WAVE MODELING AT THE BELGIAN COAST BY MEANS OF THE HYPAS * MODEL

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ABSTRACT

A summary is presented of the activities in the framework of a collaboration project between MUMM and the Ministry of Public Works, concerning the set up of a wave model for the Belgian coast. A detailed description of the model is given together with a presentation of some results of the North Sea application of the model.

INTRODUCTION

In 1987, the Ministry of Public Works (Administration of Waterways, Coastal Service) charged the Management Unit of Mathematical Models of the North Sea (MUMM) with the set up of an operational routine forecasting of the surface wave spectra along the navigation routes towards the Belgian sea harbours. This forecasting will be performed by means of a mathematical model, which integrates the transport equation for surface waves energy in the region of interest (in this case : the North Sea and the English Channel, see fig. 1). The model requires the (predicted) time history of the surface windfield as an input item, to estimate the atmospheric input to the local wave energy. A coupling with meteorological predictions will be necessary. Moreover, the quality of the wave predictions will depend on the quality of the meteorological predictions. In this

* The HYPAS model is a product of GKSS Forschungszentrum Geesthacht GmbH, Institut für Physik (Dr. W. Rosenthal, Dr. H. Günther).

** The author was a collaborator with MUMM from January till November 1988.

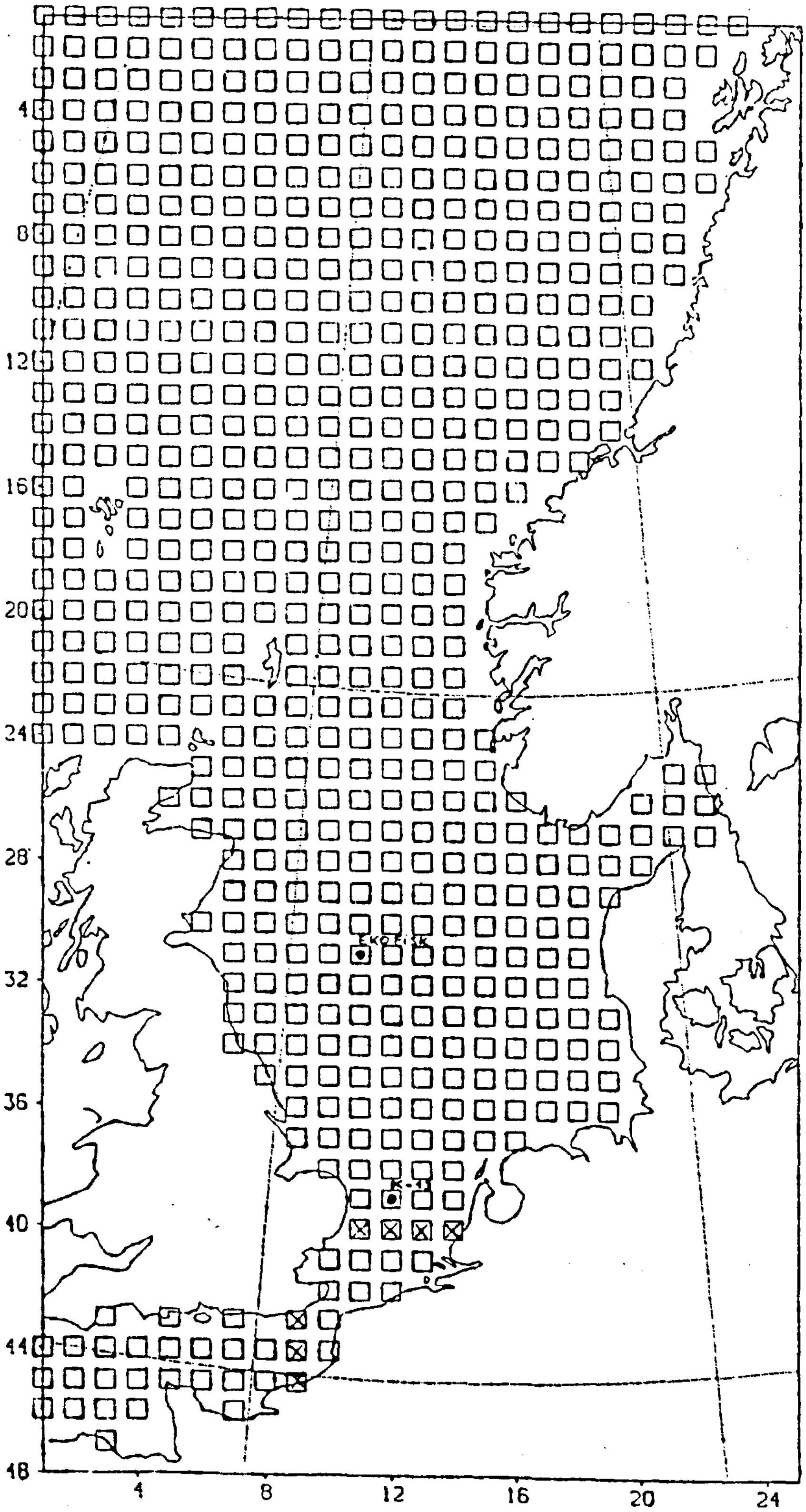

Fig. 1. Model area and coarse grid definition.

paper, only the wave prediction is discussed.

In the first chapter we will introduce the quantities, involved in the model computations. The next chapters describe the method of integration of the wave energy balance equation as well as the elements, which are necessary to perform this integration in a certain region. These elements, in fact, constitute the mathematical model. Different approaches of wave modeling are possible. This paper only deals with the options taken for the construction of the model, which is used for the present research : the HYPAS model of GKSS Forschungszentrum Geesthacht GmbH (West Germany). This model was retained after a study of several wave models by MUMM, ref. Phlips, 1987 and Adam et al., 1988. Finally, some results are shown of the North Sea application, worked out in the framework of the present research.

1 SOME ELEMENTS OF WAVE HYDRAULICS

1.1 Basic laws and physical properties

Surface waves may be described by the vertical displacement of the water surface relative to the averaged water surface (fig. 2). The seabottom depth is denoted h. Consider a simple plane sinusoidal wave, propagating in a certain horizontal (x-)direction :

$$\eta(x,t) = a \cdot cos(kx - \omega t) \tag{1}$$

where x = horizontal spatial coordinate; t =time; a =amplitude; k =the wave number and ω = angular frequency.

Following relations hold :

$$k = 2\pi/L \tag{2.a}$$

$$\omega = 2\pi f = 2\pi/T \tag{2.b}$$

where L = wave length; f = frequency and T = period of the wave. The wave height H is defined as :

$$H = 2a \tag{3}$$

The disturbance of the sea surface, caused by the wave, involves a certain current distribution which can be described by the basic hydrodynamical equations : the equation of conservation of mass and the equation of conservation of momentum, both with the necessary boundary conditions. If irrotational (potential) flow is assumed, together with incompressibility of the water and small displacements compared with the waterdepth h (which should be nearly constant), the dispersion relation for surface gravity waves can be derived from the basic hydrodynamical equations (e.g. Kinsman, 1965, pp. 117-125) :

$$\omega^2 = gk \cdot tanh(kh) \tag{4}$$

where g = acceleration of gravity.
In (4), no capillary effects are taken in account.

Furthermore we define the phase velocity c :

$$c = \frac{L}{T} = \frac{\omega}{k} \tag{5}$$

With (4) we have :

$$c^2 = (g/k)tanh(kh) \tag{6}$$

From (6) it can be seen that the propagation velocity c of a wave is depending on its wave number k and thus on its wave length L. Since the frequency can be derived from the wave length by (4), we can also say that the propagation velocity depends on the frequency. Moreover it appears that the propagation velocity of a wave is depending on the local waterdepth h. This will cause a refraction effect on waves running into shoaling water, according to Snells laws.

The propagation velocity v_g of a group of waves with almost the same frequency ω (the so-called "group velocity") is given by

$$v_g = \frac{d\omega}{dk} = n \cdot \frac{\omega}{k} = n \cdot c \tag{7.a}$$

where

$$n = \frac{1}{2}(1 + \frac{2kh}{sinh(2kh)}) \tag{7.b}$$

The relations (7) can also be written as (see (4)) :

$$v_g = \frac{g}{2\omega}[tanh(kh) + kh \cdot (1 - tanh^2(kh))] \tag{8}$$

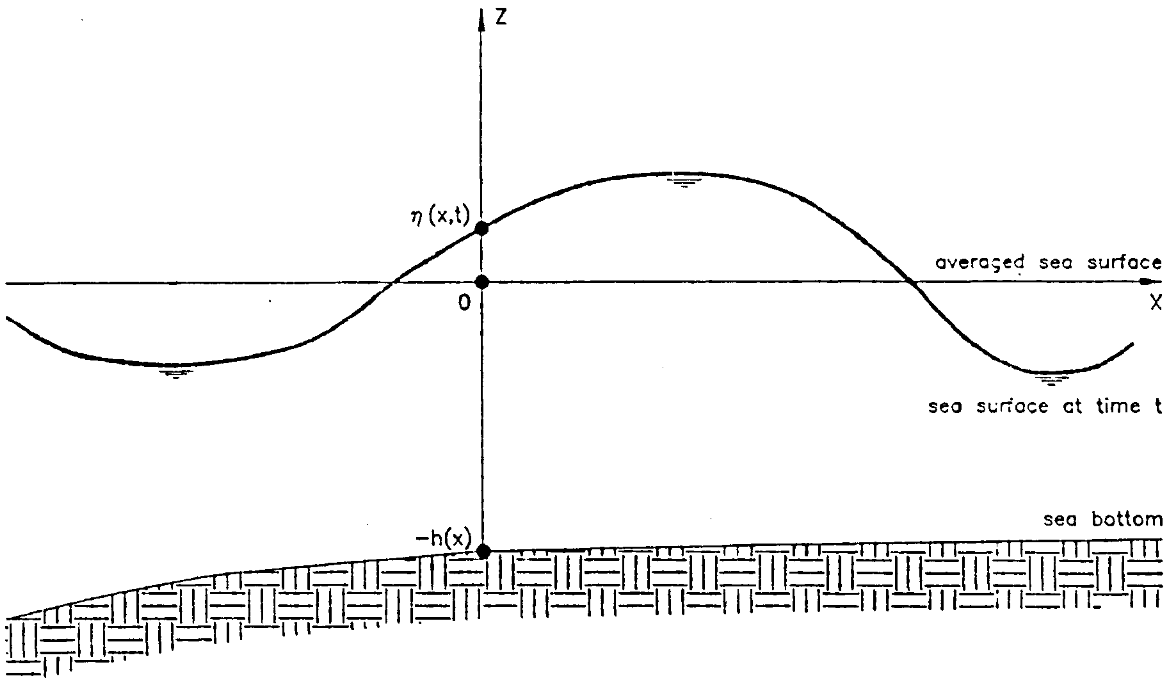

Fig. 2. Introduction of basic notations.

The dispersion relation (4), which is the result of the linearized problem of surface gravity waves, will be assumed to hold for arbitrary (non-sinusoidal and non-plane) waves, which can be considered as a Fourier composition of simple sinusoidal waves, with different frequencies (a "spectrum" of waves). Still some non-lineair effects should be considered.

(a) A spectral component with a certain frequency cannot be considered as an independent wave : other components, with different frequencies, can contribute to this component through the so-called "non-linear wave-wave interactions". Low frequency components act more or less uncoupled, however.

(b) Not every amplitude a in (1) is possible : when the inclination of the water surface increases continuously, it will reach a limit value at which "wave breaking" will occur.

Finally we introduce the notions "swell" and "sea". The surface gravity waves under consideration (period less than 30 s) are generated by wind blowing over the sea surface. Waves under active generation by local wind are called sea or wind waves. Contrary, swell is defined as waves originated from a distant area and being no longer under the influence of windforcing.

1.2 Energy of surface waves; energy spectra

Since surface waves involve a vertical displacement of the sea surface, some potential energy is contained in these waves. The potential energy ϵ_p equals the work necessary to distort a horizontal sea surface (area A) into the wave profile:

$$\epsilon_p = \frac{1}{2}\rho g \int\int_A \eta^2 \, dA \tag{9}$$

As we mentioned in the preceding paragraph, surface waves also entail water motion (currents) and thus kinetic energy. With the assumptions of the preceding paragraph, which led to the dispersion relation (4), it can be shown (see Kinsman, 1965; pp. 145-148) that there is an equipartition of the total energy ϵ between potential and kinetic energy of a sinusoidal progressive wave, thus :

$$\epsilon = 2\epsilon_p = \rho g \int\int_A \eta^2 \, dA \tag{10}$$

The quantity which will be used for wave predictions is the total energy per unit weight (of a layer of unit thickness), the (spatial) energy density E :

$$E = \frac{\epsilon}{\rho g A} \tag{11}$$

With (10) we have :

$$E = \frac{1}{A}\int\int_A \eta^2\, dA = \overline{\eta^2} \tag{12}$$

Apparently, the average of the squared surface displacement over a certain area is a measure for the (local) wave energy (density). If the sinusoidal progressive wave (1) is inserted in (12), and A contains an integer multiple of wave lengths L, we get :

$$E = \frac{a^2}{2} = \frac{H^2}{8} \tag{13}$$

In practice, a sea can be regarded as composed of Fourier components with different frequencies (wave numbers), different directions and randomly distributed faseshifts. This is what we call a "random sea". We define the following spectra of the energy density E, used to describe a random sea :

$$F(\vec{k}) = F(k_x, k_y) \quad ; \quad \int\int F(\vec{k})\, dk_x\, dk_y = E \tag{14}$$

$$E(f,\theta) \quad ; \quad \int_o^{2\pi} d\theta \int_o^{\infty} E(f,\theta)\, df = E \tag{15}$$

$$E(f) \quad ; \quad \int_o^{\infty} E(f)\, df = E \tag{16.a}$$

$$E(\theta) \quad ; \quad \int_o^{2\pi} E(\theta)\, d\theta = E \tag{16.b}$$

where k_x, k_y = components of the wave number vector (per definition parallel with the direction of wave propagation); θ = angular coordinate, indicating the direction of propagation (θ=0 being the y-axis). All spectra depend on space and time. Following relations hold :

$$k_x = k \cdot sin\theta \qquad ; \qquad k_y = k \cdot cos\theta \tag{17}$$

$$E(f,\theta) = F(\vec{k}) \cdot k |\frac{dk}{df}| \tag{18}$$

$$E(f) = \int_o^{2\pi} E(f,\theta) d\theta \tag{19.a}$$

$$E(\theta) = \int_o^{\infty} E(f,\theta) df \tag{19.b}$$

The energy-density spectrum $F(\vec{k})$, or $F(\vec{k}, x, y, t)$, in k-space, fulfills the energy transport equation :

$$\frac{\partial F}{\partial t} + \frac{\partial \vec{x}}{\partial t} \cdot \vec{\nabla} F + \frac{\partial \vec{k}}{\partial t} \cdot \vec{\nabla}_k F = S(\vec{k}, \vec{x}, t) \tag{20}$$

where in $\vec{x} = (x, y)$; $\vec{\nabla}_k$ = the gradient operator in k-space = $(\frac{\partial}{\partial k_x}, \frac{\partial}{\partial k_y})$; S= the source function, taking in account generation and dissipation processes.

The third term in the left hand side of (20) takes in account refraction effects (changing of wave length and/or changing of propagation direction).

For wave groups, following relations hold :

$$\frac{\partial \vec{x}}{\partial t} = \vec{\nabla}_k \omega = \frac{d\omega}{dk} \cdot \frac{\vec{k}}{k} = \vec{v}_g \tag{21.a}$$

$$\frac{\partial \vec{k}}{\partial t} = -\vec{\nabla}\omega = -\frac{d\omega}{dh}\vec{\nabla}h \tag{21.b}$$

It appears that the speed with which energy propagates equals the group velocity.

1.3 The JONSWAP spectrum

From practical experience, it is known that energy-density spectra of wind waves (generated by local winds) have more or less similar shapes which can be parametrized, e.g. according to the JONSWAP study (Hasselmann et al., 1973):

$$E(f) = \frac{\alpha g^2}{(2\pi)^4 f^5} e^{-\frac{5}{4}\left(\frac{f}{f_m}\right)^{-4}} \gamma^{e^{-\frac{\left(\frac{f}{f_m}-1\right)^2}{2\sigma^2}}}$$

$$\sigma = \sigma_a \quad (f \leq f_m)$$

$$\sigma = \sigma_b \quad (f \geq f_m) \tag{22}$$

A full 2D-spectrum can now be obtained, multiplying by a directional distribution function, e.g.:

$$E(f,\theta) = \frac{2}{\pi} cos^2(\theta - \theta_m) \cdot E(f) \quad ; \quad |\, \theta - \theta_m \,| \leq \frac{\pi}{2}$$

$$=0 \qquad\qquad ; \text{ otherwise}$$

The parameters which are introduced are :

f_m =the peak frequency of the integrated spectrum $E(f)$

α =a parameter, related to the total energy density E ("Phillips'parameter")

γ =the peak enhancement factor (or "overshoot"factor)

σ_a, σ_b=spread parameters of the peak enhancement

θ_m =mean wave-propagation direction.

For $\gamma = 1$ the JONSWAP spectrum (22) is called a Pierson-Moskowitz (PM) spectrum, which describes a fully developed sea state (cfr.infra). In this case, the peak frequency f_m tends towards the so called Pierson-Moskowitz frequency f_{PM} :

$$f_{PM} = \frac{0,13 \cdot g}{W_{10}^{\parallel}} \tag{24}$$

where $W_{10}^{\parallel}$ = the component of the local windspeed at 10 m height above the sea surface, which is parallel to the mean wave-propagation direction. The total energy of a Pierson-Moskowitz spectrum ($\gamma = 1$) above a certain frequency f_o can be expressed analytically as :

$$E_{PM}(f \geq f_o) = \frac{\alpha g^2}{5(2\pi f_m)^4} \cdot [1 - e^{-\frac{5}{4}(\frac{f_o}{f_m})^{-4}}] \tag{25.a}$$

For $f_o = 0$ we find the total energy content of this spectrum :

$$E_{PM}^{tot} = \frac{\alpha g^2}{5(2\pi f_m)^4} \tag{25.b}$$

In the general case ($\gamma \neq 1$), the total energy is expressed as :

$$E_{\gamma}^{tot} = c(\gamma) \cdot E_{PM}^{tot} \tag{25.c}$$

where $c(\gamma)$ is a numerically determined function, satisfying the condition

$$c(1) = 1 \tag{25.d}$$

2 THE HYPAS MODEL EQUATIONS

2.1 Transformation from k-space to (f,θ)-space

The HYPAS model integrates the energy transport equation (20) for the two-dimensional energy density spectrum, converted to (f,θ)- space. From (2.b), (4), (7.a), (17) and (18) we have :

$$f = \frac{1}{2\pi}(gk \cdot tanh(kh))^{\frac{1}{2}} \tag{26.a}$$

$$\theta = tan^{-1}(k_x/k_y) \tag{26.b}$$

$$E(f,\theta,\vec{x},t) = F(\vec{k},\vec{x},t)\big|_{\vec{k}=\vec{k}(f,\theta)} \cdot \frac{2\pi k}{v_g}\Big|_{k=k(f)} \tag{27}$$

The dispersion relation (26.a) is inverted as follows. Define the dimensionless quantities ω_h and χ by the following equations :

$$\omega_h = 2\pi f\sqrt{\frac{h}{g}} \tag{28}$$

$$\chi \cdot tanh(\omega_h^2 \chi) = 1 \tag{29}$$

Note that for deep water, χ tends to 1. The quantities ω_h and χ are a function of f and h only. If (29) is solved for χ (for given f, h and thus ω_h), (26.a) becomes:

$$k = \chi\omega_h^2/h \qquad ; \qquad tanh(kh) = 1/\chi \tag{30}$$

The group velocity v_g, as given by (8), can be written :

$$v_g = \frac{g}{4\pi f} \cdot \frac{1 + \omega_h^2 \cdot (\chi^2 - 1)}{\chi} \tag{31}$$

Since $\vec{v}_g$ has the same direction as $\vec{k}$, we have (see(17)) :

$$v_{gx} = v_g \cdot sin\theta \quad ; \quad v_{gy} = v_g \cdot cos\theta \tag{32}$$

The transport equation (20) can now be written :

$$\frac{\partial F}{\partial t} + \vec{v}_g \cdot \vec{\nabla} F + (\frac{\partial \vec{k}}{\partial t} \cdot \vec{\nabla}_k \theta)\frac{\partial F}{\partial \theta} = S(f, \theta, \vec{x}, t) \tag{33}$$

where in (see (21.b) and (26.b)) :

$$\frac{\partial \vec{k}}{\partial t} \cdot \vec{\nabla}_k \theta = -\frac{d\omega}{dh}\vec{\nabla} h \cdot \frac{(cos\theta, -sin\theta)}{k} \tag{34}$$

with (4) and (30) we have

$$\frac{\partial \vec{k}}{\partial t} \cdot \vec{\nabla}_k \theta = -\pi f \cdot \frac{\chi^2 - 1}{\chi} \cdot (cos\theta \frac{\partial h}{\partial x} - sin\theta \frac{\partial h}{\partial y}) \tag{35}$$

Finally, we give the expression of the transformation factor between the spectra in k-space and (f,θ)-space (see(27)) :

$$\frac{2\pi k}{v_g}\Big|_{k=k(f)} = \frac{(2\pi)^4 f^3}{g^2} \cdot \frac{2\chi^2}{1+\omega_h^2(\chi^2-1)}$$

(36)

To derive the transport equation for $E(f,\theta,\vec{x},t)$, one inserts (27) in (33) noting that the transformation factor (36) doesn't depend on t and θ ; it depends however on $\vec{x}$.

2.2 The shallow water correction on the spectrum

In Bouws, 1985 is described how following expression is derived for the theoretical limit sea state spectrum in (f,θ)-space

$$E(f,\theta) = E(f,\theta)_{deep\quad water} \cdot \frac{1}{\chi^2} \cdot \frac{1}{1+\omega_h^2(\chi^2-1)}$$

(37)

$$E(f,\theta)_{deep\quad water} = \frac{2B\cdot g^2}{(2\pi)^4 f^5} \cdot \varphi(\theta)$$

(38)

and B a certain constant; $\varphi(\theta)$ being an arbitrary directional distribution function, which satisfies :

$$\int_{-\pi}^{\pi} \varphi(\theta)d\theta = 1$$

(39)

The limit expression for shallow water $(h \to o)$ of (37) is :

$$E(f,\theta)_{shallow\quad water} = \frac{B\cdot gh}{(2\pi)^2 f^3} \cdot \varphi(\theta)$$

(40)

Since the expressions (38) and (40) seem to fit experimental data for a fixed value of B, it is assumed that the relation (37) also holds for real spectra (i.e.: of growing seas). Therefore the HYPAS model spectrum (of the windwaves) is defined as :

$$E(f,\theta) = \frac{2}{\pi} cos^2(\theta - \theta_m) \cdot E(f) \cdot \frac{1}{\chi^2} \cdot \frac{1}{1 + \omega_h^2(\chi^2 - 1)}; |\theta - \theta_m| \leq \frac{\pi}{2}$$
$$= 0 \qquad ; \text{otherwise} \qquad (41)$$

where $E(f)$ is the JONSWAP spectrum (22).

Note again that χ and ω_h are functions of the frequency f and the waterdepth h (see (28) and (29)). It seems that by applying the transformation factor between spectra for finite waterdepth and for deep water, a certain dissipation is accomplished.

2.3 The hybrid concept of parametrization and discretization : wind sea and swell

The wave-energy spectrum which is computed by the HYPAS model, is assumed to be the superposition of a "wind sea" part and a "swell" part, each part being computed in a seperate way. The "wind sea" part of the spectrum is the parametrized function given by (41). Its introduction guarantees the maintenance of the shape of the spectrum over a wide range of fetches and wind conditions, which is a result of the dynamic balance between the energy input from the atmosphere and the transfer of energy due to non-linear wave-wave interactions. This dynamic balance does not hold for the lower frequency wave components because wave-wave interactions and atmospheric input are almost non-existent. Therefore one introduces a so-called "swell" part of the spectrum, described by discrete frequency-direction bins containing energy packets which will be propagated independently along so-called ray paths. Since no refraction is considered, these ray paths are straight lines (in fact : great circles). The distance of propagation during a fixed timestep depends, through the group velocity (see (7) and (8)), on the frequency and the local waterdepth. In this context, the HYPAS model adopts the concept of "cans" to label the energy packets at different places, not coinciding with the gridpoints of the model. Apart from a dissipation effect, due to bottom friction, no other source terms will be considered for the "swell" part of the spectrum although "swell" energy can occur in principle at all frequencies (in the model).

Finally, a coupling between the "wind sea" and the "swell" part of the spectrum is established through the "wind sea" - "swell" interaction. This coupling becomes active in case of varying windspeed at a certain location; it takes in account the non-linear wave-wave interactions between the "wind sea" part of the spectrum and the "swell" part of the spectrum at higher frequencies. In the swell scheme this is accomplished by simply adding to or substracting from the swell energy packets. For the parametrical wind sea, the energy is updated by altering the peak-frequency, the Phillips parameter and the peak enhancement factor (see (22)). For further details we refer to §2.6.

2.4 Model equations for the "wind sea" part of the spectrum

The energy-balance equation (33) for the energy-density spectrum (in fact one uses the transport equation for the momentum-density spectrum) is transformed into prognostic equations for the parameters of the "wind sea" spectrum (41) by means of linear projection operators. This procedure is described in Hasselmann, et al., 1976, and is called "mapping onto parameter space".

Finally one obtains equations of the form :

$$\frac{\partial a_i}{\partial t} + \sum_{j=1}^{6}(D_{ijx}\frac{\partial a_j}{\partial x} + D_{ijy}\frac{\partial a_j}{\partial y}) = S_i + L_i \qquad for \quad i = 1,...,6$$

(42)

where $a_i(\vec{x},t), i = 1,6 =$ the set of introduced parameters (see(41)); $D_{ijx}, D_{ijy} =$ generalised propagation velocity components; S_i = source function for a_i; L_i = component of projected refraction terms in parameter space.

In the HYPAS model, prognostic equations are used for :

$$a_1 = f_m$$

$$a_2 = \alpha$$

$$a_3 = \gamma$$

$$a_4 = \theta_m$$

(43)

For $a_5 = \sigma_a$ and $a_6 = \sigma_b$ following diagnostic expressions are used :

$$\sigma_a = (\frac{4}{\gamma + 0,7})^2 \cdot 0,07$$

(44.a)

$$\sigma_b = (\frac{4}{\gamma + 0,7})^2 \cdot 0,09$$

(44.b)

The source functions S_i in (42) are derived in the JONSWAP study : Hasselmann, et al., 1973. The source function for θ_m is constructed such that θ_m relaxes back to the local wind direction. The relaxation rate is proportional to f_m and the magnitude of the wind component perpendicular to the mean direction of wave propagation ($\theta = \theta_m$). In the source functions for the "wind sea" part of the spectrum only the atmospheric input and the non-linear wave-wave interactions are accounted for. The dissipation due to bottom friction will be introduced in the "swell" part of the spectrum (cfr. infra).

2.5 Model equations for the "swell" part of the spectrum

Following energy balance equation will be integrated to yield the "swell" part of the spectrum :

$$\frac{\partial E}{\partial t} + \vec{v}_g \cdot \vec{\nabla} E = S_b \tag{45}$$

where $E(f, \theta, \vec{x}, t)$ = the wave-energy-density spectrum in frequency-direction space; S_b = sink term due to bottom friction, parametrized according to the JONSWAP study :

$$S_b = -\lambda E \quad ; \quad \lambda = \frac{\Gamma}{c^2 cosh^2 kh} \quad ; \quad \Gamma = 0,038 \quad m^2/s^2 \tag{46}$$

Introducing again ω_h and χ according to (28) and (29), the decay coefficient λ can be expressed as :

$$\lambda = \Gamma \frac{\omega_h^2}{gh}(\chi^2 - 1) \tag{47}$$

It appears that $\lambda \cdot h$ only depends on ω_h. In the HYPAS model, an approximative expression is implemented which contains the full dissipation of swell energy of extreme shallow water waves ($\omega_h \rightarrow o$) due to wave breaking.

From (45) it appears that all refraction effects on the "swell" part of the spectrum are neglected. This reveals the more or less artificial distinction between "wind sea" and "swell" in the model approach, that doesn't coincide with ones physical intuition. Refraction effects are taken in account in the "wind sea" part of the spectrum however.

2.6 Interaction between the "wind sea" and the "swell" part of the spectrum.

The total energy-density spectrum (being the superposition of the "wind sea" part and the "swell" part, as described above) has to "obey" some physical "rules".

First the higher frequency part of the spectrum is controlled by the dynamic balance between atmospheric energy input and energy transfer due to wave-wave interactions, which stabilizes the shape of the spectrum. The possible shapes can therefore be parametrized, with a reasonable accuracy, by expressions like the JONSWAP spectrum (41), which contain a very restricted number of parameters. Since such a parametrization is adopted by the "wind sea" part of the model spectrum, it is necessary that the "high" frequency energy is contained in this part of the spectrum, and not in the "swell" part. "High" frequencies are frequencies which exceed the Pierson-Moskowitz frequency (f_{PM}) for the local wind condition at a certain time. This means that "low" frequencies (at which "swell" energy is contained) can become "high" frequencies as the windspeed increases (and f_{PM} decreases, see (24)). Therefore a formalism is implemented to transfer energy from the "swell" part to the "wind sea" part of the spectrum when the Pierson-Moskowitz frequency decreases. The amount of energy E_{SWWS} which will be picked up by the "wind sea" part of the spectrum is given by :

$$E_{SWWS} = \int_{f_{PM}}^{\infty} df \int_{o}^{2\pi} E_{swell}(f,\theta)d\theta \qquad (48)$$

and will be removed consequently from the "swell" part of the spectrum :

$$E'_{swell}(f,\theta) = 0 \qquad for \qquad f \geq f_{PM} \quad ; \quad all \quad \theta \qquad (49)$$

(the prime indicates updated quantities after transfer).

The peak frequency f_m of the original "wind sea" part of the spectrum will be decreased to f'_m, such that the total energy content (see (25)) increases with the amount E_{SWWS} given by (48) :

$$f'_m = f_m \left(1 + \frac{5(2\pi f_m)^4}{c(\gamma) \cdot \alpha g^2} \cdot E_{SWWS}\right)^{-1/4} \tag{50}$$

Note that through this mechanism refraction can affect the low frequencies in the spectrum, although it doesn't affect the "swell" part of the spectrum.

The second phenomenon which requires an additional modeling concerns the behaviour of a random sea when the local windspeed decreases, such that the PM frequency (24) increases. In this case the domain of so-called "low" frequencies (smaller than f_{PM}) expands. For these frequencies the atmospheric input is insufficient to maintain the dynamic balance with the wave-wave interactions. The wave energy at these frequencies should not necessarily be contained in the "wind sea" part of the spectrum any longer. Still a part of the (wind sea) spectrum remains, which can just be supported by the lower level of atmospheric input. This is the spectrum of a so-called "fully developed" sea. It is assumed to take the Pierson-Moskowitz form (i.e. $\gamma = 1$ and $f_m = f_{PM}$ in (22)). In the HYPAS model this is accomplished in two steps. Starting from the original "wind sea" part of the spectrum $E_{wind}(f, \theta)$, first an intermediate spectrum $E^*_{wind}(f, \theta)$ is determined by $\gamma = 1$ and the conservation of total energy content by adjusting the Phillips parameter α to α^* (see (25)) :

$$\alpha^* = c(\gamma) \cdot \alpha \tag{51}$$

This transformation causes a certain spreading of the energy, away from the spectral peak (which is maintained at the original f_m for the time being).

The intermediate spectrum $E^*_{wind}(f,\theta)$ is now transformed into a true PM spectrum by adjusting the peak frequency to the PM frequency :

$$f'_m = f_{PM} \tag{52.a}$$

Moreover, the Phillips parameter α' of the new (PM) "wind sea" spectrum $E'_{wind}(f,\theta)$ is determined such that the energy in the "high" frequency part of the "spreaded" spectrum $E^*_{wind}(f,\theta)$ (with $\gamma = 1$) is conserved in the high-frequency part of the new (PM)-spectrum. From (25) and (52.a) we now derive:

$$\alpha' = \alpha^* \left(\frac{f'_m}{f_m}\right)^4 \frac{1 - e^{-\frac{5}{4}\left(\frac{f'_m}{f_m}\right)^{-4}}}{1 - e^{-5/4}} \tag{52.b}$$

The energy at the "low" frequencies in this new "wind sea" spectrum has now been decreased. The energy difference between the "spreaded" spectrum and the new spectrum will be transfered to the "swell" part of the spectrum :

$$E'_{swell}(f,\theta) = E_{swell}(f,\theta) + \left(E^*_{wind}(f,\theta) - E'_{wind}(f,\theta)\right)$$

$$for \quad f \leq f_{PM} \quad ; \quad all \quad \theta \tag{53}$$

No additional growth mechanisms are implemented for "swell" energy at such frequencies, that it is sensitive to the (low-level) atmospheric input. Still this energy is not absorbed in the "wind sea" part because it is outside the range of the non-linear interactions, as we already pointed out. Finally we remark that the described interactions are determined by local and instantanuous characteristics. Therefore the direction of interaction ("swell to wind sea" or "wind sea to swell") can vary from point to point and from time to time.

2.7 Boundary conditions

No distinction is made between open and closed boundaries in the HYPAS model : every boundary is assumed to be a closed boundary. At a boundary where the wind is blowing offshore, it is assumed that the parameters of the "wind-sea" part of the spectrum satisfy the fetch limited relations for growing seas as determined in the JONSWAP study and modified in Hasselmann et al., 1976 :

$$f_m = 2,84 \cdot x^{-0,3} \cdot \frac{g}{W_{10}^{\parallel}} \quad (but \quad minimum \quad f_m = f_{PM}) \tag{54.a}$$

$$\alpha = 0,066 \cdot x^{-0,2} \tag{54.b}$$

$$\begin{aligned} \gamma &= 3,3 && for \quad \nu_m \geq 0,16 \\ &= 76,76 \cdot \nu_m - 8,967 && for \quad \nu_m < 0,16 \end{aligned} \tag{54.c}$$

$$\theta_m = \theta_{wind}, \text{ the local wind direction } (W_{10}^{\parallel} = W_{10}) \tag{54.d}$$

where in :

$$x = \frac{g \cdot F}{W_{10}^{\parallel}}, \text{ the dimensionless fetch} \tag{55}$$

$$\nu_m = \frac{f_m \cdot W_{10}^{\parallel}}{g}, \text{ the dimensionless frequency} \tag{56}$$

In (55) the fetch F is taken the gridspacing which will be used for the integration of the model equations (cfr. infra). The "swell" part of the spectrum is assumed to be non existent at an "inblowing" boundary. Therefore it is necessary to define a model area that is large enough to generate all the swell energy at a point of interest.

At "out blowing" boundaries (onshore blowing wind) all information is determined by advection from "upwind" regions only. This guarantees that no "reflection of information" occurs.

3 INTEGRATION OF THE MODEL EQUATIONS

3.1 Introduction : model grid, bins, rays

The model equations, described in the preceding paragraph, will be integrated numerically to yield quantities at discrete grid points in space and at discrete timesteps. This requires the definition of a timestep Δt and a model grid, i.e. : the model area (topography) and the gridspacings $\Delta x, \Delta y$. In the HYPAS applications for the North Sea, used by MUMM, $\Delta x = \Delta y$ is taken 50 km for a so called "coarse grid" model (see fig. 1) and $\Delta x = \Delta y = 5$ km for the "fine grid model" covering the southern part of the North Sea near the Belgian coast (see fig. 3). At every grid point the value of the local (average) waterdepth (bathymetry) has to be specified. In both cases a stereographic projection is used to generate the grid points. This allows the use of a rectangular coordinate system to express the model equations in.

The specification of a discrete space-time grid is sufficient to integrate the parameters of the "wind-sea" part of the spectrum (equations (42), (44) with boundary conditions (54)), by means of a finite differences scheme. For the HYPAS model the second order Lax-Wendroff method is implemented. For a full description of this method we refer to Günther, 1981. In this chapter we will only deal with the integration of the "swell" part of the spectrum.

As the "swell" part of the spectrum is not parametrized, an additional discretisation is required to resolve the frequency-direction dependency of this part of the spectrum at a certain grid point and at a certain timestep. With this respect a grid in (f, θ)-space has to be specified, i.e. : the upper and lower boundary on f (f_{max}, f_{min}) and the resolutions Δf and $\Delta\theta$:

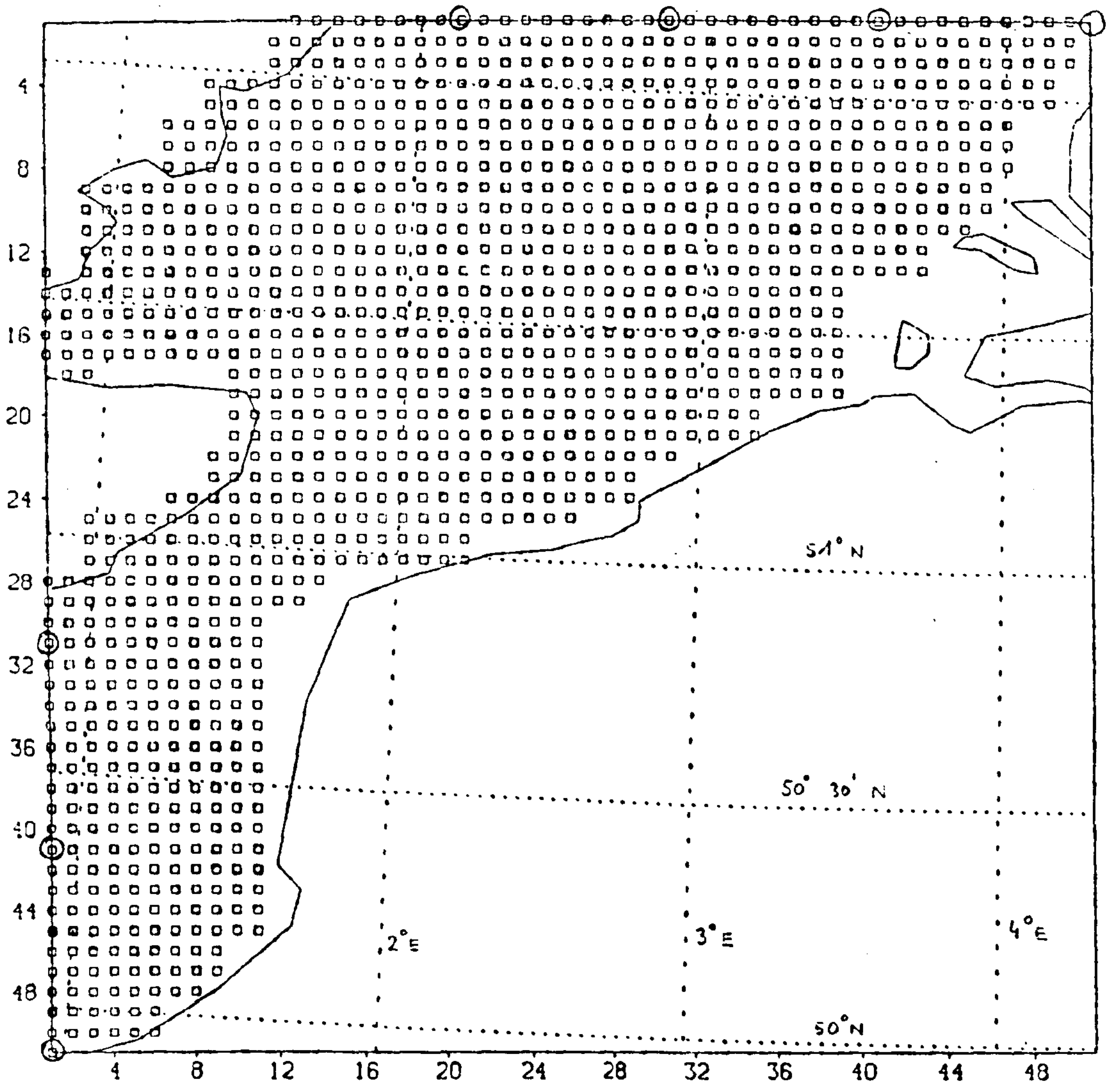

Fig. 3. Fine grid model

$$\Delta f = \frac{f_{max} - f_{min}}{m} \quad ; \quad \Delta\theta = \frac{2\pi}{n} \tag{57}$$

m and n being integer numbers.

The "swell" part of the spectrum ($E_{swell}(f,\theta)$) is now represented by discrete values E_{ij} defined as

$$E_{ij} = \int_{f_i - \frac{\Delta f}{2}}^{f_i + \frac{\Delta f}{2}} df \int_{\theta_j - \frac{\Delta\theta}{2}}^{\theta_j + \frac{\Delta\theta}{2}} E_{swell}(f,\theta) d\theta \tag{58.a}$$

where in

$$f_i = f_{min} + i \cdot \Delta f \quad ; \quad 0 \leq i \leq m \tag{58.b}$$

$$\theta_j = (j-1) \cdot \Delta\theta \quad ; \quad 1 \leq j \leq n \tag{58.c}$$

The integration domain of (58.a) in (f,θ)-space is called a frequency-direction "bin"; E_{ij} contains all swell energy in a certain bin. Since no refraction is considered for the "swell" part of the spectrum, it is possible to integrate (45) seperately for every frequency-direction bin : no couplings exist in (f,θ)-space. The advective term in (45) contains the group velociy, that depends on frequency and local waterdepth. To deal with the large variety of possible values of this propagation velocity, (45) will be integrated using the method of characteristics instead of a finite differences scheme. A characteristic is a set of points (x,y) along which the swell energy propagates; it is called a "ray path" or (characteristic) "ray". In the absence of refraction the ray paths are nearly straight lines (in the stereographic projection used in the model) : there is no change in the propagation direction of a certain wave component (with a certain frequency).

3.2 Atmospheric input

As the source functions S_i in (42) are depending on the local windspeed at a certain timestep, the windfield has to be specified over the model area (at every gridpoint) during the period of simulation. In fact these windfields are the forcing factor for the dynamics of the local wave-energy spectra.

One has to make sure that the windspeeds at 10 m height are put in the source functions since these windspeeds are rather dependent on height above the average sea-surface, and the source functions are calibrated to the windspeeds at this height. Moreover the expression (24) for the Pierson-Moskowitz frequency (transition form "low" to "high" frequencies, cfr. §2.6) has to be adapted in case another reference height is used to calculate the windfields. The windfields at different timesteps will be provided by a meteorological model when forecasting wave-energy spectra. Eventual time interpolation of the windfiels will be carried out in Fourier space.

3.3 Integration of the "swell" part of the spectrum

As already mentioned in §3.1., the model equations (45) for the "swell" part will be integrated, for every frequency-direction bin, by means of the method of characteristics. A (characteristic) ray is defined for a certain frequency f_i and direction θ_j by the set of points $\vec{x}_i(t)$ satisfying

$$\frac{d\vec{x}_i}{dt} = \vec{v}_g(f_i, h(\vec{x}_i)) \tag{59}$$

In the absence of refraction, the direction of $\vec{v}_g$ remains unaltered and therefore the ray paths are straight lines.

Integrating (45) and (46) over a frequency-direction bin (see (58)), together with (59), yields :

$$\frac{dE_{ij}(\vec{x}_i(t),t)}{dt} = -\lambda(f_i, h(\vec{x}_i)) \cdot E_{ij}(\vec{x}_i(t),t) \tag{60}$$

where in λ is assumed to be constant over the frequency-direction bin. Moreover, if λ is assumed to be constant over a time period Δt_p, (60) can be integrated by seperation of variables to yield an exponential decay (dissipation) of swell energy along a ray path :

$$E_{ij}(\vec{x}_i(t+\Delta t_p), t+\Delta t_p) = E_{ij}(\vec{x}_i(t),t) \cdot e^{-\lambda_i(t)\cdot\Delta t_p} \tag{61}$$

where in $\lambda_i(t) = \lambda(f_i, h(\vec{x}_i(t)))$ given by (47) and from (59) :

$$\vec{x}_i(t+\Delta t_p) \approx \vec{x}_i(t) + \vec{v}_g(f_i, h(\vec{x}_i(t))) \cdot \Delta t_p \tag{62}$$

The expression (62) depends on the direction θ_j of the bin under consideration since $\vec{v}_g$ is assumed to have this direction.

A problem arises from the fact that the positions $\vec{x}_i(t)$, at which the "swell" energies E_{ij} will be calculated, do not coincide in general with the specified grid points to integrate the "wind sea" part of the spectrum. The aim of the model is however, to compute the total wave-energy spectrum at these specified grid points. Therefore a certain coupling has to be established between the model grid and the "swell" grids $\vec{x}_i(t)$ (t = running over a certain simulation period divided into timesteps Δt_p), which are dependent on frequency and direction. This will be pointed out in the next paragraphs.

3.4 "Swell" propagation and dissipation in the HYPAS model

In the preceding paragraph it was pointed out that the "swell" part of the spectrum can be determined by propagation and dissipation of energy packets E_{ij} along well-defined ray paths, depending on frequency and direction. This mechanism is controlled by the timestep Δt_p (see (61) and (62)), which does not have to coincide with the timestep Δt of the finite differences scheme to integrate the model equations for the "wind sea" part of the spectrum. The expressions (61) and (62) are valid if the waterdepth remains constant over the propagation path which is followed between t and $t + \Delta t_p$. Since the model bathymetry is a set of waterdepth values which are assumed to be constant over a grid cell around a point of the model grid, the maximum value of the timestep Δt_p of the propagation and dissipation (for a given frequency and direction) is determined by

$$||\vec{x}_i(t + \Delta t_p) - \vec{x}_i(t)|| \leq \Delta s_j \tag{63.a}$$

where in

$$\Delta s_j = min\{\frac{\Delta x}{sin\theta_j}, \frac{\Delta y}{cos\theta_j}\} \tag{63.b}$$

the maximum travel distance within a grid cell in a direction θ_j (see fig. 4).

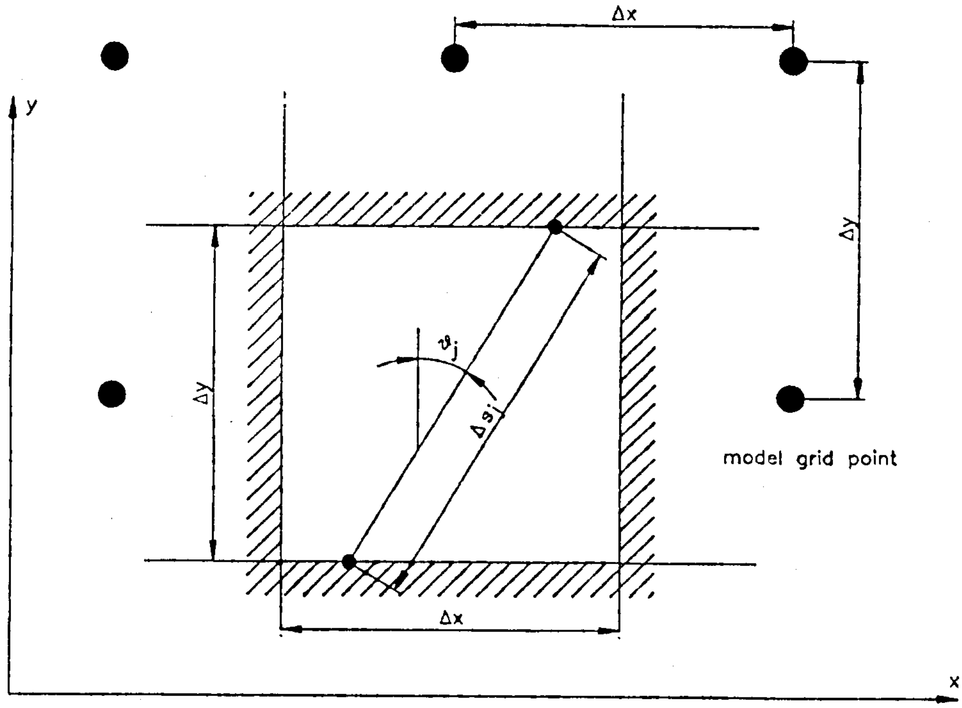

Fig. 4. Dimensions of a model grid cell.

From (63) we have, with (62) and $\Delta x = \Delta y$:

$$\Delta t_p \leq \frac{\Delta x}{v_g(f_i, h(\vec{x}_i)) \cdot K_j} \quad ; \quad K_j = max\{cos\theta_j, sin\theta_j\} \tag{64}$$

In order to minimize the number of propagation and dissipation computations it is interesting to take Δt_p as big as possible. In the HYPAS model, this timestep is taken an integer multiple of the "wind sea"-scheme timestep Δt (for a given frequency f_i and direction θ_j) :

$$\Delta t_p = N_{ij} \cdot \Delta t \tag{65.a}$$

where in the multiplicator N_{ij} follows from (64) :

$$N_{ij} = INT\left[\frac{\Delta x}{v_{gm_i} \cdot K_j \cdot \Delta t}\right]$$

(65.b)

v_{gm_i} being the maximum possible value of the group velocity, at a given frequency f_i, for all possible waterdepths h. From (8) it can be found that this value occurs for $\omega_h = 1$ (see (28)) and thus from (31) and (29) :

$$v_{gm_i} = \chi_m \frac{g}{4\pi f_i} \quad ; \quad \chi_m \tanh \chi_m = 1$$

(66)

Solving the equation for χ_m yields $\chi_m = 1.20$. The waterdepth h_i at which this maximum value occurs is given by :

$$h_i = \frac{g}{(2\pi f_i)^2}$$

(67)

For $f_i = 0.1$ Hz we have $h_i = 24.8$ m, revealing the presence of this effect in the southern North Sea. Since Δt_p is limited to the value given by (65), for a certain frequency f_i and direction θ_j, the propagation distance (during the time interval Δt_p) will not equal Δs_j in every grid cell at a model grid point $\vec{x}$ $(h(\vec{x}) \neq h_i)$. For an arbitrary grid cell it may take several propagation steps to travel through this grid cell to an adjacent one (normally not more than 4 steps are required).

3.5 Construction of swell propagation rays on the model grid; the "can" concept

In order to determine the contribution of "swell" to the total wave-energy spectrum at every model grid point, the ray paths for swell propagation and dissipation are defined as arrays of model grid points. The order of the grid points in these arrays is depending on the propagation direction (θ_j). Two successive grid points in the array for a certain direction θ_j are determined in such a way that the second point is the nearest after propagation in the direction θ_j starting from the first point over a distance Δs_j given by (63). Note that every array contains all (active) grid points.

In the preceding paragraph it was pointed out that, in general, an energy packet E_{ij} does not travel from one grid cell to another in one propagation step. The number of propagation steps required to do so depends on the local waterdepth $h(\vec{x})$, as well as on frequency and direction. Therefore it is necessary that more than one energy packet can be assigned to one model grid point for a certain frequency and direction, together with information about its state of propagation within the grid cell of this grid point.

In the HYPAS model a grid cell of a certain model grid point is divided into so-called "cans" to deal with this problem. Every can contains an energy packet E_{ij} that propagates to the adjacent can at every propagation step (time interval Δt_p), for a certain frequency and direction. The subdivision of a grid cell of a swell-propagation ray in one or more cans depends on frequency and direction and the model bathymetry; it can be carried out by a preprocessing program before the actual model computations are performed.

As far as boundary conditions are concerned : a swell ray starts and ends on land or the edge of the model grid. In the starting point of a ray a zero swell energy is initialized; all swell energy in the model is generated by the interactions with the "wind sea" part of the spectrum (§2.6). Where a swell ray ends, the energy content of the last can is assumed to be fully dissipated at the next propagation step. No reflection of swell energy is considered.

4 ALGORITHMIC DESCRIPTION OF THE HYPAS PROGRAM

4.1 Preprocessing : set up of the model grid and the "swell" rays.

The GITTER module of the HYPAS program reads all user-defined parameters concerning the grid and the simulation periods, and converts it to suitable data for the programs. In this first step the topography and the bathymetry of the model area is defined. Moreover the user specifies the spacings Δf and $\Delta\theta$ for the frequency-direction bins, which may be variable. Finally some numerical integrations and solutions are computed concerning the shallow water effects. These computations only need to be carried out once.

Consequently the UEBER module sets up the swell rays. For every direction the sequence of the grid points is determined that fits the propagation direction. Moreover the frequency-dependent subdivision in cans is coded for every sequence of grid points after determining the swell-propagation timestep Δt_p for every combination of frequency and direction.

4.2 Preparation of windfields and initial conditions for the "wind sea"

The VORBE module generates the windfields at the timesteps of the model computations by interpolation of the given windfields in time. This is accomplished by interpolating the spatial Fourier transforms of two given windfields. This guarantees a proper propagation of the spatial distribution of windspeeds. If the given windfields (e.g.: from a meteorological model) are organized on a grid, which is different from the model grid, a spatial interpolation is carried out.

The BELEG module initializes the "wind sea" and the "swell" parameters. For the "wind sea" parameters the boundary conditions (54) are used as initial conditions. All cans are initialized with a zero swell energy.

4.3 Main program : development of the sea state; output.

Starting from either the initial conditions provided by the BELEG module ("cold start") or a sea state previously calculated ("warm start"), the HYPAS module performs the integration of the wave-energy spectrum at every gridpoint for a certain number of timesteps and prepares the necessary output. Following steps can be distinguished ("cold start").

1. Reading of the windfield for the actual timestep (module AWEI).
2. Transfer of energy from the "wind sea" part of the spectrum to the "swell" part of the spectrum at the gridpoints where $f_m < f_{PM}$ (module WSSW).
3. Propagation and dissipation of "swell" energy from one can to the adjacent one on a ray path for every frequency and direction; absorption of "swell" energy in the "wind sea" spectrum at gridpoints where $f_m > f_{PM}$ (module SWPROP).
4. Eventual output of "wind sea" and "swell" part of the spectrum; preparation of computations for a new timestep (eventually end of computations).
5. Computation of "wind sea" parameters at the next timestep according to the Lax-Wendroff differences scheme, at every gridpoint; installation of boundary conditions (module STEUER).
6. Reading of the next windfield; repeat from 1.

In case of a "warm start", the computations start at step 5 after reading the necessary restart data (windfields, sea state at every gridpoint).

4.4 Postprocessing

Several modules are included in the HYPAS program to process the results of the computations such as

- HOEHE : computation of significant wave height at every grid point at different timesteps;

- ERS/PERS : printing of the results of the "wind sea" parameters and the discrete "swell" part of the spectrum at different gridpoints and at different timesteps;

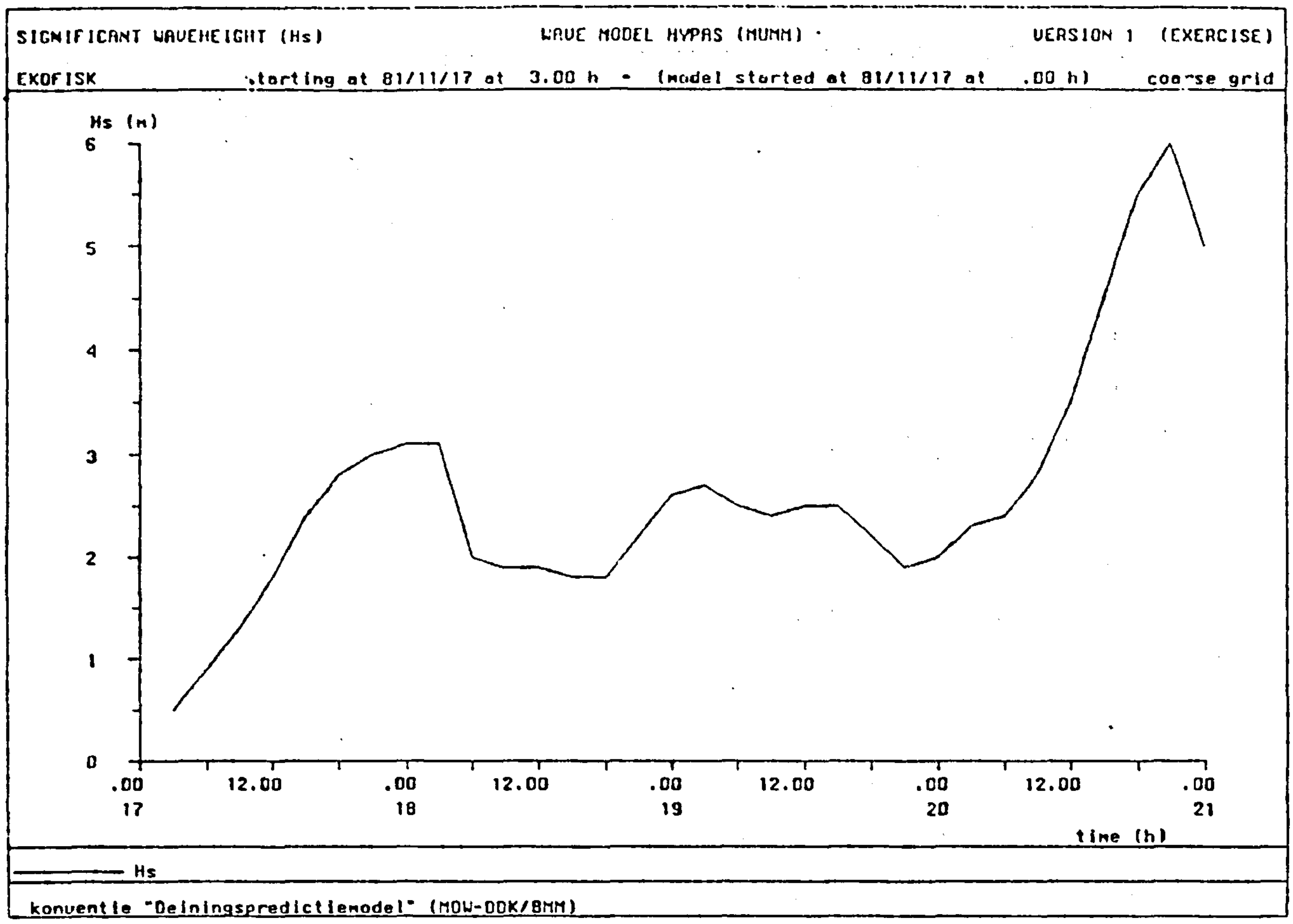

Fig. 5. Computed significant wave height time series

- VSPEC : reconstruction of the total wave energy spectrum at different gridpoints;

- VORBEP : printing of the model windfields at different timesteps, as prepared by the VORBE module.

5 APPLICATION OF THE HYPAS MODEL TO THE NORTH SEA

At the time of writing of this paper, only preliminary results of the North Sea application were available. A detailed presentation of the results will be subject of a publication later on. Still we present some results in this paper. In fig. 1 and fig. 3 the model grid is shown for the "coarse grid" and the "fine grid" model respectively (see §3.1.). In fig. 5 the evolution is shown of the computed significant wave height, compared with measurements at the grid point which is situated near the Ekofisk oil platform. A good qualitative agreement is already obtained. In fig. 6 a typical one-dimensional energy spectrum is shown (integrated over all directions), as computed by the model.

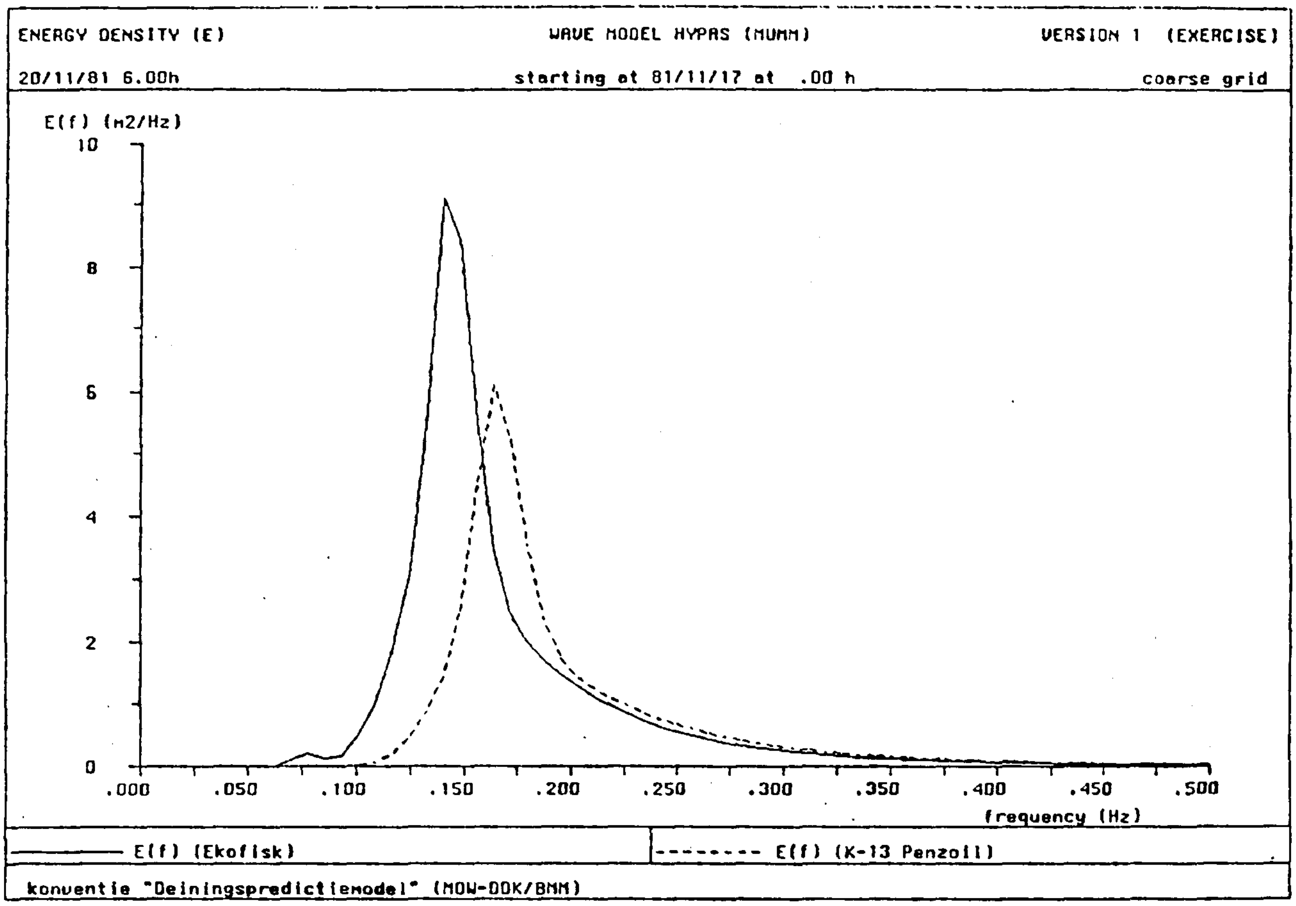

Fig. 6. Computed wave-energy spectra

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