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EFFECTS OF LENTH/AGE VARIABILITY ON LCA

by

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ABSTRACT

Cohorts in a fishery were simulated by generating normal distributions that represented distributions of length at age. The normal distribution for a given age was defined by a mean length that came from a v.Bertalanffy growth equation, and a standard deviation arbitrarily chosen by the authors, that was maintained constant, independen of age or increased with mean length as to maintain the coefficient of variation constant. Different values of K were used to simulate low and fast growing cohorts. Fishing was simulated by imposing a fishing rate, and catch at length was obtained under the assumption that fishing occurred by instantaneous pulses at the mean point of each quarter. Under the assumption of steady-state,this catch would be equivalent to that on a stock along the year. LCA was then carried out on this catch at length , and resulting F estimates were compared with the F values used to generate it. No bias arouse when the standard deviation of the normal components remained constant along the cohort lifespan. However, if s increased with age, F estimates from LCA became biased. Absolute values of bias increased with CV for a given value of K, and the sign was dependent on the way the cohort was growing (negative for low K values, positive for high ones). The influence of certain biological events such as escapement or the recruitment pattern on LCA estimates are also discussed.

Introduction

Length Cohort Analysis is a method suggested by Rodney Jones (1974a, 1974b, 1979, 1982), to estimate fishing mortality rates applied on a stock in a steady-state, when ALK's are not available, but it some function is known that expresses the relationship between mean length and age.

The method is based on the recurrent equation by Pope (1972),

$$N(i) = N(i+1)*\exp(M\Delta t) + C(i)*\exp(M\Delta t/2) \qquad (1)$$

t is then substituted by its expression from the v.Bertalanffy equation :

$$\Delta t = (1/K)*\ln((L-l(i))/(L-l(i+1))),$$

to obtain

$$N(i) = X(i)*((C(i) + N(i+1)*X(i)),$$

where $$X(i) = ((L-l(i))/(L-l(i+1))^{M/2K},$$

L = L-infinity and K are the parameters of the v. Bertalanffy eq., l(i) and l(i+1) are the lower and upper limit of the length class i, M is the natural mortality rate, C(i) is the catch from the length class i, and N(i) and N(i+1) are the abundances at age corresponding to lengths l(i) and l(i+1) respectively.

Eq.(1) is then used to obtain N(i), and that allows to estimate F as

$$F(i) = (1/\Delta t)*(\ln N(i) - \ln N(i+1) - M\Delta t)$$

The analysis of errors in the estimation by LCA was mainly developed in the eighties, together with the analysis of other techniques that also attempted to convert catches at length in catches at age . Some discussion on these methods took place during the 1985 ICES "Methods" WG meeting, and a meeting was held that same year in Mazzara del Vallo (Sicilly, Italy), sponsored by ICLARM, that was devoted to assess length-based techniques for stock assessment.

LCA raised a lot of interest for some ICES WG's, as they had to assess some stocks for which ALK's were not available (like hake, Nephrops or monkfish). Some members of these groups also analysed the technique, and the errors raised due to uncertainty on the value of the input parameters (Pereiro & Pallares, 1984; Jones, 1985, 1986; Laurec & Mesnil, 1985; Pallares & Pereiro, 1985). Addison (1989) presented a good summary of these works to the 1989 ICES Nephrops WG, and Bailey annexed to this same WG a study on "sensitivity of Length Cohort Analysis to input parameters using PAST". Errors expectable due to non accomplishment of steady-state conditions were also analysed by Pereiro & Pallares (1988) and Pallares & Pereiro (1990), that explored the consequences on the estimates of trends in fishing mortality values and fluctuations of recruitment respectively.

The authors of the papers cited above put their emphasis on the analysis of the influence on estimates obtained by LCA of a wrong choice in the value of the input parameters. This paper attempts to assess the consequences of assuming an one-to-one relationship between length and age on the estimates of F obtained by LCA, by the means of analysing some concrete illustrations, used paradigmatically to obtain some clues about the kind of error that could be raised on keeping that assumption. It is not, therefore, a systematic treatment of this complex subject, but the authors believe it can help to falsate the method, -i.e., it can help to check if the method works in order in some selected cases -.

Methods.-

An artificial catch-at-length was obtained by the following procedure :

1) The catch came from an artificial cohort that started with 1000 individuals at age 0, and its number was exponentially reduced at a constant rate. Fishing began at age 0, and the selection factor was 1 from that instant.

2) The individual length on each cohort followed a normal distribution. Mean length at age was determined by a v. Bertalanffy equation. The std. deviation was assigned by the authors through one of these two alternatives : a) it was maintained constant along the cohort lifespan, independently fron the age of individuals; or b) the coefficient of variation of length at age was maintained constant along the cohort lifespan; then, the std. deviation was obtained as the product of the CV by the mean length.

3) The number of survivors at length for a given cohort was then calculated by multiplying the total number of survivors obtained by the survival equation, and the proportion of individuals belonging to that length class, derived from the area of the normal distribution under the interval considered. The areas of the normal distribution were obtained by the procedure by Hastings (1955) as described in Rohlf & Sokal (1969).

4) The number of survivors was computed at the beginning of each quarter. It was then assumed that a fishing pulse took place in the mean point of each quarter. The number of individuals caught from a cohort was calculated by Pope's formula (1), that gives, in this case, the exact result.

5) In a steady state situation, catches obtained from the cohort along its lifespan are equivalent to the yearly catch on a stock.
The catches obtained in (4) can then be assigned to the catch at length from the stock by the addition of the quarterly catch on each cohort present. This step resulted therefore in catch at length on the stock.

The catch per length as obtained in (5) was then used to obtain estimates of the fishing rate by Length Cohort Analysis. These estimated values were then compared with those used to generate the catch at length as described above. The deviations between both values were analysed to assess the errors raised on using LCA by not taking into account the actual variability of length at age.

Input values.-

Growth.- The procedure we have just described was applied to cohorts that grew with different patterns. L-infinity was maintained constant in 100 units, and a value of 0 was always assigned to t0.

Three values were assigned to K : .1, .2 and .3 per year. Some runs were carried out on cohorts showing the growth parameters corresponding to European hake (Northern stock).

The mean length of normal components of length at age was calculated by the v.Bertalanffy eq. with those parametric values, as already explained. The normal components were defined as well by an assigned value for the std. deviation. At every case, this value was arbitrarily chosen by the authors. For runs where s was assumed to be age independent, a value of 3 or 5 was used. For those runs where the CV was maintained constant, the authors used CV values of .1, .2, .3 or .4, arbitrarily chosen too.

Mortalities.- M was assumed to be constant and equal to .2. That involves to use M/K ratios of 2, 1 or .67, up to each run.

Like VPA, LCA gives significant results only when the fishing rate is sufficiently high relative to natural mortality - at least, F should be greater than M -. Therefore, the authors chose F values of .3 and .5 per year to generate the cohorts.

Carrying out LCA.-

The length range considered to carry out the LCA on the cath at length generated from the artificial cohorts was 10 to 55 units. In order to start the procedure the right value of F was used, - i.e., the value used to generate the catch at length.

The right values of K and L-infinity were also used, in order to relate errors just with the fact that LCA ignores the variability of length at age. The length intervals used in LCA was 1 unit at every run.

The procedure was implemented on spreadsheets through a protocol "ad hoc", that included original "macros". The concrete execution, - that is relatively slow -, was made, -most of it -, by a technician especially skilled for that, and supervised by the authors. The reason to use spreadsheets is mainly that it was considered to be wise to start this work screening in a simple way for the first results, without a need of making up a program in a more powerful computer language. One of the authors is now beginning to develop an equivalent program in FORTRAN 77.

Results.-

For all runs where s was kept constant and independent of age, F was estimated practically without error, or with ver modest errors. Table RES-1 shows the relative errors (in percentage) obtained in the estimate of F by length unit. It can be seen that errors are very minor, whatever be the value of K, s and F used. Therefore, the constancy of s for the different age components leads to non biased estimates of F from LCA.

The exception to this result is the lower range of length values for K = .3. That is, at least in part, an artifact of the method used to generate the catch at length, and it will be discussed below.

When s of the normal components increased with age, -i.e., with the mean length of the individuals -, biases in the F estimates were found, and really important in some cases. The biases varied along the length range, also distorting the fishing pattern.

As the interpretation of bias was complex in some respects, we'll describe the result of runs in some extent. We'll start wit runs on cohorts generated under F = .5 .

For runs where a K value of .1 was used, biases were mostly negative, F per length class being underestimated from LCA. The exception arouse for the highest length classes (47-54) and lowest (10 and 12) for the run with CV = 10%.

Biases increased with the value of CV used, reaching sensible values for CV = 30% and 40%. Table RES-2 shows the percent value of bias for these runs with K = .1

For runs with K = .3, - or M/K = .67 -, the bias was positive, leading to overestimations of fishing mortality per unit length, -table RES-3-. The absolute value of the bias increased in general with the value of the CV used. The F values obtained for the first length-classes when the CV value was small showed very big biases with changing sign. This fact again will be discussed later.

Runs were also made for K = .2 with CV = .2 and .4. (Table RES-4). In both cases the bias produced was negative for the highest length classes, and positive for the lowest ones. In the run with CV = 20% strong fluctuations of the bias were produced for the first length classes.

In order to explore the values of bias using another F value, four runs were carried out with F = .3 instead of .5, two of them with input values of K = .1 and .3 and CV = .4, another one with K = .1 and s = 3, and the last one with K = .3, s = 5. The results of runs with s constant have already been commented, and confirm no bias in that case.

The run with K = .1 and CV = 40% resulted in general underestimations of F per length class, - as it had already happened for F = .5 -, with the exception of the first length classes, where the sign became positive. The bias increased its absolute value up to class 30, where it began to decrease, and became positive on length class 14. The maximum bias was 14%, - versus 27% obtained when F = .5 -.

In the run with K = .3, CV = 40%, the bias was positive for the whole length range, growing in absolute value from the highest length class (54) up to the length class 18, where it reached a value of 28%, then decreasing slightly and increasing again for the lowest length classes - bias of 42% for length class 10 -.

These runs with F = .3 follow in general the pattern shown by runs with F = .5, - increase in bias as CV increases; negative bias in runs with K = .1 and positive with K = .3, and very minor when s = constant -; but the concrete value of the bias on each case seems to vary in a complex way not easy to define.

The values of K and L-infinity used in the last runs were coincident with those considered correct for the Northern stock of European hake - and used by the hake WG : K = .095 and L-infinity = 114 -. Four runs were made with those parameters: CV = 40% for F = .3 and .5, and s = 3 for those same values of F. Results are shown in table RES-6.

As it was expectable after the previous runs, using a constant s resulted in no significant biases along the whole length range.

The value of K used was very close to .1, and it could be expected for a negative bias, and that really happened. The absolute value of bias increased from the length class 54 up to 37, decreasing continuously later up to the lowest length for F = .5; it increased from the beginning up to length class 39 for F = .3, then decreased and changed its sign for the very low lengths. The maximum bias for F = .3 was 45% the maximum bias for F = .5.

Discussion and Conclusions.-

Results suggest that the assumption made by LCA about the one to one correspondence between length and age can lead to significant biases of F estimates from LCA if the variance of the age components changes with age. The actual value of the bias on each concrete case is difficult to assess, that seems to depend on the way the normal components are mixed, that determines the contribution of each normal component to the cath of each length class, and that should depend on the simultaneous value of the parameters that define the normal components : K and L-infinity, their variances, and total mortality on each cohort.

Even if this paper does not try to explain such complex matters, their results point out that LCA would lead to underestimates of F for low K values, and overestimates for high ones. Fig. CON-1 shows the biases obtained on runs with CV = 40% and F = .5 for K = .1, .2 and .3, and clearly reflects how the bias changes along the length range as K increases.

Biases are inexistent in the case of constant variance . Fig. CON-2 shows the biases obtained for s=5, F= .5 and K= .1 and K = .3.

As it has been shown, serious problems in the estimation of F for the lowest length classes were raised when the value of K was .3. Sudden changes in the sign and magnitude of bias was produced mainly when low values of s were assumed.

We believe that such a behaviour is mainly due to the assumptions made in the study: normal components of length at age were distant, because the growth was very fast, s was small and fishing was made by quarterly pulses. Consequently, the mixture between components was very small, and the magnitude of catch at a given length depends significantly on the relative position of that length within the normal component where that length is included, - i.e., the proportion of the catches of two consecutive lengths will depend of the respective values of the areas enclosed by the normal distribution, and will have nothing to do with the fishing mortality on the stock. Although this result is - as we have just said -, a consequence of the approach made in by the authors, is indirectly involving that LCA will interpret the proportion between two succesive areas under the normal distribution as a survival if just one normal component is present in a given length range, in spite of the fact that it has nothing to do with a survival.

The same problem would be found if a certain escapement takes place from the fishery at a given age. In fact, every process that was disturbing the "orthodox" mixture of succesive age components could lead to serious biases in the estimates of F obtained by LCA.

Another event which could lead to errors in the estimates has to do with the youngest cohort in the fishery. If the left part of the first normal distribution is not mixed with another component of younger individuals, LCA would compare the abundance of bigger and smaller individuals belonging to the same cohort in order to assess F, resulting in wrong estimates. The existence of this effect would depend on the way recruitment and selectivity take place.

The first sensitivity results obtained seem to point out that the sensitivity to the CV would be of the same magnitude as for M and smaller than that for K. However, the problem is that LCA has been carried out with information about K, but no information has been recollected at all on CV; or, if that information was available, it has not been used.

It is very possible that our conclusions could be applied to assessments lead by artificial ALK's constructed from v.Bertalanffy equations and assumed values of the std. deviations of normal components, as made by some ICES groups.

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Table RES-1.- Percent biases in F estimates from LCA for cohorts whose age components show a constant value of s. K = growth parameter S = std. dev. ; F = fishing mortality.

K	0,1	0,1	0,3	0,3	0,2	0,3	0,1
S	3	5	3	5	3	5	5
F	0,5	0,5	0,5	0,5	0,5	0,3	0,3
LENGTHS							
10	-0,992	-7,715	-3,600	-7,610	-0,420	-7,011	-5,044
11	-0,902	-5,952	-3,576	-5,013	-0,313	-4,523	-3,416
12	-0,877	-4,711	-1,083	-3,197	-0,302	-2,798	-2,280
13	-0,875	-3,875	1,748	-1,982	-0,284	-1,652	-1,511
14	-0,879	-3,338	2,719	-1,201	-0,260	-0,923	-1,006
15	-0,885	-3,009	1,355	-0,721	-0,260	-0,479	-0,680
16	-0,890	-2,820	-0,890	-0,440	-0,278	-0,221	-0,470
17	-0,895	-2,720	-2,044	-0,283	-0,284	-0,079	-0,330
18	-0,899	-2,673	-1,333	-0,201	-0,277	-0,005	-0,231
19	-0,902	-2,658	0,320	-0,159	-0,275	0,032	-0,153
20	-0,904	-2,659	1,332	-0,138	-0,282	0,051	-0,085
21	-0,905	-2,667	0,939	-0,128	-0,287	0,060	-0,020
22	-0,904	-2,678	-0,244	-0,123	-0,287	0,064	0,047
23	-0,902	-2,689	-0,991	-0,120	-0,286	0,066	0,116
24	-0,972	-2,739	-0,706	-0,120	-0,289	0,066	0,166
25	-1,043	-2,788	0,125	-0,120	-0,292	0,065	0,217
26	-1,045	-2,796	0,593	-0,120	-0,292	0,065	0,301
27	-1,045	-2,799	0,328	-0,120	-0,292	0,064	0,391
28	-1,043	-2,798	-0,236	-0,119	-0,294	0,064	0,489
29	-1,039	-2,791	-0,463	-0,119	-0,295	0,063	0,595
30	-1,032	-2,778	-0,207	-0,118	-0,295	0,062	0,710
31	-1,022	-2,757	0,141	-0,118	-0,295	0,062	0,835
32	-1,008	-2,728	0,190	-0,117	-0,295	0,061	0,969
33	-0,991	-2,690	-0,027	-0,116	-0,295	0,060	1,116
34	-0,969	-2,642	-0,198	-0,115	-0,294	0,059	1,274
35	-0,942	-2,581	-0,148	-0,114	-0,293	0,057	1,446
36	-0,909	-2,506	-0,001	-0,113	-0,291	0,056	1,633
37	-0,869	-2,415	0,045	-0,111	-0,289	0,055	1,837
38	-0,822	-2,306	-0,029	-0,109	-0,286	0,054	2,058
39	-0,766	-2,176	-0,094	-0,108	-0,282	0,052	2,300
40	-0,700	-2,022	-0,076	-0,105	-0,278	0,050	2,562
41	-0,622	-1,841	-0,026	-0,103	-0,273	0,048	2,849
42	-0,531	-1,630	-0,017	-0,100	-0,267	0,046	3,160
43	-0,425	-1,384	-0,042	-0,097	-0,260	0,044	3,497
44	-0,301	-1,099	-0,055	-0,093	-0,252	0,041	3,860
45	-0,157	-0,773	-0,043	-0,089	-0,242	0,039	4,245
46	0,011	-0,405	-0,030	-0,085	-0,231	0,036	4,645
47	0,205	0,004	-0,032	-0,080	-0,219	0,032	5,045
48	0,430	0,443	-0,037	-0,074	-0,204	0,029	5,415
49	0,688	0,893	-0,035	-0,068	-0,188	0,025	5,710
50	0,977	1,318	-0,029	-0,061	-0,169	0,021	5,863
51	1,278	1,659	-0,026	-0,054	-0,148	0,017	5,776
52	1,535	1,830	-0,025	-0,046	-0,124	0,012	5,317
53	1,609	1,706	-0,022	-0,036	-0,097	0,007	4,323
54	1,220	1,122	-0,018	-0,026	-0,066	0,001	2,602
55	0,000	0,000	0,000	0,000	0,000	0,000	0,000

Table RES-2.- Percent biases in F estimates from LCA for cohorts growing with K = .1, fished at F = .5 . CV = coefficient of variation.

K	0,1	0,1	0,1	0,1
CV	0,1	0,2	0,3	0,4
F	0,5	0,5	0,5	0,5
LENGTHS				
10	2,083	-0,319	-1,029	-5,139
11	-2,500	-0,048	-2,384	-6,370
12	2,086	-0,498	-3,466	-7,805
13	-0,410	-1,141	-4,340	-9,254
14	-0,039	-1,558	-5,224	-10,584
15	0,245	-1,957	-6,146	-11,798
16	-0,253	-2,419	-7,050	-12,945
17	-0,156	-2,881	-7,914	-14,052
18	-0,266	-3,326	-8,748	-15,121
19	-0,393	-3,769	-9,562	-16,143
20	-0,456	-4,214	-10,357	-17,117
21	-0,555	-4,657	-11,128	-18,043
22	-0,645	-5,094	-11,876	-18,925
23	-0,730	-5,525	-12,598	-19,764
24	-0,913	-5,992	-13,310	-20,557
25	-1,097	-6,460	-14,021	-21,350
26	-1,189	-6,874	-14,669	-22,061
27	-1,279	-7,277	-15,287	-22,729
28	-1,365	-7,669	-15,877	-23,352
29	-1,446	-8,048	-16,434	-23,929
30	-1,521	-8,410	-16,959	-24,459
31	-1,589	-8,755	-17,447	-24,939
32	-1,648	-9,080	-17,898	-25,368
33	-1,697	-9,382	-18,308	-25,743
34	-1,733	-9,660	-18,675	-26,062
35	-1,755	-9,909	-18,995	-26,320
36	-1,759	-10,127	-19,264	-26,513
37	-1,744	-10,311	-19,477	-26,637
38	-1,706	-10,457	-19,631	-26,687
39	-1,642	-10,560	-19,718	-26,656
40	-1,547	-10,617	-19,733	-26,537
41	-1,418	-10,622	-19,669	-26,321
42	-1,251	-10,570	-19,516	-25,998
43	-1,042	-10,456	-19,264	-25,555
44	-0,788	-10,272	-18,902	-24,979
45	-0,491	-10,011	-18,416	-24,252
46	-0,151	-9,665	-17,788	-23,353
47	0,223	-9,222	-16,998	-22,257
48	0,616	-8,671	-16,022	-20,933
49	1,003	-7,998	-14,830	-19,341
50	1,347	-7,187	-13,384	-17,433
51	1,596	-6,216	-11,637	-15,146
52	1,676	-5,060	-9,528	-12,400
53	1,496	-3,688	-6,981	-9,086
54	0,941	-2,059	-3,892	-5,060
55	0,000	0,000	0,000	0,000

Table RES-3.- Percent biases in F estimates from LCA for cohorts growing with K = .3, fished at F = .5 . CV = coefficient of variation.

K	0,3	0,3	0,3	0,3
CV	0,1	0,2	0,3	0,4
F	0,5	0,5	0,5	0,5
LENGTHS				
10	-90,445	-35,537	11,265	35,270
11	-59,484	-27,046	-0,150	23,991
12	6,202	-6,850	-0,652	16,397
13	72,267	10,870	4,844	13,258
14	73,156	20,248	11,385	13,505
15	11,362	20,535	16,196	15,441
16	-39,704	14,744	18,466	17,628
17	-42,550	7,559	18,512	19,224
18	-14,824	2,703	17,131	19,926
19	14,799	1,449	15,182	19,787
20	26,623	2,898	13,345	19,023
21	17,154	5,285	12,002	17,898
22	-0,687	7,168	11,252	16,645
23	-10,929	7,938	10,983	15,436
24	-8,613	7,712	10,991	14,371
25	-0,062	6,965	11,071	13,488
26	6,248	6,173	11,081	12,775
27	6,729	5,625	10,949	12,196
28	3,310	5,387	10,673	11,700
29	-0,073	5,374	10,286	11,243
30	-1,072	5,449	9,838	10,788
31	0,011	5,497	9,372	10,314
32	1,519	5,462	8,919	9,812
33	2,198	5,339	8,492	9,283
34	1,921	5,158	8,091	8,735
35	1,290	4,951	7,712	8,176
36	0,882	4,747	7,342	7,617
37	0,852	4,556	6,973	7,065
38	1,019	4,376	6,600	6,525
39	1,149	4,199	6,217	6,002
40	1,143	4,018	5,826	5,497
41	1,044	3,825	5,426	5,008
42	0,934	3,618	5,022	4,535
43	0,862	3,397	4,614	4,078
44	0,821	3,165	4,207	3,635
45	0,787	2,923	3,800	3,207
46	0,740	2,671	3,395	2,793
47	0,676	2,411	2,994	2,394
48	0,603	2,142	2,596	2,013
49	0,527	1,864	2,202	1,649
50	0,450	1,575	1,813	1,307
51	0,370	1,277	1,430	0,987
52	0,285	0,969	1,055	0,692
53	0,192	0,651	0,687	0,425
54	0,093	0,323	0,331	0,189
55	0,000	0,000	0,000	0,000

Table RES-4.- Percent biases in F estimates from LCA for cohorts growing with K = .2, fished at F = .5 . CV = coefficient of variation.

K	0,2	0,2
CV	0,2	0,4
F	0,5	0,5
LENGTHS		
10	20,455	9,651
11	11,244	12,759
12	1,758	14,270
13	-0,265	13,954
14	2,959	12,486
15	6,150	10,648
16	6,849	8,984
17	5,752	7,724
18	4,462	6,849
19	3,862	6,219
20	3,892	5,679
21	4,095	5,122
22	4,138	4,506
23	3,967	3,836
24	3,695	3,141
25	3,438	2,454
26	3,239	1,797
27	3,079	1,178
28	2,926	0,600
29	2,757	0,057
30	2,572	-0,458
31	2,379	-0,948
32	2,186	-1,416
33	1,997	-1,861
34	1,811	-2,283
35	1,626	-2,678
36	1,443	-3,044
37	1,260	-3,378
38	1,079	-3,678
39	0,902	-3,943
40	0,730	-4,169
41	0,565	-4,354
42	0,406	-4,496
43	0,256	-4,592
44	0,117	-4,637
45	-0,011	-4,628
46	-0,125	-4,560
47	-0,223	-4,426
48	-0,302	-4,222
49	-0,359	-3,939
50	-0,391	-3,569
51	-0,395	-3,104
52	-0,366	-2,531
53	-0,299	-1,839
54	-0,190	-1,011
55	0,000	0,000

Table RES-5.- Percent biases in F estimates from LCA for cohorts fished fished at F = .3. CV = coefficient of variation. K = growth parameter.

K	0,1	0,3
CV	0,4	0,4
F	0,3	0,3
LENGTHS		
10	4,780142	42,07027
11	3,707292	31,03216
12	2,438651	23,83957
13	1,165105	21,07086
14	0,007025	21,55362
15	-1,04584	23,57458
16	-2,04019	25,73284
17	-2,99935	27,23841
18	-3,92256	27,83843
19	-4,80353	27,61507
20	-5,64013	26,80140
21	-6,43451	25,65944
22	-7,18940	24,41407
23	-7,90649	23,22289
24	-8,58139	22,17263
25	-9,25630	21,28933
26	-9,86172	20,55568
27	-10,4293	19,93103
28	-10,9589	19,36785
29	-11,4499	18,82332
30	-11,9016	18,26614
31	-12,3129	17,67816
32	-12,6828	17,05284
33	-13,0097	16,39258
34	-13,2919	15,70438
35	-13,5275	14,99693
36	-13,7143	14,27825
37	-13,8495	13,55442
38	-13,9302	12,82951
39	-13,9530	12,10519
40	-13,9139	11,38149
41	-13,8083	10,65724
42	-13,6311	9,930945
43	-13,3763	9,201078
44	-13,0371	8,466291
45	-12,6054	7,725557
46	-12,0720	6,978294
47	-11,4260	6,224444
48	-10,6550	5,464134
49	-9,74390	4,697634
50	-8,67501	3,925354
51	-7,42723	3,147851
52	-5,97523	2,365514
53	-4,28818	1,578966
54	-2,32841	0,788431
55	0	0

Table RES-6.- Percent biases in F estimates from LCA for cohorts of hake (Northern stock). K = .095, L-inf. = 114. s = std. dev. CV = coeff. variation. F = fishing mortality.

S	3	3		
CV			0,4	0,4
F	0,3	0,5	0,3	0,5
LENGTHS				
10	-0,626	-0,921	7,156	-2,667
11	-0,544	-0,830	6,295	-3,676
12	-0,519	-0,808	5,347	-4,747
13	-0,515	-0,808	4,227	-5,997
14	-0,516	-0,816	3,097	-7,267
15	-0,519	-0,826	2,066	-8,435
16	-0,522	-0,837	1,135	-9,491
17	-0,525	-0,847	0,268	-10,474
18	-0,528	-0,858	-0,564	-11,416
19	-0,531	-0,868	-1,365	-12,324
20	-0,533	-0,879	-2,130	-13,192
21	-0,535	-0,889	-2,856	-14,015
22	-0,537	-0,899	-3,542	-14,792
23	-0,539	-0,909	-4,189	-15,526
24	-0,540	-0,919	-4,801	-16,216
25	-0,541	-0,928	-5,378	-16,866
26	-0,542	-0,937	-5,920	-17,475
27	-0,542	-0,945	-6,427	-18,041
28	-0,542	-0,953	-6,900	-18,564
29	-0,541	-0,961	-7,337	-19,043
30	-0,540	-0,967	-7,738	-19,478
31	-0,538	-0,973	-8,103	-19,865
32	-0,536	-0,978	-8,431	-20,205
33	-0,533	-0,981	-8,721	-20,494
34	-0,529	-0,984	-8,973	-20,731
35	-0,524	-0,985	-9,184	-20,913
36	-0,518	-0,985	-9,353	-21,035
37	-0,511	-0,982	-9,480	-21,096
38	-0,503	-0,978	-9,560	-21,090
39	-0,494	-0,971	-9,594	-21,012
40	-0,483	-0,962	-9,577	-20,857
41	-0,471	-0,949	-9,508	-20,618
42	-0,458	-0,933	-9,382	-20,288
43	-0,442	-0,913	-9,197	-19,858
44	-0,424	-0,889	-8,948	-19,317
45	-0,405	-0,859	-8,630	-18,653
46	-0,382	-0,824	-8,239	-17,853
47	-0,358	-0,783	-7,768	-16,899
48	-0,330	-0,733	-7,211	-15,771
49	-0,299	-0,676	-6,558	-14,445
50	-0,264	-0,609	-5,802	-12,892
51	-0,226	-0,531	-4,932	-11,074
52	-0,183	-0,440	-3,935	-8,949
53	-0,136	-0,336	-2,798	-6,458
54	-0,085	-0,215	-1,503	-3,533
55	0,000	0,000	0,000	0,000

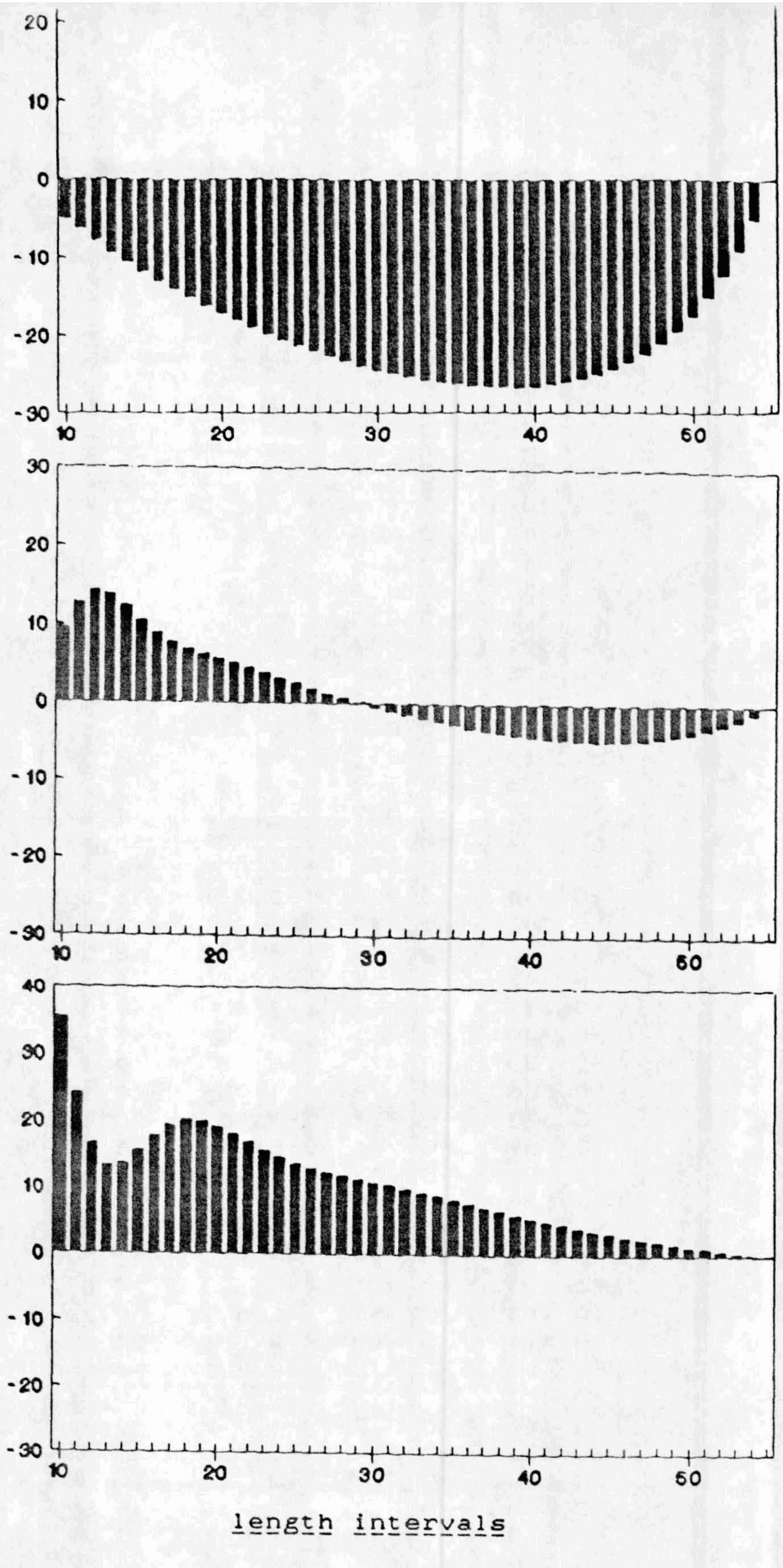

Fig. CON-1.- Percent biases of F estimates from LCA for cohorts with CV = .4, being fished at F = .5; a) K = .1 ; b) K = .2; c) K = .3. X-axis: length intervals; Y-axis: bias in % .

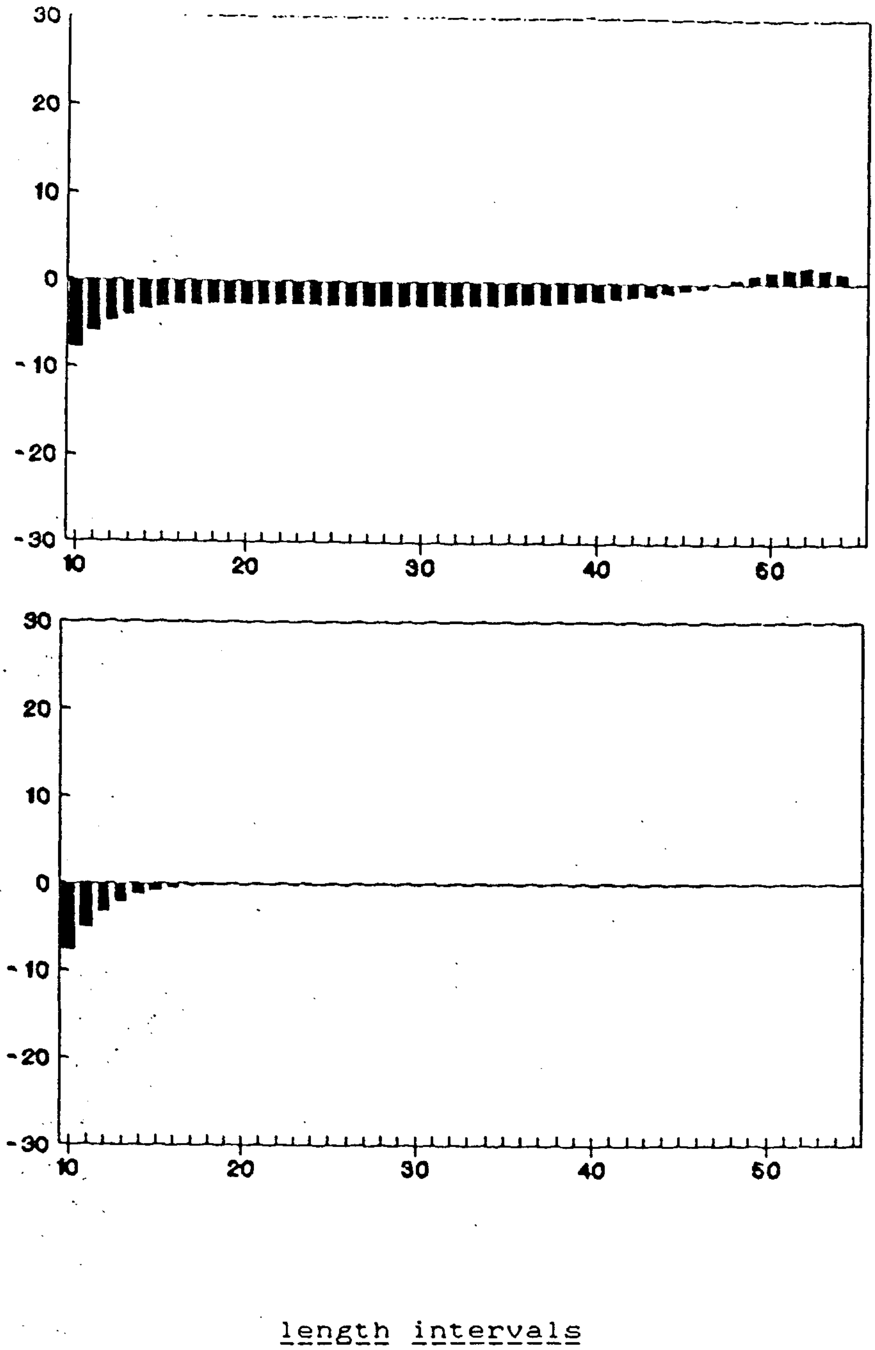

Fig. CON-2.- Biases of estimates from LCA for cohorts with s = 5, fished at F = .5. a) K = .1; b) K = .3 . X-axis: length intervals; Y-axis: bias in % .