Upwelling and upsloping in three-dimensional marine models

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The so-called σ-transformation is a change of coordinates widely used in three-dimensional marine modelling. A new vertical velocity, $\bar{u}_3$, is usually associated with this transformation. It is shown that the vertical velocity may be split into two contributions. The first one, called upsloping velocity, is merely induced by the geometry of the basin. The other one is due to intrinsic upwelling mechanisms and is proportional to $\bar{u}_3$. Examples are provided by the results of the three-dimensional model of Geohydrodynamics and Environment Research.

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Introduction

To take into account the effect of the bottom and surface topography in three-dimensional models, several authors[1–5] use a change of coordinates in which the surface and the bottom of the sea are coordinate surfaces. Although such a change of coordinates may be expressed in different ways, its formulation will remain very similar to the one proposed by Phillips[6] for the purpose of meteorological simulations. According to Nihoul *et al.*,[3] one writes

$$(\bar{t}, \bar{x}_1, \bar{x}_2, \bar{x}_3) = \left(t, x_1, x_2, L\frac{x_3 + h}{\eta + h} = L\sigma\right) \quad (1)$$

where the new variables are in the left-hand member of equation (1). The time is denoted by t, and the vertical coordinate by x_3. The horizontal coordinates are represented by x_1 and x_2. The variables η and h stand for the sea surface elevation and the depth of the sea, respectively, with respect to a reference sea level as shown in *Figure 1*. The domain of interest of the real space, whose total height,

$$H = \eta + h \quad (2)$$

is variable in space and time, is thus transformed into a domain of the σ-space whose height, L, is constant. This leads to considerable simplifications in the numerical algorithm that is used to solve the equations of the mathematical model at stake.

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One usually introduces a new vertical velocity defined as

$$\bar{u}_3 \equiv \frac{d\bar{x}_3}{dt} = L\frac{d\sigma}{dt} \quad (3)$$

where d/dt is the material derivative operator,

$$\frac{d}{dt} = \frac{\partial}{\partial t} + \mathbf{u}\cdot\nabla + u_3\frac{\partial}{\partial x_3} \quad (4)$$

with

$$\nabla = \mathbf{e}_1\frac{\partial}{\partial x_1} + \mathbf{e}_2\frac{\partial}{\partial x_2} \quad (5)$$

Unit vectors $\mathbf{e}_1$ and $\mathbf{e}_2$ are horizontal, and $\mathbf{e}_3$ is vertical. The horizontal velocity is denoted by $\mathbf{u}$, whereas u_3 represents the vertical velocity. By using definition (3) it is readily seen that

$$\frac{d}{dt} = \frac{\partial}{\partial \bar{t}} + \mathbf{u}\cdot\bar{\nabla} + \bar{u}_3\frac{\partial}{\partial \bar{x}_3} \quad (6)$$

with

$$\bar{\nabla} = \mathbf{e}_1\frac{\partial}{\partial \bar{x}_1} + \mathbf{e}_2\frac{\partial}{\partial \bar{x}_2} \quad (7)$$

which means that it is worthwhile to use $\bar{u}_3$ instead of u_3 when writing the equations of the model in the σ-space.

Upwelling and upsloping

According to (1), (2), (3) and (4), one has

$$u_3 = \frac{H}{L}\bar{u}_3 + \sigma\frac{\partial\eta}{\partial t} - \mathbf{u}\cdot[(1-\sigma)\nabla h - \sigma\nabla\eta] \quad (8)$$

The physical interpretation of the different terms in (8) will be given.

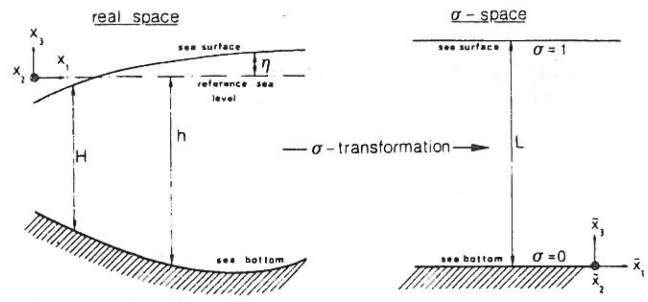

Figure 1. Illustration of the σ-transformation

First, one considers a particle whose position is fixed in the σ-space. In the real space this particle moves vertically in such a way that its relative position in the water column remains constant. Its velocity reads

$$w_1 \mathbf{e}_3 = \frac{\partial x_3}{\partial \bar{t}} \mathbf{e}_3 \tag{9}$$

According to (1), one gets

$$w_1 = \sigma \frac{\partial \eta}{\partial t} \tag{10}$$

Next, one assumes that the iso-σ surfaces are at rest. One considers a particle moving on an iso-σ surface with the horizontal velocity $\mathbf{u}$. Since this particle does not cross the iso-σ surfaces, its velocity, $\mathbf{u} + w_2\mathbf{e}_3$, is orthogonal to the unit vector $\mathbf{n}$, which is normal to the iso-σ surface at the point where the particle is, that is,

$$(\mathbf{u} + w_2\mathbf{e}_3) \cdot \mathbf{n} = 0 \tag{11}$$

It may be shown that

$$\mathbf{n} = \frac{H\nabla\sigma + \mathbf{e}_3}{(H^2\|\nabla\sigma\|^2 + 1)^{1/2}} \tag{12}$$

so that

$$w_2 = -\mathbf{u} \cdot [(1 - \sigma)\nabla h - \sigma\nabla\eta] \tag{13}$$

Finally, combining (10) and (13), one obtains the vertical velocity of a particle (whose horizontal velocity is $\mathbf{u}$) moving without crossing any iso-σ surface,

$$\begin{aligned} w_{us} &= w_1 + w_2 \\ &= \sigma \frac{\partial \eta}{\partial t} - \mathbf{u} \cdot [(1 - \sigma)\nabla h - \sigma\nabla\eta] \end{aligned} \tag{14}$$

Since the sea surface and the sea bottom are iso-σ surfaces, one may say that w_{us} is the vertical velocity of a particle that adapts its motion to the geometry of the basin. Therefore it is suggested to call w_{us} the vertical geometrical velocity[7] or the upsloping velocity.[8] The introduction of (14) into (8) yields

$$u_3 = \frac{H}{L}\bar{u}_3 + w_{us} \tag{15}$$

so that the real vertical velocity is the sum of the upsloping velocity and

$$w_{uw} = \frac{H}{L}\bar{u}_3 \tag{16}$$

The latter velocity is not directly induced by the surface and bottom topography. It is thus the intrinsic upwelling (or downwelling) velocity.

This interpretation is illustrated by *Figure 2*.

The upsloping velocity may also be interpreted as a generalization of the vertical velocity of the shallow water theory,[9] w. Indeed, if one replaces $\mathbf{u}$ by the depth-averaged horizontal velocity in equation (14), one identically obtains w.

Since the real vertical velocity plays no role in the σ-space, the computation of u_3 is part of the postprocessing of the results of a numerical simulation carried out in the σ-space. If one uses a staggered-C grid, equation (8) is not very well suited to this computation. Indeed, it is desirable to have a formula containing only conservative forms in the σ-space so that it is easy to take advantage of the fact that there are no water fluxes across the surface, the bottom, and the coast. Using the transformed continuity equation,

$$\frac{\partial H}{\partial t} + \bar{\nabla} \cdot H\mathbf{u} + \frac{\partial H\bar{u}_3}{\partial \bar{x}_3} = 0 \tag{17}$$

one gets, after some calculations,

$$\begin{aligned} u_3 = w_{uw} &+ [(1 - \sigma)h - \sigma\eta]\left[\bar{\nabla} \cdot \mathbf{u} + \frac{\partial \bar{u}_3}{\partial \bar{x}_3}\right] \\ &- \left[\bar{\nabla} \cdot h\mathbf{u} + \frac{\partial h\bar{u}_3}{\partial \bar{x}_3}\right] \end{aligned} \tag{18}$$

A practical example

The results of the three-dimensional (3-D) model of Geohydrodynamics and Environment Research (GHER)[10,11,12] provide an excellent illustration of the present discussion. This model is used to investigate the general circulation in the region of the Bering Strait.[8,12] The general (or macroscale) circulation is defined as the averaged flow "over a time T of several weeks (that is, much larger than the characteristic time

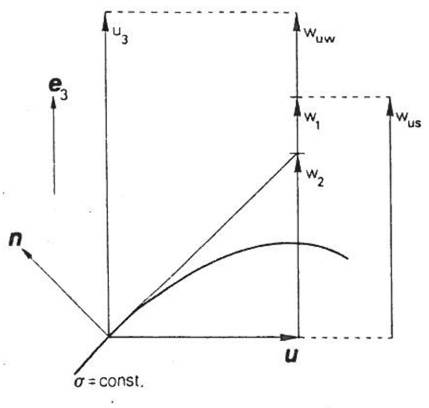

Figure 2. Physical interpretation of the upwelling (w_{uw}) and upsloping (w_{us}) velocity

of tides, passing storms and other mesoscales processes)."[12] At such a time scale the mean wind stress is small. It is thus not the dominant forcing compared to the inflows and outflows of the regional shelf currents; the mean wind stress has been neglected. However, the mean production of turbulence is strongly affected by mesoscale phenomena. Hence a suitable residual turbulence production by the shear stress has been added to the turbulence closure scheme of the model.[8,12]

Figure 3 shows the bathymetry of the domain of interest and the path of two vertical sections in the Anadyr Strait (*A* and *B*). The steady depth-averaged (from the 3-D values) macroscale horizontal velocity, $\bar{\mathbf{u}}$, induced by the open sea boundary conditions for a typical summer situation is displayed in *Figure 4*. The velocity in the plane of section *A* is shown in panel 1 of *Figure 5*, which means that the vectors of this figure are the sum of $u_3\mathbf{e}_3$ and the horizontal velocity in this plane of section, $\mathbf{u}_{SA}$. Panel 2 of *Figure 5* shows the sum of $w_{us}\mathbf{e}_3$ and $\mathbf{u}_{SA}$. Panel 3 of *Figure 5* shows the sum of $w_{uw}\mathbf{e}_3$ and $\mathbf{u}_{SA}$. Hence the great vertical velocity near the Siberian coast (on the left of panels 1, 2, and 3 of *Figure 5*) is mainly due to the effect of the bathymetry. Indeed, in this region the water is forced over a huge bump, as confirmed by panel 4 of *Figure 5*, which displays $\mathbf{u}_{SB} + u_3\mathbf{e}_3$.

Although the upwelling velocity is smaller here than the upsloping one, w_{uw} does play a significant role. The initial temperature is set in accordance with the typical seasonal features of the Northern Bering Sea described by Coachman *et al.*[13] At this instant the temperature is horizontally uniform, and its vertical profile presents a sharp thermocline at 15 m below the sea surface.

Figure 3. Map displaying the bathymetry of the domain where the 3-D model has been applied. The paths of two planes of section (*A* and *B*) are also shown

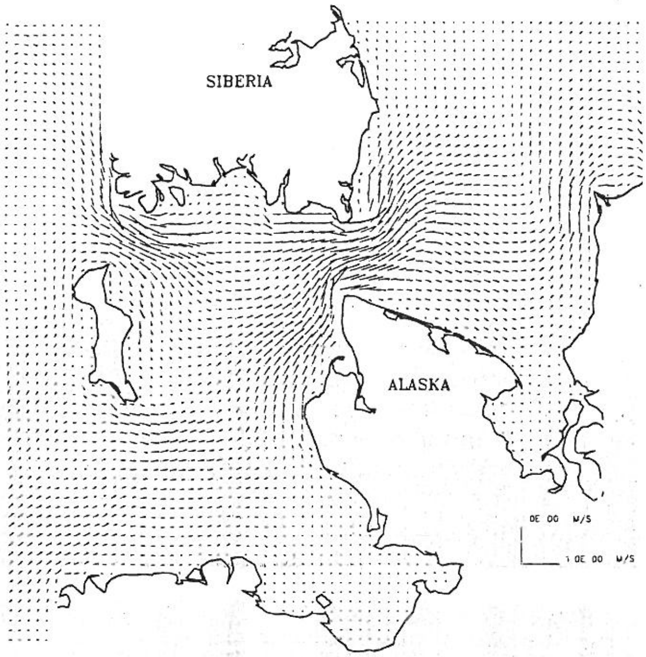

Figure 4. Steady depth-averaged macroscale current induce by the open sea boundary conditions for a typical summer si uation

When a quasi-steady state is reached, *Figure 6* show that the initial temperature field has been deeply moc ified. A thorough examination of the results of th present simulation shows that this evolution is due t the action of the upsloping velocity but also to th upwelling velocity and the vertical eddy diffusion. Thes processes result in the formation of a cold water plum in the lee of Anadyr Strait, which is confirmed b satellite imagery.[12]

The very mechanism generating the upwelling ve locity of panel 3 of *Figure 5* is not completely eluc dated. It is not induced by the wind, since the stres imposed at the sea surface has been neglected. Th velocity veering is such that the horizontal velocity on the left (right) of $\bar{\mathbf{u}}$ near the bottom (surface) of th sea. This veering is roughly equivalent to the Ekma deflection of the horizontal velocity produced by th bottom stress. Since $\mathbf{u} - \bar{\mathbf{u}}$ is directed toward the S berian coast near the bottom and in the opposite d rection just below the sea surface (see *Figure 7*), or may hypothesize that the present upwelling is drive by the bottom stress, which plays a role here simil to that of the surface stress in the classical theory wind-induced coastal upwelling. This hypothetic scenario could probably account for the positive u welling velocity taking place in the grid vertical adj cent to the Siberian coast in panel 3 of *Figure 5*. Hov ever, it cannot explain why w_{uw} has the same order magnitude several grid points away from the coas Preliminary investigations suggest that the bathymet could play an important role in this phenomenon. Fu ther theoretical and numerical study of these process is obviously needed.

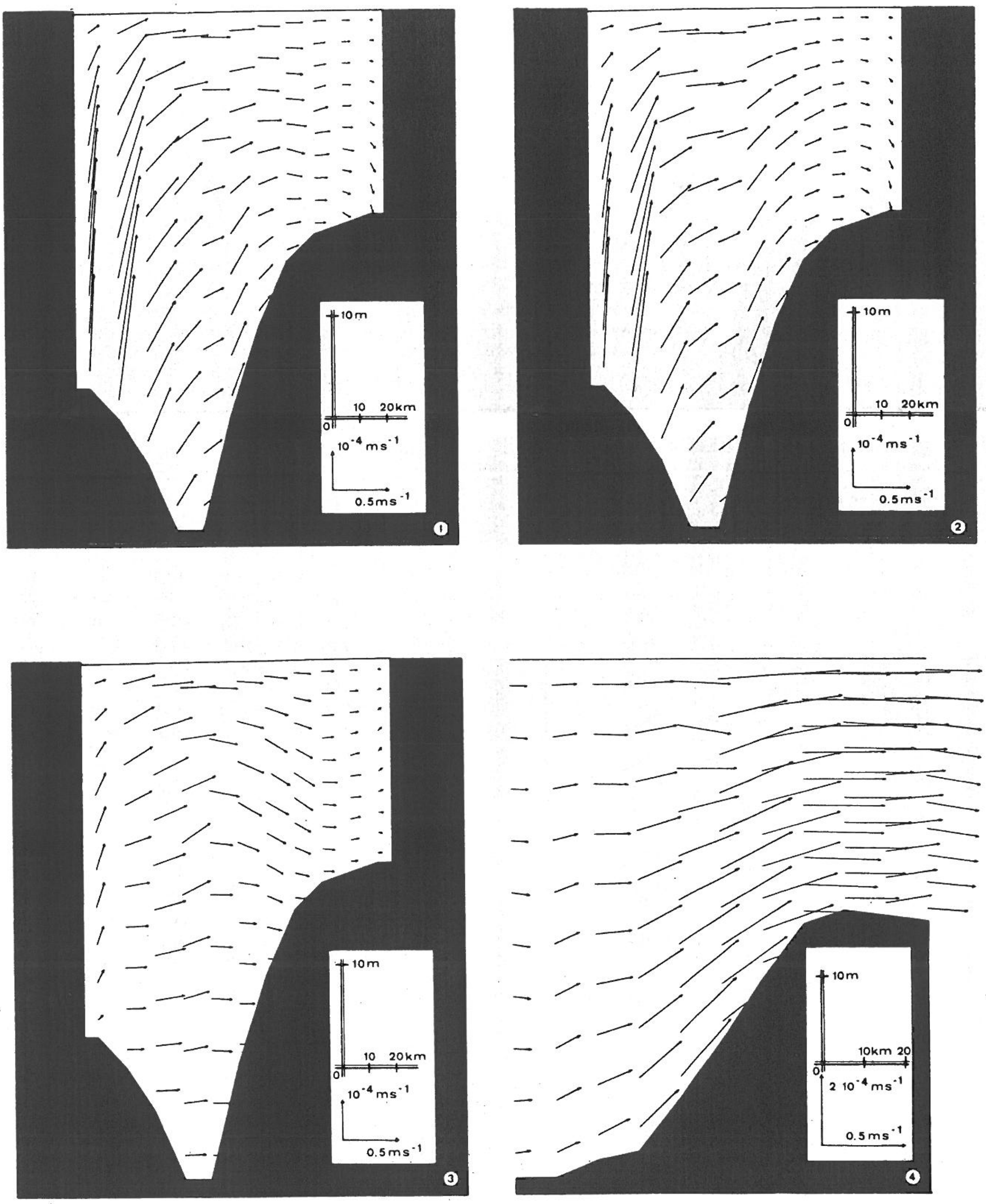

Figure 5. The velocity in the plane of section *A* is shown in panel 1. The velocity constructed with the upsloping (upwelling) component of the vertical velocity is displayed in panel 2 (panel 3). Panel 4 shows the velocity in the plane of section *B*

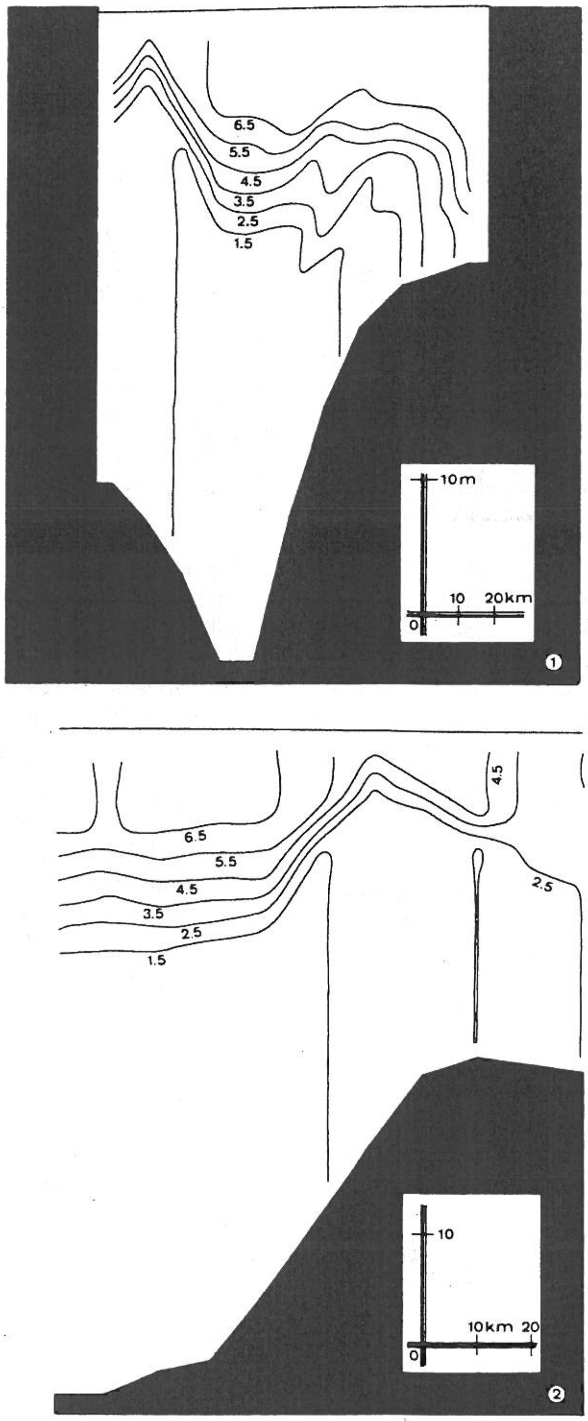

Figure 6. The temperature (Celsius) in the plane of section *A* (*B*) is shown in panel 1 (panel 2)

Conclusion

It has been shown that in the σ-space the marine models deal only with the vertical velocity $\tilde{u}_3$, which is proportional to the intrinsic upwelling velocity. A formula designed to accurately determine the real vertical velocity has been derived. Velocities $\tilde{u}_3$ and u_3 being known, the upsloping velocity is readily computed so that one has an excellent tool to decide whether an upwelling is induced mainly by the geometry of the domain, by an inherent upwelling mechanism, or by the combined action of these two effects.

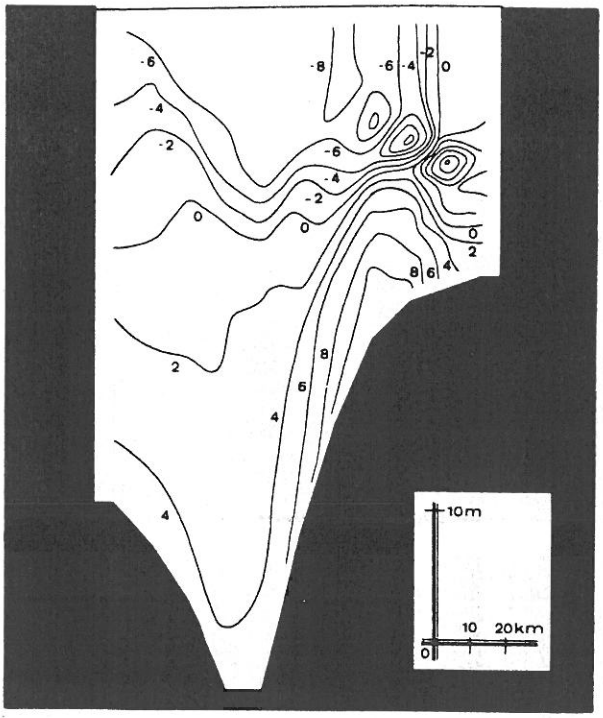

Figure 7. Horizontal velocity veering (degree) in the plane of section *A*. The veering is defined as the angle between **u** and $\bar{\mathbf{u}}$. (The veering is positive when **u** is on the left of $\bar{\mathbf{u}}$)

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