VARIATIONS ON THE MACDONALD-PFITZER THEME: MATHEMATICAL MODELS AND A PALEOLIMNOLOGICAL RECORD

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In D. Jewson's (1992) variation on the classic MacDonald-Pfitzer model, the increment of size reduction for diatom offspring decreases as a linear function of parental cell size. The consequences of this for the distribution of cell sizes in diatom populations were investigated using analytical and numerical models and the results applied to a detailed paleolimnological record. In the analytical model, a recurrence equation was developed describing the relationship between parental cell size (l_n) and size of the smaller offspring cell (l_{n+1}) using parameters of minimum cell length, maximum cell length and initial increment of decrease. A closed form solution to the recurrence equation was found, which was an affine function and invertible. The inverse function describes the relationship between cell size-class number (n) and measurable cell size (l). These functions were used to model patterns of cell size distributions in single strain and multi-strain populations, with the assumption that the underlying size-class distribution in single-strain populations is inherently binomial, and in multi-strain populations can be treated as Gaussian. These models were then applied to a 70-year annual-resolution paleolimnological record of a centric diatom population from the varved sediments of Foy Lake, Montana, USA. Three hundred valves from each year were measured, a total of over 21,000 valves, ranging in diameter from 14 to 44 micrometers, with modal diameter of 21 and mean diameter of 22.15 micrometers. Five hundred initial valves from 33 to 45 micrometers in diameter were also observed throughout the record. Single and multiple normal (Gaussian) and log-normal (Galton) distributions were fitted numerically to the data, both directly and following transformation by the model. For most years, the data were best described by single Gaussian distributions following transformation. This result agrees with the life-cycle length and frequency of significant sexual reproduction indicated by the record of initial valves, and supports the predictions of Jewson's variation on the classic MacDonald-Pfitzer theory.