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HYDRODYNAMIC MODELLING OF THE NORTH SEA

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ABSTRACT

In this paper, the recent developments of our hydrodynamic numerical simulation model for the North Sea region, will be presented. The model was established based on the finite difference method (FDM) solution of the depth integrated two dimensional shallow water equations. Instead of using the conventional semi-implicit ADI scheme, a splitting-up in time and space has been used to solve the governing partial differential equations. The falsified alternating direction implicit (FADI) scheme will be introduced briefly. The FADI model has been applied to different parts of the North Sea, in particular the north-west European continental shelf area, the west Belgian coast area and the area of the southern North Sea and the English Channel. The models of these regions will be discussed in this paper.

1 INTRODUCTION

Nobody can deny the importance of the North Sea and its coastal regions, e.g. for navigation, for fishing, and for the exploration of natural resources, etc. Beside these, it has long been and is still considered as a dumping place for different kind of waste from human activities, but its absorbtion capacity is limited. Hydrodynamic numerical (HN) models help to understand and are able to simulate the oceanographical processes, they can thus be applied as a decision making tool for various human activities which alter the natural environment of the seas. Using HN-models to predict storm surges makes it possible to take actions to protect human lives and properties in advance.

From a scientific point of view, the simulation of hydrodynamic phenomena with numerical models is a challenge. The quick developement of computers and numerical techniques enables to increase the accuracy and the efficiency of the calculations and thus improves the quality and the reliability of the models.

The purposes of this paper are : (1) to introduce the FADI (falsified alternating direction implicit) model; (2) to review some of the real world applications of the North Sea region and compare the model results with measurements; (3) to conclude some numerical modelling experiences from these applications.

2 THE MODEL FORMULATION

2.1 The governing equation

The conventional depth integrated two dimensional shallow water equations have been used to describe the flow motions in the horizontal plane in the following form:

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = fv - g\frac{\partial z}{\partial x} + \frac{1}{\rho H}\left(\tau_x^s - \tau_x^b\right) \qquad (1)$$

$$\frac{\partial v}{\partial t} + u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} = -fu - g\frac{\partial z}{\partial x} + \frac{1}{\rho H}\left(\tau_y^s - \tau_y^b\right) \qquad (2)$$

and the continuity

$$\frac{\partial z}{\partial t} + \frac{\partial (Hu)}{\partial x} + \frac{\partial (Hv)}{\partial y} = 0 \qquad (3)$$

where the notations are

t : time
x,y : spatial coordinates
u,v : depth integrated horizontal velocity components in x and y direction, respectively
z : the free surface elavation above datum (mean sea level)
h : the bathymetry
H : (= h + Z) the total water depth
g : the acceleration due to gravity
f : the Coriolis parameter
C_D : the drag coefficient at water surface

where the stress at the water surface due to wind forcing can be expressed as:

$$\tau^{s}_{x,y} = C_D \rho_a w^2 \sin \Theta \tag{4}$$

and the bottom friction is formulated as:

$$\tau^{b}_{x,y} = \frac{\rho g}{C^2} u(u^2 + v^2)^{1/2} \tag{5}$$

2.2 The FADI model

The conventional ADI (alternating direction implicit) scheme has been widely applied after Leendertse (1967). The implicit solution algorithm is not restricted by the CFL stability criterion, therefore, the scheme is allowed to use bigger time-step sizes and can thus gain computational efficiency. Different modifications (e.g. Stelling, 1980) have been introduced in order to stabilize the scheme when the nonlinear terms are included in the equations. Although some of these modifications are suitable to enlarge the time-step size, the physical nature of time series forcing terms may not be disregard, e.g. the tidal forcing is a non linear function of time which means that too big a time-step decreases the accuracy of the solution except when iteration is used.

The fractional step method (Yanenko, 1971) has been applied by the conventional ADI scheme, where the difference equations are split in the computations by means of a mixing of the implicit and the explicit calculations in the time and spatial domain. The complete set of solutions consists of six equations in two time levels. The FADI algorithm has been developed with the emphasis not on expanding the computational time-step size but on the simplification of the difference form of the differential equations. The difference form of the continuity equation is further split in the following form:

$$\frac{1}{2}\frac{\partial z}{\partial t} + \frac{\partial (Hu)}{\partial x} = 0, \tag{6a}$$

and

$$\frac{1}{2}\frac{\partial z}{\partial t} + \frac{\partial (Hv)}{\partial y} = 0, \tag{6b}$$

The individual equations of this split form are not consistent with the ordinary differential equation. A falsified solution is proposed by integrating one of the split equations together with one of the momentum equations in the time domain. By means of this falsified process a set of quasi-one dimensional equations are thus solved in each time level which consists of only the implicit solution of the two equations for the x direction. The difference equations are described on a C-grid (see Fig.1) in the following forms:

$$\frac{z^{n+1}_{i,j} - z^{n-1}_{i,j}}{2\Delta T} + \frac{Hu^{n+1}_{i+1/2,j} - Hu^{n+1}_{i-1/2,j}}{\Delta x} = 0 \tag{7}$$

and

$$\frac{u^{n+1}_{i+1/2,j} - u^{n-1}_{i+1/2,j}}{2\Delta T} + g\frac{z^{n+1}_{i+1,j} - z^{n+1}_{i,j}}{\Delta x} - f\bar{v} + Ru^{n+1}_{i+1/2,j} + \frac{1}{\rho}W_{x_{i+1/2,j}} = 0 \tag{8}$$

where

$$\bar{v} = \frac{1}{4}(v^{n}_{i,j-1/2} + v^{n}_{i+1,j-1/2} + v^{n}_{i,j+1/2} + v^{n}_{i+1,j+1/2}) \tag{9}$$

and

$$R = g\frac{\sqrt{(u^{n-1}_{i+1/2,j})^2 + \bar{v}^2}}{C^2 H^{n}_{i+1/2,j}} \tag{10}$$

The difference equations for the next time-step in the y direction for the variables v and z are treated similarly. The integral solutions of these two directions are consistent with what has been investigated by Yanenko (1971). This algorithm has been applied by the FADI model. The complete solution consists of four variables in two time levels from four equations. Because the variables have been treated in central difference form, both in time and space, the second order accuracy of the solution can be obtained without using iterations. Computational efficiency is gained from the falsification process. This saves half of the computer time, without the computation of advective terms, compared with conventional ADI schemes.

3 APLICATIONS

The model has been validated by comparing the results with the analytical solution of a constant wind forcing in a closed basin (YU et al., 1988a). By doing so only simplified linear problems can be treated. Tidal flows in shallow waters are nonlinear. The model has been verified against measurements because analytical solutions are not available. Three examples which are situated in the North Sea area and which are applying the FADI scheme are reviewed hereafter.

3.1 The English Channel and the southern North Sea

In 1988 a data base containing bathymetry, open boundary forcing and measurements in the area covering the southern part of the North Sea and the English Channel was made available (Werner and Lynch, 1988). This was the test-case data set for the "II Tidal Flow Forum" which was held on June, 1988, at MIT during the VII International Conference on Computational Method in Water Resources.

As a test-case, the mesh size and some parameters were predefined in order to make intercomparisons between different models. The results of the FADI model using the given parameters and some sensitivity analysis for different dominant parameters were discussed by YU et al. (1989). The model area and the co-tidal and co-range chart of the semi-diurnal tide are shown in figure 2. Some remarks are summarized in the following:

(1) The model results are sensitive to the geographical approximations of the land-sea boundaries in the computational grid. For example, in the computed tidal elevation at the station St. Malo a phase lag occurres (Fig.3) when the mesh includes the Channel Islands in the computations. The phase lag disappeares (Fig.3) when the islands are neglected in the computations. The grid size of 9 kilometres square is

too big to give an appropriate resolution of the geometry and bathymetry in this region.

(2) Proper use of friction factors help to improve the results. An unique Chézy coefficient for the whole model area is suitable for areas with a more or less flat bottom. For areas with a complex bathymetry, such as the test-case area, the use of depth dependent friction coefficient is strongly suggested, since friction itself is depth dependent. A distinct improvement in the results of the station Zeebrugge, see figure 4, can be obtained when Manning's friction law is used instead of Chézy's law. This is only an example which shows the sensitivity of the friction on the model results, it can however not be considered as a complete calibration process as was performed by Gerritsen and Bijlsma (1988).

(3) Special care must be taken, not only to the geographical position of the boundary but also to the number of harmonic constituents which will be used to reconstruct the tidal forcing at the open boundary in coastal seas. In the test-case, 11 constituents (01, K1, M2, S2, N2, K2, M4, MS4, MN4, M6 and 2MS6) are specified to reconstruct the time series of the vertical tide along the open boundaries (Fig.2). These 11 constituents are insufficient to represent the complete nature of the tidal forcing in this area. The positions of the open boundaries are not adequate because they are too close to the amphidromic systems (see Fig.2), and because the northern boundary is situated in an area with many sand banks and gullies. In such an environment the tidal physics cannot be represented by only 11 constituents. The non-linear features and the complexity of the tides in the English Channel have been widely discussed. Le Provost (1981) indicates that 29 components should be introduced in the open boundary forcing, in order to reach a RMS error of less than twenty centimeter at Le Havre. Choosing a proper position with sufficient constituents in such an area seems to us to be rather abritrary. Shifting the open boundary to the continental shelf edge where only the astronomical tides are active, is probably a more adequate solution.

3.2 The north-west European continental shelf model

The shallowness of the bathymetry and the complexity of the tides in the North Sea make it difficult to set up a good open boundary condition for a numerical model, therefore the model boundaries have been extended to the continental shelf edges (see Fig.5). Only the astronomical tides have been considered to reconstruct the boundary forcing. Six tidal constituents (O1, K1, M2, N2, S2, K2) have been included in the boundary forcing because they contribute the most to the tidal signal in this part of ocean. The details have been discussed by Yu et al. (1988b). The M2 co-tidal and co-amplitude chart is presented in figure 5. This is the uncalibrated result of the continental shelf model. Computed tidal levels in some stations are compared, in figure 6, to the same reference levels as in the test-case. The sensitivity of the model to surface stresses, i.e. a constant north-west wind, is presented in figure 7 for the station Zeebrugge.

3.3 The west Belgian coast model

In order to understand the tidal currents along the west Belgian coast the continental shelf model has been used to generate the open boundaries for the finer grid model by means of grid refinement techniques. The finest grid size is 300 metres square which allows the model to resolve the complex bathymetry of this area due to the existance of many sand banks and gullies. The detailed procedure and the step-wise calibrated results have been discussed by Yu et al. (1988c). In figure 8 the computed tidal currents of some points are presented and are compared with measurements.

4 CONCLUSIONS

The FADI model and its applications of hydrodynamic modelling of the North Sea have been presented. From these applications some conclusions can be made:

- the FADI model is suitable for 2-D tidal flow simulations with stable results.
- the FADI model is more efficient when compared with conventional ADI models.

- Friction is an important parameter for numerical simulation in the North Sea area. Depth dependent friction laws are more effective than constant ones.
- Shallow water effects are important in the simulation and for setting up the boundary forcing in coastal seas.
- Shifting the open boundary to the continental shelf edges reduces the tidal forcing to the astronomical constituents only.
- Calibration is (probably) the most important procedure to make the model suitable for forcasting purposes.

5 ACKNOWLEDGEMENTS

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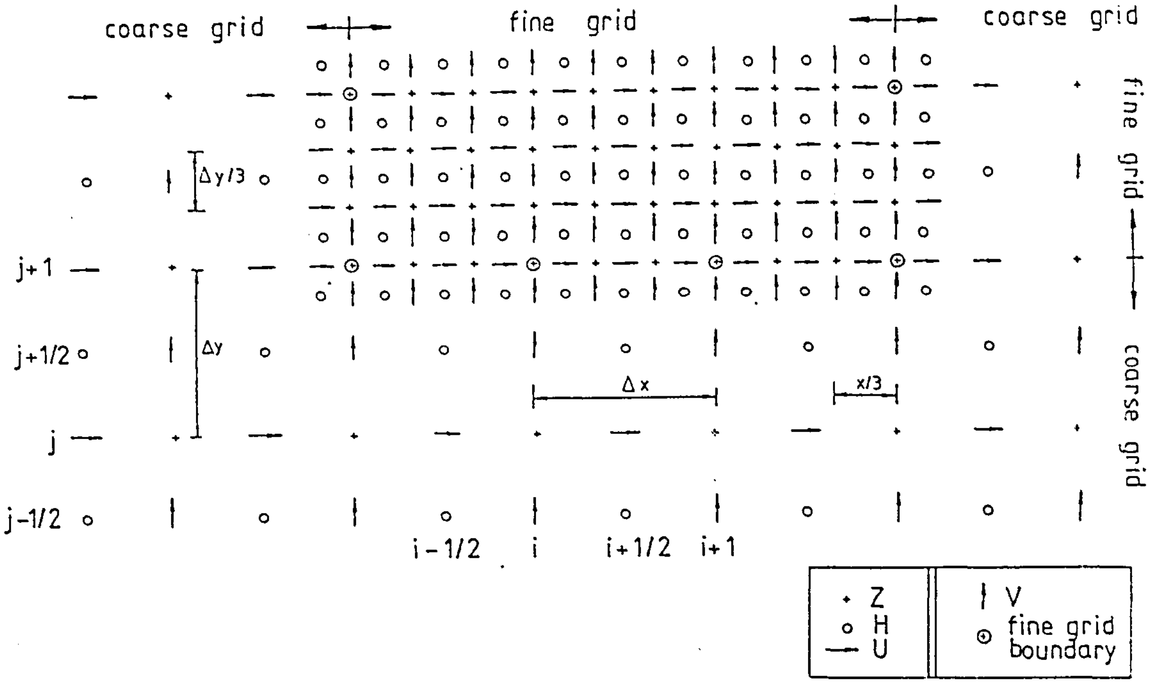

Fig. 1 : Finite Difference Grid (C-Grid)

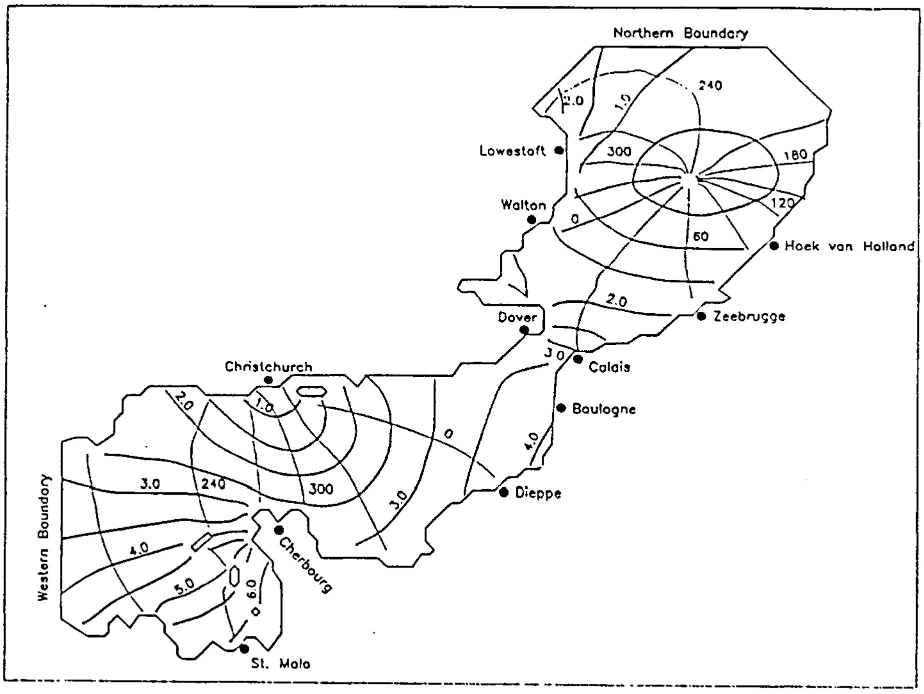

Fig. 2 : Co-tidal and Co-range map of the semi-diurnal tidal constituents in the English Channel and the southern North Sea

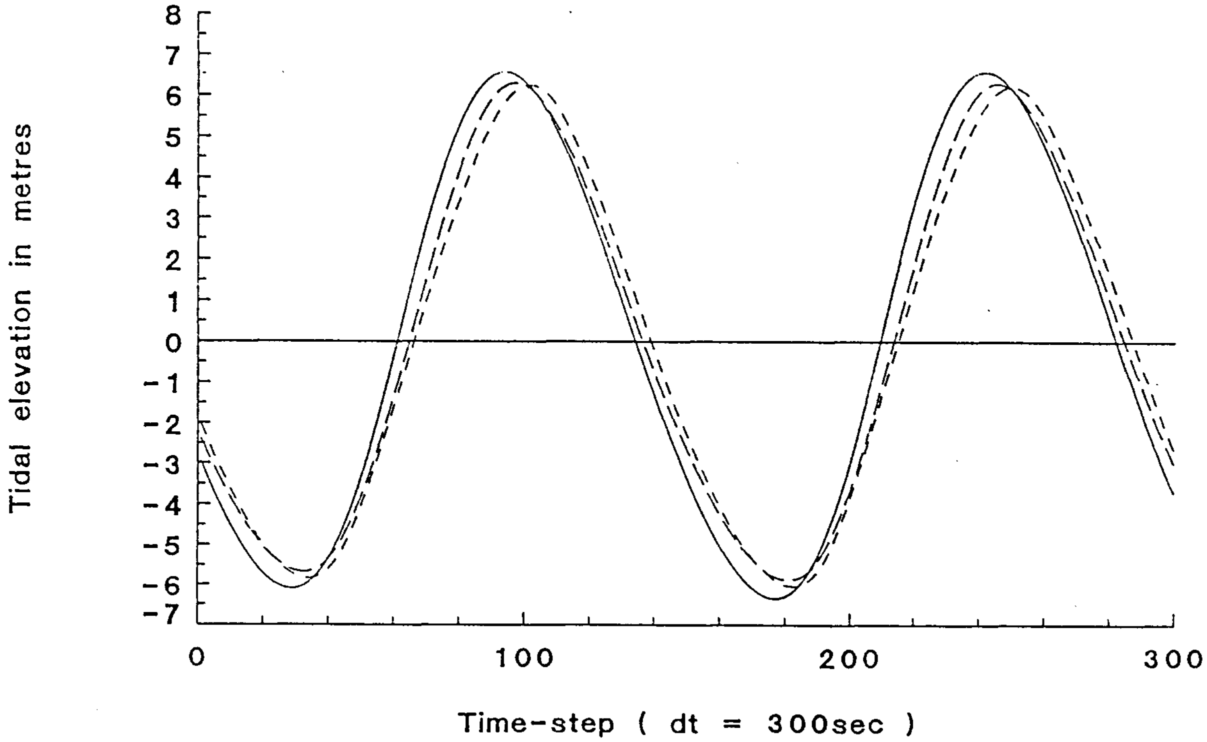

Fig. 3 : Computed tidal elevations at St. Malo compared with the reference level. Effect of the Channel Islands (——) reference, (----) with islands (– – – –) without islands

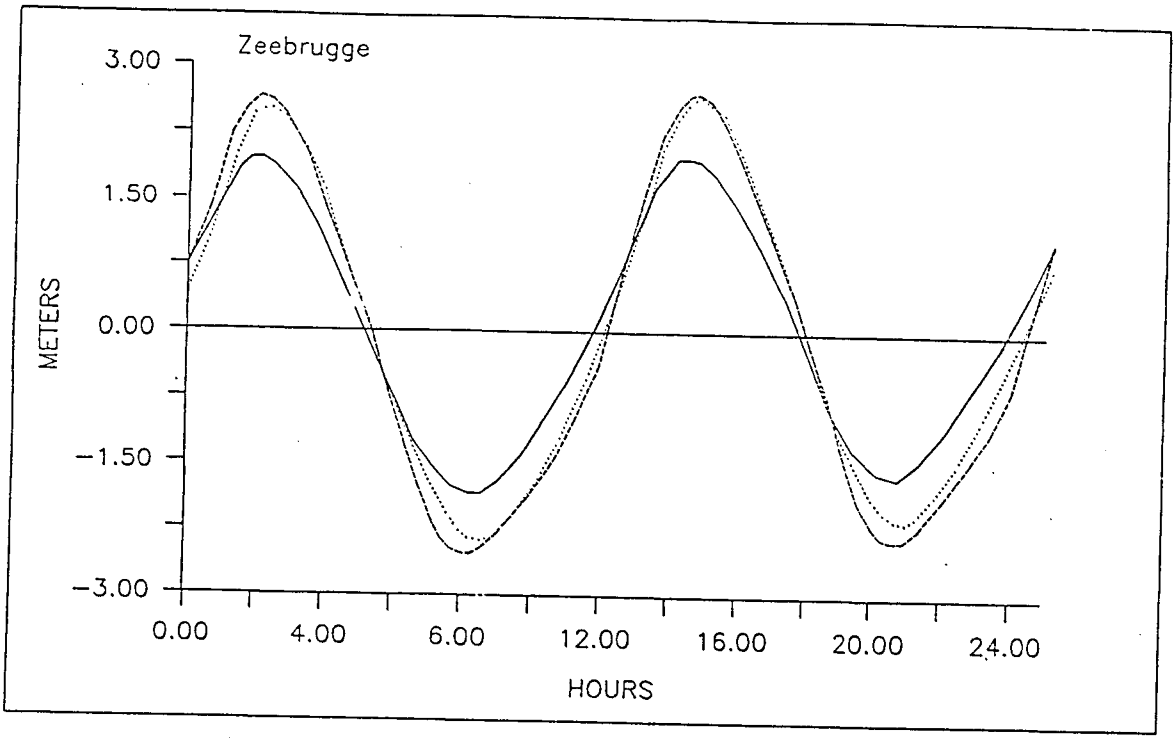

Fig. 4 : Computed tidal elevations at Zeebrugge compared with the reference level (----) reference, (.....) Manning friction law (n = 0.02), (——) Chézy friction law (C = 65).

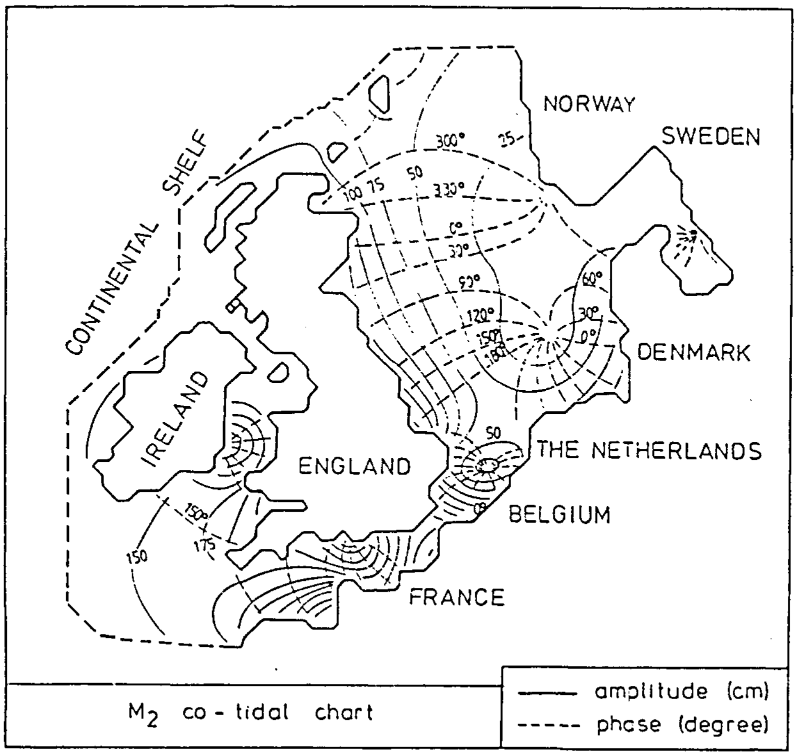

Fig. 5 : M2 co-tidal and co-range chart of the Continental shelf model

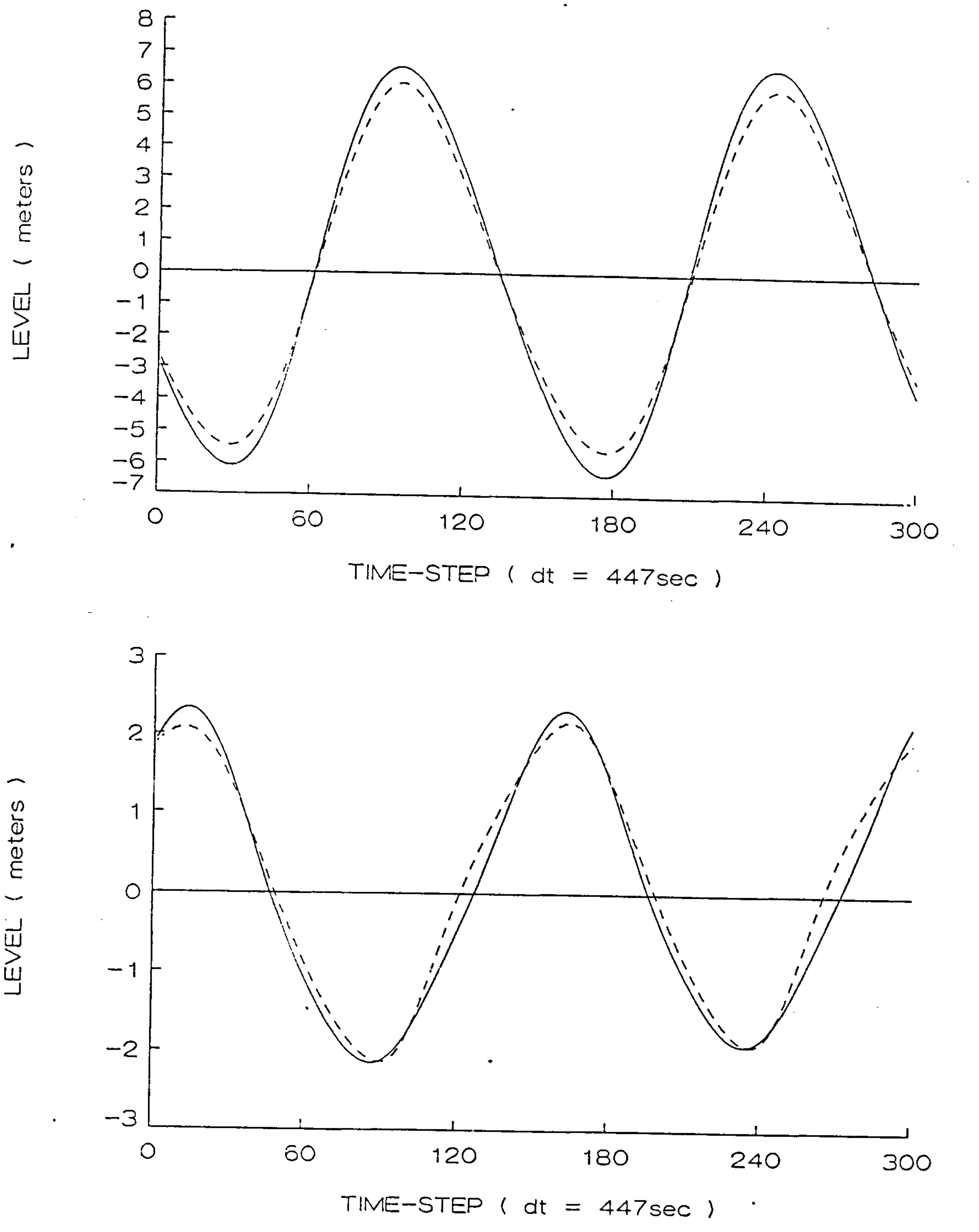

Fig. 6 : Tidal levels at St. Malo (a) and Walton (b) computed with the Continental Shelf Model (----) and compared with the reference level (——) of the Tidal Forum

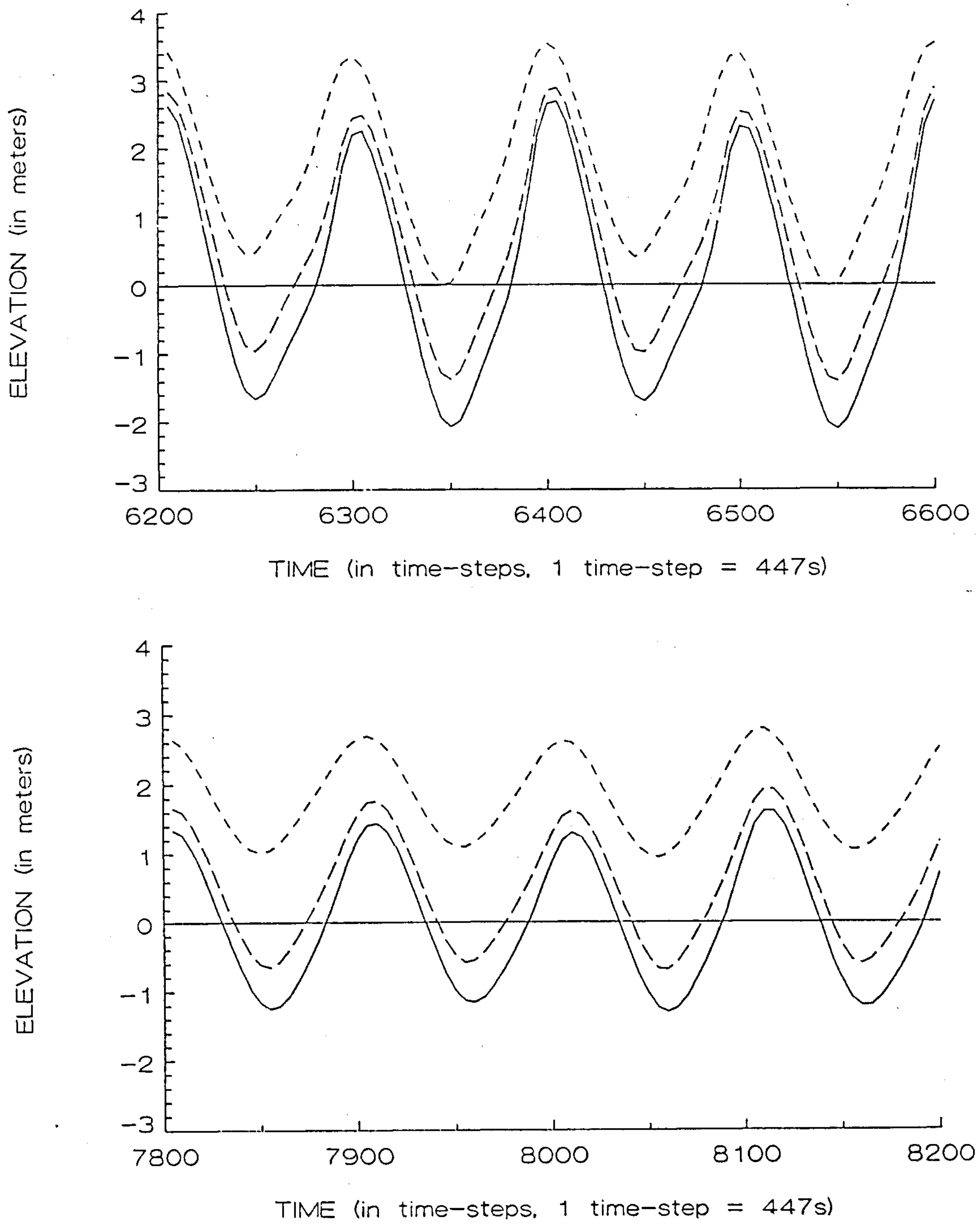

Fig. 7 : Storm surge at Zeebrugge Computed with the Continental Shelf Model (——) no wind, (----) 30 m/s NW, (— — —) 15 m/s NW (a) spring tide and (b) neap tide

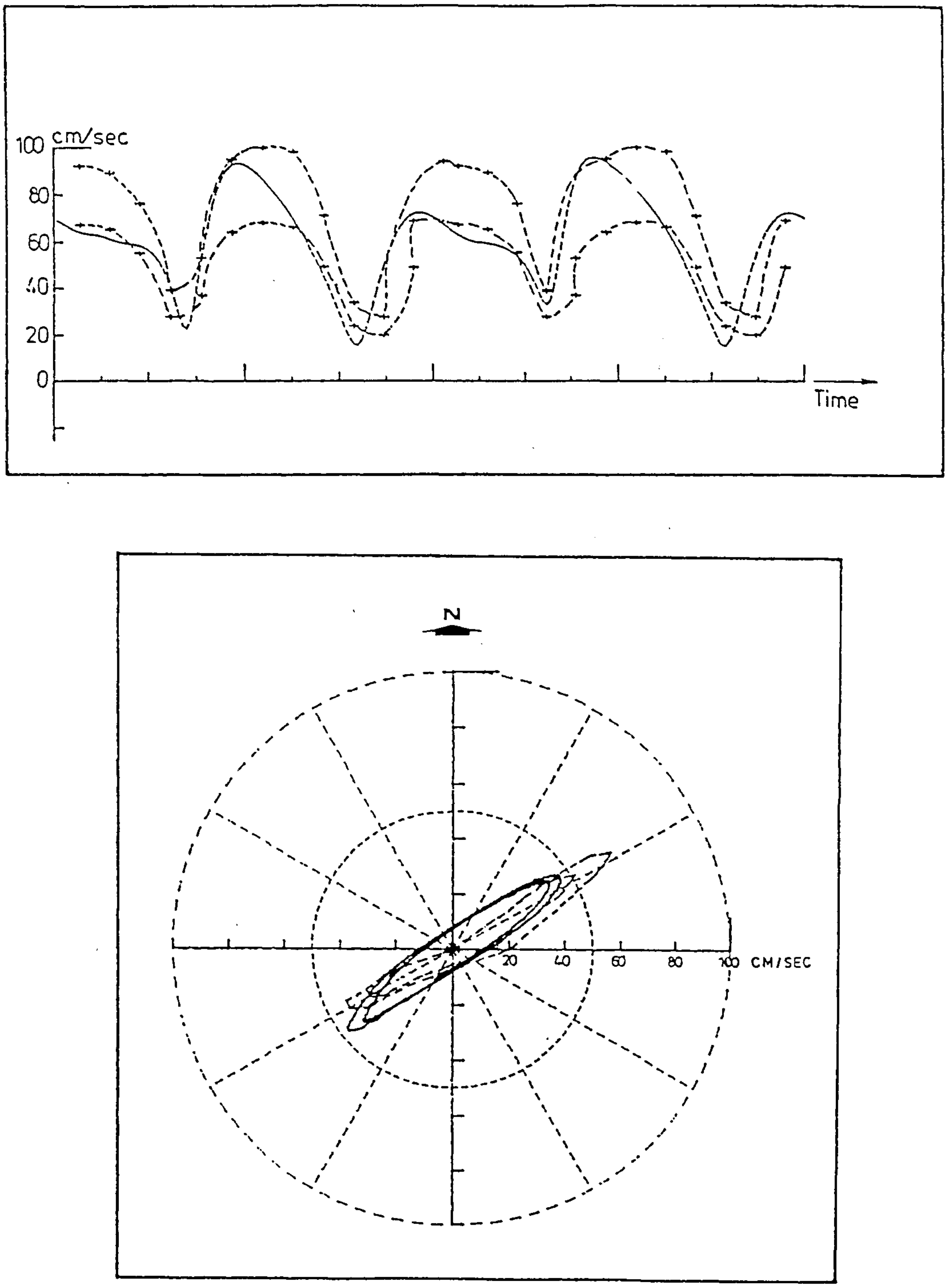

Fig. 8 : (a) Comparison between computed (----) and measured (--+--) velocities (top and bottom velocities) at the Outer Ratel Bank during spring tide.
(b) Comparison between computed (——) and measured (----) velocities outside Nieuwpoort in the Westdiep.