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A Simple Model of the Tracer Flux from the Mururoa Lagoon to the Pacific

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Abstract—It is seen that a simple ordinary differential equation may be sufficient to predict the mass of tracers that may be present in the Mururoa lagoon, as well as the flux from the lagoon to the Pacific. This model depends on a key parameter, namely the lagoon turnover time, which is determined from the results of a complex, three-dimensional, hydrodynamic model.

Keywords—Pacific, Mururoa Atoll lagoon, Residence time, Turnover time, Tracer transport.

1. INTRODUCTION

Mururoa Atoll is located in the tropical Pacific at longitude 138°55' west and latitude 21°50' south. It is a cone of volcanic rocks emerging from the ocean bottom, at a depth of approximately 3,500 m. The volcanics are topped by a layer of carbonates several metres thick. An almost impermeable coral rim delimits the boundary between the Pacific and the lagoon at sea level. The latter is a semienclosed, shallow-water body connected to the Pacific via a single pass (Figure 1). The mean depth and the volume of the lagoon are about 33 m and $4.5\times10^9\,\mathrm{m}^3$, respectively.

From 1976 to 1996, the French army detonated nuclear weapons in the volcanic rocks. Consequently, a significant amount of radioactive material is now stored in this rock layer. The carbonates and the volcanics are porous. Therefore, radioactive elements may be in contact with water, which may progressively dissolve some of them, turning them into "tracers". Since the water is believed to circulate mostly upward through the volcanics and the carbonates [1–3], it may be imagined that, sometime, radioactive tracers—RTs for short—will be released into the ocean or lagoon waters. Then, RTs will be involved in a variety of complex physical, chemical,

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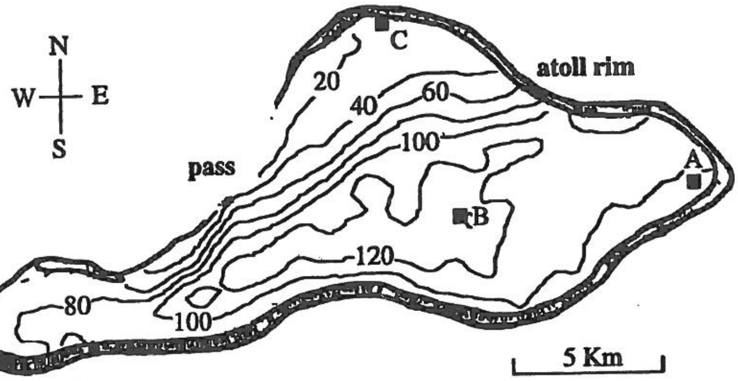

Figure 1. Isolines of the depth-mean residence time, with a contour interval of 20 days. The residence time is almost veritcally homogeneous, implying that little information is lost through the vertical averaging procedure.

and biological processes, which may eventually result in radioactive contamination on Mururoa Atoll, in French Polynesia or perhaps even far away from this region [4].

It is desirable that these hazards be assessed. Two of the numerous issues that must be addressed in this respect are the transport of RTs over distances much larger than the atoll size in the Pacific, and the evaluation of radiation doses that atoll dwellers may be exposed to. For the former, the lagoon may be considered a point source of RTs, while, for the latter, it is sufficient to know the lagoon-averaged concentration of RTs. Below, it is suggested that a simple, zero-dimensional model may be sufficient to predict the RT flux from the lagoon to the Pacific, and the mean RT concentration in the lagoon.

2. ZERO-DIMENSIONAL MODEL

It is assumed that the radioactive decay of the RTs being considered is much slower than the hydrodynamic processes taking place in the lagoon, implying that, within the domain of interest, the RTs may be regarded as passive. So, in the lagoon waters, it is assumed that the variations of the tracer concentration are caused by advection and turbulent diffusion only. Therefore, in the present study the particular nature of the tracer considered is unimportant, which amounts to considering that all tracers behave in the same way.

Let t denote time. Assume that a source, be it pointwise or not, is releasing RTs into the lagoon at rate $s(t)$. If $q(t)$ represents the mass flux leaving the domain of interest through the pass, then the tracer mass present in the lagoon at time t, $m(t)$, obeys the following differential equation:

$$\frac{dm}{dt} = s - q. \tag{1}$$

It seems reasonable to assume that the source term s is independent of the lagoon hydrodynamics. In fact, s is a forcing function to be prescribed in accordance with appropriate data sets.

By contrast, the outgoing mass flux depends on the tracer concentration and the hydrodynamic conditions, implying that q cannot be given *a priori* and must be parameterized appropriately. It is assumed that the hydrodynamics exhibits a well-established periodic regime—similar to that obtained in the numerical experiments of Tartinville *et al.* [5]—the period of which is much smaller than the timescale associated with tracer transport toward the Pacific. Further hypothesizing that the lagoon is well mixed, so that the RT concentration is almost homogeneous over the lagoon, the flux toward the Pacific may be estimated as

$$q = \frac{m}{\theta}, \tag{2}$$

where θ is a constant timescale to be determined later.

Combining (1) and (2), assuming that the initial tracer mass, $m(0)$, is known, an ordinary differential problem is obtained, the solution of which is

$$m(t) = m(0) \exp\left(\frac{-t}{\theta}\right) + \int_0^t s(\mu) \exp\left(\frac{\mu - t}{\theta}\right) d\mu. \tag{3}$$

The lagoon-averaged tracer concentration is obviously the ratio of $m(t)$ to the lagoon volume.

Expression (3) clearly indicates that the key parameter of the simplified model developed here is the timescale θ. Prior to evaluating the latter, it is appropriate to understand its physical meaning.

The residence time at a given point in the lagoon is the period of time that a RT parcel needs to leave the lagoon through the pass and enter the Pacific [5,6]. The turnover time is obtained by averaging the residence time over the volume of the lagoon. It may be shown [5] that the timescale θ is actually the turnover time—if the hypotheses underlying the present simplified model are valid.

For a constant-rate release, i.e., $s =$ constant, the RT mass present in the lagoon tends to θs as time tends to infinity. In other words, the larger the turnover time, the larger the RT mass in the lagoon. The outgoing flux, however, tends asymptotically to s, and thus tends to be independent of θ as $(\theta^{-1}t) \to \infty$.

It is noteworthy that, in studies unrelated to Mururoa Atoll, several authors have suggested using expressions bearing some similarity with (3) (e.g., [7,8]). The turnover time of the lagoon is estimated and the simplified model is briefly assessed in the following section.

3. DISCUSSION

It is impossible to determine the turnover time on the basis of simple physical reasoning alone. To do so, appropriate field data or model results are needed. Since the former are unavailable, it is necessary to use the three-dimensional model of Tartinville *et al.* [5] in order to estimate the turnover time. The lagoon flow is forced by the oceanic tide and the surface wind stress—according to the specifications "TW" of [5]. At the initial instant, RT parcels are uniformly distributed in the lagoon. Then, the RT parcels tend to move away from their initial positions and progressively leave the lagoon through the pass, causing the simulated mass of tracer remaining in the lagoon to drop with time. Ascribing to the point of departure of each RT parcel the time at which it leaves the lagoon, the residence time field is progressively constructed (Figure 1). The tracer mass present in the lagoon decays almost exponentially, as may be seen in Figure 2, allowing us to estimate that the turnover time is about 94 days. This exponential behaviour of the RT mass is a very encouraging result, but it is not sufficient to fully validate the simplified model. Other investigations are necessary.

Figure 1 indicates that the residence time varies significantly over the domain of interest. In fact, the ratio of the standard deviation of the residence time to the turnover time is 0.37, indicating that the lagoon is far from well-mixed, a finding which contradicts the basic assumption of the simplified approach. Therefore, it is necessary to assess whether or not the somewhat weak mixing is actually sufficient for the simplified model to produce acceptable results for any type of source and initial distribution of the RT parcels.

It has already been shown that, for an instantaneous release of RT particles uniformly distributed in the lagoon, the RT mass evolution predicted by (3) is in very good agreement with the three-dimensional model of Tartinville *et al.* [5]. Achieving a similar comparison for point-wise releases is obviously a key test of the quality of the simplified model. This is why the three-dimensional model is used to simulate the fate of RT parcels initially concentrated near the lagoon bottom at points A, B, and C, as indicated in Figure 1. For A and B, the three-dimensional model results are very close to (3) (Figure 2). However, the modelled evolution of the mass of the RT parcels initially located a C exhibits a significant descrepancy with (3)

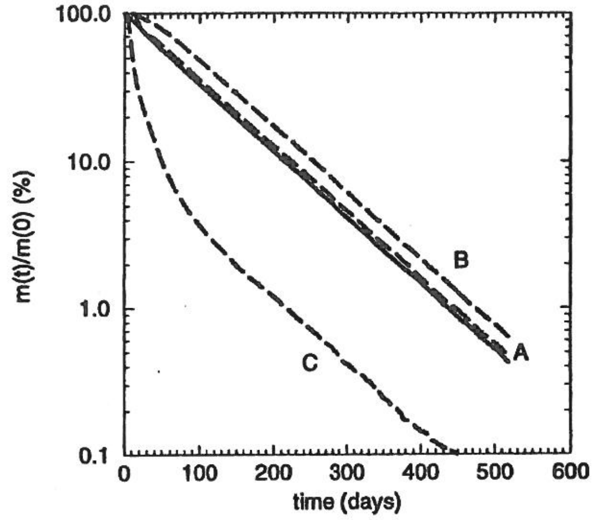

Figure 2. Ratio of the mass of the RT parcels left in the lagoon to the initial mass, $m(t)/m(0)$, as a function of time. At the initial instant, $t = 0$, the particles are either uniformly distributed in the lagoon (solid curve) or concentrated near the bottom at points A, B, and C (dashed curves), the location of which is indicated in Figure 1.

(Figure 2), which is due to the residence time being much shorter than the turnover time in the vicinity of C (Figure 1) (see [5] for a detailed discussion of the spatial distribution of the residence time).

The latter numerial experiment may be generalised easily. At $t = 0$, a large number of RT parcels are released in the center of each grid box adjacent to the bottom of the lagoon. The fate of these parcels is simulated by means of the the three-dimensional model. The temporal evolution of the tracer mass originating from each grid box is calculated and compared with expression (3), allowing us to define pointwise relative error measures:

$$(\varepsilon_1, \varepsilon_2) = \frac{1}{\theta} \int_0^\infty \left[\frac{m(t)}{m(0)} - \exp\left(\frac{-t}{\theta}\right), \left| \frac{m(t)}{m(0)} - \exp\left(\frac{-t}{\theta}\right) \right| \right] \exp\left(\frac{-t}{\theta}\right) dt. \tag{4}$$

In most of the lagoon, $|\varepsilon_1| \approx \varepsilon_2$, so that it is sufficient to analyse ε_1 (Figure 3).

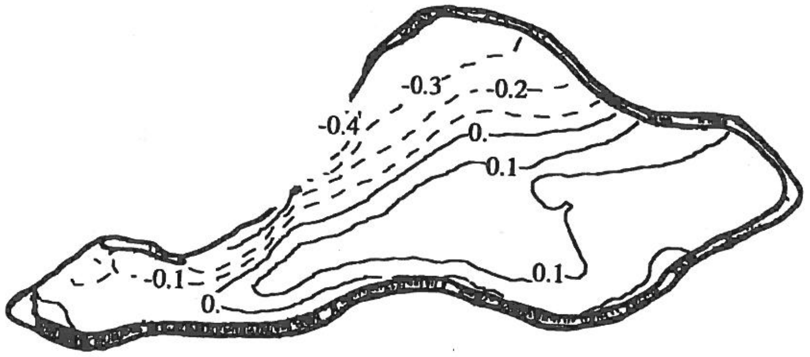

Figure 3. Isolines of ε_1, as defined in (4), with a contour interval of 0.1.

The lagoon average of $|\varepsilon_1|$ is equal to 0.12, i.e., a value sufficiently small to lend considerable credence to the simple model developed above. In other words, the relative error affecting the predictions of the simple model—with respect to the three-dimensional model—is expected to be of the order of 10% on average, which is probably acceptable for many applications.

Three-dimensional numerical experiments, the discussion of which is outside the scope of the present note, suggest that an improved estimate of the temporal evolution of the RT mass originating from a pointwise source may be obtained by replacing, in expression (3), the turnover time, θ, with the residence time at the location of the source, τ. If so, for RT parcels released at a given point at the initial time, the RT mass present in the lagoon is approximately $m(0)\exp(-t/\tau)$, so that ε_1 is close to $(\tau-\theta)/[2(\tau+\theta)]$. It is readily seen that, over a large fraction of the lagoon area, $|\tau-\theta|/\theta \ll 1$, which implies that $\varepsilon_1 \approx (\tau-\theta)/(4\theta)$. This is the reason why the isolines of ε_1 exhibit patterns somewhat similar to those of the iso-τ. By virtue of the definition of the turnover time, the lagoon average of $(\tau-\theta)(4\theta)$ is zero, so that it is no surprise that, in the numerical experiment discussed above, the absolute value of the lagoon-average of ε_1 $(= 0.015)$ is much smaller than the lagoon average of the absolute value of ε_1 $(= 0.12)$.

The relative difference between the residence time and the turnover time, $(\tau-\theta)/\theta$, is 4 times larger than the asymptotic value of the relative error measure defined above, which is why the simplified model is deemed to work rather well although the lagoon is seen to be far from well-mixed.

To conclude, it is believed that the zero-dimensional modelling strategy is a valid one, especially for problems in which a simple, pragmatic approach is sufficient.

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