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On the Predictability of the ^{137}Cs Distribution in the Severn Estuary

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The observed distribution of ^{137}Cs resulting from known sources in the Severn Estuary permits the testing of simple predictive models for the one-dimensional distribution of non-conservative substances in an estuary. These models either use a known distribution of salinity to infer corresponding ones for other substances, or else directly solve mass balance equations utilizing previously determined dispersion coefficients. Both methods are shown to provide results comparing favorably with observations.

Stokes drift plays an important role in the circulation of the Severn. A modified densimetric Froude number, including this drift, is therefore used to estimate the circulation and stratification of the estuary.

A new equation is used for calculating the ^{137}Cs distribution from the observed salinity distribution. It generalizes previous formulations to permit variation of the net runoff with position along the estuary.

Introduction

The Severn Estuary, because of its weak stratification, frequently has been used as a prototype, for which a one-dimensional salt balance equation is applicable. Stommel (1953) and Bowden (1963) have demonstrated the methodology for calculating, from observed distributions of salinity and river runoff, dispersion coefficients based upon the balance of the outward salt flux due to the net runoff with upstream tidal dispersion. Interestingly, their resulting values of the dispersion coefficients did not agree with theoretical estimates obtained from either the mixing length theories of Ketchum (1951) and Arons & Stommel (1951) or the shear effect due to the tidal currents (Bowden, 1963). More recently Uncles & Radford (1980) have determined the dependence of the dispersion coefficients for each location in the Severn Estuary upon the river flow and the tidal ranges. In a more detailed study of the salt flux terms at two stations [(a) and (b) in Figure 1], Uncles & Jordan (1979) found the salt transport due to freshwater discharge to be balanced primarily by the contribution from tidal pumping and possibly transverse shear, with the gravitational circulation playing only a very minor role in the total balance. The classification scheme presented by Hansen & Rattray (1966) provides a basis for interpreting the above observational data and for explaining the essential one-dimensional nature of the salt (and other constituent) balance in this estuary. They compared their earlier results (Hansen & Rattray, 1965), giving the

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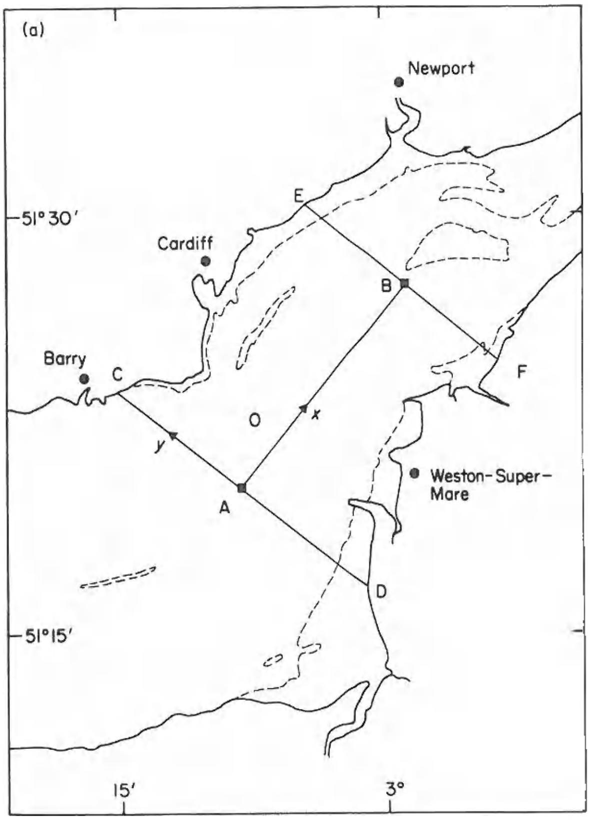

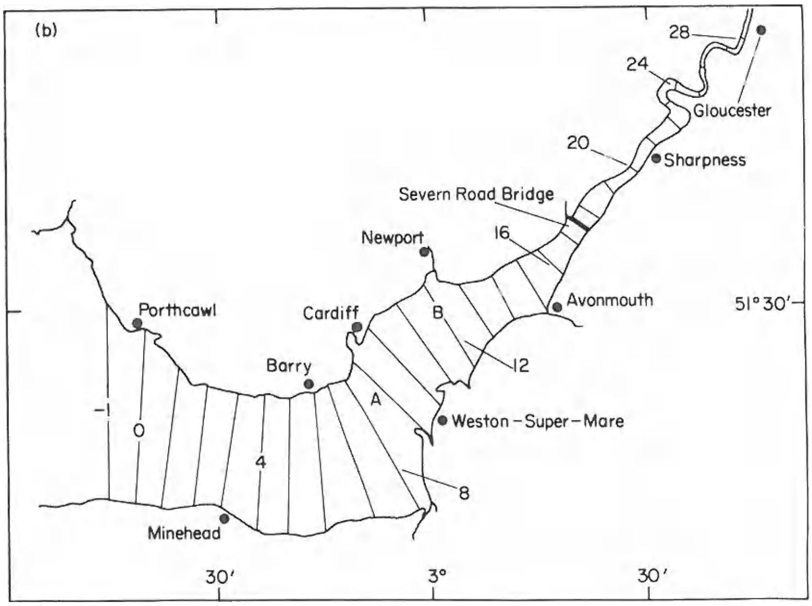

Figure 1.(a) Sketch chart of the Severn Estuary. Positions of stations A and B, together with the locations of the horizontal axes (x,y) and the transverse cross-sections CD and EF;– – – denotes the low water line for mean spring tides. Figure from Uncles & Jordon (1979). (b) Sketch chart of the whole Severn Estuary, showing the discrete representation $(k, \Delta x')$, with $k = -1(1)28$ and $\Delta x' = 5$ km. Figure from Uncles & Jordon (1979).

dependence of the vertical stratification and gravitational circulation on theoretical dimensionless parameters, to data obtained from actual estuaries under various conditions of river runoff and tidal currents. For coastal plain estuaries they were able to show a direct correspondence between these theoretical dimensionless parameters and dimensionless velocity rates obtained from the external conditions describing the estuary and its inputs. They showed that when wind effects are unimportant, two independent dimensionless parameters are both necessary and sufficient to specify the circulation and stratification in a coastal plain estuary. Thus the circulation and stratification in an estuary are not directly related but instead they depend in different ways upon the estuary geomorphology and its inputs.

By use of data from Uncles & Jordan (1979) and the Hansen & Rattray (1966) stratification–circulation diagram, the conditions in the Severn Estuary can be understood in comparison to other estuaries by the magnitudes of the parameters P, the ratio of discharge to tidal velocities, and F_m, a densimetric Froude number, determined from: U_s = the cross-sectional mean Stokes drift velocity; U_f = the river discharge per unit cross-sectional area; U_t = the rms tidal current speed; $U_d = \sqrt{gH\,\Delta\rho/\rho}$, the densimetric velocity, with H = depth; $\Delta\rho$ = the density difference between river water and seawater.

Because the Stokes drift velocity is important in the Severn as opposed to many other estuaries, it permits formation of a third dimensionless parameter for this estuary. However, there are no data nor theory to show the dependence of the circulation and stratification upon a third parameter, so instead of forming a third parameter we incorporate U_s into the parameters P and F_m based on our understanding of their roles in the momentum and salt balances.

Hansen & Rattray (1966) show empirically that the theoretical parameters of: ν, the tidally induced fraction of the upstream salt flux; Ra, an estuarine Rayleigh number; and M, a tidal mixing parameter, are related to P and F_m by the equations

$$3\,\frac{(\nu Ra)}{48} = F_m{}^{-3/4},$$
$$20\,(M/\nu) = P^{-7/5}.$$

Their variables are defined as follows: $\langle u\rangle$ = the longitudinal time–mean velocity; $\langle u\rangle_{\text{surface}}$ = the longitudinal time–mean velocity at the surface; g = the acceleration due to gravity; W,H = the width and depth of the estuary; A_v = the vertical turbulent viscosity; K_h, K_v = the horizontal and vertical turbulent diffusivities; K_{h_0} = a reference value of K_h; $\langle s\rangle$ = the time–mean salinity; $\langle\bar{s}\rangle$ = the sectional mean of $\langle s\rangle$; ρ, ρ_f = the densities of estuarine water and freshwater; $k = (1/\rho_f)(\partial\rho/\partial S)$; Ra = the estuarine Rayleigh number $= gk\langle\bar{s}\rangle H^3/A_vK_{h_0}$; M = the tidal-mixing parameter $= K_vK_{h_0}W^2/R^2$; ν = the tidally induced fraction of the total upstream salt flux.

They also give the theoretical results (for no wind stress):

$$1680\,(M/\nu)(1-\nu) = 32 + 76\left(\frac{\nu Ra}{48}\right) + \frac{152}{3}\left(\frac{\nu Ra}{48}\right)^2,$$

$$\frac{\langle \delta s\rangle}{\langle\bar{s}\rangle} = \frac{\langle s\rangle_{\text{bottom}} - \langle s\rangle_{\text{surface}}}{\langle\bar{s}\rangle} = \frac{\nu}{M}\left(\frac{1}{8} + \frac{3}{20}\,\frac{\nu Ra}{48}\right),$$

$$\frac{\langle u\rangle_{\text{surface}}}{\langle\bar{u}\rangle} = \frac{3}{2} + \frac{\nu Ra}{48},$$

where the overbar indicates a cross-sectional average and the angle brackets indicate a time average. *Ra and M enter directly only the equations for the momentum and salt balance*, respectively. Thus, we associate F_m with the momentum balance and the circulation, while P is associated with the salt balance and the stratification, although F_m enters these latter through the advection term. The Stokes drift requires an additional net outward Eulerian flow in addition to that derived from the river inflow. Therefore, the total mean velocity U_s+U_f enters into the current dynamics and thus we replace U_f by (U_s+U_f) in the Hansen and Rattray definition of F_m. On the other hand, the Stokes drift, since it provides no net horizontal displacement of the water particles in the cross-section, has no direct effect on the salt balance nor on (ν/M) which is a measure of the relative strength of vertical mixing. Therefore we hypothesize to a first approximation that P will not depend upon U_s. From these arguments we redefine F_m and P, to include the effects of Stokes drift, as follows:

$$F_m = (U_f+U_s)/U_d,$$
$$P = U_f/U_t.$$

These bulk parameters are used to estimate $\langle\delta S\rangle/\langle\bar{S}\rangle$, $\langle u\rangle_{\text{surface}}/\langle\bar{u}\rangle$ and the tidally induced fraction of the upstream salt flux ν. The calculated values of the parameters and the observed salinity and velocity ratios are shown in Table 1. Because only one station is taken in each cross-section, and channel curvature combined with strong tidal currents creates significant cross-sectional inhomogeneity in the flow patterns, the width-averaged vertical circulation is poorly determined from the observed current meter data. There is marked variation in the vertical salinity distributions between averaging periods which with the extrapolation required to infer the bottom and surface values leads to considerable uncertainty in the observed top to bottom salinity differences. Nevertheless, the redefined

TABLE 1. Theoretical values at Stations A and B in the Severn Estuary for stratification, circulation and tidal diffusive fraction of the upstream salt flux with the observed vertical stratification of salinity

	Station A	Station B
U_f (cm s^{-1})	0·05[a]	0·07[a]
U_t (cm s^{-1})	68[a]	85[a]
U_d (cm s^{-1})	180[a]	150[a]
U_s (cm s^{-1})	1·6[a]	4·3[a]
$\frac{<\delta S>}{<\bar{S}>}$, calculated	0·001[a]	0·0008[a]
$\frac{<u>\ \text{surface}}{<\bar{u}>}$, calculated	13[a]	6[a]
ν, calculated	1·00[a]	1·00[a]
$\frac{<\delta S>}{<\bar{S}>}$, observed	0·005	0·003
$\frac{<u>\ \text{surface}}{<\bar{u}>}$, observed	9[b]	2[b]

[a]Cross-section averaged values.
[b]Computed by replacing the observed depth mean currents at Stations A and B by $(U_s + U_f)$—the cross-sectionally averaged Eulerian currents.

forms of F_m and P used here give results consistent in magnitude with the observed features of the circulation and stratification in the Severn Estuary. This agreement is sufficiently good to provide an explanation for the estuary's unique nature as well as the relative importance of the mechanisms governing its horizontal salt balance. To make a significantly improved comparison would require a much more comprehensive data base as well as a possible improvement in the empirical relationships between the theoretical and observational parameters for the range of values found in this study.

Compared to other well studied estuaries, the Severn is notable for its very small vertical stratification, even though it has a significant vertical circulation. This results from the combination of small P values (7–8×10^{-4}) and of moderate F_m values (1–3×10^{-2}) compared to other estuaries. Fjords on the west coast of North America which have comparable values for P, have smaller F_m values; some coastal plain estuaries have similar values for F_m but then have large P values. The distinctive features of the Severn Estuary are thus due to a combination of moderate depths and strong tidal currents relative to its river-inflow-induced net outward velocity. This creates a significant gravitational circulation but as a result of the small vertical salinity stratification, the upstream salt flux is almost totally accomplished by tidal diffusion or dispersion as confirmed by the results of Uncles & Jordan (1979).

Because of the simple one-dimensional nature of the salt balance, one expects a similar one-dimentional balance for any substance that does not stratify in the vertical. This hypothesis was confirmed for the ^{137}Cs distribution by Uncles (1979). In this paper two methods are used to calculate the ^{137}Cs distribution and their results are compared. First, following the methods of earlier workers, the equation for the ^{137}Cs balance is integrated directly using dispersion coefficients obtained from the observed salinity distribution by Uncles & Radford (1980) and applied for the particular river runoff with average tides. Second, the ^{137}Cs distribution is determined by direct integration of a Green's function derived from the observed salinity distribution and river runoff. The Green's function used is a new result which generalizes the formulation of Rattray & Officer (1979) to include a non-uniform river flux. Because of the successful comparison of these results with the observed values, the analyses are extended further upstream to predict the ^{137}Cs distribution where there are no direct observations.

The governing equations

The one-dimensional equations governing the distributions of salinity, s, and a non-conservative constituent, c, are:

$$E \frac{\mathrm{d}\langle \bar{s} \rangle}{\mathrm{d}x} - R\langle \bar{s} \rangle = 0, \tag{1}$$

for no net salt flux (zero salinity for inflowing river water) with the boundary condition at the ocean:

$$x = l, \ \langle \bar{s} \rangle = s_l, \text{ a constant,}$$

and

$$\frac{\mathrm{d}}{\mathrm{d}x}\left(E \frac{\mathrm{d}\langle \bar{c} \rangle}{\mathrm{d}x}\right) - \frac{\mathrm{d}}{\mathrm{d}x}\left(\langle R\bar{c} \rangle\right) = \langle \bar{B} \rangle \langle A \rangle \equiv -\frac{\mathrm{d}Q}{\mathrm{d}x}, \tag{2}$$

with the boundary conditions: at $x = 0$, $\langle \bar{c} \rangle = c_0$, a constant; at $x = l$, $\langle \bar{c} \rangle = c_l$, a constant.

$E = D\langle A \rangle$,

where D = the local dispersion coefficient; A = the local cross-sectional area of the estuary; R = the upstream freshwater inflow rate; $\bar{s}$ = the local cross-sectionally averaged salinity; $\bar{c}$ = the local cross-sectionally averaged concentration of the non-conservative quantity; $\bar{B}$ = the local rate of utilization of $\bar{c}$; s_0 = the value of $\langle \bar{s} \rangle$ at $x = 0$; Q = the total rate of increase of $\bar{c}$ upstream of x.

The angle brackets, $\langle\ \rangle$, denote a time-average over tidal cycles so as to produce a negligible time dependence in the above balance equations.

The solution to equation (2) for the distribution of $\langle \bar{c} \rangle$, given in terms of a known salinity distribution satisfying equation (1) with constant R, is (Rattray & Officer, 1979):

$$\langle \bar{c}(x) \rangle = \frac{c_0 f(x)}{[1 - s_0/s_l]} + c_l \left[\frac{\langle \bar{s}(x) \rangle - s_0}{s_l - s_0}\right] - f(x) \int_0^x \left[\frac{\langle \bar{s}(\xi) \rangle - s_0}{1 - s_0/s_l}\right] \frac{B(\xi)\mathrm{d}\xi}{\bar{u}(\xi)\langle \bar{s}(\xi) \rangle}$$
$$- \left[\frac{\langle \bar{s}(x) \rangle - s_0}{1 - s_0 s_l}\right] \int_x^l \frac{f(\xi)B(\xi)d\xi}{\bar{u}(\xi)\langle \bar{s}(\xi) \rangle}, \tag{3}$$

with $f(x) = \langle s(x) \rangle / s_l$. However, in general, and for the Severn Estuary in particular the water flux through each cross-sectional area is not constant and therefore a relationship more general than equation (3) must be derived for application to this estuary.

Determination of the one-dimensional distribution of a non-conservative substance from an observed salinity distribution

When $R(x)$ is a function of cross-section position, $f(x)$ is no longer a solution to the homogeneous part of equation (2), although of course $\langle s(x) \rangle$ still is. In terms of the observed $\langle s(x) \rangle$ distribution the two independent solutions of the homogeneous equation, most convenient for our purposes, are:

$$g_1(x) = \frac{\langle \overline{s(x)} \rangle}{s_l} \frac{P(x)}{P(l)}, \tag{4}$$

$$g_2(x) = \frac{\langle \overline{s(x)} \rangle}{s_0} [1 - P(x)/P(l)], \tag{5}$$

where $$P(x) = \int_0^x \frac{\mathrm{d}\langle \bar{s}(\xi) \rangle / \mathrm{d}\xi}{R(\xi)\langle \bar{s}(\xi) \rangle^2} \mathrm{d}\xi = \int_{s_0}^{\langle \overline{s(x)} \rangle} \frac{\mathrm{d}\langle \bar{s} \rangle}{R(x)\langle \overline{s(x)} \rangle^2}. \tag{6}$$

The generalizations of the salinity and freshwater terms occurring in equation (3) are $g_l(x)$ and $g_2(x)$, respectively. Neither of them, however, satisfy equation (1). The generalization to equation (3), obtained from equations (4), (5) and (6) is:

$$\langle \overline{c(x)} \rangle = c_0 g_2(x) + c_l g_1(x)$$
$$- s_0 s_l P(l) \left[g_2(x) \int_0^x \frac{\overline{B(\xi)A(\xi)}}{\langle \overline{s(\xi)} \rangle} g_1(\xi)\mathrm{d}\xi + g_1(x) \int_x^l \frac{\overline{B(\xi)A(\xi)}}{\langle \overline{s(\xi)} \rangle} g_2(\xi)\mathrm{d}\xi \right], \tag{7}$$

which is valid for any x-dependence of $R(x)$ and $E(x)$.

Calculation of the s and ^{137}Cs distributions from the dispersion equations

Figure 1 shows the discrete representation of the Severn Estuary, with $x_{k+1} - x_k = \Delta = 5$ km, used in carrying out the calculations. The finite-difference forms of equations (1) and (2) [as shown for equation (2) only] are:

$$C_{k+1} = \frac{[Q_{k+1/2} + C_k\,(D_{k+1/2}\,A_{k+1/2}/\Delta - R_{k+1/2}/2)]}{[D_{k+1/2}\,A_{k+1/2}/\Delta + R_{k+1/2}/2]}. \tag{8}$$

The calculations for s_k and c_k were carried out sequentially starting with the boundary values $s_{-1} = 33{\cdot}00$ and $c_{-1} = 0{\cdot}90$ pCi l^{-1}. The input data are given in the first five columns of Table 2 and the results in the first three columns of Table 3. There are two ^{137}Cs sources:

TABLE 2. Data used in calculating S_k and C_k

Input data for calculations using dispersion equations					Input salinity distribution	
k	R (m^3 s^{-1})	D (m^2 s^{-1})	A (10^3 m^2)	Q (Ci s^{-1})	k	s
−0·5	144	104	590·00	0·914E−06	−1	32·65
0·5	144	104	520.00	0·914E−06	0	32·61
1·5	144	104	470·00	0·914E−06	1	32·59
2·5	144	104	460·00	0·914E−06	2	31·87
3·5	144	104	450·00	0·914E−06	3	31·40
4·5	140	113	420·00	0·913E−06	4	31·01
5·5	140	114	390·00	0·913E−06	5	30·46
6·5	140	146	360·00	0·913E−06	6	29·58
7·5	140	189	310·00	0·300E−06	7	29·42
8·5	127	225	260·00	0·299E−06	8	29·10
9·5	125	246	220·00	0·299E−06	9	28·70
10·5	112	196	180·00	0·298E−06	10	28·42
11·5	111	175	140·00	0·298E−06	11	27·89
12·5	89	162	110·00	0·296E−06	12	27·28
13·5	89	224	74·00	0·296E−06	13	26·59
14·5	89	250	56·00	0·296E−06	14	25·88
15·5	82	271	43·00	0·296E−06	15	25·07
16·5	82	380	30·00	0·296E−06	16	24·20
17·5	50	510	21·00	0·293E−06	17	23·33
18·5	50	590	13·00	0·293E−06	18	22·79
19·5	50	460	9·20	0·149E−06	19	22·06
20·5	50	540	5·90	0·400E−08	20	20·76
21·5	50	276	3·70	0·400E−08	21	19·20
22·5	50	302	2·00	0·400E−08	22	15·00
23·5	50	548	0·67	0·400E−08	23	9·88
24·5	50	491	0·53	0·400E−08	24	4·82
25·5	50	630	0·36	0·400E−08	25	1·68
26·5	50	716	0·20	0·400E−08	26	0·50
27·5	50	763	0·12	0·400E−08	27	0·03
28·5	50	775	0·15	0·400E−08	28	0·00
29·5	50	700	0·00	0·000E+01	29	0·00

one at $k = 7$ and the other at $k = 19$–20. The results are plotted as dashed lines for caesium and salinity in Figures 2 and 3, respectively. The observed values of caesium and salinity also are shown in the figures for comparison with the computed curves. With the exception of the caesium values at $k = 10$, 11 the agreement is within observational error and the limits imposed by the steady-state assumption. The observed caesium values at $k = 10$, 11 were derived using data from a single station in the cross-section near Section 12 which was

TABLE 3. Predicted s_k and c_k distributions

Distributions predicted from dispersion equation			Distributions predicted from salinity equation	
k	s	c	c	c^a
−1	33·00	0·90	0·97	0·97
0	32·62	0·96	0·98	0·98
1	32·18	1·03	0·98	0·98
2	31·71	1·11	1·10	1·11
3	31·24	1·19	1·18	1·19
4	30·76	1·27	1·24	1·26
5	30·31	1·35	1·34	1·37
6	29·84	1·43	1·49	1·53
7	29·44	1·49	1·51	1·55
8	29·09	1·50	1·51	1·56
9	28·78	1·51	1·53	1·58
10	28·45	1·52	1·54	1·59
11	28·00	1·54	1·56	1·62
12	27·37	1·57	1·59	
13	26·70	1·61	1·63	
14	25·99	1·65	1·68	
15	25·18	1·71	1·74	
16	24·31	1·77	1·81	
17	23·45	1·84	1·88	
18	22·91	1·93	1·98	
19	22·17	2·06	2·06	
20	20·90	2·11	2·04	
21	19·32	1·96	1·91	
22	15·11	1·55	1·56	
23	9·93	1·04	1·13	
24	4·88	0·55	0·69	
25	1·71	0·25	0·40	
26	0·50	0·13	0·28	
27	0·03	0·08	0·08	

[a]Boundary conditions applied at k = 11.

sampled differently than were all the other sections. Furthermore, the longitudinal distance between sampling locations in this interval was twice as great as it was elsewhere. Therefore, it is quite possible that the discrepancy between the calculated and observed values at these sections is a result of this sampling difference. In the next section we will demonstrate the effects of different river runoff and caesium input rates on the calculated caesium distribution in this reach of the estuary.

Calculation of the ^{137}Cs distribution directly from the observed salinity distribution

The input salinity distribution for these calculations shown in the last two columns of Table 2 comprises observed values for $-1 \leqslant k \leqslant 11$ and for $k > 11$ values determined from the results of the regression relationships obtained by Uncles & Radford (1980). The calculated results for caesium in the range $-1 \leqslant k \leqslant 11$ are insensitive to moderate changes in the salinity values in the range of $k > 11$. The finite difference forms used to evaluate the integrals in equations (6) and (7) [illustrated for equation (6)] are:

$$P_{k+1} = P_k + \frac{4(s_{k+1} - s_k)}{R_{k+1/2}(s_{k+1} + s_k)^2}, \tag{9}$$

where the value of s for $k+1/2$ is taken as the average of the values at k and $k+1$.

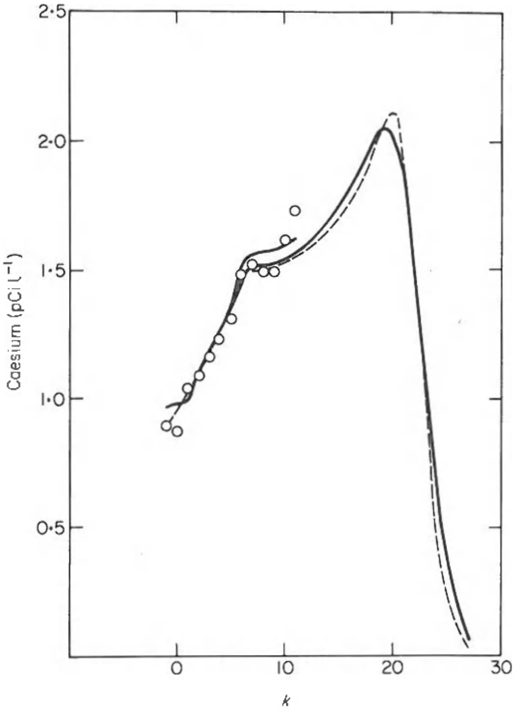

Figure 2. Caesium distribution: – – –, predicted from dispersion coefficients; ———, predicted from salinity distribution; ○, observed.

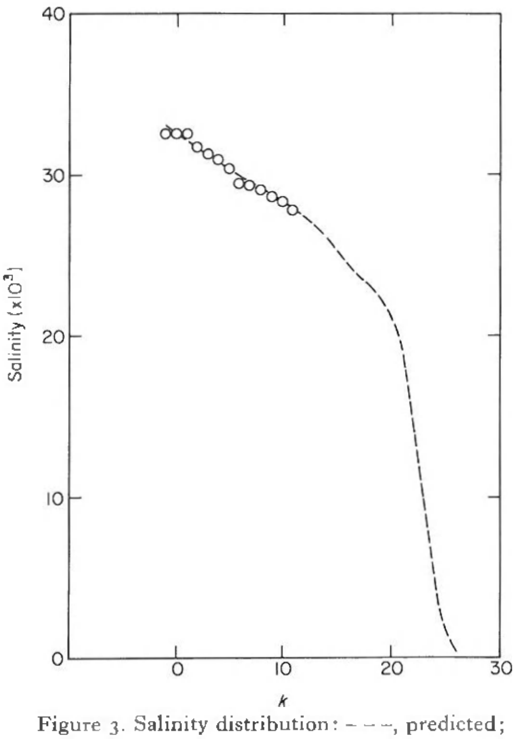

Figure 3. Salinity distribution: – – –, predicted; ○, observed.

The results of two calculations are shown in the last two columns of Table 3 and by the solid curves in Figure 2. The downstream boundary value was taken to be 0·97 pCi l^{-1} for both calculations. For one, the upstream boundary value was taken to be 1·62 pCi l^{-1} at $k = 11$ while for the other it was taken to be 0·08 pCi l^{-1} at $k = 27$. The former computation was carried out to determine how well the observed values in the range $6 \leqslant k \leqslant 11$ could be fit by results independent of the salinity values used for $k > 11$. It is apparent that the general level of the ^{137}Cs concentration can be increased in this range of k by an increase in the value imposed at $k = 11$; however, no overall change occurs in the shape of the resulting curve. Thus, the difference between computed and observed values of ^{137}Cs at $k = 10$, 11 cannot be attributed to the values assumed for s upstream of this location.

The ^{137}Cs distributions computed directly from the dispersion equation using the given dispersion coefficients and that computed by forming the Green's function from the simultaneously observed s distribution agree remarkably well. The overall shape of the curves can be understood qualitatively most simply from Ketchum's (1955) result for a single source in an estuary with constant net river runoff. For zero concentrations of the constituent in both the river and the ocean, he obtains, with the subscript k indicating values at the source:

$$c = \frac{\delta Q}{R} f_k \frac{f}{f_k}$$

downstream of the source, and

$$c = \frac{\delta Q}{R} f_k \frac{s}{s_k}$$

upstream of the source. The maximum value of c occurs at the source, decaying as the salinity upstream and as the so-called 'freshwater fraction' downstream. The distribution resulting from two sources is merely the superposition of these results with, under these restricted conditions, the relative magnitudes of the peaks depending not only on the ratio of source strength to net river runoff but also on the local salinity. This latter effect shows up strongly in the Severn with the maximum ^{137}Cs concentration occurring at the $k = 19$–20 source.

The dispersion calculation yields a smoother than observed distribution for ^{137}Cs as it does for s also. The s derived ^{137}Cs distribution is shaped more like the one observed although there is a displacement from observed values near the mouth at $k = -1, 0, 1$. The difference in the two calculations at $k = 20$, 21, the location of a ^{137}Cs source, is a reflection merely that the source input is on the $1/2$ k intervals for one calculation and on the integer k intervals for the other.

As a further investigation on the discrepancy between observed and calculated ^{137}Cs values at $k = 10$, 11 two additional computations were carried out. In one the distribution of the river inflow was changed and in the other the input rate of the upstream source was increased. Figure 4 shows the calculated distributions of ^{137}Cs when the total river flux through the sections at $k = 8{\cdot}5$–$11{\cdot}5$ inclusive have been reduced to 90 m^3 s^{-1}. The ^{137}Cs values calculated using the original salinity distribution have been increased up to 0·05 pCi l^{-1} over their previously determined values. The effects of this change in river flow are observed upstream all the way to $k = 25$ but not downstream from the region over which the changes were made. A similar distribution of effects occurs when the dispersion model is applied but in this case the maximum increase in ^{137}Cs concentration is 0·02 pCi l^{-1}. This lesser effect occurs because the salinity distribution also is modified rather than being held

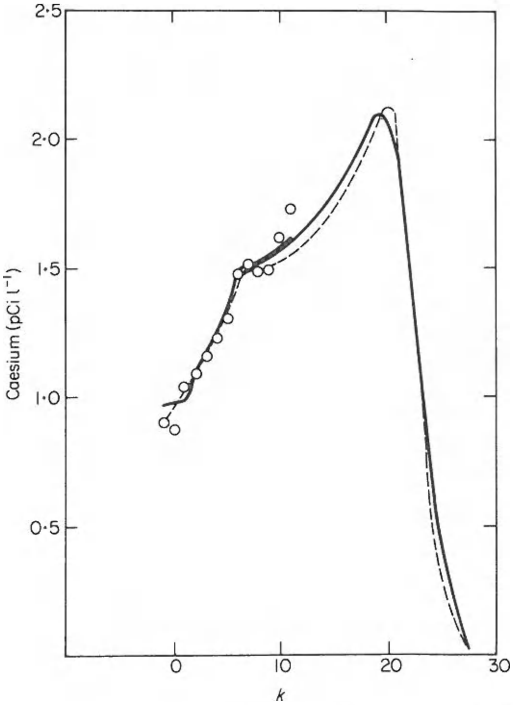

Figure 4. Caesium distribution with $R = 90\ m^3s^{-1}$ from $k = 8{\cdot}5$ to $11{\cdot}5$: – – –, predicted from dispersion coefficients; ———, predicted from salinity distribution; ○, observed.

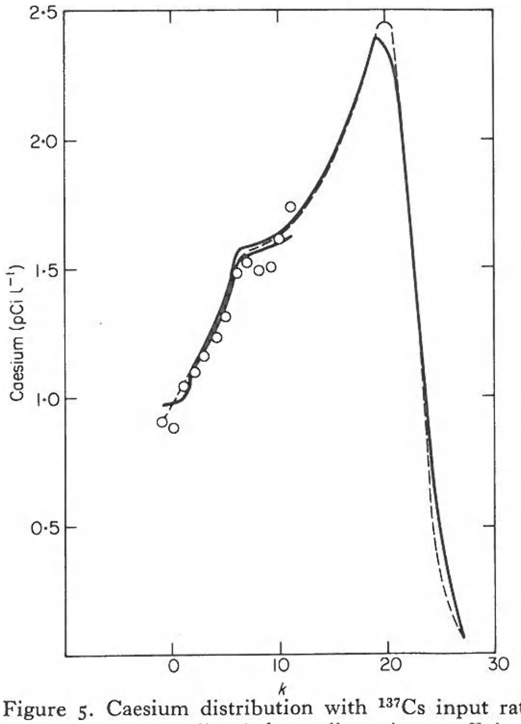

Figure 5. Caesium distribution with ^{137}Cs input rate of $0{\cdot}24 \times 10^{-6}\ Ci\ s^{-1}$ at $k = 19$: – – –, predicted from dispersion coefficients; ———, predicted from salinity distribution; ○ observed.

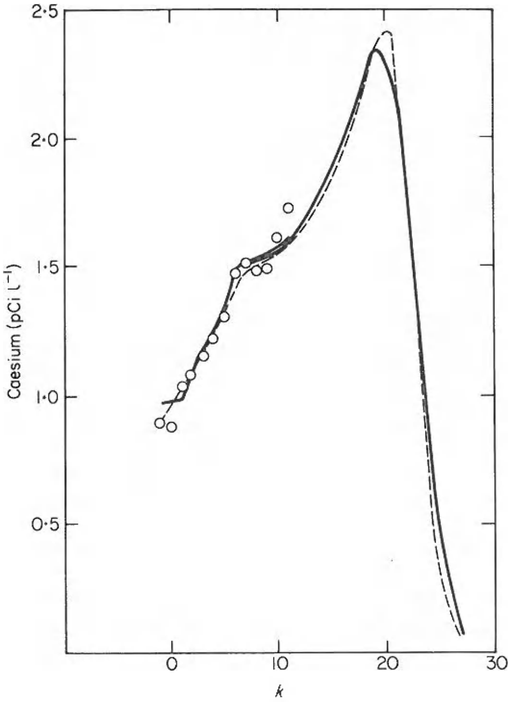

Figure 6. Caesium distribution with ^{137}Cs input of $0{\cdot}24 \times 10^{-6}$ Ci s^{-1} at $k = 19$ and $0{\cdot}51 \times 10^{-6}$ Ci s^{-1} at $k = 7$: – – –, predicted from dispersion coefficients; ———, predicted from salinity distribution; ○, observed.

fixed as in the other analysis. The increase in concentrations found here results from the decreased rate of advective outward transport with the diminished river flux through sections 8·5–11·5.

The ^{137}Cs distribution that results from an increase in the ^{137}Cs input rate from $0{\cdot}14 \times 10^{-6}$ to $0{\cdot}24 \times 10^{-6}$ Ci s^{-1} at $k = 19$ are illustrated in Figure 5. This increases the level of ^{137}Cs throughout with a maximum increase of 0·34–0·37 pCi l^{-1} near the source, 0·12 pCi l^{-1} at $k = 12$, and 0·08–0·09 pCi l^{-1} at $k = 8$. While this improves the fit to observed values at $k = 10$, 11, there is a correspondingly poorer fit at $k = 6$, 7, 8, 9. Next, the ^{137}Cs input at $k = 7$ is diminished by $0{\cdot}10 \times 10^{-6}$ Ci s^{-1} to keep the total input constant with merely a reapportionment of the relative inputs between locations $k = 7$ and 19. The resulting distribution shown in Figure 6 approaches a better least squares fit to the observed data points, but the realistic shape of the curve with the strong changes in slope in the region between $k = 6$ and 10 is diminished. These results can be understood qualitatively as simple changes following those in the source strengths, with the effects in general diminishing away from the source regions.

Discussion

The Severn Estuary is very well described by one-dimensional dispersion equations for salinity and other properties. Distributions of ^{137}Cs predicted either by direct integration of the dispersion equation or by utilization of a Green's function based upon the observed salinity distribution compare favourably with each other and with observations other than the two end sections $k = 10, 11$. Because the sampling carried out at $k = 12$, upon which the observed values depend, was different than elsewhere it seems likely that this is the major cause of these differences between observed and computed values at $k = 10, 11$.

The dispersion coefficients used in the dispersion equation are unusually well known. Their dependence upon tidal range and total rate of input of freshwater has been determined from a total of 29 sets of salinity distributions by Uncles & Radford (1980). The resulting predicted distributions give good but smoothed comparisons with the data. The use of a Green's function derived from the observed salinity distribution yields a ^{137}Cs distribution showing more of the observed small-scale structure than is obtained from the dispersion equation. Otherwise the results from the two methods are interchangeable.

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