nor lose, but be 90 lb.

EXAMPLE 6.

A sum of 324 lb is to be paid in 6 years, to wit each year 54 lb. What is their present value, subtracting compound interest at the 16th penny a year?

NOTE.

For all such questions as this example, to wit where payments are made in successive years, one year as much as the other, the last column in each table serves. Therefore in order to obtain from our tables numbers proportional to those under consideration, the root of the tables, to wit 10,000,000, will always have to be multiplied by as many years as are under consideration, for the product then has to the number corresponding to the year in question the same ratio as the Principal in question to this Principal with its interest, as will all be clearer from the examples.

PROCEDURE [1]).

Look up in the table of the 16th penny what number in the last column corresponds to the 6th year. This is found to be 48,789,356. Thereafter, on account of the 6 years, take 6 times 10,000,000, that is 60,000,000; and say: 60,000,000 gives 48,789,356; what does 324 lb give? The solution is $263\frac{27751344}{60000000}$ lb.

EXAMPLE 7.

A man owes 800 lb, to wit at the end of six years 50 lb and further every year thereafter 50 lb until payment is complete, which will extend (counting from the beginning) to the twenty-second year. What is the present value, charging compound interest at the 18th penny?

PROCEDURE.

It has to be found what is the value of 800 lb at the beginning of the sixth year, which is as much as if it were sought what is the present value of that 800 lb, to be paid in 16 years. By the foregoing 6th example this is found to be $521\frac{13152}{160000}$ lb, and this is the value of that 800 lb at the beginning of the sixth year, or (which is the same) at the end of the fifth year.

Thereafter it has to be found by the first example of this proposition what is the present value of the said $521\frac{13152}{160000}$ lb to be paid in five years. The solution is found to be $397\frac{1039615363808}{1600000000000}$ lb.

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1) The French edition of 1585 omits, as everywhere else, the reference to Trenchant (See the Introduction, p. 19). A note was added, which will be found in the Supplement.

NOTA.

De volghende exempelen dependeren *ex alterna vel inuersa proportione* deser propositien.

EXEMPEL 8.

Eenen is schuldich ten eynde van 17 iaeren 700. lb. zijn crediteur schelt hê quijte met 292. lb. ghereedt. De vraeghe is teghen wat ghecomponeerde interestsreden dat afghetrocken waere.

CONSTRUCTIE.

Men zal zegghen 700. lb. gheuê 10000000. wat 292. lb? facit 4171429. t'zelfde ghetal zalmen zoecken ten naesten door alle de tafelen op het zeuenthiende iaer; wordt bevonden in de tafel van den penninck 19. daermen vindt 4181203. waer door men zegghen zal dese interestsreden te zijne tegê den penninck 19. t'siaers bycants / maer want 4171429. wat minder is dan 4181203. soo salmen segghen desen penninck een weynich minder te zijne dan 19. te weten den penninck 18. met eenich ghebroken.

Maer tot een perfecte solutie deser ende dergelijcke questien / ist noodich datmen onder zijn tafelen hebbe een tafel van alzulcken interest reden als daer questie af is / dies niet / zoo en kanmê de solutie maer bycants zegghen / t'welck in de practijcke oock dickmael ghenoech is.

EXEMPEL 9.

Eenen is schuldich te betaelen teenemael binnen zekere iaeren 1400. lb. ende betaelt die ghereedt met 107. lb. aftreckende ghecomponeerden interst teghen 13. ten 100. t'siaers. De vraeghe is binnen hoe veel iaeren die 1400. lb. te betaelen waeren.

CONSTRUCTIE.

Men zal zegghen 1400. lb. gheuê 10000000. wat 107. lb? facit 764286. t'zelfde ghetal salmen soecken ten naestê ende meerder in de tafel van 13. ten 100. wordt bevonden 767985. responderende op het 21. iaer. Ergo binnen 21. iaeren waeren de 1400. lb. te betaelen. Ende om nu te vinden wat deel des iaers datter bouen de voor noemde 21. noch was / salmen zegghen 764286. gheuen 767985. wat 100? facit $100\frac{369900}{764286}$ van welcken facit men trecken zal 100. de reste is $\frac{369900}{764286}$ welcker resten $\frac{1}{13}$ ($\frac{1}{13}$ van weghen 13. ten 100.) is het deel des iaers datter noch bouen de 21. iaeren was / te weten t'saemen $21\frac{369900}{9935718}$ iaeren.

EXEMPEL 10.

Eenen ontfangt 1100. lb. ende hem was afghetrocken ghecomponeerden interest teghen den penninck 16. voor 18. iaeren. De vraeghe is / wat d'Hooft-somme was.

CONSTRUCTIE.

Men zal zien in de tafel van den penninck 16. wat ghetal datter respôdeert op het 18. iaer / wordt bevonden 3357988. Daer naer zalmen zegghen 3357988. comen van 10000000. waer van comen 1100. lb? facit d'Hooft-somme $3275\frac{2589300}{3357988}$ lb.

NOTE.

The following examples depend on this proposition in alternate or inverse proportion.

EXAMPLE 8.

A man owes 700 lb, to be paid at the end of 17 years. His creditor quits him with 292 lb present value. What is the rate of compound interest at which the subtraction has been made?

PROCEDURE.

Say as follows: 700 lb gives 10,000,000; what does 292 lb give? This is 4,171,429. Seek a number as close as possible to this throughout the tables at the seventeenth year. It is found in the table of the 19th penny, where is found 4,181,203, for which reason it has to be said that this rate of interest is almost at the 19th penny a year, but because 4,171,429 is slightly less than 4,181,203, it has to be said that this penny is a little less than 19, to wit the 18th penny with a fraction.

But for a perfect solution of this and similar questions it is necessary to have among one's tables a table of such a rate of interest as the one in question. If this is not the case, the solution can only be given approximately, which in practice is often sufficient.

EXAMPLE 9.

A man owes 1,400 lb, to be paid at once after a certain number of years, and pays their present value of 107 lb, subtracting compound interest at 13 per cent a year. After how many years was this 1,400 lb to be paid?

PROCEDURE.

Say as follows: 1,400 lb gives 10,000,000; what does 107 lb give? This is 764,286. Seek a number as close as possible to and higher than this in the table of 13 per cent. There is found 767,985, corresponding to the 21st year. Therefore the 1,400 lb had to be paid in 21 years. And in order to find what part of a year there was over and above the aforesaid 21, say: 764,286 gives 767,985; what does 100 give? This is $100\frac{369900}{764286}$, from which 100 has to be subtracted. The remainder is $\frac{369900}{764286}$, $\frac{1}{13}$ of which remainder ($\frac{1}{13}$ on account of 13 per cent) is the part of a year there was over and above the 21 years, to wit: together $21\,\frac{369900}{9935718}$ years.

EXAMPLE 10.

A man receives 1,100 lb, compound interest at the 16th penny for 18 years having been subtracted. What was the Principal?

PROCEDURE.

Look up in the table of the 16th penny what number corresponds to the 18th year. This is found to be 3,357,988. Thereafter say: 3,357,988 comes from 10,000,000; what does 1,100 lb come from? The Principal is $3{,}275\frac{2589300}{3357988}$ lb.

EXEMPEL 11.

Eenen wordt afghetrocken 2022. lb. voor ghecomponeerden interest van 13. iaeren tegen 9. ten 100. t'siaers. De vraeghe is wat d'Hooft-somme was.

CONSTRUCTIE.

Men zal zien in de tafel van 9. ten 100. wat ghetal datter respondeert op het 13. iaer / wordt bevonden 3261786. t'zelfde zalmen trecken van 10000000. lb. rest 6738214. Daer naer salmen segghen 6738214. heeft Hooft-somme 10000000. wat Hooft-somme zal hebben 2022. lb? facit $3000\frac{5358000}{6738214}$ lb.

NOTA.

Dese dry volghende exempelen worden ghesolueert door de laetste columne der tafelen.

EXEMPEL 12.

Eenê is schuldich 33000. lb. alle iaere 1500. lb. tot 22. iaeren toe/ende zijn crediteur schelt hem quijte met 15300. lb. ghereedt ghelt. De vraeghe is teghen wat ghecomponeerde interestsreden dat afghetrocken is.

CONSTRUCTIE.

Men zal zeggen 33000. lb. gheuê 220000000. (te weten 10000000. ghemultipliceert met 22. iaeren) wat gheuê 15300. lb? facit 102000000. dit ghetal salmen ten naesten soecken door alle de tafelê in de laetste columne op het 22. iaer / wordt bevondê in de tafel van 8. ten 100. al waermen vindt 102007429. waer door men zegghen zal dese interests reden te zijne van 8. ten 100. t'siaers bycants / maer want 102007429. wat meerder is dan 102000000. soo salmen seggen dese interests reden wat meerder te zijne dan teghen 8. ten 100. te weten 8. met eenich zeer cleyn ghebroken.

Maer tot een perfecte solutie deser ende der gelijcke questien / ist noodich datmen onder zijn tafelen hebbe een tafel van alzulcken interests reden als daer questie af is.

EXEMPEL 13.

Eenen is schuldich te betaelen zeker somme / te weten alle iaere een zesten deel der zeluer sommen / zes iaeren lanck gheduerende; veraccordeert met zijnen crediteur die te betaelen ghereedt / midts aftreckende ghecomponeerden interest den penninck 16. ende gheeft hem ghereedt 263. lb. De vraeghe is wat d'Hooft-somme was.

CONSTRUCTIE.

Men zal zien in de tafel van de penninck 16. in de laetste columne wat ghetal datter respondeert op het 6. iaer / wordt bevonden 48789356. Daer naer salmen zegghen 48789356. comen van 60000000. (te weten 10000000. ghemultipliceert met 6. iaeren) waer van comen 263. lb? facit Hooft-somme $323\frac{21038012}{48789356}$ lb.

EXAMPLE 11.

A man receives a sum of money, 2,022 lb of compound interest at 9 per cent a year for 13 years having been subtracted. What was the Principal?

PROCEDURE.

Look up in the table of 9 per cent what number corresponds to the 13th year. This is found to be 3,261,786. Subtract this from 10,000,000 lb. The remainder is 6,738,214. Thereafter say: 6,738,214 has for Principal 10,000,000; what Principal will 2,022 lb have? This is $3{,}000\frac{5358000}{6738214}$ lb.

NOTE.

The three following examples are solved by means of the last columns of the tables.

EXAMPLE 12.

A man owes 33,000 lb, 1,500 lb every year, up to 22 years, and his creditor quits him with 15,300 lb present value. At what rate of compound interest was the subtraction made?

PROCEDURE.

Say as follows: 33,000 lb gives 220,000,000 (to wit: 10,000,000 multiplied by 22 years); what does 15,300 lb give? This is 102,000,000. Seek a number as close as possible to this throughout the tables in the last column at the 22nd year. It is found in the table of 8 per cent, where is found 102,007,429, so that it has to be said that this rate of interest is approximately 8 per cent a year; but because 102,007,429 is a little more than 102,000,000, it has to be said that this rate of interest is slightly more than 8 per cent, to wit 8 *plus* a very small fraction.

But for a perfect solution of this and similar questions it is necessary to have among one's tables a table of such a rate of interest as the one in question.

EXAMPLE 13.

A man owes a certain sum, to wit that every year one sixth of this sum has to be paid, during six years. He agrees with his creditor to pay its present value, subtracting compound interest at the 16th penny; and he gives him 263 lb present value. What was the Principal?

PROCEDURE.

Look up in the table of the 16th penny in the last column what number corresponds to the 6th year. This is found to be 48,789,356. Thereafter say: 48,789,356 comes from 60,000,000 (to wit 10,000,000 multiplied by 6 years); what does 263 lb come from? The Principal is $323\ \frac{21038012}{48789356}$ lb.

EXEMPEL 14.

Eenen is schuldich te betaelen zeker somme / te weten alle iaere het $\frac{1}{27}$ der zeluer sommen 27. iaeren lanck gheduerende / veraccordeert met zijn crediteur die te betaelen ghereedt / mits aftreckêde 4010. lb. voor ghecomponeerden interest tegen 14. ten 100. De vraeghe is wat d'Hooft-somme was.

CONSTRUCTIE.

Men zal zien in de tafel van 14. ten 100. wat ghetal datter respondeert in de laetste columne op het 27. iaer / wordt bevonden 69351565. t'zelfde zalmen aftrecken van 270000000. (te weten 10000000. ghemultipliceert met 27. iaeren) rest 200648435. Daer naer salmê seggê 200648435. compt van 270000000. waer van zal comen 4010. lb? facit voor solutie $5396\frac{1044470}{200648435}$ lb.

DEMONSTRATIE.

Aenghesien int ierste exempel deser propositien gheseyt is 700. lb. te betaelen ten eynde van 10. iaeren / ghereet ghelt weerdich te zijne $225\frac{38124}{100000}$ lb. aftreckende ghecomponeerden interest teghen 12. ten 100. t'siaers / volght daer wt dat indien mê de zelue $225\frac{38124}{100000}$ lb. terstont op interest leyde teghen den voornoemden interest van 12. ten 100. dat de selue Hooft-somme met haeren interest (zoo d'operatie goedt is) zullen moeten t'saemen bedraeghen 700. lb.

Alsoo dan rekenende dien interest naer de leeringhe des ierstê exempels der tweeder propositien / zal bedraeghen met haere Hooft-somme 700. lb. waer wt besloten wordt de constructie goedt te zijne.

S'ghelijcks sal ook zijn de demonstratie van d'ander exempelen deser propositiê / welcke wy of de cortheydt achter laeten.

Alsoo dan wesende verclaert Hooft-somme tijt ende intersts reden van ghecomponeerden schaedelicken interest / hebben wy ghevonden wat die gereedt weerdich is/t'welck gheproponeert was alsoo ghedaen te worden.

APPENDIX.

Ten laetsten heeft mij goedt ghedocht een generale reghel hier te beschrijuen / om van twee ofte meer conditien de profijtelickste te kennen / ende hoe veel zy profijtelicker is dan d'ander / want hier in is by ghevalle de principaele nutbaerheydt deser tafelen gheleghen / ende dat ouermidts trafiquerende persoonen malckanderen daghelicks conditien voorstellen / welcker conditien de beste dickmael gheen van beyden bekent en is.

Om dan metten cortsten dien reghel te verclaeren / zegghe ick / dat men zien zal wat elcke gheproponeerde conditie ghereedt weerdich is in respect van eenighe interests reden /ende dat door de leeringhe van eenighe der voorgaende exempelen / welcker ghereeder sommen differentie betoont hoe veel d'een conditie beter is dâ d'ander, t'welck door exempel claerder zijn zal.

EXEMPEL.

Eenen is schuldich 32500. lb. te wetê 12000. lb. ghereedt ende 6500. lb. binnen 3. iaeren / ende de resterende 14000. lb. aldus / te weten op het vierde iaer 500. lb.

EXAMPLE 14.

A man owes a certain sum, to wit that every year he has to pay $\frac{1}{27}$ of this sum, during 27 years. He agrees with his creditor to pay its present value, subtracting 4,010 lb for compound interest at 14 per cent. What was the Principal?

PROCEDURE.

Look up in the table of 14 per cent what number corresponds in the last column to the 27th year. This is found to be 69,351,565. Subtract this from 270,000,000 (to wit 10,000,000 multiplied by 27 years); the remainder is 200,648,435. Thereafter say: 200,648,435 comes from 270,000,000; what will 4,010 lb come from? The solution is $5{,}396\frac{1044740}{200648435}$ lb.

PROOF.

Since in the first example of this proposition it has been said that the present value of 700 lb to be paid at the end of 10 years is $225\frac{38124}{100000}$ lb, subtracting compound interest at 12 per cent a year, it follows that if this $225\frac{38124}{100000}$ lb is put out at interest at once at the aforesaid rate of 12 per cent, the said Principal with its interest (if the operation is correct) will have to amount together to 700 lb.

Thus, charging the interest according to the first example of the second proposition, with its Principal it will amount to 700 lb, from which it is concluded that the procedure was correct.

The same will also be the proof of the other examples of this proposition, which we omit for brevity's sake.

Hence, given the Principal, the time, and the rate of compound detrimental interest, we have found the present value, which had been proposed to be done.

APPENDIX.

Finally it seemed suitable to me to describe here a general rule for finding which is the most profitable of two or more conditions, and by how much it is more profitable than the other, for in this consists perhaps the principal usefulness of these tables, such because businessmen will daily propose conditions to one another, while frequently neither of the two knows which condition is the best.

In order to set forth this rule as shortly as possible, I say that it has to be found what is the present value of each proposed condition in respect to a given rate of interest, such in accordance with one of the foregoing examples, the difference between these present values showing by how much one condition is better than the other, which will be clearer from an example.

EXAMPLE.

A man owes 32,500 lb, to wit 12,000 lb present value and 6,500 lb in 3 years, and the remaining 14,000 lb as follows: the fourth year 500 lb, and further ende voorts alle iaere daer naer 500. lb. totte volle betaelinghe / welcke aenloopen sal 28. iaeren. Ende hem wordt ghepresenteert te betaelen ghereedt 6000. lb. ende ten eynde van 4. iaeren noch 5000. lb. ende resterende 21000. lb. aldus; te weten op het vijfde iaer 3000. lb. ende voort alle iaere daer naer 3000. lb. tot de volle betaelinghe / hetwelck aenloopen zal 7. iaeren. De vraeghe is welcke conditie de beste is voor den crediteur / ende hoe vele sy beter is dan d'ander / rekenende ghecomponeerden interest den penninck 16.

CONSTRUCTIE.

De 12000. lb. die ghereedt te betaelen zijn ghereedt weerdt	12000 lb.
Ende de 6500. lb. die binnen 3. iaerê te betaelê zijn / zijn reedt weerdich door het 1. exempel der 4. prop.	$5419\frac{9225}{100000}$ lb.
Ende de 14000. lb. die te betaelen zijn alle iaere 500. lb. tot 28. iaeren toe / beghinnende van het vierde iaer tot het 32. iaer / zijn ghereedt weerdich door het 7. exempel der 4. prop.	$5448\frac{42830451195}{280000000000}$ lb.
Welcke dry sommê voor de weerde in ghereeden ghelde van d'ierste conditie bedraeghen	$22867\frac{68660451195}{280000000000}$ lb.
Nu volght de calculatie vande tweede conditie.	
De 6000. lb. ghereedt te betaelen zijn ghereet weerdt	6000. lb.
Ende de 5000. lb. te betaelen ten eynde van het 4. iaer / zijn ghereedt weerdt door het 1. exemp. der 4. prop.	$3923\ \frac{3245}{10000}$ lb.
Ende de 21000. lb. die te betaelen zijn alle iaere 3000. lb. tot 7. iaeren toe beginnende van het vijfde iaer tot het 12. iaer zullen ghereedt weerdt zijn door het 7. exempel der 4. prop.	$13024\frac{647522600753}{700000000000}$ lb.
Welcke dry sommê voor de weerde in gereeden ghelde van de tweede conditie bedraeghen	$22948\frac{174672600753}{700000000000}$ lb.

Alsoo dan de tweede conditie (want zy meer bedraecht dan d'ierste) is beter voor den crediteur dan d'ierste. Nu dan afghetrocken de ghereede weerde der ierste conditie van de ghereede weerde der tweeder conditien/rester $81\ \frac{12085891062}{2800000000000}$ lb. ende soo veel is de laetste conditie beter voor den crediteur dan d'ierste / welcke solutie met veel anderen dier ghelijcke daghelicks in praxi te voren comende en zouden zonder t'behulp van dese tafelê niet dan door eenen onestimeerlicken aerbeydt connen ghegheuen worden.

ANDER EXEMPEL.

Men begheert te weten hoe veel 2000. lb. ghereedt op 7. iaeren beter zijn / rekenende ghecomponeerden interest teghen 4. ten 100. alle vierendeel iaers: dan de zelue 2000. lb. ghereedt op zeuen iaeren rekenende gecomponeerden interest dê penninck 16. t'siaers.

every succeeding year 500 lb until payment is complete, which will take 28 years. An offer is made to him that he shall pay 6,000 lb present value and at the end of 4 years 5,000 lb more, and the remaining 21,000 lb as follows: the fifth year 3,000 lb and further every succeeding year 3,000 lb until payment is complete, which will take 7 years. The question is which condition is the best for the creditor, and by how much it is better than the other, charging compound interest at the 16th penny.

PROCEDURE.

The present value of the 12,000 lb to be paid at present is 12,000 lb.

And the present value of the 6,500 lb which has to be paid in 3 years, by the 1st example of the 4th proposition, is $5,419 \frac{9225}{100000}$ lb

And the present value of the 14,000 lb, of which every year 500 lb has to be paid, during 28 years, beginning from the fourth year up to the 32nd year, by the 7th example of the 4th proposition, is $5,448\frac{42830451195}{280000000000}$ lb

The present value of the three sums on the first conditions amounts to $22,867\frac{68660451195}{280000000000}$ lb

Now follows the computation of the second condition.

The present value of the 6,000 lb to be paid at present is 6,000 lb.

And the present value of the 5,000 lb to be paid at the end of the 4th year, by the 1st example of the 4th proposition, is $3,923\frac{3245}{10000}$ lb

And the present value of the 21,000 lb, of which every year 3,000 lb has to be paid, up to 7 years, beginning from the fifth year to the 12th year, by the 7th example of the 4th proposition, will be $13,024\frac{647522600753}{700000000000}$ lb

The present value of the three sums on the second condition amounts to $22,948\frac{174672600753}{700000000000}$ lb

The second condition therefore (because it is more than the first) is better for the creditor than the first. And when the present value on the first condition is subtracted from the present value on the second condition, there remains $81\frac{12085891062}{2800000000000}$ lb, and by so much the last condition is better for the creditor than the first; and this solution and many other similar cases, which are of daily occurrence in practice, could not be given without the aid of these tables unless with incalculable exertion.

OTHER EXAMPLE.

It is required to know by how much 2,000 lb present value is better in 7 years, charging compound interest at 4 per cent every quarter of a year, than the said 2,000 lb present value in seven years would be, charging compound interest at the 16th penny a year.

NOTA.

Dese conditien zouden in simpelen interest gelijck zijn / maer in ghecomponeerden interest is de differentie groot.

CONSTRUCTIE.

Men zal zien in de tafel van 4. ten 100. wat 2000. lb. ghereedt met haeren interest bedraegen op 28. termijnen / (want 28. zulcke termijnen maken 7. iaeren) wordt bevonden $5997\frac{1372316}{3334772}$ lb.

Daer naer salmen zien wat 2000. lb. met haeren interest bedraegen op zeuen iaeren teghen 16. ten 100. t'siaers / wordt bevonden door het 1. exempel der 3. propositiê $5652\frac{1556660}{3538295}$ lb. Nu dâ afgetrockê $5652\frac{1556660}{3538295}$ lb. van $5997\frac{1372316}{3334772}$ lb. rest bycants 345. lb. ende soo veel bedraecht den interest van d'ierste conditie meer dan den interest van de laetste.

Alsoo dan alsvoren gheseyt is / salmê in alle anderen dier ghelijcken de ghereede weerde zoecken van verscheyden conditiê / ende haere differentien zullen de profijtelickste conditie betoonen.

NOTA.

Soo iemandt te opereren hadde in cleyne sommen / zoude moghen twee oft dry cijffer letterê van de ghetaelen der tafelen min ghebruycken / die van achteren af cortende / midts der ghelijcke menichte van letteren / oock afcortende van de wortel der tafel / als dies ghelijcke in *tabula sinuum* ende meer andere oock de ghebruyck is / wantet op cleyne sommen gheen merckelicke differentie en can brengen / iae dickmael veel minder dan de weerde van den minsten penninck die der ghemunt wordt: Maer op groote sommen zoudet merckelijcker zijn. Daer om hebbê wy onse tafelen ghemaeckt dienende zoo wel tot groote notabele sommen / ghelijck dickmael zijn penninghen van Banckiers / Potentatê / Prouincien ende dierghelijcke / als tot cleyne sommen.

FINIS.

NOTE.

These conditions would be equal in simple interest, but in compound interest the difference is great.

PROCEDURE.

Look up in the table of 4 per cent what 2,000 lb present value with its interest amounts to in 28 terms (for 28 such terms make 7 years); this is found to be $5{,}997\frac{1372316}{3334772}$ lb.

Thereafter it has to be found what 2,000 lb with its interest amounts to in seven years at 16 per cent a year; by the 1st example of the 3rd proposition this is found to be $5{,}652\frac{1556660}{3538295}$ lb. When $5{,}652\frac{1556660}{3538295}$ lb is subtracted from $5{,}997\frac{1372316}{3334772}$ lb, there remains approximately 345 lb, and by so much the interest on the first condition is more than the interest on the last one.

Thus, as has been said above, in all similar cases the present value on different conditions has to be sought, and their difference will show which is the most profitable condition.

NOTE.

If a man had to operate with small sums, he might omit two or three digits from the numbers of the tables, abbreviating them on the right, provided the root of the table were also abbreviated by the same number of digits, such as is also commonly done in sine tables and more such tables, for on small sums this cannot make any appreciable difference, yea, often much less than the value of the smallest coin that is minted. But on large sums it would be more perceptible. For that reason we have made our tables so that they may serve for large and notable sums (such as often occur with Bankers, Potentates, Provinces, and the like) as well as small sums.

END

SUPPLEMENT (1590)

EXEMPEL 7.

Het zijn 324 lb / te betalê binnê 6 iaren / te wetê 54 lb t' siaers. Vrage wat de selue weerdich zijn gereet gelt / aftreckêde simpelê interest tegê 12 tê 100?

CONSTRUCTIE.

Men sal sien wat ghelt datmen nae dese conditie verschiet / datmen na d'eerste niet en soude verschoten hebben. Nu dan wantmen na dese conditie verschiet 54 lb die te betalen waren binnen 1 iaer daer nae / soo moetmen sien hoe veel de selue 54 lb te betalen binnen een iaer / weerdich zijn ghereedt gelt / ende wort bevonden deur het voorgaende eerste exempel van dese propositie / $48\frac{3}{14}$ lb. Ende om der gelijcke redenen sullen ander 54 lb / op 2 iaeren weerdich zijn gereet $43\frac{17}{31}$ lb. Eñ de derde 54 lb op 3 iaren $39\frac{12}{17}$ lb. Eñ de vierde 54 lb op 4 iarê $36\frac{18}{37}$ lb. Ende de vijfde 54 lb op de 5 iarê $33\frac{3}{4}$ lb. Eñ de laetste 54 lb op 6 iaren $31\frac{17}{43}$ lb. Eñ de somme der bouê schreuen ses partien is voor solutie $233\frac{2356847}{23476796}$ lb.

NOTA.

Maer want dit moeyelijck is voor elck termijn een bysonder reeckeninge te maken / *als hier bouen / voornamelick* alst van veel iaeren of termijnen is / soo machmen tafelen maken / deur welcke mê sulcx sal moghen solveren met een werckinghe aldus:

Om te maken eê tafel van 12 ten 100 / men sal nemen eenich groot getal / waer af d'eerste letter sy 1 / ende al d'ander 0; als by voorbeelt 10000000, t'welck wy noemê wortel des tafels: seggende 112 (te weten capitael 100 / metten interest van een iaer) geuen 100, wat 10000000? compt 8928571 / als ghetal dienende voor t'eerste iaer. Aengaende het ouerschot / dat laetmen verloren gaen / als van geender acht zijnde. Voorts om te vinden t'getal van 2 iaeren / mê sal seggen 124 (te weten 100 capitael / metten interest van twee iaren) geuen 100. wat 10000000? compt 8064516 / de selue vergaert tot 8928571 / maken 16993087 / voor t'getal der twee iaren. Daer nae om te vinden het ghetal der drie iaren / men

SUPPLEMENT

ADDITIONS AND MODIFICATIONS, FOUND IN THE EDITION OF 1590

Modified text of Example 7, p. 45.

A sum of 324 lb is to be paid in 6 years, to wit 54 lb a year. What is the present value of this sum, subtracting simple interest at 12 per cent?

PROCEDURE.

It has to be found what money one disburses on this condition that one would not have disbursed on the first condition. Thus, because on this condition one disburses 54 lb, which was to be paid 1 year later, it has to be found what is the present value of this 54 lb to be paid in one year; by the preceding first example of this proposition this is found to be $48\frac{3}{14}$ lb. And for the same reasons the present value of the second 54 lb, to be paid in 2 years, will be $43\frac{17}{31}$ lb. And that of the third 54 lb, to be paid in 3 years, $39\frac{12}{17}$ lb. And that of the fourth 54 lb, to be paid in 4 years, $36\frac{18}{37}$ lb. And that of the fifth 54 lb, to be paid in 5 years, $33\frac{3}{4}$ lb. And that of the last 54 lb, to be paid in 6 years, $31\frac{17}{43}$ lb. And the sum of the above-mentioned six amounts is $233\frac{2356847}{23476796}$ lb, which is the solution.

NOTE.

But because it is difficult to make a separate calculation for each term, as above, especially in the case of many years or terms, one can make tables by means of which such problems can be solved by the following operation:

In order to make a table of 12 per cent, take some large number, of which the first figure is to be 1 and all the others 0, *e.g.* 10,000,000, which we call the root of the table, saying: 112 (to wit the principal of 100 with the interest of one year) gives 100; what does 10,000,000 give? This is 8,928,571, being the number serving for the first year. As to the remainder, this is neglected as being of no account. Further, in order to find the number of 2 years, say: 124 (to wit the principal of 100 with the interest of two years) gives 100; what does 10,000,000 give? This is 8,064,516. When this is added to 8,928,571, this makes 16,993,087 for the number of the two years. Thereafter, in order to find the number of the sal segghen 136 (te weten capitael 100 / metten interest van 3 iaeren) geuen 100 / wat 10000000? compt 7352941 / de selue vergaert tot de 16993087 / maken 24346028 / voor t'getal der drie iaren. Ende alsoo machmen voort varen met soo veel iaeren alsmen wil / welcke wy in dese tafel tot 8 termijnen veruolcht hebben / in deser voegê.

Tafel van simpelen schadelicken interest / van 12 ten 100.

1	8928571.
2	16993087.
3	24346028.
4	31102785.
5	37352785.
6	43166738.
7	48601521.
8	53703562.

Nu om deur dese Tafel te solverê de questie van die seuende exempel / men sal Multiplicerê de wortel des tafels 10000000 / met de iaren daer questie af is / te weten / met 6 / maeckt 60000000 / daer na salmen seggen / 60000000 gheuen 43166738 (t'welck het ghetal is ouercomende inde tafel tegen de 6 Jaren) wat 324 lb? compt 233$\frac{6023112}{60000000}$ lb. die doen 233 lb 2 ß 0 $\frac{5546880}{60000000}$ gr. eñ d'ander solutie was 233 $\frac{2356847}{23476796}$lb doende 233 lb 2 ß 0$\frac{2200176}{23476796}$gr. welcke solutien alleenlijck verschil hebben van een seer cleyn ghedeelte van 1 gr. / dat van gheender achte en is / deur oorsaeck dat de uyterste volmaecktheyt inde tafel niet en is / om de resten diemen int maecken der tafelen verloren laet.

EXEMPEL 9.

Een is schuldich 200 lb te betaelen in 5 iaeren / wat sullen die weerdich zijn in 2 iaeren /rekenende simpelen interest teghen 10 ten hondert.

CONSTRUCTIE.

Men sal trecken 2 iaeren van 5 iaeren / blijft 3. iaeren / op de welcke de voornoemde 200 lb weerdich sullen zijn (deur het tweede exempel van dese propositie) 153$\frac{11}{13}$ lb.

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three years, say: 136 (to wit the principal of 100 with the interest of 3 years) gives 100; what does 10,000,000 give? This is 7,352,941. When this is added to the 16,993,087, this makes 24,346,028 for the number of the three years. And thus one may go on with as many years as one wishes, which we have continued in this table up to 8 terms, as follows:

Table of simple detrimental interest of 12 per cent.

1.	8928571
2.	16993087
3.	24346028
4.	31102785
5.	37352785
6.	43166738
7.	48601521
8.	53703562

Now in order to solve by means of this table the question of the seventh example, multiply the root of the table (10,000,000) by the years in question, to wit by 6. This makes 60,000,000. Thereafter say: 60,000,000 gives 43,166,738 (which is the number corresponding in the table to the 6th year); what does 324 lb give? This is $233 \frac{6023112}{60000000}$ lb, which makes 233 lb 2 sh. $0 \frac{5546880}{60000000}$ d.; and the other solution was $233 \frac{2356847}{23476796}$ lb, which makes 233 lb 2 sh. $0 \frac{2200176}{23476796}$ d.; these solutions only differ by a very small part of 1 d., which is of no account, because there is no extreme perfection in the table, because of the remainders that are neglected during the making of the tables.

Modified text of Example 9, p. 47.

A man owes 200 lb to be paid in 5 years. What is their value in 2 years, charging simple interest at 10 per cent a year?

PROCEDURE.

Subtract 2 years from 5 years; the remainder is 3 years, in which the value of the aforesaid 200 lb will be (by the second example of this proposition) $153 \frac{11}{13}$ lb.

EXEMPEL 10.

Eenen is schuldich binnen 3 iaren 420 lb / ende binnen 6 iaren daer na noch 560 lb: de vraghe is wat dese partien weert zijn te betalen t'samen op 2 iaren / rekenende simpelen interest tegen 10 tê 100?

CONSTRUCTIE.

De 420 lb te betalen binnê 3 iaren / zijn weerdich binnen 2 iarê / deur het voorgaende 9 exempel / $381\frac{9}{11}$ lb; ende de 560 lb te betalen op 6 iaren daer nae / dats binnen 9 iaeren / zijn weerdich binnen 2. iaeren /deur het voornoemde 9. exempel / $329\frac{7}{17}$ lb. welcke met de voorsz. $381\frac{9}{11}$ lb / maecken voor solutie $711\frac{43}{187}$ lb.

NOTA.

So de somme eñ plaets van 324 lb. een ander gheweest hadde niet passende op 6. euen iaeren /ick neme van 330. lb. te betalen met 54 lb. tsiaers 6. iaer lanck eñ op seuende iaer noch 6 lb. men soude eerst vinden de weerde in ghereet ghelt vande 324. lb. na de leeringhe wt dit seste exempel. Daer nae de weerde in ghereedt ghelt vande 6. lb. (die te betaelen zijn in 7. iaer) naer de leeringhe van teerste exempel van dese propositie. Ende desomme van deze twee partyen soude t'begheerde zijn.

Merckt oock cortheyts haluen / dat sooder effen 100. lb. te betalen waeren ettelijcke iaeren achter malcander ende dattet niet noodich en waer de menichte der ß ende gr. te weten gelijckt somwijlen wel te passe coemt. So wijsen d'eerste letteren in de derde tafel de menichte der ponden / sonder datmen behoeft eenige rekeninghe te maecken. Als by gelijckenis 100. lb iaerlijcx 12. iaerê lâck / wat zijn die gereed weert / af te trecken tegê dê penninck 16?

Ick sien in de tafel van dê penninck 16. al waer ick deerste letter van het 12. iaer vinde 827. daerô 827. lb. (wel verstaende dat daer toch noch ß ende gr. gebrekê) is de solutie. Maer waerêt geweest 200. lb. iaerlijcx / 12 iaeren lanck / soo en soudemen die 827. lb. maer te dobbeleren hebben /bedraghende voor solutie 1654. lb. waer wt de gemeenen regel te verstaê is / hoemê de somme met 300. lb. 400. lb. oock mede met effen duysent en dierghelijcke.

Modified text of Example 10, p. 49.

A man owes 420 lb to be paid in 3 years, and 6 years later 560 lb more. What will these sums be worth, if paid together after 2 years, charging simple interest at 10 per cent a year?

PROCEDURE.

The value in 2 years of the 420 lb, to be paid in 3 years, by the preceding 9th example is $381\frac{9}{11}$ lb, and the value in 2 years of the 560 lb, to be paid 6 years later, *i.e.* in 9 years, by the preceding 9th example is $329\frac{7}{17}$ lb, which together with the aforesaid $381\frac{9}{11}$ lb makes $711\frac{43}{178}$ lb, which is the solution.

Note, added after Example 6, p. 101.

NOTE.

If instead of 324 lb the amount had been another, not divisible in 6 equal yearly terms, I assume 330 lb, to be paid with 54 lb a year during 6 years and the seventh year 6 lb more, the present value of the 324 lb would first have to be found, in accordance with this sixth example. Thereafter the present value of the 6 lb (which is to be paid in 7 years) in accordance with the first example of this proposition. And the sum of these two amounts would be the required value.

Note also, for brevity's sake, that if precisely 100 lb were to be paid several years in succession and if it were not necessary to know the amount of the sh. and the d., as sometimes happens, the first figures in the third table indicate the amount of the pounds without any calculation having to be made. For example: What is the present value of 100 lb to be paid yearly during 12 years, interest at the 16th penny to be subtracted?

I look it up in the table of the 16th penny, where I find the first figures of the 12th year to be 827; therefore 827 lb (on the understanding that this number still lacks sh. and d.) is the solution. But if the amount had been 200 lb, to be paid yearly during 12 years, one would merely have to double this 827 lb, the solution thus being 1,654 lb, from which can be inferred the general rule how to find the sum with 300 lb, 400 lb, and also with precisely one thousand and the like.

PROBLEMATA GEOMETRICA

INTRODUCTION

§ 1

Stevin's contributions to geometry illustrate the fundamental position of Euclid's *Elements* in the work of sixteenth-century mathematicians. The *Elements* were their main source of reference, to which they constantly returned for knowledge, method, and inspiration. The typically "Greek" reasoning of Euclid, which was also basic to the demonstrations of Apollonius and Archimedes, geometrical to the core, was an essential element in the mathematical thinking of sixteenth-century Europe.

This Greek influence was gradually undermined by the adaptation of the arithmetical-algebraic methods traditional in the Orient, which reached Europe almost entirely through authors originally writing in the Arabic language. Stevin's *Tables of Interest* present a good example of how the practice of life itself compelled mathematicians to become proficient in these methods. On a higher theoretical level we see the same influence at work in Stevin's *Arithmétique*. Even the *Problemata Geometrica,* though fundamentally a series of papers based on the "Greek" approach, shows the influence of the Arabic tradition in several places.

Several printed editions of the *Elements* existed in Stevin's days. One of Stevin's favourites was the Latin edition by Clavius, the Jesuit astronomer at the Vatican. It was a thorough piece of work, first published in 1574, consisting of two volumes in a rather handy quarto size. It had the advantage of introducing the reader to related work by other mathematicians, explained in *Scholia* to the text [1]). Other books used by Stevin in preparing his *Problemata* were Dürer's *Underweysung* of 1525 and Commandinus' Latin edition of the principal works of Archimedes of 1558.

The *Problemata* consists of five books, each with a topic of its own [2]). The first book, after an introduction on proportions of lines, solves problems concerning the division of polygons into parts of a given ratio. The second book contains the application of the so-called *regula falsi* to certain constructions or, in other words, shows how certain constructions can be performed with the aid of similarity of figures. In the third book we find Stevin's studies on regular and semi-regular polyhedra. The fourth book deals with the construction of a polyhedron of a given volume similar to a given polyhedron, the fifth book with the construction of a polyhedron similar to two (similar) polyhedra and equal to either their sum or their difference. While the first and second books are based on Euclid, the third is based on Dürer; the last two are the result of Stevin's study of Archimedes.

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[1]) *Euclidis Elementorum libri XV, . . ., accessit XVI. De solidorum regularium comparatione . . . Auctore Christophoro Clavio* (Rome, 1574, 2 vols, several later editions). See footnote [2]) to the Introduction to *L'Arithmétique.*

[2]) A full description and analysis of the *Problemata* has been given by N. L. W. A. Gravelaar, *Stevin's Problemata Geometrica,* Nieuw Archief voor Wiskunde, (2) 5 (1902), pp. 106–191.

§ 2

The first book opens with a classification of ratios and proportions, based on the fifth book of Euclid's *Elements*. This classification, in its attempt to give special names to particular proportions, strikes us as clumsy and pedantic, but Stevin merely followed an ancient tradition. All this labelling was fundamentally due to a serious desire to understand Euclid, though it was encumbered with relics from the works of medieval latinists [3]). The following list may explain some of the terminology for ratios in a modern fashion:

$(n+1):n$	superparticularis	$n:(n+1)$	subsuperparticularis
$(n+l):n$	superpartiens	$n:(n+l)$	like the corresponding terms to the left, with "sub" prefixed: "subsuperpartiens", etc.
$kn:n$	multiplex	$n:kn$	
$(kn+1):n$	multiplex superparticularis	$n:(kn+1)$	
$(kn+l):n$	multiplex superpartiens	$n:(kn+l)$	

In accordance with the Greek precedent the cases $l = 1$ and $l > 1$ are treated separately, since unity was not considered a number. Stevin was later to break with this concept (see the introduction to *L'Arithmétique*).

There are terms for special ratios in accordance with the general scheme. For instance:

$2:1$ dupla, $3:1$ tripla

$(n+1):n$	$n = 2$	sesquialtera, hence $3:2$
	$n = 3$	sesquitertia, hence $4:3$
$n:(n+1)$	$n = 2$	subsesquialtera, hence $2:3$
	$n = 3$	subsesquitertia, hence $3:4$
$(n+2):n$	$n = 3$	superbipartienstertias, hence $5:3$
	$n = 5$	superbipartiensquintas, hence $7:5$
$(kn+1):n$	$k = 2,\ n = 4$	duplasesquiquarta, hence $9:4$
	$k = 3,\ n = 6$	triplasesquisexta, hence $19:6$.

There are more terms in Stevin's text, which are not all to be found in Clavius, but which all formed part of the regular curriculum of the universities. The figures, with the simple numerical illustrations, are similar to those in Clavius.

The next part consists of the application of this theory of proportions to the problem of the division of figures into parts of a given ratio. Stevin found an example of this problem in an appendix by Clavius to the 6th book of the *Elements*, where it is shown how to divide a triangle into two parts in a given

[3]) This terminology is a Latin translation of Greek terms used by Nicomachus in his *Introduction to Arithmetic* and passed into the language of the regular quadrivium of the Medieval and Renaissance schools, primarily through the study of Boetius, who used it in his *Arithmetica* and in his *Musica*; see *A. M. T. S. Boetii De Institutione Arithmeticae libri duo. De Institutione Musicae libri quinque* . . . edidit G. Friedlein (Leipzig, 1867, VIII + 492 pp.), esp. Lib. I of the *Arithmetica*; T. L. Heath, *A Manual of Greek Mathematics* (Oxford, 1931), p. 69. English translation of Nicomachus' *Arithmetica* by M. L. D'Ooge, New-York, 1926.

ratio by a line passing through a point on a side 4). This was not, however, an original idea of Clavius. As he sets forth himself in the Prolegomena to his translation, he found the problem in a book published by Commandinus and John Dee in 1570, which, he says, though ascribed to a certain Mahomed of Bagdad 5), may have been Euclid's book on *Divisions* 6). Stevin, who did not know this book, took Clavius' problem and discussed aspects of it in his first set of three problems. Then he himself added five more problems, which he thought to be novel. All eight problems deal with the division of polygons in a given ratio either by a line through a vertex, or by a line through a point on a side, or by a line parallel to a side.

They were not so very novel after all, as Stevin would have discovered if he had found an opportunity to consult Mahomed of Bagdad-Commandinus. We do not know if he ever did. But after the *Problemata* had been published and Stevin had found the time to catch up in his reading, he discovered some other authors who had dealt with the division of figures 7). Stevin mentions Cardan, Ferrari, and especially Tartaglia in a part of his *General Trattato* (1560) 8). These authors took their inspiration directly or indirectly (through Paciolo's *Summa* of 1494) from Leonardo Pisano's *Practica Geometriae* (1220), and through this book from Arabic sources. We now know that the text of Mahomed of Bagdad and that of Leonardo Pisano are different versions of the lost book of Euclid on *Divisions of Figures*. It has been possible to reconstruct the lost book from these different versions, together with another one, found by Woepcke in 1851 in a manuscript text 9). This book, as the title indicates, contains a large number of problems of the same nature as those of Stevin in the first book of his *Problemata*.

Stevin also mentions in the *Meetdaet* that after his book had been published, Benedetti published a treatise in which the division of figures was taken up 10). However, despite all this competition, Stevin's work was excellent enough to be preferred by Clavius, who in 1604 praised his treatment of the division of figures above the others 11). Stevin himself was not too satisfied with his work,

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4) Clavius, *l.c.* 1), p. 230 r., Problema XIII: "A dato puncto in latere trianguli lineam rectam ducere quae triangulum dividat in duo segmenta secundam proportionem datam."

5) *De superficierum divisionibus libri Machometo Bagdedino ascriptus nunc primum Joannis Dee Londinensis et Federici Commandini Urbinatis opera in lucem editus* (Pesaro, 1570). There was an Italian translation of 1570 and an English one of 1660.

6) Clavius, *l.c.* 1), p. 4, dealing with Euclid: "*Opus de Divisionibus*, quod nunnulli suspicantur esse libellum illum acutissimum de superficierum divisionibus, Machometo Bagdedino ascriptum, qui nuper Ioannis Dee Londinensis et Federici Commandini Urbinatis opera in lucem est editus".

7) *Meetdaet*, p. 144. See our Introduction.

8) N. Tartaglia, *La quinta parte del general trattato de' numeri et misure*. Venetia 1560, fol. 23 v–44 r.

9) R. C. Archibald, *Euclid's Book on Divisions of Figures ... with a restoration based on Woepcke's text and on the Practica Geometriae of Leonardo Pisano* (Cambridge, 1915, VIII + 88 pp.) — This book has an extensive bibliography, in which the references to Leonardo, Cardan, and Ferrari can be found.

10) G. B. Benedetti, *Diversarum speculationum mathematicarum et physicarum liber* (Taurini, 1585), esp. pp. 304–307.

11) Clavius, *Opera mathematica* II (Mainz, 1611), p. 417; after having mentioned the Dee-Commandinus edition as "acutissimus et eruditione resertissimus", Clavius continues: "Idem vero postea argumentum alia via agressus est, et meo certo iudicio, faciliori, et

and in his *Meetdaet* improved in several ways on his *Problemata*. In particular he generalized the problem of the division of figures by taking the point through which the line of division has to be constructed, inside and outside the polygon in any position [12].

§ 3

The second book of the *Problemata* contains problems involving the so-called „regula falsi", the rule of the false supposition [13]. It is a device to solve problems leading to the linear equation $ax = b$ by first substituting for x an arbitrary number $x = x_0$; if $ax_0 = b_0$, then $x : x_0 = b : b_0$ and x is found by means of proportion. It is a method used even now by people unfamiliar with algebra — or, in the language of the sixteenth century, unfamiliar with "coss". The device also functions for problems which lead to an equation of the form $ax + b = c$; in this case we need two "false suppositions" $x = x_1$, $x = x_2$; if $ax_1 + b = c_1$, $ax_2 + b = c_2$, then $(x - x_1) : (x - x_2) = (c - c_1) : (c - c_2)$, and $x = \frac{x_2 (c - c_1) - x_1 (c - c_2)}{c_2 - c_1}$. This is the "regula falsi duplicis positionis". Both rules are standard in all sixteenth-century books on arithmetic, and Stevin also teaches them in his *La Pratique D'Arithmétique* [14]. In the *Problemata* Stevin introduces this "regula falsi" in accordance with this desire to bring about as close a relation as possible between arithmetical and geometrical proportions. Applying the "regula falsi" to problems in geometry, he has to consider proportions, and this amounts to the solution of certain geometrical problems by means of similarity. If, for instance (Ex. II), we have to construct a square when the difference between diagonal and side is given, we start with any square (this is the false supposition), determine for this square the difference between diagonal and side, and then find the side of the required square by means of a proportion. All that Stevin now requires is Euclid's theory of proportions, which he finds in Books V and VI of the *Elements*.

§ 4

The third book is by far the most interesting part of the *Problemata*. It contains a theory not only of the regular solids, but also of certain semi-regular solids and of polyhedra which Stevin calls "augmented regular solids". All Stevin had to go by was Euclid's *Elements*, Book XIII, the so-called XIVth, XVth, and XVIth books, which Clavius also had translated, and Dürer's *Underweysung der*

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magis generali, Simon Stevinius Brugensis: sed in qua aliquid desiderari videatur, ut omnibus superficiebus rectilineis (quod ipse velle videtur) convenire possit, quod facile iudicabant, qui illius problemata Geometrica attente perlegerint . . . Deinde superficiarum rectilinearum divisionem aggrediemur, insistentes eiusdem Stevinii vestigiis, nisi quando generalius rem oportebit demonstrare."

[12]) *Meetdaet*, p. 144. These cases, though new to Stevin, had already been treated by Euclid and Leonardo Pisano, see R. C. Archibald, *l.c.*[9])

[13]) On the *regula falsi*, see J. Tropfke, *Geschichte der Elementar-Mathematik* III (Berlin-Leipzig, 3e Aufl., 1937), p. 152; D. E. Smith, *History of Mathematics* II (Boston, 1925), p. 437.

[14]) *La Pratique D'Arithmétique*, p. 122.

Rechnung mit dem Zirckel und Richtscheyt of 1525[15]). From Euclid-Clavius Stevin obtained his information on the five regular solids, from Dürer the method of obtaining semi-regular solids (as well as the regular ones) by paper-folding. To understand these different achievements, we shall denote a polyhedron with m faces which are regular polygons of a sides, n faces which are regular polygons of b sides, etc., by $\{m_a, n_b, \ldots\}$. Then the five regular solids are the tetrahedron $\{4_3\}$, the cube $\{6_4\}$, the octahedron $\{8_3\}$, the dodecahedron $\{12_5\}$, and the icosahedron $\{20_3\}$. A semi-regular solid or, as Stevin calls it, a "truncated regular solid" is defined (Def. 11) as a solid inscribed in a sphere, of which all the solid angles are equal, of which the faces are regular polygons which are not all congruent, and of which all the edges are equal. Dürer had the models of seven such solids: $\{4_3, 4_6\}$, $\{8_3, 6_8\}$, $\{6_4, 8_3\}$, $\{8_6, 6_4\}$, $\{18_4, 8_3\}$, $\{6_4, 32_6\}$, $\{6_8, 8_6, 12_4\}$. Dürer had two more models, but one of these, $\{6_4, 12_3\}$, has some isosceles triangles, while the other $\{6_{12}, 32_3\}$, as Stevin showed, is impossible as a closed polyhedron[16]).

Stevin reconstructed these solids, not only from plane diagrams by folding, but also by finding the method by which these solids are generated by cutting off (truncating) parts of the regular solids. He found three types not described by Dürer. We can give a survey of his results and those of others in the following way.

The five regular solids can be divided into two pairs of dually related bodies, the pair $\{6_4\}$ and $\{8_3\}$, and the pair $\{12_5\}$ and $\{20_3\}$, and the tetrahedron $\{4_3\}$, which is dual to itself. By duality is meant one-to-one correspondence of vertices and faces, edges corresponding to themselves. For instance, the cube $\{6_4\}$ has 8 vertices and 6 faces, while the octahedron $\{8_3\}$ has 6 vertices and 8 faces; both have 12 edges. The polyhedra $\{12_5\}$ and $\{20_3\}$ both have 30 edges.

Semi-regular solids can be obtained from the regular solids by truncation, as follows:

1) cutting off pyramids at the vertices up to the centre of the adjacent edges, so that the original edges disappear:
   a) $\{4_3\}$ passes into a smaller $\{4_3\}$.
   b) $\{6_4\}$ and $\{8_3\}$ pass into $\{6_4, 8_3\} = \{8_3, 6_4\}$. Described by Stevin in Def. 13, Section 15 (also Def. 17). Kepler later called this solid *cuboctahedron*.
   c) $\{12_5\}$ and $\{20_3\}$ pass into $\{12_5, 20_3\} = \{20_3, 12_5\}$. Described by Stevin in Def. 21, Section 18 (also Def. 19). Kepler later called this solid *icosidodecahedron*. Wanting in Dürer.

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15) A. Dürer, *Underweysung der Messung mit dem Zirckel und Richtscheyt* (Nuremberg, 1525). Latin edition: *Albertus Durerus Nuremburgensis . . . adeo exacte quatuor his suarum Institutionum geometricarum libris* (Paris, 1533), 2nd German edition: *Underweysung der Messung* . . . Nurenberg 1538).

16) *Problemata*, p. 46. On the possibility of constructing closed polyhedra from plane patterns by paper-folding and on convex polyhedra in general, see A. D. Aleksandrov, *Vypuklye mnogogranniki* (Convex Polyhedra, Moskow-Leningrad, 1950, 428 pp.). German: *Innere Geometrie der konvexen Flächen* (Berlin, 1954). The main theorem is: To every closed, directable, plane diagram with given identification of edges and vertices, for which the sum of the angles at the same vertex is at most 2π and which satisfies Euler's condition on the vertices, angles, and faces, there exists one convex polyhedron. See also W. Blaschke, *Griechische und anschauliche Geometrie* (München, 1953), p. 22.

2) cutting off pyramids at the vertices till the original faces have become regular polygons with twice the number of sides:
  a) $\{4_3\}$ passes into $\{4_3, 4_6\}$. Described by Stevin in Def. 12, Section 11.
  b) $\{6_4\}$ passes into $\{6_8, 8_3\}$. Described by Stevin in Def. 14, Section 12.
  c) $\{8_3\}$ passes into $\{8_6, 6_4\}$. Described by Stevin in Def. 16, Section 18.
  d) $\{12_5\}$ passes into $\{12_{10}, 20_3\}$. Described by Stevin in Def. 20, Section 17. Wanting in Dürer.
  e) $\{20_3\}$ passes into $\{20_6, 12_5\}$. Described by Stevin in Def. 22, Section 19. Wanting in Dürer.

3) letting faces shrink into similar ones. At the edges squares are formed, instead of vertices there appear regular triangles, squares or pentagons:
  a) $\{4_3\}$ passes into $\{8_3, 6_4\} = 1^{b)}$
  b) $\{6_4\}$ and $\{8_3\}$ pass into $\{18_4, 8_3\} = \{8_3, 18_4\}$. Described by Stevin in Def. 15, Section 13.
  c) $\{12_5\}$ and $\{20_3\}$ pass into $\{12_5, 20_3, 30_4\}$. Wanting in Stevin and Dürer, but to be found in Archimedes.

4) letting faces shrink and be transformed into regular polygons with twice the number of sides. At the edges squares are formed:
  a) $\{6_4\}$ and $\{8_3\}$ pass into $\{6_8, 8_6, 12_4\}$. Described by Stevin in Def. 16, Section 14.
  b) $\{12_5\}$ and $\{20_3\}$ pass into $\{12_{10}, 20_6, 30_4\}$. Wanting in Stevin and Dürer, but to be found in Archimedes.

Apart from these solids there exist two more semi-regular bodies. One, $\{32_3, 6_4\}$, was found by Stevin, Appendix, p. 83, and Stevin remarks that it does not seem possible to obtain it from one of the regular bodies by truncation. It is asymmetric in the sense that there are two forms, distinguishable by the epithets left and right. Kepler called this solid *cubus simus,* snub cube. There also exists a semi-regular body $\{12_5, 80_3\}$ with the same type of symmetry, wanting in Stevin, which Kepler called snub dodecahedron.

Stevin thus obtained, apart from the seven Dürer types, the additional solids $\{20_3, 12_5\}$, $\{12_{10}, 20_3\}$, and $\{20_6, 12_5\}$. He was one of the first, if not the first, in Renaissance days to find all these ten.

However, shortly after he had published his *Problemata,* the *Collectiones mathematicae* of Pappus appeared in print for the first time (1588), and this book contained an account of Archimedes' work on the semi-regular solids [17]. It was then found, not only that Archimedes had listed all of Stevin's polyhedra, but that he even had three additional ones, which we have marked $\{12_5, 20_3, 30_4\}$, $\{12_{10}, 20_6, 30_4\}$, and $\{12_5, 80_3\}$. There is no sign in the *Meetdaet* to show that Stevin became aware of this contribution by Archimedes, nor is there any sign that he ever knew of any other student of semi-regular solids besides Dürer [18].

Pappus' enumeration of Archimedes' solids was made the subject of a study

[17]) *Pappi Alexandrini mathematicae Collectiones,* ed. F. Commandinus. (Venice, 1588, reissued Pesaro, 1602).
[18]) see footnote [23])

by Kepler in 1619 [19]). Kepler derived them systematically, illustrated his description by figures, and gave them the names by which they are still known. Kepler was also the first to pay attention to the polar figures of the "Archimedean solids", as he called them. He described two of them, the polar of $\{8_3, 6_4\}$, called the *rhombic dodecahedron,* and the polar of $\{20_3, 12_5\}$, called the *rhombic triacontahedron.*

Besides the thirteen Archimedean solids described by Kepler there exist two more, but they are rather trivial ones. They are obtained by taking two regular polygons of n sides in two parallel planes, and placing them in such a way that they are either the bases of a rectangular prism with square faces, or the bases of an antiprism (prismoid) with equilateral faces. Their symbols are $\{2_n, n_4\}$, $\{2_n, 2n_3\}$.

The third book of the *Problemata* also contains a description of what Stevin called "augmented regular solids". These are polyhedra obtained by placing on top of each face of a regular polyhedron as base a pyramid with equal edges. Stevin lists all five of them. He was led to the consideration of these solids by a discovery of Frans Cophart, leader of the Collegium Musicum at Leiden. Cophart had taken a cube and cut out twelve tetrahedra, each having the end points of an edge and the midpoints of the faces through this edge as vertices. The solid thus obtained by "faceting" the cube is what is now called the *stella octangula;* it is bounded by twenty-four congruent equilateral triangles. Cophart had claimed it as a sixth regular solid.

Stevin, while admiring the discovery, had to deny this claim. He pointed out that the vertices of Cophart's solid do not all lie on one sphere, but are distributed on two spheres, six on one sphere and eight on a concentric one. At the same time he discovered another construction of the solid by starting, not from a cube and then faceting it, but from an octahedron and then "augmenting" it by placing a regular tetrahedron on each face with this face as base. He now saw that this procedure could be applied to all regular bodies, and in this way he obtained four new polyhedra.

Of all these five solids of Stevin we only call the *stella octangula* a regular star-polyhedron. The reason is that regular star-polyhedra are obtained from the regular polyhedra by the process of "stellating", *i.e.* by producing the planes of the faces and allowing non-adjacent faces to intersect in such a way that the faces of the new solid are regular star-polygons (polygons obtained by allowing non-adjacent sides of regular polygons to intersect). This procedure does not yield a new body in the case of the regular tetrahedron and the cube, but gives us the *stella octangula* in the case of the regular octahedron. We also obtain regular star-polyhedra by stellating the regular dodecahedron and icosahedron; for each of these solids we obtain two possible star-polyhedra. But whereas these four bodies are single, the *stella octangula* is found to be the intersection of two regular tetrahedra. We may thus speak of nine regular solids: five ordinary (Platonic) and four stellated ones.

These solids can be obtained, not only by stellating, but also by "faceting" the five Platonic bodies, *i.e.* by taking solid pieces out of them in accordance with definite directives. The Copland solid was obtained by faceting a cube. We

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[19]) J. Kepler, *Harmonices mundi libri V* (Linz, 1619), Lib. II. (*Gesammelte Werke*, herausg. von Max Caspar, Band VI).

now see that Stevin was on the way to show how to replace faceting by stellating; unfortunately, he missed the final step. He also missed the other fundamental property of the *stella octangula*, viz. that it is decomposed into two regular tetrahedra. Instead of this he continued to construct other augmented solids by placing equal-edged pyramids on top of the faces of the other Platonic bodies [20].

Stevin does not seem to have been aware that Pacioli, in his *Divina proportione* of 1509, had enumerated a large number of solids obtained from regular solids by "truncating" and "augmenting" — procedures called by Pacioli "abscindere" and "elevare" [21]. However, Pacioli did not show how these solids are to be obtained from Platonic bodies by Monsignor Daniel Barbaro. The *La Pratica Della* that he actually constructed models of some, if not all, of his polyhedra). It was Dürer who stressed the method of paper-folding, and it was from him that Stevin obtained his ideas. It also seems to have escaped Stevin's attention that Dürer's ideas had been applied to many semi-regular and other polyhedra obtained from Platonic bodies by Monsignor Daniel Barbaro. The *Pratica Della Perspectiva* of this Patriarch of Aquileia, published in 1568 [22], contained not only the description of a large number of polyhedra obtained by truncating or augmenting the Platonic bodies (many of them are non-Archimedean solids), but also their construction by paper-folding, as well as a representation of them in perspective drawing [23].

§ 6

The fourth and fifth books of the *Problemata Geometrica* present Stevin to us as a student of Archimedes. The *editio princeps* of Archimedes' works had appeared at Basle in 1544; it contained not only the original Greek text and a Latin translation, but also the precious commentaries of Eutocius, again both in Greek and in Latin [24]. Another useful, though limited, edition was the Latin trans-

[20]) On regular and semi-regular solids see further M. Brückner, *Vielecke und Vielfläche, Theorie und Geschichte* (Leipzig, 1900, VIII + 227 pp.); H. S. M. Coxeter, *Regular Polytopes* (London, 1948, XVIII + 321 pp.).

[21]) *Fra Luca Pacioli Divina Proportione. Die Lehre vom Goldenen Schnitt. Nach der venezianischen Ausgabe vom Jahre 1509 neu herausgegeben, übersetzt und erläutert von C. Winterberg* (Wien, 1896), VI + 367 pp.

[22]) D. Barbaro, *La pratica della perspettiva* (Venice, 1568; there is also an edition Venice 1569, 195 pp.).

[23]) There were other authors of the sixteenth century who shared Pacioli's and Barbaro's interest in truncated and augmented bodies and who seem to have remained unknown to Stevin. Pre-eminent among them is the Nuremberg goldsmith Wenzel Jamnitzer, whose *Perspectiva corporum regularium* (Nuremberg, 1568) contains beautiful illustrations. The solids $\{ 6_4, 8_3 \}$ and $\{ 20_3, 12_5 \}$ appear in the French Euclid translation by Bishop François de Foix, comte de Candala (1566, 2nd ed., 1578). Moreover, R. Bombelli, in a chapter of his *Algebra*, which remained unpublished until 1929, also discussed some of these bodies and their plane schemes: *L'Algebra Opera di R. Bombelli di Bologna Libri IV e V* ..., publ. a cura di E. Bortolotti (Bologna, 1929). – On the further history of star-polyhedra see Kap. I of S. Günther, *Vermischte Untersuchungen zur Geschichte der mathematischen Wissenschaften* (Leipzig, 1876, VII + 352 pp.), pp. 1–92. The modern theory of these polyhedra opens with Poinsot, *Mémoire sur les polygones et les polyèdres*, Journ. Ec. Polytechnique, 10e cah., tome 4 (1810), pp. 16–46.

[24]) *Archimedis opera quaequidem extant omnia, nunc primum et graece et latine in lucem edita; adjecta sunt Eutocii Ascalonitae in eosdem Archimedis libros commentaria, item graece et latine* (Basel, 1544); editor was Thomas Gechauff (Venatorius).

lation of five of Archimedes' treatises with Eutocius' commentary on one of them, prepared by Commandinus and published in 1558 [25]). Stevin quotes Commandinus' edition, but he must also have known the *editio princeps,* since he shows himself to be acquainted with material which is to be found in the publication of 1544, but not in that of 1558 [26]).

Archimedes, in the book *On the Sphere and Cylinder,* the book in which he determines the area and the volume of the sphere, solves some problems which involve the finding of the two mean proportionals between two given lines. An example is formed by the problem: "given two spherical segments, to find a third segment similar to the one and having its volume equal to that of the other"; another example consists in the problem of finding a sphere equal in volume to a given cone or cylinder. These problems have in common that they lead up to what we call a cubic equation, and in particular to an equation of the form $x^3 = ar^3$, where r is a given line and a a given number. Thus the second problem, in the case of a given cone of height h and base radius r, leads to the equation $x^3 = \frac{h}{4}r^2 = \frac{h}{4r}r^3$ for the radius x of the sphere. The classical example of such problems is the duplication of a cube, where $a = 2$. A common Greek method of solving such problems was that by means of two mean proportionals x,y between two given lines a,b; if

$$a : x = x : y = y : b,$$

then $x^3 : a^3 = b : a$. In the case mentioned above we might write, for instance: $r : x = x : y = y : \frac{h}{4}$. However, two mean proportionals between two given lines cannot be constructed with compasses and straightedge alone. It is one of the merits of Eutocius (6th cent. A.D.) that he preserved in his commentaries a large number of solutions for this problem; they bear the names of Hero, Diocles, Eratosthenes, Apollonius and Plato, and of several others [27]). They solve the problem either by the intersection of certain curves or by the use of some special instrument ("mechanice" — "tuighwerckelyk", as Stevin was to translate it). About all this Stevin could find information in the *editio princeps.* Moreover, in Commandinus' edition, though it does not contain the book *On the Sphere and Cylinder* with its commentaries, there are several problems which belong to the same group [28]). The first is the problem: "Given any two cones (or cylinders), to find a third cone (or cylinder), equal in volume to the first and similar to the second"; the second replaces the full cone (or cylinder) by segments. The

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[25]) *Archimedis opera non nulla a Federigo Commandino Urbinate nuper in Latinum conversa et commentariis illustrata* (Venice, 1558). This edition contains *Circuli dimensio, De lineis spiralibus, Quadratura parabolae, De conoidibus et sphaeroidibus, De arenae numero,* and Eutocius' commentary on the *Circuli dimensio.*

[26]) On Archimedes, apart from the edition by J. Heiberg, *Archimedis opera omnia cum commentariis Eutocii* (Lipsiae, 1910–1915), see the following books:

P. Ver Eecke, *Les oeuvres complètes d'Archimède* (Paris, Bruxelles, 1921) LIX + 553 pp..
E. J. Dijksterhuis, *Archimedes* (Copenhagen, 1956), 422 pp.

[27]) On these methods, apart from the books mentioned *sub* ([26]), see Th. Heath, *History of Greek Mathematics* (Oxford, 1921) I, pp. 244-270, or *id., A Manual of Greek Mathematics* (Oxford, 1931), pp. 154-170.

[28]) Commandinus, *l.c.* ([25]), pp. 52 r, v.

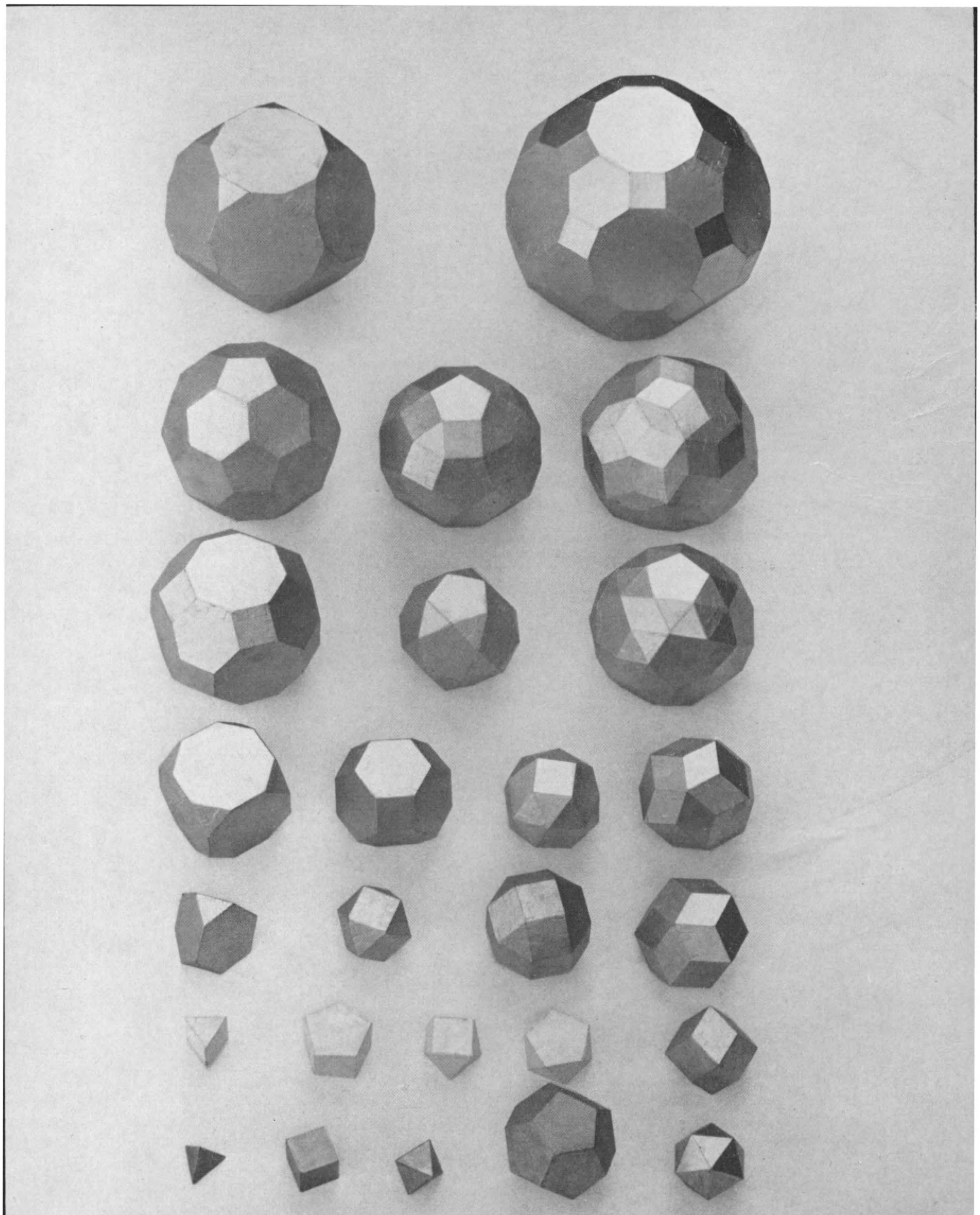
others replace cone and cylinder by ellipsoids and paraboloids of revolution and their segments.

Stevin, in his fourth book, casts the problem into its general form: "Given two solids S_1, S_2, to find a third solid S_3, equal to S_1 and similar to S_2". As such it is the generalization of the problem discussed in plane geometry by Euclid in *Elements* VI, 25. Euclid states the problem for arbitrary polygons. Stevin uses the general term "solid" and then makes use of the theorem that any solid can be changed into a cone of equal volume; he actually applies the theorem to the solids in which Euclid was interested; polyhedra, spheres, circular cones, cylinders, and to segments of cones. Stevin shows, for instance, how a spherical segment can be changed into a cone with equal base. Later, in the *Meetdaet*, he gives some more examples [29]).

Stevin's procedure is as follows: a) he changes S_1 into a circular cone C_1, and S_2 into a circular cone C_2; b) he then changes C_1 into a cone C_1' of the same altitude as C_2; c) then constructs a cone C_3, similar to C_2 and equal to C_1'; d) he then changes C_3 into an equal solid S, reversing the process by which S was changed into C_2. The steps a), b), d) only involve ordinary proportions, step c) involves the construction of two mean proportionals; for this purpose Stevin mentions Hero's construction, on which Eutocius reports.

It is difficult to say how far the material provided by Stevin in his fourth book has any originality. Stevin seems to have felt this also, and therefore, in the last book of the *Problemata,* solved another problem leading to two mean proportionals between two lines, which appears to be a new one. Given two similar solids S_1 and S_2, $S_1 > S_2$, Stevin asked to find a solid similar to S_1 and S_2, and equal to a) the sum of, b) the difference between S_1 and S_2. The problem was again solved by reducing the solids to circular cones.

[29]) *Meetdaet* VI, Props. 31–34.



3.10^2 $\{20_3,12_{10}\}$				$4.6.10$ $30_4,20_6,12_{10}$
5.6^2 $\{12_5,20_6\}$		$3.4.5.4$ $\{20_3,30_4,12_5\}$		X
$4.6.8$ $\{6_8,8_6,12_4\}$		$(3.5)^2$ $\{12_5,20_3\}$		$3^4.5$ $\{80_3,12_5\}$
3.8^2 $\{8_3,6_8\}$	4.6^2 $\{8_6,6_4\}$		$3^4.4$ $\{6_4,32_3\}$	X
3.6^2 $\{4_3,4_6\}$	$(3.4)^2$ $\{6_4,8_3\}$		3.4^3 $\{18_4,8_3\}$	X
X	X	X	X	X
X	X	X	X	X

Inside the dodded contour: the 13 Archimedean solids.
Each solid is designated by two notations: one of W. W. R. Ball *(History of Mathematics*, 11th edition, p. 136), and another, used in the text of this introduction.
See also: Cundry-Rollett, *Mathematical Models*, p. 94, 120. L. F. Toth, *Lagerungen in der Ebene*, p. 20.

PROBLEMATUM
GEOMETRICORUM

In gratiam D. MAXIMILIANI, DOMINI A CRUNINGEN &c. editorum, Libri V.

Auctore

SIMONE STEVINIO BRUGENSE.

ANTVERPIAE,
Apud Ioannem Bellerum ad insigne
Aquilæ aureæ.

ILLVSTRISSIMO
HEROI, D. MAXIMILIANO, DOMINO CRVNINGAE, CREVECVEVR, HEENVLIET, HASERVVOVDE, STEENKERCKEN, VICECOMITI ZELANDIAE &c.

SVPREMO MACHINARVM BELLICARVM INFERIORIS GERMANIAE PRAEFECTO.

SIMON STEVINIVS.
S. P.

GEOMETRIAE, mediusfidius, vtilitas magna, imo vero necesſitas. Et vero, quid tandem non illi feremus acceptum? Ponamus nobis ante oculos pauca quædam ex multis, ſine quibus certè neque commodè, neque omnino benè viuitur. An non hinc domicilia, an non & vrbes? an non veſtes, omniſque ſuppellex? an non omnia cum pacis tum belli inſtrumenta? Hic mihi tu ipſe teſtis locupletiſsimus, tu inquam Heros clariſs. qui nobilitate cuius par, ingenio ſuperas omnes. Neque enim potes, neque, credo, vis celare tuos in hac arte profectus: fama hinc tibi magna: &, quod noſtro ſeculo inſolens, inculpata.

PROBLEMATA GEOMETRICA

To the illustrious hero, Maximilian, Lord of Cruningen, Crevecueur, Heenvliet, Haserwoude, Steenkercken, Viscount of Zeeland, etc. Supreme Superintendent of the Implements of War of the Low Countries. *)

SIMON STEVIN.

S. P.

Great indeed is the usefulness, nay the indispensability of Geometry. For indeed, what good thing do we not owe to it after all? Let us bear in mind a few things out of many without which life certainly cannot be lived so comfortably, nay, even not at all well. Do not the houses and the towns result from it, clothes and all furniture? And all implements, both of peace and of war? In this respect Thou Thyself art a most reliable witness, Thou, I say, most famous hero, who art the equal of anyone in nobility and excellest all in spirit. For Thou art neither able nor, I believe, desirous to conceal Thy proficiency in this art. Thou hast gained in it a great end, what is unusual in our age: an unblemished fame.

*) Maximilian van Cruyninghen was born on July 29, 1555. By resolution of the States General of December 27, 1579 he was appointed General of the Artillery of the Army of the States General by anticipation, and by resolution of the States General of January 13, 1581 this appointment was made permanent. In 1597 he became a member of the Council of State for Zeeland and in 1600 Governor of Ostend, a post which he held only a short time. He died on January 5, 1612 (see: F. J. G. ten Raa en F. de Bas, *Het Staatsche Leger*, 1568-1795. Deel I (Breda 1911), pp. 151, 159, 240. Deel II (Breda 1918) pp. 275, 278).

IN GEOMETRICA PROBLE-MATA SIMONIS STEVINII,

Lucæ Belleri I. F. Carmen.

CVR Opifex rerum Cælos, cur pondera Terræ
Cur Maris vndisoni tractus, cur AEthera fecit?
Quid Polus? & quorsum cælo radiantia fixit
Lumina? cur Lunæ cursus, Solisq; labores?
Scilicet vt rerum moles, & congruus ordo,
Ad se animos trahat humanos: propiusque videre
Artificem per tanta suum miracula possent.
Hinc Deus in paucis sublimius organa vexit
Ingenij, per quæ manuum structura suarum,
Et formæ decor, & diuina pateret imago.
Verè igitur Diuam veteres dixère Mathesin,
Cuius ab arte labor, superas cognoscere sedes,
Terrarum, pelagiq; vias, & operta tenebris
Naturæ secreta dedit: coramque tueri.
Quæque vigore suo reliquas exuscitat Artes,
Viuificam inspirans animam: fragilesq; per artus
Lapsa, fouet, iuuat, & toto se corpore miscet.
Qualis vbi stagnans exæstuat aggere Nilus,
Impatiens freni, & laxis iam liber habenis
Per Pharios spaciatur agros: omnemq; benigno
Diluuio fœcundat humum: iam vertice læto,
Stant fruges, grauibusq; tumet iam campus aristis.
Ergo age, qui tantas oculis vis cernere moles,
Aut veri te ducit amor, doctasq; per artes
Si facili cupis ire via: te Diua Mathesis
Instruet, & reliquas ibit comes vna per omnes;
Doctrinaq; alto & rerum te cardine sistet.

IN EIVSDEM GEOMETRICA PROBLE-mata Henricus Vuithemius.

AEgyptus celerem septena per ostia Nilum
Dum videt in varias ire, redire, plagas:
Dum videt obductos violento gurgite campos,
Arvaq; limitibus cuncta carere suis:
Arte Geometrica (mendax nisi sacra vetustas)
Fluminis aduersi publica damna mouet.
Non satis esse putans, distingui limite terras
Internoscendi ni quoq; norma foret.
Haud aliter SIMON *confusa Mathemata cernens*
STEVINIVS: *numeris nec bene iuncta suis*
Docta GEOMETRIAE *tractat.* PROBLEMATA: *nodos*
Dissoluit: facilem monstrat in Arte viam.

Equidē meritiſsimò Reſpublica ſibi gaudeat, ego illi gratuler, delatam hanc tibi provinciam, vt bellicarum machinarum cura penes te ſumma ſit. Vota, fateor, ſint omnium noſtrum non iis opus eſſe, ſed pace pacta, firma, ſtabilique bello belliciſque inſtrumentis ſemel, ſimulque interdici, te quoque tua præfectura abdicari. Quod ſi fata duint, neq; te ſententia vertit, ſi non maiora, iocondiora tamen à te pace, quàm bello expectaremus. Neque enim dubitamus quin ingenium illud tuum, bellicis iam negotiis occupatum, ſi Deus Patriam quandoque ſaluam pacatam velit, tum vero vel minimè in ipſo otio futurum ſit otioſum.

Nunc certè, vt optimo iure Reſpublica ſuas Machinas, ſuam Salutem, ita ego mea Problemata, meos labores, tibi Patrono digniſsimo commendo, dedicoque. Profint foris, profint domi. Tu, ea, ſi mereantur, foue, ſtudiiſque, vt ſoles, faue.

The Commonwealth therefore may rightly rejoice at this, and I congratulate them on the fact that Thou hast been entrusted with the whole task of attending to the implements of war. Let everyone, I declare, hope that they will not be needed, but that, once a firm and lasting peace has been concluded and war, along with the implements of war, is forbidden once and for all, Thou wilt also be able to forgo Thy command. If Fate ordains this and Thy disposition does not alter, in peacetime we might expect from Thee, if not greater, at any rate surely even more pleasant things than in wartime. For we do not doubt but Thy mind, which is now engrossed by military matters, will, once God preserves our country in peace, by no means become idle in retirement.

However, as the State by the best of rights is now doing with regard to its implements of war, its welfare, I entrust to Thee, as a most worthy protector, my Problemata, my work, and I dedicate them to Thee. I hope that they may benefit both public and private life. If they so deserve, mayest Thou be pleased to give them Thy approval and to promote them in Thy own studies, as Thou art wont to do.

LIBER PRIMVS

IN QVO DEMONSTRABITVR QVOmodo à dato puncto in latere cuiuſcunque rectilinei, recta linea Geometricè ducenda ſit verſus partem petitam, quæ rectilineum diuidat ſecundum rationem datam.

ITEM QVOMODO IN QVOCVNQVE RECTIlineo ducenda erit linea recta & parallela cum latere ipſius quæſito, quæ rectilineum diuidat verſus partem petitam ſecundum rationem datam.

QVONIAM *in quibuſdam demonſtrationibus ſequentium Problematum, habebimus rationum ac proportionum quædam inuſitata vocabula, vt transformatæ proportionis &c. Vtile duxi ante ipſorum Problematum deſcriptionem, aperire quid cum ipſis vocabulis velimus. Item quid de ratione ac proportione ſentiamus.*

Dico enim rationem Geometricam, quam alij ſub duobus terminis limitant, admittere terminos quotlibet, quia tam inter tres quatuor vel plures, quam inter duas magnitudines, eſt ipſarum ſecundum quantitatem mutua habitudo. Hoc igitur vtilitatis (vt ſuo loco apparebit) & (quia res ita ſe habet) neceſſitatis gratia vt concedatur petimus.

FIRST BOOK,

in which it is to be demonstrated how from a given point on the side of any rectilinear figure a straight line is to be drawn geometrically towards a required part, which line divides the rectilinear figure in a given ratio.

Also how in any rectilinear figure is to be drawn a line parallel to a required side of said figure, which line divides the rectilinear figure towards a required part in a given ratio.

Since in certain proofs of the following problems we shall have certain unusual terms of ratios and proportions, *e.g.* of transformed proportion, etc., I have deemed it expedient to reveal before the description of the problems themselves what we mean by these terms. Also what are our views on ratio and proportion.

In fact, I say that Geometrical ratio, which others limit to two terms, admits of any desired number of terms, because between three, four or more as well as between two magnitudes there is a mutual relation in respect of quantity. We therefore request the reader to concede us this for the sake of utility (as will appear in its place) and necessity (because the matter is like this).

Ex quo concesso opus erit, talis rationis ac proportionis & reliquorum ex ipsis dependentium definitiones describere. Ipsarum autem summa in subiecto schemate exhibetur.

Magnitudinum, quæ & termini dicuntur, comparatio est in

- *Ratione quæ est*
  - *Regularis*
    - *Positiua defi.* 2.
    - *Mutata*
      - *Transformata defi.* 8.
      - *Inversa defi.* 9.
  - *Irregularis, vt perturbata defi.* 10.
- *Proportione quæ est*
  - *Regularis*
    - *Positiua. defi.* 12.
    - *Mutata*
      - *Transformata defi.* 19.
      - *Inversa defi.* 20.
      - *Alterna defi.* 21.
  - *Irregularis, vt perturbata defi.* 22.

When this has been conceded, it will be necessary to describe the definitions of such ratio and proportion and the other things dependent on them. Now the sum of these is shown in the following scheme.

Magnitudes, which are also called terms, are compared in

- ratio, which is
  - regular
    - positive Def. 2
    - changed
      - transformed Def. 8
      - inverted Def. 9
  - irregular, *e.g.* perturbed Def. 10
- proportion, which is
  - regular
    - positive Def. 12
    - changed
      - transformed Def. 19
      - inverted Def. 20
      - alternated Def. 21
  - irregular, *e.g.* perturbed Def. 22

Definitio 1.

Terminus eſt vna finita magnitudo.

Explicatio.

Vt linea A *dici poteſt terminus, eodemque modo vna ſuperficies aut vnum corpus terminus dicitur quaſi diſtinguens (potentialiter ſaltem) partes rationis aut proportionis: quare notandum eſt hic ſenſum eſſe de alio termino quam habetur in 14. prop. lib. 1. Euclid. nam ibi de extremitate vel extremitatibus magnitudinum loquitur: hic vero conſideramus totam magnitudinem, quatenus eſt limes, vt diximus, terminans partes rationis ſeu proportionis.*

A

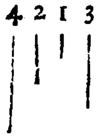
Definitio 2.

Ratio magnitudinum eſt diuerſorum terminorum eiuſdem generis magnitudinis mutua quædam ſecundum quantitatem habitudo.

Explicatio.

A B C D

4 2 1 3



Sint quidam termini magnitudinis eiuſdem generis (termini autem diuerſorum generum magnitudinum non habent inter ſe Geometricam comparationem) vt lineæ A B C D. *Igitur illarum linearum mutua habitudo ſecundum quantitatem, vt* A *duplum ipſius* B, *&* B *duplum ipſius* C, *&* D *ſeſquialterum ipſius* B *&c. dicitur ratio.*

Definitio 3.

Ratio in duobus terminis paucißimis conſiſtit.

Expli-

Definition 1.

A term is one finite magnitude.

Explanation.

Thus, the line *A* may be called a term, and in the same way a figure or a body is called a term, as if to distinguish (at least potentially) the parts of a ratio or a proportion: for which reason it is to be noted that term is used here in another sense than in the 14th proposition *) of Euclid's 1st book, for there the extremity or extremities of magnitudes are referred to; here, however, we consider the whole magnitude, as far as it is a limit, as we have said, which terminates the parts of a ratio or a proportion.

Definition 2.

A ratio of magnitudes is a certain mutual relation in respect of quantity of different terms of the same kind of magnitude.

Explanation.

Let there be certain terms of a magnitude of the same kind (indeed, the terms of different kinds of magnitudes are not susceptible of Geometrical comparison among each other), such as the lines *A, B, C, D*. Then the mutual relation in respect of quantity of those lines, *e.g. A* being twice *B*, and *B* twice *C*, and *D* one and a half times *B*, etc., is called their ratio.

Definition 3.

A ratio consists of at least two terms.

*) Stevin means Definition 13: A boundary is that which is an extremity of anything.

Explicatio.

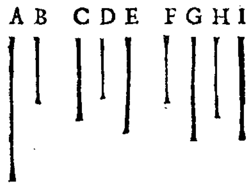
Res clara est cum in omni comparatione ad minimum sint duæ quantitates quarum sit similitudo.

Definitio 4.

Binaria ratio est, quæ in duobus terminis consistit. Ternaria vero ratio quæ in tribus terminis: Et sic pari ordine secundum multitudinem terminorum vocabitur ratio.

Explicatio.

Vt duorum terminorum A B *comparatio, dicitur à binis illis terminis binaria ratio. Eodem modo dicetur* C D E *ternaria, &* F G H I *quaternaria ratio &c.*



Definitio 5.

AEquales rationes sunt, quarum termini sunt multitudine pares, & vt vnius rationis primi termini quantitas, ad secundi termini quantitatem: sic alterius rationis primi termini quantitas, ad secundi termini quantitatem. Si vero rationes essent ternariæ: tunc vt vnius rationis primi termini quantitas, ad secundi, & secundi ad tertii: sic alterius rationis primi termini quantitas, ad secundi, & secundi ad tertii: & sic deinceps pari ordine in omnibus rationibus secundum multitudinem terminorum.

Expli-

Explanation.

This is clear, since in every comparison there are at least two quantities which are similar to one another.

Definition 4.

A binary ratio is a ratio which consists of two terms. A ternary ratio, however, is a ratio which consists of three terms. And thus in the same way the ratio is called after the number of its terms.

Explanation.

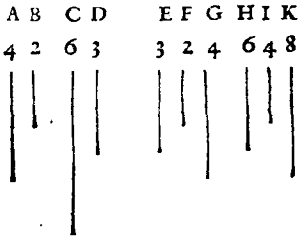
Thus, the comparison of two terms A, B is called, after those two terms, a binary ratio. In the same way $C : D : E$ will be called a ternary, and $F : G : H : I$ a quaternary ratio, etc.

Definition 5.

Equal ratios are ratios whose terms are the same in number, and as in the one ratio the quantity of the first term is to the quantity of the second term, so in the other ratio is the quantity of the first term to the quantity of the second term. But if the ratios are ternary ratios, then as in the one ratio the quantity of the first term is to that of the second term, and that of the second term to that of the third term, so in the other ratio is the quantity of the first term to that of the second term, and that of the second term to that of the third term; and so on in the same way with all the ratios according to the number of the terms.

Explicatio.

Sit binaria ratio A B, *cuius primi termini quantitas* A, *sit duplum secundi termini* B: *Sit & altera ratio binaria* C D, *cuius primi termini quantitas* C, *sit quoque duplum secundi* D. *Igitur ratio* A B, *æqualis dicitur rationi* C D. *Si verò esset ternaria ratio vt* E F G, *cuius primi termini quantitas* E, *sit sesquialtera secundi* F, *& secundi termini* F *quantitas, sit subduplum tertij* G. *Essetque & altera ternaria ratio* H I K, *cuius primi termini quantitas* H, *sit quoque sesquialtera secundi, & secundi termini* I *quantitas, sit quoque subduplum tertij* K: *Igitur ratio* E F G *æqualis dicitur rationi* H I K.



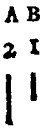
Idem intelligendum erit de rationibus æqualibus inexplicabilium magnitudinum.

Definitio 6.

Explicabilis ratio est quæ explicabili numero explicari potest.

Explicatio.

Vt habitudo secundum quantitatem rectæ A *ad rectum* B, *sit dupla. Quare, quia talis ratio explicabili numero explicatur (sumus autem in sententia illorum qui radices inexplicabiles numerum vocant, de quo aliàs in nostra Algebra latiùs dicetur) nempe hoc vocabulo dupla, dicitur* A B *explicabilis ratio: Idemque intelligendum est in ternaria & quaterna ratione, &c. Huc pertinent explicabilis binariæ rationis species & subdiuisiones, quas in subscripta tabula complectemur hoc modo:*



Explanation.

Let there be a binary ratio $A : B$, the quantity of whose first term A is twice the second term B. Let there also be another binary ratio $C : D$, the quantity of whose first term C is also twice the second term D. Then the ratio $A : B$ is said to be equal to the ratio $C : D$. If, however, there is a ternary ratio, such as $E : F : G$, the quantity of whose first term E is one and a half times the second term F, and the quantity of the second term F is one half the third term G, and if there is another ternary ratio $H : I : K$, the quantity of whose first term H is also one and a half times the second term, and the quantity of the second term I is also one half the third term K, then the ratio $E : F : G$ is said to be equal to the ratio $H : I : K$.

The same is to be understood with regard to equal ratios of irrational magnitudes.

Definition 6.

A rational ratio is a ratio which can be expressed by means of a rational number.

Explanation.

Thus, let the relation in respect of quantity of the line A to the line B be double. Therefore, because such a ratio is expressed by means of a rational number (we are of the opinion of those who call irrational roots numbers, about which we shall speak more fully elsewhere in our Algebra) *), to wit by the word "double", $A : B$ is called a rational ratio. And the same is to be understood with regard to a ternary and a quaternary ratio, etc. Here belong the kinds and subdivisions of a rational binary ratio, which we include in the table below, as follows:

*) See *L'Arithmétique*, Def. XXXI.

Ratio eſt aut

- *Æqualis*
- *Inæqualis*
  - *Maioris inæqualitatis*
    - *Simplex*
      - *Superparticularis*
      - *Superpartiens*
    - *Compoſita*
      - *Quæ per integros numeros tantum explicatur vt multiplex.*
      - *Quæ per integros & fractos numeros explicatur vt*
        - *Multiplex ſuper particularis*
        - *Multiplex ſuper partiens*
  - *Minoris inæqualitatis*
    - *Simplex*
      - *Subſuperparticularis*
      - *Subſuperpartiens*
    - *Compoſita*
      - *Quæ per integros numeros tantũ explicatur, vt ſubmultiplex.*
      - *Quæ per integros & fractos numeros explicatur vt*
        - *Submultiplex ſuper particularis*
        - *Submultiplex ſuper partiens.*

A ratio is either

- equal
- or
- unequal
  - of major inequality
    - simple
      - superparticular
      - superpartiens
    - compound
      - which is expressed by means of integers only, such as multiple
      - which is expressed by means of integers and fractions, such as
        - multiple superparticular
        - multiple superpartiens
  - of minor inequality
    - simple
      - subsuperparticular
      - subsuperpartiens
    - compound
      - which is expressed by means of integers only, such as submultiple
      - which is expressed by means of integers and fractions, such as
        - submultiple superparticular
        - submultiple superpartiens

Definitio 7.

Inexplicabilis ratio est, quæ explicabili numero explicari non potest.

Explicatio.

Vt est inter infinitas alias magnitudines latus quadrati ad eiusdem quadrati diagonalem.

Definitio 8.

Transformata ratio est, in qua per resumptionem fit termini vel terminorum transfiguratio.

Explicatio.

Sit data ratio quæcunque vt binaria A B, *ad* B C, *Igitur si tota* A C, *sumatur pro vno termino, & comparetur ad alterutrum datum terminum, vt* A B, *pro altero termino, dicetur illa sumptio* A C, *ad* A B, *(propter terminorum transfigurationem) transformata ratio (quam alij compositam rationem vocant) datæ rationis* A B, *ad* B C.

Aut alio modo secetur à recta B C, *recta* B D, *æqualis rectæ* A B, *sitque reliquum* D C: *Igitur si* D C, *sumatur pro vno termino, & comparetur ad alterutrum datum terminum, vt* A B, *pro altero termino, dicetur illa sumptio* D C, *ad* A B, *(propter terminorum transfigurationem) transformata ratio (quam alij quoque disiunctam rationem vocant) datæ rationis* A B, *ad* B C.

Aut alio modo si sumatur pars quæcunque rectæ A B, *vt recta* A E, *& comparetur ad totam* A C, *vel ad quandam partem dicetur talis resumptio (propter terminorum transfigurationem) transformata ratio datæ rationis* A B, *ad* B C.

Aut si multiplicetur aliquis terminus vel termini pars, & comparetur ad totam vel aliquam partem rationis, dicetur talis resumptio (propter terminorum transfigurationem) transformata ratio datæ rationis.

In summa omnem resumptionem per transfigurationem termini vel terminorum ex data ratione originem trahentem, quæ multis ac pene infinitis modis fieri potest, vocamus ipsius datæ rationis transformatam rationem.

Complectimurque hoc modo sub hac definitione Coniunctam, Disiunctam, & Conversam rationem, simul & omnes alias rationes, quarum terminorum vt supra diximus fit per transfigurationem mutatio. Qualis vero sit

Definition 7.

An irrational ratio is a ratio which cannot be expressed by means of a rational number.

Explanation.

Thus, among an infinite number of other magnitudes, the ratio of the side of a square to the diagonal of said square.

Definition 8.

A transformed ratio is a ratio in which by re-association a transfiguration of a term or of terms is effected.

Explanation.

Let any ratio be given, *e.g.* the binary ratio *AB* : *BC*. Then, if the whole *AC* be taken as one term and compared with one of the two given terms, *e.g.* *AB*, as the other term, this association of *AC* and *AB* (on account of the transfiguration of the terms) will be called a transformed ratio (which others call a compound ratio) of the given ratio *AB* : *BC*.

Or, in another way, let there be cut from the line *BC* the line *BD*, equal to the line *AB*, and let the rest be *DC*. Then, if *DC* be taken as one term and compared with one of the two given terms, *e.g.* *AB*, as the other term, this association of *DC* and *AB* (on account of the transfiguration of the terms) will be called a transformed ratio (which others also call disjunct ratio) of the given ratio *AB* : *BC*.

Or, in another way, if a part of any line *AB*, *e.g.* the line *AE*, be taken and compared with the whole line *AC* or with any part, such a re-association (on account of the transfiguration of the terms) will be called a transformed ratio of the given ratio *AB* : *BC*.

Or if a multiple be taken of some term or part of a term and compared with the whole or some part of the ratio, such a re-association (on account of the transfiguration of the terms) will be called a transformed ratio of the given ratio.

In general, we call every re-association through transfiguration of a term or of terms originating from a given ratio, which can be effected in many and almost an infinite number of ways, a transformed ratio of the said given ratio.

And we thus include in this definition the Conjunct, the Disjunct, and the Converse ratio, and likewise all other ratios whose terms have been changed, as we have said above, through transfiguration. However, which practical use is *in praxi huius definitionis vſus, in demonſtrationibus quorundam Problematum huius libri ſatis erit manifeſtum.*

Definitio 9.

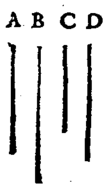
Inverſa ratio eſt ſumptio conſequentis termini ad antecedentem.

Explicatio.

Sit data ratio A *ad* B, *in qua comparetur* A *ad* B, *Igitur ſi comparemus vice verſa conſequentem* B, *ad antecedentem* A, *dicetur talis ſumptio* B *ad* A, *inverſa ratio rationis* A *ad* B.

Definitio 10.

Perturbata ratio eſt, comparatio ſecundi termini ad tertium, & primi ad ſecundum: ſi verò plurium terminorum fuerit ratio, tum ſecundi ad tertium, & tertij ad quartum, & ſic deinceps quamdiu ratio extiterit: tandemque primi ad ſecundum.



Explicatio.

Sint termini A B C D, *Igitur ſi comparemus* B *ad* C *&* C *ad* D *tamdemque* A *ad* B *talis comparatio dicitur perturbata ratio.*

Definitio 11.

Perturbata ratio in tribus terminis paucisſimis conſiſtit.

Definitio 12.

Proportio magnitudinum eſt duarum æqualium rationum ſimilitudo.

Expli-

made of this definition will be sufficiently shown in the proofs of some Problems of this book.

Definition 9.

Inverted ratio is an association of the consequent with the antecedent term.

Explanation.

Let there be given the ratio $A : B$, in which A is compared with B. Then, if we compare *vice versa* the consequent term B with the antecedent term A, such an association of B with A is called the inverted ratio of the ratio $A : B$.

Definition 10.

Perturbed ratio is the comparison of the second term with the third, and of the first with the second; but if the ratio should comprise more terms, then of the second with the third, and of the third with the fourth, and so on as far as the ratio goes, and finally of the first with the second.

Explanation.

Let the terms be A, B, C, D. Then, if we compare B with C, and C with D, and finally A with B, such a comparison is called a perturbed ratio.

Definition 11.

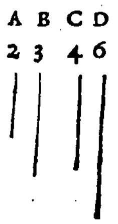
A perturbed ratio consists of at least three terms.

Definition 12.

A proportion of magnitudes is the similarity of two equal ratios.

Explicatio.

Sint duæ quæcunque æquales rationes, vt binariæ A B, *&* C D, *Illarum verò comparatio nempe vt se habet* A *ad* B, *sic se habet* C *ad* D *dicitur proportio: Vel termini* A B *dicuntur proportionales cum terminis* C D.

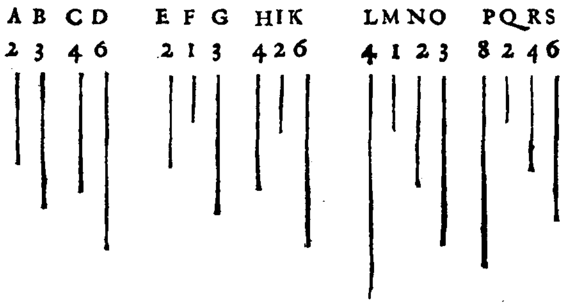


Definitio 13.

Binaria proportio est quæ ex duabus æqualibus binarijs rationibus consistit. Ternaria verò proportio quæ ex duabus æqualibus ternarijs rationibus consistit, & sic pari ordine secundum species rationum vocabitur proportio.

Explicatio.

Vt duæ æquales binariæ rationes A B *&* C D, *dicuntur binaria proportio: Similiter duæ æquales ternariæ rationes* E F G *&* H I K, *dicuntur ternaria proportio: Similiter duæ æquales quaternariæ rationes* L M N O, *&* P Q R S, *dicuntur quaternaria proportio. Idem de reliquis, vt quinaria, senaria &c. proportione intelligendum est.*



Explanation.

Let there be any two equal ratios, such as the binary ratios $A : B$ and $C : D$. Now their comparison, *viz.* as A is to B, so is C to D, is called a proportion; or the terms A and B are said to be proportional to the terms C and D.

Definition 13.

A binary proportion is a proportion which consists of two equal binary ratios. But a ternary proportion is a proportion which consists of two equal ternary ratios, and thus in the same way the proportion will be called after the kind of the ratios.

Explanation.

Thus, two equal binary ratios $A : B$ and $C : D$ are called a binary proportion. Similarly, two equal ternary ratios $E : F : G$ and $H : I : K$ are called a ternary proportion. Similarly, two equal quaternary ratios $L : M : N : O$ and $P : Q : R : S$ are called a quaternary proportion. The same is to be understood of the others, *viz.* of a quinary, a senary, etc. proportion.

Definitio 14.

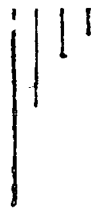
Continua proportio est, cum quisque intermedius terminus vice antecedentis & consequentis sumitur.

Explicatio.

Sit proportio A B C D, *sitque terminus* A *duplus ipsi* B, *&* B *duplus ipsi* C, *&* C *duplus ipsi* D: *igitur quia intermedij termini, vt* B *&* C, *vice antecedentis & consequentis sumi possunt (nam si sumatur* B *pro antecedenti termino, & dicamus vt* B *ad* C, *sic* C *ad* D) *erunt* B C C D *termini proportionales.*

Similiter si idem terminus B, *sumatur pro consequenti termino, & dicatur, vt* A *ad* B, *sic* B *ad* C, *erunt* A B B C, *termini proportionales. Eodem modo inuenietur* C *posse sumi pro antecedenti & consequenti termino, diceturque* A B C D *proportio continua.*

A B C D
8 4 2 1



Definitio 15.

Continua proportio in tribus terminis paucissimis consistit.

Explicatio:

Vt continua proportio A B C, *in qua dicimus vt* A *ad* B, *sic* B *ad* C, *ex minoribus terminis quam tribus constare non potest.*

A B C
8 4 2

Defi-

Definition 14.

Continuous proportion is if each of the mean terms may be taken as the antecedent and the consequent term.

Explanation.

Let the proportion be $A : B = C : D$, and let the term A be twice the term B, and B twice the term C, and C twice the term D, then because the mean terms, *viz.* B and C, can be taken as the antecedent and the consequent term (for if B is taken as the antecedent term, we also say: as B is to C, so is C to D), then B, C, C, D will be proportional terms.

Similarly, if the same term B is taken as the consequent term and it is said: as A is to B, so is B to C, then A, B, B, C will be proportional terms. In the same way it will be found that C can be taken as the antecedent and the consequent term, and $A : B = C : D$ will be called a continuous proportion.

Definition 15.

A continuous proportion consists of at least three terms.

Explanation.

Thus, the continuous proportion $A : B = B : C$, in which we say: as A is to B, so is B to C, cannot consist of fewer than three terms.

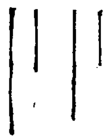
Definitio 16.

Discontinua proportio est cum quisque intermedius terminus vice antecedentis & consequentis sumi non potest.

Explicatio.

Sit proportio A B C D, *sitque terminus* A *duplus ipsi* B, *&* B, *in subsequi altera ratione ad* C, *&* C *duplus ipsi* B: *Igitur quia intermedij termini, vt* B *&* C *vice antecedentis & consequentis sumi non possunt (nam vt* B *ad* C, *sic non est* C *ad* D *&c.) dicitur* A B C D, *discontinua proportio.*

A B C D
6 3 4 2



Definitio 17.

Discontinua proportio in quatuor terminis paucissimis consistit.

Explicatio.

Vt discontinua proportio A B C D *præcedentis decimæ sextæ definitionis, consistit in quatuor terminis, neque ex minoribus constare potest.*

Definitio 18.

Proportionis Homologi termini dicuntur, primus primæ rationis, cùm primo secundæ rationis. Similiter dicuntur Homologi termini secundus primæ rationis, cum secundo secundæ rationis, & sic pari ordine in reliquis secundum multitudinem terminorum.

Explicatio.

Sit proportio quæcunque, vt ternaria A B C, *ad* D E F: *Igitur primus terminus* A *primæ rationis, cum primo termino* D, *secundæ rationis, dicuntur Homologi termini: Eodemque modo dicuntur* B E *Homologi termini, similiter &* C F *Homologi termini.*

ABC

Definition 16.

Discontinuous proportion is if each of the mean terms cannot be taken as the antecedent and the consequent term.

Explanation.

Let the proportion be $A : B = C : D$, and let the term A be twice the term B, and let B be to C in the ratio of $3 : 4$, and let C be twice the term D, then because the mean terms, *viz.* B and C, cannot be taken as the antecedent and the consequent term (for as B is to C, so is C not to D, etc.), $A : B = C : D$ is called a discontinuous proportion.

Definition 17.

A discontinuous proportion consists of at least four terms.

Explanation.

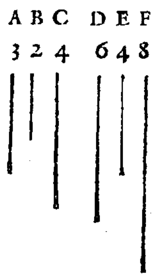
Thus, the discontinuous proportion $A : B = C : D$ of the foregoing sixteenth definition consists of four terms, and cannot consist of fewer terms.

Definition 18.

Homologous terms of a proportion are the first of the first ratio to the first of the second ratio. Similarly, homologous terms are the second of the first ratio to the second of the second ratio, and thus in the same way with the rest, according to the number of terms.

Explanation.

Let there be any proportion, *e.g.* the ternary proportion $A : B : C = D : E : F$, then the first term A of the first ratio with the first term D of the second ratio are called homologous terms. And in the same way B and E are called homologous terms; similarly also C and F are called homologous terms.

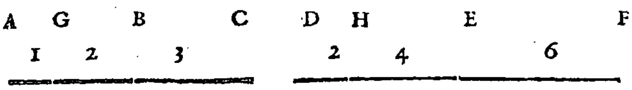


Definitio 19.

Transformata proportio eſt quæ ex duabus æqualibus transformatis rationibus conſiſtit.

Explicatio.

Sint duæ rationes quarum prima A B *ad* B C, *Secunda* D E *ad* E F, *ſitque rationis* A B *ad* B C *transformata ratio per* 8. *definitionem* A G *ad* G C, *cui æqualis ratio ſit, transformata ratio* D H *ad* H F: *Igitur proportio ex rationibus* A G *ad* G C, *&* D H *ad* H F, *dicitur transformata proportio datæ proportionis* A B, B C, D E, E F. *Poteſtque hæc transformata proportio tam varijs modis accidere, quàm ſunt transformatarum rationum octauæ definitionis differentiæ.*



NOTA.

Hac definitione transformatæ proportionis conceſſa, ſuperflua videntur theoremata 1. 2. 3. 4. 5. 6. 12. 15. 17. 18. 19. 20. 22. *&* 24. *lib.* 5. *Euclid. quæ omnia cùm multis alijs ſimilibus (cùm omnia ſub hac vnica definitione comprehendantur) in vno theoremate poſſent explicari.*

Definitio 20.

Inverſa proportio eſt quæ ex duabus æqualibus inverſis rationibus conſiſtit.

Expli-

Definition 19.

Transformed proportion is a proportion which consists of two equal transformed ratios.

Explanation.

Let there be two ratios, of which the first is $AB : BC$, the second $DE : EF$, and let the transformed ratio of the ratio $AB : BC$ by the 8th definition be $AG : GC$, to which let the transformed ratio $DH : HF$ be equal. Then the proportion consisting of the ratios $AG : GC$ and $DH : HF$ is called a transformed proportion of the given proportion $AB : BC = DE : EF$. And this transformed proportion may occur in as varied ways as there are different kinds of transformed ratios of the eighth definition.

NOTE.

Once this definition of a transformed proportion has been conceded, the theorems 1, 2, 3, 4, 5, 6, 12, 15, 17, 18, 19, 20, 22, and 24 of Euclid's 5th book would seem to be superfluous; they can all, with many other and similar theorems (since they are all included in this one definition), be expressed in one theorem *).

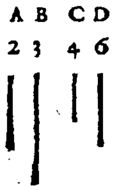
Definition 20.

Inverted proportion is a proportion which consists of two equal inverted ratios.

*) This is due to the general way in which Stevin has defined transformed ratio: if $a : b$ is a given ratio, and $p, q, \ldots\ldots$ are given numbers, then any ratio $(k_1a + l_1b + m_1p + n_1q + \ldots\ldots):(k_2a + l_2b + m_2p + n_2q + \ldots\ldots)$, where $k_1 \ldots\ldots n_2$ are positive or negative numbers, is a transformed ratio.

Explicatio.

Sit data proportio A B C D, *sitque* D C *rationis* C D *inverſa ratio per nouam definitionem : Similiter* B A *rationis* A B *inverſa ratio: Igitur ſi inferatur, vt* D *ad* C *ſic* B *ad* A, *vel, vt* B *ad* A, *ſic* D *ad* C, *dicetur hoc argumentari ab inverſa proportione, proportionis* A B C D.

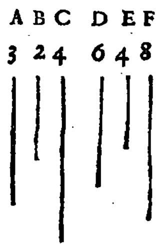


Definitio 21.

Alterna proportio eſt ſimilis ſumptio homologorum terminorum ad homologos terminos.

Explicatio.

Sit proportio quæcunque, vt ternaria A B C, D E F *ſumanturque homologi termini, vt* A D *&* C F: *Igitur proportio* A D C F *dicitur alterna proportio datæ proportionis, poteſtque hoc modo alterna proportio ex ipſa data proportione varijs modis ſumi.*



NOTA.

Dicitur in hac definitione ſimilis ſumptio, *hoc eſt, ſi antecedens terminus primæ alternæ rationis, fuerit ex prima data ratione, requiritur vt antecedens terminus ſecundæ alternæ rationis, ſumatur quoque ex prima data ratione, & ita de conſequentibus terminis, vt ſupra factum eſt, nam* A *&* C *ſunt alternæ proportionis antecedentes termini, & ex eadem prima data ratione.*

Hæc igitur ſimilis ſumptio *obſervatu neceſſaria eſt, nam etſi* D A *ſunt homologi termini, ſimiliter &* C F, *Tamen vt* D *ad* A *ſic non eſt* C *ad* F: *Quare vt in definitione, termini ſunt ſimiliter ſumendi.*

C Notandum

Explanation.

Let there be given the proportion $A : B = C : D$, and let $D : C$ be the inverted ratio of the ratio $C : D$ by the ninth definition. Similarly, let $B : A$ be the inverted ratio of the ratio $A : B$. Then, if it is inferred that as D is to C, so is B to A, or as B is to A, so is D to C, it will be said that this is proved from the inverted proportion of the proportion $A : B = C : D$.

Definition 21.

Alternated proportion is a similar association of homologous terms with homologous terms.

Explanation.

Let there by any proportion, *e.g.* the ternary proportion $A : B : C = D : E : F$, and let the homologous terms be taken, such as A, D and C, F. Then the proportion $A : D = C : F$ is called an alternated proportion of the given proportion, and thus an alternated proportion can be taken from the said given proportion in various ways.

NOTE.

In this definition a *similar association* is spoken of, *i.e.* if the antecedent term of the first alternated ratio is taken from the first given ratio, it is required that the antecedent term of the second alternated ratio be also taken from the first given ratio, and thus with the consequent terms, as has been done above, for A and C are antecedent terms of an alternated proportion and have been taken from the same first given ratio.

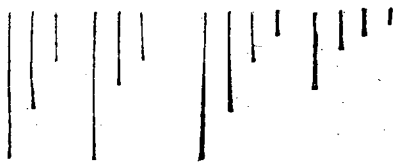
Hence it is necessary that this *similar association* be observed, for though D and A are homologous terms, and similarly C and F, nevertheless as D is to A, so is C not to F. And therefore, as in the definition, the terms have to be associated in a similar way.

Notandum est quoque alternam proportionem non posse sumi nisi ex duabus rationibus eiusdem generis magnitudinis.

Definitio 22.

Perturbata proportio est similitudo duarum æqualium rationum quarum altera est perturbata.

A B C D E F G H I K L M N O
6..42. 6.3.2. 12.8.4.2. 6.3.2.1.



Explicatio.

Sint duæ rationes quarum prima A B C *regularis per* 12.*defi.* & *altera* D E F *perturbata per* 10. *defi.* & *æqualis primæ rationi, id est,* *vt* A *ad* B, & B *ad* C, *sic* E *ad* F & D *ad* E. *Idemque intelligendum est, vbi rationes fuerint ex pluribus quàm tribus terminis, vt ratio regularis* G H I K *cum perturbata* L M N O. *Igitur talis comparatio dicitur perturbata proportio, ad differentiam regularis proportionis definitæ in præcedenti* 12. *defi.*

Vtilitas huius definitionis in praxi nempe in propositionum demonstratione est, quod rationum extremi termini sunt proportionales, hoc est, si est vt A *ad* B, & B *ad* C, *sic* E *ad* F & D *ad* E, *Ergo concluditur vt* A *ad* C, *sic* D *ad* F, *aut si est, vt* G *ad* H & H *ad* I. & I *ad* K, *sic* M *ad* N & N *ad* O & L *ad* M, *Ergo vt* G *ad* K, *sic* L *ad* O. *Illamque perpetuò esse necessariam consequentiam, colligitur ex* 23. *prop. lib.* 5. *Euclid.*

Definitio 23.

Perturbata proportio in sex terminis paucissimis consistit.

Definitio 24.

Cum tres termini proportionales fuerint: Primus ad tertium duplicatam rationem habere dicitur eius, quam habet ad secundum. At cum quatuor termini continuè proportionales fuerint, primus ad quartum triplicatam rationem habere dicitur eius, quam habet ad secundum: Et semper deinceps vno amplius quamdiu proportio extiterit:

It is also to be noted that an alternated proportion cannot be associated unless from two ratios of the same kind of magnitude.

Definition 22.

Perturbed proportion is the similarity of two equal ratios, one of which is perturbed.

Explanation.

Let there be two ratios, the first of which, $A : B : C$, is regular by the 12th definition, while the other, $D : E : F$, is perturbed by the 10th definition and equal to the first ratio, *i.e.* as A is to B, and B to C, so is E to F, and D to E. And the same is to be understood where there should be ratios of more than three terms, such as the regular ratio $G : H : I : K$ with the perturbed ratio $L : M : N : O$. Then such a comparison is called a perturbed proportion, to distinguish it from the regular proportion defined in the foregoing 12th definition.

The utility of this definition in practice indeed is shown in the proof of the propositions, because the extreme terms of the ratios are proportional, *i.e.* if as A is to B, and B to C, so is E to F, and D to E, it is concluded that as A is to C, so is D to F; or if as G is to H, and H to I, and I to K, so is M to N, and N to O, and L to M, then as G is to K, so is L to O. And that this is perpetually a necessary consequence, is inferred from the 23rd proposition of Euclid's 5th book *).

Definition 23.

A perturbed proportion consists of at least six terms.

Definition 24.

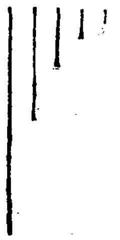
When three terms are proportional, the first is said to be to the third in the duplicate ratio of that in which it is to the second. And when four terms are continuously proportional, the first is said to be to the fourth in the triplicate ratio of that in which it is to the second. And so always on, one more than the proportion goes.

*) Euclid V Prop. 23: If there be three magnitudes, and others equal to them in multitude, which taken two and two together are in the same ratio, and the proportion of them be perturbed, they will also be in the same ratio at the corresponding places.

Explicatio.

Sint termini continuè proportionales A B C D E: *Igitur primus terminus* A *ad tertium* C *duplicatam dicitur habere rationem eius, quam habet ad secundum* B. *Et primus terminus* A, *ad quartum* D *triplicatam dicitur habere rationem eius, quam habet ad secundum* B: *Similiter primus terminus* A *ad quintum* E, *quadruplicatã dicitur habere rationem eius, quam habet ad secundum* B.

A B C D E
16 8 4 2 1



Notandum est hic non esse quæstionem (vt multi putant) de magnitudine terminorum (nam dici potest E *ad* C *duplicatam habere rationem eius quam habet ad* D *&c. Sed de ipsorum nomine proportionis duplicatæ triplicatæ &c. vt recte hunc locum sub* 10. *definitione lib.* 5. *Euclid. explicat doctissimus Mathematicus Christophorus Clauius, cui placebit legat ipsum.*

PROBLEMA I.

Datis rectilinei triangulis: Rectas lineas invenire inter se in ea ratione ac ordine vt sunt trianguli.

Explicatio dati.

Sint dati rectilinei A B C D E F *quatuor trianguli* A B C, A C D, A D E, A E F.

Explicatio quæsiti.

Oporteat quatuor rectas lineas invenire, inter se in ea ratione ac ordine, quo sunt ipsi trianguli, hoc est, cum triangulis proportionales.

Constructio.

Describatur per 45. *prop. lib.* 1. *Euclid. Parallelogrammum* G H I K *æquale toti rectilineo dato* A B C D E F, *Sitque parallelogrammum* G L *æquale triangulo* A B C, *& parallelogrammum* M N *æquale triangulo* A C D, *& parallelogrammum* O P *æquale triangulo* A D E, *& parallelogrammum* Q I *æquale triangulo* A E F,

 Dico

Explanation.

Let the continuously proportional terms be $A : B = B : C = C : D = D : E$. Then the first term A is said to be to the third term C in the duplicate ratio of that in which it is to the second term B. And the first term A is said to be to the fourth term D in the triplicate ratio of that in which it is to the second term B. Similarly, the first term A is said to be to the fifth term E in the quadruplicate ratio of that in which it is to the second term B.

It is to be noted here that there is no question (as many people think) of the magnitude of the terms (for it may be said that E is to C in the duplicate ratio of that in which it is to D, etc.), but of the name "duplicate, triplicate, etc. proportion" of these terms, as this passage is rightly explained under the 10th definition of Euclid's 5th book by the most learned Mathematician Christophorus Clavius; let he whom it pleases read him *).

PROBLEM I.

Given the triangles of a rectilinear figure, to find lines which are to one another in the same ratio and order as the triangles are.

Given.

Let the four triangles ABC, ACD, ADE, AEF of a rectilinear figure $ABCDEF$ be given.

Required.

Let it be required to find four lines which are to one another in the same ratio and order as the triangles are, *i.e.* proportional to the triangles.

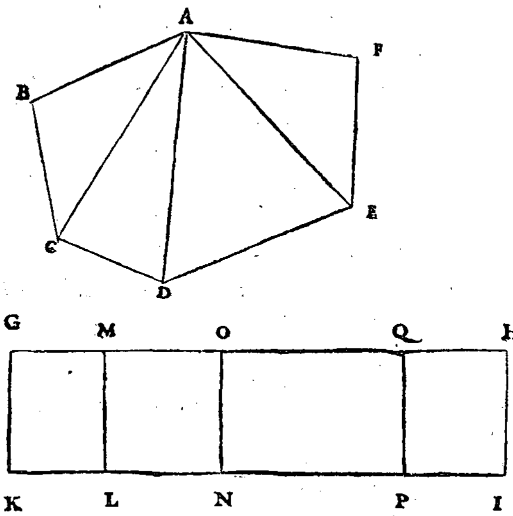
Construction.

By the 45th proposition of Euclid's 1st book construct a Parallelogram $GHIK$ equal to the whole given rectilinear figure $ABCDEF$, and let the parallelogram GL be equal to the triangle ABC, and the parallelogram MN equal to the triangle ACD, and the parallelogram OP equal to the triangle ADE, and the parallelogram QI equal to the triangle AEF.

---

*) When $a:b = b:c$, then $a:c$ has the duplicate ratio of $a:b$; when $a:b = b:c = c:d$, then $a:d$ has the triplicate ratio of $a:b$; hence $a/c = (a/b)^2$; $a/d = (a/b)^3$. As to Clavius, see Introduction, footnote 1.

Dico quatuor rectas lineas K L, L N, N P, P I, *esse inventas, inter se in ea ratione ac ordine qua sunt trianguli dati, hoc est, vt triangulus* A B C *ad* A C D, *&* A C D *ad* A D E, *&* A D E *ad* A E F: *Sic* K L *ad* L N, *&* L N *ad* N P, *&* N P *ad* P I, *vt erat quæsitum.*



Demonstratio.

Vt recta K L *ad rectam* L N, *sic parallelogrãmum* M N *per* 1 *prop. lib.* 6. *Eucl.* *Et triangulus* A B C *æqualis est parallelogrammo* G L, *& triangulus* A C D *æqualis est parallelogrammo,* M N, *per constructionem: Quare vt recta* K L *ad rectam* L N, *Sic triangulus* A B C, *ad triangulum* A C D. *Eodemque modo ostendetur rectam* N P *correspondere triangulo* A D E, *& rectam* P I *triangulo* A E F.

Conclusio.

Igitur datis rectilinei triangulis, rectæ lineæ inventæ sunt inter se in ea ratione ac ordine, vt sunt trianguli, quod erat faciendum.

Idem alio modo.

Explicatio dati.

Sint iterum dati rectilinei trianguli A B C, A C D, A D E, A E F.

Explica-

I say that four lines *KL*, *LN*, *NP*, *PI* have been found which are to one another in the same ratio and order in which the given triangles are, *i.e.* as triangle *ABC* is to *ACD*, and *ACD* to *ADE*, and *ADE* to *AEF*, so is *KL* to *LN*, and *LN* to *NP*, and *NP* to *PI*; as was required.

Proof.

As the line *KL* is to the line *LN*, so is the parallelogram *LG* to the parallelogram *MN* by the 1st proposition of Euclid's 6th book. And the triangle *ABC* is equal to the parallelogram *GL*, and the triangle *ACD* is equal to the parallelogram *MN* by the construction. And therefore, as the line *KL* is to the line *LN*, so is the triangle *ABC* to the triangle *ACD*. And in the same way it will be shown that the line *NP* corresponds to the triangle *ADE*, and the line *PI* to the triangle *AEF*.

Conclusion.

Therefore, given the triangles of a rectilinear figure, lines have been found which are to one another in the same ratio and order as the triangles are; which was to be performed.

The same in another way.

Given.

Let the triangles *ABC*, *ACD*, *ADE*, *AEF* of a rectilinear figure again be given.

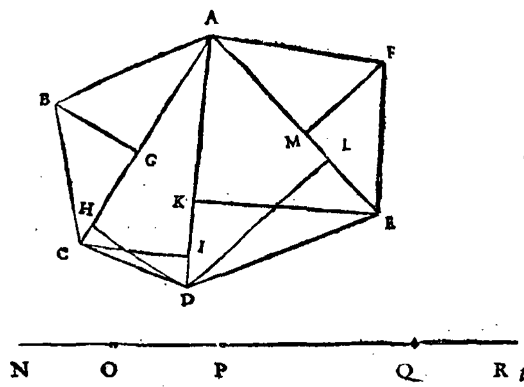
Explicatio quæsiti.

Oporteat quatuor rectas lineas invenire, inter se in ea ratione ac ordine quo sunt ipsi trianguli.

Constructio.

Ducatur recta B G *perpendicularis ad rectam* A C, *& recta* D H *perpendicularis ad eandem rectam* A C, *& recta* C I *perpendicularis ad rectam* A D, *& recta* E K *perpendicularis ad rectam* A D, *& recta* D L *perpendicularis ad rectam* A E, *& recta* F M *perpendicularis ad rectam* A E, *& recta* N O *æqualis rectæ* B G, *& recta* O P *æqualis rectæ* H D: *inveniaturque per* 12. *prop. lib.* 6. *Euclid. quarta linea proportionalis, quarum prima* C I, *secunda* K E, *tertia* H D *vel* O P, *sitque quarta* P Q: *Inveniatur deinde quarta linea proportionalis quarum prima* D L, *secunda* F M, *tertia* P Q, *sitque quarta* Q R: *Eodemque modo continuandum esset si plures essent trianguli.*

Dico quatuor rectas lineas N O, O P, P Q, Q R *esse inventas inter se in ea ratione ac ordine, in quo sunt dati trianguli, hoc est, vt triangulus* A B C *ad* A C D, *&* A C D *ad* A D E, *&* A D E *ad* A E F: *Sic* N O *ad* O P, *&* O P *ad* P Q, *&* P Q *ad* Q R, *vt erat quæsitum.*



Demonstratio.

Distinctio I.

Vt recta B G *ad rectam* H D, *sic triangulus* A B C *ad triangulum* A C D, *vt colligitur ex* 1 *prop. lib.* 6. *Euclid. Quod inter alios explicauit Christophorus Clauius ad dictam* 1. *prop. lib.* 6. *Euclid. Nam sunt trianguli quorum basis eadem* A C, *quare ita se habent vt altitudines: Et recta* B G *æqualis est*

 rectæ

Required.

Let it be required to find four lines which are to one another in the same ratio and order as the triangles are.

Construction.

Draw the line *BG* perpendicular to the line *AC*, and the line *DH* perpendicular to the same line *AC*, and the line *CI* perpendicular to the line *AD*, and the line *EK* perpendicular to the line *AD*, and the line *DL* perpendicular to the line *AE*, and the line *FM* perpendicular to the line *AE*, and the line *NO* equal to the line *BG*, and the line *OP* equal to the line *HD*. And by the 12th proposition of Euclid's 6th book find the fourth proportional, the first term being *CI*, the second *KE*, the third *HD* or *OP*; and let the fourth be *PQ*. And subsequently find the fourth proportional, the first term being *DL*, the second *FM*, the third *PQ*; and let the fourth be *QR*. And this would have to be continued in the same way if there were more triangles.

I say that four lines *NO*, *OP*, *PQ*, *QR* have been found, which are to one another in the same ratio and order in which the given triangles are, *i.e.* as the triangle *ABC* is to *ACD*, and *ACD* to *ADE*, and *ADE* to *AEF*, so is *NO* to *OP*, and *OP* to *PQ*, and *PQ* to *QR*; as was required.

Proof.

Section 1.

As the line *BG* is to the line *HD*, so is the triangle *ABC* to the triangle *ACD*, as is inferred from the 1st proposition of Euclid's 6th book, which has been explained, among others, by Christophorus Clavius in his commentary on the 1st proposition of Euclid's 6th book. For they are triangles with the same base *AC*, on which account they are to one another in the ratio of their heights. Moreover the line *BG* is equal to the line *NO*, and the line *HD* to the line *OP* *rectæ* N O, *&* *recta* H D *rectæ* O P *per constructionem: Ergo vt recta* N O *ad* O P, *sic triangulus* A B C *ad triangulum* A C D.

Distinctio 2.

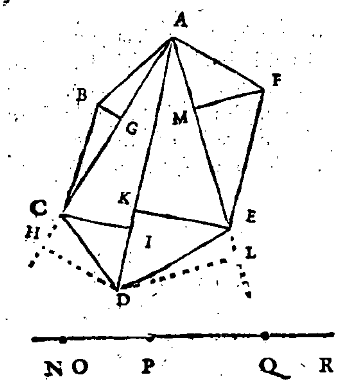
Vt recta C I *ad rectam* K E, *sic triangulus* A C D *ad triangulum* A D E *per locum citatum in* 1 *distinctione, nam sunt trianguli ad eandem basin* A D. *& per constructionem vt recta* C I *ad rectam* K E, *sic recta* O P *ad* P Q, *Ergo vt recta* O P *ad* P Q, *sic triangulus* A C D *ad triangulum* A D E.

Distinctio 3.

Vt recta D L *ad rectam* M F, *sic triangulus* A D E *ad triangulum* A E F *per locum citatum in* 1 *distinctione, nam sunt trianguli ad eandem basin* A E. *Et vt recta* D L *ad rectam* M F, *sic recta* P Q *ad* Q R *per constructionem, Ergo vt recta* P Q *ad* Q R, *sic triangulus* A D E *ad triangulum* A E F.

Conclusio.

Igitur datis rectilinei triangulis rectæ lineæ inventæ sunt inter se in ea ratione ac ordine vt sunt trianguli. Quod per hunc secundum modum erat faciendum.



NOTA.

Si perpendiculares quædam caderent extra rectilineum, vt perpendicularis D H *ad rectam* A C, *tunc producenda esset recta* A C, *& ducenda* D H *perpendicularis ad productam* A C: *Similiterque* D L *ad productam* A E *vt in hac figura patet, cui præcedens constructio ac demonstratio applicari potest.*

PROBLEMA II.

A quouis angulo trianguli rectam lineam ducere, quæ diuidat triangulum versus partem petitam secundum rationem datam.

Expli-

by the construction. Consequently, as the line *NO* is to *OP*, so is the triangle *ABC* to the triangle *ACD*.

Section 2.

As the line *CI* is to the line *KE*, so is the triangle *ACD* to the triangle *ADE* by the passage cited in the 1st section, for they are triangles with the same base *AD*, and by the construction: as the line *CI* is to the line *KE*, so is the line *OP* to *PQ*. Consequently, as the line *OP* is to *PQ*, so is the triangle *ACD* to the triangle *ADE*.

Section 3.

As the line *DL* is to the line *MF*, so is the triangle *ADE* to the triangle *AEF* by the passage cited in the 1st section, for they are triangles with the same base *AE*. Moreover, as the line *DL* is to the line *MF*, so is the line *PQ* to *QR* by the construction. Consequently, as the line *PQ* is to *QR*, so is the triangle *ADE* to the triangle *AEF*.

Conclusion.

Therefore, given the triangles of a rectilinear figure, lines have been found which are to one another in the same ratio and order as the triangles are. Which was to be performed by this second method.

NOTE.

If any of the perpendiculars should fall outside the rectilinear figure, such as the perpendicular *DH* to the line *AC*, then the line *AC* would have to be produced and *DH* would have to be drawn perpendicular to *AC* produced. And similarly, *DL* to *AE* produced, as is visible in this drawing, to which the preceding construction and proof can be applied.

PROBLEM II.

To draw from any angle of a triangle a line which divides the triangle in a given ratio such that required parts are towards given vertices.

Explicatio dati.

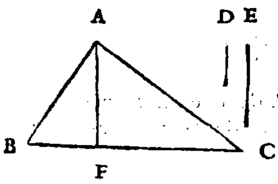
Sit datus triangulus A B C, *& data ratio recta* D *ad* E.

Explicatio quæsiti.

Oporteat ab angulo A *rectam lineam ducere, quæ diuidat triangulum* A B C *hoc modo vt pars versus* B *ad partem versus* C *eam habeat rationem quam* D *ad* E.

Constructio.

Diuidatur per 10. *prop. lib* 6. *Euclid. oppositum latus dati anguli* A *vt latus* B C *hoc modo, vt pars versus* B *ad partem versus* C *eam habeat rationem, quam recta* D *ad* E *sitque in* F, *& ducatur* A F.



Dico ab angulo A *rectam lineam* A F, *esse ductam quæ diuidit triangulum* A B C, *hoc modo vt triangulus* A F B *pars scilicet versus* B, *ad triangulum* A F C *partem versus* C, *eam habeat rationem quam* D *ad* E *vt erat quæsitum.*

Demonstratio.

Vt recta B F *ad rectam* F C, *sic triangulus* A F B *ad triangulum* A F C *per* 1. *prop. lib.* 6. *Euclid. Et vt* B F *ad* F C, *sic* D *ad* E *per constructionem, Ergo vt* D *ad* E *sic triangulus* A F B *ad triangulum* A F C.

Conclusio.

Igitur à quouis angulo trianguli &c. Quod erat faciendum.

PROBLEMA III.

A dato puncto in latere trianguli, rectam lineam ducere quæ diuidat triangulum versus partem petitam secundum rationem datam.

Explicatio dati.

Sit datus triangulus A B C, *& datum punctum* D *in recta* A B, *data vero ratio recta* E *ad* F.

Explicatio quæsiti.

Oporteat à puncto D *rectam lineam ducere, quæ diuidat triangulum* A B C *hoc modo vt pars versus* B *ad partē versus* A, *eam habeat rationē quā* E *ad* F.

Con-

Given.

Let a triangle *ABC* and the direct ratio *D* : *E* be given.

Required.

Let is be required to draw from the angle *A* a line which divides the triangle *ABC* in such a way that the part towards *B* is to the part towards *C* in the same ratio as *D* to *E*.

Construction.

By the 10th proposition of Euclid's 6th book divide the side opposite to the given angle *A*, *viz.* the side *BC*, in such a way that the part towards *B* is to the part towards *C* in the same ratio as the line *D* to *E*, and let this be in *F*, and draw *AF*.

I say that from the angle *A* a line *AF* has been drawn, which divides the triangle *ABC* in such a way that the triangle *AFB*, *i.e.* the part towards *B*, is to the triangle *AFC*, the part towards *C*, in the same ratio as *D* to *E*; as was required.

Proof.

As the line *BF* is to the line *FC*, so is the triangle *AFB* to the triangle *AFC*, by the 1st proposition of Euclid's 6th book. Moreover, as *BF* is to *FC*, so is *D* to *E* by the construction. Consequently, as *D* is to *E*, so is the triangle *AFB* to the triangle *AFC*.

Conclusion.

Therefore, from any angle of a triangle etc. Which was to be performed.

PROBLEM III.

From a given point on a side of a triangle to draw a line which divides the triangle in a given ratio such that required parts are towards given vertices.

Given.

Let a triangle *ABC* be given, and a point *D* on the line *AB*, and let the direct ratio of *E* : *F* be given.

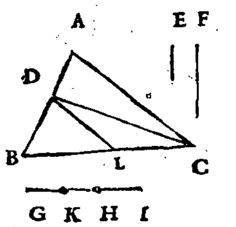
Required.

Let it be required to draw from the point *D* a line which divides the triangle *ABC* in such a way that the part towards *B* is to the part towards *A* in the same ratio as *E* to *F*.

Constructio.

Ducatur recta D C, *hoc est, à dato puncto* D *in angulum oppositum lateri* A B. *Inveniantur deinde per præcedens primum Problema duæ rectæ lineæ* G H, *&* H I, *inter se in ea ratione ac ordine, vt sunt trianguli* D B C *&* A D C: *Diuidatur deinde recta* G I *per* 10. *prop. lib.* 6. *Euclid. hoc modo vt pars versus* G *ad partem versus* I, *eam habeat rationem quam* E *ad* F, *sitque in* K. *Animaduertatur deinde in vtrum terminum rationis* G H *ad* H I *cadat punctum* K, *nempe an in* G H *an in* H I, *cadit autem in hoc exemplo in rectam seu terminum* G H, *cuius sumendus est Homologus terminus triangulorum nempe* D C B (*est enim triangulus* D C B *Homologus terminus cum* G H, *nam vt triangulus* D C B *ad triangulum* D C A, *sic recta* G H *ad rectam* H I *per constructionem*) *cuius latus oppositum angulo* B D C *vt latus* B C, *diuidatur per* 10. *prop. lib.* 6. *Euclid. in* L, *hoc modo vt* B L *ad* L C *eam habeat rationem, Quam* G K *ad* K H, *ducaturque recta* D L.

Dico à dato puncto D *rectam lineam* D L *esse ductam, quæ diuidit triangulum* A B C *hoc modo, vt pars versus* B *nempe triangulus* D L B, *ad partem versus* A *nempe trapezium* D L C A, *eam habeat rationem quam* E *ad* F *vt erat quæsitum.*



Demonstratio.

Vt recta B L *ad rectam* L C, *sic triangulus* D L B *ad triangulum* D L C *per* 1 *prop. lib.* 6. *Euclid. Et per constructionem, vt recta* B L *ad rectam* L C *sic recta* G K *ad rectam* K H: *Ergo vt triangulus* D L B *ad triangulum* D L C, *sic recta* G K *ad rectam* K H, *Quare triangulus* D L B *&* *recta* G K *sunt transformatæ proportionis homologi termini* (*nam vt supra dictum est, triangulus* D C B *est homologus terminus cum* G H) *Igitur per transformatam proportionem, vt recta* G K *ad reliquum suæ rationis* K I, *sic triangulus* D L B *ad reliquum suæ rationis*

Construction.

Draw the line *DC*, *i.e.* from the given point *D* to the angle opposite to the side *AB*. Subsequently, by the foregoing first Problem, find two lines *GH* and *HI* which are to one another in the same ratio and order as the triangles *DBC* and *ADC* are. Then by the 10th proposition of Euclid's 6th book divide the line *GI* in such a way that the part towards *G* is to the part towards *I* in the same ratio as *E* to *F*, and let this be in *K*. Then note in which term of the ratio *GH* to *HI* falls the point *K*, to wit: whether in *GH* or in *HI*. Now in this example it falls in the line or term *GH*. Now take the homologous term of this in the triangles, *viz.* *DCB* (in fact, triangle *DCB* is the term homologous to *GH*, for as triangle *DCB* is to triangle *DCA*, so is the line *GH* to the line *HI* by the construction), and by the 10th proposition of Euclid's 6th book divide its side opposite to the angle *BDC*, *viz.* the side *BC*, in *L* in such a way that *BL* is to *LC* in the same ratio as *GK* to *KH*, and draw the line *DL*.

I say that from the given point *D* a line *DL* has been drawn, which divides the triangle *ABC* in such a way that the part towards *B*, *viz.* the triangle *DLB*, is to the part towards *A*, *viz.* the quadrangle *DLCA*, in the same ratio as *E* to *F*; as was required.

Proof.

As the line *BL* is to the line *LC*, so is the triangle *DLB* to the triangle *DLC*, by the 1st proposition of Euclid's 6th book. Moreover, by the construction, as the line *BL* is to the line *LC*, so is the line *GK* to the line *KH*. Consequently, as the triangle *DLB* is to the triangle *DLC*, so is the line *GK* to the line *KH*; on which account the triangle *DLB* and the line *GK* are homologous terms of a transformed proportion (for as has been said above, the triangle *DCB* is the term homologous to *GH*). Therefore, by the transformed proportion, as the line *GK* is to the rest of its ratio, *KI*, so is the triangle *DLB* to the rest of its ratio, *viz.* to the quadrangle *DLCA*. But *GK* is to *KI* in the same ratio, by the con-

nis, nempe ad trapezium D L C A. *Sed* G K *ad* K I *habet eam rationem per constructionem quam* E *ad* F. *Ergo triangulus* D L B, *ad trapezium* D L C A, *eam habet rationem quam recta* E *ad* F.

Conclusio.

Igitur à dato puncto in latere &c. Quod erat faciendum.

NOTA.

Alius est modus constructionis huius Problematis apud varios authores, cuius inter alios meminit Christophorus Clauius in fine lib. 6. Euclid. Sed vt sequentia Problemata essent apertiora, secuti sumus hic, nostram generalem inventionem omnium rectilineorum.

Sequentia quinque Problemata sunt ea quæ ante hac nunquam descripta putamus.

PROBLEMA IIII.

A quovis angulo quadranguli, rectam lineam ducere, quæ diuidat quadrangulum versus partem petitam secundum rationem datam.

Explicatio dati.

Sit datum quadrangulum quodcunque A B C D, *data verò ratio* E *ad* F.

Explicatio quæsiti.

Oporteat ab angulo D A B *rectam lineam ducere, quæ diuidat quadrangulum* A B C D *hoc modo vt pars versus* D, *ad partem versus* B, *eam habeat rationem quam* E *ad* F.

Constructio.

Ducatur ab angulo D A B *in angulum oppositum* B C D *recta* A C:

struction, as E to F. Consequently, the triangle DLB is to the quadrangle $DLCA$ in the same ratio as the line E to F.

Conclusion.

Therefore, from a given point on a side etc. Which was to be performed.

NOTE.

Among several authors there is another method for the construction of this Problem, mention of which is made, among others, by Christophorus Clavius at the end of Euclid's 6th book. But in order that the following Problems might be clearer, we have here followed our general invention concerning all rectilinear figures.

The following five Problems are such as we deem have never been described before.

PROBLEM IV.

To draw from any angle of a quadrangle a line which divides the quadrangle in a given ratio such that required parts are towards given vertices.

Given.

Let any quadrangle $ABCD$ be given, and let the ratio of E to F be given.

Required.

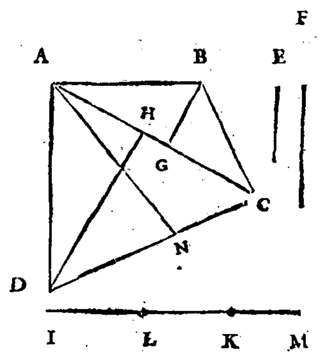
Let it be required to draw from the angle DAB a line which divides the quadrangle $ABCD$ in such a way that the part towards D is to the part towards B in the same ratio as E to F.

Construction.

From the angle DAB draw the line AC to the opposite angle BCD; sub-

Deinde recta B G *perpendicularis ad rectam* A C, *& recta* D H *perpendicularis ad rectam* A C, *& recta* I K *æqualis rectæ* D H, *& producatur* I K *in* L, *ita vt* K L *æqualis sit ipsi* B G, *diuidatur deinde recta* I L *in* M *per* 10 *prop. lib.* 6. *Euclid. Hoc modo vt* I M *ad* M L *eam habeat rationem quam* E *ad* F. *Animaduertatur deinde in quem terminum rationis* I K *ad* K L *cadat punctum* M, *scilicet an in* I K *vel in* K L: *cadit autem in hoc exemplo in rectan seu terminum* I K. *Quare eius capiendus est homologus terminus triangulorum nempe* A C D (*est enim triangulus* A C D *homologus terminus cum* I K, *nam vt triangulus* A C D *ad triangulum* A C B, *sic recta* H D *ad rectam* G B *per locum citatum in* I *distinctione secundi modi præcedentis primi Problematis, & recta* I K *æqualis est rectæ* D H, *& recta* K L *æqualis rectæ* G B. *Ergo vt triangulus* A C D *ad triangulum* A C B *sic recta* I K *ad rectam* K L) *cuius latus oppositum angulo* D A C *vt latus* D C, *diuidatur per* 10. *prop. lib.* 6. *Euclid. in* N, *hoc modo vt recta* D N *ad* N C, *eam habeat rationem quam recta* I M *ad* M K, *ducaturque recta* A N.

Dico ab angulo D A B, *rectam lineam* A N *esse ductam, quæ diuidit quadrangulum* A B C D *hoc modo, vt pars versus* D *nempe triangulus* A N B, *ad partem versus* B *nempe trapezium* A N C B, *eam habeat rationem quam* E *ad* F, *vt erat quæsitum.*



Demonstratio.

Vt recta D N *ad rectam* N C, *sic triangulus* A N D *ad triangulũ* A N C *per* 1. *prop. lib.* 6. *Euclid. Et per constructionem vt recta* D N *ad rectam* N C, *sic recta* I M *ad rectam* M K: *Ergo vt triangulus* A N D *ad triangulum* A N C, *sic recta* I M *ad rectam* M K, *quare triangulus* A N D *& recta* I M *sunt transformatæ proportionis homologi termini* (*nam vt supra* sequently the line *BG* perpendicular to the line *AC*, and the line *DH* perpendicular to the line *AC*, and the line *IK* equal to the line *DH*, and produce *IK* in *L* in such a way that *KL* be equal to the said *BG*, and then, by the 10th proposition of Euclid's 6th book, divide the line *IL* in *M* in such a way that *IM* is to *ML* in the same ratio as *E* to *F*. Then note in which term of the ratio *IK* to *KL* the point *M* falls, *viz.* in *IK* or in *KL*. Now in this example it falls in the line or term *IK*. Therefore we have to take its homologous term of the triangles, *viz.* *ACD* (indeed, the triangle *ACD* is the term homologous to *IK*, for as the triangle *ACD* is to the triangle *ACB*, so is the line *HD* to the line *GB*, by the passage cited in the 1st section of the second method of the foregoing first Problem; and the line *IK* is equal to the line *DH*, and the line *KL* is equal to the line *GB*; consequently, as the triangle *ACD* is to the triangle *ACB*, so is the line *IK* to the line *KL*), and by the 10th proposition of Euclid's 6th book divide its side opposite to the angle *DAC*, *viz.* the side *DC*, in *N* in such a way that the line *DN* is to *NC* in the same ratio as the line *IM* to *MK*, and draw the line *AN*.

I say that from the angle *DAB* a line *AN* has been drawn, which divides the quadrangle *ABCD* in such a way that the part towards *D*, *viz.* the triangle *ANB*, is to the part towards *B*, *viz.* the quadrangle *ANCB*, in the same ratio as *E* to *F*; as was required.

Proof.

As the line *DN* is to the line *NC*, so is the triangle *AND* to the triangle *ANC*, by the 1st proposition of Euclid's 6th book. And by the construction: as the line *DN* is to the line *NC*, so is the line *IM* to the line *MK*. Consequently, as the triangle *AND* is to the triangle *ANC*, so is the line *IM* to the line *MK*; therefore the triangle *AND* and the line *IM* are homologous terms of a transformed *pra dictum est triangulus* A C D *est homologus terminus cum recta* I K) *Quare per transformatam proportionem, vt recta* I M *ad reliquum suæ rationis* M L, *sic triangulus* A N D *ad reliquum suæ rationis nempe ad trapezium* A N C B. *Et* I M *ad* M L *habet eam rationem per constructionem quam recta* E *ad* F: *Ergo triangulus* A N D *ad trapezium* A N C B, *eam habet rationem quam recta* E *ad* F.

Conclusio.

Igitur à quouis angulo quadranguli &c. Quod erat faciendum.

PROBLEMA V.

A dato puncto in latere cuiuscunque rectilinei, rectam lineam ducere quæ diuidat rectilineum versus partem petitam secundum rationem datam.

Explicatio dati.

Sit datum rectilineum pentagonum A B C D E, *datumq́ punctum* F *in latere* A B: *Data verò ratio recta* G *ad* H.

Explicatio quæsiti.

Oporteat à puncto F *rectam lineam ducere, quæ diuidat rectilineum datum, hoc modò vt pars versus* B *ad partem versus* A, *eam habeat rationem quam* G *ad* H.

Constructio.

Ducantur tres rectæ F E, F D, F C. *Inveniantur deinde quatuor rectæ lineæ per præcedens primum Problema* I K, K L, L M, M N, *inter se in ea ratione ac ordine, vt sunt quatuor trianguli* F B C, F C D, F D E, F E A. *Diuidatur deinde recta* I N *per* 10. *prop. lib.* 6. *Euclid. in* O, *hoc modo vt* I O *ad* O N *eam habeat rationem quam* G *ad* H, *animaduertatur deinde* proportion (for, as has been said above, the triangle *ACD* is the term homologous to the line *IK*). Therefore, by the transformed proportion, as the line *IM* is to the rest of its ratio *ML*, so is the triangle *AND* to the rest of its ratio, *viz.* to the quadrangle *ANCB*. And *IM* is to *ML* in the same ratio, by the construction, as the line *E* to *F*. Consequently, the triangle *AND* is to the quadrangle *ANCB* in the same ratio as the line *E* to *F*.

Conclusion.

Therefore, from any angle of a quadrangle etc. Which was to be performed.

PROBLEM V.

From a given point on the side of any rectilinear figure to draw a line which divides the rectilinear figure in a given ratio such that required parts are towards given vertices.

Given.

Let a pentagonal rectilinear figure *ABCDE* be given, and also a point *F* on the side *AB*; and let the direct ratio of *G* to *H* be given.

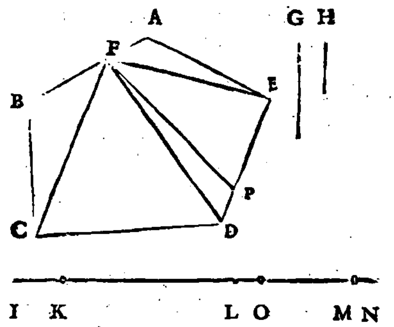
Required.

Let it be required to draw from the point *F* a line which divides the given rectilinear figure in such a way that the part towards *B* is to the part towards *A* in the same ratio as *G* to *H*.

Construction.

Draw the three lines *FE, FD, FC*. Subsequently, by the foregoing first Problem, find four lines *IK, KL, LM, MN*, which are to one another in the same ratio and order as the four triangles *FBC, FCD, FDE, FEA*. Then, by the 10th proposition of Euclid's 6th book, divide the line *IN* at *O* in such a way that *IO* is to *ON* *in quem terminum quaternariæ rationis* I K, K L, L M, M N *cadat punctum* O, *cadit autem in hoc exemplo in tertium terminum* L M *cuius sumendus est homologus terminus triangulorum nempe* F D E (*est autem triangulus* F D E *homologus terminus cum* L M *per præcedentem 16. defini.*) *cuius latus oppositum dato puncto* F, *vt latus* E D, *diuidatur per* 10. *prop. lib.* 6. *Euclid. in* P, *hoc modo, vt* D P *ad* P E *eam habeat rationem quam* L O *ad* O M *ducaturque recta* F P.

Dico *à dato puncto* F, *rectam lineam* F P *esse ductam, quæ diuidit rectilineum datum hoc modo, vt pars versus* B *nempe pentagonum* F P D C B, *ad partem versus* A *nempe trapezium* F P E A, *eam habeat rationem quam* G *ad* H *vt erat quæsitum.*



Demonstratio.

Vt recta D P *ad rectam* P E, *sic triangulus* F P D *ad triangulum* F P E *per* 1. *prop. lib.* 6. *Euclid.* & *per constructionem, vt recta* D P *ad rectam* P E, *sic recta* L O *ad rectam* O M: *Ergo vt triangulus* F P D *ad triangulum* F P E, *sic recta* L O *ad rectam* O M: *Quare triangulus* F P E & *recta* O M *sunt transformatæ proportionis homologi termini: Igitur per transformatam proportionem vt recta* I O, *ad reliquum suæ rationis* O N, *sic pentagonum* F P D C B *ad reliquum suæ rationis nempe ad quadrangulum* F P E A. *Sed* N O *ad* O I, *eam habet rationem per constructionem quam* H *ad* G: *Ergo pentagonum* F P D C B *ad quadrangulum* F P E A, *eam habet rationem quam recta* G *ad* H.

Conclusio.

Igitur à dato puncto in latere &c. Quod erat faciendum.

NOTA.

in the same ratio as *G* to *H*. Then note in which term of the quaternary ratio *IK* : *KL* : *LM* : *MN* the point *O* falls. Now in this example it falls in the third term *LM*. Take the homologous term to this of the triangles , *viz. FDE* (indeed, the triangle *FDE* is the term homologous to *LM* by the preceding 16th definition), and by the 10th proposition of Euclid's 6th book divided the side opposite to the given point *F*, *viz.* the side *ED*, at *P* in such a way that *DP* is to *PE* in the same ratio as *LO* to *OM*, and draw the line *FP*.

I say that from the given point *F* a line *FP* has been drawn, which divides the given rectilinear figure in such a way that the part towards *B*, *viz.* the pentagon *FPDCB*, is to the part towards *A*, *viz.* the quadrangle *FPEA*, in the same ratio as *G* to *H*; as was required.

Proof.

As the line *DP* is to the line *PE*, so is the triangle *FPD* to the triangle *FPE*, by the 1st proposition of Euclid's 6th book. And by the construction: as the line *DP* is to the line *PE*, so is the line *LO* to the line *OM*. Consequently, as the triangle *FPD* is to the triangle *FPE*, so is the line *LO* to the line *OM*. Therefore the triangle *FPE* and the line *OM* are homologous terms of a transformed proportion. Therefore, by the transformed proportion, as the line *IO* is to the rest of its ratio *ON*, so is the pentagon *FPDCB* to the rest of its ratio, *viz.* to the quadrangle *FPEA*. But *NO* is to *OI* in the same ratio, by the construction, as *H* to *G*. Consequently, the pentagon *FPDCB* is to the quadrangle *FPEA* in the same ratio as the line *G* to *H*.

Conclusion.

Therefore, from a given point on the side etc. Which was to be performed.

NOTA.

Si punctum O *cecidisset in* M, *tunc recta* F E *sine alia inquisitione fuisset recta quæsita. Si verò punctum* O *cecidisset in* L, *tunc recta* F D *absque alia inquisitione fuisset recta quæsita, & sic de ceteris.*

Hucusque dictum est de diuisione rectilineorum à dato puncto in latere ipsorum: Sequitur nunc vt in principio promisimus, vt dicatur de rectilineorum diuisione versus partem petitam sub ratione data, & cum linea parallela cum latere quæsito.

PROBLEMA VI.

In dato triangulo rectam lineam ducere parallelam cum latere trianguli quæsito, quæ triangulum diuidat versus partem quæsitam secundum rationem datam.

Explicatio dati.

Sit datus triangulus A B C, *& data ratio recta* D E *ad* E F.

Explicatio quæsiti.

Oporteat rectam lineam ducere parallelam cum latere A C, *quæ triangulum* A B C *diuidat, hoc modo vt pars trianguli versus* B, *ad reliquam partem eam habeat rationem quam* D E *ad* E F.

Constructio.

Diuidatur per 10. *prop. lib.* 6. *Euclid. alterutrum latus* B C *vel* B A, *sitque* B C *in* G, *hoc modo vt* B G, *ad* G C *eam habeat rationem quam recta* D E *ad* E F: *Inueniaturque per* 13. *prop. lib.* 6. *Euclid. media linea proportionalis inter* B G *&* B C, *sitque recta* B H: *ducaturque recta* H I *parallela ipsi* A C.

NOTE.

If the point *O* had fallen in *M*, then the line *FE* would have been the required line without any other investigation. But if the point *O* had fallen in *L*, then the line *FD* would have been the required line without any other investigation; and thus with the others.

Up to this point we have spoken about the division of rectilinear figures from a given point on a side of the latter. As we promised at the beginning, we shall now speak about the division of rectilinear figures towards a required part in a given ratio, *viz.* by a line parallel to a required side.

PROBLEM VI.

In a given triangle to draw a line, parallel to a required side of the triangle, which divides the triangle in a given ratio such that required parts are towards given vertices.

Given.

Let the triangle *ABC* and the direct ratio of *DE* to *EF* be given.

Required.

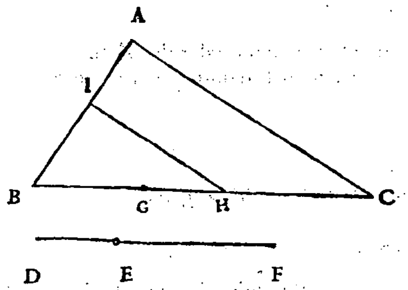
Let it be required to draw a line, parallel to the side *AC*, which divides the triangle *ABC* in such a way that the part of the triangle towards *B* is to the remaining part in the same ratio as *DE* to *EF*.

Construction.

By the 10th proposition of Euclid's 6th book divide one of the sides *BC* or *BA* — and let it be *BC* in *G* — in such a way that *BG* is to *GC* in the same ratio as the line *DE* to *EF*. And by the 13th proposition of Euclid's 6th book find the mean proportional between *BG* and *BC*, and let this be the line *BH*. And draw the line *HI* parallel to the side *AC*.

Dico rectam lineam H I *esse ductam parallelam cum latere* A C, *diuidentem triangulum* A B C *hoc modo, vt pars trianguli versus* B *nempe triangulus* I B H, *ad reliquam partem nempe trapezium* A C H I *eam habeat rationem quam* D E *ad* E F, *vt erat quæsitum.*

Demonstratio.



Recta B C *trianguli* A B C *est latus homologum cum* B H *trianguli* I B H, *& recta* B G *ad rectam* B C *duplicatam eam habet rationem quam ipsa* B G *ad rectam* B H *per constructionem: Ergo per 20. prop. lib. 6. Euclid. vt recta* B G *ad rectam* B C, *sic triangulus* I B H *ad triangulum* A B C (*sunt enim similia poligona*) *Et* B G *ad* B C *per constructionem eam habet rationem quam* D E *ad* D F: *Ergo vt* D E *ad* D F, *sic triangulus* I B H *ad triangulum* A B C, *& per disiunctam proportionem vt recta* D E *ad rectam* E F, *sic triangulus* I B H *ad trapezium* A C H I.

Conclusio.

Igitur in dato triangulo &c. Quod erat faciendum.

PROBLEMA VII.

In dato trapezio rectam lineam ducere parallelam cum latere trapezij quæsito quæ trapezium diuidat versus partem quæsitam secundum rationem datam.

NOTA.

I say that a line *HI* has been drawn parallel to the side *AC*, dividing the triangle *ABC* in such a way that the part of the triangle towards *B*, *viz.* the triangle *IBH*, is to the remaining part, *viz.* the quadrangle *ACHI*, in the same ratio as *DE* to *EF*; as was required.

Proof.

The line *BC* of the triangle *ABC* is the side homologous to *BH* of the triangle *IBH*, and the line *BG* is to the line *BC* in the duplicate ratio of that of the line *BG* to the line *BH*, by the construction. Consequently, by the 20th proposition of Euclid's 6th book, as the line *BG* is to the line *BC*, so is the triangle *IBH* to the triangle *ABC* (for they are similar polygons). And by the construction *BG* is to *BC* in the same ratio as *DE* to *DF*. Consequently, as *DE* is to *DF*, so is the triangle *IBH* to the triangle *ABC*, and by the disjunct ratio: as the line *DE* is to the line *EF*, so is the triangle *IBH* to the quadrangle *ACHI*.

Conclusion.

Therefore, in a given triangle etc. Which was to be performed.

PROBLEM VII.

In a given quadrangle to draw a line, parallel to a required side of the quadrangle, which divides the quadrangle in a given ratio such that required parts are towards given vertices.

NOTA.

Omnis linea recta parall. cum latere diuidens trapezium, tangit suis limitibus duo latera eundem angulum trapezij continentia, aut à latere in angulum lateri oppositum cadit aut duo latera tangit ipsius opposita. Igitur quoniam operatio in ipsis est diuersa dabuntur duo exempla.

Explicatio dati primi modi.

Vbi linea diuidens trapezium cadit in duo latera eundem angulum continentia.

Sit datum trapezium A B C D, *& data ratio* E F *ad* F G.

Explicatio quæsiti.

Oporteat rectam lineam ducere parallelam cum B C *quæ trapezium diuidat hoc modo, vt pars trapezij versus* A, *ad reliquam partem, eam habeat rationem quam* E F *ad* F G.

Constructio.

Describatur per 45. prop. lib. 6. Euclid. parallelogrammum H I K L *æquale trapezio* A B C D, *secetur& latus* H I *in* M *per* 10. *prop. lib.* 6. *Euclid. hoc modo, vt* H M *ad* M I *eam habeat rationem quam* E F *ad* F G, *ducaturque* M N *parallela ipsi* H L, *ducaturque* D O *parallela rectæ* B C: *describaturque per 25. prop. lib. 6. Euclid. triangulus* A P Q *æqualis parallelogrammo* H N, *& similis triangulo* A O D.

Dico rectam lineam P Q *parallelam cum* B C *esse ductam, quæ trapezium* A B C D *diuidit hoc modo, vt pars trapezij versus* A *nempe triangulus* A P Q *ad reliquam partem nempe rectilineum* P B C D Q, *eam habeat rationem quam* E F *ad* F G *vt erat quæsitum.*

Demon-

NOTE.

Any line, parallel to a side, which divides a quadrangle, touches with its extremities two sides including the same angle of the quadrangle, or it falls from one side to the angle opposite to the side, or it connects two opposite sides of the quadrangle. Therefore, since the operation in the said cases is different, two examples will be given.

Given according to the first manner.

Where the line dividing the quadrangle connects two sides containing the same angle.

Let the quadrangle *ABCD* and the ratio of *EF* to *FG* be given.

Required.

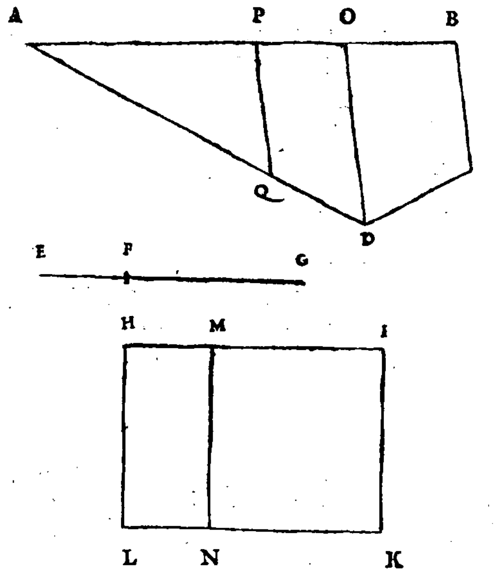
Let it be required to draw a line, parallel to *BC*, which divides the quadrangle in such a way that the part of the quadrangle towards *A* is to the remaining part in the same ratio as *EF* to *FG*.

Construction.

By the 45th proposition of Euclid's 1st book construct the parallelogram *HIKL*, equal to the quadrangle *ABCD*, and by the 10th proposition of Euclid's 6th book cut the side *HI* at *M*, in such a way that *HM* is to *MI* in the same ratio as *EF* to *FG*. And draw *MN* parallel to the side *HL*, and draw *DO* parallel to the line *BC*. And by the 25th proposition of Euclid's 6th book construct the triangle *APQ*, equal to the parallelogram *HN* and similar to the triangle *AOD*.

I say that a line *PQ* has been drawn, parallel to *BC*, which divides the quadrangle *ABCD* in such a way that the part of the quadrangle towards *A*, *viz.* the triangle *APQ*, is to the remaining part, *viz.* the rectilinear figure *PBCDQ*, in the same ratio as *EF* to *FG*; as was required.

Demonstratio.



Vt E F *ad* F G *sic* H M *ad* M I *per constructionem*, *& vt* H M *ad* M I *sic parallelogrammum* H N *ad parallelogrammũ* M K *per* I. *prop. lib.* 6. *Euclid. Ergo vt* E F *ad* F G *sic parallelogrammum* H N, *ad parallelogrammũ* M K, *& triangulus* A P Q *æqualis est parallelogrammo* H N *per constructionem*: *Quare* (*quia totum trapezium* A B C D *æquale est toti parallelogrammo* H K *per constructionem*) *rectilineum* P B C D Q *æquale est parallelogrammo* M K: *Ergo vt* E F *ad* F G, *sic triangulus* A P Q *ad rectilineum* P B C D Q. *Est præterea* P Q *parallela ipsi* O D *per constructionem*, *&* O D *parallela cum* B C *per constructionem*: *Quare per* 30. *prop. lib.* I. *Euclid.* P Q *est parallela ipsi* B C.

Conclusio.

Igitur in dato trapezio recta linea ducta est &c. Quod erat faciendum.

NOTA.

Si in hoc exemplo recta P Q *caderet à latere in angulum lateri oppositum, tunc*

Proof.

As *EF* is to *FG*, so is *HM* to *MI*, by the construction, and as *HM* is to *MI*, so is the parallelogram *HN* to the parallelogram *MK*, by the 1st proposition of Euclid's 6th book. Consequently, as *EF* is to *FG*, so is the parallelogram *HN* to the parallelogram *MK*. And the triangle *APQ* is equal to the parallelogram *HN* by the construction. Therefore (because the whole quadrangle *ABCD* is equal to the whole parallelogram *HK*, by the construction) the rectilinear figure *PBCDQ* is equal to the parallelogram *MK*. Consequently, as *EF* is to *FG*, so is the triangle *APQ* to the rectilinear figure *PBCDQ*. Moreover *PQ* is parallel to *OD* by the construction and *OD* parallel to *BC* by the construction. Therefore, by the 30th proposition of Euclid's 1st book, *PQ* is parallel to *BC*.

Conclusion.

Therefore, in a given quadrangle a line has been drawn etc. Which was to be performed.

NOTE.

If in this example the line *PQ* fell from one side to the angle opposite to this *tunc recta* O D *eſſet recta quæſita, quare operatio eadem eſſet vt ſupra.*

Explicatio dati ſecundi modi.

Vbi linea diuidens trapezium cadit in duo latera trapezij oppoſita.

Sit datum trapezium A B C D *& data ratio recta* E F *ad* F G.

Explicatio quæſiti.

Oporteat rectam lineam ducere parallelam cum A B, *quæ trapezium* A B C D *diuidat, hoc modo vt pars trapezij verſus* D C, *ad reliquam partem, eam habeat rationem quam* E F *ad* F G.

Conſtructio.

Producantur ea trapezij latera quæ concurrere poſſunt, quod per 5. poſtulatum lib. 1. *Euclid. in omni trapezio eſt poſſibile, ſintq́ latera* A D *&* B C, *quæ producta concurrant in* H: *deſcribatur deinde per 45. prop. lib.* 1. *Euclid. parallelogrammum* I K L M *æquale triangulo* H C D, *& parallelogrammum* K N O L *æquale trapezio* A B C D, *diuidaturq́ue* K N *in* P *per* 10. *prop. lib.* 6. *Euclid. hoc modo, vt* K P *ad* P N *eam habeat rationem quam* E F *ad* F G, *ducaturq́ue* P Q: *Diuidatur deinde triangulus* A H B *per præcedens* 6. *Problem. recta* R S *parallela ipſi* A B (*quæ in hoc exemplo cadit in duo latera trapezij oppoſita*) *hoc modo vt triangulus* R H S, *ad trapezium* A B S R *eam habeat rationem quam* I P *ad* P N.

Dico rectam lineam R S *parallelam cum* A B *eſſe ductam, quæ trapezium* A B C D *diuidit, hoc modo, vt pars trapezij verſus* D C *nempe trapezium* D R S C, *ad reliquam partem nempe trapezium* R A B S, *eam habeat rationem quam* E F *ad* F G *vt erat quæſitum.*

side, then the line *OD* would be the required line; therefore the operation would be the same as above.

Given according to the second manner.

Where the line dividing the quadrangle connects two opposite sides of the quadrangle.

Let the quadrangle *ABCD* and the direct ratio of *EF* to *FG* be given.

Required.

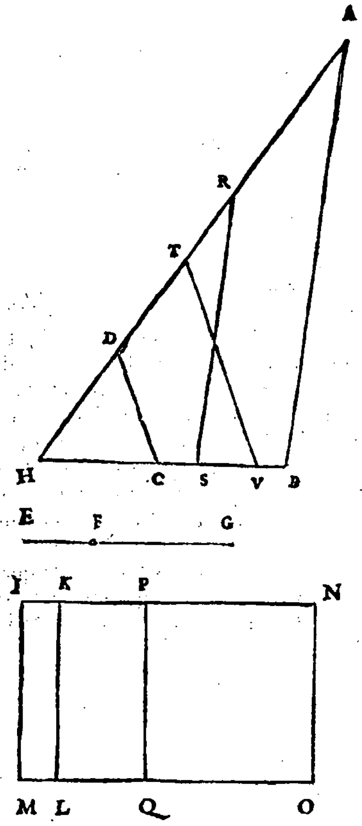
Let it be required to draw a line, parallel to *AB*, which divides the quadrangle *ABCD* in such a way that the part of the quadrangle towards *DC* is to the remaining part in the same ratio as *EF* to *FG*.

Construction.

Produce those sides of the quadrangle which can meet, which by the 5th postulate of Euclid's 1st book is possible in every quadrangle, and let it be the sides *AD* and *BC* which, when produced, meet in *H*. Subsequently, by the 45th proposition of Euclid's 1st book, construct a parallelogram *IKLM*, equal to the triangle *HCD*, and a parallelogram *KNOL*, equal to the quadrangle *ABCD*. And, by the 10th proposition of Euclid's 6th book, divide *KN* at *P*, in such a way that *KP* is to *PN* in the same ratio as *EF* to *FG*, and draw *PQ*. Subsequently, by the preceding 6th Problem, divide the triangle *AHB* by the line *RS* parallel to *AB* (which in this example connects two opposite sides of the quadrangle) in such a way that the triangle *RHS* is to the quadrangle *ABSR* in the same ratio as *IP* to *PN*.

I say that a line *RS* has been drawn, parallel to *AB*, which divides the quadrangle *ABCD* in such a way that the part of the quadrangle towards *DC*, *viz.* the quadrangle *DRSC*, is to the remaining part, *viz.* the quadrangle *RABS*, in the same ratio as *EF* to *FG*; as was required.

Demonstratio.



Triangulus R H S *æqualis est parallelogrammo* I Q *& triangulus* D H C *æqualis est parallelogrammo* I L *per constructionem: Quare per* 3. *Axioma lib.* 1. *Euc. trapezium* D R S C *æquale est parallelogrammo* K Q: *Et per consequens parallelogrammũ* P O *æquale est trapezio* R A B S (*nam parallelogrammũ* K O *æquale est per constructionem trapezio* D A B C) *parallelogrammum autem* K Q *ad parallelogrammum* P O *eam habeat rationem per constructionem quam* E F *ad* F G: *Ergo trapezium* D R S C *ad trapezium* R A B S *eam habeat rationem quam* E F *ad* F G. *Est præterea* R S *parallela ipsi* A B *per constructionem.*

Conclusio.

Igitur in dato trapezio recta linea ducta est &c. Quod erat faciendum.

NOTA.

Proof.

The triangle *RHS* is equal to the parallelogram *IQ*, and the triangle *DHC* is equal to the parallelogram *IL* by the construction. Therefore, by the 3rd Axiom of Euclid's 1st book, the quadrangle *DRSC* is equal to the parallelogram *KQ*. And in consequence the parallelogram *PO* is equal to the quadrangle *RABS* (for the parallelogram *KO* is equal, by the construction, to the quadrangle *DABC*). Now the parallelogram *KQ* is to the parallelogram *PO* in the same ratio, by the construction, as *EF* to *FG*. Accordingly, the quadrangle *DRSC* is to the quadrangle *RABS* in the same ratio as *EF* to *FG*. Moreover *RS* is parallel to *AB* by the construction.

Conclusion.

Therefore, in a given quadrangle a line has been drawn etc. Which was to be performed.

NOTA.

Si quæsitum fuisset, rectam lineam secantem trapezium ducere parallelam cum D C, *describendus fuisset per* 25. *prop. lib.* 6. *Euclid. triangulus* H T V *æqualis parallelogrammo* I Q, *& similis triangulo* H D C, *demonstrareturq́ ut supra trapezium* D T V C, *ad trapezium* T A B V *eam habere rationem quam* E F *ad* F G. *Si verò illa linea fuisset ducenda parallela cum* D A *vel cum* C B, *tunc essent latera* A B *&* D C *producenda versus partes* C B, *& appositus triangulus qualis est* H D C *esset ad partem trapezij versus* C B, *quare operatio ex ea parte eadem esset ut supra.*

PROBLEMA VIII.

In dato quocunque rectilineo rectam lineam ducere parallelam cùm latere rectilinei quæsito, quæ rectilineum diuidat versus partem quæsitam secundum rationem datam.

Explicatio dati.

Sit datum rectilineum A B C D E F G, *& data ratio recta* H I *ad* I K.

Explicatio quæsiti.

Oporteat rectam lineam ducere parallelam cum A B, *quæ rectilineum* A B C D E F G *diuidat, hoc modo ut pars rectilinei versus* E, *ad reliquam partem eam habeat rationem quam* H I *ad* I K.

Constructio.

Producantur F E *&* C D *concurrentes ad* L; *describaturque per* 45. *prop. lib.* 1. *Euclid. parallelogrammum* M N O P *æquale triangulo* L E D, *& similiter parallelogrammum* N Q R O *æquale rectilineo* A B C D E F G: *diuidaturque* N Q *in* S *per* 10. *prop. lib.* 6. *Euclid. hoc modo ut* N S *ad* S Q *eam habeat rationem quam* H I *ad* I K: *ducaturq́* S T *parallela cum* M P *ducaturque recta* F V *parallela cum* A B, *describatur deinde per* 25. *prop.*

NOTE.

If it had been required to draw a line intersecting a quadrangle, parallel to *DC*, by the 25th proposition of Euclid's 6th book a triangle *HTV*, equal to the parallelogram *IQ* and similar to the triangle *HDC*, would have had to be constructed and it will then be proved as above that the quadrangle *DTVC* is to the quadrangle *TABV* in the same ratio as *EF* to *FG*. But if the said line had had to be drawn parallel to *DA* or to *CB*, then the sides *AB* and *DC* would have to be produced towards the parts *C* and *B*, and a triangle such as *HDC* would have to be added to the part of the quadrangle towards *CB*; therefore the operation from this part would be the same as above.

PROBLEM VIII.

In any given rectilinear figure to draw a line, parallel to a required side of the figure, which divides the figure in a given ratio such that required parts are towards given vertices.

Given.

Let the rectilinear figure *ABCDEFG* and the direct ratio of *HI* to *IK* be given.

Required.

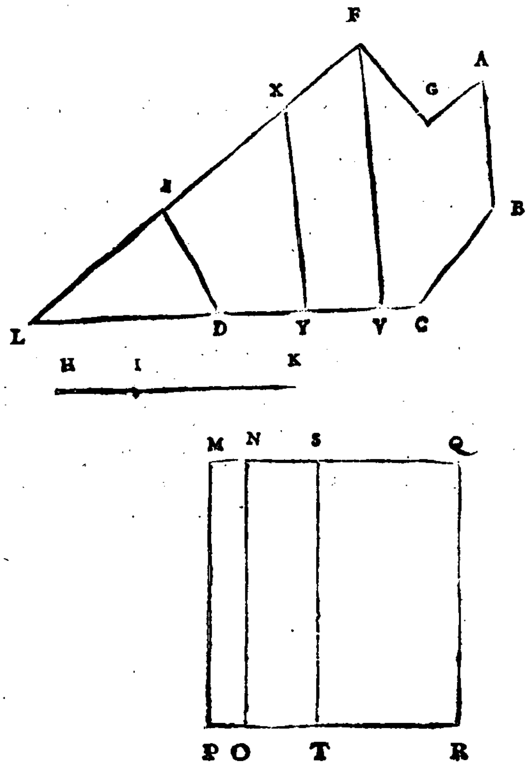
Let it be required to draw a line, parallel to *AB*, which divides the rectilinear figure *ABCDEFG* in such a way that the part of the figure towards *E* is to the remaining part in the same ratio as *HI* to *IK*.

Construction.

Produce *FE* and *CD* until they meet at *L*; and by the 45th proposition of Euclid's 1st book construct the parallelogram *MNOP*, equal to the triangle *LED*, and similarly the parallelogram *NQRO*, equal to the rectilinear figure *ABCDEFG*. And divide N*Q* at *S*, by the 10th proposition of Euclid's 6th book, in such a way that *NS* is to *SQ* in the same ratio as *HI* to *IK*. And draw *ST* parallel to *MP*, and draw the line *FV* parallel to *AB*. Subsequently, by the 25th proposition *lib. 6. Euclid. triangulus* L X Y *æqualis parallelogrammo* M T *& similis triangulo* L F V.

Dico rectam lineam X Y *esse ductam parallelam cum* A B, *diuidentem rectilineum datum* A B C D E F G, *hoc modo, vt pars rectilinei versus* E, *nempe trapezium* X Y D E, *ad reliquam partem nempe rectilineum* X F G A B C Y, *eam habeat rationem quam* H I *ad* I K *vt erat quæsitum.*

Demonstratio.



Distinct. 1.

Triangulus L X Y *æqualis est parallelogrammo* M T, *& triangulus* L E D *æqualis est parallelogrãmo* M O *per constructionem, quare per 3. axioma lib. 1. Euclid. trapezium* X Y D E *æquale est parallelogrãmo* N T.

Distinct. 2.

Rectilineum A B C D E F G *æquale est parallelogrã-mo* of Euclid's 6th book, construct the triangle *LXY*, equal to the parallelogram *MT* and similar to the triangle *LFV*.

I say that a line *XY* has been drawn, parallel to *AB*, dividing the given rectilinear figure *ABCDEFG* in such a way that the part of the rectilinear figure towards *E*, *viz.* the quadrangle *XYDE*, is to the remaining part, *viz.* the rectilinear figure *XFGABCY*, in the same ratio as *HI* to *IK*; as was required.

Proof.

Section 1.

The triangle *LXY* is equal to the parallelogram *MT*, and the triangle *LED* is equal to the parallelogram *MO* by the construction; therefore, by the 3rd axiom of Euclid's 1st book, the quadrangle *XYDE* is equal to the parallelogram *NT*.

Section 2.

The rectilinear figure *ABCDEFG* is equal to the parallelogram *NR* by the *mo* N R *per constructionem, & trapezium* X Y D E *æquale est parallelogrammo* N T *per primam distinctionem, quare per tertium axioma lib.* I. *Euclid. rectilineum* X F G A B C Y *æquale est parallelogrammo* S R.

Distinctio 3.

Vt H I *ad* I K, *sic* N S *ad* S Q *per constructionem, & per* 1. *prop. lib.* 6. *Euclid. vt recta* N S *ad* S Q, *sic parallelogrammum* N T *ad parallelogrammum* S R: *Ergo vt* H I *ad* I K *sic parallelogrammum* N T *ad parallelogrammum* S R. *Parallelogrammum autem* N T *æquale est trapezio* X Y D E *per* I. *distinctionem & parallelogrammum* S R *æquale est rectilineo* X F G A B C Y *per secundam distinctionem: Ergo vt* H I *ad* I K, *sic trapezium* X Y D E *ad rectilineum* X F G A B C Y. *Est præterea linea recta* X Y *parallela ipsi* F V, *&* F V *parallela ipsi* A B *per constructionem: Quare per* 30. *prop. lib.* I. *Euclid.* X Y *est parallela ipsi* A B.

Conclusio.

Igitur in dato rectilineo recta linea ducta est parallela cum &c. Quod erat faciendum.

NOTA.

Si terminus Y *rectæ* X Y *caderet in latus* E D, *operatio tunc foret facilior, vt ex primo exemplo præcedentis septimi problematis facilè colligi potest, nam opus non esset triangulum* E L D *construere. Si verò terminus antedictus caderet in* B C *aut alias (quod varijs modis contingere potest) tum operatio ex antecedentibus esset collectu facilis.*

Primi libri finis.

construction, and the quadrangle *XYDE* is equal to the parallelogram *NT* by the first section; therefore, by the third axiom of Euclid's 1st book, the rectilinear figure *XFGABCY* is equal to the parallelogram *SR*.

Section 3.

As *HI* is to *IK*, so is *NS* to *SQ* by the construction, and, by the 1st proposition of Euclid's 6th book, as the line *NS* is to *SQ*, so is the parallelogram *NT* to the parallelogram *SR*. Consequently, as *HI* is to *IK*, so is the parallelogram *NT* to the parallelogram *SR*. Now the parallelogram *NT* is equal to the quadrangle *XYDE* by the 1st section, and the parallelogram *SR* is equal to the rectilinear figure *XFGABCY* by the second section. Accordingly, as *HI* is to *IK*, so is the quadrangle *YXDE* to the rectilinear figure *XFGABCY*. Moreover, the line *XY* is parallel to *FV*, and *FV* is parallel to *AB* by the construction. Therefore, by the 30th proposition of Euclid's 1st book, *XY* is parallel to *AB*.

Conclusion.

Therefore, in a given rectilinear figure a line has been drawn, parallel to etc. Which was to be performed.

NOTE.

If the extremity *Y* of the line *XY* fell on the side *ED*, then the operation would have been easier, as can readily be inferred from the first example of the preceding seventh problem, for it would not then be necessary to construct the triangle *ELD*. But if the aforesaid extremity fell on *BC* or somewhere else (which may happen in various ways), then the operation would easily be inferred from the preceding constructions.

END OF THE FIRST BOOK.

LIBER SECVNDVS

DE CONTINVAE QVANTITATIS regula Falsi.

Quid sit regula Falsi.

QVONIAM *geometriam (quam breuiter speramus nos edituros) in Methodum Arithmeticæ methodo similem digessimus (quod naturalis ordo videtur requirere propter magnam convenientiam continuæ & discontinuæ quantitatis vbi quodcunque genus magnitudinis, vt sunt linea, superficies, corpus, per quatuor species, vt Additionem, Subtractionem, Multiplicationem & Diuisionem, præterea per regulas, vt proportionum &c. tractabimus) offerebat se quoque ex ordine Problema quoddam, vbi per falsam positionem veram solutionem petitam Geometricè inveniremus: Quare vt continuæ & discontinuæ quantitatum correspondentiam tantò manifestius redderemus (nam vulgaris quædam regula in Arithmetica habetur quæ regula Falsi dicitur) Regulam Falsi continuæ quantitatis nominauimus, non quod falsum docet, sed quia per falsam positionem pervenitur ad cognitionem veri.*

Vtilitas huius regulæ inter alia hæc est, Quod eam quasi quoddam generale Problema citare possimus, quoties alicuius occultæ magnitudinis quantitatem & formam operæpretium erit invenire, id enim iubebitur tantum per regulam Falsi expediri. Itaque sæpe non opus erit in Problematum constructionibus quarundam occultarum magnitudinum inventionem copiosius describere.

PROBLEMA.

Ex datæ lineæ explicata tantum qualitate, superficiem describere æqualem & similem superficiei in qua ipsa linea existit.

NOTA.

Sensus Problematis est de superficiebus Geometricis, hoc est, de ijs quæ

SECOND BOOK
OF THE *REGULA FALSI* OF CONTINUOUS QUANTITY.

What the *Regula Falsi* Is.

Since we have arranged geometry (which we shortly hope to publish) in a Method similar to the method of Arithmetic *) (which the natural order of things seems to require because of the great agreement between continuous and discontinuous quantities, where we shall deal with any kind of magnitude, such as a line, a plane figure, a solid, by the four operations, *viz.* Addition, Subtraction, Multiplication, and Division, and also by rules, *viz.* of proportions, etc.), a certain Problem presented itself also in due time, where by a false position we could find by Geometrical means the true solution sought. Therefore, in order to render the correspondence between continuous and discontinuous quantities the more evident (for in Arithmetic there is a certain common rule which is called *regula falsi*), we have called it the *Regula Falsi* of a continuous quantity, not because it teaches false things, but because knowledge of true things is arrived at by a false position.

The usefulness of this rule is, *inter alia,* that we can cite it as a general Problem whenever it is worth while to find the quantity and form of some unknown magnitude, for it will be prescribed to find this out only by the *regula falsi.* And thus it will frequently not be necessary in the constructions of Problems to describe the finding of unknown magnitudes more fully.

PROBLEM.

If only the kind of line is given, to construct a figure equal and similar to the figure in which said line occurs.

NOTE.

The meaning of the Problem relates to Geometrical figures, *i.e.* such as can

*) See Introduction to the *Meetdaet.*

quæ Geometrica quadam lege describi poßunt.

Exemplum primum.

Explicatio dati.

Sit data recta A, *linea cuiusdam occulti æquilateri trianguli, talis vt si linea æqualis perpendiculari ab angulo in medium oppositi lateris secetur à latere, & reliquo addatur recta æqualis rectæ à centro trianguli in medium lateris Summa Additionis sit ipsa* A.

Explicatio quæsiti.

Oporteat ex huiusmodi lineæ A *explicata qualitate, æquilaterum triangulum describere, æqualem æquilatero occulto in quo existit* A.

Constructio.

Fingamus (vt fit in regula Falsi Arithmetica) quæsitum eße inventum, describaturque æquilaterus triangulus quicunque B C D *(sed, quamvis demonstratio in omnibus est eadem, tamen propter oculorum & manuum errorem, certior euadet operatio si positiua figura sumatur semper maior quam data occulta) quasi eßet æquilaterum quæsitum, fiatque operatio in eo secundum quæstionem supra adhibitam hoc modo: Ducatur perpendicularis* B E, *& recta* F G *à centro trianguli* F *in medium lateris* B C, *seceturq; à latere* C D *recta* C H *æqualis ipsi* B E, *sitque reliquum* H D, *cui addatur recta* D I *æqualis rectæ* F G: *Igitur si positio fuisset vera, recta* H I *æqualis eßet ipsi* A: *Sed ei inæqualis & maior est, ergo & falsa est: Quare nunc vera positio invenienda erit hoc modo: Inveniatur quarta linea proportionalis per* 12. *prop. lib.* 6. *Euclid. Quarum prima* H I *secunda latus quodlibet trianguli positi vt* B C, *tertia recta* A *sitque quarta* K L, *ex qua construatur æquilaterum triangulum* L K M.

Dico *ex lineæ* A *explicata qualitate, æquilaterum triangulum* L K M *esse descriptum, æqualem occulto æquilatero triangulo in quo recta* A *existit vt erat quæsitum.*

Præbe constructed by means of some Geometrical law.

FIRST EXAMPLE.

Given.

Let a line *A* be given, a line of some unknown equilateral triangle, such that if a line equal to the perpendicular from an angle to the mid-point of the opposite side be cut off from a side, and to the rest be added a line equal to the line from the centre of the triangle to the mid-point of a side, the Sum Total of the Addition shall be the said line *A*.

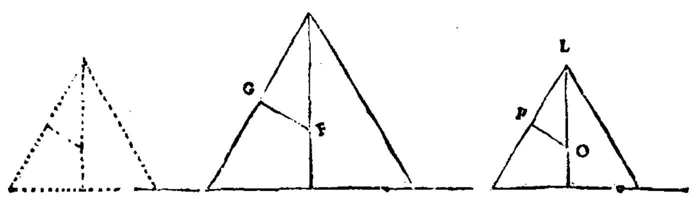
Required.

Let it be required, if only the kind of such a line *A* is given, to construct an equilateral triangle equal to the unknown equilateral triangle in which *A* occurs.

Construction.

Imagine (as is done in the Arithmetical *regula Falsi)* that what is required has been found, and construct any equilateral triangle *BCD* (but, though the proof is the same in all cases, nevertheless because of errors of eyes and hands the operation turns out more certain if the figure assumed is always taken larger than the unknown figure) as if it were the required equilateral triangle, and perform the operation in this triangle according to the problem set above, in the following way: Draw the perpendicular *BE*, and the line *FG* from the centre of the triangle *F* to the mid-point of the side *BC*, and from the side *CD* cut off the line *CH*, equal to the said *BE*, and let the rest be *HD*, to which add the line *DI* equal to the line *FG*. Therefore, if the position were true, the line *HI* would be equal to *A*. But it is unequal thereto and larger; consequently it is also false. And therefore the true position will have to be found in the following way. By the 12th proposition of Euclid's 6th book find the fourth proportional, the first term being *HI*, the second the side of any assumed triangle, *viz. BC,* the third the line *A*, and let the fourth be *KL*, in accordance with which construct the equilateral triangle *LKM*.

I say that, if the kind of the line *A* is given, an equilateral triangle *LKM* has been constructed, equal to the unknown equilateral triangle in which the line *A* occurs; as was required.



Præparatio demonſtrationis.

Ducatur perpendicularis L N, *&* *recta* O P *à centro* O *in medium lateris* L K, *appliceturque intervallum perpendicularis* L N *à* K *in rectam* K M, *ſitque* K Q, *addaturque in directum ipſius* Q M *recta* M R *æqualis ipſi* O P.

Demonſtratio.

Vt H I *ad* B C *ſic* A *ad* K L *per conſtructionem: Similiter vt* H I *ad* B C *ſic* Q R *ad* K L *vt colligitur ex 4. prop. lib. 6. Euclid. Igitur* A *&* Q R *ad eandem* K L *eandem habent rationem, quare ſunt inter ſe æquales per 9. prop. lib. 5. Euclid. Et proinde ſunt æquales homologæ lineæ. Sed æquales homologæ lineæ exiſtunt in æqualibus & ſimilibus figuris, Ergo æquilaterum* L K M *æqualis eſt occulto æquilatero in quo exiſtit recta* A.

Concluſio.

Igitur ex datæ lineæ explicata tantum &c. Quod erat faciendum.

Exemplum ſecundum.

Explicatio dati.

Sit data recta A, *linea cuiuſdam occulti quadrati talis, vt ſi linea æqualis lateri ipſius quadrati ſecetur à diagonali, reliquum ſit ipſa* A.

Expli-

Preparation of the Proof.

Draw the perpendicular *LN*, and the line *OP* from the centre *O* to the midpoint of the side *LK*, and mark off the length of the perpendicular *LN* from *K* on the line *KM*, and let this be *KQ*; and produce *QM* to *R* so that the line *MR* is equal to *OP*.

Proof.

As *HI* is to *BC*, so is *A* to *KL* by the construction. Similarly, as *HI* is to *BC*, so is *QR* to *KL*, as is inferred from the 4th proposition of Euclid's 6th book. Therefore *A* and *QR* are to the same *KL* in the same ratio; consequently they are equal to one another by the 9th proposition of Euclid's 5th book. And accordingly they are equal homologous lines. But equal homologous lines occur in equal and similar figures. Consequently the equilateral triangle *LKM* is equal to the unknown equilateral triangle in which the line *A* occurs.

Conclusion.

Therefore, if only the kind of line is given, etc. Which was to be performed.

SECOND EXAMPLE.

Given.

Let a line *A* be given, a line of some unknown square such that if a line equal to the side of said square be cut off from a diagonal, the rest shall be the said line *A*.

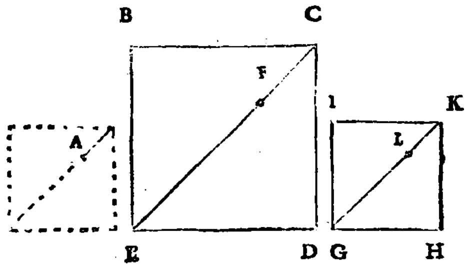
Explicatio quæſiti.

Oporteat ex huiuſmodi lineæ A *explicata qualitate, quadratum deſcribere, æquale quadrato occulto in quo* A *exiſtit.*

Conſtructio.

Fingamus quæſitum eſſe inventum, deſcribaturque quadratum quodcunque B C D E *quaſi eſſet quadratum quæſitum, opereturque in eo ſecundum quæſtionem ſupra adhibitam, hoc modo: Ducatur eius diagonalis* E C, *à qua ſecetur recta* E F *æqualis lateri* E D, *ſitque reliquum* F C: *Igitur ſi poſitio fuiſſet vera recta* F C *æqualis eſſet ipſi* A, *ſed ei inæqualis & maior eſt, ergo poſitio eſt falſa, quare nunc vera poſitio invenienda erit hoc modo: Inveniatur quarta linea proportionalis per* 12. *prop. lib.* 6. *Euclid. quarum prima* F C, *ſecunda* E D, *tertia* A, *ſitque quarta* G H, *ex qua conſtruatur quadratum* I K H G.

Dico ex lineæ A *explicata qualitate, quadratum* I K H G *inventum eſſe, æquale occulto quadrato in quo* A *exiſtit vt erat quæſitum.*



Præparatio demonſtrationis.

Ducatur diagonalis G K, *appliceturque intervallum* G H *à* G *in rectam* G K *ad* L.

Required.

Let it be required, if only the kind of such a line *A* is given, to construct a square equal to the unknown square in which *A* occurs.

Construction.

Imagine that what is required has been found, and construct any square *BCDE* as if it were the required square, and perform the operation in this square according to the problem set above, in the following way: Draw a diagonal *EC* of this square, from which cut off the line *EF* equal to the side *ED*, and let the rest be *FC*. Therefore, if the position were true, the line *FC* would be equal to *A*, but it is unequal thereto and larger. Consequently the position is false; therefore now the true position will have to be found in the following way. By the 12th proposition of Euclid's 6th book, find we fourth proportional, the first term being *FC*, the second *ED*, the third *A*, and let the fourth be *GH*, in accordance with which construct the square *IKHG*.

I say that, if the kind of the line *A* is given, a square *IKHG* has been found, equal to the unknown square in which *A* occurs; as was required.

Preparation of the Proof.

Draw the diagonal *GK* and mark off the length *GH* from *G* on the line *GK* to *L*.

Demonstratio.

Vt F C *ad* E D, *sic* A *ad* G H *per constructionem. Similiter vt* F C *ad* E D *sic* L K *ad* G H, *vt colligitur ex* 4. *prop. lib.* 6. *Euclid. Igitur* A *&* L K *ad eandem* G H *eandem habent rationem, quare sunt inter se æquales per* 9. *prop. lib.* 5. *Euclid. Et proinde sunt æquales homologæ lineæ, sed æquales homologæ lineæ existunt in æqualibus & similibus figuris, ergo quadratum* I K H G *æquale est occulto quadrato in quo existit recta* A.

Conclusio.

Igitur ex lineæ explicata tantum &c. Quod erat faciendum.

Exemplum tertium.

Explicatio dati.

Sit data recta A *linea perpendicularis cuiusdam occulti pentagoni æquilateri & æquianguli, ab angulo in medium ipsius oppositi lateris.*

Explicatio quæsiti.

Oporteat ex ciusmodi lineæ A *explicata qualitate pentagonum describere, æquale est simile pentagono occulto in quo* A *existit.*

Constructio.

Fingamus quæsitum esse inventum describaturque pentagonum B C D E F *quodcunque, quasi esset pentagonum quæsitum: Opereturque in eo secundum petitionem adhibitam, hoc modo: Ducatur recta* B G *à* B *in medium oppositi lateris* E D. *Igitur si positio esset vera recta* B G *æqualis esset ipsi* A, *sed ei inæqualis & maior est, ergo illa positio est Falsa, quare nunc vera positio invenienda erit hoc modo. Inveniatur quarta linea proportionalis per* 12. *prop. lib.* 6. *Euclid. Quarum prima* B G, *secunda* B C, *tertia* A, *sitque quarta* H I, *à qua construatur pentagonum* H I K L M.

Dico

Proof.

As *FC* is to *ED*, so is *A* to *GH* by the construction. Similarly, as *FC* is to *ED*, so is *LK* to *GH*, as is inferred from the 4th proposition of Euclid's 6th book. Therefore *A* and *LK* are to the same *GH* in the same ratio; consequently they are equal to one another by the 9th proposition of Euclid's 5th book. And accordingly they are equal homologous lines; but equal homologous lines occur in equal and similar figures, consequently the square *IKHG* is equal to the unknown square in which the line *A* occurs.

Conclusion.

Therefore, if only the kind of line is given, etc. Which was to be performed.

THIRD EXAMPLE.

Given.

Let the line *A* be given, a perpendicular line of any unknown equilateral and equiangular pentagon, from an angle to the mid-point of the opposite side thereof.

Required.

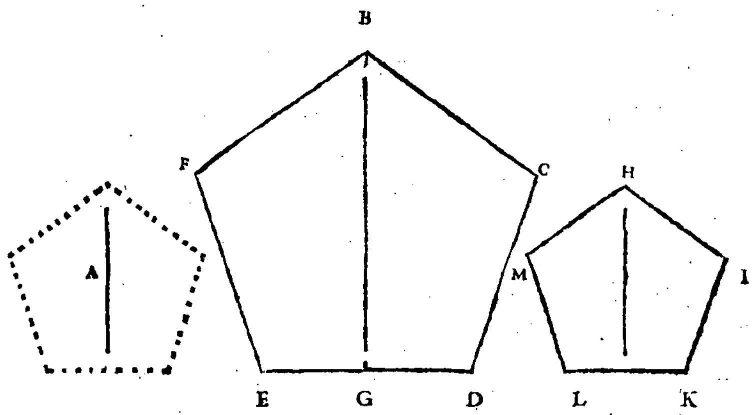
Let it be required, if the kind of such a line *A* is given, to construct a pentagon equal and similar to the unknown pentagon in which *A* occurs.

Construction.

Imagine that what is required has been found, and construct any pentagon *BCDEF* as if it were the required pentagon. And perform the operation in this pentagon according to the problem set, in the following way: Draw the line *BG* from *B* to the mid-point of the opposite side *ED*. Therefore, if the position were true, the line *BG* would be equal to *A*. But it is unequal thereto and larger, consequently the position is false; therefore the true position will now have to be found in the following way. By the 12th proposition of Euclid's 6th book, find the fourth proportional, the first term being *BG*, the second *BC*, the third *A*, and let the fourth be *HI*, in accordance with which construct the pentagon *HIKLM*.

I say that, if the kind of the line *A* is given, the pentagon *HIKLM* has been constructed, equal and similar to the unknown pentagon in which *A* occurs; as was required.

Dico ex lineæ A *explicata qualitate pentagonum* HIKLM *descriptum esse, æquale & simile occulto pentagono in quo* A *existit ut erat quæsitum.*



Demonstratio.

Demonstratio ex primo & secundo exemplo est manifesta.

Conclusio.

Igitur ex lineæ explicata tantum &c. Quod erat faciendum.

Exemplum quartum.

Explicatio dati.

Sit data recta A, *linea cuiusdam occulti rectilinij similis rectilineo* BCDEF, *ita ut linea æqualis occulti rectilinei homologæ lineæ cum* BC, *secta ab occulti rectilinei homologa linea cum* FC, *& reliquo addita occulti rectilinei homologa linea cum* FE, *sit in directum unius lineæ ipsa data linea* A.

Proof.

The proof is evident from the first and the second example.

Conclusion.

Therefore, if only the kind of a line is given, etc. Which was to be performed.

FOURTH EXAMPLE.

Given.

Let a line *A* be given, a line of some unknown rectilinear figure similar to the rectilinear figure *BCDEF*, such that if the line equal to the line of the unknown figure that is homologous to *BC* be cut off from the line of the unknown figure that is homologous to *FC*, and the line of the unknown figure that is homologous to *FE* be added to the rest, the said given line *A* shall lie on one and the same line.

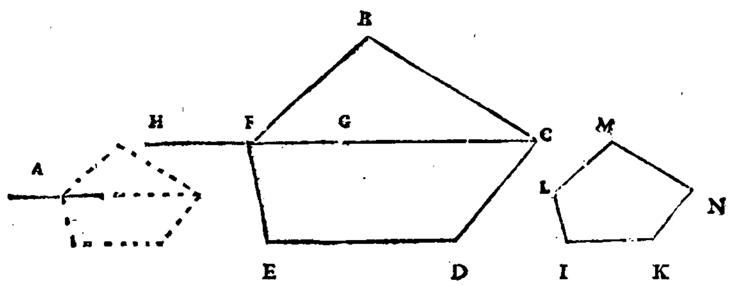
Explicatio quæſiti.

Oporteat ex huiuſmodi lineæ A *explicata qualitate rectilineum deſcribere, æquale & ſimile ſimiliterque poſitum rectilineo occulto in quo* A *exiſtit.*

Conſtructio.

Fingamus quæſitum eſſe inventum atque ipſum datum rectilineum B C D E F *eſſe petitum rectilineum: Opereturque in eo ſecundum petitionem ſupra exhibitam hoc modo: Secetur à recta* C F *recta* C G *æqualis rectæ* C B, *ſitque reliquum* F G, *cui in directum addatur recta* F H *æqualis rectæ* F E. *Igitur ſi poſitio eſſet vera recta* H G *æqualis eſſet ipſi* A *ſed illi inæqualis & maior eſt, ergo poſitio erat Falſa: Quare nunc vera poſitio invenienda erit hoc modo: Inveniatur quarta linea proportionalis per* 12. *prop. lib.* 6. *Euclid. quarum prima* H G, *ſecunda* E D, *tertia* A, *ſitque quarta* I K, *à qua vt homologa linea cum* E D *conſtruatur per* 18. *prop. lib.* 6. *Euclid. rectilineum* L M N K I, *ſimile ſimiliterque poſitum rectilineo* B C D E F.

Dico ex lineæ A *explicata qualitate rectilineum* L M N K I *eſſe deſcriptum æquale & ſimile ſimiliterque poſitum rectilineo occulto in quo* A *exiſtit vt erat quæſitum.*



Demon-

Required.

Let it be required, if the kind of such a line *A* is given, to construct a rectilinear figure equal and similar and similarly placed to the unknown rectilinear figure in which *A* occurs.

Construction.

Imagine that what is required has been found, and that the said given rectilinear figure *BCDEF* is the required rectilinear figure. And perform the operation in this figure, according to the problem set above, in the following way: From the line *CF* cut off the line *CG* equal to the line *CB*, and let the rest be *FG*, and produce *FG* so that the line *FH* is equal to the line *FE*. Therefore, if the position were true, the line *HG* would be equal to *A*. But it is unequal thereto and larger, consequently the position was false. Therefore the true position will now have to be found in the following way. By the 12th proposition of Euclid's 6th book, find the fourth proportional, the first term being *HG*, the second *ED*, the third *A*, and let the fourth be *IK*, in accordance with which, as being the line homologous to *ED*, construct by the 18th proposition of Euclid's 6th book the rectilinear figure *LMNKI*, similar and similarly placed to the rectilinear figure *BCDEF*.

I say that, if the kind of the line *A* is given, the rectilinear figure *LMNKI* has been constructed, equal and similar and similarly placed to the unknown rectilinear figure in which *A* occurs; as was required.

Demonſtratio.

Demonſtratio ex demonſtratione primi & ſecundi exempli eſt manifeſta.

Concluſio.

Igitur ex lineæ tantum explicata æqualitate &c. Quod erat faciendum.

Varia poſſent per hanc regulam deſcribi exempla tam in ſolidis quàm in planis, vt in quadrato vel pentagono, maximus æquilaterus triangulus, aut in quocunque regulari poligono, maximum quodcunque regulare poligonum, Similiter & minimum cuius anguli tangant latera circumſcripti. &c. Sed cum regulæ ſenſus ex præcedentibus quatuor, ſatis videatur explicatus, commendamus reliqua quibus illæ Geometricæ ſpeculationes erunt cordi.

Secundi Libri

FINIS.

Proof.

The proof is evident from the proof of the first and the second example.

Conclusion.

Therefore, if only the kind of a line is given, etc. Which was to be performed.

By this rule various examples might be constructed, both in solid and in plane figures, *e.g.* in a square or a pentagon the greatest equilateral triangle, or in any regular polygon any greatest regular polygon. Similarly also the smallest figure whose angles touch the sides of the circumscribed figure, etc. But since the meaning of the rule seems to be sufficiently clear from the preceding four examples, we recommend the others to those to whom these Geometrical speculations are dear.

END OF THE SECOND BOOK.

LIBER TERTIVS

DE QVINQVE REGVLARIVM, QVINque auctorum Regularium & nouem Truncatorum regularium corporum eidem sphæræ inscriptibilium descriptione.

PRAETER *quinque corpora regularia quorum Mathematici meminerunt, animaduertimus alia quædam corpora quæ quamvis talem non haberent regularitatem vt in quinque illis regularibus requiritur (nam demonstratur quinque tantum talia corpora posse inveniri) nihilominus Geometricarum speculationum essent plena, ac mirabilis dispositionis correlatiuarum superficierum. Horum autem corporum sex meminit Albertus Durerus, in sua Geometria (sunt quidem in eadem Alberti descriptione & alia duo corpora quæ ex complicatis planis componuntur quorum alterum non potest plicari, ratio est quia ad vnum angulum solidum construendum compositi sunt tres anguli plani æquales quatuor rectis, qui angulum solidum per* 21. *prop. lib.* 11. *Euclid. non constituunt. Alterum verò corpus non continetur intra metas quæ in sequenti* 11. *definit. sunt positæ, quare illa duo corpora reliquimus) sed cum talium corporum originem vel nomina apud neminem inveniremus tamen existimaremus non sine aliquo certo fundamento consistere, vidimus tandem regularia corpora ipsorum esse scatebram, nam illorum vnum, erat tetraedrum truncatum, altera tria, truncati cubi, & quintum, truncatum octoedrum: Sexti verò corporis truncatio hæc scribentibus nobis erat ignota, quamvis ex truncato cubo originem habere non dubitamus. Cumque hæc nobis eßent nota invenimus (nam tale quid sæpe fit cum rerum causas cognoscimus) alia tria corpora non minoris elegantiæ nempe ex truncatis Dodecaedro & Icosaedro. Quorum definitiones sunt sequentes defi.* 20. 21. 22. *Et ipsorum planorum dispositiones in sequenti secundo Problemate distinct.* 17. 18. 19. *inveniendæ*

venien-

THIRD BOOK.

Of the construction of the five regular, the five augmented Regular, and the nine Truncated regular solids that can be inscribed in the same sphere.

Besides the five regular solids mentioned by mathematicians, we draw attention to some other solids which, though they do not have so great regularity as is required in these five regular solids (for it is proved that only five such solids are to be found), nevertheless would be full of Geometrical speculations and of a remarkable arrangement of the correlative faces. Now six of these solids have been mentioned by Albert Dürer, in his Geometry (indeed, in the said description of Albert there are also two other solids which are composed by folding of planes, one of which cannot be folded; the reason is that for the construction of one solid angle, three equal plane angles equal to four right angles have been put together, which do not constitute a solid angle by the 21st proposition of Euclid's 11th book. And the other solid is not included between the boundaries which are set in the following 11th definition, for which reasons we have omitted those two solids), but since we have not found the origin or the names of such solids in any author, and yet judged, not without a certain foundation, that they exist, at last we have found that the regular solids are their source, for one of them was a truncated tetrahedron, three others were truncated cubes, and the fifth was a truncated octahedron, but the truncation of the sixth solid was unknown to us when we were writing this, though we did not doubt that it derived from a truncated cube. And when these things were known to us, we found (for such a thing frequently happens when we become acquainted with the causes of things) three other solids of no less elegance, *viz.* from a truncated Dodecahedron and Icosahedron; their definitions are the following definitions 20, 21, 22; and the dispositions of their faces will be found in the following second Problem, sections 17, 18, 19. If by any chance they have *venientur. Si forte ab alio ante nos sunt inventa (de quo ferè non dubitarem propter magnam diligentiam veterum in formarum inquisitione) fatemur hoc nos ignorare. Quare vt pro nostro invento talia edimus.*

Postea verò factum est (recitamus hæc quia aliquando non iniucundum est inventionum occasiones non ignorare) vt Franciscus Cophart Archimusicus nostri Leidensis Musicorum collegij, & Geometriæ singularis amator, vellet mihi persuadere se casu quodam sextum corpus regulare vidiße, cuius constructio talis erat:

Ducantur omnes Diagonales lineæ omnium quadratorum cubi, ducantur deinde plana ab omnibus angulis solidis cubi per duas diagonales lineas vsque ad ipsarum diagonalium medietates, exscindanturque hoc modo omnia superficierum cubi latera, cum subiecta solida parte ipsius cubi inter duo secantia plana comprehensa. Erunt itaque cubo (quoniam duodecim habet latera) duodecim crenæ inscisæ: relinqueturque elegans corpus in viginti & quatuor æqualibus triangulis æquilateris contentum. Quare ille argumentabatur hoc modo:

Corpora sphæræ inscriptibilia quorum superficies sunt omnes æquales & similes, sunt corpora regularia:

Corpus hoc est corpus sphæræ inscriptibile, cuius superficies sunt omnes æquales & similes:

Ergo est corpus regulare, & per consequens sextum.

Sed negabamus partem antecedentem aßumptionis, quoniam tale corpus non est corpus sphæræ ita inscriptibile, vt in regularium corporum inscribilitate requiritur, nam sensus ibi est omnes angulos solidos corporum debere existere in superficie sphæræ circumscriptæ, huius verò corporis duæ sunt species solidorum angulorum, nam alterius speciei anguli sunt externi, alterius interni. Verum quidem est omnes angulos externos eidem sphæræ eße inscriptibiles: Similiter & omnes angulos internos eidem sphæræ inscriptibiles: Sed non omnes eidem, nam alia est sphæra externorum angulorum alia internorum.

Igitur

been found by someone else before us (which I should hardly doubt, considering the great diligence of the Ancients in the study of forms), we confess that we ignore this. We therefore publish them as our invention.

Afterwards it happened (we mention this because sometimes it is not unpleasant not to ignore the moments at which something was discovered) that Frans Cophart *), the leader of our Leiden society of musicians and an extraordinary lover of Geometry, wanted to persuade me that he happened to have found a sixth regular solid, whose construction was as follows:

Draw all the diagonals of all the squares of a cube, and then draw planes from all the solid angles of the cube through two diagonals up to the midpoints of said diagonals, and in this way cut off all the sides of the faces of the cube, with the adjacent solid part of the cube included between two intersecting planes. And thus the cube (since it has twelve edges) will have twelve incisions; there remains an elegant solid included by twenty-four equal equilateral triangles. Therefore he argued as follows:

Solids that can be inscribed in a sphere and whose faces are all equal and similar are regular solids.

This solid is a solid that can be inscribed in a sphere and whose faces are all equal and similar.

Therefore, it is a regular solid, and in consequence the sixth.

But we denied the first part of the assumption, since this solid is not a solid that can be inscribed in a sphere in such a way as is required with regular solids, for the meaning is there that all the solid angles of the solids must lie on the surface of the circumscribed sphere, but with this solid there are two kinds of solid angles, for the angles of one kind are exterior and those of the other, interior. Indeed, it is true that all the exterior angles can be inscribed in the same sphere. Similarly also all the interior angles can be inscribed in the same sphere. But not all in the same, for there is one sphere for the exterior angles

*) It has not been possible to trace any biographical details about this musician.

Igitur quia hoc corpus non habebat omnes proprietates quæ in regularibus corporibus requiruntur, concludebamus illud non esse sextum corpus regulare. Postea verò vidimus tale corpus esse octoedrum cui opposita erant octo tetraedra, quorum basses erant octoedri octo superficies. Cumque hoc animaduerteremus vnà cum elegantia ipsius, atque Geometricis rationibus in eo consistentibus, adplicauimus talem constructionem ad cetera quatuor regularia corpora, quæ omnia regularia aucta vocauimus, quorum constructio & eidem sphæræ inscriptio vnà cum ceteris, est materia de qua nunc agetur.

Primò igitur describemus horum corporum Definitiones. Secundò illorum laterum inventiones, ita vt eidem sphæræ sint inscriptibilia. Tertiò demonstrabitur, ex inventis lateribus, corporum constructio eidem sphæræ inscriptibilium. At notandum est id quod de quinque regularibus corporibus dicetur pro nostro invento non exhiberi, sed ordinis ac necessitatis gratia suis locis commemorare.

Definitiones quinque corporum regularium.

Definitio 1.

Tetraedrum est corpus sub quatuor triangulis æqualibus & æquilateris contentum.

Definitio 2.

Cubus est corpus sub sex quadratis æqualibus contentum.

Definitio 3.

Octoedrum est corpus sub octo triangulis æqualibus & æquilateris contentum.

Definitio 4.

Dodecaedrum est corpus sub duodecim pentagonis æqualibus & æquilateris & æquiangulis contentum.

Defi-and another for the interior angles. Therefore, because this solid did not have all the properties which are required in regular solids, we concluded that this is not the sixth regular solid. Later, however, we found that such a solid is an octahedron against which had been placed eight tetrahedra, whose bases were the eight faces of the octahedron. And when we noted this, along with its elegance and the Geometrical ratios present in it, we applied this construction to the other four regular solids, all of which we called augmented regular solids, whose construction and inscription in the same sphere along with the others is the subject matter now to be dealt with.

In the first place therefore we are going to describe the Definitions of these solids. Secondly, the finding of their edges such that they can be inscribed in the same sphere. Thirdly, the construction, from the edges found, of the solids that can be inscribed in the same sphere will be proved. But it is to be noted that what is said about the five regular solids is not set forth as our invention, but is called to mind in its place for the sake of good order and necessity.

Definitions of the Five Regular Solids.

Definition 1.

A tetrahedron is a solid included by four equal and equilateral triangles.

Definition 2.

A cube is a solid included by six equal squares.

Definition 3.

An octahedron is a solid included by eight equal and equilateral triangles.

Definition 4.

A dodecahedron is a solid included by twelve equal and equilateral and equiangular pentagons.

Definitio 5.

Icosaedrum est corpus sub viginti triangulis æqualibus & æquilateris contentum.

Definitiones quinque auctorum corporum regularium.

Definitio 6.

Si cuicunque superficiei tetraedri apponatur tetraedrum habens superficiem illam pro basi: Corpus ex illis compositum duodecim triangulis æqualibus & æquilateris contentum vocatur tetraedrum auctum.

Definitio 7.

Si cuicunque superficiei hexaedri apponatur pyramis habens superficiem illam pro basi, & reliquas superficies triangula æquilatera: Corpus ex illis compositum vigintiquatuor triangulis æqualibus & æquilateris contentum, vocatur Hexaedrum auctum.

Definitio 8.

Si cuicunque superficiei octoedri apponatur tetraedrum habens superficiem illam pro basi: Corpus ex illis compositum viginti & quatuor triangulis æqualibus & æquilateris contentum, vocatur octoedrum auctum.

Definitio 9.

Si cuicunque superficiei dodecaedri apponatur pyramis habens superficiem illam pro basi, & reliquas superficies triangula æquilalatera: Corpus ex illis compositum sexaginta triangulis æqualibus & æquilateris contentum, vocatur dodecaedrum auctum.

Definitio 10.

Si cuicunque superficiei icosaedri apponatur tetraedrum habens superficiem ipsam pro basi: Corpus ex illis compositum sexaginta triangulis æqualibus & æquilateris contentum, vocatur icosaedrum auctum.

Definition 5.

An icosahedron is a solid included by twenty equal and equilateral triangles.

Definitions of the Five Augmented Regular Solids.

Definition 6.

If against every face of a tetrahedron is placed a tetrahedron having said face for its base, the solid composed of these, included by twelve equal and equilateral triangles, is called augmented tetrahedron.

Definition 7.

If against every face of a hexahedron is placed a pyramid having said face for its base, while the other faces are equilateral triangles, the solid composed of these, included by twenty-four equal and equilateral triangles, is called an augmented hexahedron.

Definition 8.

If against every face of an octahedron is placed a tetrahedron having said face for its base, the solid composed of these, included by twenty-four equal and equilateral triangles, is called an augmented octahedron.

Definition 9.

If against every face of a dodecahedron is placed a pyramid having said face for its base, while the other faces are equilateral triangles, the solid composed of these, included by sixty equal and equilateral triangles, is called an augmented dodecahedron.

Definition 10.

If against every face of an icosahedron is placed a tetrahedron having said face for its base, the solid composed of these, included by sixty equal and equilateral triangles, is called an augmented icosahedron.

Definitiones nouem truncatorum corporum regularium.

Definitio 11.

Solidum ſphæræ inſcriptibile cuius anguli ſolidi ſunt omnes æquales, & cuius plana non ſunt omnia ſimilia, & quodcunque planum eſt æquiangulum & æquilaterum, & omnium planorum latera ſunt inter ſe æqualia: vocatur truncatum corpus regulare.

Definitio 12.

Si omnia latera tetraedri diuidãtur in tres partes æquas, & plano ſingulus angulus ſolidus tetraedri abſcindatur, per trium laterum diuiſiones ipſi angulo proximas; Reliquum ſolidum vocatur truncatum tetraedrum per laterum tertias.

NOTA.

Habet hoc corpus quatuor plana hexagona, & quatuor triangularia, duodecim angulos ſolidos, & decem & octo latera.

Superuacaneum exiſtimamus tum hic, tum in ſequentibus notis, horum planorum formas ex qualitate laterum, angulorum æqualitate, & ſimilitudine planorum exprimere, vt exempli gratia, cùm ſupra dicatur de quatuor planis hexagonis, non dicimus quatuor plana hexagona, æquilatera, & æquiangula, æqualia & ſimilia: ſed tantum quatuor hexagona. Similiter non dicimus duodecim angulos ſolidos æquales, & octodecim latera æqualia, ſed tantum duodecim angulos ſolidos, & decem & octo latera: Quoniam reliqua vt colligitur ex 11. definitione ſequuntur neceſſario.

NOTA.

Si tetraedri anguli ſolidi ſimiliter abſcindantur per laterum media, reliquum ſolidum erit octoedrum.

Definitio 13.

Si omnia latera cubi diuidantur in duas partes æquas, & plano ſin-

Definitions of the Nine Truncated Regular Solids.

Definition 11.

A solid that can be inscribed in a sphere, whose solid angles are all equal and whose faces are not all similar, while all the faces are equiangular and equilateral, and the sides of all the faces are equal to one another, is called a truncated regular solid.

Definition 12.

If all the edges of a tetrahedron are divided into three equal parts, and each solid angle of the tetrahedron is cut off by a plane through those points of division of the three edges which are adjacent to said angle, the remaining solid is called a tetrahedron truncated through the third parts of the edges.

NOTE.

This solid has four hexagonal and four triangular faces, twelve solid angles, and eighteen edges.

We consider it superfluous, both here and in the following notes, to express the forms of these faces in the kind of their sides, the equality of the angles, and the similarity of the faces; *e.g.* when above four hexagonal faces are referred to, we do not say four hexagonal equilateral and equiangular, equal and similar faces, but only four hexagonal faces. Similarly, we do not say twelve equal solid angles, and eighteen equal edges, but only twelve solid angles and eighteen edges, since the rest, as is inferred from the 11th definition, follows by necessity.

NOTE.

If the solid angles of a tetrahedron are similarly cut off through the midpoints of the edges, the remaining solid will be an octahedron.

Definition 13.

If all the edges of a cube are divided into two equal parts, and all the solid ſinguli anguli ſolidi cubi abſcindantur, per trium laterum diuiſiones ipſi angulo proximas: Reliquum ſolidum vocatur Truncatus Cubus per laterum media.

NOTA.

Habet hoc corpus ſex plana quadrata, & octotriangularia, & duodecim angulos ſolidos, & 24. latera.

NOTA.

Hoc corpus ſimile eſt truncato octoedro per laterum media ſequentis 17. Definitionis.

Definitio 14.

Si omnia latera cubi diuidantur in tres partes, hoc modo vt ſingulæ mediæ partes ſe habeāt ad vtramque alteram partem ipſius lateris vt diagonalis quadrati ad ſuum latus, & plano ſinguli anguli ſolidi ipſius cubi abſcindantur per trium laterum diuiſiones ipſi angulo proximas: Reliquum ſolidum vocatur truncatus cubus per laterum diuiſiones in tres partes.

NOTA.

Habet hoc corpus ſex plana octogona, octo triangularia, & viginti quatuor angulos ſolidos, & triginta & ſex latera.

Definitio 15.

Si omnia latera cubi diuidantur in tres partes, hoc modo vt ſingulæ mediæ partes ſe habeāt ad vtramque alteram partem ipſius lateris, vt diagonalis quadrati ad ſuum latus, & plano ſingula latera abſcindātur per quatuor laterum diuiſiones in ipſis abſcindendis lateribus, non exiſtentibus & ipſis lateribus proximas, relinquetur corpus habens ſex quadrata, & octo angulos ſolidos in æquidiſtantia à centro cubi, & ab eodem centro remotiores quàm reliqui anguli ſolidi: Si deinde ſinguli anguli illorum octo, plano abſcindātur per tres proximos angles of the cube are cut off by planes through those points of division of the three edges which are adjacent to said angle, the remaining solid is called a cube truncated through the mid-points of the edges.

NOTE.

This solid has six square and eight triangular faces, and twelve solid angles, and 24 edges.

NOTE.

This solid is similar to the octahedron truncated through the mid-points of the edges, of the following 17th Definition.

Definition 14.

If all the edges of a cube are divided into three parts, in such a way that all the middle parts are to the two other parts of said edge as the diagonal of a square to its side, and if all the solid angles of said cube are cut off by planes through those points of division of the three edges which are adjacent to said angle, the remaining solid is called a cube truncated through the divisions of the edges into three parts.

NOTE.

This solid has six octagonal and eight triangular faces, and twenty-four solid angles, and thirty-six edges.

Definition 15.

If all the edges of a cube are divided into three parts, in such a way that all the middle parts are to the two other parts of said edge as the diagonal of a square to its side, and the edges are cut off by planes through those points of division of the four edges which do not lie on said edges to be cut off and are adjacent to said edges, a solid remains which has six squares and eight solid angles at the same distance from the centre of the cube and further distant from said centre than the other solid angles; if subsequently all the eight solid angles are cut off by planes through three adjacent plane angles of the mos angulos planos trium quadratorum ipsis solidis angulis proximorum: Reliquum solidum vocatur bistruncatus cubus primus.

NOTA.

Habet hoc corpus octodecim quadrata, octo plana triangularia, viginti quatuor angulos solidos, quadraginta latera.

Definitio 16.

Si omnia latera cubi diuidantur in quinque partes, hoc modo vt mediæ partes se habeant ad quamcunque partem reliquarum quatuor partium ipsius lateris, vt diagonalis quadrati ad suum latus, & plano singula latera abscindantur, per quatuor laterum diuisiones in vnoquoque abscindendo latere non existentes, & ipsi lateri proximas, relinquaturque hoc modo corpus habens sex quadrata & octo angulos solidos in æquidistantia à centro, & ab eodē centro remotiores quàm reliqui anguli solidi: Si deinde omnia latera illorum sex quadratorum diuidantur in tres partes, hoc modo vt singulæ mediæ partes se habeant ad vtramque alteram partem ipsius lateris, vt diagonalis quadrati ad suum latus, & plano singuli anguli solidi illorum octo angulorum abscindantur, per sex diuisiones illorum laterum quadratorum ipsis angulis solidis proximas: Reliquum solidum vocatur bistruncatus cubus secundus.

NOTA.

Habet hoc corpus sex plana octogona, & octo hexagona, & duodecim quadrata, & quadraginta & octo angulos solidos, & septuaginta & duo latera.

Definitio 17.

Si omnia latera octoedri diuidantur in duas partes æquas, & plano singuli anguli solidi octoedri abscindantur per quatuor laterum diuisiones ipsis angulis proximas: Reliquum solidum vocatur truncatum octoedrum per laterum media.

NOTA.

three squares adjacent to said solid angles, the remaining solid is called the first twice-truncated cube.

NOTE.

This solid has eighteen squares, eight triangular faces, twenty-four solid angles, and forty edges.

Definition 16.

If all the edges of a cube are divided into five parts, in such a way that all the middle parts are to each of the four other parts of said edges as the diagonal of a square to its side, and the edges are cut off by planes through those points of division of the four edges which do not lie on the edges to be cut off and are adjacent to said edges, and in this way a solid remains which has six squares and eight solid angles at the same distance from the centre and further distant from said centre than the other solid angles; if subsequently all the sides of those six squares are divided into three parts, in such a way that all the middle parts are to the two other parts of said sides as the diagonal of a square to its side, and all the eight solid angles are cut off by planes through those six points of division of the sides of the squares which are adjacent to said solid angles, the remaining solid is called the second twice-truncated cube.

NOTE.

This solid has six octagonal, eight hexagonal, and twelve square faces, forty-eight solid angles, and seventy-two edges.

Definition 17.

If all the edges of an octahedron are divided into two equal parts, and the solid angles of the octahedron are cut off by planes through those points of division of the four edges which are adjacent to said angles, the remaining solid is called an octahedron truncated through the mid-points of the edges.

NOTA.

Habet hoc corpus ſex plana quadrata, & octo triangularia, & duodecim angulos ſolidos, & viginti quatuor latera.

NOTA.

Hoc corpus ſimile eſt truncato cubo per laterum media 13. definitionis.

Definitio 18.

Si omnia latera octoedri diuidantur in tres partes æquas, & plano ſinguli anguli ſolidi octoedri abſcindantur, per quatuor laterum diuiſiones ipſis angulis proximas: Reliquum ſolidum vocatur octoedrum truncatum per laterum tertias.

NOTA.

Habet hoc corpus ſex quadrata, & octo plana hexagona, & viginti & quatuor angulos ſolidos, & triginta & ſex latera.

Definitio 19.

Si omnia latera dodecaedri diuidantur in duas partes æquas, & plano ſinguli anguli ſolidi abſcindantur per trium laterum diuiſiones ipſis angulis proximas: Reliquum ſolidum vocatur truncatum dodecaedrum per laterum media.

NOTA.

Habet hoc corpus duodecim plana pentagona, viginti triangularia, & triginta angulos ſolidos, & ſexaginta latera.

NOTA.

Hoc corpus ſimile eſt truncato Icoſaedro per laterum media ſequentis 21. definitionis.

NOTE.

This solid has six square and eight triangular faces, twelve solid angles, and twenty-four edges.

NOTE.

This solid is similar to a cube truncated through the mid-points of the edges, of the 13th definition.

Definition 18.

If all the edges of an octahedron are divided into three equal parts, and the solid angles of the octahedron are cut off by planes through those points of division of the four edges which are adjacent to said angles, the remaining solid is called an octahedron truncated through the third parts of the edges.

NOTE.

This solid has six square and eight hexagonal faces, twenty-four solid angles, and thirty-six edges.

Definition 19.

If all the edges of a dodecahedron are divided into two equal parts, and the solid angles are cut off by planes through those points of division of the three edges which are adjacent to said angles, the remaining solid is called a dodecahedron truncated through the mid-points of the edges.

NOTE.

This solid has twelve pentagonal and twenty triangular faces, thirty solid angles, and sixty edges.

NOTE.

This solid is similar to an icosahedron truncated through the mid-points of the edges, of the following 21st definition.

Definitio 20.

Si omnia latera dodecaedri diuidantur in tres partes, hoc modo vt singulæ mediæ partes ad vtramque alteram partem ipsius lateris se habeant vt chorda arcus duarum quintarum peripheriæ circuli ad chordam arcus vnius quintæ eiusdem peripheriæ & plano singuli anguli solidi dodecaedri abscindantur per trium laterum diuisiones ipsis angulis proximas: Reliquum solidum vocatur truncatum dodecaedrum per laterum diuisiones in tres partes.

NOTA.

Habet hoc corpus duodecim plana decagona, & viginti triangularia, & sexaginta angulos solidos, & nonaginta latera.

Definitio 21.

Si omnia latera Icosaedri diuidantur in duas partes æquas, & plano singuli anguli solidi icosaedri abscindantur per quinque laterum diuisiones ipsis angulis proximas; Reliquum solidum vocatur truncatum icosaedrum per laterum media.

NOTA.

Habet hoc corpus duodecim plana pentagona, & viginti triangularia, & triginta angulos solidos, & sexaginta latera.

NOTA.

Hoc corpus simile est truncato dodecaedro per laterum media præcedentis 19. definitionis.

Definitio 22.

Si omnia latera icosaedri diuidantur in tres partes æquas, & plano singuli anguli solidi icosaedri abscindantur per quinque laterum diuisiones ipsis angulis proximas; Reliquum solidum vocatur icosaedrum per laterum tertias.

NOTA.

Definition 20.

If all the edges of a dodecahedron are divided into three parts, in such a way that all the middle parts are to the two other parts of said edges as the chord of an arc of two-fifths of the circumference of a circle to the chord of an arc of one-fifth of the same circumference, and the solid angles of the dodecahedron are cut off by planes through those points of division of the three edges which are adjacent to said angles, the remaining solid is called a dodecahedron truncated through the division of the edges into three parts.

NOTE.

This solid has twelve decagonal and twenty triangular faces, sixty solid angles, and ninety edges.

Definition 21.

If all the edges of an icosahedron are divided into two equal parts, and the solid angles of the icosahedron are cut off by planes through those points of division of the five edges which are adjacent to said angles, the remaining solid is called an icosahedron truncated through the mid-points of the edges.

NOTE.

This solid has twelve pentagonal and twenty triangular faces, thirty solid angles, and sixty edges.

NOTE.

This solid is similar to a dodecahedron truncated through the mid-points of the edges, of the preceding 19th definition.

Definition 22.

If all the edges of an icosahedron are divided into three equal parts, and the solid angles of the icosahedron are cut off by planes through those points of division of the five edges which are adjacent to said angles, the remaining solid is called an icosahedron truncated through the third parts of the edges.

NOTA.

Habet hoc corpus viginti plana hexagona, & duodecim pentagona, & sexaginta angulos solidos, & nonaginta latera.

PROBLEMA .I

Dato maximo circulo sphæræ: latera quinque regularium corporum, quinque auctorum corporum, & nouem truncatorum corporum regularium, ipsi sphæræ inscriptibilium, invenire.

Explicatio dati.

Sit datus maximus circulus sphæræ A B C D *cuius diameter sit* A C *& centrum* E.

Explicatio quæsiti.

Oporteat invenire latera quinque regularium corporum, quinque auctorum corporum regularium, & nouem truncatorum regularium corporum, sphæræ, cuius A B C D *est maximus circulus, inscriptibilium.*

Constructio.

Distinctio I.

Abscindatur à recta E C *tertia pars ipsius vt* E F, *ducaturque recta* F G *perpendicularis ad rectam* E C, *& terminus* G *sit in circuli peripheria: ducatur deinde recta* A G *pro latere tetraedri.*

Distinctio 2.

Ducatur recta C G *pro latere cubi.*

Distinctio 3.

Ducatur semidiameter E B *perpendicularis ad* A C, *ducaturque recta* B C *pro latere octoedri.*

Distinctio 4.

Ducatur recta H C *æqualis rectæ* C A *efficiens angulum* H C A *rectum, duca-*

NOTE.

This solid has twenty hexagonal and twelve pentagonal faces, sixty solid angles, and ninety edges.

PROBLEM I.

Given the great circle of a sphere: to find the edges of the five regular solids, the five augmented solids, and the nine truncated regular solids that can be inscribed in said sphere.

Given.

Let the great circle *ABCD* of a sphere be given, whose diameter shall be *AC* and the centre *E.*

Required.

Let it be required to find the edges of the five regular solids, the five augmented regular solids, and the nine truncated regular solids that can be inscribed in the sphere, of which *ABCD* is the great circle.

Construction.

Section 1.

From the line *EC* cut off one third, *viz. EF*, and draw a line *FG* perpendicular to the line EC, and let the extremity *G* be on the circumference of the circle. Subsequently draw the line *AG* as the edge of the tetrahedron.

Section 2.

Draw the line *CG* as the edge of the cube.

Section 3.

Draw the semi-diameter *EB* perpendicular to *AC*, and draw the line *BC* as the edge of the octahedron.

Section 4.

Draw the line *HC* equal to the line *CA*, making the angle *HCA* right, and *ducaturque recta* E H *secans peripheriam ad* I, *ducaturque* I C *pro latere icosaedri.*

Distinctio 5.

Diuidatur per 30. *prop. lib.* 6. *Euclid.* G C *per extremam ac mediam rationem in* K *sitque maior pars* C K *pro latere dodecaedri.*

Distinctio 6.

Applicetur intervallum G A *à* G *in productam* A C *ad punctum* L, *ducaturque* G L, *&* E G, *&* C M, *parallela cum* G L *& terminus ipsius* M *in recta* E G, *eritque ipsa* C M *pro latere tetraedri aucti.*

Distinctio 7.

Ducatur recta E N *ad angulos rectos cum* A G, *secans* A G *in* O, *& peripheriam in* P, *& intervallum* G C *applicetur ab* A *in rectam* P N *nempe ad punctum* Q, *ducaturque* A Q, *& recta* P R *parallela cum* Q A, *& terminus eius* R *in recta* A E, *eritque ipsa* P R *pro latere aucti cubi.*

Distinctio 8.

Describatur triangulus aequilaterus cuius latus aequale sit ipsi B C, *sitque ipsius trianguli perpendicularis ab angulo in medium oppositi lateris recta* S, *applicetur que intervallum ipsius à* B *in rectam* E A *sitque ad* R, *ducaturq; recta* B R *ducatur item recta* E T *ad angulos rectos cum* R B, *secans rectam* R B *ad* V, *& peripheriam ad* X, *applicetur deinde intervallum* B C, *à* B *in rectam* X T *sitq; ad* Y, *ducaturq; recta* Y B, *& eius parallela* X Z, *sitq; terminus* Z *in recta* B E, *eritq; ipsa* X Z *pro latere aucti octoedri.*

Aut alio modo quod facilius & idem est (sed demonstrationis gratia quae infra sequentur, sunt antedicta descripta) accipiatur A O, *nempe medium rectae* A G *pro latere antedicti aucti octoedri.*

Distinctio 9.

Applicetur in dato circulo recta A I, *aequalis rectae* C K, *inveniatur deinde* draw the line *EH*, cutting the circumference at *I*, and draw *IC* as the edge of the icosahedron.

Section 5.

By the 30th proposition of Euclid's 6th book divide *GC* in extreme and mean ratio at *K*, and let the larger section *CK* be the edge of the dodecahedron.

Section 6.

Mark off the length *GA* from *G* on *AC* produced up to the point *L*, and draw *GL* and *EG*, and *CM* parallel to *GL*, with its extremity *M* on the line *EG*, then said *CM* will be the edge of the augmented tetrahedron.

Section 7.

Draw the line *EN* at right angles to *AG*, cutting *AG* at *O* and the circumference at *P*, and mark off the length *GC* from *A* towards the line *PN*, *viz.* at the point *Q*, and draw *AQ*, and the line *PR* parallel to *QA*, with its extremity *R* on the line *AE*, then the said *PR* will be the edge of the augmented cube.

Section 8.

Construct an equilateral triangle whose side shall be equal to the side *BC*, and let the perpendicular in the said triangle from an angle to the mid-point of the opposite side be the line *S*, and mark off its length from *B* towards the line *EA*, and let this be at *R*, and draw the line *BR*, and also the line *ET* at right angles to *RB*, cutting the line *RB* at *V* and the circumference at *X*; subsequently mark off the length *BC* from *B* towards the line *XT*, and let this be at *Y*, and draw the line *YB*, and its parallel *XZ*, and let the extremity *Z* lie on the line *BE*, then this line *XZ* will be the edge of the augmented octahedron.

Or in another way, which is easier and the same (but, for the sake of the proof which will follow below, the aforesaid things have been described), take *AO*, *viz.* one half of the line *AG*, as the edge of the aforesaid augmented octahedron.

Section 9.

In the given circle mark off the line *AI*, equal to the line *CK*, and subse-

inde per 12. *prop. lib.* 6. *Euclid. recta linea in ea ratione ad* K C, *vt est recta linea ab angulo pentagoni æquilateri & æquianguli in medium oppositi lateris ad latus eiusdem pentagoni sitque recta* 1, 2, *quo interuallo describatur centro* 1 *arcus circa* 2, *idemque interuallū applicetur à* C *in ipsum arcum ad* 2, *ducanturque rectæ* 1, 2, & 2 C & *recta* E 3 *secans peripheriam ad* 4, & *ad angulos rectos cum* C 2, *appliceturque interuallum* C K *ex* C *in rectam* 4, 3, *vtpote ad* 5 *ducaturque recta* C 5 & *recta* 4, 6, *parallela cum* C 5 *sitque terminus* 6 *in recta* F C *eritque ipsa* 4, 6, *pro latere aucti dodecaedri.*

Distinctio 10.

Applicetur in dato circulo recta A 7, *æqualis rectæ* C 1, *describatur deinde triangulus æquilaterus cuius latus æquale sit rectæ* C 1, & *recta eiusdem trianguli ab angulo in medium oppositi lateris sit* 8: *deinde interuallo rectæ* 8 *describatur centro* 7 *arcus circa* 9: *idemque interuallum applicetur à* C *in ipsum arcum ad* 9 *ducanturque rectæ* 7, 9 & 9 C, & *recta* E 10 *secans peripheriam ad* 11, & *angulos efficiens rectos cum recta* 7, 9: *Appliceturque interuallum* C 1, *à* 7 *in rectam* 11, 10 *nempe ad* 12, *ducaturque recta* 7, 12, & E 7, & 11, 13, *parallela cum* 12. 7, *sitque terminus* 13 *in recta* E 7, *eritq; ipsa recta* 11, 13 *pro latere aucti icosaedri.*

Distinctio 11.

Producatur E A *ad* 14 & *notetur tertia pars rectæ* A G, *sitque* A 15, *ducaturque recta* E 15, *quæ producatur in peripheriam ad* 16, *ducaturq; recta* 16, 17, *parallela cum* G A, *sitq; terminus* 17 *in recta* A 14 *eritque ipsa recta* 16, 17 *pro latere tetraedri truncati per laterum tertias.*

Distinctio 12.

Diuidatur recta G C *per* 10. *prop. lib.* 6. *Euclid. in tres partes hoc modo vt media pars* 18, 19, *ad vtramq; extremam partem eam habeat rationem quàm diagonalis quadrati ad latus eiusdem, tum ducatur recta* E 18, & *producatur in peripheriam ad punctum* 20, *similiter ducatur* E 19 & *producatur in peripheriam ad punctum* 21; *ducaturq; recta* 20, 21 *pro latere truncati cubi per laterum diuisiones in tres partes.*

quently, by the 12th proposition of Euclid's 6th book, find a line which is to *KC* in the same ratio as a line from an angle of an equilateral and equiangular pentagon to the mid-point of the opposite side to the side of the said pentagon, and let this line be 1,2; with this length describe from the centre 1 an arc about 2, and mark off the same length from *C* towards the said arc at 2, and draw the lines 1,2 and 2*C*, and the line *E*3, cutting the circumference at 4 and at right angles to *C*2, and mark off the length *CK* from *C* towards the line 4,3, *viz.* at 5, and draw the line *C*5 and the line 4,6 parallel to *C*5, and let its extremity 6 lie on the line *FC*; then this line 4,6 will be the edge of the augmented dodecahedron.

Section 10.

In a given circle mark off the line *A*7, equal to the line *CI*; subsequently construct the equilateral triangle whose side is equal to the line *CI*, and let the line in this triangle from an angle to the mid-point of the opposite side be 8; then with the length 8 describe an arc from the centre 7 in the neighbourhood of 9, and mark off the same length from *C* to the said arc at 9, and draw the lines 7,9 and 9*C*, and the line *E*10, cutting the circumference at 11, and making right angles with the line 7,9. And mark off the length *C*1 from 7 towards the line 11,10, *viz.* at 12, and draw the lines 7,12, and *E*7, and 11,13 parallel to 12,7, and let the extremity 13 lie on the line *E*7, then the said line 11,13 will be the edge of the augmented icosahedron.

Section 11.

Produce *EA* to 14 and mark off one third of the line *AG*, and let this be *A*15; draw the line *E*15, and produce it to the circumference at 16, and draw the line 16,17 parallel to *GA*, and let its extremity 17 lie on the line *A*14, then the said line 16,17 will be the edge of the tetrahedron truncated through the third parts of the edges.

Section 12.

By the 10th proposition of Euclid's 6th book, divide the line *GC* into three parts in such a way that the middle part 18,19 is to the extreme parts in the same ratio as the diagonal of a square to its side. Then draw the line *E*18 and produce it to the circumference at the point 20. Similarly draw *E*19 and produce it to the circumference at the point 21. And draw the line 20,21 as the edge of the cube truncated through the divisions of the edges into three parts.

Distinctio 13.

Diuidatur quadrans seu peripheria A B, *in duo æqualia in puncto* 22, *ducaturque* B 22: *Similiter* B 23 *æqualis* B 22 *efficiens angulum* E B 23 *rectum, ducaturq; recta* E 23 *secans peripheriam ad* 24, *ducatur item recta* 24, 25 *parallela cum* 23, B *sitq; terminus* 25 *in recta* B E, *eritque recta* 24, 25 *pro latere cubi bistruncati primi.*

Distinctio 14.

Describatur quadratum 26, 27, 28 *quodcunque, cuius latus sit* 26, 27, *diagonalis verò* 26, 28, *ducaturq; recta* 29, 30, *in qua notetur interuallum* 29, 31 *æquale rectæ* 26, 27: & *recta* 31, 32, *æqualis rectæ* 26, 27: & *recta* 32, 33, *æqualis rectæ* 26, 28: & *recta* 33, 34, *æqualis rectæ* 26, 27, & *recta* 34, 30 *æqualis rectæ* 26, 27, *ducaturque recta* 31, 35 *æqualis rectæ* 29, 30 *efficiens angulum* 30, 31, 35 *rectum: Ducatur recta* 35, 34, & *recta* 34, 36 *æqualis rectæ* 26, 28 *efficiens angulum* 36, 34, 35 *rectum: ducatur recta* 35, 36: *Applicetur que interuallum diametri dati circuli* A C, *à puncto* 35 *in rectum* 35, 36, *sitque recta* 35, 37 *ducaturq; recta* 37, 38 *parallela cum recta* 36, 34, *sitque punctum* 38 *in recta* 35, 34 *eritque recta* 37, 38 *pro latere bistruncati cubi secundi.*

Distinctio 15.

Semidiameter E B *est pro latere octoedri truncati per laterum media.*

Distinctio 16.

Producatur E B *ad* 39. *ducaturque ab* I *recta* I, 40 *parallela cum* C B, & *terminus* 40 *in recta* B 39, *eritque recta* I, 40 *pro latere octoedri truncati per laterum tertias.*

Distinctio 17.

Diuidatur recta A I *per* 10. *prop. lib.* 6. *Euclid. in tres partes tales vt media pars* 41, 42 *ad vtramque extremam partem eam habeat rationem quam chorda arcus duarum quintarum peripheriæ ad chordam arcus vnius quintæ: Ducaturque recta* E 42 *quæ producatur in peripheriam ad punctum*

Section 13.

Divide the quadrant or circumference *AB* into two equal parts at the point 22, and draw *B*22. Similarly *B*23, equal to *B*22, making the angle *EB*23 right, and draw the line *E*23, cutting the circumference at 24. Also draw the line 24,25 parallel to 23,*B*, and let its extremity 25 lie on the line *BE*. Then the line 24,25 will be the edge of the first twice-truncated cube.

Section 14.

Construct any square 26,27,28, whose side shall be 26,27, and the diagonal 26,28. And draw the line 29,30, on which mark off the length 29,31, equal to the line 26,27; and the line 31,32, equal to the line 26,27; and the line 32,33, equal to the line 26,28; and the line 33,34, equal to the line 26,27; and the line 34,30, equal to the line 26,27. And draw the line 31,35, equal to the line 29,30, making the angle 30,31,35 right. Draw the line 35,34 and the line 34,36, equal to the line 26,28, making the angle 36,34,35 right. Draw the line 35,36. And mark off the length of the diameter of the given circle *AC* from the point 35 on the line 35,36, and let this be the line 35,37; and draw the line 37,38, parallel to the line 36,34, and let the point 38 lie on the line 35,34. Then the line 37,38 will be the edge of the second twice-truncated cube.

Section 15.

The semi-diameter *EB* is the edge of the octahedron truncated through the mid-points of the edges.

Section 16.

Produce *EB* to 39, and from 1 draw the line 1,40, parallel to *CB*, and let its extremity 40 lie on the line *B*39; then the line 1,40 will be the edge of the octahedron truncated through the third parts of the edges.

Section 17.

By the 10th proposition of Euclid's 6th book, divide the line *A*1 into three parts such that the middle part 41,42 is to the extreme parts in the same ratio as the chord of an arc of two fifths of the circumference to the chord of an arc of one fifth. And draw the line *E*42; produce this to the circumference at the *punctum 43: Similiter ducatur* E 41 *& producatur in peripheriam ad punctum 44: ducatur deinde 43, 44 pro latere dodecaedri truncati per laterum diuisiones in tres partes.*

Distinctio 18.

Diuidatur I C *in duo æqualia in puncto 45, ducaturque recta* E 45 *quæ producatur in peripheriam ad punctum 46, ducaturq́ recta 46, 47 parallela cum* I C, *& terminus 47 in recta* C L, *eritq́ recta 46, 47 pro latere icosaedri truncati per laterum media.*

Distinctio 19.

Sumatur tertia pars rectæ C I, *vt* C 48, *ducatur recta* E 48 *quæ producatur vsq́ in peripheriam ad 49, ducaturque recta 49, 50 parallela cum* I C *& terminus 50 existens in recta* C L, *eritque recta 49, 50 pro latere icosaedri truncati per laterum tertias.*

Dico latera supra petita esse inventa vt supra in fine cuiuscunq́ distinctionis explicata sunt vt erat quæsitum.

NOTA.

In definitionibus præcedentibus 21. corpora sunt definita, hic verò tantum 19. constructa: ratio est quia cubus truncatus per laterum media & octoedrum truncatum per laterum media sunt similia, vt in ipsorum definitionibus notatum est, quare hæc duo corpora vnius tantum corporis constructione egent. Similiter & vna constructione egent truncatum dodecaedrum per laterum media & truncatum Icosaedrum per laterum media.

point 43. Similarly draw *E*41 and produce it to the circumference at the point 44. Subsequently draw 43,44 as the edge of the dodecahedron truncated through the divisions of the edges into three parts.

Section 18.

Divide *IC* into two equal parts at the point 45, and draw the line *E*45; produce this to the circumference at the point 46; and draw the line 46,47, parallel to *IC*, and let its extremity 47 lie on the line *CL*. Then the line 46,47 will be the edge of the icosahedron truncated through the mid-points of the edges.

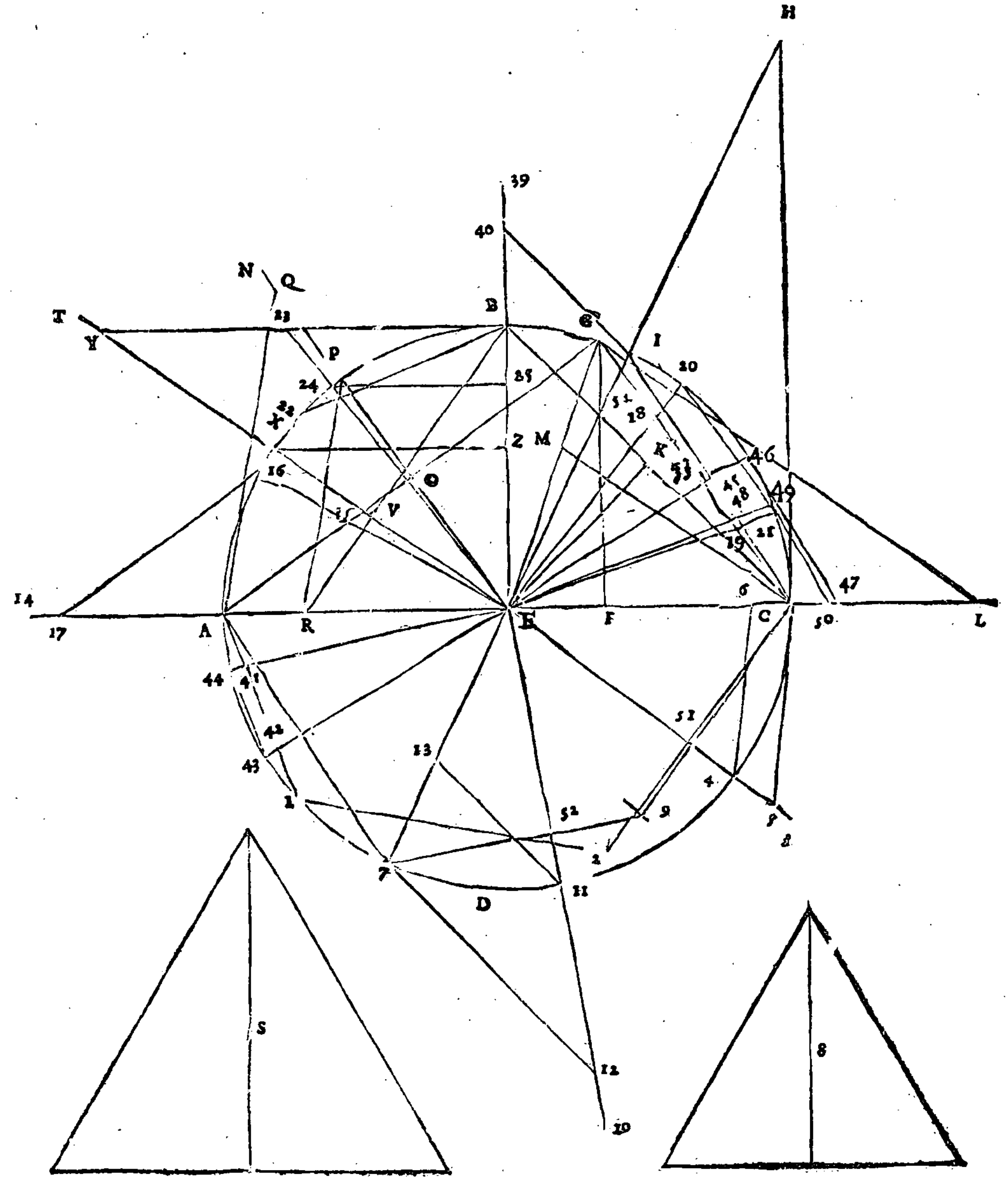
Section 19.

Take one third of the line *CI*, *viz.* *C*48. Draw the line *E*48; produce this to the circumference at 49, and draw the line 49,50, parallel to *IC*, with its extremity 50 lying on the line *CL*. Then the line 49,50 will be the edge of the icosahedron truncated through the third parts of the edges.

I say that the edges required above have been found as explained above at the end of each section; as was required.

NOTE.

In the preceding definitions 21 solids have been defined, but here only 19 have been constructed. The reason is that the cube truncated through the mid-points of the edges and the octahedron truncated through the mid-points of the edges are similar, as has been mentioned in their definitions, so that these two solids call for the construction of only one solid. Similarly only one construction also is called for in the case of the dodecahedron truncated through the mid-points of the edges and the icosahedron truncated through the mid-points of the edges.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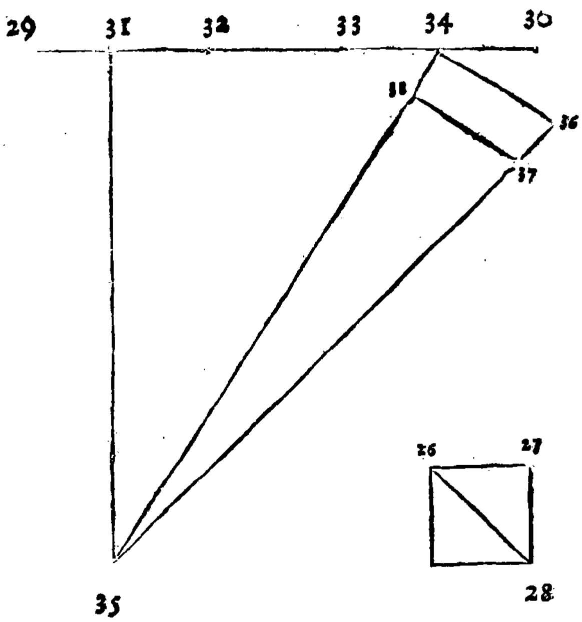


Demonstratio.

Distinctio 1.

Latera quinque regularium corporum ea esse quæ quinque prioribus distinctionibus constructionis explicata sunt, per 18. prop. lib. 13. Euclid. est manifestum.

Distinctio 2.

Quoniam A G *est latus tetraedri, &* E *ipsius centrum &* F *repræsentat centrum basis, sequitur rectam* A F *esse tetraedri perpendicularem seu altitudinem: Sed* A F *per constructionem æqualis est ipsi* F L, *quare* E L *est semidiameter aucti tetraedri cuius latus æquale est rectæ* L G: *Sed vt* E L *ad* L G *sic* E C *ad* C M *per 4. prop. lib. 6. Euclid. nam trianguli*

Proof.

Section 1.

That the edges of the five regular solids are those which have been described in the first five sections of the construction, is evident by the 18th proposition of Euclid's 13th book.

Section 2.

Since *AG* is the edge of a tetrahedron, and *E* its centre, and *F* represents the centre of the base, it follows that the line *AF* is the perpendicular or altitude of the tetrahedron. But by the construction *AF* is equal to the line *FL*, so that *EL* is the semi-diameter of the augmented tetrahedron, whose edge is equal to the line *LG*. But as *EL* is to *LG*, so is *EC* to *CM*, by the 4th proposition of Euclid's

ELG & ECM *sunt similes: Quare vt dictum est in constructione, distinctione* 6. *recta* CM *est latus aucti tetraedri, cuius circumscriptibilis sphæræ semidiameter est dati circuli semidiameter* EC.

Distinctio 3.

Quoniam AG *repræsentat cubi quadrati diagonalem, &* E *cubi centrum, erit recta* EO (*est autem* O *communis sectio linearum* AG & QE) *intervallum à centro cubi, ad centrum quadrati cubi, & quoniam* AQ *est latus trianguli triangulorum pyramidis supra quadratum cubi erectorum, erit* QO *ipsius pyramidis altitudo, quare tota* EQ *erit semidiameter aucti cubi cuius latus* AQ. *Vt verò* AQ *ad* QE *sic* RP *ad* PE *per* 4. *prop. lib.* 6. *Euclid.* (*sunt enim trianguli* AQE & RPE *similes*) *quare vt dictum est in constructione, distinct.* ~~text.~~ RP *est latus aucti cubi cuius circumscriptibilis sphæræ semidiameter est dati circuli semidiameter* EP.

Distinctio 4.

Quoniam BR *est perpendicularis trianguli octoedri per constructionem, & recta* EV *ad angulos rectos ipsi* BR, *erit* EV *intervallum à centro octoedri ad centrum sui trianguli: Et quia* BY *est latus triangulorum superpositæ pyramidis nempe æqualis ipsi* BC, *erit* YV *ipsius pyramidis altitudo, quare tota recta* EY *est talis aucti octoedri semidiameter, cuius latus* YB. *Sed vt* BY *ad* YE *sic* ZX *ad* XE *per* 4. *prop. lib.* 6. *Eucl.* (*nam trianguli* BYE & ZXE *sunt similes*) *quare vt dictum est in constructione, dist.* 8. *recta* XZ *est latus aucti octoedri cuius circumscriptibilis sphæra semidiameter est dati circuli semidiameter* EX.

Distinctio 5.

Quoniam recta C2 *est recta cadens ab angulo pentagoni dodecaedri in medium oppositi lateris eiusdem pentagoni, præterea recta* E51 *ad angulos rectos ipsi* C2, *erit punctum* 51 *centrum pentagoni dodecaedri, quare* E51 *erit intervallum à centro dodecaedri in centrum ipsius basis, est præterea re-*

cta

6th book, for the triangles *ELG* and *ECM* are similar; therefore, as has been said in the construction, section 6, the line *CM* is the edge of the augmented tetrahedron for which the semi-diameter of the sphere that can be circumscribed about it is the semi-diameter *EC* of the given circle.

Section 3.

Since *AG* represents the diagonal of the square of a cube and *E* the centre of the cube, the line *EO* (*O* being the point of intersection of the lines *AG* and *QE*) will be the distance from the centre of the cube to the centre of the square of the cube, and since *AQ* is the side of one of the triangles of the pyramid which have been erected on the square of the cube, *QO* will be the altitude of said pyramid; therefore the whole line *EQ* will be the semi-diameter of the augmented cube whose edge is *AQ*. But as *AQ* is to *QE*, so is *RP* to *PE*, by the 4th proposition of Euclid's 6th book (for the triangles *AQE* and *RPE* are similar); therefore, as has been said in the construction, third section, *RP* is the edge of the augmented cube for which the semi-diameter of the sphere that can be circumscribed about it is the semi-diameter *EP* of the given circle.

Section 4.

Since *BR* is the perpendicular of a triangle of an octahedron by the construction, and the line *EV* is at right angles to the said line *BR*, *EV* will be the distance from the centre of the octahedron to the centre of its triangle. And because *BY* is the side of the triangles of the superposed pyramid, *viz.* equal to the line *BC*, *YV* will be the altitude of the said pyramid; therefore the whole line *EY* is the semi-diameter of such an augmented octahedron, whose edge is *YB*. But as *BY* is to *YE*, so is *ZX* to *XE*, by the 4th proposition of Euclid's 6th book (for the triangles *BYE* and *ZXE* are similar); therefore, as has been said in the construction, section 8, the line *XZ* is the edge of the augmented octahedron, for which the semi-diameter of the sphere that can be circumscribed about it is the semi-diameter *EX* of the given circle.

Section 5.

Since the line *C*2 is a line falling from an angle of a pentagon of a dodecahedron to the mid-point of the opposite side of the same pentagon, and moreover the line *E*51 is at right angles to the said line *C*2, the point 51 will be the centre of the pentagon of the dodecahedron; therefore *E*51 will be the distance from the centre of the dodecahedron to the centre of its base; moreover the line *C*5 *cta* C 5 *latus trianguli additæ pyramidis, quare recta* 51, 5 *erit ipsius pyramidis altitudo, quare tota* E 5 *est talis aucti dodecaedri semidiameter. Sed vt recta* C 5 *ad rectam* 5 E, *sic recta* 6, 4. *ad rectam* 4 E *per* 4. *prop. lib.* 6. *Euclid.* (*nam trianguli* C 5 E *&* 6, 4, E *sunt similes*) *quare vt dictum est in constructione, distinct.* 9. *recta* 6, 4. *est latus aucti dodecaedri cuius circumscriptibilis sphæræ semidiameter est dati circuli semidiameter* E 4.

Distinctio 6.

Quoniam recta 7, 9 *est recta cadens ab angulo trianguli icosaedri in medium oppositi lateris eiusdem, præterea recta* E 5 2 *ad angulos rectos ipsi* 7, 9, *erit punctum* 52 *centrum trianguli seu basis icosaedri, quare recta* E 52 *erit intervallum à centro icosaedri in centrum ipsius basis: est præterea recta* 7, 12 *latus additæ pyramidis, quare recta* 52, 12 *erit ipsius pyramidis altitudo, quare tota* E 12 *talis aucti dodecaedri semidiameter. Sed vt* 7, 12 *ad* 12 E, *sic recta* 13, 11 *ad rectam* 11 E *per* 4. *prop. lib.* 6. *Euclid.* (*nam trianguli* 7, 12 E *&* 13, 11 E *sunt similes*) *quare vt dictum est in constructione dist.* 9. *recta* 13, 11 *est latus aucti icosaedri cuius circumscriptibilis sphæræ semidiameter est dati circuli semidiameter* E 11.

Distinctio 7.

Quoniam A 15 *est tertia pars lateris* A G *tetraedri, erit* E 15 *semidiameter circumscriptibilis sphæræ truncati tetraedri cuius latus* 15 A. *Sed vt recta* 15 A *ad rectam* A E, *sic recta* 16, 17 *ad rectam* 17 E *per* 4. *prop. lib.* 6. *Euclid.* (*nam trianguli* 15 A E *&* 16, 17, E *sunt similes*) *quare vt dictum est in constructione dist.* 11, *recta* 16, 17 *est latus truncati tetraedri per laterum tertias, cuius circumscriptibilis sphæræ semidiameter est dati circuli semidiameter* 16 E.

Distinctio 8.

Quoniam recta 19, 18 *est pars lateris cubi respondens lateri ipsius truncati* is the side of the triangle of the added pyramid; therefore the line 51, 5 will be the altitude of the said pyramid; therefore the whole line *E*5 is the semi-diameter of such an augmented dodecahedron. But as the line *C*5 is to the line 5*E*, so is the line 6,4 to the line 4*E*, by the 4th proposition of Euclid's 6th book (for the triangles *C*5*E* and 6,4*E* are similar); therefore, as has been said in the construction, section 9, the line 6,4 is the edge of the augmented dodecahedron for which the semi-diameter of the sphere that can be circumscribed about it is the semi-diameter *E*4 of the given circle.

Section 6.

Since the line 7,9 is a line falling from an angle of a triangle of an icosahedron to the mid-point of its opposite side, and moreover the line *E*52 is at right angles to the said line 7,9, the point 52 will be the centre of the triangle or the base of the icosahedron; therefore, the line *E*52 will be the distance from the centre of the icosahedron to the centre of its base; moreover the line 7,12 is the side of the added pyramid; therefore the line 52, 12 will be the altitude of the said pyramid; therefore the whole line *E*12 will be the semi-diameter of such an augmented dodecahedron. But as 7,12 is to 12*E*, so is the line 13,11 to the line 11*E*, by the 4th proposition of Euclid's 6th book (for the triangles 7,12*E* and 13,11*E* are similar); therefore, as has been said in the construction, section 9, the line 13, 11 is the edge of the augmented icosahedron for which the semi-diameter of the sphere that can be circumscribed about it is the semi-diameter *E*11 of the given circle.

Section 7.

Since *A*15 is one third of the edge *AG* of a tetrahedron, *E*15 will be the semi-diameter of the sphere that can be circumscribed about the truncated tetrahedron whose edge is 15*A*. But as the line 15*A* is to the line *AE*, so is the line 16,17 to the line 17*E*, by the 4th proposition of Euclid's 6th book (for the triangles 15*AE* and 16,17,*E* are similar); therefore, as has been said in the construction, section 11, the line 16,17 is the edge of the tetrahedron truncated through the third parts of its edges for which the semi-diameter of the sphere that can be circumscribed about it is the semi-diameter 16*E* of the given circle.

Section 8.

Since the line 19,18 is a part of the edge of a cube corresponding to the edge *cati cubi per laterum diuisiones in tres partes, erit* 18 E *semidiameter ipsius truncati cubi cuius latus est recta* 19,18. *Sed vt recta* 19,18 *ad rectam* 18 E, *sic recta* 21, 20 *ad rectam* 20 E *per* 4. *prop. lib.* 6. *Euclid.* (*nam trianguli* 19, 18, E *&* 21, 20, E *sunt similes*) *quare vt dictum est in constructione, distinct.* 12. *recta* 21, 20 *est latus truncati cubi per laterum diuisiones in tres partes cuius circumscriptibilis sphæræ semidiameter est dati circuli semidiameter* 20 E.

Distinctio 9.

Quoniam B 23 *est latus cubi bistruncati primi cuius semidiameter est* 23 E (*hoc autem petimus hic breuitatis gratia concedi cùm primò aspectu in solido corpore sit manifestum*) *erit recta* 25, 24 *latus cubi bistruncati primi cuius semidiameter* 24 E: *nam vt recta* B 23 *ad rectam* 23 E *sic recta* 25, 24 *ad rectam* 24 E *per* 4. *prop. lib.* 6. *Euclid.* (*sunt autem trianguli* B 23 E *&* 25, 24 E *similes*) *quare vt dictum est in constructione, dist.* 13. *recta* 24, 25 *est latus cubi bistruncati primi cuius circumscriptibilis sphæræ semidiameter dati circuli est semidiameter* 24 E.

Distinctio 10.

Quoniam recta 34, 36 *est linea correspondens lateri bistruncati cubi secundi cuius circumscriptibilis sphæræ diameter est recta* 36, 35 (*hoc autem petimus hic breuitatis gratia concedi cùm primò aspectu in solido corpore sit manifestum*) *erit recta* 38, 37 *linea correspondens lateri bistruncati cubi secundi cuius circumscriptibilis sphæræ diameter est recta* 35, 37, *nam vt recta* 38, 37 *ad rectam* 37, 35 *sic recta* 34, 36, *ad rectam* 36, 35 *per* 4. *prop. lib.* 6. *Euclid.* (*sunt autem trianguli* 34, 36, 35, *&* 38, 37, 35, *similes*) *sed recta* 37, 35 *æqualis est diametro* A C, *quare vt dictum est in constructione, dist.* 14, *recta* 38, 37 *est latus bistruncati cubi secundi cuius circumscriptibilis sphæræ diameter est dati circuli diameter* A C.

Distinctio 11.

Diuidatur (*demonstrationis gratia*) B C *hoc est latus octoedri, in duo æqua-*

of the said cube truncated through the divisions of the edges into three parts, 18*E* will be the semi-diameter of the said truncated cube whose edge is the line 19,18. But as the line 19,18 is to the line 18*E*, so is the line 21,20 to the line 20*E*, by the 4th proposition of Euclid's 6th book (for the triangles 19,18,*E* and 21,20, *E* are similar); therefore, as has been said in the construction, section 12, the line 21,20 is the edge of the cube truncated through the divisions of the edges into three parts for which the semi-diameter of the sphere that can be circumscribed about it is the semi-diameter 20*E* of the given circle.

Section 9.

Since *B*23 is the edge of the first twice-truncated cube, whose semi-diameter is 23*E* (this we ask the reader to concede us here for brevity's sake, because it is evident at the first glance in a solid), the line 25,24 will be the edge of the first twice-truncated cube whose semi-diameter is 24*E*. For as the line *B*23 is to the line 23*E*, so is the line 25,24 to the line 24*E*, by the 4th proposition of Euclid's 6th book (for the triangles *B*23*E* and 25,24*E* are similar); therefore, as has been said in the construction, section 13, the line 24,25 is the edge of the first twice-truncated cube for which the semi-diameter of the sphere that can be circumscribed about it is the semi-diameter 24*E* of the given circle.

Section 10.

Since the line 34,36 is a line corresponding to the edge of the second twice-truncated cube for which the diameter of the sphere that can be circumscribed about it is the line 36,35 (this we ask the reader to concede us here for brevity's sake, because it is evident at the first glance in a solid), the line 38,37 will be a line corresponding to the edge of the second twice-truncated cube for which the diameter of the sphere that can be circumscribed about it is the line 35,37; for as the line 38,37 is to the line 37,35, so is the line 34,36 to the line 36,35, by the 4th proposition of Euclid's 6th book (for the triangles 34,36,35 and 38,37,35 are similar); but the line 37,35 is equal to the diameter *AC*; therefore, as has been said in the construction, section 14, the line 38,37 is the edge of the second twice-truncated cube for which the diameter of the sphere that can be circumscribed about it is the diameter *AC* of the given circle.

Section 11.

(For the sake of the proof) divide *BC*, *i.e.* the edge of an octahedron, into *æqualia ad punctum 53 ducaturque* E 53. *Igitur triangulus* B 53 E *est isosceles per 8. prop. lib. 6. (nam* E 53 *est perpendicularis ad rectam* B C *in trianglou rectangulo isoscele* B E C *quare* E 53 B *similis triangulo* B E C *est isosceles) cuius latera* E 53 & 53 B *sunt inter se æqualia. Quare (quoniam* B 53 *repræsentat latus truncati octoedri per laterum media cuius semidiameter circumscriptibilis sphæræ est* E 53) *latus truncati octoedri per laterum media, æqualis est suæ circumscriptibilis sphæræ semidiametro. Sed in hac propositione est circumscriptibilis sphæræ semidiameter* E B, *quare vt dictum est in constructione, dist.* 15. *recta* B E *est latus octoedri truncati per laterũ media cuius circumscriptibilis sphæræ semidiameter est dati circuli semidiameter* E B.

Distinctio 12.

Notetur demonstrationis gratia communis sectio rectarum B C & F G *hac nota* 54. *Igitur recta* B 54 *est tertia pars lateris octoedri* B C *(nam per* 4. *prop. lib.* 6. *Euclid. vt recta* E F *ad rectam* F C: *Sic recta* B 54 *ad rectam* 54 C, *quia triangulus* E C B *similis est triangulo* F C 54, & *recta* E F *est tertia pars rectæ* E C *per primam distinctionem constructionis, quare* B 54 *est tertia pars rectæ* B C) *quare recta* B 54 *æqualis est lateri truncati octoedri per laterum tertias, cuius semidiameter* E 54, *sed vt recta* B 54 *ad rectam* 54 E, *sic recta* 40, I *ad rectam* I E *per* 4. *prop. lib.* 6, *Euclid. (nam trianguli* B 54 E & 40 I E *sunt similes) quare vt dictum est in constructione, dist.* 16, *recta* 40 I *est latus truncati octoedri per laterum tertias, cuius circumscriptibilis sphæræ semidiameter est dati circuli semidiameter* E I.

Distinctio 13.

Quoniam recta 41, 42 *est pars correspondens lateri truncati dodecaedri per laterum diuisiones in tres partes, cuius circumscriptibilis sphæræ semidiameter est recta* E 41, *erit recta* 44, 43 *pars respondens lateri truncati dodecaedri per laterum diuisiones in tres partes cuius circumscriptibilis sphæræ semidiameter est* E 44, *Nam vt recta* E 41 *ad rectam* 41, 42, *sic recta* E 44, *ad rectam* 44, 43 *per* 4. *prop. lib.* 6. *Euclid. (sunt enim trianguli* E 41, 42 & E 44, 43 *similes) quare vt dictum est in constructione, dist.* 17, *recta* 41, 43 *est latus dodecaedri truncati per laterum diuisiones in tres partes, cuius circumscriptibilis sphæræ semidiameter est dati circuli semidiameter* E 44.

two equal parts at the point 53, and draw *E*53. Therefore the triangle *B*53*E* is isosceles, by the 8th proposition of the 6th book (for *E*53 is the perpendicular to the line *BC* in the right-angled isosceles triangle *BEC*; therefore *E*53*B*, similar to the triangle *BEC*, is isosceles), while its sides *E*53 and 53*B* are equal to one another. Therefore (since *B*53 represents the edge of an octahedron truncated through the mid-points of the edges for which the semi-diameter of the sphere that can be circumscribed about it is *E*53) the edge of the octahedron truncated through the mid-points of the edges is equal to the semi-diameter of the sphere that can be circumscribed about it. But in this proposition the semi-diameter of the sphere that can be circumscribed about it is *EB*; therefore, as has been said in the construction, section 15, the line *BE* is the edge of the octahedron truncated through the mid-points of the edges for which the semi-diameter of the sphere that can be circumscribed about it is the semi-diameter *EB* of the given circle.

Section 12.

For the sake of the proof mark the point of intersection of the lines *BC* and *FG* by the mark 54. Therefore the line *B*54 is one third of the edge of the octahedron *BC* (for, by the 4th proposition of Euclid's 6th book, as the line *EF* is to the line *FC*, so is the line *B*54 to the line 54*C*, because the triangle *ECB* is similar to the triangle *FC*54, and the line *EF* is one third of the line *EC* by the first section of the construction; therefore *B*54 is one third of the line *BC*); therefore the line *B*54 is equal to the edge of the octahedron truncated through the third parts of the edges, whose semi-diameter is *E*54, but as the line *B*54 is to the line 54*E*, so is the line 40,*I* to the line *IE*, by the 4th proposition of Euclid's 6th book (for the triangles *B*54*E* and 40*IE* are similar); therefore, as has been said in the construction, section 16, the line 40*I* is the edge of the octahedron truncated through the third parts of the edges for which the semi-diameter of the sphere than can be circumscribed about it is the semi-diameter *EI* of the given circle.

Distinctio 14.

Quoniam recta C 45 *est linea correspondens lateri truncati icosaedri per laterum media, cuius circumscriptibilis sphæræ semidiameter est* 45 E, *erit recta* 47,46 *linea correspondens lateri truncati icosaedri per laterum media cuius circumscriptibilis sphæræ semidiameter est recta* 46, E. *Nam vt recta* C 45 *ad rectam* 45 E, *sic recta* 47, 46. *ad rectam* 46 E *per* 4. *prop. lib.* 6. *Euclid.* (*sunt enim trianguli* C 45 E *&* 47, 46 E *similes*) *quare vt dictum est in constructione dist.* 18, *recta* 47,46 *est latus truncati icosaedri, per laterum media, cuius circumscriptibilis sphæræ semidiameter est dati circuli semidiameter* 46 E.

Distinctio 15.

Quoniam recta C 48 *est linea correspondens lateri truncati icosaedri per laterum tertias, cuius circumscriptibilis sphæræ semidiameter est* 48 E, *erit recta* 50, 49 *linea correspondens lateri truncati icosaedri per laterum tertias, cuius circumscriptibilis sphæræ semidiameter est* 49 E, *nam vt recta* C 48 *ad rectam* 48 E *sic recta* 50, 49 *ad rectam* 49 E *per* 4. *prop. lib.* 6. *Euclid.* (*sunt enim trianguli* C 48 E *&* 50,49 E *similes*) *quare vt dictum est in constructione dist.* 19. *recta* 50, 49 *est latus truncati icosaedri per laterum tertias cuius circumscriptibilis sphæræ semidiameter est dati circuli semidiameter* 49 E.

Conclusio.

Igitur dato maximo circulo sphæræ latera quinque &c. Quod erat faciendum.

PROBLEMA II.

Datis lateribus quinque corporum regularium, & quinque auctorum regularium corporum, & nouem truncatorū regularium corporum, eidem sphæræ inscriptibilium, plana construere ac disponere, quæ si ritè complicentur efficiant ipsa corpora.

Expli-

Section 13.

Since the line 41,42 is a part corresponding to the edge of the dodecahedron truncated through the divisions of the edges into three parts for which the semi-diameter of the sphere that can be circumscribed about it is the line *E*41, the line 44,43 will be the part corresponding to the edge of the dodecahedron truncated through the divisions of the edges into three parts for which the semi-diameter of the sphere that can be circumscribed about it is *E*44, for as the line *E*41 is to the line 41,42, so is the line *E*44 to the line 44,43, by the 4th proposition of Euclid's 6th book (for the triangles *E*41,42 and *E*44,43 are similar); therefore, as has been said in the construction, section 17, the line 41,43 is the edge of the dodecahedron truncated through the divisions of the edges into three parts for which the semi-diameter of the sphere that can be circumscribed about it is the semi-diameter *E*44 of the given circle.

Section 14.

Since the line *C*45 is a line corresponding to the edge of the icosahedron truncated through the mid-points of the edges for which the semi-diameter of the sphere that can be circumscribed about it is 45*E*, the line 47,46 will be the line corresponding to the edge of the icosahedron truncated through the mid-points of the edges for which the semi-diameter of the sphere that can be circumscribed about it is the line 46,*E*. For as the line *C*45 is to the line 45*E*, so is the line 47,46 to the line 46*E*, by the 4th proposition of Euclid's 6th book (for the triangles *C*45*E* and 47,46*E* are similar); therefore, as has been said in the construction, section 18, the line 47,46 is the edge of the icosahedron truncated through the mid-points of the edges for which the semi-diameter of the sphere that can be circumscribed about it is the semi-diameter 46*E* of the given circle.

Section 15.

Since the line *C*48 is a line corresponding to the edge of the icosahedron truncated through the third parts of the edges for which the semi-diameter of the sphere that can be circumscribed about it is 48*E*, the line 50,49 will be the line corresponding to the edge of the icosahedron truncated through the third parts of the edges for which the semi-diameter of the sphere that can be circumscribed about it is 49*E*, for as the line *C*48 is to the line 48*E*, so is the line 50,49 to the line 49*E*, by the 4th proposition of Euclid's 6th book (for the triangles *C*48*E* and 50,49*E* are similar); therefore, as has been said in the construction, section 19, the line 50,49 is the edge of the icosahedron truncated through the third parts of the edges for which the semi-diameter of the sphere that can be circumscribed about it is the semi-diameter 49*E* of the given circle.

Conclusion.

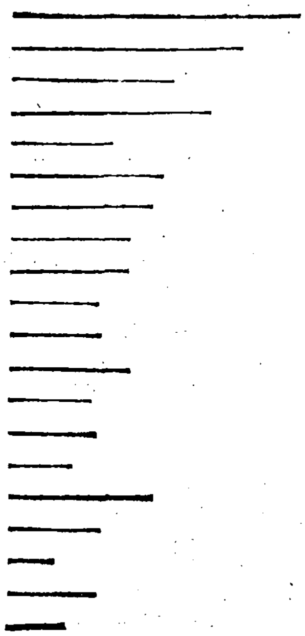
Therefore, given the great circle of a sphere: the edges of the five, etc. Which was to be performed.

PROBLEM II.

Given the edges of the five regular solids and the five augmented regular solids and the nine truncated regular solids that can be inscribed in the same sphere, to construct and dispose plane figures which, if properly folded together, form said solids.

Explicatio dati.

Sint data latera antedictorum corporum (inventa per primum præcedens Problema) eidem sphæræ inscriptibilium talia:



A	*Diameter circumscriptibilis sphæræ.*
B	*Latus tetraedri.*
C	*Cubi.*
D	*Octoedri.*
E	*Dodecaedri.*
F	*Icosaedri.*
G	*Aucti tetraedri.*
H	*Aucti cubi.*
I	*Aucti octoedri.*
K	*Aucti dodecaedri.*
L	*Aucti icosaedri.*
M	*Truncati tetraedri.*
N	*Trũcati cubi per laterũ divisio. in tres partes.*
O	*Bistruncati cubi primi.*
P	*Bistruncati cubi secundi.*
Q	*Truncati octoedri per laterum media.*
R	*Truncati octoedri per laterum tertias.*
S	*Truncati dodecaedri per laterũ divis. in tres (partes.*
T	*Trũcati icosaedri per laterũ media.*
V	*Truncati icosaedri per laterum tertias.*

Explicatio quæsiti.

Oporteat ex datis illis lineis constructa plana, disponere quæ si rite complicentur efficiant antedicta corpora eidem sphæræ (cuius diameter æqualis sit rectæ A*) inscriptibilia.*

Given.

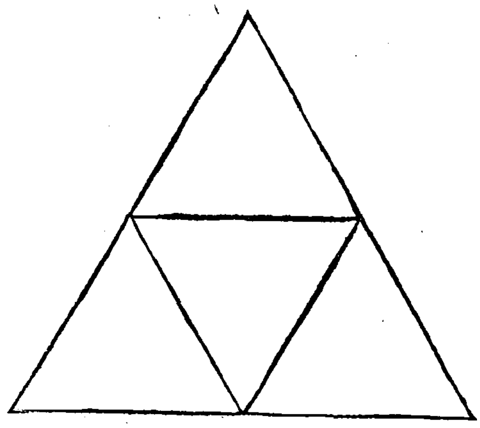
Let the edges of the aforesaid solids (found by the first preceding Problem) that can be inscribed in the same sphere be given, as follows:

A	the diameter of the sphere that can be circumscribed about the figures
B	the edge of the tetrahedron
C	of the cube
D	of the octahedron
E	of the dodecahedron
F	of the icosahedron
G	of the augmented tetrahedron
H	of the augmented cube
I	of the augmented octahedron
K	of the augmented dodecahedron
L	of the augmented icosahedron
M	of the truncated tetrahedron
N	of the cube truncated through the divisions of the edges into three parts
O	of the first twice-truncated cube
P	of the second twice-truncated cube
Q	of the octahedron truncated through the mid-points of the edges
R	of the octahedron truncated through the third parts of the edges
S	of the dodecahedron truncated through the divisions of the edges into three parts
T	of the icosahedron truncated through the mid-points of the edges
V	of the icosahedron truncated through the third parts of the edges

Required.

Let it be required to dispose the plane figures constructed from these given lines in such a way that, when properly folded together, they form the aforesaid solids that can be inscribed in the same sphere (whose diameter shall be equal to the line *A*).

Constructio.

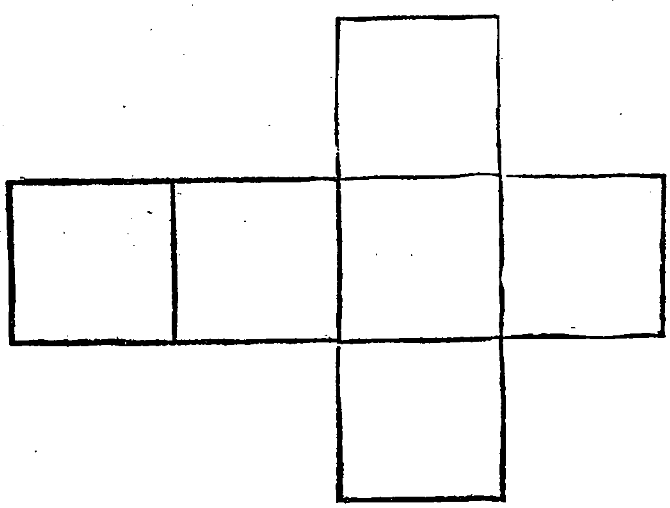


Distinctio 1.

Ex apta quadam plicabili materia disponantur vt infra pro tetraedro, quatuor trianguli quorum singula latera æqualia sint rectæ B.

Distinctio 2.

Disponantur vt infra pro cubo sex quadrata quorum singula latera æqualia sint rectæ C.



Construction.

Section 1.

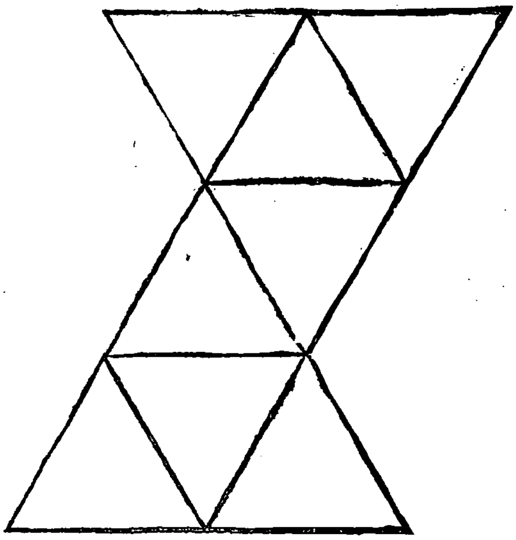
In order to form the tetrahedron dispose from some suitable foldable material, as shown below, four triangles, each of whose sides be equal to the line *B*.

Section 2.

In order to form the cube dispose, as shown below, six squares, each of whose sides be equal to the line *C*.

Diſtinctio 3.

Diſponantur vt infra pro octoedro, octo trianguli quorum ſingula latera æqualia ſint rectæ D.

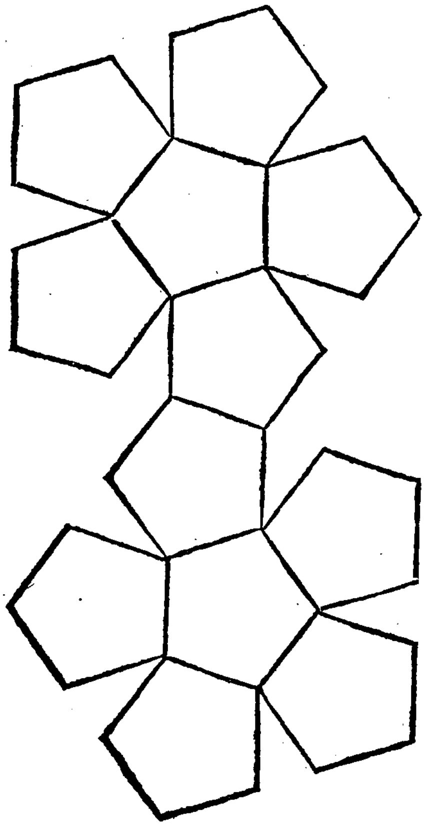


Section 3.

In order to form the octahedron dispose, as shown below, eight triangles, each of whose sides be equal to the line D.

Distinctio 4.

Disponantur vt infra pro dodecaedro 12 pentagona quorum singula latera æqualia sint rectæ E.

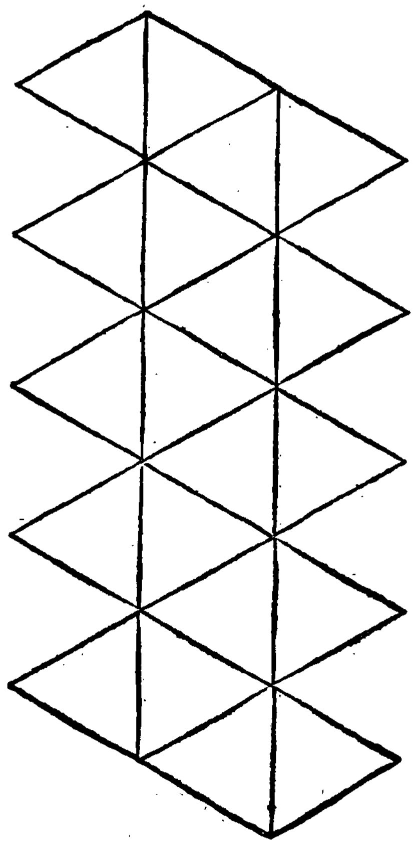


Section 4.

In order to form the dodecahedron dispose, as shown below, 12 pentagons, each of whose sides be equal to the line *E*.

Distinctio 5.

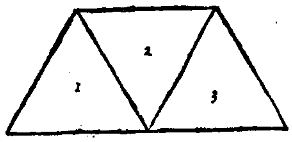
Disponantur ut infra pro Icosaedro viginti trianguli quorum singula latera æqualia sint rectæ F.



Section 5.

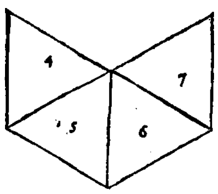
In order to form the icosahedron dispose, as shown below, twenty triangles, each of whose sides be equal to the line *F*.

Diſtinctio 6.



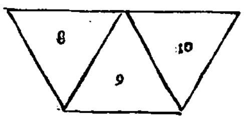
Diſponantur pro aucto tetraedro quatuor trianguli vt in præcedenti prima diſtinctione quorum ſingula latera æqualia ſint rectæ G. *Deinde quater tres trianguli vt ſunt tres trianguli* 1, 2, 3, *quorum ſingula latera æqualia ſint ipſi* G.

Diſtinctio 7.



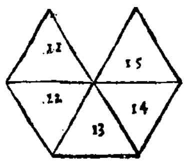
Diſponantur pro aucto cubo ſex quadrata, vt in præcedenti ſecunda diſtinctione quorum ſingula latera æqualia ſint rectæ H. Deinde *ſexies quatuor trianguli vt ſunt* 4, 5, 6, 7, *quorum ſingula latera æqualia ſint ipſi* H.

Diſtinctio 8.



Diſponantur pro aucto octoedro octo trianguli vt in præcedenti tertia diſtinctione, quorum ſingula latera æqualia ſint rectæ I. Deinde *octies tres trianguli vt ſunt tres trianguli* 8, 9, 10, *quorum ſingula latera æqualia ſint ipſi* I.

Diſtinctio 9.



Diſponantur pro aucto dodecaedro duodecim pentagona vt in præcedenti quarta diſtinctione, quorum ſingula latera æqualia ſint rectæ K: Deinde *duodecies quinque trianguli vt ſunt quinque trianguli* 11, 12, 13, 14, 15, *quorum ſingula latera æqualia ſint ipſi* K.

Section 6.

In order to form the augmented tetrahedron, dispose four triangles, as in the preceding first section, each of whose sides be equal to the line *G*. Subsequently four times three triangles such as the three triangles 1, 2, 3, each of whose sides be equal to the said *G*.

Section 7.

In order to form the augmented cube, dispose six squares, as in the preceding second section, each of whose sides be equal to the line *H*. Subsequently six times four triangles such as 4, 5, 6, 7, each of whose sides be equal to the said *H*.

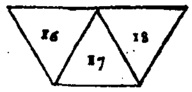
Section 8.

In order to form the augmented octahedron, dispose eight triangles as in the preceding third section, each of whose sides be equal to the line *I*. Subsequently eight times three triangles such as the three triangles 8, 9, 10, each of whose sides be equal to the said *I*.

Section 9.

In order to form the augmented dodecahedron, dispose twelve pentagons as in the preceding fourth section, each of whose sides be equal to the line *K*. Subsequently twelve times five triangles such as the five triangles 11, 12, 13, 14, 15, each of whose sides be equal to the said *K*.

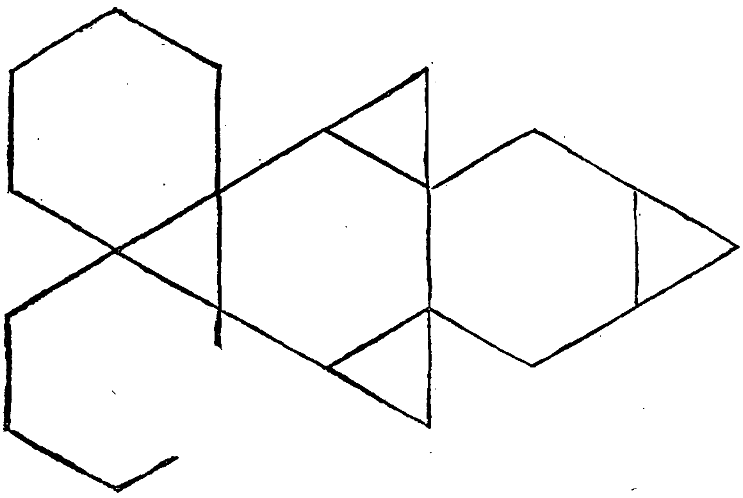
Distinctio 10.



Disponantur pro aucto icosaedro viginti trianguli vt in praecedenti qninta dist. quorum singula latera aequalia sint rectae L. *Deinde vicies tres trianguli vt sunt tres trianguli* 16, 17, 18, *quorum singula latera aequalia sint ipsi* L.

Distinctio 11.

Disponantur vt infra pro truncato tetraedro per laterum tertias, quatuor hexagona, & quatuor trianguli quorum singula latera aequalia sint rectae M.



Section 10.

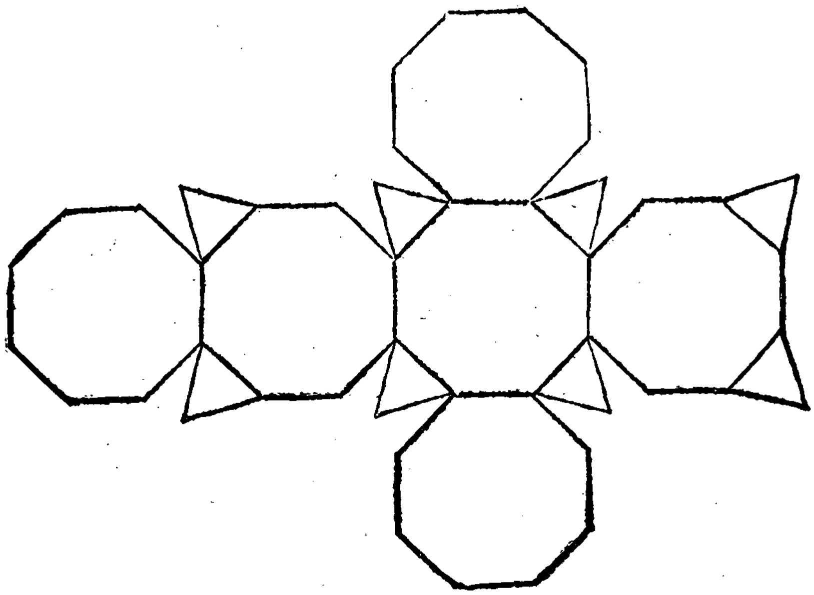
In order to form the augmented icosahedron, dispose twenty triangles as in the preceding fifth section, each of whose sides be equal to the line *L*. Subsequently twenty times three triangles such as the three triangles 16, 17, 18, each of whose sides be equal to the said *L*.

Section 11.

In order to form the tetrahedron truncated through the third parts of the edges, dispose, as shown below, four hexagons and four triangles, each of whose sides be equal to the line *M*.

Distinctio 12.

Disponantur vt infra pro truncato cubo per laterum diuisiones in tres partes, sex octogona, & octo trianguli, quorum singula latera æqualia sint rectæ N.

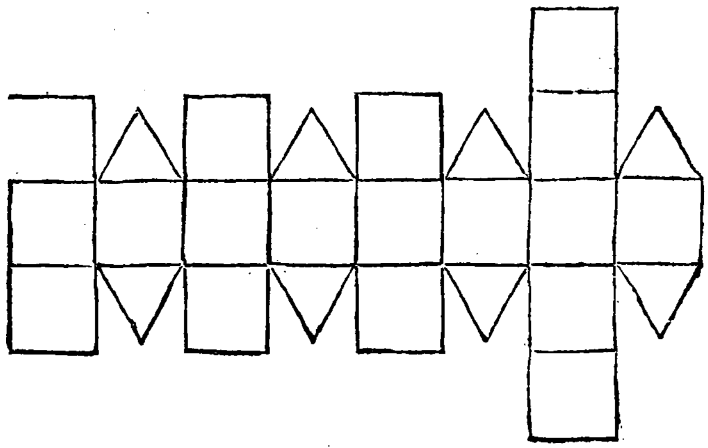


Section 12.

In order to form the cube truncated through the divisions of the edges into three parts, dispose, as shown below, six octagons and eight triangles, each of whose sides be equal to the line *N*.

Distinctio 13.

Disponantur vt infra pro bistruncato cubo primo, octodecim quadrata, & octo trianguli quorum singula latera æqualia sint rectæ O.

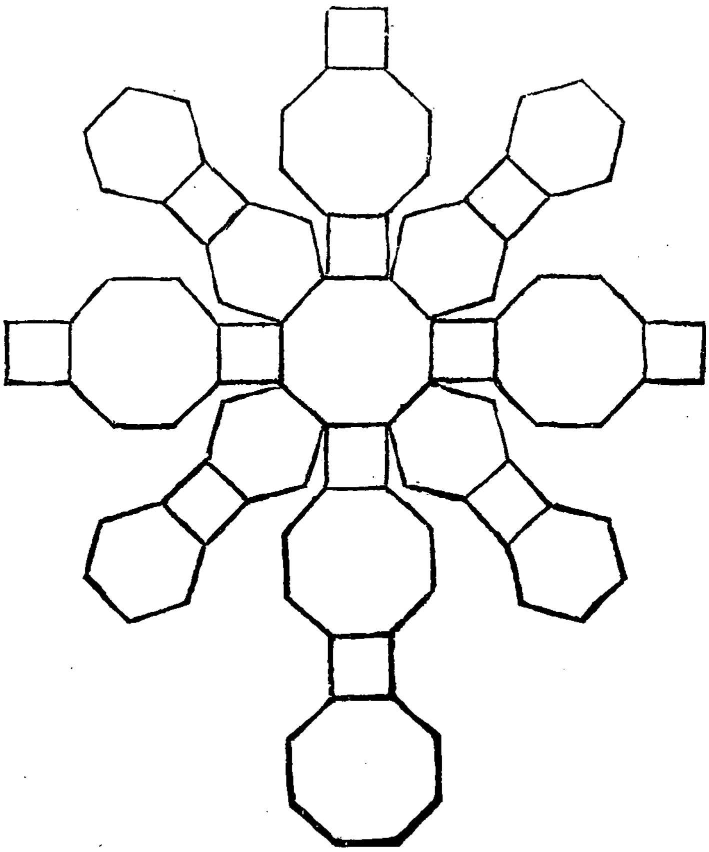


Section 13.

In order to form the first twice-truncated cube, dispose, as shown below, eighteen squares and eight triangles, each of whose sides be equal to the line *O*.

Diſtinctio 14.

Diſponantur vt infra pro biſtruncato cubo ſecundo ſex octogona, octo hexagona, & duodecim quadrata, quorum ſingula latera æqualia ſint rectæ P.

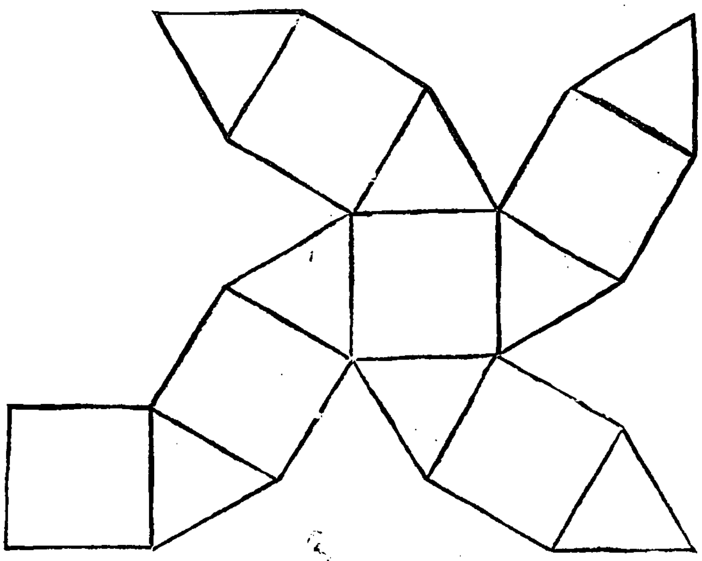


Section 14.

In order to form the second twice-truncated cube, dispose, as shown below, six octagons, eight hexagons, and twelve squares, each of whose sides be equal to the line P.

Distinctio 15.

Disponantur vt infra pro truncato Octoedro per laterum media, sex quadrata, & octo trianguli, quorum singula latera æqualia sint recta Q.

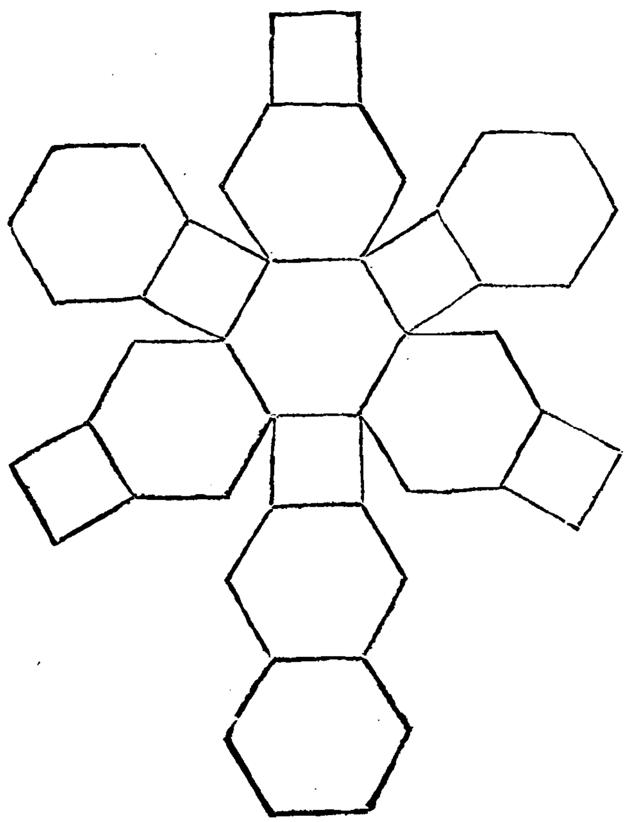


Section 15.

In order to form the Octahedron truncated through the mid-points of the edges, dispose, as shown below, six squares and eight triangles, each of whose sides be equal to the line Q.

Distinctio 16.

Disponantur vt infra pro truncato octoedro per laterum tertias, sex quadrata, & octo hexagona, quorum singula latera æqualia sint rectæ R.

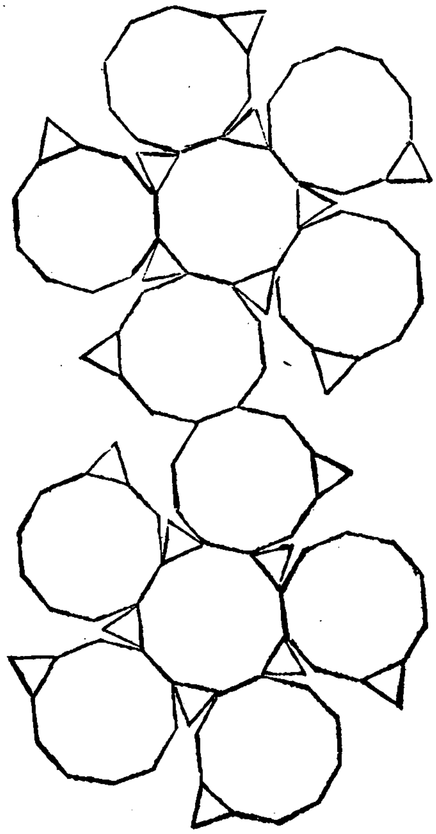


Section 16.

In order to form the octahedron truncated through the third parts of the edges, dispose, as shown below, six squares and eight hexagons, each of whose sides be equal to the line R.

Distinctio 17.

Disponantur ut infra pro truncato dodecaedro per laterum diuisiones in tres partes duodecim decogona, & viginti trianguli quorum singula latera æqualia sint recta 8.

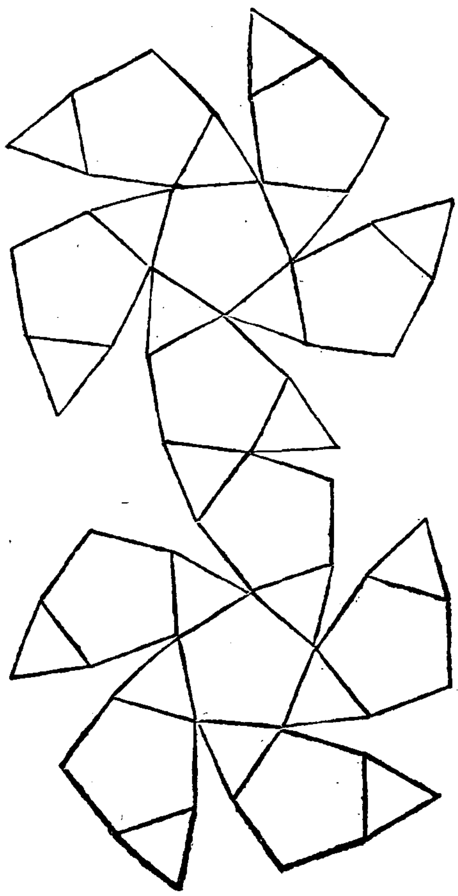


Section 17.

In order to form the dodecahedron truncated through the divisions of the edges into three parts, dispose, as shown below, twelve decagons and twenty triangles, each of whose sides be equal to the line *S*.

Distinctio 18.

Disponantur vt infra pro truncato icosaedro per laterum media duodecim pentagona, & viginti trianguli, quorum singula latera æqualia sint rectæ T.

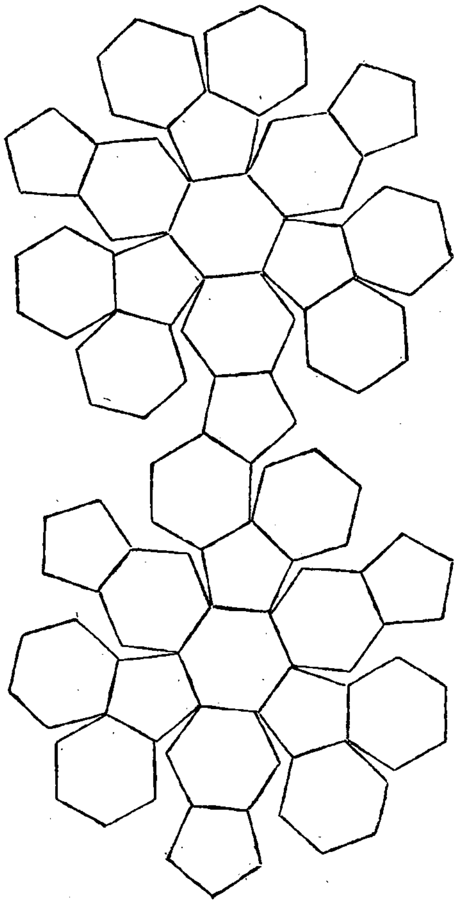


Section 18.

In order to form the icosahedron truncated through the mid-points of the edges, dispose, as shown below, twelve pentagons and twenty triangles, each of whose sides be equal to the line T.

Distinctio 19.

Disponantur, ut infra, pro truncato icosaedro per laterum tertias, viginti hexagona, & duodecim pentagona, quorum singula latera æqualia sint recta V.



Section 19.

In order to form the icosahedron truncated through the third parts of the edges, dispose, as shown below, twenty hexagons and twelve pentagons, each of whose sides be equal to the line V.

Post talem planorum dispositionem erunt plana rite inter se complicanda: Quomodo verò fiet hæc complicatio in singulis corporibus describere supervacaneum videtur, cum res per se satis sit nota. Post verò talem complicationem, erunt planorum latera, vbi opus fuerit, conglutinanda, si ex papyro, ligno vel simili fuerint plana: Aut ferruminanda, si fuerint ex aliquo metallo.

Auctorum verò corporum constructio talis erit: Primò complicentur ac perficiantur quinque corpora regularia quæ in principijs sextæ, septimæ, octauæ, nonæ, & decimæ distinctionum sunt recitata: Deinde cuicunque superficiei ipsorum applicetur sua pyramis absq; basi: exempli gratia, aucti tetraedri sextæ distinctionis tetraedrum primum complicetur: deinde complicentur & tres illi trianguli his notis 1, 2, 3, signatis, ita vt efficiant pyramidem sine base: Appliceturque ipsa pyramis cum parte vbi basis deficit, cuidam superficiei tetraedri, ipsique conglutinetur, vel conferruminetur.

Eodemq; modo applicentur tres tales pyramides reliquis tribus tetraedri superficiebus, eritque auctum tetraedrum exactum. Similiter agetur in reliquis quatuor auctis corporibus.

Dico ex talibus datis lineis plana esse constructa ac disposita, quæ si ita vt dictum est complicentur, & conglutinentur, efficient antedicta petita corpora eidem sphæræ inscriptibilia vt erat quæsitum.

Demonstratio.

Demonstratio ex demonstratione præcedentis primi Problematis est manifesta.

Appendix.

Planorum verò dispositio corporis truncati (cuius est facta mentio in principio huius 3. lib.) cuius truncandi modus hac scribentem me latebat talis est:

After this disposition of the plane figures, they will have to be folded together properly. It seemed, however, superfluous to describe for each of the solids how this folding takes place, since this matter is sufficiently known in itself. But after this folding, the sides of the faces will have to be glued together, wherever necessary, if the plane figures are made of paper, wood or the like, or to be joined together if they are made of some metal.

But the construction of the augmented solids will be as follows. First fold and complete the five regular solids which are mentioned at the beginning of the sixth, seventh, eighth, ninth, and tenth sections. Subsequently place against every face of those solids its pyramid without a base, *e.g.* of the augmented tetrahedron fold first the tetrahedron of the sixth section; next fold also those three triangles which are marked 1, 2, 3, in such a way that they form a pyramid without a base. And place the said pyramid with the part where the base is lacking against a face of the tetrahedron, and glue or join it thereto.

In the same way place three such pyramids against the remaining three faces of the tetrahedron; then the exact augmented tetrahedron will be completed. Proceed similarly with the other four augmented solids.

I say that from such given lines plane figures have been constructed and disposed which, if they are so folded and glued together as has been said, form *the aforesaid required solids that can be inscribed in the same sphere*; as was required.

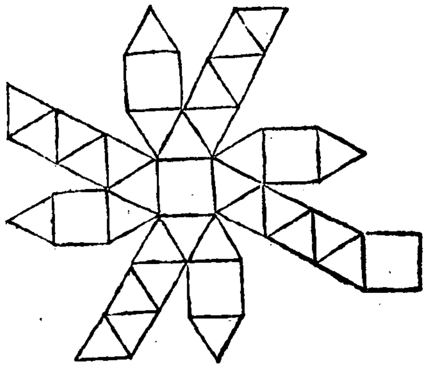
Proof.

The proof is clear from the proof of the preceding first Problem.

Appendix.

However, the disposition of the faces of a truncated solid (of which mention has been made at the beginning of this 3rd book), of which I ignored the way *est: Disponantur, vt infra, sex quadrata & 36. trianguli.*

Sed propter ipsius truncationis, seu veræ originis ignorantiam non potuimus hoc Geometricè antedictæ sphæræ inscriptibile cum ceteris construere.



Tertii Libri

FINIS.

of truncating when I wrote this, is as follows. Dispose, as shown below, six squares and 32 triangles.

But on account of lack of knowledge of the said truncation or of its true origin we have not been able to construct this solid that can be inscribed in the aforesaid sphere Geometrically along with the others.

END OF THE THIRD BOOK.

LIBER QVARTVS

IN QVO DEMONSTRABITVR QVOmodo datis duobus corporibus Geometricis, tertium corpus describi potest, alteri datorum simile, alteri vero æquale.

PROBLEMA *quoddam eximium Clariß. vir, à veteribus inventum est, & ab Euclide prop. 25. lib. 6. descriptum, cuius sensus talis est:* Datis duobus rectilineis, tertium rectilineum describere, alteri datorum simile, alteri vero æquale. *Cumque in planis tale Problema inventum animaduerteremus, tamen in solidis non esse simile generale Problema descriptum (dico generale) quoniam Archimedis inventio in chordis segmentis sphæralibus ad 5. prop. lib. 2. de sphæra & cylindro est in eo specialis: præterea cum consideraremus magnam sympathiam inter magnitudinem superficialem & corpoream (nam quemadmodum triangula & parallelogramma quorum eadem est altitudo, ita se habent inter se vt bases per 1. prop. lib. 6. Euclid. Sic parallelepipeda, pyramides, coni, & cylindri, quorum eadem est altitudo, ita se habent inter se vt bases, per 32. prop. lib. 11. & per 5, 6, & 11. prop. lib. 12. Euclid. Præterea quemadmodum triangula & parallelogramma quorum bases & altitudines reciprocantur, sunt inter se æqualia: Sic parallelepipeda, pyramides, coni, & cylindri, quorum bases & altitudines reciprocantur sunt inter se æquales per 34. prop. lib. 11. & per 9, & 15, prop. lib. 12. Euclid. Præterea quemadmodum similia rectilinea duplicatam eam habent inter se rationem, quam latus homologum ad homologũ latus per 20. prop. lib. 6. Euc. Sic similia corpora triplicatam eam habent rationem, quam latus homologum ad homologum latus per 33. prop. lib. 11. & per 8. 12 & 18. prop. lib. 12. Euclid.) adplicauimus animum ad simile Problema inveniendum in solidis. Idque fæliciter esse inventum, atque ita generale in solidis, vt est supradictum Problema ad 25. prop. lib. 6. Euclid. in planis, complectens tum* Archi-

FOURTH BOOK

in which it is to be proved how, when two Geometrical solids are given, a third solid can be constructed, similar to one of the given solids and equal to the other.

A very beautiful problem, o illustrious lord, was found by the Ancients and described by Euclid in the 25th proposition of the 6th book, the sense of which is as follows: Given two rectilinear figures, to construct a third rectilinear figure, similar to one of the given figures and equal to the other. And since we noted that this Problem had been found for plane figures, yet that for solids no similar general Problem had been described (I say: general, since Archimedes' invention in the matter of segments of spheres in the 5th proposition of book 2 on the sphere and cylinder *) is of a particular character in this field); and since moreover we considered there was great similarity between a plane and a solid magnitude (for as triangles and parallelograms whose altitude is the same are to one another as their bases, by the 1st proposition of Euclid's 6th book, so parallelepipeds, pyramids, cones, and cylinders whose altitude is the same are to one another as their bases, by the 32nd proposition of Euclid's 11th book and by the 5th, 6th, and 11th propositions of his 12th book) — moreover, as triangles and parallelograms whose bases and altitudes are inversely proportional are equal to one another, so parallelepipeds, pyramids, cones, and cylinders whose bases and altitudes are inversely proportional are equal to one another, by the 34th proposition of Euclid's 11th book and by the 9th and 15th propositions of his 12th book; moreover, as similar rectilinear figures are to one another in the duplicate ratio of that of a homologous side to a homologous side, by the 20th proposition of Euclid's 6th book, so similar solids are to one another in the triplicate ratio of that of a homologous edge to a homologous edge, by the 33rd proposition of Euclid's 11th book and by the 8th, 12th, and 18th propositions of his 12th book — we applied our minds to the finding of a similar Problem for solids. And that it has fortunately been found, and even as general for solids as the above-mentioned Problem in the 25th proposition of Euclid's 6th book is for plane figures, comprehending Archimedes' aforesaid invention

*) To construct a segment of a sphere similar to a given segment of a sphere and equal to another given segment of a sphere.

Archimedis antedictum inventum, tum omnia similia in alijs formis magnitudinum, venit in hac secunda parte demonstrandum.

Sed antequam ad rem propositam perveniamus, tria Problemata describentur ad quæsitæ propositionis constructionem necessaria, quorum primum est.

PROBLEMA I.

Datis duabus rectis lineis duas medias proportionales invenire.

NOTA.

Etsi hoc Problema (quamvis non Geometricè) per diversa instrumenta multifariam à veteribus sit inventum, dabitur tamen hic tantum vnicum exemplum per lineas, secundum modum Heronis. Reliquos modos qui per instrumenta expediuntur, in nostra Geometria suis instrumentis accommodatis breviter speramus nos edituros.

Explicatio dati.

Sint igitur duæ datæ lineæ AB, *&* CD.

Explicatio quæsiti.

Oporteat ipsis duas medias lineas proportionales invenire.

Constructio.

Ducantur rectæ EF, *&* EG, *efficientes angulum* GEF *rectum: Applicetur que intervallum* AB, *ab* E, *in recta* EF, *sitque* EH, *ducaturque* HI, *æqualis ipsi* CD *& ad angulos rectos ipsi* EF, *Similiter ducatur recta à puncto* I, *in rectam* EG, *& parallela ipsi* EH, *sitq;* IK, *ducaturque recta* EI, *cuius medium punctum notetur ad* L, *deinde adiumento circini, pede fixo in* L, *signentur pede mobili duo puncta vt* M, *in recta* KG, *alterum vt* N, *in recta* HF: *Si verò illa puncta ita contingerent vt linea recta* MN *ducta, contineret in se punctum* I, *bene esset; Si verò ita minimè accideret, ponenda essent talia puncta, qualia sunt* M, *&* N, *ad* as well as all similar ones for other forms of magnitudes, is shown in this second part.

But before we come to the matter proposed, three Problems will be described which are necessary for the construction of the proposition in question, the first of which is as follows.

PROBLEM I.

Given two lines, to find the two mean proportionals.

NOTE.

Though this Problem was found (though not by a Geometrical method) by the Ancients in different ways by means of different instruments, yet only a single example for lines will here be given, according to the manner of Hero. The other methods, which are carried out by means of instruments, we hope to publish shortly in our Geometry by means of the appropriate instruments *).

Given.

Therefore let there be two given lines *AB* and *CD*.

Required.

Let it be required to find for these lines the two mean proportionals.

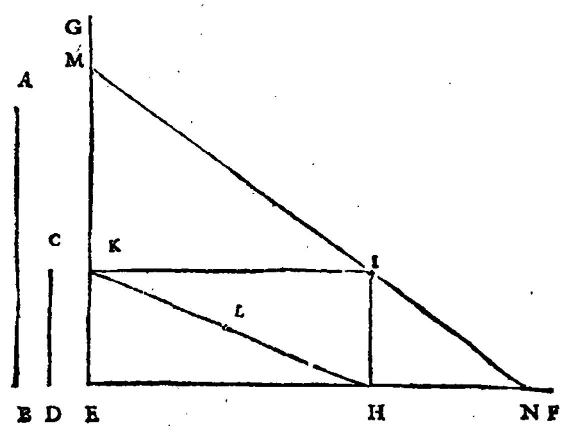
Construction.

Draw the lines *EF* and *EG*, making the angle *GEF* right. Mark off the length *AB* from *E* on the line *EF*, and let this be *EH*. And draw *HI*, equal to *CD* and at right angles to *EF*. Similarly draw a line from the point *I* to the line *EG* and parallel to *EH*, and let this be *IK*; and draw the line *EI*, whose mid-point shall be marked at *L*; subsequently with the aid of the compasses, the fixed leg being at *L*, mark with the movable leg two points, *viz.* *M* on the line *KG* and the other, *viz.* *N*, on the line *HF*. Now if these points fell so that, when the line *MN* is drawn, it would contain the point *I*, this would be al right. But if this did not happen at all, it would be necessary to mark points such as *M* and *N* at a greater

*) Stevin indeed returned to this subject in Book IV of the *Meetdaet.*

maius aut minus intervallum à puncto L, *quo ad ducta recta* M N, *in se punctum* I *contineret*, *vt in hoc exemplo*, *vbi ponitur punctum* I, *in recta* M N *existere.*

Dico datis rectis A B, *&* C D, *duas medias lineas proportionales* K M, *&* H N *esse inventas (quarum prima* A B *secunda* K M *tertia* H N *quarta* C D*) vt erat quæsitum.*



Demonstratio.

Demonstratio habetur apud Eutochium commentatorem in secundum librum de sphæra & cylindro Archimedis.

Conclusio.

Igitur datis duabus rectis lineis duæ mediæ proportionales inventæ sunt: Quod erat faciendum.

PROBLEMA II.

Dato cono æqualem conum sub data altitudine describere.

Expli-

or smaller distance from the point *L* till the line *MN*, being drawn, would contain the point *I*, as in this example, where the point *I* is supposed to lie on the line *MN*.

I say that, given the lines *AB* and *CD*, the two mean proportionals *KM* and *HN* have been found (the first term being *AB*, the second *KM*, the third *HN*, the fourth *CD*); as was required.

Proof.

The proof will be found in Eutocius the commentator, in the second book on the sphere and cylinder of Archimedes.

Conclusion.

Therefore, given two lines, the two mean proportionals have been found; which was to be performed.

PROBLEM II.

To construct a cone equal to a given cone, with a given altitude.

Explicatio dati.

Sit datus conus A B C, *cuius altitudo* A D, *&* *diameter basis* B C: *Data verò altitudo* E F.

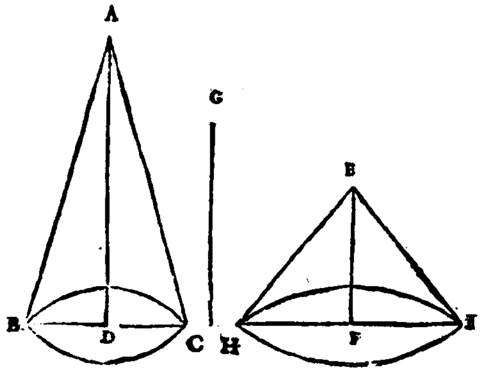
Explicatio quæsiti.

Oporteat alterum conum describere æqualem cono A B C, *&* *sub data altitudine* E F.

Constructio.

Inveniatur media linea proportionalis inter A D, *&* E F, *per* 13. *prop. lib.* 6. *Euclid. sitque* G: *inveniatur deinde quarta linea proportionalis per* 12. *prop. lib.* 6. *Euclid. quarum prima* G, *secunda* A D, *tertia* B C, *sitque quarta* H I: *Deinde ad circulum cuius diameter* H I *&* *ad altitudinem* E F, *construatur conus* E H I.

Dico conum E H I, *esse constructum, æqualem cono dato* A B C *&* *sub data altitudine* E F, *vt erat quæsitum.*



NOTA.

Ante Problematis demonstrationem, existimamus aliquid notari hic opera-

Given.

Let the cone *ABC* be given, whose altitude is *AD*, and the diameter of the base *BC*. And let the altitude *EF* be given.

Required.

Let it be required to construct another cone, equal to the cone *ABC*, and with the given altitude *EF*.

Construction.

By the 13th proposition of Euclid's 6th book, find the mean proportional between *AD* and *EF*, and let this be *G*. Then, by the 12th proposition of Euclid's 6th book, find the fourth proportional, the first term being *G*, the second *AD*, the third *BC;* and let the fourth be *HI*. Subsequently on the circle whose diameter is *HI* and with the altitude *EF* construct the cone *EHI*.

I say that a cone *EHI* has been constructed, equal to the given cone *ABC* and with the given altitude *EF;* as was required.

NOTE.

Before the proof of the Problem we think it worth while that something *operæpretium, nempe quoniam in sequentibus demonstrationibus sæpe dicetur de duplicata ac triplicata ratione terminorum, lectori necesse esse (si demonstrationes velit intelligere) scire, & rectè intelligere quid sit duplicata & triplicata ratio:* Definitur *quidem ipsa in decima definitione lib.* 5. *Eucl. Sed multos interpretes elementorum Euclidis invenimus, hunc locum (quamvis sit magnæ consequentiæ) non ritè explicantes, exceptò doctissimo Mathematico* Christophoro Clauio Bambergensi *commentatore in elementa Euclidis.*

Demonstratio.

Distinctio 1.

A D, *ad* E F, *duplicatam eam habet rationem quam* A D, *ad* G, *nam* G *est illarum media proportionalis per constructionem.*

Distinctio 2.

Circulus H I, *ad circulum* B C, *duplicatam eam habet rationem quam homologa linea* H I, *ad homologam lineam* B C, *vt colligitur ex* 20. *prop. lib.* 6. *Euclid. Sed vt recta* H I, *ad rectam* B C, *sic* A D, *ad* G, *per inversam rationem constructionis: Ergo circulus* H I, *ad circulum* B C, *duplicatam eam habet rationem quam recta* A D, *ad* G. *Sed* A D, *ad* E F, *demonstrata est distinctione* 1, *eandem habere duplicatam rationem quam* A D, *ad* G: *Ergo vt recta* A D, *ad rectam* E F, *sic circulus* H I, *ad circulum* B C. *Igitur sunt coni quorum bases & altitudines reciprocantur, quare per* 15. *prop. lib.* 12. *Euclid. coni* A B C, *&* E H I, *sunt inter se æquales. Præterea conum* E H I, *constructum esse ad datam altitudinem* E F, *ex ipsa constructione manifestum est.*

Conclusio.

Igitur dato cono æqualis conus sub data altitudine descriptus est: Quod erat faciendum.

NOTA.

Antedicta conorum constructio ac demonstratio applicari potest ad subscriptos should be noted here, *viz.* that since in the following proofs there will often be question of the duplicate and triplicate ratio of the terms, the reader must know (if he is to understand the proofs) and understand aright what is the duplicate and the triplicate ratio. This is indeed defined in the tenth definition of Euclid's 5th book. But we have found many interpreters of the elements of Euclid who do not properly explain this passage (though it is of great consequence), except the most learned Mathematician *Christophorus Clavius Bambergensis,* the commentator of Euclid's elements.

Proof.

Section 1.

AD has to *EF* the duplicate ratio of that of *AD* to *G,* for *G* is their mean proportional by the construction.

Section 2.

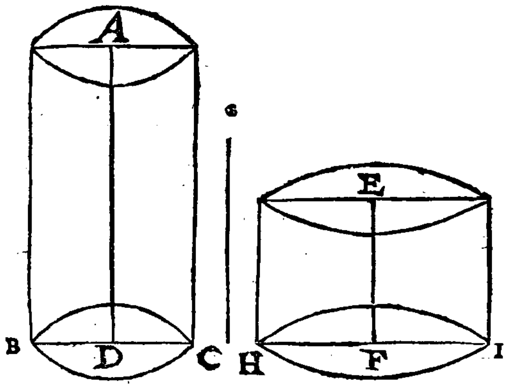
The circle *HI* is to the circle *BC* in the duplicate ratio of that of the homologous line *HI* to the homologous line *BC,* as is inferred from the 20th proposition of Euclid's 6th book. But as the line *HI* is to the line *BC,* so is *AD* to *G,* by the inverted ratio of the construction. Consequently, the circle *HI* is to the circle *BC* in the duplicate ratio of that of the line *AD* to *G.* But it has been proved in section 1 that *AD* is to *EF* in the same duplicate ratio of that of *AD* to *G.* Consequently, as the line *AD* is to the line *EF,* so is the circle *HI* to the circle *BC.* Therefore the solids are cones whose bases and altitudes are inversely proportional, so that, by the 15th proposition of Euclid's 12th book, the cones *ABC* and *EHI* are equal to one another. Moreover, it is evident from the construction itself that the cone *EHI* has been constructed with the given altitude *EF.*

Conclusion.

Therefore, given a cone, a cone equal thereto has been constructed with a given altitude; which was to be performed.

NOTE.

The aforesaid construction and proof of cones can be applied to the cylinders *ſcriptos cylindros, quorum baſes & altitudines quia reciprocantur, concludetur per 15. prop. lib. 12. Euclid. eſſe aequales.*



PROBLEMA III.

Dato chordæ ſegmento ſphærali, æqualem conum deſcribere, habentem baſin cum chordæ ſegmento eandem.

NOTA.

Chordæ ſegmentum ſphærale vocamus partem ſphæræ plano à ſphæra ſectam, ratio huius appellationis vna cum ratione nominis diametralis ſegmenti ſphæralis in noſtra Geometria dicetur.

Explicatio dati.

Sit igitur datum chordæ ſegmentum ſphærale A B C, *cuius diameter baſis* A B *centrum baſis* D, *vertex ſegmenti* C, *& ſegmenti maximus ſphæræ circulus ſit* A E B C, *cuius diameter* C E, *& ſemidiameter* F E.

 Expli-

shown below, for because their bases and altitudes are inversely proportional, it is concluded by the 15th proposition of Euclid's 12th book that they are equal.

PROBLEM III.

Given a chordal segment of a sphere *), to construct a cone equal thereto, having the same base as the chordal segment of a sphere.

NOTE.

A chordal segment of a sphere we call the part of a sphere cut from the sphere by a plane, the reason of which name will be explained in our Geometry along with the reason of the name of diametrical segment of a sphere **).

Given.

Let therefore the segment of a sphere *ABC* be given, whose diameter of the base is *AB*, the centre of the base *D*, the vertex of the segment *C*, and let the great circle of the segment of the sphere be *AEBC*, whose diameter is *CE* and whose semi-diameter is *FE*.

*) In the sequel the usual term *segment of a sphere* will be used.
**) Stevin, in his *Meetdaet*, pp. 93, 187, speaks of „halfmiddellijnsne" and „clootcoordsne".

Explicatio quæsiti.

Oporteat ipsi segmento A B C, *æqualem conum describere, habentem basin cum segmento eandem.*

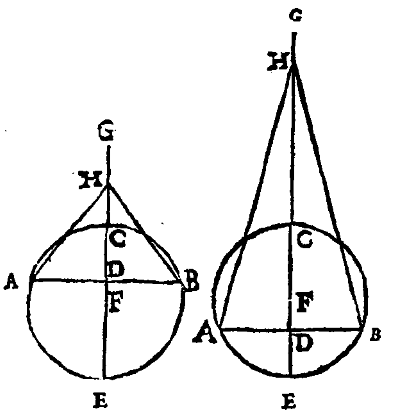
Constructio.

Producatur E C, *in directum ad* G: *Inveniatur deinde quarta linea proportionalis per* 12. *prop. lib.* 6. *Euclid. quarum prima est* E D, *secunda eadem* E D, *&* E F, *in directum vnius lineæ, tertia* D C, *sitque quarta* D H: *Deinde ad circulum cuius diameter est* A B, *& ad altitudinem* D H, *construatur conus* H A B.

Dico chordæ segmento sphærali A B C, *æqualem conum* H A B, *esse constructum, habentem basin cum dato chordæ segmento eandem, vt erat quæsitum.*

Demonstratio.

Demonstratio habetur ad 2. *prop. lib.* 2. *de sphæra & cylindro Archimedis.*



Conclusio.

Igitur dato chordæ segmento sphærali &c. Quod erat faciendum.

Hucusq; descripta sunt quæ ad constructionem huius inventionis sunt necessaria. Nunc ad rem.

PROBLEMA IIII.

ac quæsitum huius Quarti libri:

D atis

Required.

Let it be required to construct a cone, equal to this segment *ABC*, having the same base as the segment.

Construction.

Produce *EC* to *G*. Then, by the 12th proposition of Euclid's 6th book, find the fourth proportional, the first term being *ED*, the second the same *ED* and *EF* on one and the same line, the third *DC*; and let the fourth be *DH*. Subsequently construct the cone *HAB* on the circle whose diameter is *AB* and with the altitude *DH*.

I say that a cone *HAB* has been constructed, equal to the segment of a sphere *ABC*, having the same base as the given segment of a sphere; as was required.

Proof.

The proof will be found in the 2nd proposition of book 2 on the sphere and cylinder of Archimedes.

Conclusion.

Therefore, given a chordal segment of a sphere, etc. Which was to be performed.

Hitherto have been described the things which are necessary for the construction of this invention. Now let us come to the point.

Datis quibuscunque duobus corporibus Geometricis, tertium corpus describere, alteri datorum simile, alteri vero æquale.

NOTA.

Geometricum corpus vocamus quod Geometrica lege construitur, vt est sphæra, Chordæ segmentum sphæræ, Diametrale segmentum sphæræ, Sphæroides, Segmentum sphæroidis, Conoidale, Segmentum conoidale, Columna, cuius duæ sunt species, vt Cylindrus, & Prisma: Pyramis, Corpora regularia, aucta corpora regularia, truncata corpora regularia: de quorum omnium constructione in nostra Geometria abunde dicetur.

Hæc inquam corpora & alia quæ Geometricè construuntur vocamus corpora Geometrica ad differentiam corporum, vt sunt plærunque silices, fragmenta lapidum & similia.

Explicatio dati.

Exempli 1.

Sint duo corpora quæcunque, nempe duo coni A B C, *&* D E F, *sitque coni* D E F *altitudo, recta* D G, *& basis diameter* E F.

Explicatio quæsiti.

Oporteat tertium conum construere, cono D E F *similem & cono* A B C *æqualem.*

Constructio.

Describatur cono A B C, *æqualis conus* H I K, *sub altitudine altitudini* D G *æquali, per præcedens secundum Problema, eius basis diameter sit* I K: *Inveniatur deinde tertia linea proportionalis per* 11. *prop. lib.* 6. *Euc. quarum prima* E F, *secunda* I K, *sitq; tertia* L: *Inveniantur deinde duæ mediæ lineæ proportionales, per præcedens primum Problema, inter* E F, *&* L, *quarum mediarum sequens ipsam* E F, *sit* M N: *Inveniatur deinde quarta linea proportionalis per* 12. *prop. lib.* 6. *Euclid. quarum prima* E F, *secunda* D G, *tertia* M N, *sitque quarta* O P; *Deinde ad circulum cuius diame-*

PROBLEM IV,

and what is sought in this Fourth book:

Given any two Geometrical solids, to construct a third solid, similar to one of the given solids and equal to the other.

NOTE.

We call Geometrical solid a solid which is constructed by a Geometrical law, such as sphere, a segment of a sphere, a sector of a sphere, Spheroids, a Segment of a spheroid, a Conoid, a conoidal Segment, a Column, of which there are two kinds, *viz.* the Cylinder and the Prism, a Pyramid, the regular Solids, the augmented regular solids, the truncated regular solids; the construction of all of which will be dealt with fully in our Geometry.

Indeed, we call these solids and others which are constructed Geometrically Geometrical solids to distinguish them from bodies such as, generally, stones, fragments of stones, and the like.

Given.

of Example 1.

Let there be any two solids, *viz.* the two cones *ABC* and *DEF*, and let the altitude of the cone *DEF* be the line *DG*, and the diameter of the base *EF*.

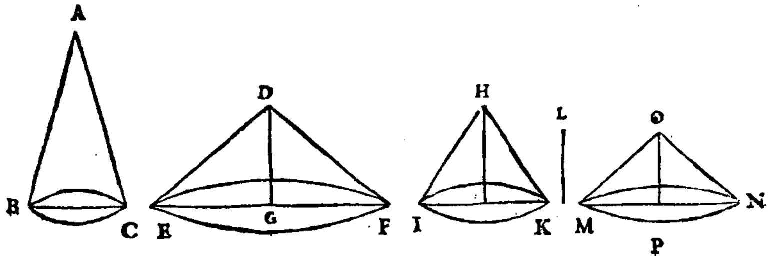
Required.

Let it be required to construct a third cone, similar to the cone *DEF* and equal to the cone *ABC*.

Construction.

Construct the cone *HIK* equal to the cone *ABC*, with an altitude equal to the altitude *DG*, by the preceding second Problem; let the diameter of its base be *IK*. Then, by the 11th proposition of Euclid's 6th book, find the third proportional, the first term being *EF*, the second *IK*; and let the third be *L*. Subsequently, by the preceding first Problem, find the two mean proportionals between *EF* and *L*, and let that one of these mean proportionals which follows *EF* be *MN*. Then, by the 12th proposition of Euclid's 6th book, find the fourth proportional, the first term being *EF*, the second *DG*, the third *MN*; and let the fourth be *diameter* M N, *& sub altitudine* O P, *construatur conus* O M N.

Dico tertium conum O M N, *esse descriptum cono* A B C *æqualem, & cono* D E F *similem, vt erat quæsitum.*



Demonstratio.

Distinctio 1.

Vt basis diameter E F, *ad sui coni altitudinem* D G, *sic basis diameter* M N, *ad sui coni altitudinem* O P, *per constructionem, quare per* 24. *definitionem lib.* 11. *Euclid. coni* D E F, *&* O M N, *sunt similes, quod primò erat demonstrandum.*

Sequitur nunc demonstrari conum O M N *æqualem esse cono* A B C, *hoc modo.*

Distinctio 2.

Recta E F, *ad rectam* L, *duplicatam eam habet rationem quam recta* E F, *ad* I K, *nam* I K, *est illarum media proportionalis per constructionem: sequitur vt colligitur ex* 20. *prop. lib.* 6. *Eucl. (nam rectæ* E F, *&* I K, *sunt homologæ lineæ in similibus planis) rationem circuli* I K, *ad circulum* E F, *æqualem esse rationi rectæ* L, *ad rectam* E F: *Sed per* 11. *prop. lib.* 11. *Euc. vt basis seu circulus* I K, *ad circulum* E F, *sic conus* H I K, *ad conum* D E F, *nam sunt per constructionem coni sub æqualibus altitudinibus:*

Distin-

OP. Subsequently on the circle whose diameter is *MN* and with the altitude *OP* construct the cone *OMN*.

I say that a third cone *OMN* has been constructed, equal to the cone *ABC* and similar to the cone *DEF*; as was required.

Proof.

Section 1.

As the diameter of the base *EF* is to the altitude of its cone *DG*, so is the diameter of the base *MN* to the altitude of its cone *OP*, by the construction; therefore, by the 24th definition of Euclid's 11th book, the cones *DEF* and *OMN* are similar, which was to be proved in the first place.

Next it will be proved that the cone *OMN* is equal to the cone *ABC*, in the following way.

Section 2.

The line *EF* is to the line *L* in the duplicate ratio of that of the line *EF* to *IK*, for *IK* is their mean proportional by the construction. It follows, as is inferred from the 20th proposition of Euclid's 6th book (for the lines *EF* and *IK* are homologous lines in similar plane figures), that the ratio of the circle *IK* to the circle *EF* is equal to the ratio of the line *L* to the line *EF*. But, by the 11th proposition of Euclid's 11th book, as the base or the circle *IK* is to the circle *EF*, so is the cone *HIK* to the cone *DEF*, for by the construction they are cones with equal altitudes.

Distinctio 3.

Ergo vt recta L, ad rectam E F, sic conus H I K, ad conum D E F.

Distinctio 4.

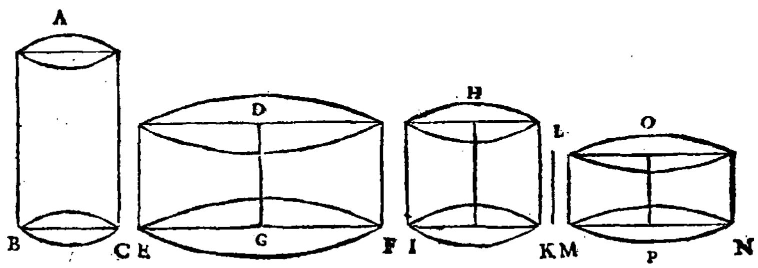
Deinde recta E F, ad L, triplicatam eam habet rationem quam ipsa E F, ad rectam M N (nam quatuor continue proportionalium linearum E F, est prima, M N secunda, & L quarta) quare per 12. prop. lib. 12. Euclid. vt recta L, ad rectam E F, sic (quia coni O M N, & D E F, sunt per primam distinctionem similes) conus O M N, ad conum D E F: Sed distinctione tertia ostensum est eandem rationem esse à cono H I K, ad eundem conum D E F. Ergo (quoniam quorum rationes ad idem æquales sunt ea inter se sunt æqualia) conus O M N, æqualis est cono H I K. Deinde per constructionem conus A B C, est cono H I K æqualis: Ergo (quia quæ eidem æqualia & inter se sunt æqualia) conus O M N, cono A B C, æqualis est.

Conclusio.

Igitur datis quibuscunque &c. Quod erat faciendum.

Exemplum secundum.

Antedicta conorum constructio ac demonstratio applicari potest ad subscriptos cylindros:



Section 3.

Consequently, as the line *L* is to the line *EF*, so is the cone *HIK* to the cone *DEF*.

Section 4.

Next, the line *EF* is to *L* in the triplicate ratio of that of *EF* to the line *MN* (for of the four lines in continuous proportion *EF* is the first, *MN* the second, and *L* the fourth); therefore, by the 12th proposition of Euclid's 12th book, as the line *L* is to the line *EF*, so (because the cones *OMN* and *DEF* are similar by the first section) is the cone *OMN* to the cone *DEF*. But in the third section it has been shown that the same ratio exists between the cone *HIK* and the same cone *DEF*. Consequently (since things whose ratios to the same thing are equal are also equal to one another), the cone *OMN* is equal to the cone *HIK*. Then, by the construction, the cone *ABC* is equal to the cone *HIK*. Consequently (because things which are equal to the same thing are also equal to one another), the cone *OMN* is equal to the cone *ABC*.

Conclusion.

Therefore, given any etc. Which was to be performed.

Second Example.

The aforesaid construction and proof of cones can be applied to the cylinders shown below.

Explicatio dati.

Exempli tertii.

Sint duo chordæ segmenta sphæralia A B C D, *&* E F G H, *sitque chordæ segmenti* E F G H, *altitudo* H F, *&* *basis diameter* E G.

Explicatio quæsiti.

Oporteat tertium chordæ segmentum sphærale construere, segmento E F G H *simile, & segmento* A B C D *æquale.*

Constructio.

Describatur conus I A C, *æqualis chordæ segmento sphærali* A B C D: *Similiter & conus* K E G, *æqualis segmento* E F G H, *per præcedens tertium Problema: describatur deinde cono* I A C, *æqualis conus* L M N, *sub altitudine altitudini* K F *æquali per præcedens secundum Problema, eius basis diameter sit* M N *inveniatur deinde tertia linea proportionalis per* 11. *prop. lib.* 6. *Euclid. quarum prima* E G, *secunda* M N, *sitque tertia* O P: *deinde inveniantur duæ mediæ lineæ proportionales per præcedens primum Problema inter* E G, *&* O P, *quarum mediarum sequens ipsam* E G, *sit* Q R: *Inveniatur deinde quarta linea proportionalis per* 12. *prop. lib.* 6. *Euclid. quarum prima* E G, *secunda* H F, *tertia* Q R, *sitq; quarta* S T: *Deinde ad circulum cuius diameter* Q R, *& sub altitudine* S T, *construatur chordæ segmentum sphærale* Q T R S.

Dico tertium chordæ segmentum sphærale Q T R S, *esse constructum chordæ segmento sphærali* E F G H *simile, & segmento* A B C D *æquale, vt erat quæsitum.*

Præ-

Given.

of the third Example.

Let there be two segments of a sphere *ABCD* and *EFGH*, and let the altitude of the segment *EFGH* be *HF* and the diameter of its base *EG*.

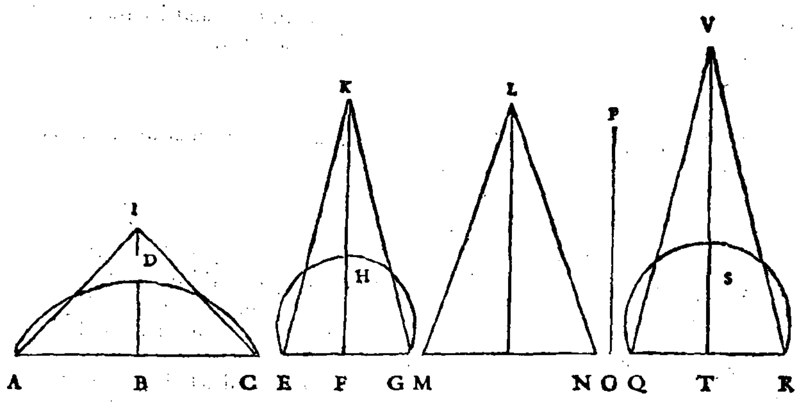
Required.

Let it be required to construct a third segment of a sphere, similar to the segment *EFGH* and equal to the segment *ABCD*.

Construction.

Construct a cone *IAC*, equal to the segment of a sphere *ABCD*. Similarly also a cone *KEG*, equal to the segment *EFGH*, by the preceding third Problem. Then construct a cone *LMN*, equal to the cone *IAC*, with an altitude equal to the altitude *KF* by the preceding second Problem, and let the diameter of its base be *MN*. Subsequently, by the 11th proposition of Euclid's 6th book, find the third proportional, the first term being *EG*, the second *MN*; and let the third be *OP*. Then, by the preceding first Problem, find the two mean proportionals between *EG* and *OP*, and let that one of these mean proportionals which follows *EG* be *QR*. Then, by the 12th proposition of Euclid's 6th book, find the fourth proportional, the first term being *EG*, the second *HF*, the third *QR*; and let the fourth be *ST*. Subsequently, on the circle whose diameter is *QR* and with the altitude *ST* construct the segment of a sphere *QTRS*.

I say that a third segment of a sphere *QTRS* has been constructed, similar to the segment of a sphere *EFGH* and equal to the segment *ABCD*; as was required.



Præparatio demonstrationis.

Describatur conus V Q R, *æqualis chordæ segmento sphærali* Q T R S, *per præcedens 3. problema.*

Demonstratio.

Segmentorum Q T R S *&* E F G H *altitudines, & basium diametri, sunt per constructionem proportionales, quare segmenta sunt similia. Quod primo erat notandum.*

Sequitur nunc demonstrari segmentum Q T R S, *æquale esse segmento* A B C D *hoc modo:*

Conus V Q R *per demonstrationem præcedentis primi exempli, æqualis est cono* I A C, *ergo & æqualis est segmento* A B C D, *(nam segmentum* A B C D, *& conus* I A C, *sunt per constructionem æquales) & cono* V Q R, *æquale est segmentum* Q T R S, *per præparationem demonstrationis: Ergo segmentum* Q T R S, *æquale est segmento* A B C D.

Conclu-

Preparation of the Proof.

Describe a cone *VQR*, equal to the segment of a sphere *QTRS*, by the preceding 3rd problem.

Proof.

The altitudes of the segments *QTRS* and *EFGH* and the diameters of the bases are proportional by the construction; therefore the segments are similar. Which was to be noted in the first place.

Next, it will be proved that the segment *QTRS* is equal to the segment *ABCD*, in the following way:

The cone *VQR*, by the proof of the preceding first example, is equal to the cone *IAC*; consequently, it is also equal to the segment *ABCD* (for the segment *ABCD* and the cone *IAC* are equal by the construction), and the segment *QTRS* is equal to the cone *VQR*, by the preparation of the proof; consequently, the segment *QTRS* is equal to the segment *ABCD*.

Conclusio.

Igitur datis quibuscunque &c. Quod erat faciendum.

NOTA.

Non importune videtur huic Problemati applicari modus constructionis Archimedis eiusdem Problematis, ex propositione 5. lib. 2. de sphæra & cylindro sumptus, vt cuivis concordantia particularis descriptionis problematis Archimedis, cum vniversali hac nostra constructione sit manifesta.

Explicatio dati.

Sit datum chordæ segmentum sphærale A B C D, *cuius vertex* D, *& sui totius sphæræ diameter* D E, *& ipsius sphæræ semidiameter* F D: *Sitque alterum datum segmentum sphærale* G H I K, *cuius vertex* K, *& sui totius sphæræ diameter* K L, *& ipsius sphæræ semidiameter* M K, *(sint præterea hæc data segmenta æqualia & similia datis segmentis præcedentis exempli, in eum finem vt comparemus solutionem præcedentis exempli, ad solutionem huius, quæ debent esse æquales cum sint æqualium quæsitorum solutiones.)*

Explicatio quæsiti.

Oporteat per modum Archimedis tertium segmentum construere, segmento, G H I K *simile, & segmento* A B C D *æquale.*

Constructio.

Inveniatur quarta linea proportionalis per 12. *prop. lib.* 6. *Euclid. quarum prima* B E, *secunda* B E, *&* E F, *in directum vnius lineæ, tertia* B D, *sitque quarta* N: *Similiter inveniatur per eandem* 12. *prop. lib.* 6. *Euclid. quarta linea proportionalis quarum prima* H L, *secunda* H L, *&* M L, *in directum vnius lineæ, tertia* H K, *sitque quarta* O: *Deinde inveniatur quarta linea proportionalis per eandem* 12. *prop. lib.* 6. *Euclid. quarum prima* O, *secunda* G I, *tertia* N, *sitque quarta* P: *Deinde inveniantur duæ mediæ*

Conclusion.

Therefore, given any etc. Which was to be performed.

NOTE.

It does not seem inappropriate to apply to this Problem the construction method of Archimedes of the same Problem, taken from the 5th proposition of the 2nd book on the sphere and cylinder, in order that the agreement of Archimedes' particular description of the problem with this general construction of ours may be evident to anyone.

Given.

Let the segment of a sphere *ABCD* be given, whose vertex is *D*, and the diameter of its total sphere *DE*, and the semi-diameter of the said sphere *FD*. And let the other given segment be *GHIK*, whose vertex is *K*, and the diameter of its total sphere *KL*, and the semi-diameter of the said sphere *MK* (moreover let these given segments be equal and similar to the given segments of the preceding example, in order that we may compare the solution of the preceding example with the solution of this one, for they must be equal, since they are the solutions of equal requirements).

Required.

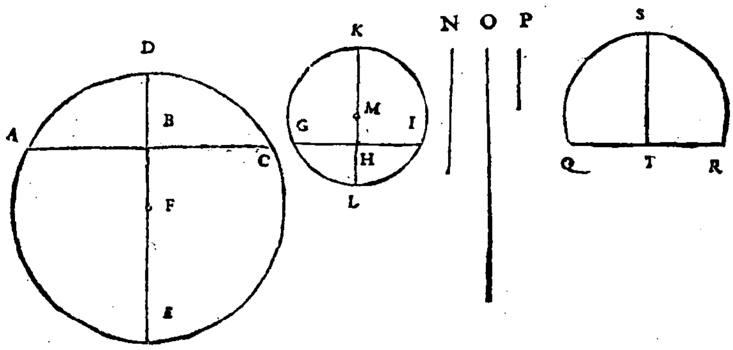
Let it be required to construct, in the manner of Archimedes, a third segment similar to the segment *GHIK* and equal to the segment *ABCD*.

Construction.

By the 12th proposition of Euclid's 6th book, find the fourth proportional, the first term being *BE*, the second *BE* and *EF* on one and the same line, the third *BD*; and let the fourth be *N*. Similarly, by the same 12th proposition of Euclid's 6th book, find the fourth proportional, the first term being *HL*, the second *HL* and *ML* on one and the same line, the third *HK*; and let the fourth be *O*. Then, by the same 12th proposition of Euclid's 6th book, find the fourth proportional, the first term being *O*, the second *GI*, the third *N*; and let the fourth be *P*. Sub-

diæ lineæ proportionales per primum præcedens Problema inter A C, *&* P, *harum autem mediarum sequens ipsam* A C, *sit* Q R: *Inveniatur deinde quarta linea proportionalis per* 12. *prop. lib.* 6. *Euclid. quarum prima* G I, *secunda* H K, *tertia* Q R, *sitq́ quarta* S T: *Deinde ad circulum cuius diameter* Q R, *& sub altitudine* S T, *construatur segmentum* Q T R S.

Dico tertium chordæ segmentum sphærale Q T R S, *esse constructum, chordæ segmento sphærali* G H I K *simile, & segmento* A B C D *æquale, vt erat quæsitum.*



Demonstratio.

Demonstratio habetur ad 5. *prop. lib.* 2. *de sphæra & cylindro Archimedis.*

Conclusio.

Igitur Problema hoc secundum Archimedem expeditum est, quod erat faciendum.

NOTA.

Dico præterea chordæ segmentum sphærale Q T R S *huius constructionis, æquale & simile esse chordæ segmento sphærali* Q T R S *nostræ præ-*

 cedensequently, by the first preceding Problem, find the two mean proportionals between *AC* and *P*, but let that one of these mean proportionals which follows *AC* be *QR*. Then, by the 12th proposition of Euclid's 6th book, find the fourth proportional, the first term being *GI*, the second *HK*, the third *QR*; and let the fourth be *ST*. Subsequently, on the circle whose diameter is *QR* and with the altitude *ST* construct the segment *QTRS*.

I say that a third segment of a sphere *QTRS* has been constructed, similar to the segment of a sphere *GHIK* and equal to the segment *ABCD*; as was required.

Proof.

The proof will be found in the 5th proposition of book 2 on the sphere and cylinder of Archimedes.

Conclusion.

Therefore this Problem has been carried out according to Archimedes, which was to be performed.

NOTE.

I say moreover that the segment of a sphere *QTRS* of this construction is equal and similar to the segment of a sphere *QTRS* of our preceding construction, for *cedentis constructionis, nam per hypothesin data segmenta huius æqualia & similia sunt datis segmentis illius: Deinde quæsitum huius & illius est idem, quare requiruntur æquales solutiones: Sed probata est ab Archimede constructio huius, & à nobis probata est constructio illius, ergo segmentum* Q T R S *huius, & segmentum* Q T R S *illius, sunt similia & æqualia. Quarum constructionum convenientiam propositum erat exhibere.*

Potest quoque hoc nostrum Problema per numeros demonstrari, quod in maiorem declarationem efficiatur hoc modo:

Explicatio dati.

Sit pyramis A B C, *cuius basis sit quadratum, & latus* B C *eiusdem quadrati 2 pedum, altitudo vero pyramidis* A D *sit 12 pedum, quare ipsius pyramidis magnitudo 16 pedum: Sit deinde pyramis* E F G, *cuius basis sit quadratum, & latus* F G *eiusdem quadrati 8 pedum, altitudo vero ipsius pyramidis* E H *3 pedum.*

Explicatio quæsiti.

Oporteat per numeros eo ordine, vt supra per lineas factum est, tertiam pyramidem describere, pyramidi E F G *similem & pyramidi,* A B C *æqualem.*

Constructio.

Describatur pyramidi A B C, *æqualis pyramis* I K L, *sub altitudine* I M, *altitudini* E H *æquali, nempe 3 pedum quare eius basis (vt fiat pyramis cuius magnitudo sit 16 pedum) erit quadratum cuius latus* K L *erit 4 pedum: Inveniatur deinde tertia linea proportialis, quarum prima* F G *8, secunda* K L *4, eritque tertia* N *2 pedum: Inveniantur deinde duæ mediæ lineæ proportionales inter* F G *8, &* N *2, quarum mediarum sequens ipsam* F G, *erit* O P, *radix cubica de 128, probatur quia 8, & radix cubica de 128, & radix cubica de 32, & 2, sunt quatuor numeri in continua proportione; Inveniatur deinde quarta linea proportionalis, quarum prima* F G *8, secunda* E H *3, tertia* O P *radix cubica de 128, eritque quarta pro altitudine* Q R *radix cubica de* $\frac{3456}{512}$, *deinde ad quadratum cuius* by the hypothesis the given segments of the latter are equal and similar to the given segments of the former. Next, the requirements of the latter and the former are the same, so that equal solutions are required. But the construction of the latter has been proved by Archimedes, and the construction of the former has been proved by us; consequently the segment *QTRS* of the latter and the segment *QTRS* of the former are similar and equal; the agreement between which constructions it had been proposed to set forth.

This Problem of ours can also be demonstrated by means of numbers, which may be effected, for greater clarity, in the following way.

Given.

Let there be a pyramid *ABC*, whose base be a square, and let the side *BC* of said square be 2 feet, and the altitude of the pyramid *AD* 12 feet, so that the volume of this pyramid is 16 feet. Further let there be a pyramid *EFG*, whose base be a square, and let the side *FG* of this square be 8 feet, and the altitude of said pyramid *EH* 3 feet.

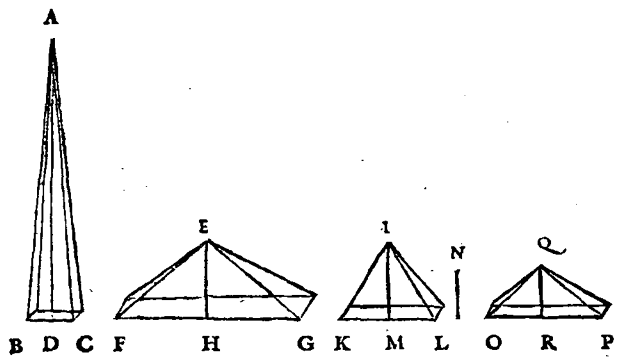
Required.

Let it be required to construct by means of numbers, in the same order as has been done above by lines, a third pyramid similar to the pyramid *EFG* and equal to the pyramid *ABC*.

Construction.

Construct a pyramid *IKL* equal to the pyramid *ABC*, with the altitude *IM* equal to the altitude *EH*, *viz.* 3 feet, so that its base (in order to make a pyramid whose volume be 16 feet) will be a square whose side *KL* will be 4 feet. Then find the third proportional, the first term being $FG = 8$, the second $KL = 4$; then the third will be $N = 2$ feet. Subsequently find the two mean proportionals between $FG = 8$ and $N = 2$, the one of these mean proportionals which follows *FG* being *OP*, the cube root of 128; this is proved because 8, and the cube root of 128, and the cube root of 32, and 2 are four numbers in continuous proportion. Then find the fourth proportional, the first term being $FG = 8$, the second $EH = 3$, the third $OP =$ the cube root of 128; then the fourth, *viz.* the altitude *QR*, will be the cube root of $\frac{3456}{512}$. Subsequently, on the square *cuius latus* O P, *& sub altitudine* Q R, *construatur pyramis* Q O P, *eius magnitudo erit* 16 *pedum: ratio est quia quadratum cuius latus est* O P *radix cubica de* 128, *erit radix cubica de* 16384, *quod multiplicatum per altitudinem* Q R *radicem cubicam de* $\frac{3456}{512}$, *facit productum radicem cubicam de* $\frac{56623104}{512}$, *cuius tertia pars pro magnitudine pyramidis* Q O P, *est radix cubica de* $\frac{56623104}{13824}$, *hoc est radix cubica de* 4096, *facit vt supra dictum est* 16 *pedes.*

Dico tertiam pyramidem Q O P, *per numeros eo ordine vt supra per lineas factum est, esse descriptam pyramidi* E F G *similem, & pyramidi* A B C *æqualem, vt erat quæsitum.*



Demonstratio.

Demonstratio ex eo manifesta est quod pyramis Q O P, *est pyramidi* E F G *similis, & pyramidi* A B C *æqualis, per ipsam numerorum constructionem.*

Conclusio.

Igitur quod primo in continua quantitate erat ostensum, hic per numeros similiter demonstratum est, quod in maiorem declarationem erat faciendum.

N 2 *Potest*

whose side is *OP* and with the altitude *QR* construct a pyramid *QOP*; its volume will be 16 feet. The reason is that the square whose side is *OP* = the cube root of 128 will be the cube root of 16384, which, when multiplied by the altitude *QR* = the cube root of $\frac{3456}{512}$, gives the product = the cube root of $\frac{56623104}{512}$; the third part of which, *viz.* the volume of the pyramid *QOP*, is the cube root of $\frac{56623104}{13824}$, *i.e.* the cube root of 4096, which makes, as said above, 16 feet.

I say that a third pyramid *QOP* has been constructed, by means of numbers, in the same order as has been done above by lines, similar to the pyramid *EFG* and equal to the pyramid *ABC*; as was required.

Proof.

The proof is evident from the fact that the pyramid QOP is similar to the pyramid *EFG* and equal to the pyramid *ABC*, by the numerical construction itself.

Conclusion.

Therefore, what had first been shown in continuous quantity has here been similarly proved by means of numbers, which was to be done for greater clarity.

Potest hoc exemplum quoque fieri per regulam quæ Algebra dicta est, sed illa cum vulgaris sit, non necessarium duximus hic exhiberi.

NOTA.

Requirebatur quidem Problemate præcedenti quarto, datis quibuscunque duobus corporibus Geometricis &c. Sed exempla supra exhibita existimamus pro quibuscunque datis corporibus sufficere, quia omni corpori Geometrico de quibus supra est facta mentio, æqualis conus potest describi (quarum descriptionum Problemata in nostra Geometria ordine collocabimus) vnde operatio in alijs datis formis corporum non erit dissimilis ab operatione præcedentium exemplorum.

His ita demonstratis, applicabimus præcedenti quarto Problemati quoddam theorema tale:

THEOREMA.

Si fuerit diametrorum basium tertia linea proportionalis, duorum rectorum conorum æqualis altitudinis, fueritque prima linea media proportionalis, duarum mediarum proportionalium, inter primam diametrum & tertiam, fueritque quædam recta linea in ea ratione ad illam primam mediam, vt primi coni altitudo ad suam diametrum basis: Conus rectus cuius diameter basis fuerit illa prima media, altitudo vero illa recta linea, similis erit primo cono, æqualis vero alteri cono.

Explicatio dati.

Sit (in figura primi exempli præcedentis quarti Problematis) diametrorum basium EF, *&* IK, *tertia linea proportionalis* L, *duorum rectorum conorum* DEF, *&* HIK, *æqualis altitudinis; sitque prima media linea proportionalis* MN, *duarum mediarum proportionalium inter primam diametrum* EF, *& tertiam lineam* L: *sitque recta linea* OP *in ea*

ratione

This example can also be dealt with by means of the rule called Algebra, but since this is common knowledge, we have not thought it necessary to set it forth here.

NOTE.

It was indeed required in the preceding fourth Problem that, given any two Geometrical solids, etc. But we think the examples set forth above suffice for any given solids, because it is possible to construct a cone equal to any Geometrical solid mention of which is made above (the description of the Problems of which constructions we shall include in due order in our Geometry), whence the operation with other given types of solids will not be dissimilar from the operation of the preceding examples.

These therefore having been proved, we shall apply to the preceding fourth Problem a certain theorem, as follows:

T H E O R E M.

If there were a third proportional to the diameters of the bases of two right cones of equal altitude, and if there were a first mean proportional of the two mean proportionals between the first and the third of the diameters, and if there were a line in the same ratio to said first mean proportional as the altitude of the first cone to its diameter of the base; then the right cone whose diameter of the base should be the said first mean proportional, and its altitude the said line, will be similar to the first cone and equal to the other cone.

Given.

(In the figure of the first example of the preceding fourth Problem) let there be a third proportional *L* to the diameters of the bases *EF* and *IK* of two right cones *DEF* and *HIK* of equal altitude; and let there be a first mean proportional *MN* of two mean proportionals between the first diameter *EF* and the third line *ratione ad illam primam mediam* MN, *vt primi coni* DEF *altitudo* DG, *ad suam diametrum basis* EF.

Dico conum rectum cuius diameter basis est illa prima media MN, *altitudo vero illa recta linea* OP, *similem esse primo cono* DEF, *aequalem vero alteri cono* HIK.

Demonstratio.

Demonstratio habetur ad primum exemplum praecedentis quarti Problematis.

Conclusio.

Igitur si fuerit diametrorum &c. Quod erat demonstrandum.

Quarti Libri
FINIS.

L; and let the line *OP* be to the said first mean proportional *MN* in the same ratio as the altitude *DG* of the first cone *DEF* to its diameter of the base *EF*.

I say that the right cone whose diameter of the base is the said first mean proportional *MN*, and whose altitude is the said line *OP*, is similar to the first cone *DEF* and equal to the other cone *HIK*.

Proof.

The proof will be found in the first example of the preceding fourth Problem.

Conclusion.

Therefore, if there were etc. Which was to be proved.

END OF THE FOURTH BOOK.

LIBER QVINTVS

IN QVO DEMONSTRABITVR QVOMOdo datis quibuscunque duorum similium Geometricorum corporum homologis lineis, tertium corpus construi potest datis duobus æquale, & alteri datorum simile.

Item quomodo datis quibuscunque duobus similium & inæqualium Geometricorum corporum homologis lineis, tertium corpus construi potest tanto minus dato maiore, quantum est datum minus, & alteri datorum simile.

ANTEQVAM *explicetur Problematis constructio huius Quinti libri, dicetur prius quid & quale sit, & quomodo inventum sit Problema.*

Inprimis igitur notandum est varios esse modos, quibus datis duobus planis similibus, tertium planum describimus, duobus datis æquale, & alteri datorum simile, quos modos exemplis explicare non videtur inutile.

Complectemur igitur antedictum Problemate tali.

PROBLEMA I.

Datis duobus planis similibus: tertium planum describere datis duobus æquale & alteri datorum simile.

Explicatio dati.

Sint duo similia triangula A B, *&* C D, *quorum homologa latera* A, *&* C.

Expli-

FIFTH BOOK,

in which it will be shown how, given any homologous lines of two similar Geometrical solids, a third solid can be constructed, equal to the two given solids and similar to one of the given solids.

Likewise how, given any two homologous lines of similar and unequal Geometrical solids, a third solid can be constructed, as much smaller than the larger of the given solids as the smaller of the given solids, and similar to one of them.

Before the construction of the Problem of this Fifth book is explained, it is first to be stated what and how it is, and how the Problem has been found.
To begin with therefore it is to be noted that there are various ways in which, given two similar plane figures, we construct a third plane figure, equal to the two given figures and similar to one of the given figures; it does not seem inappropriate to explain these ways by examples.
Let us therefore comprehend the above in the following Problem.

PROBLEM I.

Given two similar plane figures: to construct a third plane figure, equal to the two given figures and similar to one of the given figures.

Given.

Let there be two similar triangles *AB* and *CD*, whose homologous sides are *A* and *C*.

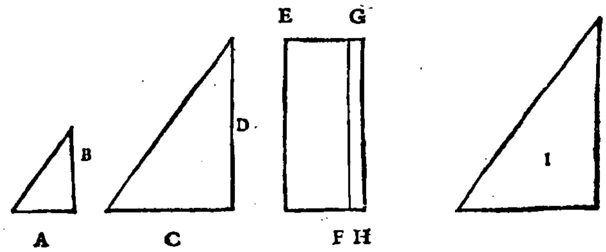
Explicatio quæsiti.

Oporteat tertium triangulum describere, duobus A B, *&* C D *æquale & alteri vt* A B *simile.*

Constructio primi modi.

Describatur per 45. prop. lib. 1. Eucl. parallelogrammum E F, *æquale triangulo* C D: *Similiterq; parallelogrammum* G H, *æquale triangulo* A B *eiusdemque altitudinis cum parallelogrammo* E F: *Describatur deinde per 25. prop. lib. 6, triangulum* I, *æquale toti parallelogrammo* E H, *& simile triangulo* A B.

Dico tertium triangulum I *esse descriptum, æquale duobus triangulis* A B, *&* C D, *& ipsi* A B *simile, vt erat quæsitum.*



Demonstratio.

Demonstratio ex constructione est manifesta.

Conclusio.

Igitur datis duabus planis &c. Quod erat faciendum.

Constructio secundi modi.

Secundus modus multo est facilior atque generalior quam primus: fit autem

Required.

Let it be required to construct a third triangle, equal to the two, *AB* and *CD*, and similar to one of them, *viz. AB*.

Construction According to the First Manner.

By the 45th proposition of Euclid's 1st book construct a parallelogram *EF*, equal to the triangle *CD*. And similarly, a parallelogram *GH*, equal to the triangle *AB* and having the same altitude as the parallelogram *EF*. Then, by the 25th proposition of the 6th book, construct a triangle *I*, equal to the whole parallelogram *EH* and similar to the triangle *AB*.

I say that a third triangle *I* has been constructed, equal to the two triangles *AB* and *CD*, and similar to *AB*; as was required.

Proof.

The proof is evident from the construction.

Conclusion.

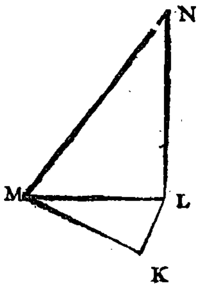
Therefore, given two plane figures etc. Which was to be performed.

Construction According to the Second Manner.

The second manner is much easier and more general than the first. The first, *autem primus tantum Geometricè in rectilineis planis, sed hic secundus modus in circulis, & circulorum partibus habet locum, est autem talis:*

Ducatur recta K L, *æqualis rectæ* A, *& recta* K M, *æqualis rectæ* C, *efficientes angulum* M K L *rectum, ducaturque recta* L M, *cui per* 18 *.prop. lib.* 6. *Euclid. vt homologæ lineæ cum* A *construatur triangulus* N M L, *simile triangulo* A B.

Dico tertium triangulum N M L, *esse descriptum æquale duobus triangulis* A B, *&* C D, *& ipsi* A B *triangulo simile, vt erat quæsitum.*



Demonstratio.

Demonstratio habetur ad 31. *prop. lib.* 6. *Euclid.*

His de planis intellectis, sciendum est simile generale Problema hocusque in solidis non fuisse editum (dixi generale, quoniam Problema illud de duplicatione cubi speciale in ea re est) hoc tamen à nobis esse inventum in hac tertia parte demonstrabitur.

Primo notandum est talis Problematis constructionem in solidis, iuxta primum modum supra in planis recensum, ex præcedenti Quarti libri quarto Problemate esse notum, nam post datorum similium corporum additionem, nihil aliud deest, quam per antedictū 4. *Problema corpori ex additis corporibus composito, æquale corpus describere simile dato corpori.*

Tamen cum videremus antedictum secundum modum in planis multo elegantiorem, generaliorem, atq; faciliorem esse priore, incidimus in eam opinionem simile in solidis fieri posse, propter magnam sympathiam inter magnitudinem corpoream, & superficialem, vt supra dictum est: neque fefellit nos in ea re opinio, nam per solas lineas absque corporum conversione in alias formas however, is performed Geometrically only with rectilinear plane figures, but this second manner takes place with circles and parts of circles; it is as follows.

Draw a line *KL* equal to the line *A*, and a line *KM* equal to the line *C*, making the angle *MKL* right; and draw the line *LM*, on which, by the 18th proposition of Euclid's 6th book, as being the line homologous to *A*, construct the triangle *NML*, similar to the triangle *AB*.

I say that a third triangle *NML* has been constructed, equal to the two triangles *AB* and *CD*, and similar to the triangle *AB*; as was required.

Proof.

The proof will be found in the 31st proposition of Euclid's 6th book.

These things having been understood for plane figures, it is to be known that a similar general Problem has not so far been published for solids (I have said: general, since the Problem of the duplication of the cube is particular in this respect); however, it will be proved in this third part that this has been found by us.

In the first place it is to be noted that the construction of this Problem for solids, along with the first manner shown above for plane figures, is known from the fourth Problem of the preceding Fourth book, for after the addition of the given similar solids nothing else remains to be done but to construct, by the aforesaid 4th Problem, a solid equal to the solid composed of the added solids, and similar to the given solid.

However, when we saw that for plane figures the aforesaid second manner is much more elegant, more general, and easier than the first, we hit on the idea that a similar thing could be done for solids, on account of the great agreement between a solid and a plane magnitude, as has been said above. Nor were we mistaken in this opinion about the matter, for the construction of the similar Problem for solids will here be demonstrated by lines alone, without the con-

formas corporum, quemadmodum in planis in supradicto secundo exemplo factum est, demonstrabitur hic similis Problematis constructio in solidis.

Sed vt vna causam inventionis aperiamus, demonstrabimus ante quomodo tertium modum invenerimus constructionis præcedentis problematis, præcedenti secundo modo similem, scilicet per solas lineas, hoc modo;

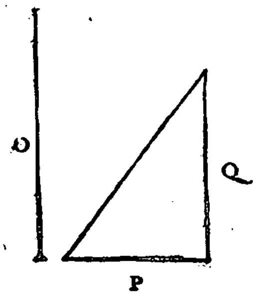
Constructio tertii modi.

Inveniatur tertia linea proportionalis per 11. *prop. lib.* 6. *Euclid. quarum prima* A, *secunda* C, *sitque tertia* O: *Inveniatur deinde media linea proportionalis per* 13. *prop. lib.* 6. *Euclid. inter* A, *& lineam æqualem duabus lineis* A, *&* O, *in directum vnius lineæ, sitque illa media proportionalis* P, *ex qua per* 18. *prop. lib.* 6. *Euclid. vt homologa linea cum* A, *construatur triangulus* P Q, *similis triangulo* A B.

Dico tertium triangulum P Q, *esse descriptum æquale duobus triangulis* A B, *&* C D, *& triangulo* A B *simile, vt erat quæsitum.*

Demonstratio.

Distinctio 1.



Recta A, *ad rectam* O, *duplicatam eam habet rationem quam recta* A, *ad rectam* C, *per constructionem: quare vt recta* A, *ad rectam* O, *sic triangulus* A B, *ad triangulum* C D, *per* 20. *prop. lib.* 6. *Euclid.*

Distinctio 2.

Quare per compositam rationem, vt duæ rectæ A *&* O *simul, ad rectam* A, *sic duo trianguli* A B, *&* C D *simul, ad triangulum* A B.

version of solids into other types of solids, as has been done for plane figures in the above-mentioned second example.

But in order to reveal at the same time the cause of the discovery, we shall show first how we found the third manner of the construction of the preceding problem, similar to the preceding second manner, *viz.* by lines alone, in the following way:

Construction According to the Third Manner.

By the 11th proposition of Euclid's 6th book, find the third proportional, the first term being *A*, the second *C;* and let the third be *O*. Then, by the 13th proposition of Euclid's 6th book, find the mean proportional between *A* and a line equal to the two lines *A* and *O,* on one and the same line; and let this mean proportional be *P,* from which, by the 18th proposition of Euclid's 6th book, as being the line homologous to *A,* construct a triangle *PQ,* similar to the triangle *AB.*

I say that a third triangle *PQ* has been constructed, equal to the two triangles *AB* and *CD,* and similar to the triangle *AB;* as was required.

Proof.

Section 1.

The line *A* is to the line *O* in the duplicate ratio of that of the line *A* to the line *C,* by the construction; therefore, as the line *A* is to the line *O,* so is the triangle *AB* to the triangle *CD,* by the 20th proposition of Euclid's 6th book.

Section 2.

Therefore, by the compound ratio, as the two lines *A* and *O* together are to the line *A,* so are the two triangles *AB* and *CD* together to the triangle *AB.*

Distinctio 3.

Recta A, *ad rectam æqualem duabus rectis* A *&* O *simul, duplicatam eam habet rationem, quam recta* A, *ad rectam* B, *per constructionem, quare vt recta* A, *ad duas rectas* A *&* O, *sic triangulus* A, *ad triangulum* P Q, *& per ipsius inversam rationem, vt duæ rectæ* A *&* O *simul, ad rectam* A, *sic triangulus* P Q, *ad triangulum* A B: *Sed ostensum est distinctione secunda, eandem esse rationem duorum triangulorum* A B *&* C D *simul, ad eundem triangulum* A B: *Ergo (quia quorum rationes ad idem sunt æquales, ea inter se sunt æqualia) triangulus* P Q *æqualis est duobus triangulis* A B *&* C D.

Præterea similem esse triangulo A B, *ex constructione apparet.*

Conclusio.

Igitur datis duabus planis &c. quod erat faciendum.

Potest quoque constructio antedicti Problematis per numeros demonstrari: Quod in maiorem euidentiam efficiatur hoc modo.

Explicatio dati.

Sint trianguli A B *&* C D, *rectanguli, sitque latus* A 3 *pedum, &* B 4 *pedum, vnde superficies trianguli* A B, *(quia rectangulus est per hypothesin) erit* 6 *pedum: Sit præterea latus* C, 6 *pedum, quare latus* D *(quia trianguli* A B *&* C D *sunt similes) erit* 8 *pedum; vnde superficies trianguli* C D *erit* 24 *pedum.*

Explicatio quæsiti.

Oporteat per numeros tertium triangulum invenire eo ordine vt supra in constructione tertij modi per lineas factum est datis duobus triangulis A B *&* C D *æqualem, & triangulo* A B *similem.*

Constructio.

Procedatur per numeros eo ordine vt in præcedenti tertia constructione per

Section 3.

The line *A* is to the line equal to the two lines *A* and *O* together in the duplicate ratio of that of the line *A* to the line *B*, by the construction; therefore, as the line *A* is to the two lines *A* and *O*, so is the triangle *A* to the triangle *PQ*, and by the inverted ratio of this: as the two lines *A* and *O* together are to the line *A*, so is the triangle *PQ* to the triangle *AB*. But it has been shown in the second section that the ratio of the two triangles *AB* and *CD* together to the same triangle *AB* is the same. Consequently (because things whose ratios to the same thing are equal are equal to one another) the triangle *PQ* is equal to the two triangles *AB* and *CD*.

Further it appears from the construction that it is similar to the triangle *AB*.

Conclusion.

Therefore, given two plane figures etc.; which was to be performed.

The construction of the aforesaid Problem can also be proved by means of numbers, which may be done, for greater evidence, in the following way.

Given.

Let the triangles *AB* and *CD* be right-angled, and let the side A be 3 feet and B 4 feet, whence the area of the triangle *AB* (because it is right-angled by the hypothesis) will be 6 feet. Further let the side *C* be 6 feet; therefore the side *D* (because the triangles *AB* and *CD* are similar) will be 8 feet, whence the area of the triangle *CD* will be 24 feet.

Required.

Let it be required to find, by means of numbers, a third triangle in the same order as has been done above in the construction according to the third manner by means of lines, equal to the two given triangles *AB* and *CD*, and similar to the triangle *AB*.

Construction.

Proceed by means of numbers in the same order as has been done in the *per lineas factum est, inveniaturque tertia linea proportionalis quarum prima* A 3, *secunda* C 6, *quare tertia* O *erit* 12. *Inveniatur deinde media linea proportionalis inter* A 3, *& lineam æqualem duabus lineis* A 3, *&* O 12, *hoc est inveniatur media linea proportionalis inter* A 3 *&* 15, *facit pro* P *radicem quadratam de* 45, *ex qua vt homologa linea cum* A, *construatur triangulus* P Q *similis triangulo* A B, *quod fiet hoc modo: Inveniatur quarta linea proportionalis per regulam proportionis, quarum prima* A 3, *secunda* B 4, *tertia* P *radix quadrata de* 45, *eritque quarta pro recta* Q *radix quadrata de* 80, *quare superficies trianguli* P Q *(quoniam radix quadrata de* 45 *multiplicata per radicem quadratam de* 80, *dat productum* 60, *cuius medium* 30*) erit* 30.

Dico per numeros tertium triangulum P Q, *inventum esse, eo ordine vt supra per lineas factum est, datis duobus triangulis* A B, *&* C D *æqualem, & triangulo* A B *similem, vt erat quæsitum.*

Demonstratio.

Demonstratio ex eo manifesta est, quod triagulus A B 6 *pedum, & triangulus* C D 24 *pedum, simul efficiunt (vt supra de triangulo* P Q *ostensum est,)* 30 *pedes. Igitur triangulus* P Q *æqualis est duobus triangulis* A B *&* C D.

Præterea latera P *&* Q *trianguli* P Q, *proportionalia sunt lateribus* A, *&* B, *trianguli* A B, *per numerorum constructionem: habent præterea angulum angulo æqualem, nempe ambo angulum rectum: Ergo per* 7. *prop. lib.* 6. *Euclid. sunt similes.*

Conclusio.

Igitur quod primò in continua quantitate erat ostensum, hic per numeros similiter ostensum est, quod in maiorem declarationem erat faciendum.

Cum vero huius tertiæ constructionis inventio ita certò sit comprobata, preceding third construction by means of lines, and find the third proportional, the first term being $A = 3$, the second $C = 6$, so that the third O will be 12. Then find the mean proportional between $A = 3$ and a line equal to the two lines $A = 3$ and $O = 12$, *i.e.* find the mean proportional between $A = 3$ and 15. This makes, for P, the square root of 45, from which, as being the line homologous to A, construct a triangle PQ similar to the triangle AB, which is done in the following way. Find the fourth proportional by the rule of proportions, the first term being $A = 3$, the second $B = 4$, the third $P =$ the square root of 45; then the fourth, for the line Q, will be the square root of 80, so that the area of the triangle PQ (since the square root of 45, multiplied by the square root of 80, gives the product 60, one half of which is 30) will be 30.

I say that, by means of numbers, a third triangle PQ has been found in the same order as has been done above by means of lines, equal to the two given triangles AB and CD, and similar to the triangle AB; as was required.

Proof.

The proof is evident from the fact that the triangle $AB = 6$ feet and the triangle $CD = 24$ feet make together (as has been shown above for the triangle PQ) 30 feet. Therefore the triangle PQ is equal to the two triangles AB and CD.

Further, the sides P and Q of the triangle PQ are proportional to the sides A and B of the triangle AB, by the construction by means of numbers. Moreover, each has one angle equal to that of the other, *viz.* both a right angle. Consequently, by the 7th proposition of Euclid's 6th book they are similar.

Conclusion.

Therefore, what had first been shown in continuous quantity, has here been similarly shown by means of numbers, which was to be done for greater clarity.

Now since the invention of this third construction has thus been proved for *vtile videtur ipsi constructioni suum Theorema adijcere tale.*

THEOREMA.

Si tertia linea proportionalis duarum homologarum linearum existentium in similibus planis, addatur primæ lineæ: Media linea proportionalis inter primam & illam compositam, est potentialiter homologa linea, cum illis homologis lineis, cuiusdam plani quod simile est alteri datorum, & æquale ambobus.

Cum vero hunc tertium modum invenissemus in planis, patefacta nobis est via similis inventionis in solidis, nam quicquid in similibus planis factum est per duplicatam rationem, id fiet in solidis per triplicatam rationem, neque aliud (si quis rectè animaduertat) invenietur discrimen. Igitur ad rem nunc accedamus.

PROBLEMA II.

Datis quibuscunque duorum similium Geometricorum corporum homologis lineis, tertium corpus construere datis duobus æquale, & alteri datorum simile.

Explicatio dati.

Sint duo data similia Geometrica corpora A B *&* C D, *quorum homologæ lineæ sint* A, *&* C.

Explicatio quæsiti.

Oporteat tertium corpus describere datis duobus A B *&* C D *æquale, & corpori* A B *simile.*

Concertain, it seems useful to add to this construction its Theorem, as follows:

THEOREM.

If the third proportional to two homologous lines occurring in similar plane figures be added to the first line, the mean proportional between the first and the said composite line is potentially a line homologous to those homologous lines of a certain plane figure which is similar to one of the two given figures and equal to both.*)

When we had found this third manner for plane figures, a way to find a similar manner for solids occurred to us, for whatever has been done for similar plane figures by the duplicate ratio will also be done for solids by the triplicate ratio, and no other difference will be found (if one attends well).

Therefore let us now come to the point.

PROBLEM II.

Given any homologous lines of two similar Geometrical solids, to construct a third solid, equal to the two given solids and similar to one of the given solids.

Given.

Let two similar Geometrical solids *AB* and *CD* be given, whose homologous lines shall be *A* and *C*.

Required

Let it be required to construct a third solid, equal to the two given solids *AB* and *CD*, and similar to the solid *AB*.

*) If two similar areas A_1 and A_2 are to each other as the squares of homologous lines p_1 and p_2, or $A_1 : A_2 = p_1^2 : p_2^2$, then $A_1 : (A_1 + A_2) = p_1^2 : (p_1^2 + p_2^2)$. The third proportional c to two lines a and b satisfies the equation $a{:}b = b{:}c$.

Constructio.

Inveniatur tertia linea proportionalis per 11. *prop. lib.* 6. *Euclid. quarum prima* A, *secunda* C, *sitq; tertia* E: *inveniatur deinde quarta linea proportionalis per* 12. *prop. lib.* 6. *Euclid. quarum prima* A, *secunda* C, *tertia* E, *sitque quarta* F; *Inveniantur deinde duæ mediæ lineæ proportionales per primum Problema præcedentis* 4. *lib. inter rectam* A, & *rectam æqualem duabus rectis scilicet* A & F, *quarum mediarum linearum sequens rectam* A, *sit recta* G, *ex qua vt homologa linea cum linea* A, *construatur corpus* G H, *corpori* A B *simile.*

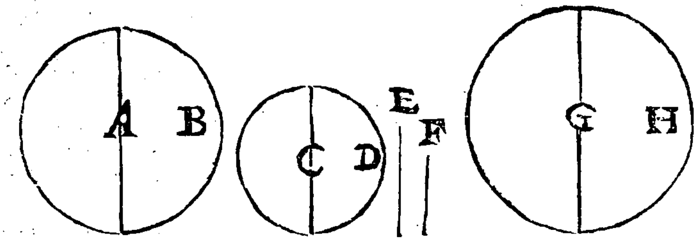
Dico tertium corpus G H, *inventum esse datis duobus* A B, & C D *æquale*: & *corpori* A B, *simile, vt erat quæsitum,*

Construction.

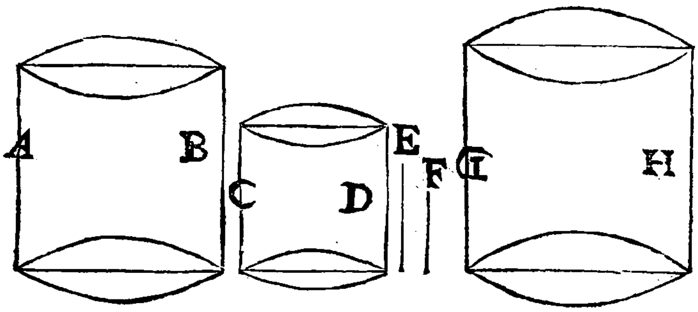
By the 11th proposition of Euclid's 6th book, find the third proportional, the first term being *A*, the second *C;* and let the third be *E*. Then, by the 12th proposition of Euclid's 6th book, find the fourth proportional, the first term being *A*, the second *C*, the third *E;* and let the fourth be *F*. Subsequently, by the first Problem of the preceding 4th book, find the two mean proportionals between the line *A* and the line equal to the two lines, *viz.* *A* and *F*, and let that one of these mean proportionals which follows the line *A* be the line *G*, from which, as being the line homologous to the line *A*, construct the solid *GH*, similar to the solid *AB*.

I say that a third solid *GH* has been found, equal to the two given solids *AB* and *CD*, and similar to the solid *AB*; as was required.

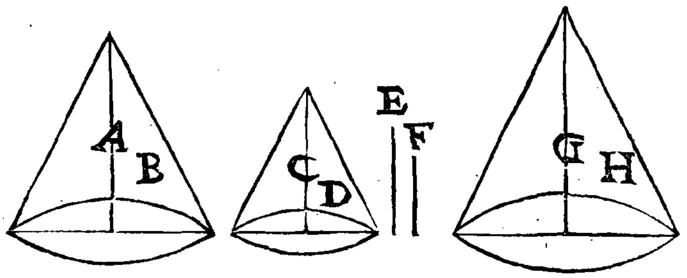
Exemplum de sphæris.



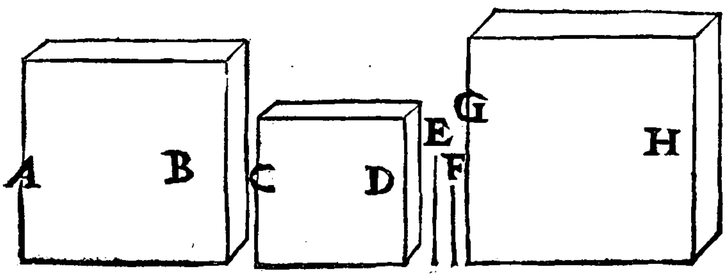
Exemplum de cylindris



Exemplum de conis.



Exemplum de cubis.



Example of Spheres
Example of Cylinders
Example of Cones
Example of Cubes

Demonstratio.

Distinctio 1.

Recta A, *ad rectam* F, *triplicatam eam habet rationem quam recta* A, *ad rectam* C, *per constructionem: Quare vt recta* A, *ad rectam* F, *sic corpus* A B, *ad corpus* C D, *per* 33. *prop. lib.* 11. *& per* 8, 12, *&* 18. *prop. lib.* 12. *Euclid.*

Distinctio 2.

Quare per compositam rationem, vt duæ rectæ A, *&* F *simul, ad rectam* A, *sic duo corpora* A B, *&* C D *simul, ad corpus* A B.

Distinctio 3.

Recta A, *ad rectam æqualem duabus rectis* A *&* F, *triplicatam eam habet rationem quam recta* A, *ad rectam* G, *per constructionem: Quare vt recta* A, *ad duas rectas* A *&* F, *sic corpus* A B, *ad corpus* G H: *Et per ipsius inversam rationem, vt duæ rectæ* A *&* F *simul, ad rectam* A, *sic corpus* G H, *ad corpus* A B: *Sed ostensum est distinctione secunda, eandem esse rationem duorum corporum* A B, *&* C D, *ad idem corpus* A B: *Ergo (quia quorum rationes eidem sunt æquales, ea inter se sunt æqualia) corpus* G H, *æquale est duobus corporibus* A B *&* C D.

Præterea corpus G H, *simile esse corpori* A B, *ex constructione est manifestum.*

Conclusio.

Igitur datis quibuscunque duobus &c. Quod erat faciendum.

Possumus quoque (in maiorem declarationem veritatis constructionis huius inventionis) constructionem antedicti Problematis per numeros exhibere, quod efficiatur hoc modo:

Explicatio dati.

Sit cubus A B, *cuius magnitudo* 8 *pedum, quare eius latus* A, *radix cubica*

Proof.

Section 1.

The line *A* is to the line *F* in the triplicate ratio of that of the line *A* to the line *C*, by the construction. Therefore, as the line *A* is to the line *F*, so is the solid *AB* to the solid *CD*, by the 33rd proposition of Euclid's 11th book and by the 8th, 12th, and 18th propositions of his 12th book.

Section 2.

Therefore, by the compound ratio, as the two lines *A* and *F* together are to the line *A*, so are the two solids *AB* and *CD* together to the solid *AB*.

Section 3.

The line *A* is to the line equal to the two lines *A* and *F* in the triplicate ratio of that of the line *A* to the line *G*, by the construction. Therefore, as the line *A* is to the two lines *A* and *F*, so is the solid *AB* to the solid *GH*. And by the inverted ratio of this: as the two lines *A* and *F* together are to the line *A*, so is the solid *GH* to the solid *AB*. But it has been shown in the second section that the ratio of the two solids *AB* and *CD* to the same solid *AB* is the same. Consequently (because things whose ratios to the same thing are equal are equal to one another) the solid *GH* is equal to the two solids *AB* and *CD*.

Further it is evident from the construction that the solid *GH* is similar to the solid *AB*.

Conclusion.

Therefore, given any two etc. Which was to be performed.

We can also (for greater revelation of the correctness of the construction of this invention) set forth the construction of the aforesaid Problem by means of numbers, which may be done in the following manner:

Given.

Let there be a cube *AB*, whose volume is 8 feet; therefore its side *A* will be *cubica de 8 erit 2: Sit deinde alter cubus* C D, *cuius magnitudo 19 pedum, quare eius latus* C, *erit radix cubica de 19.*

Explicatio quæsiti.

Oporteat per numeros tertium cubum describere (eo ordine vt supra per lineas descriptus est) datis duobus cubis A B. & C D *æqualem.*

Constructio.

Procedamus eo ordine vt supra factum est, inveniaturque tertia linea proportionalis quarum prima A 2, *secunda* C *radix cubica de 19, quare tertia linea* E *erit radix cubica de* $\frac{361}{8}$, *inveniatur deinde quarta linea proportionalis quarum prima* A 2, *secunda* C *radix cubica de 19, tertia* E *radix cubica de* $\frac{361}{8}$: *Igitur quarta erit* F *radix cubica de* $\frac{6859}{64}$, *id est* $\frac{19}{4}$: *Inveniantur deinde duæ mediæ lineæ proportionales inter rectam* A 2, & *rectam æqualem duabus rectis* A 2 & F $\frac{19}{4}$, *hoc autem in numeris ita proponendum est: Inveniantur duo medij numeri proportionales inter* 2, & $\frac{27}{4}$, *illorum autem mediorum numerorum numerus, sequens numerum* 2, *erit pro recta* G 3, *probatur: quia* 2, 3, $\frac{9}{4}$, $\frac{27}{4}$, *sunt in continua proportione.*

Igitur ex recta seu latere G 3 *construatur cubus* G H, *cuius magnitudo erit 27, pedum.*

Dico tertium cubum G H, *per numeros esse inventum eo ordine vt supra per lineas factum est, & datis duobus cubis* A B & C D *æqualem, vt erat quæsitum.*

Demon-

the cubic root of 8 = 2. Let there also be another cube *CD*, whose volume is 19 feet; therefore its side *C* will be the cube root of 19.

Required.

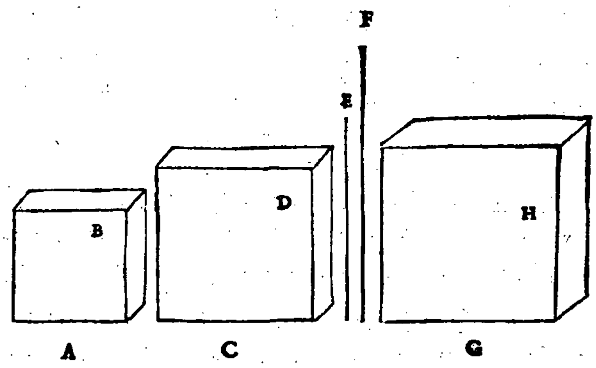
Let it be required to construct, by means of numbers, a third cube (in the same order as has been constructed above by means of lines), equal to the two given cubes *AB* and *CD*.

Construction.

Let us proceed in the same order as has been done above, and find the third proportional, the first term being $A = 2$, the second $C =$ the cube root of 19; therefore the third line E will be the cube root of $\frac{361}{8}$ Then find the fourth proportional, the first term being $A = 2$, the second $C =$ the cube root of 19, the third $E =$ the cube root of $\frac{361}{8}$. Therefore the fourth will be $F =$ the cube root of $\frac{6859}{64}$, *i.e.* $\frac{19}{4}$. Subsequently find the two mean proportionals between the line $A = 2$ and the line equal to the two lines $A = 2$ and $F = \frac{19}{4}$; however, this has to be exposed in numbers as follows: Find the two mean proportionals between 2 and $\frac{27}{4}$; however, the number of these mean proportionals which follows the number 2 will be, for the line G, 3; this is proved by the fact that 2, 3, $\frac{9}{4}$, $\frac{27}{4}$ are in continuous proportion.

Therefore, from the line or side $G = 3$ construct a cube *GH*, whose volume will be 27 feet.

I say that a third cube *GH* has been found by means of numbers in the same order as has been done above by means of lines, and equal to the two given cubes *AB* and *CD*; as was required.



Demonſtratio.

Demonſtratio ex eo manifeſta eſt, quod cubus A B 8 *pedum, & cubus* C D 19 *pedum, efficiunt ſimul vt ſupra de cubo* G H *oſtenſum eſt,* 27 *pedes.*

Concluſio.

Igitur quod primò in continua quantitate erat oſtenſum, hic per numeros ſimiliter oſtenſum eſt, quod in maiorem declarationem erat faciendum.

His ita demonſtratis applicabimus præcedenti primo Problemati ſuum Theorema tale.

THEOREMA.

Si quarta linea proportionalis duarum homologarum linearum exiſtentium in ſimilibus corporibus, addatur primæ lineæ: Antecedens linea duarum mediarum proportionalium inter primam & illam compoſitam, eſt potentialiter homoga linea cum illis homologis lineis, cuiuſdam corporis quod ſimile eſt alteri datorum, & æquale ambobus.

P Expli-

Proof.

The proof is evident from the fact that the cube $AB = 8$ feet and the cube $CD = 19$ feet together make 27 feet, as has been shown above for the cube GH.

Conclusion.

Therefore, what had first been shown in continuous quantity has here been similarly shown by means of numbers, which was to be done for greater clarity.
These things thus having been proved, we shall add to the preceding first Problem its Theorem, as follows.

THEOREM.

If the fourth proportional to two homologous lines occurring in similar solids be added to the first line, the antecedent of the two mean proportionals between the first and this composite line is potentially a line homologous to those homologous lines of a certain solid which is similar to one of the given solids and equal to both *).

*) If two similar solids S_1 and S_2 are to each other as the cubes of homologous lines p_1 and p_2, or $S_1 : S_2 = p_1^3 : p_2^3$, then $S_1 : (S_1 + S_2) = p_1^3 : (p_1^3 + p_2^3)$.

Explicatio dati.

Sit (in figuris præcedentis Problematis) quarta linea proportionalis F, *duarum linearum* A *&* C, *existentium in similibus corporibus* A B *&* C D, *quæ linea* F, *addatur primæ lineæ* A.

Dico antecedentem lineam G, *duarum mediarum proportionalium inter primam* A, *& illam compositam, nempe ex* F *&* A, *esse potentialiter homologam lineam cum illis homologis lineis* A *&* C, *cuiusdam corporis vt* G H, *quod simile est alteri datorum, vt ipsi* A B, *& æquale ambobus corporibus* A B *&* C D.

Demonstratio.

Demonstratio habetur in præcedentibus demonstrationibus, tum per lineas, tum per numeros.

Conclusio.

Igitur si quarta linea &c. quod erat demonstrandum.

PROBLEMA III.

Datis quibuscunque duorum similium & inæqualium Geometricorum corporum homologis lineis, tertium corpus construere, tanto minus dato maiore, quantum est datum minus, & alteri datorum simile.

Explicatio dati.

Sit datum corpus minus A B, *maius vero corpus* C D, *ipsi* A B *simile, quorum homologæ lineæ sint* A *&* C.

Explicatio quæsiti.

Oporteat tertium corpus construere tanto minus dato maiore C D, *quantum est datum corpus* A B: *Præterea vt sit corpori* C D *simile.*

NOTA.

Given.

(In the figure of the preceding Problem) let the fourth proportional F to two lines A and C, occurring in the similar solids AB and CD be given, which line F shall be added to the first line A.

I say that the antecedent line G of the two mean proportionals between the first A and this composite line, *viz.* composed of F and A, is potentially a line homologous to those homologous lines A and C of a certain solid, *viz.* GH, which is similar to one of the given solids, *viz.* to AB, and equal to both solids AB and CD.

Proof.

The proof will be found in the preceding proofs, both by means of lines and by means of numbers.

Conclusion.

Therefore, if the fourth line etc.; which was to be proved.

PROBLEM III.

Given any homologous lines of two similar and unequal Geometrical solids, to construct a third solid as much smaller than the larger of the given solids as the smaller of the given solids and similar to one of the given solids.

Given.

Let the given smaller solid be AB, and the larger solid CD, similar to AB, whose homologous lines shall be A and C.

Required.

Let it be required to construct a third solid as much smaller than the given larger solid CD as the given solid AB; and further that it be similar to the solid CD.

NOTA.

Hoc Problema (ab antecedenti primo Problemate dependens) ita se habet ad antecedens primum Problema, vt in Arithmetica subtractio ad additionem: Quare si præcedens Problema vocaretur similium corporum additio, posset eadem ratione hoc Problema dici similium corporum subtractio. Igitur vt Problematis sensus dilucidior fiat, dicimus quæsitum ferè nil aliud esse quam sublata à corpore C D *quadam parte corpori* A B *æquali, quod à reliquo oporteat corpus construere toti corpori* C D *simile.*

Constructio.

Inveniatur tertia linea proportionalis per 11. *prop. lib.* 6. *Euclid. quarum prima* A, *secunda* C, *sitque tertia* E: *Inveniatur deinde quarta linea proportionalis per* 12. *prop. lib.* 6. *Euclid. quarum prima* A, *secunda* C, *tertia* E, *sitq; quarta* F: *Inveniantur deinde duæ mediæ lineæ proportionales per primum Problema præcedentis quarti libri inter rectam* A, *& alteram rectam, æqualem reliquo rectæ* F, *subducta recta* A, *quarum mediarum linearum sequens rectam* A, *sit recta* G, *ex qua vt homologa linea cum linea* A *construatur corpus* G H *corpori* C D *simile.*

Dico tertium tertium corpus G H *esse inventum, tanto minus dato corpore* C D, *quantum est corpus* A B, *& corpori* C D *simile, vt erat quæsitum.*

NOTE.

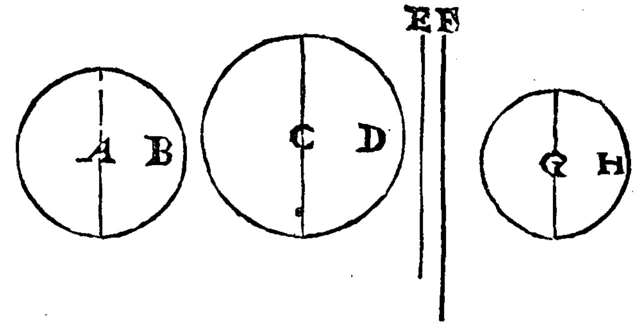
This Problem (depending on the antecedent first Problem) is in the same relation to the antecedent first Problem as subtraction to addition in Arithmetic. Therefore, if the preceding Problem were called the addition of similar solids, this Problem might for the same reason be called the subtraction of similar solids. Therefore, in order that the sense of the Problem may become clearer, we say that what is required is hardly anything else but that, after a certain part equal to the solid *AB* has been taken from the solid *CD*, it is required to construct from the rest a solid similar to the whole solid *CD*.

Construction.

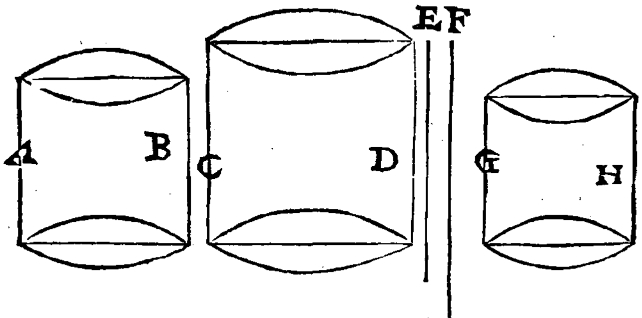
By the 11th proposition of Euclid's 6th book, find the third proportional, the first term being *A*, the second *C*; and let the third be *E*. Then, by the 12th proposition of Euclid's 6th book, find the fourth proportional, the first term being *A*, the second *C*, the third *E*; and let the fourth be *F*. Subsequently, by the first Problem of the preceding fourth book, find the two mean proportionals between the line *A* and another line, equal to the rest of the line *F* when the line *A* has been taken from it, and let that one of these mean proportionals which follows the line *A* be the line *G*, from which, as being a line homologous to the line *A*, construct a solid *GH* similar to the solid *CD*.

I say that a third solid *GH* has been found, as much smaller than the given solid *CD* as the solid *AB*, and similar to the solid *CD*; as was required.

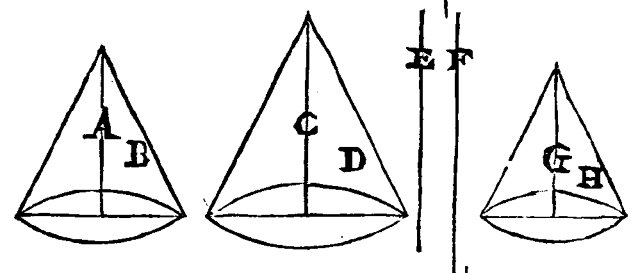
Exemplum de ſphæris.



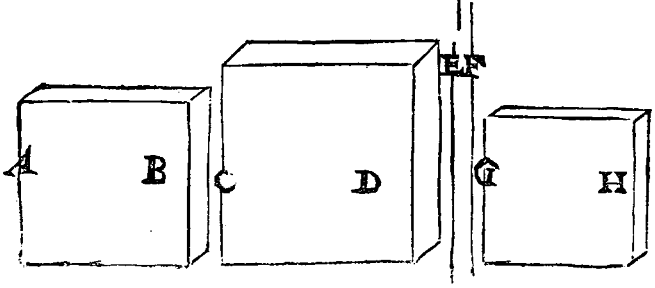
Exemplum de cylindris



Exemplum de conis.



Exemplum de cubis.



Example of Spheres
Example of Cylinders
Example of Cones
Example of Cubes

Demonſtratio.

Diſtinctio 1.

Recta A, *ad rectam* F, *triplicatam eam habet rationem quam recta* A, *ad rectam* C, *per conſtructionem: Quare vt recta* A, *ad rectam* F, *ſic corpus* A B, *ad corpus* C D, *per* 33. *prop. lib.* 11. *& per* 8, 12, *&* 18. *prop. lib.* 12. *Euclid.*

Diſtinctio 2.

Quare per diſiunctam proportionem, vt recta F *minus recta* A, *ad rectam* A, *ſic corpus* C D, *minus corpore* A B, *ad corpus* A B.

Diſtinctio 3.

Recta A, *ad rectam* F, *minus recta* A, *triplicatam eam habet rationem quam recta* A, *ad rectam* G, *per conſtructionem: Quare vt recta* A, *ad rectam* F, *minus recta* A, *ſic corpus* A B, *ad corpus* G H: *Et per ipſius inverſam proportionem, vt recta* F, *minus recta* A, *ad rectam* A, *ſic corpus* G H, *ad corpus* A B: *Sed oſtenſum eſt diſtinctione ſecunda, eandem eſſe rationem corporis* C D, *minus corpore* A B, *ad idem corpus* A B: *Ergo (quia quorum rationes eidem ſunt æquales ea inter ſe ſunt æqualia) corpus* G H, *æquale eſt corpori* C D, *minus corpore* A B, *hoc eſt corpus* G H, *tanto minus eſt dato corpori* C D, *quantum eſt datum corpus* A B.

Concluſio.

Igitur datis quibuſcunque duorum &c. Quod erat faciendum.

His ita demonſtratis applicabimus hoc Problemati ſuum Theorema tale.

THEOREMA

Si à quarta linea proportionali duarum homologarum linearum exiſtentium in ſimilibus inæqualibus corporibus, auferatur minor

 homo-

Proof.

Section 1.

The line A is to the line F in the triplicate ratio of that of the line A to the line C, by the construction. Therefore, as the line A is to the line F, so is the solid AB to the solid CD, by the 33rd proposition of Euclid's 11th book and by the 8th, 12th, and 18th propositions of his 12th book.

Section 2.

Therefore, by the disjunct proportion: as the line F minus the line A is to the line A, so is the solid CD minus the solid AB to the solid AB.

Section 3.

The line A is to the line F minus the line A in the triplicate ratio of that of the line A to the line G, by the construction. Therefore, as the line A is to the line F minus the line A, so is the solid AB to the solid GH. And, by the inverted proportion of this: as the line F minus the line A is to the line A, so is the solid GH to the solid AB. But it has been shown in the second section that the ratio of the solid CD minus the solid AB to the same solid AB is the same. Consequently (because things whose ratios to the same thing are equal are equal to one another) the solid GH is equal to the solid CD minus the solid AB, *i.e.* the solid GH is as much smaller than the given solid CD as the given solid AB.

Conclusion.

Therefore, given any etc. Which was to be performed.

These things thus having been proved, we shall add to this Problem its Theorem, as follows.

THEOREM.

If from the fourth proportional of two homologous lines occurring in similar unequal solids the smaller of the homologous lines be taken away, the ante-

homologarum: Antecedens linea duarum mediarum proportionalium, inter minorem homologam lineam, & illius lineæ reliquum, est potentialiter homologa linea cum illis homologis, cuiusdam corporis quod simile est alteri datorum corporum, & tanto minus dato maiore, quantum est datum minus.

Explicatio.

Sit in figuris præcedentis secundi Problematis quarta linea proportionalis F, *duarum homologarum linearum* A *&* C, *ex similibus inæqualibus corporibus* A B, *&* C D, *à qua linea* F, *auferatur minor homologarum* A, *dico minorem lineam* G, *duarum mediarum proportionalium linearum inter minorem homologam lineam* A, *& illius lineæ reliquum (hoc est reliquum subductæ* A *ab* F*) esse potentialiter homologam lineam cum illis homologis lineis* A, *&* C, *cuiusdam corporis vt ipsius* G H, *quod simile est alteri datorum corporum, vt ipsi* C D, *& tanto minus dato maiore* C D, *quantum est datum minus corpus* A B.

Demonstratio.

Demonstratio huius supra exhibita est.

Adhiberemus huic secundo Problemati demonstrationem per numeros, vt ad primum Problema factum est, nisi rem satis claram existimaremus.

Quinti Libri
FINIS.

cedent of the two means proportionals between the smaller homologous line and the rest of the said line is potentially a line homologous to those homologous lines of a certain solid which is similar to one of the given solids, and as much smaller than the given larger solid as the given smaller solid.

Explanation.

In the figures of the preceding second Problem let the third proportional *F* to two homologous lines *A* and *C* from similar unequal solids *AB* and *CD* be given, from which line *F* let the smaller of the homologous lines *A* be taken; I say that the smaller line *G* of the two mean proportionals between the smaller homologous line *A* and the rest of this line *(i.e.* the rest after *A* has been taken from *F*) is potentially the line homologous to these homologous lines *A* and *C* of a certain solid, *viz.* of *GH*, which is similar to one of the given solids, *viz.* *CD*, and as much smaller than the given larger solid *CD* as the given smaller solid *AB*.

Proof.

The proof of this has been set forth above.

We should have added to this second Problem a proof by means of numbers, as has been done for the first Problem, if we had not considered the matter clear enough.

END OF THE FIFTH BOOK.

Epilogus.

Hæc ſunt Generoſiſs. D. quæ tibi dicare deſtinauimus, quæ ſi A. T. grata eſſe ſentiemus, alia habemus Matheſium arcana ſub tui Nominis auſpicijs proditura: Interim hæc qualiacunque boni conſules. Vale. Ego tibi me quam officioſiſsimè commendabo Lugduni Batauorum.

ERRATA.

Pagin. 9. in explicationis linea prima. ad rectū, lege ad rectam. 14. in explicationis lin. 1. ex minoribus, lege ex paucioribus. 15. in explicationis 17. definitionis lin. 2. ex minoribus, lege ex paucioribus. 16. in explicationis lin. 4. A G ad G B, lege A G ad G C. 19. lin. 3. ſecundam lege ſecundum. Et in conſtructionis lin. 2. A B C D E E lege A B C D E F. 20. in demonſtrationis lin. 3. ſic parallelogrammum M N, lege ſic parallelogrammū L G ad parallelogrammum M N. 26. in figura pro L pone M, & pro M pone L. 27. in conſtructionis lin. 1. inueniūtur lege inveniantur. 31. in conſtructionis lin. 1. diſcribatur lege deſcribatur. Et lin. 5. A B Q lege A P Q. Et lin. 9. A P lege A P Q. 33. in conſtructionis lin. 3. quæ productæ lege quæ producta. 34. in demonſtrationis lin. 17. habeat lege habet. Et lin. 21. habeat lege habet. 38. lin. 1. quā breuiter lege quam breui. 44. in conſtructionis 4. lin. ſique lege ſitque. 48. lin. 4. oppoſita lege appoſita. Et lin. 5. baſes lege baſes. 54. lin. vltima, icoſaedrum per laterum tertias, lege icoſaedrum truncatum per laterum tertias. 56. diſt. 8. lin. 7. Y Z lege X Z. Et lin. 10. ſequentur lege ſequetur. 58. diſt. 14. lin. 9. in rectam lege in rectam. 60. in figura pro 35 pone 53. 62. diſtin. 3. lin. 8. text lege tertiæ. Et diſt. 5. lin. 1. dodecaedri medium lege dodecaedri in medium. Et lin. 2. ad angū ad angulos. Et lin. 3. quare E lege quare E 51. 63. lin. 1. cta lege recta. 103. in concluſione duabus lege duobus. 113. in theorematis lin. 4. homoga lege homologa.

EPILOGUE

These are, most noble Lord, the problems which we resolved to dedicate to Thee, and if we learn that they please Thee, we shall publish other secrets of Mathematics under the protection of Thy Name. Meanwhile Thou will take all these, without distinction, in good part. Farewell. I commend myself to Thee most respectfully, at Leiden.

DE THIENDE

THE TENTH

INTRODUCTION

§ 1.

Stevin published his essay on the decimal division in 1585, when he had lived for at least four years in Holland, probably most of the time, if not all, at Leiden. It was a period of intensive work, in which between 1582 and 1586 he prepared for publication the whole series of books, mentioned on pp. 26-27 of Vol. I. Once he had settled down after his peregrinations, he used the opportunity to publish, one after the other, the results of his experience and reflection.

De Thiende, English *The Tenth,* or *The Disme*, is by far the best known of Stevin's publications; it earned him the title of inventor of the decimal fractions. The title, if taken with a grain of salt, is deserved. It is true that decimal fractions appeared long before Stevin, but it was largely through his efforts that they eventually became common computational practice. It was also Stevin who first showed the advantage of a systematic decimal division of weights and measures.

By 1585 the system of Hindu-Arabic numerals, decimal and in positional notation, was in common use throughout Europe. Even the particular shapes of the ten digits 0, 1, 2, 3, 4, 5, 6, 7, 8, 9 had been more or less standardized and did not substantially differ from the shapes familiar to us at the present time. This system supplemented, but did not supplant, the older system which made use of counters on lines (Stevin's "legpenninghen", p. 34 of *De Thiende,* French "gettons") (1). Schools and schoolbooks often taught both methods. On the advanced front of learning the decimal positional system had been fully accepted. The great sixteenth-century progress in computational technique would have been impossible without it. However, there were still many who avoided fractions as hard to handle, and those who did use them often worked with different notations. This lack of consistency has never been completely removed, so that even now we write $4\frac{1}{4}$ also in the form 4¼ or 4.25 (4,25; 4·25), and in angular notation in the form 4°15′. The notation 4.25 is clearly the result of applying the decimal method to fractions with cold consistency. It is Stevin's merit that he demonstrated the simplicity of this approach, even though his own particular notation was still clumsy.

The textbooks of the sixteenth century usually presented fractions with the aid of numerator and denominator, as it is still done. There were variations in the way these two parts of the fraction were distinguished from each other, sometimes with, sometimes without a fractional bar, sometimes by placing one above, sometimes beside the other. A special symbol might be introduced for

• In the bibliographical quotations, (H) means: Harvard Library; (Hu) means: Huntington Library.

(1) On the use of these counters see A. Nagel, *Die Rechenpfennige und die operative Arithmetik.* Numismatische Zeitschrift (Wien) 19 (1887) pp. 309–368; F. P. Barnard, *The Casting Counter and the Counting Board*, Oxford, 1916; C. P. Burger, *ABC penningen of rekenpenningen*, Het Boek 18 (1929), pp. 193–202, also *ib.* 19 (1930), p. 222; L. C. Karpinski, *The History of Arithmetic*, Chicago–New York 1925, pp. 33–37.

some simple fractions, such as ½ (2). For comparison with large denominators sexagesimal fractions were widely used, usually without explicitly expressing these denominators. This method dates back to ancient Mesopotamia, was used by Ptolemy in his *Almagest*, and is still in use for angular measurement; in it a symbol such as $4°21'33''14'''$ means $4 + \frac{21}{60} + \frac{33}{3600} + \frac{14}{216000}$. Another method, favoured by table-makers, was to eliminate fractions altogether by the choice of a sufficiently large unit. Here was a vital case where thinking in decimal rather than in sexagesimal terms became more and more common when the sixteenth century advanced.

Ptolemy's chord tables in the *Almagest* had been composed for a circle with radius $R = 60$, and both angles and chords were expressed in the sexagesimal system, so that chord $60° = 60^p$, and chord $176° = 119^p 55'38''$, which is equivalent to sin $30° = 30^p$, sin $88° = 59^p\ 57'49''$ (3). George Peurbach, the Viennese astronomer (1423-1416), left a table of sines computed for $R = 60{,}000$, but with the sines expressed decimally, so that sin 30° = 30,000; sin 88° = 59,964 (sines were conceived as line segments—semi-chords of the double arc—, not as ratios) (4). This method was taken over by Peurbach's pupil Regiomontanus (1436—1476), the Ian van Kuenincxberghe of *De Thiende*, p. 20, Jehan de Montroial of the French edition, p. 148, who not only used the unit $R = 6.10^4$ in the sine table of his *Tabula directionum*, but also the unit $R = 6.10^6$ in his Supplement to Peurbach's *Tractatus* on Ptolemy's propositions on sines and chords. In other tables Regiomontanus took a new step in the direction of the decimal system. In the same *Tabula directionum* we find a tangent table based on $R = 10^5$ and in the Supplement to Peurbach's *Tractatus* computations based on $R = 10^7$ (5). These tables, published many years after Regiomontanus' death,

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(2) F. Cajori, *A History of Mathematical Notations* I, Chicago, 1926, pp. 309. ff; J. Tropfke, *Geschichte der Elementar-Mathematik* I, 3e Aufl. Berlin–Leipzig, 1930, pp. 172 ff.; D. E. Smith, *History of Mathematics* II, Boston–New York, pp. 235 ff. These books contain much information on the history of common and of decimal fractions. For decimal fractions see also G. Sarton, *The First Explanation of Decimal Fractions and Measures*, Isis 33 (1935), pp. 153–244.

(3) The chord table is found in Book I, Ch. 2, of the *Almagest*. It is equivalent with a table of sines, for angles ascending by 15', the results are accurate to 5 decimals. We write *p* (partes) to express lengths in sexagesimal units. A transcription of the chord tables in the familiar Hindu-Arabic notation *e.g.* in *Des Claudius Ptolemäus Handbuch der Astronomie*, Erster Band... übersetzt... von K. Manitius. Leipzig, 1912, pp. 37–40.

(4) These tables by Peurbach are in the *Österreichische Nationalbibliothek: Cod. Vindob.* 5277, fol. 287^r–289^v. They are mentioned in J. Tropfke, *l.c.* 2) p. 175.

(5) We have consulted the 1559 edition of the *Tabulae directionum: Ioannis de Monteregio Mathematici clarissimi tabulae directionum projectionumque... eiusdem Regiomontani tabula sinuum, per singula minuta extensa...* Tubingae, 1559, 156 double pages (H). The tangent table is the one page *Tabula foecunda* (p. 29^r), which lists for tan 45° the value 100,000, hence $R = 10^5$. The sine table (pp. 139 ff) lists 30,000 for sin 30°, hence $R = 6.10^4$. The first edition of this work was published at Augsburg 1490. – The tables with $R = 6.10^6$ and $R = 10^7$ in *Tractatus Georgii Purbachii super propositiones Ptolemai de sinubus et chordis*, Nuremberg, 1541, according to J. Tropfke, *Geschichte der Elementar Mathematik* V, 2e Aufl., Berlin–Leipzig, 1923, pp. 178, 179. See also A. v. Braunmühl, *Vorlesungen über Geschichte der Trigonometrie* I, Leipzig, 1900, p. 120; M. Cantor, *Vorlesungen über Geschichte der Mathematik* II, Leipzig, 1892, Ch. 55.

enjoyed considerable authority during the sixteenth century, and helped to establish the decimal system as the basis for the computation of trigonometrical tables. The great tables of that period, such as the *Opus Palatinum,* were all based on a radius equal to a power of 10.

The basis $R = 1$ remained unpopular for a long time, because of the lack of a convenient notation for decimal fractions. Stevin himself, in his *Tables of Interest,* which antedate *De Thiende,* and in his trigonometrical tables, published afterwards, used 10^7 as his unit. The basis $R = 1$ gained acceptance mainly through the influence of the logarithmic tables, and it was here that Stevin's suggestions fell on willing ears.

There are indications that in the period before the appearance of Stevin's booklet mathematicians began to appreciate the use of a decimal notation in working with fractions. There is an early — though not the earliest — example in a Hebrew manuscript written by Rabbi Immanuel Bonfils of Tarascon about 1350. Here we find a proposal for a system in which the unit is divided "into ten parts which are called Primes, and each Prime is divided into ten parts which are called Seconds, and so into infinity". For such fractional quantities Bonfils gives some rules of multiplication and division, which result from what we now call the exponential law $10^a.10^b = 10^{a+b}$ (a, b positive integers). These rules are applicable to denominators as well as numerators, a fact we express by allowing a, b to be positive as well as negative. The manuscript has no numerical examples (6). It is of some importance because Tarascon, in 1350, was an important trading and cultural centre close to the Papal Court at Avignon.

At about the same time the Paris astronomer John of Meurs (Iohannes de Muris) computed $\sqrt{2}$ by means of decimal magnification; in our present notation his reasoning may be transcribed as follows:

$$\sqrt{2} = \frac{1}{1000}\sqrt{2{,}000{,}000} = \frac{1}{1000}\ 1414.$$

He remarks that the result, $1^0 24' 50'' 24'''$ in sexagesimal fractions, can also be expressed by considering $\sqrt{2}$ as equal to 1414, if the first digit is taken as an integer, the next one as a tenth, etc. (7).

In sixteenth-century printed mathematical texts we find some play with decimal fractions, written either with denominators as common fractions, or in some positional notation without denominator. For example, in the *Exempel Büchlein* of 1530 we find the author Christoff Rudolff teaching the compound interest calculus with a table for $375\ (1 + \frac{5}{100})^n$, $n = 1, 2, \ldots, 10$. He writes the results in a notation which only differs from our notation of the decimal fractions in the use of a vertical dash as the decimal separatrix, *e.g.* 413|4375 for the case $n = 2$ (8). Another case is presented by François Viète in his *Canon*

(6) S. Gandz, *The Invention of the Decimal Fractions and the Application of the Exponential Calculus by Immanuel Bonfils of Tarascon* (c. 1350), Isis 25 (1936), pp. 16–35.

(7) In *Quadripartitum numerorum* (c. 1325), see L. C. Karpinski, *The Decimal Point,* Science, 25 (1917), pp. 663–665. The quotation is from Vienna Ms. 4770, fol. 224^v. This method of decimal magnification is much older, see J. Tropfke *l.c.*[2]) p. 173. On this method see also J. Ginsburg, *Predecessors of Magini,* Scripta Mathematica (1932), pp. 168–169.

(8) *Exempel Bůchlin Rechnung belangend darbey... durch Christoffen Růdolff,* Augsburg, 1530, Aufg. 71. Reproduced in G. Sarton, *l.c.*[2]) p. 225 and in D. E. Smith *l.c.*[2]) p. 241, see also J. Tropfke *l.c.*[2]) p. 177, where we also find a quotation from another book by Rudolff: *Behend und hubsch Rechnung durch die kunstreichen regeln Algebre,* Strassburg 1525.

mathematicus of 1579, where we occasionally find decimal fractions without denominator, and the fractional part of the number in smaller type than the integral part and underlined. Writing 100,000$\underline{^{000,00}}$ for the radius of a circle, Viète places the semi-perimeter between 314,159,$\underline{^{265,35}}$ and 314,159,$\underline{^{265,37}}$. The commas are used to arrange the digits into groups of three. In another place in the same book we find for sin 60° the value 86,602|540,37, in another place again we find fractions with numerator and denominator (9).

§ 2.

Stevin's achievement consists in divesting the decimal fractions of their casual character. In doing this, he appealed to the learned as well as the practical world, to the reckonmaster as well as to the merchant and the wine gauger. He advertised the advantages of his decimal notation on the very title page of his pamphlet, proclaiming that he was "teaching how to perform with an ease, unheard of, all computations necessary between men by integers without fractions".

At a time when the fractional calculus and division in general were considered difficult operations (10), this computation by integers without fractions must have appealed to many. Stevin in particular appealed to the man of practice, for whose benefit he wrote in the vernacular and endeavoured to be as simple and clear as possible.

Stevin's claim that he could perform all computations by integers without fractions strikes us as rather odd, since he is supposed to have contributed more than anybody else to the introduction of decimal fractions. Yet he claimed that he had done away with fractions. It is true that, historically speaking, the result of Stevin's work was that the fractional calculus became as easy as the calculus with integers. But it is also true that Stevin was thinking primarily of the elimination of fractions. He accomplished this by introducing the tenth part of a unit, ①, the hundredth part of a unit, ②, as new units, so that for instance the fraction which we write 47.58 and Stevin 47⓪5①8② was regarded by Stevin as 4758② — a notation which he also used —, or 4758 items of the second unit. We do a similar thing when we express miles in feet, hectares in ares, or gallons in pints. However, especially after Napier introduced the notation 47.58 with special reference to Stevin, Stevin's method was understood as that of decimal fractions.

Stevin's notation seems to us clumsy and also less elegant than that which Rudolff used more than fifty years earlier. The notation 32⓪5①7② reminds us of the sexagesimal notation, where a symbol such as 5°7′26″34‴ can only be understood if the 5, 7, 26, 34 are separated by certain marks. This results from the fact that this sexagesimal notation is already mingled with a decimal one, since the number of units, minutes, seconds, etc. is expressed decimally (26 means

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(9) Viète's book is a treatise on plane and spherical trigonometry entitled: *Canon mathematicus seu ad triangula cum adpendicibus*, Lutetiae, 1579 (H). The explanatory text of 6 + 75 pp, entitled *Francisci Vietae Universalium inspectionum ad canonem mathematicum liber singularis*, has five appendices, all tables. Our examples are on p. 15 and p. 64. See also K. Hunrath, *Zur Geschichte der Decimalbrüche*, Zeitschr. f. Mathem. u. Physik 38 (1893), Hist. lit. Abt, pp. 25–27.

(10) See *e.g.* H. E. Timerding, *Die Kultur der Gegenwart* III, Erste Abteilung, Die mathematischen Wissenschaften, Zweite Lieferung, Leipzig–Berlin, 1914, p. 92A.

2 . 10 + 6, not 2 : 60 + 6). Stevin's notation, however, is purely decimal. He might have written 32°5′7″ (as some of his successors have done), but he preferred the 0-notation which he had also used elsewhere to indicate powers, albeit not necessarily powers of ten. This was probably due to the influence of the Bolognese mathematician Bombelli, who, in his *Algebra* of 1572, had used half circles with numbers as insets to indicate powers of the variable, an improvement on the current Coss notation (11). Stevin had studied Bombelli, whom he quotes in *L'Arithmétique* (12). This notation therefore means that 32⓪5①7② = $32 \cdot 10^0 + 5 \cdot 10^{-1} + 7 \cdot 10^{-2}$, to use our modern way of writing. Stevin was not too orthodox in his commitment to his own notation; in other books he wrote $\overset{\textcircled{0}}{5}\ \overset{\textcircled{1}}{7}\ \overset{\textcircled{2}}{8}\ \overset{\textcircled{3}}{9}$, or the simpler 732② for what we write as 7.32 (13). An advantage of his method was that he could add 7⓪5①8② to 4⓪7①5② and get 11⓪12①13② = 11⓪13①3② = 12⓪3①3②, which may have been helpful to inexperienced reckoners, who could thus keep track of intermediate stages in the process of calculation. Stevin was also able to do away with zeros: 5②4(5) means 0.05004 in our notation.

Stevin gives a proof that his method allows the handling of decimal fractions as if they were integers by rewriting these fractions in the form $\frac{a}{b}$, where b is an appropriate power of ten, and then applying the rules for the computation with these fractions as explained in *L'Arithmétique*. The result is then again cast into the decimal 0-notation. This proof is substantially the same we use, though, in accordance with his time, Stevin gives numerical examples where we should express ourselves in algebraic notation (14). He gives his demonstrations in the classical way with the terms *Given, Required, Construction, Demonstration, Conclusion*, which shows that he realized that careful proofs are as necessary in arithmetic as in geometry, an unusual thing for his day, and for many days to come.

§ 3.

After the appearance of *De Thiende* decimal fractions appeared more and more frequently in print. It is safe to assume that Stevin's work contributed to this growing popularity without ascribing the success exclusively to him. With all the table-making and other reckoning in progress decimal fractions were "in the

(11) R. Bombelli, *L'Algebra*, Bologna 1572, 1579. On Bombelli's symbolism see E. Bortolotti, *Sulla rappresentazione simbolica della incognita e delle potenze di essa introdotta dal Bombelli*, Archivio di Storia della Scienza 8 (1927) pp. 49–63. On the Coss-notation see F. Cajori *l.c.*[2]), Ch III, and J. Tropfke, *Geschichte der Elementar-Mathematik* III, Berlin–Leipzig, 3e Aufl., pp. 31–32.

(12) *L'Arithmétique*, see *e.g.* p. 28. At other places Stevin uses his o-notation to indicate sexagesimal fractions: *Weereltschrift* I p. 59, III p. 18.

(13) *Meetdaet*, see *e.g.* p. 32.

(14) For this he was rebuked – posthumously – by A. Tacquet, in *Arithmeticae theoria et praxis*, Lovani, 1646 (pp. 177–179 of the 2d ed., Antwerp, 1665): "Mais (Stevin) n'a pas exposé son invention avec toute l'exactitude, ni l'ampleur nécessaire et il ne l'a pas démontrée, car, ce qu'il nomme démonstration ne consiste qu' à donner un example". French transl. of H. Bosmans, *André Tacquet (S. J.) et son traité d'Arithmétique théorique et pratique*, Isis 9 (1927), pp. 66–82. - For a modern introduction to decimal fractions see *e.g.* G. H. Hardy – E. M. Wright, *An Introduction to the Theory of Numbers*, Oxford, 2d ed., 1945, Ch. IX.

air". Stevin himself saw to it that a French translation of his pamphlet under the name of *La Disme* was published in the same year 1585 in which *De Thiende* appeared. This translation was added to the collection of essays which he published under the title of *La Pratique d'Arithmétique,* a collection always bound together with Stevin's large treatise *L'Arithmétique* (15). *De Thiende* itself was reprinted in 1626, after Stevin's death, this time as an appendix to a book compiled by De Decker, who published it again in 1630 as part of another book (16). When Girard, in 1634, published the *Oeuvres* of Stevin, he included the French version (17). There were thus several ways by which mathematicians could become acquainted with the ideas of our pioneering engineer.

They were not the only opportunities. Two English versions appeared, a literal one by Robert Norton, published in 1608 (18), and a freer one by Henry Lyte, published in 1619 (19). We have taken the Norton translation to serve as the English version of *De Thiende* in the present edition of Stevin's works. In the mean time decimal fractions had appeared in books not, or only partly, influenced by Stevin, often rather casually, as if the authors were not sufficiently aware of the fundamental importance of the innovation. Clavius (1537-1612), the influential Vatican astronomer and a prolific textbook writer, used the notation 34.4 for the *partes proportionales* in his sine table of 1593, though the sines themselves appear as integers to the radius $R = 10^7$ (20). Less casual was the

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(15) *La Disme, enseignant facilement expédier par nombres entiers sans rompuz, tous comptes se rencontrans aux affaires des Hommes*... Leiden, Plantin, 1585, pp. 132–160 of the *Pratique d'Arithmétique.*

(16) *Eerste Deel van de Nieuwe Telkunst, inhoudende verscheyde manieren van rekenen... door Ezechiel De Decker... Noch is hier achter bygevoeght de Thiende van Symon Stevin van Brugghe.* Ter Goude, by Pieter Rammaseyn... 1626. – See on this book M. van Haaften, *De Decker's Eerste deel van de Nieuwe Telkonst,* De Verzekeringsbode, 25 Sept. 1920 (pp. 406–410). De Decker's book of 1630 is called *Nieuwe Rabattafels,* Gouda, Rammaseyn.

(17) In the reprint of *L'Arithmétique* and *La Pratique d'Arithmétique,* discussed further on in this volume.

(18) *Dime: The Art of Tenths, or Decimall Arithmetic, teaching how to perform all computations whatsoever by whole numbers without fractions, by the four principles of common arithmetic, namely: addition, subtraction, multiplication, and division. Invented by the excellent mathematician, Simon Stevin. Published in English with some additions by Robert Norton, Gentleman. Imprinted at London by S.S. for Hugh Astley, and are to be sold at his shop at St. Magnus' Corner.* 1608 (H.). – The title page is reproduced in L. C. Karpinski, *The History of Arithmetic,* Chicago, New York, 1925, p. 132.

(19) *The Art of tenths, or decimall arithmeticke... exercised by Henry Lyte Gentleman... London, printed by Edward Griffin,* 1619. (Hu.) (The title page is reproduced in Isis 33 (1935), p.223. This booklet contains an exposition of Stevin's method. The author has, to use his own words, 'sometime, now ten yeeres sithence (gentle reader) bin intreated by divers to publish my Exercises of Decimall...'; he acknowledges that this 'art of tenths' was 'devised first by the excellent Mathematitian Mr. Simon Steven'. Lyte writes $1\ \overset{(0)}{7}\ \overset{(1)}{5}\ \overset{(2)}{8}\ \overset{(3)}{2}$ (brackets instead of circles) and explains $3\ \overset{(0)}{4}\ \overset{(3)}{5}$ as $3400\overset{(3)}{5}$ (information from Dr. Frank Weymouth). On Lyte, who was born in 1573 as a son of Henry Lyte, translator of Dodoens' *Cruydeboeck,* see R. E. Ockenden, *Apropos of Henry Lyte,* Isis 25 (1936). pp. 135–136.

(20) Clavius' table appeared with his *Astrolabium* (1593). It also appears as a separate item in Tome I of Clavius' *Opera mathematica,* Moguntiae, 1612, 5 vols. (H.). The title is: *Sinus vel semisses rectarum in circulo subtensarum: lineae tangentes atque secantes.* On p. 54 the sexagesimal division is compared with the decimal one, the names Peurbach, Regiomontanus, and Appianus are mentioned, but not Stevin. See also J. Ginsburg, *On the Early History of the Decimal Point,* Amer. Mathem. Monthly 35 (1928), pp. 347–349.

Bolognese astronomer G. A. Magini, who in his text on plane triangles of 1592 taught that, in writing decimal fractions, integer and fractional part should be separated by a comma, such as 6822,11 (21). In Magini and Clavius we probably have the first authors to use our present notation (22).

Jost Bürgi (1552-1632), of Kassel and Prague, who with Napier is considered the inventor of logarithms, has a claim to the invention of decimal fractions; at any rate, Kepler thought so. We know that Bürgi used them after 1592, but his book *Arithmetica* remained in manuscript. He wrote $141_0 4$ for 141.4. On the title page of Bürgi's published *Progress Tabulen* of 1620 we find 230270˙022 for our 230270.022 (23). Kepler, who claimed in 1616 that "this kind of fractional calculus has been invented by Jost Bürgi for the calculus of sines", used the notation 3(65 for our 3.65. With these fractions, he wrote, we can perform all arithmetical operations just as with ordinary numbers (24).

Another claimant for the title of inventor of decimal fractions is Johann Hartmann Beyer (1563-1625) of Frankfurt a. M. He called his calculus the *dekarithmos* and in his *Logistica decimalis* of 1619 wrote that he invented this method of reckoning with decimal fractions in 1597 under the influence of astronomers, or "star-artisans". There are reasons for not taking these claims too seriously, since Beyer's notation and nomenclature is rather reminiscent of Stevin; he quotes the Dutch surveyor Sems, who recognized Stevin's influence, and when, in 1619, he dedicated another book to Prince Maurice of Orange, he may well have been aware who was Maurice's principal mathematical adviser. It is, of course, possible that this indirect acquaintance with Stevin only came about after Beyer had had his happy inspiration in 1597. In his book we find such notations as $12\overset{0}{3}\,\overset{\text{I}}{4}\,\overset{\text{II}}{5}\,\overset{\text{III}}{9}\,\overset{\text{IV}}{3}\,\overset{\text{V}}{7}\,\overset{\text{VI}}{2}$ or $12\overset{0}{3}\,459.37\overset{\text{VI}}{2}$ (25).

(21) *Ioanni Magini Patavini... de planis triangulis liber unicus...* Venetiis 1592, p. 47: 'Separabis virgulas duas quoque ultimas notas ad dextram sic 6822, 11 et 3117, 82: hoc autem processu illi numeri divisi erunt per numerum 100 prior quidem intelligetur 6822 cum 11 centesimis'. Decimal fractions also appear in Magini's *Tabulae primi mobilis*, 1604. See J. Ginsburg, *On the Early History of the Decimal Point*, Scripta Mathematica 1 (1932), pp. 84–85.

(22) Some writers list among the books of this period in which decimal fractions are found Thomas Masterson, *His first booke of Arithmeticke...* London, 1592, 4 + 21 pp. But all we find here is an occasional decimal separatrix in a division by a power of 10, *e.g.* 984735 divided by 100 is written: $\begin{array}{c|c}\text{quotient} & \text{remainder}\\ 9847 & 35\end{array}$. Masterson's opinion of Stevin was not high, he refers to a correspondence he had with Stevin and with Coignet on certain errors he had found in their work and which he proposed to correct; they seem to deal with problems of compound interest. See our introduction to *Tables of Interest*, footnote [29]).

(23) Bürgi's *Arithmetica* was completed some time after 1592. It remained in manuscript, inspected by Kepler, see footnote [24]). On Bürgi's *Arithmetische und geometrische Progress Tabulen*, Prag 1610, see M. Cantor *l.c.* [5]) p. 567; O. Mautz, *Zur Stellung des Dezimalkommas in der Bürgischen Logarithmentafel*, Verhandl. Naturforscher Ges. Basel 32 (1901–02), pp. 104–106.

(24) J. Kepler, *Auszzug ausz der Uralter Messe Kunst Archimedis* (known as *Oesterreichisches Wein-Visier-Büchlein*), Lintz, 1616, repr. in *Kepleri Opera omnia*, ed. Frisch V, 1864, pp. 498–613. On p. 547: 'Diese Art der Bruchrechnung ist von Jost Bürgen zu der Sinusrechnung erdacht... Indessen lesset sich also die gantze Zahl und der Bruch mit einander durch alle species arithmeticae handeln wie nur ein Zahl'.

(25) *Logistica decimalis: das is Kunst Rechnung der Zehentheyligen Brüchen... beschrieben durch Johann Hartmann Beyern...* 1619, *Frankfurt*, 230 pp. On p. 22: 'Zu der Invention dieser

A somewhat controversial figure in the present-day literature on the history of decimal fractions is Bartholomeus Pitiscus (1561-1625), of Heidelberg, who wrote a *Trigonometria,* published in editions of 1595, 1600, 1609, and 1612. This is the first book to use the term *trigonometry.* There is no doubt that Pitiscus knew decimal fractions; he used them freely. The controversial question is whether he used a decimal point (as did Clavius). It seems that while he used notations such as 02679 492 for our 0.2679492, and 13|00024 for our 13.00024, and also $29\left|\frac{95}{100}\right.$, it cannot be claimed that the dots he used for breaking up large numbers into groups to facilitate reading can be considered as decimal points (26).

§ 4.

The man who must rank with Stevin in his influence on the development of decimal fractions was no less a person than the Laird of Merchiston, John Napier (1550-1617), the inventor (or co-inventor with Jobst Bürgi) of logarithms. Napier is also primarily responsible for our present notation with the point as decimal separatrix. The first edition of Napier's *Descriptio* (1614) is decimal in so as far that it contains sines based on $R = 10^7$, but there are no decimal fractions yet. We find them, with a point as separatrix, in a passage in Edward Wright's English translation of the *Descriptio* (1616) (27). Napier's *Rabdologia* of 1617 hails Stevin and adopts his principle, it also proposes the notation 1993,273 (with point or comma) for $1993\frac{273}{1000}$ — using also 821,2′5″ for $821\frac{25}{100}$, as well as a decimal fraction with 10^{14} fully written out in the denominator (28).

Zehentheiligen Brüchen ist mir erstlichen Anno 1597... von den Gestirnkünstlern folgender gestalt Anlasz gegeben werden'. The whole page is reproduced on p. 221 of G. Sarton, *l.c.*2). There exists an earlier version of the *Logistica decimalis: Eine neue und schöne Art der Vollkommenen Visierkunst... Frankfurt* 1603, 191 pp., with a Latin version, also of Frankfurt 1603: *Stereometriae inanium nova et facilis ratio.* Here Beyer published his ideas on decimal fractions for the first time. See G. Sarton, *l.c.*2), pp. 178–180, with facsimile titlepage reproductions. On Beyer's relation to Sems, see K. Hunrath *l.c.*9).

(26) On the different editions and translations of Pitiscus' *Trigonometria*, see R. C. Archibald, *Mathem. Tables and Other Aids to Computation* 3 (1949) pp. 390–397, with full bibliography. The title of the 1600 ed. is *Trigonometria sive De dimensione Triangulorum Libri Quinque*... Augsburg 1600, VIII + 371 pp. (H). Of the literature on Pitiscus and the decimal point in general we mention: N. L. W. A. Gravelaar, *Pitiscus' Trigonometria,* Nieuw Archief v. Wiskunde (2) 3 (1898), pp. 253–278; *De notatie der decimale breuken, ib.* (2) 4 (1899), pp. 54–73; F. Cajori *l.c.*2) pp. 317–322, with full discussion; J. W. L. Glaisher, *On the Introduction of the Decimal Point into Arithmetic*, Report 43d Meeting British Assoc. Adv. Science, London, 1874, pp. 13–17; J. D. White, London Times Liter. Suppl., Sept. 9, 1909; D. E. Smith, *The Invention of Decimal Fractions,* Teachers College Bulletin (New York) 5 (1910), pp. 11–21.

(27) *Mirifici Logarithmorum Canonis descriptio... Authore ac Inventore Ioanne Nepero...* Edinburgi, 1614, 8 + 57 + 91 pp. Transl.: *A description of the admirable table of logarithmes... translated into English by the late... Edward Wright...* London, 1616, 22 + 89 + 91 + 8 pp. The so-called decimal point may not have been intended as such, see F. Cajori *l.c.*2) p. 323.

(28) *Rabdologiae, seu numerationis per virgulas libri duo... Authore et Inventore Ioanne Nepere... Edinburgi,* 1617, 12 + 154 + 2 pp. On p. 21 is the 'Admonitio pro Decimali Arithmetica' with the words... 'quas doctissimus ille Mathematicus Simon Stevinus in sua Decimali Arithmetica sic notat, et nominat ① primas, ② secundas, ③ tertias...' The notation 821, 2′ 5″ is on p. 39.

In the posthumous *Constructio* (1619) we find the Laird more consistent; at the very beginning, in Prop. 5, we find the principle clearly stated: "*whatever is written after the period is a fraction*". Hence 25.803 means $25\frac{803}{1000}$ (29). The appendix to the *Constructio*, written by Henry Briggs, uses the notation $2\underline{5118865}$.

We now enter the period of the great tables of logarithms, in which the decimal notation for fractions is taken for granted. Briggs, in the *Arithmetica Logarithmica* of 1624, which lists logarithms to the base 10, uses the comma as decimal separatrix (also for other purposes, as in 4,40141,77793 for log 25201). Vlacq, who completed Briggs' tables, continued this practice of using the comma, so that we find in his work such familiar expressions as 0.47712 for log 3 (30). The separation of mantissa and characteristic by some symbol such as a point or comma is a natural result of the listing of logarithms in tables, and leads naturally to decimal fractions in our modern notation when 10 is accepted as the base.

Stevin's, Napier's, and Briggs' contributions were combined in the *Eerste Deel van de Nieuwe Telkonst* (1626) and the *Tweede Deel van de Nieuwe Telkonst* (1627) by the Gouda surveyor Ezechiel De Decker (31). Here we find together Stevin's *Thiende,* Vlacq's translation of the *Rabdologia*, and the Briggsian logarithms of all numbers from 1 to 10^6. These two *Telkonst* books testify to the triumph of the decimal system. They stress three essential aspects of this victory: the Hindu-Arabic notation with the modern digits, the decimal fractions, and the logarithms to the base of 10. One change was still to come, though it was implicit in the frame of the decimal system: the rewriting of the trigonometric tables to a unit $R = 1$. Other deviations from the decimal method, such as the measurement of angles, of weights, of lengths, of volumes, continued to form a subject of discussion and disputation for many years and even now have not been removed to everybody's satisfaction.

§ 5.

The further history of the decimal fractions is not without a certain interest. There were loyal Stevin followers, followers who preferred some modification of

(29) *Mirifici Logarithmorum Canonis Constructio... una cum Annotationibus aliquot doctissimi D. Henrici Briggii... Authore et Inventore Ioanne Nepero...* Edinburgi 1619, 68 pp. On p. 6: 'In numeris periodo sic in se distinctis, quisquid post periodum notatur fractio est, cuius denominator est unitas cum tot cyphris post se, quot sunt figurae post periodum'. – There exists an edition printed in Lyons, 1620, 64 pp. in which Briggs' notation by mistake is rendered $\underline{25118865}$. There also exists an English translation: *The construction of the wonderful canon of logarithms by John Napier, translated by W. R. Macdonald*, Edinburgh and London, 1889, XIX + 169 pp, with bibliography.

(30) *Tweede deel van de Nieuwe Telkonst, ofte wonderlücke konstighe tafel, Inhoudende de Logarithmi, voor de getallen van 1 af tot 100.000 toe... door Ezechiel De Dekker...* Ter Goude, P. Rammaseyn, 1627. Vlacq computed those tables which Briggs had not yet published. The book was followed by *Arithmetica Logarithmica, sive Logarithmorum Chiliades Centum, pro Numeris naturali Serie crescentibus ab Unitate ad 100.000.– aucta per A. Vlacq*, Goudae, P. Rammasenius, 1628; also in a French version of the same year. See M. van Haaften, *Ce n'est pas Vlacq, en 1628, mais De Decker, en 1627, qui a publié le premier une table de logarithmes étendue et complète*, Nieuw Archief v. Wiskunde (2) 15 (1928), pp. 49–54, see also *ibid.* 31 (1942), pp. 59–64.

(31) See *l.c.* (16) and (30). This *Eerste Deel* contains the Dutch translation of the *Rabdologia.*

his system, but maintained special symbols for the primes, seconds, etc., followers of Napier and Briggs, and writers who ignored the invention. Among the loyal Stevin followers we reckon the surveyors Dou, Sems, and De Decker, Albert Girard and Professor Van Schooten at Leiden. Van Schooten, in his *Exercitationum mathematicarum liber* of 1657 preferred 525③ to 5①2②5③, but sometimes also wrote the redundant forms 27,3① or 27.3① (32). At the end of the seventeenth century we find the military engineer De la Londe with such a notation as $2',4'',3''',5^{IV}$ or 2435^{IV} or 0|2435 (33), and the mathematician Ozanam

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with 3 8 2 4 5 9 or 382459④; both authors refer to Stevin and call his method *la dixme* (34). As late as 1739 we find l'abbé Deidier teaching that decimal fractions should be written as $89.5^{I}2^{II}7^{III}6^{IV}$ or 895276^{IV}, though he used the ordinary point notation for logarithms (35). The final judgment in favour of our present notation may well be ascribed to Euler, and in particular to that book of his which established so many of our mathematical customs (including the reference of sines, tangents, etc. to the radius $R = 1$), the *Introductio in analysin infinitorum* of 1748. There remained some difference in form and position of the decimal separatrix; we still find 5,7 as well as 5.7, 5·7, and 5˙7.

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(32) Francisci à Schooten, *Exercitationum mathematicarum liber primus*... Lugd. Bat. 1657, pref. + 112 pp. On p. 19 the author expresses 'stuivers' and 'duiten' in florins and writes: '10 stufr, 8 den', as 525③ (one florin = 20 st.; 1 st. = 6 den.) The notation 27.3 ① and 27,3 ① on p. 49; on p. 99 we read $14\frac{93}{100}$.

(33) De la Londe, *Traité de l'arithmétique dixme*. Liège[?]. The author was one of Vauban's trusted engineers, commander of the corps de génie during the first siege of Philippsburg (Baden) in 1676. During 1682–83 we find him in charge of the Flemish barrière fortresses, and in 1688 again at Philippsburg during the second siege. During this siege he was killed by a canon ball. See A. Allent, *Histoire du corps impérial du génie* I, Paris, 1805, pp. 137, 164, 221, 225, 228; Lazard, *Vauban*, Thèse Paris, 1934, pp. 139, 140; R. Blomfield, *Sebastien le Prestre de Vauban*, 1633–1707, London, 1938, p. 86. – The title of De la Londe's book is given by F. T. Verhaeghe, *Spreekbeurt*, uitgegeven door de Kon. Maatsch. v. Vaderl. Taal- en Letterkunde te Brugge, 1821, p. 76; S. van de Weyer *Steviniana*, in *Simon Stevin et M. Dumortier*, Nieuport 1845, and A. J. J. van de Velde, Meded. Kon. Vlaamse Acad. voor Wetensch. 10 (1948), with differing data. Terquem, *Notice bibliographique sur le calcul décimal*, Nouvelles Annales de Mathém. 12 (1853), pp. 195–208, mentions a *L'Arithmétique des ingénieurs contenant le calcul des toises, de la maçonnerie, des terres et de la charpente, par M. de la Londe*, 1e ed. 1685, 2e ed., Paris, 1689, 144 pp. This book adopts Stevin's system. Our example of De la Londe 's notation is from L. Gougeon, *Abrégé de l'Arithmétique en Dixme*..., an appendix to *Parallèle de l'arithmétique vulgaire et d'une autre moderne inventée par M. de la Londe, ingénieur général de France*, Liège, 1695, 259 pp., a book also mentioned by Terquem.

(34) *L'usage du compas de proportion... par M. Ozanam*, La Haye, 1691, 216 pp. The *Traité de la Dixme* is on pp. 198–216.

(35) *Suite de l'Arithmétique des géomètres... par M. l'abbé Deidier*, Paris, 1739, 6 + 416 pp. The chapter 'Des fractions décimales' is at the end, pp. 411–416. Deidier was not the only one to make a difference in the notation for decimal fraction and for logarithms. In the *Eléments de mathématiques* par M. Rivard, 6e éd., Paris, 1768, Première partie, 271 pp., we find decimal fractions in the redundant form 4.25^{II}(p. 208), but logarithms without any decimal separatrix, log 57 = 17558749. For other examples of decimal notations in the 17th & 18th centuries see the literature under [2]), Terquem, *l.c.*[33]) and F. Cajori, *A List of Oughtreds' Mathematical Symbols*, with historical notes, Un. of California Publ. in Mathematics 1 (1920), pp. 171–186, esp. footnote [2]).

§ 6.

Stevin had yet another purpose with his pamphlet. He proposed to replace the confused systems of weights and measures of his day by a system based on the decimal division of one unit. He did not propose anything about the nature of the unit itself: he only pointed out the advantages of a decimal subdivision.

Attempts at the uniformization of measuring systems have been made whenever states were in process of consolidation. We know of such attempts by Frankish and French kings as far back as Charlemagne and Charles the Bald. To give an example closer to Stevin's days: in 1558 the French States General, in a request to Henry II, ordered the reduction of the weights and measures of the kingdom to those of Paris (36).

Stevin's proposal was made at a time when the Northern Netherlands, after having officially constituted themselves as an independent commonwealth in 1581, were faced with this task of consolidation. This must have seemed to Stevin an appropriate time to press his suggestions. Here was a field in which the mathematician and the engineer could collaborate with the man of business for the common weal. Stevin dedicated his work to men of practice, for whose benefit he wrote and published it in the vernacular (37). He wanted to be read outside the charmed circle of humanists and cossists.

He pointed out how useful the decimal subdivision would be to particular crafts. Let the surveyors apply it to their unit, the rod (la verge), the tapestry measurers to their unit, the ell ('aulne), the wine gaugers to the "aem" (l'ame), the astronomers to the degrees of the circle, the masters of the mint to their ducats and pounds. Stevin knew at least one precedent: at Antwerp the "aem" was already divided into 100 *"potten"*. He also knew of surveyors who, on his advice, were using yardsticks with a decimal division (38). Stevin himself declared that he would use the decimal scale in his planned treatise on astronomy. However, his later book on this subject has no such innovations.

It is well known that Stevin's proposals on the reformation of weights and measures did not meet with the same success as his proposals on the reformation of fractions. Not until the French Revolution was anything of permanent importance accomplished in the decimal uniformization of scales, and then it took place as part of a reform which also standardized the units themselves. However, some

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(36) G. Bigourdan, *Le système métrique des poids et mesures*, Paris, 1901, VI + 458 pp., see pp. 5 ff.; A. Favre, *Les origines du système métrique*, Paris, 1931, 242 pp. Neither Bigourdan nor Favre mentions Stevin.

(37) The French edition of *De Thiende*, *l.c.* [17]), carries on the front page the words: *premierement descripte en Flameng, et maintenant convertie en François, par Simon Stevin de Bruges.*

(38) H. R. Calvert, *Decimal Division of Scales before the Metric System*, Isis 25 (1936), pp. 433-436. The oldest decimal division on a scale reported by this paper is the one described in *The Mathematical Jewel* by J. Blagrave, 1585. The book by Henry Lyte, *(l.c.)* (19) contains the following advertisement: 'Those that would have either the Yard or two foote Ruler made very well according to the Arte of Tens with tables for that purpose I have set downe in this booke let them repaire to Mr. Tomson dwelling in Hosier Lane, who make Geometricall Instruments.' About an assayer's *Probierbüchlein* of 1578, written perhaps c. 1555 or earlier with a decimal system of weights see C. S. Smith, Isis 46 (1955), pp. 354-357.

attempts at decimal scales were made before that period, though they may not always have been wholly or partly due to Stevin's influence. We have already mentioned the occasional decimal division of surveyors' yardsticks. Of more importance were the attempts at a decimal division of angles. Such attempts date from far back; there exists in Munich a Latin codex of about 1450, in which a certain Ruffi proposes the division of a degree into 100 minutes, and of a minute into 100 seconds ([39]). The first printed tables with a decimal angular division are found in Briggs' *Trigonometria britannica* ([40]). Briggs acknowledges his indebtedness to an idea of Viète, but we also know that he was acquainted with Stevin's ideas ([41]). These tables still have a quadrant of 90, not of 100, degrees ([42]). This was in accordance with Stevin, who did not challenge the division of the quadrant into 90 degrees, but only the subdivision of the degrees. This reform did not meet with ready acceptance, and it was not until the period of the French Revolution that we find tables with a decimal division of angles, now also with a centesimal division of the quadrant. The continuity with the older work is preserved, since both the Borda and the Callet tables, which date from this period, refer to Briggs and to Vlacq (whose publisher, Rammaseyn, was also Briggs' Dutch publisher). Laplace, in his *Mécanique céleste,* adopted the decimal division of the degree, but not of the quadrant. The decimal division of the quadrant itself is now fairly generally accepted in surveying; in other fields it is making progress and even when the sexagesimal division is used, the fractions in the seconds are always decimal: 5°7'8.5" ([43]).

The standardization of the units themselves has also had its unsuccessful pioneers. An example is the proposal to use the seconds pendulum as the standard of length, to which in the second half of the seventeenth century such men as Mouton, Picard, Wren, and Huygens committed themselves. The system which the French committee during the Revolution accepted, and which was based on the metre as the forty-millionth part of the earth's circumference at the equator, goes back to another proposal of Mouton ([44]). But even now there is still plenty of disagreement on the subject of the standardization of units. It is a cause of satisfaction that in scientific work the C.G.S. system has been uniformly accepted. No unit in this system has as yet been called after Stevin.

([39]) See *Mathem. Tables and Other Aids to Computation* I (1943–45), p. 33.

([40]) *Trigonometria britannica sive De doctrina triangulorum libri duo... a Clar. Doct.... Henrico Briggio...* Goudae 1633. These tables are preceded by 110 pp. of trigonometry by H. Gellibrand. The unit is $R = 10^{10}$.

([41]) On Viète's influence, see R. C. Archibald and A. Pogo, *Briggs and Vieta, Mathem. Tables and Other Aids to Computation* I (1943–45), pp. 129–130. An illustration of Stevin's influence is Gellibrand's use of Stevin's (1), (2),... for x, x^2,...

([42]) A follower of Stevin was J. Verrooten: *Euclides zes eerste boekken van de beginselen der wiskonsten, in Neerduits vertaald door Jacob Willemsz. Verrooten van Haerlem...* Hamburg, 1638, 344 pp., who divides the quadrant into 10 parts or (1), every (1) into (2) ....; his unit is $R = 10^{10}$.

([43]) R. Mehmke, *Bericht über die Winkelteilung*, Jahresber. Deutsch. Math. Ver. 8, Erstes Heft (1900), pp. 139–158; P. Wijdenes, *Decimale tafels*, Euclides 13 (1936/37), pp. 193–217; R. C. Archibald, *Tables of Trigonometric Functions in Non-sexagesimal Arguments, Mathem. Tables and Other Aids to Computation* I (1943–1945), pp. 33–44, 160, 400–401.

([44]) G. Sarton *l.c.* [2]) pp. 190–192; G. Bigourdan *l.c.* [36]), pp. 6, 7.

§ 7.

Two facsimile reprints of Stevin's pamphlet have been published. The original Dutch edition of 1585 was reproduced in 1924 by H. Bosmans, the French version of the same year in 1935 by G. Sarton. A modern English translation has been published by Vera Sanford in D. E. Smith's *Source Book of Mathematics* ([45]).

NOTE. The Persian astronomer Al-Kashi, who lived for a while at the court of Ulugh Bey at Samarkand, and died in 1429, worked freely with decimal fractions. See *D. G. Al-Kaši Ključ Arifmetiki, Traktat ob Okružnosti,* translated and ed. by B. A. Rozenfel'd (Moscow 1956), 566 pp., especially 6. 62, where, in the "Key to Arithmetic", is shown how to multiply 14.3 into 25.07, answer 358.501. — According to Y. Mikami, *The Development of Mathematics in China and Japan,* (Abh. zur Gesch. d. math. Wissensch. XXX), Leipzig 1913, p. 26, Yang Hui (second half 13th century) showed that 24.68 x 36.56 = 902.308.

([45]) The Dutch text of the 1585 edition has been reproduced in fascimile in *De Thiende de Simon Stevin. Fac-simile de l'édition originale Plantinienne de 1585. Avec une introduction par H. Bosmans,* Ed. de la Soc. des Bibliophiles Anversois Nr 38, Anvers et la Haye, 1924, 41 + 36 + (1) pp. The French text of the edition of 1585 of *La Pratique d'Arithmétique* is reproduced in facsimile in G. Sarton *l.c.* [2]) pp. 230–244.

The Sanford translation can be found on pp. 20–34 of D. E. Smith, *Source Book of Mathematics,* New York-London, 1929.

Here are some other publications in which *De Thiende* is discussed: H. Bosmans, *La Thiende de Simon Stevin,* Revue des Questions Scientifiques, Louvain 77 (1920), pp. 109–139; M. van Haaften, *De Thiende van Stevin,* De Verzekeringsbode, 4 Dec. 1920, pp. 73–77; V. Sanford, *The Disme of Simon Stevin,* Mathematics Teacher, NewYork, 14 (1921), pp. 321–333; F. Cajori *l.c.*[2]) pp. 154–156; R. Depau, *Simon Stevin,* Bruxelles, 1942, pp. 58–70; E. J. Dijksterhuis, *Simon Stevin,* 's Gravenhage, 1943, pp. 65–69.

DE
THIENDE

Leerende door onghehoorde lichticheyt allen rekeninghen onder den Menschen noodich vallende, afveerdighen door heele ghetalen sonder ghebrokenen.

Beschreven door SIMON STEVIN *van Brugghe.*

TOT LEYDEN,
By Christoffel Plantijn.
M. D. LXXXV.

DIME

THE ART OF TENTHS

or

Decimal Arithmetic,
Teaching how to perform all computations whatsoever by whole numbers without fractions, by the four principles of common arithmetic, namely: addition, subtraction, multiplication, and division.

Invented by the excellent mathematician,
SIMON STEVIN.

Published in English with some additions
by
Robert Norton, Gentleman.

Imprinted at London by S.S. for Hugh Astley, and are to be sold at his shop at St. Magnus' Corner. 1608.[1])

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[1]) This English translation of *DeThiende* was prepared by Richard Norton and published in 1608; for the title see footnote [18]) of the Introduction. The booklet contains a literal translation, almost certainly from the French version, with some additions: a) a short preface "to the courteous reader", b) a table for the conversion of sexagesimal fractions into decimal ones, and c) a short exposition on integers, how to write them, to perform the main species and to work with the rule of three. This exposition is taken from Stevin's *L'Arithmétique* and we deal with it in the proper place. In using Norton's translation we have modernized the spelling and corrected some misprints.

The translator, Richard Norton, was the son of the British lawyer and poet Thomas Norton (1532–1584) and a nephew of Archbishop Cranmer. The father is remembered as the co-author of what is said to be the first English tragedy in blank verse, *Gorboduc* (acted in 1561) and as a translator of psalms and of Calvin's *Institutes*. The son, according to the *Dictionary of National Biography 41* (1895), was an engineer and gunner in the Royal service, became engineer of the Tower of London in 1627 and died in 1635. He wrote several texts on mathematics and artillery, supplied tables of interest to the 1628 edition of Robert Recorde's *Grounde of Arts* and seems to have been the author of the verses signed Ro: Norton, printed at the beginning of Captain John Smith's *Generall historie of Virginia, New England and the Summer Isles*, London, 1624.

On Norton see also E. J. R. Taylor, *The Mathematical Practitioners of Tudor and Stuart England*. Cambridge, Un. Press 1954, XI + 442 pp.

Norton calls Stevin's method both *Dime* and *The Art of Tenths* in the title, but in the text only uses the term *Dime*.

We reproduce this translation of *De Thiende* through the courtesy of the Houghton Library of Harvard University, Cambridge, Mass.

DEN STERREKYCKERS,

LANDTMETERS, Tapijtmeters, Wijnmeters, Lichaemmeters int ghemeene, Muntmeesters, ende allen Cooplieden, wenscht SIMON STEVIN Gheluck.

YEMANDT *ansiende de cleenheyt deses boucx, ende die verghelijckende met de Grootheyt van ulieden mijne* E. HEEREN *ande vvelcke het toegheeyghent vvort, sal byghevalle uyt sodanighe onevenheydt ons voornemen ongeschict achten; Maer soo hy de* Everedenheydt *insiet, vvelcke is ghelijck deses Pampiers* Weynicheyt, *tot dier Menschelicker Cranckheyt, alsoo deses groote Nutbaerheden, tot dier hooghe Verstanden, sal hem bevinden de uyterste* Palen *met malcanderen vergheleecken te hebben, vvelcke naer alle Everedenheyts verkeeringe dat niet en lijden: De derde dan tot de vierde. Maer vvat sal dit voorghestelde doch sijn? eenen vvonderlicken diepsinnighen Vondt?*

Proportionem.

Terminis.

THE PREFACE OF SIMON STEVIN.

To Astronomers, Land-meters, Measurers of Tapestry, Gaugers, Stereometers in general, Money-Masters, and to all Merchants, SIMON STEVIN wishes health.

Many, seeing the smallness of this book and considering your worthiness, to whom it is dedicated, may perchance esteem this our conceit absurd. But if the proportion *be considered, the small quantity hereof compared to human imbecility, and the great utility unto high and ingenious intendments, it will be found to have made comparison of the extreme* terms, *which permit not any conversion of proportion. But what of that? Is this an admirable invention?*

Neen voorvvaer, maer eenen handel soo gantsch slecht, datse nau Vondts name vveerdich en is, vvant ghelijck een grof Mensche vvel bygheValle eenen grooten Schadt vindt, sonder eenighe conste daer in ghelegen te sijne, alsoo ist hier oock toegheghaen: Daerom soo my yemandt om t'verclaren haerder prouffijtelickheydt, vvilde achten voor eenen Eyghenlover mijns verstandts, hy bethoont sonder tvvijffel, ofte in hem noch oirdeel noch vvetenschap des onderscheydts te sijne, van het slechte buyten het besonder, ofte dat hy een benijder is der Ghemeene vvelvaert: Maer tsy daermede hoet vvil, om diens onnutte laster, en moet deses nut niet ghelaten sijn. Ghelijck dan een Schipper by gheValle ghevonden hebbende een onbekent Eylandt, dẽ Coninck stoutelick verclaert alle de costelickhedẽ van dien, als in hem te hebbẽ Schoone Vruchtẽ, Goudtbergen, Lustige Landauvven, etc. sonder dat sulcx tot sijns selfs verheffing strect; Also sullẽ vvy hier vrymoedich spreken van deses Vonds Groote Nutbaerheydt

No certainly: for it is so mean as that it scant deserves the name of an invention, for as the countryman by chance sometime finds a great treasure, without any use of skill or cunning, so hath it happened herein. Therefore, if any will think that I vaunt myself of my knowledge, because of the explication of these utilities, out of doubt he shows himself to have neither judgment, understanding, nor knowledge, to discern simple things from ingenious inventions, but he (rather) seems envious of the common benefit; yet howsoever, it were not fit to omit the benefit hereof for the inconvenience of such calumny. But as the mariner, having by hap found a certain unknown island, spares not to declare to his Prince the riches and profits thereof, as the fair fruits, precious minerals, pleasant champions [2]*), etc., and that without imputation of self-glorification, even so*

[2]) Champion, comp. French "champagne", field, landscape. Comp. e.g. Deut. XI, 30, author. transl. of 1611: "the Canaanites which dwell in the campions".

heydt, Groote seg ick, ja Grooter dan ick dincke yemandt van ulieden verwacht, sonder dat het keeren can tot mijn Eygenroem.

Anghesien dan dat de Stoffe *deser voorghestelder Thiende (diens naems Oirsake de volgende eerste* Bepalinghe *verclaren sal) is Ghetal, wiens* Daets *nutbaerheydt yeder van ulieden door de ervaring genouch bekẽt is, so en valt daer af hier niet vele gheseyt te worden, want ist een* Sterrekijcker, *hy weet dat de* Werelt *door des* Sterreconsts Rekeningẽ, *als Maeckende Oirsaecke der constighe verre Seylaigen (want de verheffing des* Evenaers *ende* Aspunts, *leert sy den Stierman duer t'middel vande Tafel des dagelicschen afwijcksels der Sonnen; Men beschrijft door haer der plaetsen ware langden ende breeden, oock der selver veranderingẽ op yder Streecke, &c.) een prieel der wellusticheydt gewonden is, overvloedich tot velẽ plaetsen, van dies het Eertrijck daer nochtans uyt der Natueren niet voortbrenghen en can. Maer want* selden besoeten son-

Materia. Definitio. Effecti. Astrologus. Supputationes Astronomicas. Æquatore Poli.

 der

shall we speak freely of the great use of this invention; I call it great, being greater than any of you expect to come from me. Seeing then that the matter *of this Dime (the cause of the name whereof shall be declared by the first* definition *following) is number, the use and effects of which yourselves shall sufficiently witness by your continual experiences, therefore it were not necessary to use many words thereof, for the* astrologer *knows that the world is become by* computation astronomical *(seeing it teaches the pilot the elevation of the* equator *and of the* pole, *by means of the declination of the sun, to describe the true longitudes, latitudes, situations and distances of places, etc.) a paradise, abounding in some places with such things as the earth cannot bring* der besueren, *so en is hen oock de moeyelickheyt sodanigher rekeningen niet verborghen, door de lastighe Menichvuldighinghen ende Deelinghen, dieder rijsen uyt de tsestich deelige* voortganck *der Boogskens, die genoēt vvorden* Gradus, Minuta, Secunda, Tertia, etc. *Maer ist een Landtmeter, hem is bekēt de groote vveldaet, die de* Werelt *ontfangt uyt sijne Conste, door de vvelcke vele svvarichedē ende tvvisten geschouvvet vvordē, die om des Landts onbekende inhout onder de Menschen daghelijcx rijsen soudē. Beneven dit so en sijn hem oock niet verholen (voornamelick den genē die van sulcx veel te doen valt) de verdrietighe Menichvuldigingen dieder spruyten, uyt de Roeden, Voeten, ende dickmael Duymen onder malcanderen, vvelcke niet alleene moeyelick en sijn, maer (hoe vvel nochtans het meten ende dander voorgaende recht gedaen sijn) dickmael oirsaeck van dvvalinghe, streckende tot groote schade van desen of dien, oock tot verderfnis vande goede Mare des Meters:*

Progressione.

Ende

forth in other. But as the sweet is never without the sour, *so the travail in such computations cannot be unto him hidden, namely in the busy multiplications and divisions which proceed of the 60th* progression *of* degrees, minutes, seconds, thirds, etc.: *And the surveyor or land-meter knows what great benefit the world receives from his science, by which many dissensions and difficulties are avoided which otherwise would arise by reason of the unknown capacity of land; besides, he is not ignorant (especially whose business and employment is great) of the troublesome multiplications of rods, feet, and oftentimes of inches, the one by the other, which not only molests, but also often (though he be very well experienced) causes error, tending to the damage of both parties, as also to the discredit of landmeter or surveyor, and so for the money-masters, merchants, and*

Ende alſo met dẽ Muntmeeſters, Cooplieden ende yegelick int ſijne: maer ſo vele die vveerdiger, ende de vvegẽ om daer toe te commen moeyelicker ſijn, ſoo veel te meerder is deſe Groote Ontdeckte THIENDE, *vvelcke alle die ſvvaricheden gantſch te nederleght. Maer hoe? Sy leert (op dat ick met eẽ vvoort vele ſegghe) alle rekeninghen die onder de Menſchen noodich vallen, afveerdigẽ ſonder gebroken getalen: Inder vougen dat der Telconſtens vier eerſte ſlechte beghinſelen, diemen noemt Vergaderen, Aftrecken, Menichvuldighen, ende Deelen, met heele getalen tot deſen genouch doen: Dergelijcke lichticheyt oock veroirſaeckende, den genen die de legpenningẽ gebruycken, ſo hier naer opentlick blijcken ſal: Nu of hier duer ghevvonnen ſal vvorden dẽ coſtelicken oncoopelicken Tijt; Of hier duer behouden ſal vvordẽ tgene anderſins dickmael verloren ſoude gaen; Of hier duer gevveert ſal vvorden Moeyte, Dvvalinghe, Tvviſt, Schade, ende ander Ongevallen deſe gemeenelick volgende, dat ſtelle ick geer-*

each one in his business. Therefore how much they are more worthy, and the means to attain them the more laborious, so much the greater and better is this Dime, *taking away those difficulties. But how? It teaches (to speak in a word) the easy performance of all reckonings, computations, & accounts, without broken numbers, which can happen in man's business, in such sort as that the four principles of arithmetic, namely addition, subtraction, multiplication, & division, by whole numbers may satisfy these effects, affording the like facility unto those that use counters. Now if by those means we gain the time which is precious, if hereby that be saved which otherwise should be lost, if so the pains, controversy, error, damage, and other inconveniences commonly happening therein be* *ne tot ulieden oirdeele. Angaende my yemandt ſegghen mochte, dat vele ſaecken int eerſte anſien dickmael beſonder gelaten, maer alſmenſe int vverck vvil ſtellen, ſo en canmen daer mede niet uytrechten, ende ghelijct met de Vonden der Roerſouckers dickvvils toegaet, vvelcke int cleene goedt ſijn, maer int groote en duegen ſy niet. Dien verantvvoorden vvy alſulck tvvijffel hier geenſins te vveſen, overmidts het int groote, dat is inde ſaecke ſelver, nu dagelijcx metter Daet ghenouch verſocht vvort, te vveten door verſcheyden ervarẽ Landtmeters alhier in Hollandt, die vvy dat verclaert hebben, vvelcke (verlatende tghene ſy tot verlichtinghe van dien daer toe gevonden hadden, elck naer ſijn maniere) dit gebruycken tot hun groote vernouginge, ende met ſulcken vruchten, als de Nature vvijſt daer uyt nootſaeckelicken te moeten volghen: Tſelve ſal yeghelicken van ulieden mijne* E. HEEREN *vvedervarẽ, die doen ſullen als ſylieden. Vaert daerentuſſchen vvel, ende daer naer niet qualick.*

CORT

eased, or taken away, then I leave it willingly unto your judgment to be censured; and for that, that some may say that certain inventions at the first seem good, which when they come to be practised effect nothing of worth, as it often happens to the searchers of strong moving [3]*), which seem good in small proofs and models, when in great, or coming to the effect, they are not worth a button: whereto we answer that herein is no such doubt, for experience daily shows the same, namely by the practice of divers expert land-meters of Holland* [4]*), unto whom we have shown it, who (laying aside that which each of them had, according to his own manner, invented to lessen their pains in their computations) do use the same to their great contentment, and by such fruit as the nature of it witnesses the due effect necessarily follows. The like shall also happen to each of yourselves using the same as they do. Meanwhile live in all felicity.*

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[3]) This is a translation of the Dutch "roersouckers", after Stevin's French version: "chercheurs de fort mouvements". It probably stands for people who start moving things, take initiative, comp. the archaic Dutch expressions "roermaker", "roerstichter" (information from Prof. Dr. C. G. N. De Vooys). The Dutch has "vonden der roersouckers", where "vonden" stands for "findings, inventions", and the whole expression for something like "widely proclaimed innovations".

[4]) Such "expert land-meters" may have been Dou and Sems, see the Introduction, pp. 379, 382.

DE THIENDE heeft twee deelen, Bepalinghen ende Werckinghe. Int eerſte deel ſal door d'eerſte Bepalinghe verclaert wordẽ wat *Thiende* ſy, door de tweede wat *Beghin*, door de derde wat *Eerſte*, *Tweede*, &c. door de vierde wat *Thiendetal* beteeckent.

De Werckinghe ſal door vier Voorſtellen leeren der Thiendetalens Vergadering, Aftrecking, Menichvuldiging, ende Deeling; wiens ooghenſchijnelicke oirden deſe Tafel anwijſt aldus:

DE THIENDE *heeft twee deelen.*
- *Bepaling, als wat dat ſy.*
  - *Thiende.*
  - *Beghin.*
  - *Eerſte Tweede, &c.*
  - *Thiendetal.*
- Werck*ing*, *die is der Thiendetalẽs*
  - *Vergadering.*
  - *Aftrecking.*
  - *Menichvuldiging.*
  - *Deeling.*

By t'voorgaende ſal noch gevoucht worden een ANHANGSEL, wijſende des Thiendens ghebruyck door ſommighe exempelen der Saecken.

THE ARGUMENT.

THE DIME has two parts, that is Definitions & Operations. By the first definition is declared what *Dime* is, by the second, third, and fourth what *commencement, prime, second,* etc. and *dime numbers* are. The operation is declared by four propositions: the addition, subtraction, multiplication, and division of dime numbers. The order whereof may be successively represented by this Table.

THE DIME *has two parts*	*Definitions, as what is*	*Dime, Commencement, Prime, Second, etc. Dime number.*
	Operations or Practice of the	*Addition, Subtraction, Multiplication, Division.*

And to the end the premises may the better be explained, there shall be hereunto an APPENDIX adjoined, declaring the use of the Dime in many things by certain examples, and also definitions and operations, to teach such as do not already know the use and practice of numeration, and the four principles of common arithmetic in whole numbers, namely addition, subtraction, multipli-

HET EERSTE DEEL DER THIENDE VANDE BEPALINGHEN.

I. BEPALINGHE.

THIENDE *is eē specie der Telconsten, door de vvelcke men alle rekeninghen onder den Menschē noodich vallende, afveerdicht door heele ghetalen, sonder ghebrokenen, ghevonden uyt de thiende voortganck, bestaende inde cijfferletteren daer eenich ghetal door beschreven vvort.*

VERCLARINGHE.

HET sy een ghetal van Duyst een hondert ende elf, beschreven met cijfferletteren aldus 1111, inde welcke blijct, dat elcke 1, het thiende deel is van sijn naest voorgaende. Alsoo oock in 2378 elcke een vande 8, is het thiende deel van elcke een der 7, ende alsoo in allen anderen: Maer want het voughelick is, dat de saecken daermen af spreecken wil, namen hebben, ende dat dese maniere van rekeninghe ghevonden is uyt d'anmerckinghe van alsulcken thienden voortganck, ja wesentlick in thiende voortganck bestaet, als int volghende claerlick blijcken sal, soo noemen wy den cation, & division, together with the Golden Rule, sufficient to instruct the most ignorant in the usual practice of this art of Dime or decimal arithmetic.

THE FIRST PART.

Of the Definitions of the Dimes.

THE FIRST DEFINITION

Dime is a kind of arithmetic, invented by the tenth progression, consisting in characters of ciphers, whereby a certain number is described and by which also all accounts which happen in human affairs are dispatched by whole numbers, without fractions or broken numbers.

Explication

Let the certain number be one thousand one hundred and eleven, described by the characters of ciphers thus 1111, in which it appears that each 1 is the 10th part of his precedent character 1; likewise in 2378 each unity of 8 is the tenth of each unity of 7, and so of all the others. But because it is convenient that the things whereof we would speak have names, and that this manner of computation den handel van dien eyghentlick ende bequamelick, de THIENDE. Door de selve worden alle rekeninghen ons ontmoetende volbrocht met besondere lichticheyt door heele ghetalen sonder gebrokenen als hier naer opentlick bewesen sal worden.

II. BEPALINGHE.

Alle voorgestelde heel ghetal, noemen wy BEGHIN, *sijn teecken is soodanich* ⓪.

VERCLARINGHE.

ALS byghelijckenis eenich heel ghegheven ghetal van driehondert vierentsestich, wy noement driehondert vierentsestich BEGHINSELEN, die aldus beschrijvende 364⓪. Ende alsoo met allen anderen dier ghelijcken.

III. BEPALINGHE.

Ende elck thiendedeel vande eenheyt des BEGHINS, *noemen wy* EERSTE, *sijn teecken is* ①; *Ende elck thiendedeel vande eenheyt der Eerste, noemē wy* TWEEDE, *sijn teecken is* ②; *Ende soo voort elck thiendedeel der eenheyt van sijn voorgaende, altijt in d'oirden een meer.*

VER-is found by the consideration of such tenth or dime progression, that is that it consists therein entirely, as shall hereafter appear, we call this treatise fitly by the name of *Dime*, whereby all accounts happening in the affairs of man may be wrought and effected without fractions or broken numbers, as hereafter appears.

THE SECOND DEFINITION

Every number propounded is called COMMENCEMENT, whose sign is thus ⓪.

Explication

By example, a certain number is propounded of three hundred sixty-four: we call them the 364 *commencements*, described thus 364⓪, and so of all other like.

THE THIRD DEFINITION

And each tenth part of the unity of the COMMENCEMENT *we call the* PRIME, *whose sign is thus* ①, *and each tenth part of the unity of the prime we call the* SECOND, *whose sign is* ②, *and so of the other: each tenth part of the unity of the precedent sign, always in order one further.*

VERCLARINGHE.

ALS 3 ① 7 ② 5 ③ 9 ④, dat is te ſeggen 3 *Eerſten*, 7 *Tweeden*, 5 *Derden*, 9 *Vierden*, ende ſoo mochtmen oneyndelick voortgaen. Maer om van hare weerde te ſegghen, ſoo is kennelick dat naer luyt deſer Bepalinge, de voornoemde ghetalen doen $\frac{3}{10}$, $\frac{7}{100}$, $\frac{5}{1000}$, $\frac{9}{10000}$, tſamen $\frac{3759}{10000}$. Alſoo oock 8 ⓪ 9 ① 3 ② 7 ③, ſijn weert 8 $\frac{9}{10}$, $\frac{3}{100}$, $\frac{7}{1000}$, dat is t'ſamen 8 $\frac{937}{1000}$ ende ſoo met allen anderen dier ghelijcke. Het is oock te anmercken, dat wy inde THIENDE nerghens gebroken getalen en ghebruycken: Oock dat het ghetal vande menichvuldicheyt der Teeckenen, uytghenomen ⓪, nummermeer boven de 9 en comt. By exempel, wy en ſchrijven niet 7 ① 12 ② maer in diens plaetſe 8 ① 2 ②, want ſy ſoo veel weert ſijn.

IIII. BEPALINGHE.

De ghetalen der voorgaender tvveeder ende derder bepalinghe, noemen vvy int gemeen THIENDETALEN.

EYNDE DER BEPALINGHEN.

Explication

As 3 ① 7 ② 5 ③ 9 ④, that is to say: 3 *primes,* 7 *seconds,* 5 *thirds,* 9 *fourths,* and so proceeding infinitely, but to speak of their value, you may note that according to this definition the said numbers are $\frac{3}{10}$, $\frac{7}{100}$, $\frac{5}{1000}$, $\frac{9}{10000}$, together $\frac{3759}{10000}$, and likewise 8 ⓪ 9 ① 3 ② 7 ③ are worth 8, $\frac{9}{10}$, $\frac{3}{100}$, $\frac{7}{1000}$, together $8\frac{937}{1000}$, and so of other like. Also you may understand that in this *dime* we use no fractions, and that the multitude of signs, except ⓪, never exceed 9, as for example not 7 ① 12 ②, but in their place 8 ① 2 ②, for they value as much.

THE FOURTH DEFINITION

The numbers of the second and third definitions beforegoing are generally called DIME NUMBERS.

The End of the Definitions

HET ANDER DEEL DER THIENDE VANDE WERCKINGHE.

I. VOORSTEL VANDE VERGADERINGHE.

Wesende ghegeven Thiendetalen te vergaderen: hare Somme te vinden.

T'GHEGHEVEN. Het sijn drie oirdens van Thiendetalen, welcker eerste 27⓪8①4②7③, de tweede, 37⓪6①7②5③, de derde, 875⓪7①8②2③, T'BEGHEERDE. Wy moeten haer Somme vinden. WERCKING. Men sal de ghegheven ghetalen in oirden stellen als hier neven, die vergaderende naer de ghemeene maniere der vergaderinghe van heele getalen aldus:

	⓪	①	②	③
2 7		8	4	7
3 7		6	7	5
8 7 5		7	8	2
9 4 1		3	0	4

Comt in Somme (door het 1. probleme onser Franscher Arith.) 941304 dat sijn (t'welck de teeckenen boven de ghetalen staende, anwijsen) 941⓪3①0②4③. Ick segghe de selve te wesen de ware begheerde Somme. BEWYS. De ghegeven 27⓪8①4②7③, doen (door de 3e. bepaling) $27\frac{8}{10}$, $\frac{4}{100}$, $\frac{7}{1000}$, maecken t'samen $27\frac{847}{1000}$. Ende door de selve reden sullen de 37⓪6①7②5③ weerdich sijn $37\frac{675}{1000}$; Ende de 875⓪7①

8②

THE SECOND PART OF THE DIME.

Of the Operation or Practice.

THE FIRST PROPOSITION: OF ADDITION

Dime numbers being given, how to add them to find their sum.

THE EXPLICATION PROPOUNDED: There are 3 orders of dime numbers given, of which the first 27 ⓪, 8 ①, 4 ②, 7 ③, the second 37 ⓪, 6 ①, 7 ②, 5 ③, the third 875 ⓪, 7 ①, 8 ②, 2 ③.
THE EXPLICATION REQUIRED: We must find their total sum.

CONSTRUCTION

The numbers given must be placed in order as here adjoining, adding them in the vulgar manner of adding of whole numbers in this manner. The sum (by the first problem of our French Arithmetic [5])) is 941304, which are (that which the signs above the

			⓪	①	②	③
		2	7	8	4	7
		3	7	6	7	5
8	7	5	7	8	2	
9	4	1	3	0	4	

numbers do show) 941 ⓪ 5 ① 0 ② 4 ③. I say they are the sum required. Demonstration: The 27 ⓪ 8 ① 4 ② 7 ③ given make by the 3rd definition before 27, $\frac{8}{10}$, $\frac{4}{100}$, $\frac{7}{1000}$, together $27\frac{847}{1000}$ and by the same reason the 37 ⓪ 6 ① 7 ② 5 ③ shall make $37\frac{875}{1000}$ and the 875 ⓪ 7 ① 8 ② 2 ③ will make $875\frac{782}{1000}$,

5) *L'Arithmétique* (1585) Work V p. 81.

8 ② 2 ③ sullen doen 875 $\frac{782}{1000}$ welcke drie ghetalen als 27 $\frac{847}{1000}$ 37 $\frac{675}{1000}$ 875 $\frac{782}{1000}$, maecken t'samen (door het 10. probleme onser Franscher Arith.) 941 $\frac{304}{1000}$. Maer soo veel is oock weerdich de somme 941 ⓪ 3 ① 0 ② 4 ③, het is dan de ware somme, t'welck wy bewijsen moesten. BESLVYT. Wesende dan ghegheven Thiendetalen te vergaderen, wy hebben haer somme ghevonden soo wy voorghenomen hadden te doen.

MERCKT.

SOo inde ghegheven Thiendetalen eenich der natuerlicke oirden ghebraecke, men sal syn plaetse vollen met dat ghebreeckende. Laet by exempel de gegheven Thiendetalen syn 8 ⓪ 5 ① 6 ②, ende 5 ⓪ 7 ②, *in welck laetste ghebreeckt het Thiendetal der oirden* ①, *men sal in syn plaetse stellen* 0 ①, *nemende dan als voor ghegeven Thiendetal* 5 ⓪ 0 ① 7 ② *die vergaderende als vooren, in deser voughen:*

	⓪	①	②
	8	5	6
	5	0	7
1	3	6	3

Dit vermaen sal oock dienē tot de drie volgende voorstellē, alwaermē altijt d'oirden der gebreeckender Thiendetalen vervullen moet, gelijck in dit exempel gedaen is.

II. VOORSTEL VANDE AFTRECKINGHE.

Wesende ghegheven thiendetal daermen aftreckt, ende Thiendetal af te trecken: De Reste te vinden.

T'GEwhich three numbers make by common addition of vulgar arithmetic $941 \frac{304}{1000}$. But so much is the sum 941 ⓪ 5 ① 0 ② 4 ③; therefore it is the true sum to be demonstrated. Conclusion: Then dime numbers being given to be added, we have found their sum, which is the thing required.

NOTE that if in the number given there want some signs of their natural order, the place of the defectant shall be filled. As, for example, let the numbers given be 8 ⓪ 5 ① 6 ② and 5 ⓪ 7 ②, in which the latter wanted the sign of ①; in the place thereof shall 0 ① be put. Take then for that latter number given 5 ⓪ 0 ① 7 ②, adding them in this sort.

	⓪	①	②
	8	5	6
	5	0	7
1	3	6	3

This advertisement shall also serve in the three following propositions, wherein the order of the defailing figures must be supplied, as was done in the former example.

THE SECOND PROPOSITION: OF SUBTRACTION

A dime number being given to subtract, another less dime number given: out of the same to find their rest.

T'GHEGHEVEN. Het sy Thiendetal daermen aftreckt 237⓪5①7②8③, ende Thiendetal af te trecken. 59⓪7①4②9③. T'BEGHEERDE. Wy moeté haer Reste vinden. WERCKING. Men sal de ghegheven Thiendetalen in oirden stellen als hier neven, aftreckende naer de ghemeene maniere der Aftreckinge van heele ghetalen aldus:

⓪	①	②	③
2 3 7	5	7	8
5 9	7	4	9
1 7 7	8	2	9

Rest (door het 2. Probleme onser Franscher Arith.) 177829, dat sijn (twelck de teeckenen boven de ghetalen staende anwijsen) 177⓪8①2②9③. Ick segghe de selve te wesen de begheerde Reste. BEWYS. De ghegheven 237⓪5①7②8③ doen (door de 3e. Bepalinge) 237 $\frac{5}{10}$ $\frac{7}{100}$ $\frac{8}{1000}$, maecken t'samen 237 $\frac{578}{1000}$; Ende door de selve reden sullen de 59⓪7①4②9③ weerdich sijn 59 $\frac{749}{1000}$, welcke ghetrocken van 237 $\frac{578}{1000}$, rest (door het 11e. Probleme onser Franscher Arith.) 177 $\frac{829}{1000}$: Maer so veel is oock weerdich de voornoemde reste 177⓪8①2②9③, het is dan de ware Reste, twelck wy bewijsen moesten. BESLVYT. Wesende dan ghegheven Thiendetal daermen aftreckt, ende Thiendetal af te trecken, wy hebben haer Reste ghevonden, als voorghenomen was ghedaen te worden.

III. VOOR-

EXPLICATION PROPOUNDED: Be the numbers given 237 ⓪ 5 ① 7 ② 8 ③ & 59 ⓪ 7 ① 3 ② 9 ③. THE EXPLICATION REQUIRED: To find their rest.

CONSTRUCTION: The numbers given shall be placed in this sort, subtracting according to vulgar manner of subtraction of whole numbers, thus.

		⓪	①	②	③
2	3	7	5	7	8
	5	9	7	3	9
1	7	7	8	3	9

The rest is 177839, which values as the signs over them do denote 177 ⓪ 8 ① 3 ② 9 ③. I affirm the same to be the rest required.

Demonstration: the 237 ⓪ 5 ① 7 ② 8 ③ make (by the third definition of this Dime) $237\frac{5}{10}, \frac{7}{100}, \frac{8}{1000}$, together $237\frac{578}{1000}$, and by the same reason the 59 ⓪ 7 ① 3 ② 9 ③ value $59\frac{739}{1000}$, which subtracted from $237\frac{578}{1000}$, there rests $177\frac{839}{1000}$, but so much doth 177 ⓪ 8 ① 3 ② 9 ③ value; that is then the true rest which should be made manifest. CONCLUSION: a dime being given, to subtract it out of another dime number, and to know the rest, which we have found.

III. VOORSTEL VANDE MENICHVVLDIGHINGHE.

Wesende ghegheven Thiendetal te Menichvuldighen, ende Thiendetal Menichvulder: haer Vytbreng te vinden.

T GHEGHEVEN. Het sy Thiendetal te Menichvuldighen 32 ⓪ 5 ① 7 ②, ende het Thiendetal Menichvulder 89 ⓪ 4 ① 6 ②. TBEGHEERDE. Wy moeten haer Vytbreng vinden. WERCKING. Men sal de gegevẽ getalẽ in oirden stellen als hier nevẽ, Menichvuldigende naer de gemeene maniere van Menichvuldighen met heele ghetalen aldus: Gheeft Vytbreng (door het 3ᵉ. Prob. onser Fran. Arith.) 29137122: Nu om te weten wat dit sijn,

```
          ⓪ ① ②
        3  2  5  7
        8  9  4  6
       -----------
     1  9  5  4  2
   1  3  0  2  8
 2  9  3  1  3
2 6  0  5  6
------------------
2 9 1 3 7 1 2 2
      ⓪ ① ② ③ ④
```

men sal vergaderen beyde de laetste gegeven teeckenen, welcker een is ②, ende het ander oock ②, maecken tsamen ④, waer uyt men besluyten sal, dat de laetste cijffer des Vytbrengs is ④, welcke bekent wesende soo sijn oock (om haer volghende oirden) openbaer alle dander, Inder voughen dat 2913 ⓪ 7 ① 1 ② 2 ③ 2 ④, sijn het begheerde Vytbreng. BEWYS, Het ghegheven Thiendetal te menichvuldighen 32 ⓪ 5 ① 7 ②, doet (als blijct

THE THIRD PROPOSITION: OF MULTIPLICATION

A dime number being given to be multiplied, and a multiplicator given: to find their product.

THE EXPLICATION PROPOUNDED: Be the number to be multiplied 32 ⓪ 5 ① 7 ②, and the multiplicator 89 ⓪ 4 ① 6 ②.

THE EXPLICATION REQUIRED: To find the product. CONSTRUCTION: The given numbers are to be placed as here is shown, multiplying according to the vulgar manner of multiplication by whole numbers, in this manner, giving the product 29137122. Now to know how much they value, join the two last signs together as the one ② and the other ② also, which together make ④, and say that the last sign of the product shall be ④, which being known, all the rest are also known by their continued order. So that the product required is 2913 ⓪ 7 ① 1 ② 2 ③ 2 ④.

					⓪	①	②
				3	2	5	7
				8	9	4	6
			1	9	5	4	2
		1	3	0	2	8	
	2	9	3	1	3		
2	6	0	5	6			
2	9	1	3	7	1	2	2
			⓪	①	②	③	④

blijct door de derde Bepaling) $32 \frac{5}{10} \frac{7}{100}$, maecken tsamen $32 \frac{57}{100}$; Ende door de selve reden blijct den Menichvulder 89⓪4①6②, weerdich te sijne $89 \frac{46}{100}$, met de selve vermenichvuldicht de voornoemde $32 \frac{57}{100}$, gheeft Vytbreng (door het 12ᵉ. probleme onser Franscher Arith.) $2913 \frac{7122}{10000}$; Maer soo veel is oock weerdich den voornoemden Vytbreng 2913⓪7①1②2③2④, het is dan den waren Vytbreng; Twelck wy bewijsen moesten. Maer om nu te bethoonen de reden waerom ② vermenichvuldicht door ②, gheeft Vytbreng (welck de somme der ghetalen is) ④. Waerom ④ met ⑤, geeft Vytbreng ⑨, ende waerom ⓪ met ③ gheeft ③, etc. soo laet ons nemen $\frac{2}{10}$ ende $\frac{3}{100}$ (welcke door de derde Bepalinghe sijn 2①3②) hare Vytbreng is $\frac{6}{1000}$, welcke door de voornoemde derde Bepalinge sijn 6③. Vermenichvuldighende dan ① met ②, den Vytbreng sijn ③. BESLVYT. Wesende dan gegeven Thiendetal te Menichvuldighen, ende Thiendetal Menichvulder, wy hebben haren Vytbreng ghevonden; als voorghenomen was gedaen te worden.

MERCKT.

SOo het laetste teecken des Thiendetals te Menichvuldigē ende Menichvulders ongelyck waren, als by exempel deen 3④7⑤8⑥, dander 5①4②; Men sal doen als vooren, ende de ghesteltheyt der letteren vande Werckinghe sal soodanich syn:

		④	⑤	⑥	
		3	7	8	
			5	4	②
	1	5	1	2	
1	8	9	0		
2	0	4	1	2	
④	⑤	⑥	⑦	⑧	

DEMONSTRATION: The number given to be multiplied, 32 ⓪ 5 ① 7 ② (as appears by the third definition of this Dime), $32,\frac{5}{10},\frac{7}{100}$, together $32\frac{57}{100}$; and by the same reason the multiplicator 89 ⓪ 4 ① 6 ② value $89\frac{46}{100}$ by the same, the said $32\frac{57}{100}$ multiplied gives the product $2913\frac{7122}{10000}$. But it also values 2913 ⓪ 7 ① 1 ② 2 ③ 2 ④.

It is then the true product, which we were to demonstrate. But to show why ② multiplied by ② gives the product ④, which is the sum of their numbers, also why ④ by ⑤ produces ⑨, and why ⓪ by ③ produces ③, etc., let us take $\frac{2}{10}$ and $\frac{3}{100}$, which (by the third definition of this Dime) are 2 ① 3 ②, their product is $\frac{6}{1000}$, which value by the said third definition 6 ③; multiplying then ① by ②, the product is ③, namely a sign compounded of the sum of the numbers of the signs given.

CONCLUSION

A dime number to multiply and to be multiplied being given, we have found the product, as we ought.

NOTE

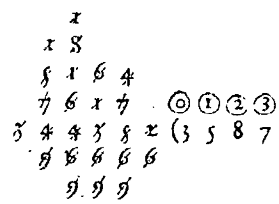
If the latter sign of the number to be multiplied be unequal to the latter sign of the multiplicator, as, for example, the one 3 ④ 7 ⑤ 8 ⑥, the other 5 ① 4 ②, they shall be handled as aforesaid, and the disposition thereof shall be thus.

		④	⑤	⑥	
		3	7	8	
			5	4	②
	1	5	1	2	
1	8	9	0		
2	0	4	1	2	
④	⑤	⑥	⑦	⑧	

IIII. VOORSTEL VANDE DEELINGHE.

Wesende ghegeven Thiendetal te Deelen, ende Thiendetal Deeler: Haren Soomenichmael te vinden.

T GHEGHEVEN. Het sy Thiendetal te deelen 3⓪ 4① 4② 3③ 5④ 2⑤, ende deeler 9① 6②. TBEGHEERDE. Wy moeten haer Soomenichmael vinden. WERCKING. Men sal de gegevē Thiendetalen deelen (achterlatende haer teeckenen) naer de ghemeene maniere van deelen met heele getalen aldus:



Geeft Somenichmael (door het vierde Probleme onser Franscher Arith.) 3587: Nu om te weten wat dit sijn, men sal af trecken het laetste teecken des Deelders, welck is ②, van t'laetste teecken des Thiendetals te deelen ⑤, rest ③, voor het teecken der laetster cijfferletter des Soomenichmaels, welcke bekent wesende, soo sijn oock (om haer volghende oirden) openbaer alle dander, inder voughen dat 3⓪ 5① 8② 7③ sijn den begheerden Soomenichmael. BEWYS, Het ghegeven Thiendetal 3⓪ 4① 4② 3③ 5④, 2⑤ doet (als blijct door de 3ᵉ Bepaling) 3 $\frac{4}{10}$ $\frac{4}{100}$ $\frac{3}{1000}$ $\frac{5}{10000}$ $\frac{2}{100000}$ maecken tsamen 3 $\frac{44352}{100000}$;

Ende

THE FOURTH PROPOSITION: OF DIVISION

A dime number for the dividend and divisor being given: to find the quotient.

EXPLICATION PROPOSED: Let the number for the dividend be 3 ⓪ 4 ① 4 ② 3 ③ 5 ④ 2 ⑤ and the divisor 9 ① 6 ②.

EXPLICATION REQUIRED: To find their quotient.

CONSTRUCTION: The numbers given divided (omitting the signs) according to the vulgar manner of dividing of whole numbers, gives the quotient 3587; now to know what they value, the latter sign of the divisor ② must be subtracted from the latter sign of the dividend, which is ⑤, rests ③ for the latter sign of the latter character of the quotient, which being so known, all the rest are also manifest by their continued order, thus 3 ⓪ 5 ① 8 ② 7 ③ are the quotient required.

DEMONSTRATION: The number dividend given 3 ⓪ 4 ① 4 ② 3 ③ 5 ④ 2 ⑤ makes (by the third definition of this Dime) 3, $\frac{4}{10}$, $\frac{4}{100}$, $\frac{3}{1000}$, $\frac{5}{10000}$, $\frac{2}{100000}$, together

Ende door de ſelve reden blijct den Deelder 9 ① 6 ② weerdich te ſijne $\frac{96}{100}$, door twelcke gedeelt de voornoemde 3 $\frac{44352}{1000000}$, gheeft Soomenichmael (door het 13. Probleme onſer Franſcher Arith.) 3 $\frac{587}{1000}$. Maer ſo veel is oock weerdich den voornomden Soomenichmael 3 ⓪ 5 ① 8 ② 7 ③, het is dan den waren Soomenichmael, Twelck wy bewijſen moeſten. BESLVYT. Weſende dan ghegheven Thiendetal te Deelen, ende Thiendetal Deeler, wy hebben haren Soomenichmael gevonden, als wy voorghenomen hadden te doen.

I. MERCKT.

S*OO de teeckenen des Deelders hoogher waren dan des Thiendetals te Deelen, men ſal by het Thiendetal te deelen ſoo veel* o *ſtellen, alſmen wil, ofte alſt noodich valt. By exempel* 7 ② *ſijn te deelen door* 4 ⑤, *ick ſtelle neven de* 7 *ettelicke* o *aldus* 7000,
die deelende als vooren ge- 3 2
daen is in deſer vougē: Geeft 7̸ 0̸ 0̸ 0̸ (1 7 5 0 ⓪
Soomenichmael 1 7 5 0 ⓪. 4̸ 4̸ 4̸ 4̸
Het ghebuert oock altemet dat den Soomenichmael met gheen heele ghetalen en can uytgheſproken worden, als 4 ①, *ghedeelt* x x x (1 ⓪ ① ②
door 3 ② *in deſer ma-* 4̸ 0̸ 0̸ 0̸ 0 0 0 (1 3 3 3
nieren: Alwaer blijct 3̸ 3̸ 3̸ 3̸
datter oneyndelicke drien uyt commen ſouden, ſonder eenichmael even uyt te gheraecken: In ſulcken ghevalle machmen ſoo naer commen als de ſaecke dat voordert, ende het overſchot verloren laten. Wel is waer

$3\frac{44352}{100000}$, and by the same reason the divisor 9 ① 6 ② values $\frac{96}{100}$, by which $3\frac{44352}{100000}$ being divided, gives the quotient $3\frac{587}{1000}$; but the said quotient values 3 ⓪ 5 ① 8 ② 7 ③, therefore it is the true quotient to be demonstrated.

CONCLUSION: A dime number being given for the dividend and divisor, we have found the quotient required.

NOTE: *If the divisor's signs be higher than the signs of the dividend, there*
may be as many such ciphers 0 joined to the 3 2
dividend as you will, or many as shall be necessary: 7 0 0 0 (1750 ⓪
as for example, 7② *are to be divided by* 4 ⑤, *I* 4 4 4 4
place after the 7 certain 0, thus 7000, dividing them as afore said, and in this sort it gives for the quotient 1750 ⓪.

It happens also sometimes that the quotient cannot be expressed by whole
numbers, as 4 ① *divided by* 3 ② *in* 1 1 1 (1 ⓪ ① ②
this sort, whereby appears that there 4 0 0 0 0 0 0 (1 3 3 3
will infinitely come 3's, and in such 3 3 3 3

dat 13⓪3①3 $\frac{1}{3}$②, ofte 13⓪3①3②3 $\frac{1}{3}$③ etc. souden het volcommen begheerde sijn, maer ons voornemen is in dese Thiende te wercken met louter heele ghetalen, want wy opsicht hebben naer t'ghene in sMenschen handel plaets houdt, alwaermen het duysenste deel van een Mijte, van een Aes, van een Graen ende dierghelijcke, verloren laet; So tselfde oock byden voornaemsten Meters ende Telders dickmael onderhouden wort, in vele rekeninghen van grooten belanghe: Als Ptolemeus ende Ian van Kueninckxberghe, en hebben hare Boogpees Tafelen *met de uyterste volmaectheyt niet beschreven, hoe wel het door* Veelnamighe Ghetalen *doenlick was, Reden dat dese onvolmaectheyt (ansiende dier dinghen Eynde) nutter is dan soodanighe volmaectheydt.*

Tabulas Arcuum & Chordarum. Multinomios numeros.

II. MERCT.

DE *Vyttreckinghen aller specien der* Wortelen *mueghen hier in oock gheschien. By exempel om te vinden den viercanten* Wortel *van 5②2③9④ (dienende tot het maecken der Boogpeez Tafelen naer Ptolomeus maniere) men sal wercken naer de ghemeene ghebruyck aldus: Ende den wortel sal sijn 2①3②, want den helft van het laetste teecken des gheghevens is altijt het laetste teecken des wortels: Daerom soo het laetste ghegheven teecken oneffen ghetal ware,*

x		
5	2	9
2		3
	4	

men

a case you may come so near as the thing requires, omitting the remainder. It is true, that 13 ⓪ 3 ① $3\frac{1}{3}$ ②, *or* 13 ⓪ 3 ① 3 ② $3\frac{1}{3}$ ③ *etc. shall be the perfect quotient required. But our invention in this Dime is to work all by whole numbers. For seeing that in any affairs men reckon not of the thousandth part of a mite, es, grain, etc., as the like is also used of the principal geometricians and astronomers in computations of great consequence, as Ptolemy and Johannes Montaregio* [6]), *have not described their tables of arcs, chords or sines in extreme perfection (as possibly they might have done by multinomial numbers), because that imperfection (considering the scope and end of those tables) is more convenient than such perfection.*

NOTE 2. *The extraction of all kinds of roots may also be made by these dime numbers; as, for example, to extract the square root of* 5 ② 2 ③ 9 ④, *which is performed in the vulgar manner of extraction in this sort, and the root shall be* 2 ① 3 ②, *for the moiety or half of the latter sign of the numbers given is always the latter sign of the root; wherefore, if the latter sign given were of a number impair, the sign of the next following shall be added, and then it shall be a number*

1
5 2 9
―――
2 3
―――
4

---

[6]) See the Introduction to *De Thiende*, esp. footnote [5]) and to the *Driehouckhandel*. Johannes Montaregio, or Ian van Kuenincxberghe, Iehan de Montroial, is best known under his latinized name Iohannes Regiomontanus (1436–1476). This craftsman, humanist, astronomer and mathematician of Nuremberg, born near Königsberg in Franconia (hence his name), influenced the development of trigonometry as an independent science for more than a century by his tables and his *De triangulis omnimodis libri quinque* (first published in 1533). The sines, for Regiomontanus as well as for Stevin, were half chords, not ratios. On Regiomontanus see E. Zinner, *Leben und Wirken des Johannes Müller von Königsberg genannt Regiomontanus*, Schriftenreihe zur bayr. Landesgesch. 31, München, 1938, XIII + 294 pp.

men salder noch een naestvolghende teecken toedoen, ende wercken dan als boven.

Insghelijcx oock int Vyttrecken des Teerlincxwortel, daer sal het laetste teecken des Wortels, altijt het derdendeel sijn van het laetste ghegheven teecken, ende alsoo voort in allen anderen specien der Wortelen.

EYNDE DER THIENDE

pair; and then extract the root as before. Likewise in the extraction of the cubic root, the third part of the latter sign given shall be always the sign of the root; and so of all other kinds of roots.

THE END OF THE DIME

AENHANGSEL.

VOORREDEN.

NADEMAEL *vvy hier vooren de Thiende beschreven hebben soo verre ter Saecken noodich schijnt, sullen nu commen tot de ghebruyck van dien, bethoonende door 6 Leden, hoe alle rekeninghen ter Menschelicker nootlickheyt ontmoetende, door haer lichtelick ende slichtelick connen afgheveerdicht vvorden met heele ghetalen, beghinnende eerst (gelijck sy oock eerst int vverck gestelt is) ande rekeninghen der Landtmeterie als volght.*

I. LIDT VANDE REKENINGHEN DER LANDTMETERIE.

MEN sal de roede anderſins ſegghen te weſen een BEGHIN, dat is 1 ⓪, die deelende in thien even deelen, welcker yder doen ſal een ***Eerſte***, ofte 1 ①; Daer naer ſalmen elcke ***Eer-***

ſte

THE APPENDIX

THE PREFACE

Seeing that we have already described the Dime, we will now come to the use thereof, showing by 6 articles how all computations which can happen in any man's business may be easily performed thereby; beginning first to show how they are to be put in practice in the casting up of the content or quantity of land, measured as follows.

THE FIRST ARTICLE: OF THE COMPUTATIONS OF LAND-METING.

Call the perch or rod [7]) also *commencement*, which is 1 ⓪, dividing that into

[7]) The English "perch" or "rod" and the Dutch "roede" are both measures of area and of length. For information on the precise meaning of the many measures mentioned in Stevin's book one may consult the *Oxford Dictionary of the English language*.

ste wederom deelen in thien even deelen, welcker yder sijn, sal 1 ②, ende soomen die deelinghen cleender begheert, soo salmen elcke 1 ②, noch eenmael deelen in thien even deelen, die elck 1 ③ doen sullen, ende soo voort by aldien het noodich viele: Hoe wel soo veel het Landtmeten belangt, de deelen in ② sijn cleen ghenouch: maer tot de saecken die nauwer mate begheeren, als Lootdaecken, Lichamen, etc. daer machmen de ③ ghebruycken.

Angaende dat de meestendeel der Landtmeters gheen roede en besighen, maer een keten van drie, vier, ofte vijf roeden lanck, teeckenende op den rock van het Rechtcruys, eenighe vijf ofte ses Voeten, met haren Duymen, sulcx mueghen sy hier oock doen, alleenelick voor die vijf ofte ses Voeten met haren Duymen, stellende vijf ofte ses *Eersten* met haren *Tweeden*.

Dit aldus sijnde men sal int meten ghebruycken dese deelen, sonder opsicht te hebben naer Voeten ofte Duymen die elcke Roede naer Landtsghebruyck inhoudt, ende t'ghene naer die mate sal moeten Vergadert, Afghetrocken, Ghemenichvuldicht, ofte Ghedeelt worden, dat salmen doen naer de leeringhe der voorgaender vier Voorstellen.

By exempel, daer sijn te vergaderen vier Driehoucken, ofte sticken Landts, welcker eerste 345 ⓪ 7 ① 2 ②, het tweede 872 ⓪ 5 ① 3 ②, het derde 615 ⓪ 4 ① 8 ②, het vierde 956 ⓪

10 equal parts, whereof each one shall be 1 ①; then divide each prime again into 10 equal parts, each of which shall be 1 ②; and again each of them into 10 equal parts, and each of them shall be 1 ③, proceeding further so, if need be. But in land-meting, divisions of seconds will be small enough. Yet for such things as require more exactness, as roofs of lead, bodies, etc., there may be thirds used, and for as much as the greater number of land-meters use not the pole, but a chain line of three, four or five perch long, marking upon the yard of their cross staff [8]) certain feet 5 or 6 with fingers, palms, etc., the like may be done here; for in the place of their five or six feet with their fingers, they may put 5 or 6 *primes* with their *seconds*.

This being so prepared, these shall be used in measuring, without regarding the feet and fingers of the pole, according to the custom of the place; and that which must be added, subtracted, multiplied or divided according to this measure shall be performed according to the doctrine of the precedent examples.

As, for example, we are to add 4 triangles or surfaces of land, whereof the first 345 ⓪ 7 ① 2 ②, the second 872 ⓪ 5 ① 3 ②, the third 615 ⓪ 4 ① 8 ②, the fourth 956 ⓪ 8 ① 6 ②.

			⓪	①	②
	3	4	5	7	2
	8	7	2	5	3
	6	1	5	4	8
	9	5	6	8	6
2	7	9	0	5	9

8) The Dutch *rechtcruys*, in Stevin's French version *croix rectangulaire*, translated *cross-staff*, was an instrument used by surveyors for setting out perpendiculars by lines of sight, crossing each other at right angles. It was also known as *surveyor's cross*. The cross was horizontal and supported by a pole, the *yard* of our text, on which Stevin wants to measure off a decimal scale. A variant of this cross was a graduated horizontal circle with a pointer (alhidade) along which sighting could be performed, but even in the variations the basic rectangular cross remained.

Surveyors also used chains for measuring distances, or setting out perpendiculars, in which case they used the so-called 6, 8 and 10 rule, a popular application of Pythagoras' theorem.

The surveyor's cross is mentioned in many books on surveying. In N. Bion, *Traité de la construction et des principaux usages des instruments de mathématique*, Nouvelle édition, La Haye 1723, p. 133 we find it referred to as "équerre d'arpenteur", with a picture (information from Dr. P. H. van Cittert).

8①6②, Dese vergadert naer de maniere int eerste voorstel verclaert in deser voughen: Hare somme sal sijn 2790⓪ ofte Roeden, 5①9②, De voornomde Roeden ghedeelt naer de ghebruyck met so veel, alsser Roeden op een Morghen ofte Ghemet gaen, men sal de Morghen ofte Ghemeten hebben.

			⓪	①	②
	3	4	5	7	2
	8	7	2	5	3
	6	1	5	4	8
	9	5	6	8	6
2	7	9	0	5	9

Maer soomen wil weten hoe veel Voeten en Duymen de 5①9② maeckē (twelck hier eens voor al gheseyt, den Landtmeter maer eenmael en behouft te doen int laetste sijnder rekeninghen, die hy den eyghenaers overlevert, hoe wel den meestendeel van haer onnut achten, aldaer van Voeten te spreecken) men sal op de Roede besien hoe veel Voeten ende Duymen (welcke neven de deelen der Thiendetalen op een ander sijde der Roeden gheteeckent staen) daer op passen.

Ten anderen, wesende van 57⓪3①2②, te trecken 32⓪5①7②, men sal wercken naer het 2^e^ Voorstel in deser voughen: Ende sullen resten 24⓪, ofte Roeden, 7①5②.

	⓪	①	②
5	7	3	2
3	2	5	7
2	4	7	5

These being added according to the manner declared in the first proposition of this Dime in this sort, their sum will be 2790 ⓪ or perches 5 ① 9 ②, the said rods or perches divided according to the custom of the place (for every acre contains certain perches), by the number of perches you shall have the acres sought. But if one would know how many feet and fingers are in the 5 ① 9 ② (that which the land-meter shall need to do but once, and that at the end of the casting up of the proprietaries, although most men esteem it unnecessary to make any mention of feet and fingers), it will appear upon the pole how many feet and fingers (which are marked, joining the tenth part upon another side of the rod) accord with themselves.

In the second, out of 57 ⓪ 3 ① 2 ② subtracted 32 ⓪ 5 ① 7 ②, it may be effected according to the second proposition of this Dime in this manner:

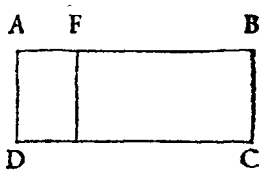
```
 5 7 3 2
 -------
 3 2 5 7
 -------
 2 4 7 5
```

Ten derden, wesende te Vermenichvuldighen van wegen de sijden eens Driehoucx ofte Vierhoucx 8⓪ 7① 3②, door 7⓪ 5① 4②: Men sal doen naer het 3ᵉ voorstel aldus:

⓪ ① ②
8 7 3
7 5 4
3 4 9 2
4 3 6 5
6 1 1 1
6 5 8 2 4 2
⓪ ① ② ③ ④

Gheven uytbreng ofte Plat 65⓪ 8①, etc.

Ten Vierden, laet A B C D, een viersijdich rechthouck sijn, waer af ghesneden moet worden 367⓪ 6①, Ende de sijde A D, doet 26⓪ 3①, De vraghe is hoe verre men van A, naer B, meten sal, om af te snijden de voornomde 367⓪ 6①.



Men sal 367⓪ 6① deelen door de 26⓪ 3①, naer het vierde voorstel aldus:

1
2 2
7 6
2 5 0 8
4 6 3 1
1 0 4 7 3 9 ⓪ ① ②
3 6 7 6 0 0 (1 3 9 7
2 6 3 3 3 3
2 6 6 6
2 2

Gheeft Soomenichmael voor de begeerde langde van A, naer B, welcke sy A F, 13⓪ 9① 7②, Ofte naerder canmen commen soomen wil (hoe wel het onnoodich schijnt) door het eerste Merct des vierden voorstels. Van

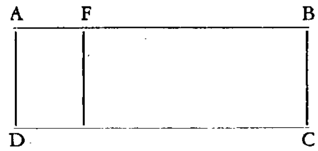
 alle

In the third (for multiplication of the sides of certain triangles and quadrangles) multiply 8 ⓪ 7 ① 3 ② by 7 ⓪ 5 ① 4 ②, & this may be performed according to the third proposition of this Dime, in this manner:

			⓪	①	②
			8	7	3
			7	5	4
		3	4	9	2
	4	3	6	5	.
6	1	1	1		
6	5	8	2	4	2
	⓪	①	②	③	④

And gives for the product or superficies 65 ⓪ 8 ① etc.

In the fourth let A, B, C, D be a certain quadrangle rectangular, from which we must cut 367 ⓪ 6 ①, and the side AD makes 26 ⓪ 3 ①: the question is how much we shall measure from A towards B to cut off (I mean by a line parallel to AD) the said 367 ⓪ 6 ①.



Divide 367 ⓪ 6 ① by 26 ⓪ 3 ① according to the fourth proposition of this Dime: so the quotient gives from A towards B 13 ⓪ 9 ① 7 ②, which is AF.

And if we will, we may come nearer (although it be needless) by the second alle welcke exempelen de Bewijsen in hare voorstellen ghedaen sijn.

II. LIDT VANDE REKENINGEN DER TAPYTMETERIE.

DEs Tapijtmeters Elle sal hem 1 ⓪ verstrecken de selve sal hy (op eenighe sijde daer de Stadtmatens deelinghen niet en staen) deelen als vooren des Landtmeters Roe ghedaen is, te weten in 10 even deelen, welcker yeder 1 ① sy, ende yder 1 ① weder in 10 even deelen, welcker yder 1 ② doe, ende soo voorts. Wat de gebruyck van dien belangt, anghesien d'exempelen in alles overcommen met het ghene int eerste Lidt vande Landtmeterie gheseyt is, soo sijn dese door die, kennelick ghenouch, inder voughen dat het niet noodich en is daer af alhier meer te roeren.

III. LIDT VANDE WYNMETERIE.

EEn Ame (welcke t'Andtwerpen 100 potten doet) sal 1 ⓪ sijn, de selve sal op diepte ende langde der wijnroede ghedeelt worden in 10 even deelen (wel verstaende even int ansien des wijns, niet der Roeden, wiens deelen der diepte oneven vallen) ende yder van dien sal 1 ① sijn, inhoudende 10 potten, wederom elcke 1 ① in thien even deelen, welcke yder 1 ② sal maecken, die note of the fourth proposition; the demonstrations of all these examples are already made in their propositions.

```
      2 2
      7 6
    2 5 0 (8
    4 6 3 1
  1 0 4 7 3 (9          1 3 9 7
  3 6 7 6 0 0             ⓪ ① ②
  2 6 3 3 3 3
    2 6 6 6
      2 2
```

THE SECOND ARTICLE: OF THE COMPUTATIONS OF THE MEASURES OF TAPESTRY OR CLOTH

The ell of the measurer of tapestry or cloth shall be to him 1 ⓪, the which he shall divide (upon the side whereon the partitions which are according to the ordinance of the town is not set out) as is done above on the pole of the land-meter, namely into 10 equal parts, whereof each shall be 1 ⓪, then each 1 ① into 10 equal parts, of which each shall be 1 ②, etc. And for the practice, seeing that these examples do altogether accord with those of the first article of land-meting, it is thereby sufficiently manifest, so as we need not here make any mention again of them.

THE THIRD ARTICLE: OF THE COMPUTATIONS SERVING TO GAUGING, AND THE MEASURES OF ALL LIQUOR VESSELS

One ame (which makes 100 pots Antwerp) shall be 1 ⓪, the same shall be divided in length and deepness into 10 equal parts (namely equal in respect of the wine, not of the rod; of which the parts of the depth shall be unequal), and each part shall be 1 ① containing 10 pots; then again each 1 ① into 10 parts equal as afore, and each will make 1 ② worth 1 pot; then each 1 ② into 10 equal parts, making each 1 ③.

die een pot weert is, ende elck van desen wederom in thienen, ende elck sal 1 ③ verstrecken. De roede alsoo ghedeelt sijnde, men sal (om te vinden het inhoudt der tonnen) Menichvuldigen ende Wercken als int voorgaende 1ᵉ Lidt ghedaen is, welck door t'selfde openbaer ghenouch sijnde, en sullen daer af hier niet wijder segghen.

Maer anghesien dees thiendeelighe voortganck der diepten niet ghemeen en is, soo mueghen wy daer af dit verclaren: Laet de Roede A B, een Ame sijn, dat is 1 ⓪ die ghedeelt sy in thien dieppunten (naer de ghebruyck) C, D, E, F, G, H, I, K, L, A, yder doende 1 ①, welcke wederom ghedeelt moeten worden in thienen, dat aldus toegaet: Men sal eerst elcke 1 ① deelen in tween in deser voughen: Men sal trecken de Linie B M, rechthouckich op A B, ende even met de 1 ① B C, ende vinden daer naer (door het 13ᵉ voorstel des sesten boucx van Euclides) de middel Evenredenighe Linie tusschen B M, ende haer helft, welcke sy B N, teeckenende B O even an B N, ende soo dan N O, even is an B C, de werckinge gaedt wel; Daer naer salmen de langde N C, teeckenen van B naer A, als B P, welcke even vallende an N C, t'werck is goedt; insghelijcx de langde D N, van B tot Q, ende soo voorts met dander. Nu rester noch elck deser lengden als B O, ende O C, etc. te deelen in vijven aldus: Men sal tusschen B M, ende haer thiendedeel, vinden de middel Everednighe linie, welcke sy B R, teeckenende B S

even

Now the rod being so divided, to know the content of the tun, multiply and work as in the precedent first article, of which (being sufficiently manifest) we will not speak here any farther.

But seeing that this tenth division of the deepness is not vulgarly known, we will explain the same. Let the rod be one ame A B, which is 1 ⓪, divided (according to the custom) into the points of the deepness of these nine: C, D, E, F, G, H, I, K, A, making each part 1 ①, which shall be again each part divided into 10, thus. Let each 1 ① be divided into two so: draw the line BM with a right angle upon AB and equal to 1 ①, BC, then (by the 13th proposition of Euclid his 6th book) [9] find the mean proportional between BM and his moiety, which is BN, cutting BO equal to BN. And if NO be equal to BC, the operation is good. Then note the length NC from B towards A, as BP, the which being equal to NC, the operation is good; likewise the length of BN from B to Q; and so of the rest.

It remains yet to divide each length as BO & OC, etc. into five, thus: Seek the mean proportional between BM & his 10th part, which shall be BR, cutting

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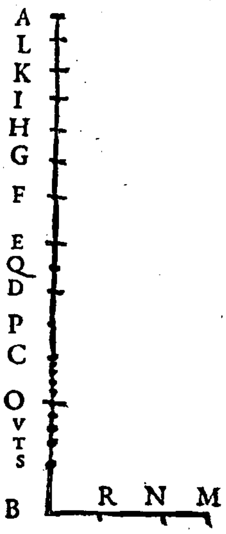
[9]) Euclid, in *Elements* VI 13, shows how to find the mean proportional to two given line segments with the aid of a circle drawn upon the sum of these line segments as diameter.

even an B R; Daer naer ſalmen de langde S R, teeckenen van B naer A, als B T, inſghelijcx de langde T R, van B tot V, ende ſoo voorts. Sghelijcx ſal oock den voortganck ſijn om de ② ofte potten als B S ende S T, etc. te deelen in ③. Ick ſegghe dat B S, ende S T, ende T V, etc. ſijn de ware begeerde ②, t'welck aldus beweſen wort: Overmidts B N, is middel Everednighe (duer *t'Gheſtelde*) tuſſchen B M, ende haer helft, ſoo is het viercant van B N (duer het 17^{e} voorſtel des ſeſten boucx van Euclides) even an den rechthouck van B M ende hare helft; Maer dien Rechthouck is den helft des viercants van B M, Het Viercant dan van B N, is even anden helft des Viercants van B M, Maer B O is (door t'Gheſtelde) even an B N, ende B C an B M, het Viercant dan van B O, is even anden helft des Viereants van B C, Sghelijcx ſal oock het bewijs ſijn dat het Viercant van B S, even is an het thiendedeel des Viercants B M, daerom, etc.

Per hypotheſin.



Het

BS equal to BR. Then the length SR, noted from B towards A as BT, and likewise the length TR from B to V, & so of the others, & in like sort proceeding to divide BS and ST, etc. into ③, I say that BS, ST, and TV, etc. are the desired ②, which is thus to be demonstrated.

For that BN is the mean proportional line (by the *hypothesis)* between BM and his moiety, the square of BN (by the 17th proposition of the sixth book of Euclid) [10]) shall be equal to the rectangle of BM & his moiety. But the same rectangle is the moiety of the square of BM; the square then of BN is equal to the moiety of the square of BM. But BO is (by hypothesis) equal to BN, and BC to BM; the square then of BO is equal to the moiety of the square of BC. And in like sort it is to be demonstrated that the square of BS is equal to the

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[10]) Euclid, in *Elements* VI 17, shows geometrically that when a, b, c are in geometrical proportion, $ac = b^2$, and conversely.

Het bewijs is cort ghemaect, overmidts wy indies niet aen Leerlinghen maer aen Meesters schrijven.

IIII. LIDT VANDE LICHAEMMETERIE INT GHEMEENE.

HET is wel waer dat alle Wijnmeterie (die wy hier vooren verclaert hebbẽ) is Lichaemmeterie, maer anmerckende de verscheyden deelinghen der roeden van d'een buyten d'ander, oock dat dit, alsulcken verschil heeft tot dat, als Gheslachte tot Specie, soo mueghen sy met reden onderscheyden worden, want alle Lichaemmeterie gheen Wijnmeterie en is. Om dan tot de Saecke te commen, den Lichaemmeter sal ghebruycken de Stadtmate, als Roede ofte Elle met hare Thiendedeelinghen, soo die int eerste ende tweede Lidt beschreven sijn, wiens gebruyck van het voorgaende weynich schillende, aldus toegaet: Ick neme datter te meten sy eenige Vierhouckige Rechthouckighe Colomme, diens Langde 3 ① 2 ②, Breede 2 ① 4 ②, Hoochde 2 ⓪ 3 ① 5 ②, Vraghe hoe veel Stoffe daer in sy, ofte van wat begrijp sodanighen lichaem is. Men sal Menichvuldigẽ naer de leering des derden Voorstels. Langde door Breede, ende dien Vytbreng weder door Hoochde in deser voughen: Geeft Vytbreng als blijct 1 ① 8 ② 4 ④ 8 ⑤.

```
          ① ②
          3 2
          2 4
       --------
        1 2 8
        6 4
       ----------
        7 6 8 ②
        2 3 5 ②
     ------------
      3 8 4 0
    2 3 0 4
  1 5 3 6
  ----------------
  1 8 0 4 8 0
  ① ② ③ ④ ⑤ ⑥
```

tenth part of the square of BM. Wherefore etc. we have made the demonstration brief, because we write not this to learners, but unto masters in their science. 11)

THE FOURTH ARTICLE: OF COMPUTATIONS OF STEREOMETRY IN GENERAL

True it is that gaugery, which we have before declared, is stereometry (that is to say, the art of measuring of bodies), but considering the divers divisions of the rod, yard or measure of the one and other, and that and this do so much differ as the genus and the species: they ought by good reason to be distinguished. For all stereometry is not gaugery. To come to the point, the stereometrian shall use the measure of the town or place, as the yard, ell, etc. with his ten partitions, as is described in the first and second articles; the use and practice thereof (as is before shown) is thus: Put case we have a quadrangular rectangular column to be measured, the length whereof is 3 ① 2 ②, the breadth 2 ① 4 ②, the height 2 ⓪ 3 ① 5 ②. The question is how much the substance or matter of that pillar is. Multiply (according to the doctrine of the 4th proposition of this Dime) the length by the breadth, & the product again by the height in this manner.
And the product appears to be 1 ① 8 ② 4 ④ 8 ⑤.

				①	②	
				3	2	
				2	4	
			1	2	8	
			6	4		
			7	6	8	④
			2	3	5	②
		3	8	4	0	
	2	3	0	4		
1	5	3	6			
1	8	0	4	8	0	
①	②	③	④	⑤	⑥	

11) Stevin's division of the unit BC is equivalent to the following interpolation of numbers between 0 and 1: 0, $\frac{1}{10}\sqrt{10}$, $\frac{1}{10}\sqrt{20}$, $\frac{1}{10}\sqrt{30}$, ..., $\frac{1}{10}\sqrt{90}$, 1, where BS : BT : .... : BO $= \frac{1}{10}\sqrt{10} : \frac{1}{10}\sqrt{20} : \ldots : \frac{1}{10}\sqrt{50}$. The squares of these numbers are 0, $\frac{1}{10}$, $\frac{2}{10}$, $\frac{3}{10}$, ... $\frac{9}{10}$, 1.

MERCT.

*Y*EMANDT *den Grondt der Lichaemmeterie niet ghenouch ervaren (want tot dien spreecken wy hier) mocht dincken waeromme men segt dat de colomme hier boven maer* 1 ①, *etc. groot en is, nademael sy over de* 180 *Teerlinghen in haer houdt, diens sijden elck van* 1 ① *lanck sijn; Die sal weten dat een Roede Lichaems niet en is van* 10 ①, *als een Roede in langde, maer van* 1000 ①, *in welcken ansien* 1 ① *doet* 100 *Teerlinghen elck van* 1 ①; *Alsoo der ghelijcke den Landtmeters int Plat ghenouch bekendt is, want alsmen segt* 2 *Roeden* 3 *Voeten Landts, dat en sijn niet* 2 *Roeden ende drie Viercante voeten, maer* 2 *Roeden ende (rekenende* 12 *Voeten voor de Roe)* 36 *viercante voeten: Daerom soo de vraghe hier boven gheweest ware van hoe veel teerlinghen elck van* 1 ①, *de voornomde colomme groot is, men soude t'besluyt daer naer moeten voughen, anmerckende dat yder* 1 ① *van dese, doet* 100 ① *van dien, ende yder* 1 ② *van dese,* 10 ① *van dien, etc. Ofte andersins, soo het thiendedeel der Roede de grootste mate is, daer op den Lichaemmeter opsicht heeft, hy mach dat Thiendeel noemen* Beghin, *dat is* ⓪, *ende voort als boven.*

V. LIDT VANDE STERRE-CONSTS REKENINGHEN.

Gradus. DE oude Sterrekijckers het Rondt ghedeelt hebbende in 360. *Trappen*, bevonden dat de

NOTE, *some, ignorant (and understanding not that we speak here) of the principles of stereometry, may marvel whereof it is said that the greatness of the abovesaid column is but 1 ①, etc., seeing that it contains more than 180 cubes, of which the length of each side is 1 ①; he must know that the body of one yard is not a body of 10 ① as a yard in length, but 1000 ①, in respect whereof 1 ① makes 100 cubes, each of 1 ①, as the like is sufficiently manifest amongst land-meters in surfaces; for when they say 2 rods, 3 feet of land, it is not barely meant 2 square rods and three square feet, but two rods (and counting but 12 feet to the rod) 36 feet square; therefore if the said question had been how many cubes, each being 1 ①, was in the greatness of the said pillar, the solution should have been fitted accordingly, considering that each of these 1 ① doth make 100 ① of those; and each 1 ② of these makes 10 ① of those, etc. or otherwise, if the tenth part of the yard be the greatest measure that the stereometrian proposes, he may call it 1 ⓪, and so as above said.*

THE FIFTH ARTICLE: OF ASTRONOMICAL COMPUTATIONS

The ancient astronomers having divided their circles each into 360 *degrees*, they saw that the astronomical computations of them with their parts was too de Sterreconſts rekeninghen der ſelver met haren onderdeelen ofte ghebroken ghetalen, veel te moeyelick vielen, Daerom hebben ſy elcken Trap willen ſcheyden in ſeecker deelen, ende de ſelve deelen andermael in alſoo veel, etc. om duer ſulcke middel altijt lichtelicker te mueghen wercken door heele ghetalen, daer toe verkieſende de t'ſeſtichdeelighe voortganck, overmidts 60 een ghetal is metelick door vele verſcheyden heele maten, namelick 1, 2, 3, 4, 5, 6, 10, 12, 15, 20, 30. Maer ſoo wy de Ervaring ghelooven (met alder eerbieding der looflicker Oudtheyt, ende door beweechniſſe tot de ghemeene nut gheſproken) voorwaer de t'eſtichdeelighe voortganck en was niet de bequaemſte, immer onder de ghene die machtelick inde Natuere beſtonden, maer de Thiendeelighe, welcke aldus toegaedt: De 360. Trappen des Rondts, noemen wy anderſins *Beghinſelen*, ende yder Trap ofte 1 ⓪ ſal ghedeelt worden in 10 even deelen, welcker yder ons een ① verſtreckt, daer naer yder 1 ①, weder in 10 ②, ende ſoo vervolghens als int voorgaende dickmael ghedaen is.

Nu deſe deylinghen alſoo verſtaen ſijnde, wy ſouden mueghen hare belooſde lichte maniere van Vergaderen, Aftrecken, Menichvuldighen, ende Deelen, door verſcheyden exempelen beſchrijven, maer angheſien ſy vande vier voorgaende voorſtellen gantſch niet en verſchillen, ſulck verhael ſoude hier ſchadelicke Tijtverlies, ende onnoodighe pampierquiſtighe ſijn, daerom

laten

laborious; and therefore they divided also each degree into certain parts, and these again into as many, etc., to the end thereby to work always by whole numbers, choosing the 60th progression because that 60 is a number measurable by many whole measures, namely 1, 2, 3, 4, 5, 6, 10, 12, 15, 20, 30; but if experience may be credited (we say with reverence to the venerable antiquity and moved with the common utility), the 60th progression was not the most convenient (at least) amongst those that in nature consist potentially, but the tenth, which is thus. We call the 360 degrees also *commencements,* expressing them so 360 $\textcircled{0}$, and each of them a degree or 1 $\textcircled{0}$ to be divided into 10 equal parts, of which each shall make 1 $\textcircled{1}$, and again each 1 $\textcircled{1}$ into 10 $\textcircled{2}$, and so of the rest, as the like hath already been often done.

Now this division being understood, we may describe more easily that we promised in addition, subtraction, multiplication, and division; but because there is no difference between the operation of these and the four former propositions of this book, it would but be loss of time, and therefore they shall serve for laten wy die voor exempelen deses Lidts verstrecken. Dit noch hier by voughende, dat wy inde Sterreconst die wy in onse Duytsche Tale (dat is inde aldercierlicste alderrijckste, ende aldervolmaeckste Spraecke der Spraecken, van wiens groote besonderheydt wy cortelick noch al veel breeder ende seeckerder betooch verwachten, dan Pieter ende Ian daer af ghedaen hebben inde Bewijsconst ofte Dialectike onlancx uytghegheven) hopen te laten uytgaen, dese maniere der deelinghe in allen Tafelen ende Rekeninghen sich daer ontmoetende, ghebruycken sullen.

VI. LIDT VANDE REKENINGHEN DER MUNTMEESTERS, *Cooplieden, ende allen Staten van volcke int ghemeene.*

OM generalick ende int cort te spreecken vanden grondt deses Lidts, soo is te weten dat alle mate, als Langhe, Drooghe, Natte, Ghelt, etc. ghedeelt sal worden door de voornoemde thiendeelighe voortganck, Ende elcke groote vermaerde Specie van dien salmen *Beghin* noemen, als Marck, *Beghin* der ghewichten daer mede men Silver ende Goudt weecht: Pondt, *Beghin* van dander ghemeene ghewichten: Pondtgroot in Vlaenderen, Ponsteerlincx in Inghelandt, Ducaet in Spaeigne, etc. *Beghin* des Ghelts.

Des

examples of this article; yet adding thus much that we will use this manner of partition in all the tables & computations which happen in astronomy [12]), such as we hope to divulge in our vulgar German [13]) language, which is the most rich adorned and perfect tongue of all other, & of the most singularity, of which we attend a more abundant demonstration than Peter and John have made thereof in the Bewysconst and Dialectique, lately divulged [14]).

THE SIXTH ARTICLE: OF THE COMPUTATIONS OF MONEY-MASTERS, MERCHANTS, AND OF ALL ESTATES IN GENERAL

To the end we speak in general and briefly of the sum and contents of this article, it must be always understood that all measures (be they of length, liquors, of money, etc.) be parted by the tenth progression, and each notable species of them shall be called *commencement:* as a mark, *commencement* of weight, by the which silver and gold are weighed, pound of other common weights, livres de gros in Flanders, pound sterling in England, ducat in Spain, etc. *commencement*

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[12]) On this see the Introduction § 6, and footnote [40]).
[13]) Concerning the use of German in the sense of Dutch, see Vol. I, p. 7, note.
[14]) Stevin here refers to his *Dialectike*, Work III (cf. the bibliography in Vol. I, p. 26). – Norton, at this place, introduces a table "for the reducing of minutes, seconds, etc. of the 60th progression into primes, seconds, etc. of the tenth progression", with an explanation.

Des Marcx hoochſte teecken ſal ſijn ④, want 1 ④ ſal ontrent een half Antwerps Aes weghen. Voor het hoochſte teecken vant Pondtgroote, ſchijnt de ③ te mueghen beſtaen, aenghesien ſoodanighen 1 ③ min doet, dan het vierendeel van 1 ß.

De onderdeelen des ghewichts om alle dinghen duer te connen weghen, ſullen ſijn (inde plaets van Halfpondt, Vierendeel, halfvierendeel, Once, Loot, Enghelſche, Grein, Aes, etc.) van elck teecken 5, 3, 2, 1; Dat is; Naer het Pondt ofte 1 ⓪, ſal volghen een ghewichte van 5 ① (doende $\frac{1}{2}$ ℔.) daer naer van 3 ①, dan van 2 ①, dan van 1 ①: Ende dergelijcke onder deelen ſal oock hebben de ① ende d'ander volghende.

Wy achtent oock nut dat elck onderdeel van wat Stoffe ſijn Grondt ſy, ghenoemt worde met name *Eerſte, TWeede, Derde*, etc. Ende dat overmidts ons kennelick is *TWeede* Vermenichvuldicht met *Derde*, te gheven Vytbreng *Vijfde*, (want 2 ende 3 maecken 5, als vooren gheſeyt is) t'welck door andere namen ſoo merckelick niet en ſoude connen gheſchieden. Maer alſmen die met onderſcheydt der Stoffen noemen wil (ghelijckmen ſegt Halfelle Halfpondt Halfpinte, etc.) ſoo mueghen wy die heeten *Marcxeerſte, MarcxtWeede, PondtſtWeede, EllenſtWeede*, etc.

Nu om van deſen exempel te gheven, Ick neme dat 1 Marck Goudt weerdich ſy 36 ℔ 5 ① 3 ②, de Vraghe is wat 8 Marck 3 ① 5 ② 4 ③ bedraghen ſullen. Men ſal 3653 vermenich-

 vuldigen

of money; the highest sign of the mark shall be ④, for 1 ④ shall weigh about the half of one Es of Antwerp, the ③ shall serve for the highest sign of the livre de gros, seeing that 1 ③ makes less than the quarter of one gr.

The subdivisions of weight to weigh all things shall be (in place of the half pound, quarter, half quarter, ounce, half ounce, esterlin, grain, Es, etc. of each sign 5, 3, 2, 1, that is to say that after the pound or 1 ⓪ shall follow the half pound or 5 ①, then the 3 ①, then the 2 ①, then the 1 ①, and the like subdivisions have also the 1 ① and the other following.

We think it necessary that each subdivision, what matter soever the subject be of, be called *prime, second, third,* etc., and that because it is notable unto us that the *second,* being multiplied by the *third,* gives in the product the *fifth* (because two and three make five, as is said before), also the third divided by the second gives the quotient prime, etc. that which so properly cannot be done by any other names; but when it shall be named for distinction of the matters (as to say, half an ell, half a pound, half a pint, etc.), we may call them *prime of mark, second of mark, second of pound, second of ell,* etc.

But to the end we may give example, suppose 1 mark of gold value 36 lbs 5 ① 3 ②, the question what values 8 marks 3 ① 5 ② 4 ③: multiply 3653 by vuldighen met 8 3 5 4, gheeft Vytbreng door het derde Voorſtel, welck oock is het begheerde Beſluyt, 3 0 5 ℔ 1 ① 7 ② 1 ③. wat de 6 ④ 2 ⑤ belangt, die en ſijn hier van gheender acht.

Andermael 2 Ellen 3 ①, coſten 3 ℔ 2 ① 5 ②, wat ſullen coſten 7 Ellen 5 ① 3 ②? Men ſal naer de ghebruyck de laetſte ghegheven Pale Vermenichvuldighen met de tweede, ende den uytbreng deelen door d'eerſte; Dat is 7 5 3 met 3 2 5, doet 2 4 4 7 2 5, die Ghedeelt door 2 3, gheeft Soomenichmael ende Beſluyt, 1 0 ℔ 6 ① 4 ②.

Wy ſouden mueghen ander exempelen gheven in alle de ghemeene Reghelen der Telconſten in ſ'Menſchen handelinghen dickmael te vooren commende, als de Reghel des Gheſelſchaps, des Verloops, van Wiſſelinge, etc. bethoonende hoe ſy alle door heele ghetalen afgheveerdicht connen worden; oock mede deſer lichte gebruyck door de Legpenninghen: Maer angheſien ſulcx uyt het voorgaende openbaer is ſullent daer by laten.

Wy ſouden oock door verghelijckinghe vande moeyelicke exempelen der ghebroken ghetalen, opentlicker hebben connen bethoonen het groote verſchil der lichticheydt van deſe buyten die, maer wy hebben ſulcx om de cortheydt overgheſleghen.

TEN laetſten moeten wy noch ſegghen van eenich onderſcheydt deſes ſeſten Lidts, met de voorgaende vijf leden, welck is, dat yghelick per-

8354, giving the product by the fourth proposition (which is also the solution required) 305 lbs 1 ① 7 ② 1 ③; as for the 6 ④ and 2 ⑤, they are here of no estimation.

Suppose again that 2 ells and 3 ① cost 3 lbs 2 ① 5 ②, the question is what shall 7 ells 5 ① 3 ② cost. Multiply according to the custom the last term given by the second, and divide the product by the first, that is to say: 753 by 325 makes 244725, which, divided by 23, gives the quotient and solution 10 lbs 6 ① 4 ②.

We should like to give other examples in all the common rules of Arithmetic occurring often in man's actions, such as the rule of society, of interest, of exchange, etc., showing how they can be all expedited by integer numbers, as well as by easy use of counters; but we shall leave it at that because it is clear from the preceding [15]).

We could also more amply demonstrate by the difficult examples of broken numbers the comparison and great difference of the facility of this more than that, but we will pass them over for brevity's sake.

Lastly it may be said that there is some difference between this last sixth article and the 5 precedent articles, which is that each one may exercise for

[15]) This paragraph is omitted by Norton.

persoon voor sijn selven de thiende deelingen van die voorgaende Leden, ghebruycken can sonder ghemeene oirdening door de Overheydt daer af ghestelt te moeten worden; maer sulcx niet soo bequamelick in dit laetste wandt d'exempelen van dien sijn ghemeene rekeninghen die allen oogenblick (om soo te segghen) te vooren commen, inde welcke het vougheliick soude sijn, dat het besluyt alsoo bevonden, by alle man voor goedt ghehouden ware: Daerom ghemerct de wonderlicke groote nutbaerheydt van dien, het ware te wenschen dat eenighe, als de ghene dier t'meeste gherief door verwachten, sulcx beneerstichden om ter Daet ghebrocht te worden; Te weten dat beneven de ghemeene deelinghen dieder nu der Maten, Ghewichten, ende des Ghelts sijn (blijvende elcke Hooftmate, Hooftghewicht, Hooftghelt, tot allen plaetsen onverandert) noch Wettelick door de Overheydt veroirdent wierde, de voornoemde thiende deelinge, op dat ygelick wie wilde, die mochte ghebruycken.

Het ware oock ter saecken voorderlick, dat de weerden des Ghelts voornamelick des geens nieu ghemunt wort, op seeckere *Eersten TWeeden*, ende *Derden* gheweerdicht wierden.

Maer of dit al schoone niet soo haest int werck ghestelt en wierde, ghelijct wel te wenschen waer, daer in sal ons ten eersten vernoughen, dat het ten minsten onsen Naercommers voorderlick sijn sal, want het is seecker, dat by aldien de Menschen in toecommenden tijt, van sulcker aert sijn als sy in themselves the tenth partition of the said precedent 5 articles, though it be not given by the magistrate of the place as a general order, but it is not so in this latter: for the examples hereof are vulgar computations, which do almost continually happen to every man, to whom it were necessary that the solution so found were of each accepted for good and lawful. Therefore, considering the so great use, it would be a commendable thing, if some of those who expect the greatest commodity would solicit to put the same in execution to effect, namely that joining the vulgar partitions that are now in weight, measures, and moneys (continuing still each capital measure, weight, and coin in all places unaltered) that the same tenth progression might be lawfully ordained by the superiors for everyone that would use the same; it might also do well, if the values of moneys, principally the new coins, might be valued and reckoned upon certain *primes, seconds, thirds,* etc. But if all this be not put in practice so soon as we could wish, yet it will first content us that it will be beneficial to our sucden voorleden gheweest hebben, dat sy soodanighen voordeel niet altijt verswijmen en sullen.

Ten anderen, soo en ist voor yghelick int besonder de vorworpenste wetenschap niet, dat hem kennelick is hoe het Menschelicke Geslachte sonder cost ofte aerbeydt, sijn selven verlossen can van soo vele groote moeyten, als sy maer en willen.

Ten laetsten; hoe wel misschien de Daet deses sesten Lidts voor eenighen Tijt lanck niet blijcken en sal, Doch soo can een yghelick de voorste vijve ghenieten, soot kennelick is dat sommighe der selver nu al deghelick int werck ghestelt sijn.

EYNDE DES AENHANGSELS.

cessors, if future men shall hereafter be of such nature as our predecessors, who were never negligent of so great advantage. Secondly, that it is not unnecessary for each in particular, for so much as concerns him, for that they may all deliver themselves when they will from so much and so great labour. And lastly, although the effects of the first article appear not immediately, yet it may be; and in the meantime may each one exercise himself in the five precedent, such as shall be most convenient for them; as some of them have already practised.

THE END OF THE APPENDIX