Sand transport over a barchan dune

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Abstract

The present work investigates an important and yet unsolved issue: the relationship between the sand flux and the fluid shear stress over a spatially varying bed of particles. It is now recognized that over such a bed, the particle flux is not in equilibrium with the shear stress: there is some lag related to the particle inertia or particle settling. A confident modelling of this relaxation phenomena and the corresponding length scales, is still lacking (Charru, Andreotti and Claudin 2013). This question is investigated here from experiments on barchan dunes in a closed-conduit water flow. From visualizations with a high-speed camera and a tracking algorithm, the particle motion over the whole dune surface is determined: particle trajectories, local velocity and surface density of the moving particles, and local particle flux. The relationship between the local particle flux and local shear stress (estimated from previous analyses) is investigated. Surprisingly, the particle flux appears to be out-of-equilibrium over the whole dune surface, with saturation length much larger than expected.

1. INTRODUCTION

When a fluid flows over an erodible bed (e. g. in a channel, river, or marine current), particles forming the bed are eroded and transported downstream. As a consequence, an intially flat bed generally does not remain flat: bedforms such as ripples and dunes are observed to grow and propagate.

Although these phenomena are known for a long time, and have been widely studied both in the natural environment and laboratory flumes, their understanding is far from complete. From the experimental point of view, wavelength measurements exhibit large scatter, over one order of magnitude (Yalin 1985). From the theoretical point of view, the modelling of the bed shear stress τ_b exerted by the fluid flow on the bed is dependent on uncertain knowledge about the turbulence over an undulated wall (Abrams and Hanratty 1985). Moreover, the sediment transport over a variable bed is not well-known, and the use of the classical laws $q_{sat}(\tau_b)$ for the particle flux, which assume q_{sat} to be in equilibrium with τ_b, is now recognized to lead to wrong predictions, i.e.

ripple lengths much to small (Charru, Andreotti and Claudin 2013).

Crucial improvements for the sand transport law have been proposed during the last decade, based on the idea that as the shear stress varies, the particle flux does not adapt immediately, but after a certain relaxation time of length. The simplest transport law accounting for this relaxation is the first-order equation (Sauermann, Kroy and Herrmann 2001; Andreotti, Claudin and Douady 2002):

$$L_{sat}\,\partial_x q = q_{sat}(\tau_b) - q. \qquad (1)$$

This law states that when the actual flux q differs from the saturated -- or equilibrium -- flux $q_{sat}(\tau_b)$, it relaxes exponentially towards $q_{sat}(\tau_b)$ over the relaxation length L_{sat} (figure 1). This equation holds for quasistatic situations where time variations of the bed are slow, i.e. when the bedform velocity is much smaller that the particle velocity, which is generally the case. For situations where such a condition would not be met, the term $T_{sat}\partial_t q$ should be added to the l.h.s. of Eq. (1), where T_{sat} is a relaxation time.

Eq. (1) has been first introduced from phenomenological considerations. A different introduction of this equation was proposed by Charru and Hinch (2006) and Charru (2006), based on the modelling of the erosion and deposition rates (Figure 2). The expression of the saturation length is discussed below.
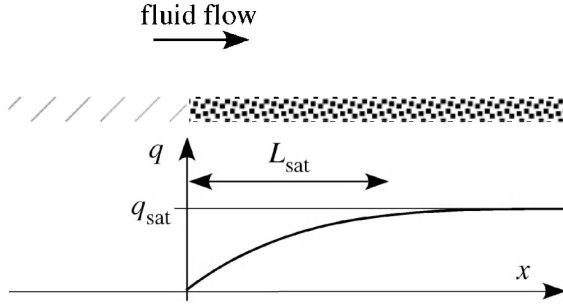

Figure 1. Fluid flow coming over an erodible bed (for $x > 0$) and corresponding particle flux increasing from zero to q_{sat} over the length L_{sat}..
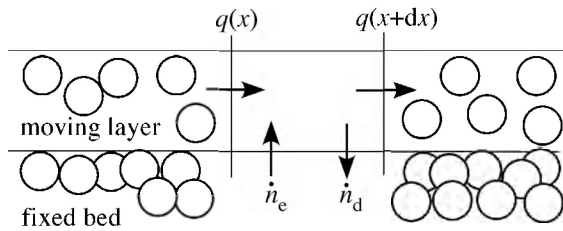

Figure 2. Sketch of the mass conservation balance between the flux divergence $\partial_x q$, and the erosion and deposition rates n_e and n_d, from which Eq. (1) may be derived.

When Eq. (1) is used in bed stability calculations (instead of the usual equilibrium law $q = q_{sat}(\tau_b)$ which corresponds to $L_{sat} = 0$), ripple length predictions are much improved (Charru 2006). Indeed, bed instability is rooted in the fact that the phase of the bed shear stress leads that of the bed deformation, due to fluid inertia. Within the equilbrium assumption, the phase of the particle flux is the same as that of the bed shear stress, so that particles are dragged from troughs to crests (figure 3a). Accounting from the stabilizing (diffusive) effect of gravity on inclined slopes, a cut-off wavenumber arises, which defines the most amplified wavelength which can be compared to

ripple length measurements. Within the new non-equilibrium model (figure 3b), the particle flux lags, which corresponds to a stabilizing effect : a relaxation length L_{sat} larger than the fluid inertia length L_τ carries the maximum particle flux downstream of the crest of the disturbance, so that the bed deformation vanishes and the flat bed is stable.
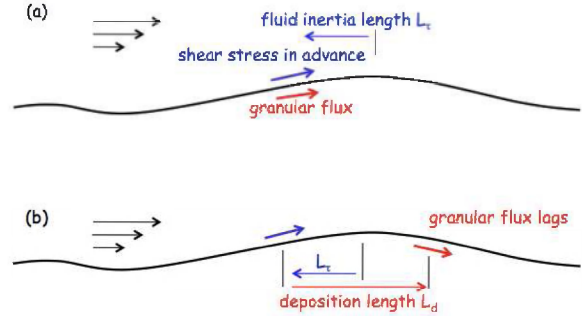

Figure 3. (a) Within the equilibrium assumption $q = q_{sat}(\tau_b)$, the shear stress and particle flux disturbances are in phase and the flat bed is unstable. (b) Accounting for relaxation effects stabilizes the bed (apart from gravity effects).

Predictions of the most amplified wavelength imply that some expression is given for L_{sat}. Two heuristic models have been proposed. When particle inertia dominates, L_{sat} scales with the acceleration length (ρ_p/ρ) d, where ρ_p and ρ are the particle and fluid density and d is the particle diameter (Andreotti, Claudin and Pouliquen 2010). When settling effects dominate, L_{sat} scales with the deposition length (u_*/V_{fall}) d, where u_* is the friction velocity (which is a scale for the particle velocity) and V_{fall} is the settling velocity of the particles (Charru 2006). The first situation is more likely relevant for aeolian transport (where the ratio ρ_p/ρ is large), whereas the second is expected to hold for aqueous transport where ρ_p/ρ is about two. Thorough measurements in water and oil, of the erosion and deposition processes at the particle scale, have allowed some determination of L_{sat}, for both laminar flow (Charru, Mouilleron and Eiff 2004) and turbulent flow (Lajeunesse, Malverti and Charru 2010). However, direct measurements of L_{sat} in non equilibrium conditions have never been performed. The aim of this paper is to present such measurements.

2. BARCHAN DUNES

A nice situation of non-equilibrium particle flux is provided by barchan dunes. Such dunes form when sand is transported by a fluid flow (gas or liquid) over a non-erodible bed. These dunes exhibit a remarkable crescentic shape with horns pointing downstream (figure 4). They are commonly observed in deserts, with height of a few meters and velocity of a few tens of meters per year. In his famous book, Bagnold (1941) first pointed out their significance for understanding the physics of blown sand. Their outstanding shape and stability properties have triggered a number of studies, aiming at understanding the conditions for their formation, their migration velocity and time evolution, and also the striking existence of a minimum length, of about ten meters in air, below which no dune is observed. A model accounting for their main properties was derived by Kroy, Sauermann and Hermann (2002) and Andreotti, Claudin and Douady (2002), from calculations of the shear stress at the dune surface by Hunt, Leibovich and Richards (1988) and Eq. (1) for the sand flux accounting for the retarding effect of grain inertia (Sauermann, Kroy and Hermann 2001).
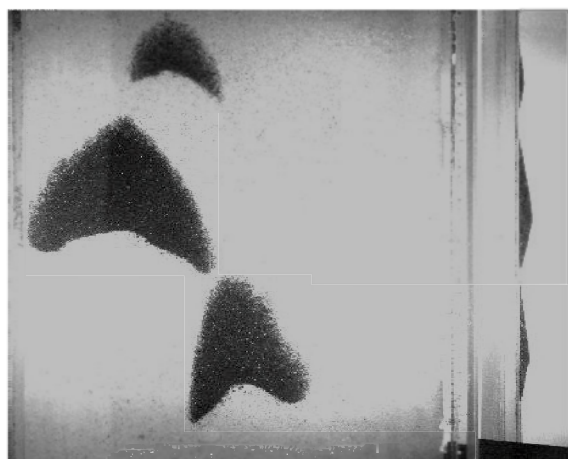

Figure 4. Three barchan dunes under a water flow, seen from above. Top of the photo: side view from an inclined mirror. The width of the largest dune is 4 cm (photograph from V. Laval, IMFT).

Similar barchan dunes have also been observed under water flows when, again, the sediment supply is limited. The size of these aquaeous dunes is however much shorter than in air, of a few centimeters, say. Such dunes formed in an open channel flow are reported by Mantz (1978), with typical width of about three centimeters; in this paper the resemblance with the aeolian dunes observed by Bagnold was noted. This resemblance was investigated further by Hersen (2005) from experiments in which the dunes were formed on a tray oscillating asymmetrically in a water tank. Barchan dunes also form in circular pipes where they may cause damages to industrial installations (Al-lababidi *et al.* 2008). An extensive study of barchan dunes in water has been performed by Franklin and Charru (2011), discussing their geometry, velocity, and long time evolution. The existence of a smallest dune size of about one centimeter was evidenced, supporting the idea that the relaxation length scsale on the deposition length $(u_*/V_{fall})\,d$ rather than the acceleration length $(\rho_p/\rho)\,d$. Again, no direct measurement of L_{sat} was provided.

3. EXPERIMENTAL RESULTS

1.1. Experimental arrangement

The experimental arrangement mainly consists of a horizontal plexiglass channel, six meters long, with rectangular cross-section of height $2\delta = 60$ mm and width $b = 120$. Water flows from a head-tank with free surface 2.5 meters above the channel, and enters the channel through a divergent-convergent device with a honeycomb section in order to suppress large eddies and homogeneize the small-scale turbulence. At the open end of the channel, particles are separated by sedimentation in a large tank, and a pump drives the water up to the head-tank. The volumetric flow rate is measured with an electromagnetic flow-meter. For more details see Franklin and Charru (2011).

The particles (glass beads, with diameter $d = 1.1$, 0.2 or 0.5 mm) were deposited in the channel, previously filled with water, with the help of a syringe through a small hole in the upper wall located at 4.15 m form the entrance. The sand settles in the water at rest and forms a conical heap. Then the flow is started up, and the heap deforms. A slip face develops on the lee-side where particles avalanche, and horns grow on both sides. The equilibrium shape of the dune is quickly

reached, as it has travelled over a distance of the order of its own size. Varying the shape of the initial heap does not change the final dune shape. When the heap is large, it may split and form several interacting barchan dunes, as shown in Figure 4.

The evolution of the shape of the dune was recorded with a video camera placed above the channel and mounted on a travelling system. The resolution of the camera was 2048x2048 pixels, with field of view at the bottom wall of about 140x140 mm^2. The side view of the dune was recorded on the same images thanks to a mirror inclined at 45°.
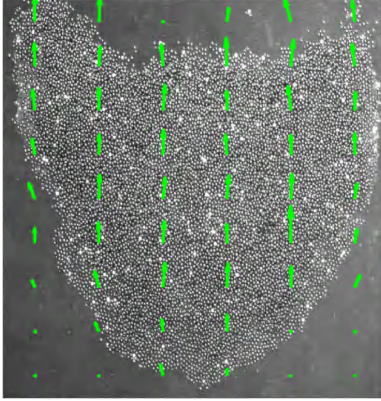
1.2. Particle motion

Figure 5. A barchan dune seen from above, and the particle velocity field averaged over a rectangular mesh (green arrows).

The motion of a few dyed particles (5%) was recorded, allowing the determination of the surface density of the moving particles (number of moving particles per unit horizontal area) and their velocity. Figure 5 displays a typical particle velocity field over a moving barchan dune, where the measured velocity of individual particles has been averaged over a rectangular area with size of 1/10 and 1/6 of the length and width of the dune, respectively. The velocity increases from the upstream foot of the dune towards the crest, and is larger in the central part of the dune than at the horns, as expected.

From the velocity $u(x, y)$ and moving particle density $n(x, y)$, the field of the particle flux, $q(x, y)$ = $n(x, y)\ u(x, y)$, can be obtained. An example of the resulting profiles is shown in figure 6, along three slices : (i) the central part of the dune, (ii) the side parts, and (iii) the horns. It can be seen that, firstly, the flux increases from the dune foot towards the crest (located at $x = 40$ mm), as expected. Secondly, the flux is the largest in the central part, and minimum over the horns. Thirdly, the flux increases everywhere, with no indication of saturation in the vicinity of the crest.
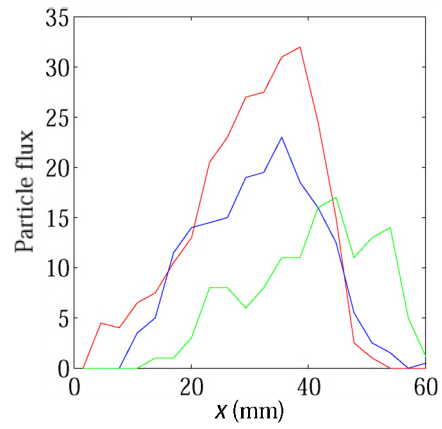

Figure 6. Sand flux along the dune (cm^{-1}), with x=0 at the dune foot. Red: central part ; blue : side part ; green: horns.

4. RELAXATION LENGTH

The above measurements may be put together by (i) shifting the x-origin of each slice, initially at $x_d(y)$, up to the foot of the slice, and (ii) normalizing the flux in each slice by a saturated flux $q_{sat}(y)$ obtained by fitting the data points with Eq. (1). The result is shown in figure 7. It can be seen that the data points nicely fall close to the exponential relaxation curve corresponding to Eq. (1) (except very near the foot), with

$$L_{sat} = 300\ (u_*/V_{fall})\ d$$

where u_* is the friction velocity on the bottom wall of the channel. This length appears to be one of magnitude larger than those obtained from measurements of the equilibrium erosion and

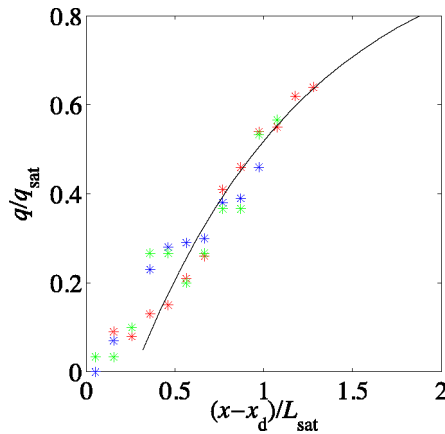deposition rates. Further investigation is needed here to clarify this point.

Figure 7. Normalized sand flux along the dune (see text for the normalizations). Red: central part ; blue : side part ; green: horns. Solid line: Eq. (1) with $L_d = 300$ (u_*/V_{fall}) d.

5. CONCLUSIONS

Understanding relaxation effects of the particle flux over a variable erodible bottom is a key point for confident prediction of the sediment transport over a variable bottom. In particular, such an understanding is necessary for predictions of the ripple length under water. We have shown here that field measurements of the particle flux $q(x, y)$ over barchan dunes allows the determination of the relaxation length. Our results show that the particle flux increases from the dune foot up to the crest, where it is not yet saturated. Fitting these measurements with the non-equilibrium model corresponding to Eq. (1) provides a relaxation length much larger than expected from previous analyses. Further analyses are in progress, which include the variations of the shear stress over the dune.

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