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Short Communication

Revisiting Nihoul's model for oil slicks transport and spreading on the sea

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ABSTRACT

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Nihoul's model for the transport and spreading of oil slicks on the sea is summarized. It is shown that the evolution equation of the model admits a similarity solution that is valid in the gravity–friction and friction–surface tension regimes. An alternative governing equation is proposed.

INTRODUCTION

Nihoul (1984a,b) has developed a model, hereafter referred to as NM (Nihoul's Model), capable of representing the transport and spreading of oil slicks on the sea. On the basis of dimensional arguments, approximate similarity solutions for axisymmetric slicks have been derived.

The purpose of the present note is to show that there exists an exact similarity solution, valid over the range of spreading regimes covered by NM. In addition, an alternative form of the governing equation of the model is proposed.

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SUMMARY OF NM

NM expresses the conservation of oil and the balance of forces acting on the oil layer by means of equations integrated over the depth of the oil layer. The mass conservation equation is

$$\frac{\partial h}{\partial t} + \nabla \cdot (h\mathbf{u}) = Q \tag{1}$$

where h is the thickness of the oil layer present at the sea surface; $\mathbf{u}$ is the oil velocity averaged over the depth of the oil layer; Q is a production/destruction term taking into account the presence of sources releasing oil and the effect of loss processes such as evaporation or sinking into the water column; t is time; and ∇ is the horizontal gradient operator.

Neglecting inertia forces, which are important only in the first phase of spreading (Nihoul, 1984a), the momentum equation, integrated over the oil layer depth, represents the balance of the gravity ($\mathbf{F}_g$), frictional ($\mathbf{F}_f$) and surface tension ($\mathbf{F}_{st}$) forces

$$\mathbf{F}_g + \mathbf{F}_f + \mathbf{F}_{st} = 0 \tag{2}$$

According to NM, the gravity force may be expressed as

$$\mathbf{F}_g = -g'h\,\nabla h \tag{3}$$

with $g' = g\,(\rho^w\text{-}\rho)/\rho$, where g, ρ^w and ρ denote gravitational acceleration, and water and oil density, respectively. The frictional forces are due to the combined effect of the stresses at the air–oil interface, τ^a, and the oil-water interface, τ^w, which is parameterized by a linear expression

$$\tau^w = -k(\mathbf{u} - \mathbf{u}^w) \tag{4}$$

where k is an appropriate drag coefficient and $\mathbf{u}^w$ is the water velocity below the oil slick. Hence, one has

$$\mathbf{F}_f = \frac{1}{\rho}[\tau^a - k(\mathbf{u} - \mathbf{u}^w)] \tag{5}$$

In NM, the surface tension force is assumed to increase from zero at the center of the slick to its maximum value at the edge. Accordingly $\mathbf{F}_{st}$ is parameterized as

$$\mathbf{F}_{st} = \frac{\gamma}{\rho}\nabla\psi^2 \tag{6}$$

In the latter expression, γ is a relevant surface tension parameter and ψ is a function that is equal to 0 (1) at the center (edge) of the slick (notice that ψ^2 is equal to function ϕ of NM).

Combining (3), (5) and (6) with (2), one obtains

$$\mathbf{u} = \mathbf{u}^{\mathrm{w}} + \frac{1}{k}\tau^{\mathrm{a}} - \frac{g'\rho}{k} h\, \nabla h + \frac{\gamma}{k} \nabla \psi^2 \tag{7}$$

which can be introduced into (1), leading to NM's governing equation

$$\frac{\partial h}{\partial t} + \nabla \cdot (h\mathbf{v}) = Q + \nabla \cdot \left(\frac{g'\rho}{k} h^2\, \nabla h - \frac{\gamma}{k} h\, \nabla \psi^2 \right) \tag{8}$$

In the above equation, $\mathbf{v} = \mathbf{u}^{\mathrm{w}} + \tau^{\mathrm{a}}/k$ may be interpreted as the oil advection velocity induced by atmospheric and oceanic forcings.

SIMILARITY SOLUTIONS

When Q is negligible and when the extent of the oil slick is small compared with the length scale of $\mathbf{v}$, one may de-couple the advection and the spreading of the slick. It follows that the center of gravity of the slick, $\mathbf{x}_{\mathrm{c}}$, moves according to

$$\frac{\mathrm{d}\mathbf{x}_{\mathrm{c}}}{\mathrm{d}t} = \mathbf{v} \tag{9}$$

Assuming that the slick is axisymmetric, using polar coordinate r having its origin at $\mathbf{x}_{\mathrm{c}}$, and introducing (9) into (8), one obtains

$$\frac{\partial h}{\partial t} = \frac{1}{r}\frac{\partial}{\partial r}\left[r \left(\frac{g'\rho}{k} h^2 \frac{\partial h}{\partial r} - \frac{\gamma}{k} h \frac{\partial \psi^2}{\partial r} \right) \right] \tag{10}$$

Nihoul (1984b) suggested looking for a similarity solution of the form

$$h = \frac{3V}{2\pi}\frac{1}{R^2}\left[1 - \left(\frac{r}{R} \right)^2 \right]^{1/2} \tag{11}$$

where V denotes the released volume of oil and R represents the radius of the slick. It is readily seen that (11) satisfies the overall mass conservation equation

$$V = 2\pi \int_0^R hr\, \mathrm{d}r \tag{12}$$

Nihoul (1984a) suggests

$$\psi = \frac{r}{R} \tag{13}$$

According to (11), (13) is equivalent to

$$\psi = \left[1 - \left(\frac{2\pi}{3V} R^2 h \right)^2 \right]^{1/2} \tag{14}$$

As shown by Deleersnijder and Loffet (1985), transforming (13) to (14) greatly facilitates the search for a similarity solution.

Combining (10) and (14), one obtains

$$\frac{\partial h}{\partial t} = \left[\frac{g'\rho}{k} + \frac{2\gamma}{k} \left(\frac{2\pi}{3V} \right)^2 R^4 \right] \frac{1}{r} \frac{\partial}{\partial r} \left(rh^2 \frac{\partial h}{\partial r} \right) \tag{15}$$

Introducing (11) into (15), one has

$$\frac{\mathrm{d}R}{\mathrm{d}t} = \frac{2\gamma}{a^2 k} \frac{1 + a^2 R^4}{R^5} \tag{16}$$

where

$$a = \frac{2\pi}{3V} \left(\frac{2\gamma}{g'\rho} \right)^{1/2} \tag{17}$$

Finally, the solution of (16) is given by

$$R^2 - \frac{1}{a} \operatorname{arctg}(aR^2) = \frac{4\gamma}{k} t \tag{18}$$

which provides the radius of the slick as a function of the released volume and the elapsed time.

It must be stressed that (18) holds for the whole range of successive spreading regimes covered by (10), i.e. the gravity–friction regime ($\mathbf{F}_g \approx \mathbf{F}_f$) and the friction–surface tension regime ($\mathbf{F}_f \approx \mathbf{F}_{st}$).

In the gravity–friction regime, $aR^2 \ll 1$ so that (18) may be approximated by

$$R \sim \left[\frac{27V^2}{2\pi^2} \frac{g'\rho}{k} \right]^{1/6} t^{1/6} \approx \left[1.37\, V^2 \frac{g'\rho}{k} \right]^{1/6} t^{1/6} \tag{19}$$

When $aR^2 \gg 1$, the slick is in the friction–surface tension regime and (18) admits the following asymptotic expansion

$$R \sim 2 \left(\frac{\gamma}{k} \right)^{1/2} t^{1/2} \tag{20}$$

Expressions (19) and (20) exhibit the same functional dependence in t as obtained by Nihoul [1984a, equations (33) and (34)] on the basis of dimensional arguments.

ALTERNATIVE FORM OF NM'S GOVERNING EQUATION

There remains a difficulty in NM's governing equation (8). Indeed, (8) relies on definition (13), which is, in principle, only valid for an axisymmetric slick. For a real oil slick, it is indeed not easy to unambiguously define r and R. When the above similarity solution holds, by virtue of (11), (13) may be rewritten as

$$\psi = \left[1 - \left(\frac{h}{h_{\max}}\right)^2\right]^{1/2} \tag{21}$$

$h_{\max}$ representing the maximum thickness of the oil layer. Assuming that (21) roughly applies in general, one may then introduce (21) into (8) to obtain

$$\frac{\partial h}{\partial t} + \nabla \cdot (h\mathbf{v}) = Q + \nabla \cdot (\alpha \nabla h) \tag{22}$$

where

$$\alpha = \left(\frac{g'\rho}{k} + \frac{2\gamma}{kh_{\max}^2}\right)h^2 \tag{23}$$

Since (22) is of a parabolic nature, one might think that the spreading rate should be infinite (Crank, 1975). This is, however, not the case, for the diffusion coefficient α is zero at the edge of the slick. More precisely, as the edge is approached $\alpha \to 0$ while $|\nabla h| \to \infty$, eventually resulting in a finite, non-zero spreading rate. As a consequence, numerical solution of (22) should be conducted carefully (Deleersnijder and Loffet, 1985).

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