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FISHING MORTALITY AND THE VARIATION OF CATCHES: A TIME SERIES APPROACH

by



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SUMMARY

The effect of fishing mortality on the variation of yield is re-examined, with particular attention to the changes of stock size from one year to the next. Using a SHOT model of the stock dynamics, we show that the year-on-year variability of the exploitable biomass increases roughly in proportion to the fishing mortality. The theory is further developed by means of an age-structured model which takes account of the exploitation pattern (the fishing mortality as a function of age). We determine the condition for minimum variability. The corresponding exploitation pattern is not unique but depends on the fishing mortality on the recruiting year-class. We discuss the relationship between the exploitation pattern and the selectivity of the fishing gears used to exploit the stock.

As an example, the theory is applied to the cod stock in the North Sea. We show how the stability of the yield depends on the fishing effort through the number of year-classes contributing to the fishery. The effect of changing the exploitation pattern is considered. It appears that the optimum gear selectivity for the most stable yield depends on the level of exploitation. When the exploitation is high, it is better to have a selectivity function which increases steadily with age, rather than the "knife-edged" selection which is commonly believed to be more desirable.

We conclude that the reduction of stability due to heavy exploitation, and the improvement available through effort reduction and/or technical measures to control the selectivity of fishing gear, are much larger than indicated by previous work which considered only the variation of yield about its mean level.

1. INTRODUCTION

The variability of fish stock size due to varying recruitment is increased at high levels of exploitation, and this leads to large variations in allowable catches from year to year. Previous analyses of this problem (Beverton and Holt, 1957; Horwood and Shepherd, 1981; MacLennan and Shepherd, 1988; Gislason, 1990) have examined the effect on the variance of yield, namely the variability around the long-term mean. In practice, it seems likely that the variations from one year to the next are more troublesome than those around the mean. While fish catchers and buyers have some interest in the prospects for the distant future, they tend to be much more concerned about the next year or two, especially when reductions in yield are forecast. The reasons for this are mainly economic, to do with the natural distaste of investors for fishing vessels and processing plant to be idle through lack of supplies. We shall not consider the economics of fishing here, but evidently there is a need to take account of the relative importance of short-term changes, in biological models of fish stocks and in the scientific advice given for their management.

In this paper, we begin by considering the relative changes of stock size from year to year, at various levels of exploitation, using a simple SHOT model of the dynamics of the exploited stock (Shepherd, 1991). We go on to develop an age-structured model which takes account of the exploitation pattern, the fishing mortality F as a function of age. For simplicity, we treat recruitment as a non-correlated random variable, of the kind sometimes described by the engineering term "white noise". This is probably an oversimplification, since recruitment time series often exhibit positive auto-correlation, or "red noise" which implies that short-term changes are less than those to be expected over long time scales (Steele, 1985). Horwood and Shepherd (1981) have described a more generally applicable theory which incorporates the recruitment as a deterministic function of the spawning population, together with a stochastic component which is considered as a perturbation of the stock-recruitment relationship. For many important stocks, however, the stock-recruitment relationship is poorly understood, and much of the variation in recruitment cannot be explained by changes in population size. This is particularly true of the cod and haddock stocks in the North Sea (Jones, 1973).

The exploitation pattern is determined by the total fishing effort and the selectivity of the gears used in the fishery. Thus the age-structured model is relevant to the important question of gear selectivity and how it might be improved through technical measures, as well as the management of effort through catch quotas or limiting the size of fishing fleets. The practical implications are illustrated by considering the effect of changes in the current exploitation pattern of the cod stock in the North Sea.

2. THEORY

2.1 Dynamics of the Exploited Stock

Shepherd (1991) proposes a simple auto-regressive model which describes the evolution of a fish stock from one year to the next, through the relationship:

$$B(y+1) = hB(y) + wR(y+1) \quad (1)$$

where B is exploited stock biomass, y indicates year, w is the typical weight of a new recruit to the fishery, and R is recruitment (to the exploited stock) in numbers. h is the hangover factor given by:

$$h = \exp(G - Z) \quad (2)$$

where G is the average log weight ratio of consecutive age groups, and Z the total mortality rate, both estimated for those age groups which contribute most of the catch. We assume that for constant fishing effort, the yield is proportional to the exploited biomass. Thus the statistics of yield follow immediately from those of B .

With this model, the average stock size is given by:

$$\bar{B} = h\bar{B} + w\bar{R}$$

and thus

$$\bar{B} = w\bar{R}/(1 - h) \quad (3)$$

Similarly, if recruitment is uncorrelated with itself and previous values of stock size, so that any stock-recruitment relationship is unimportant in comparison to other sources of variation.

$$\text{var}(B) = h^2 \text{var}(B) + w^2 \text{var}(R)$$

and

$$\text{var}(B) = w^2 \text{var}(R)/(1 - h^2) \quad (4)$$

Considering just the relative variance of stock size and recruitment (MacLennan and Shepherd, 1988):

$$\frac{\text{var}(B)}{\bar{B}^2} = \frac{(1 - h)^2}{(1 - h^2)} \frac{\text{var}(R)}{\bar{R}^2} \quad (5)$$

The coefficient of variation of stock size relative to that of recruitment may be used as a measure of long-term variability:

$$H_1 = \frac{CV(B)}{CV(R)} = \frac{(1 - h)}{(1 - h^2)^{1/2}} \quad (6)$$

We may however also consider the variance of changes of stock size, since

$$\begin{aligned} \text{var}(\Delta B) &= \text{var}[B(y + 1) - B(y)] \\ &= \text{var}[hB(y) + wR(y + 1) - B(y)] \\ &= w^2 \text{var}(R) + (1 - h)^2 \text{var}(B) \\ &= w^2 \text{var}(R) \left[1 + \frac{(1 - h)^2}{(1 - h^2)} \right] \\ &= w^2 \text{var}(R) \left[\frac{2}{(1 + h)} \right] \end{aligned} \quad (7)$$

The relative variance of changes of stock size is therefore:

$$\left(\frac{\text{var}(\Delta B)}{\overline{B}^2} \right) = 2 \frac{(1 - h)^2}{(1 + h)} \frac{\text{var}(R)}{\overline{R}^2} \quad (8)$$

A corresponding measure of the variability of short-term changes of stock relative to the variability of recruitment is:

$$H_2 = \frac{[\text{var}(\Delta B)]^{1/2}}{\overline{B} \text{CV}(R)} = \left[\frac{2(1 - h)^2}{(1 + h)} \right]^{1/2} \quad (9)$$

We call H_1 and H_2 the "variability factors" for long-term and short-term changes respectively. They indicate the extent to which changes in recruitment are transmitted to the yield time-series, taking account of the smoothing due to several year-classes being in the fishery.

Comparing equations (6) and (9), dividing out the common factor $(1-h)$ and taking squares, we see that $H_1 = H_2$ when:

$$\frac{1}{1 - h^2} = \frac{2}{1 + h} \quad (10)$$

this condition is satisfied when h , the hangover factor, equals 0.5 and therefore from equation (2):

$$G - Z = \ln(h) = -0.69 \quad (11)$$

Furthermore, if h is less than 0.5 then the short-term variability exceeds the long-term variability ($H_2 > H_1$). If h is more than 0.5, the opposite is the case.

2.2 The Age-Structured Model

Suppose there are k year-classes in the fishery. R_i is the recruitment in year i , and the yield in year y is

$$Y_y = \sum_{n=0}^{k-1} c_n R_{y-n} \quad (12)$$

where n is an index of age, not necessarily the same as the biological age, starting from $n=0$ for the recruiting year-class. k is supposed to be large enough for the contribution of the plus-group to be negligible. According to the classical model of Beverton and Holt (1957), the c_n are given by:

$$c_n = W_n (1 - M_n/z_n) \exp \left(- \sum_{j=0}^{n-1} z_j \right) \{1 - \exp(-z_n)\} \quad (13)$$

W_n = mean weight per fish at age n

M_n = natural mortality

z_n = total mortality

This model allows the mortalities to change with age. According to MacLennan and Shepherd (1988), the long-term variability factor is:

$$H_1 = \left\{ \sum_{n=0}^{k-1} c_n^2 \right\}^{1/2} / \sum_{n=0}^{k-1} c_n \quad (14)$$

Gislason (1990) showed that H_1 is minimised when the c_n are constant. In that case $c_n = c_0$ for all n and $H_1 = 1/\sqrt{k}$. The worst case occurs when c_0 is finite and the other c_n are all zero, so that only one year-class is in the fishery, when $H_1 = 1$.

We now consider the first difference of yield which is:

$$\begin{aligned} \Delta Y_y &= Y_{y+1} - Y_y \\ &= c_0 R_{y+1} + \sum_{n=0}^{k-1} (c_{n+1} - c_n) R_{y-n} - c_k R_0 \end{aligned} \quad (15)$$

The criteria for stabilising the yield and its first difference are not the same. Assuming the recruitments are uncorrelated from one year to the next and independent of the spawning biomass, the variance ratio is now:

$$\text{var}(\Delta Y)/\text{var}(R) = c_0^2 + (c_1 - c_0)^2 + \dots + (c_k - c_{k-1})^2 + c_k^2 \quad (16)$$

and the mean yield in proportion to the mean recruitment is

$$\bar{Y}/\bar{R} = c_0 + c_1 + \dots + c_k \quad (17)$$

The short-term variability factor is defined by equation (9) with Y substituted for B. Thus:

$$H_2 = \left[\frac{\text{var}(\Delta Y)/\bar{Y}^2}{\text{var}(R)/\bar{R}^2} \right]^{1/2} \\ = \left\{ \frac{c_0^2 + (c_1 - c_0)^2 + (c_2 - c_1)^2 + \dots + (c_k - c_{k-1})^2 + c_k^2}{(c_0 + c_1 + \dots + c_k)^2} \right\}^{1/2} \quad (18)$$

The most stable condition occurs when H_2 is minimum. If all the c_n are multiplied by a constant factor, H_2 is unchanged. Thus the set of c_n values corresponding to the minimum H_2 is not unique. However, there is a unique set of the ratios c_n/c_0 for which H_2 is minimum. Differentiating H_2 with respect to c_n leads to the following recurrence relationship for the minimum condition, defined by the set of simultaneous equations $\partial H_2 / \partial c_n$ for $n = 1 \rightarrow (k-1)$.

$$c_{n+1} - 2c_n + c_{n-1} = - \{H_2(\underline{c})\}^2 \sum_{i=0}^{k-1} c_i \quad (19)$$

where \underline{c} is the vector $[c_0, c_1, c_2 \dots c_k]$. Equation (19) is valid for $n=(k-1)$ if we define c_k to be zero. The solution involves complicated algebraic manipulation, but it may be verified by substitution in (19) that H_2 is minimum when

$$c_n/c_0 = 1 + n(k-n)/k \quad (20)$$

This implies that the optimum c_n/c_0 are symmetric about the middle age in the fishery. Furthermore, c_n/c_0 has a maximum value of $(k+2)/4$ at the middle age and decreases to 1 for the youngest ($n=0$) and oldest ($n=k-1$) ages. The corresponding minimum of H_2 is obtained by substituting (20) in (18) and summing over n . It is:

$$H_{2\min} = 2\sqrt{3} / \{k(k^2 + 3k + 2)\}^{1/2} \quad (21)$$

Some examples of the c_n/c_0 values which minimise H_2 are given in Table 1. Note that for $k=2$, $H_{2\min} = H_{1\min}$; but for any $k>2$, $H_{2\min} < H_{1\min}$.

TABLE 1: Values of c_n/c_0 which minimise the short-term variability factor H_2 for a fishery on k year-classes. n is the age relative to the year-class recruiting at $n=0$

n	$k=1$	$k=2$	$k=3$	$k=4$	$k=5$	$k=6$
0	1.00	1.00	1.00	1.00	1.00	1.00
1		1.00	1.33	1.50	1.60	1.67
2			1.00	1.50	1.80	2.00
3				1.00	1.60	2.00
4					1.00	1.67
5						1.00
H_2	1.414	0.707	0.447	0.316	0.239	0.189

n	$k=7$	$k=8$	$k=9$	$k=10$	$k=11$	$k=12$
0	1.00	1.00	1.00	1.00	1.00	1.00
1	1.71	1.75	1.78	1.80	1.82	1.83
2	2.14	2.25	2.33	2.40	2.45	2.50
3	2.29	2.50	2.67	2.80	2.91	3.00
4	2.14	2.50	2.78	3.00	3.18	3.33
5	1.71	2.25	2.67	3.00	3.27	3.50
6	1.00	1.75	2.33	2.80	3.18	3.50
7		1.00	1.78	2.40	2.91	3.33
8			1.00	1.80	2.45	3.00
9				1.00	1.82	2.50
10					1.00	1.83
11						1.00
H_2	0.154	0.129	0.110	0.095	0.084	0.074

The next step is to determine the exploitation pattern $[F_n] = [z_n - M_n]$ which minimises H_2 . $[F_n]$ is not unique; one of the fishing mortalities must be known to obtain a solution. Suppose F_0 , the fishing mortality on the recruiting year-class is known. Then $z_0 = M_0 + F_0$ and

$$c_0 = W_0 (F_0/z_0) \{1 - \exp(-z_0)\} \quad (22)$$

The c_n are immediately obtained from the optimum ratios (c_n/c_0). To determine z_n , equation (13) is rearranged as

$$(1 - M_n/z_n) \{1 - \exp(-z_n)\} = (c_n/W_n) \exp\left(-\sum_{j=0}^{n-1} z_j\right) \quad (23)$$

There is no analytical solution of (23), but a numerical solution by computer program is straightforward. z_1 is first calculated from the known z_0 , using equation (23) with $n=1$, then z_2 using $n=2$ and so on.

It is not always possible to find real z_n corresponding to the minimum H_2 . Since $z_n > M_n > 0$, the left hand side of (23) lies between 0 and 1. Hence there is no solution for any n for which the right hand side of (23) is greater than 1. This will occur if F_0 is greater than some limit, F_{00} say. However, if $F_0 < F_{00}$ there is always an exploitation pattern which minimises H_2 , although this optimum pattern changes with F_0 which must increase with the fishing effort. When the effort exceeds an amount such that equation (23) has no solution, H_2 will be more than H_{2min} whatever the pattern of fishing mortalities.

3. RESULTS

Figure 1 shows how H_1 and H_2 depend on the fishing mortality for a stock for which $(G-M) = -0.1$, M being the natural mortality averaged appropriately over the year-classes in the fishery. Both measures increase monotonically with F , as does the ratio H_2/H_1 . As noted earlier, $H_2=H_1$ when $h=0.5$, or $F = (G-M) + 0.69$.

We have considered the age-structured model with reference to the biological and fishing data for the North Sea cod, see Table 2 which has been extracted from Anon (1991). Eleven year-classes are included in this table, but c_n/c_0 is very small (<0.01) for ages greater than 9. H_2 is 0.378, slightly less than H_1 at 0.501. However, the minimum H_2 for nine year-classes (Table 1) is 0.110, so the exploitation pattern is far from optimum. If there were nine year-classes in the fishery being exploited optimally, the short-term variability factor could in principle be reduced by nearly four times.

TABLE 2: Biological and fishing data for North Sea cod from Anon (1991). W is the weight per fish and F is the fishing mortality, both mean values for the period 1985 to 1989. Relevant statistics calculated from equations (14, 17, 18) are $\bar{Y}/\bar{R} = 0.485$, $H_1 = 0.501$ and $H_2 = 0.378$

Age	M	W (kg)	F	c_n/c_0
1	0.80	0.50	0.14	1.00
2	0.35	0.93	0.97	3.59
3	0.25	1.93	0.964	2.06
4	0.2	3.72	0.859	1.12
5	0.2	6.01	0.817	0.60
6	0.2	8.12	0.804	0.29
7	0.2	9.90	0.809	0.13
8	0.2	11.57	0.812	0.05
9	0.2	12.83	0.777	0.02
10	0.2	14.15	0.752	0.00
11	0.2	14.75	1.293	0.00

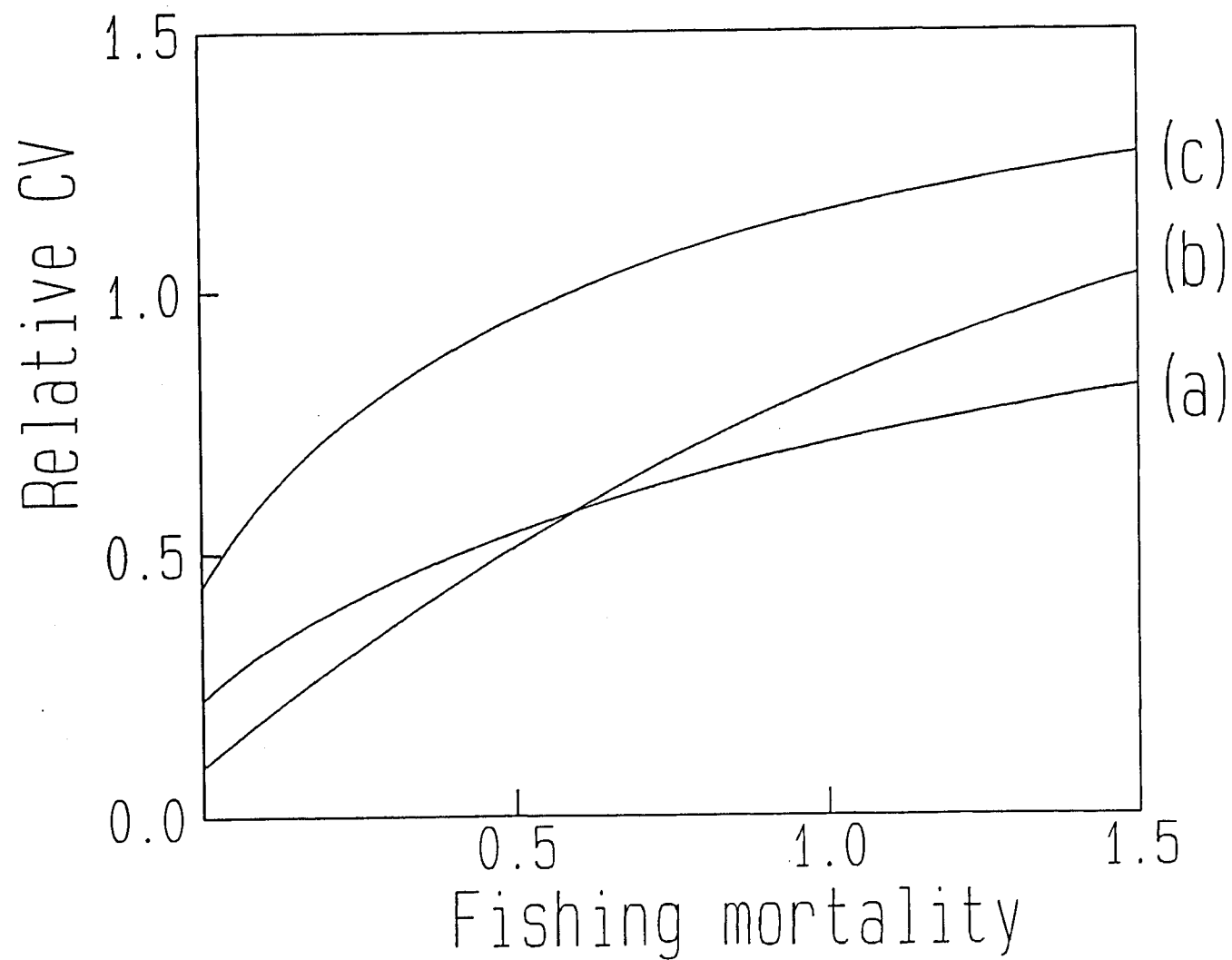


FIGURE 1

Dependence of the variability factors H_1 and H_2 on the fishing mortality for a stock with natural growth $G-M = -0.1$; (a) H_1 ; (b) H_2 ; (c) H_2/H_1

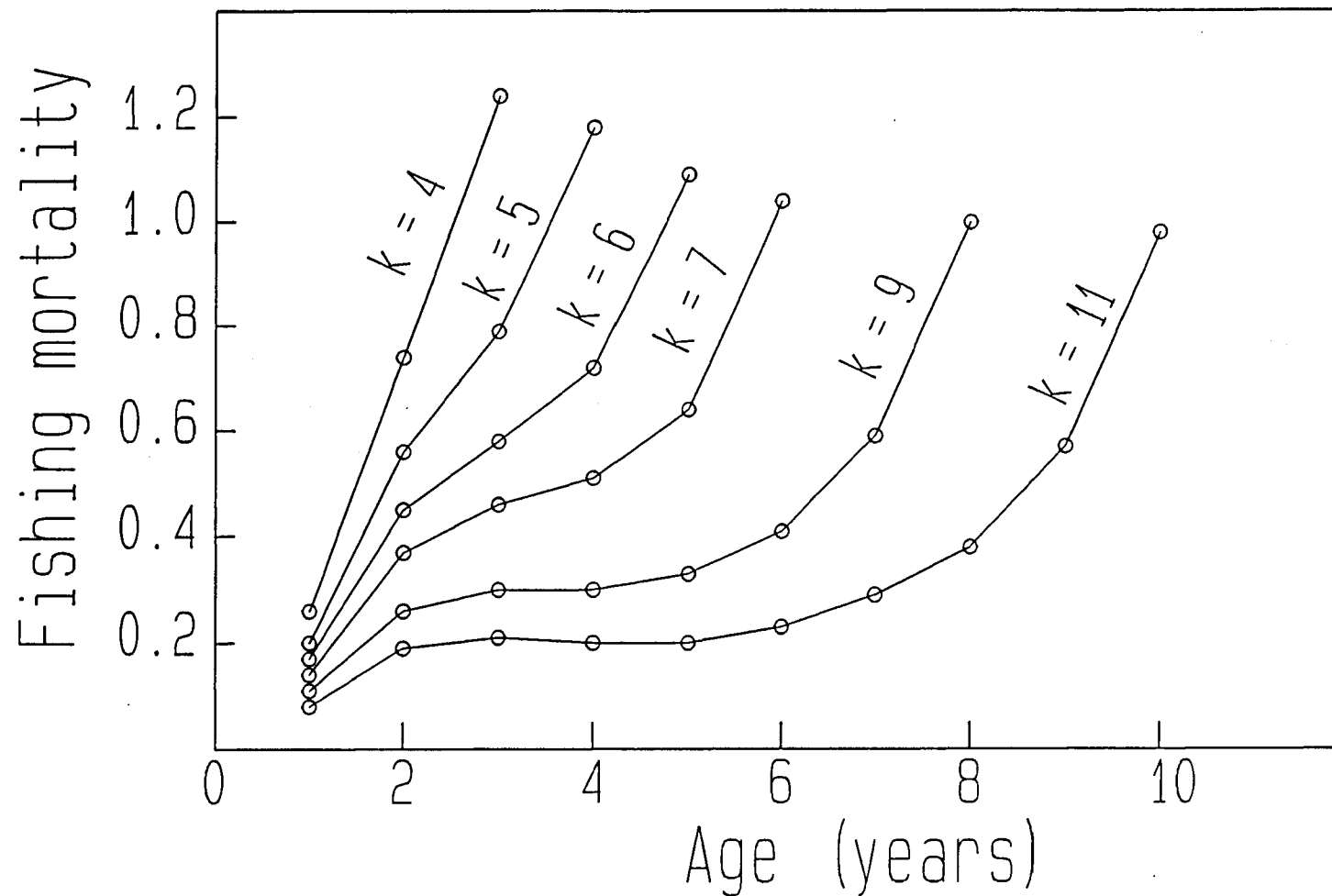


FIGURE 2

Exploitation patterns for minimum H_2 and maximum F at age 1 with k year-classes in the fishery, using biological data for North Sea cod.

TABLE 3: North Sea cod, dependence of the variability factors H_1 and H_2 on fishing effort. F_a is the actual mean fishing mortality at age in the period 1985-1989. F/F_a is the proportional change applied equally to all age groups

F/F_a	H_1	H_2
0.3	0.338	0.122
0.4	0.356	0.147
0.5	0.377	0.180
0.6	0.401	0.218
0.7	0.426	0.258
0.8	0.452	0.298
0.9	0.477	0.339
1.0	0.501	0.378
1.2	0.544	0.450
1.4	0.581	0.512
1.7	0.623	0.583
2.0	0.651	0.630

Reduction of the fishing effort would lead to more year-classes in the fishery. Table 3 shows how H_1 and H_2 change with the fishing mortality, supposing that F is changed by the same factor for all age groups relative to F_a , the mean values for the current fishery given in Table 2. The results for small F/F_a are approximate because the contribution of the plus-group may then be important. However, the calculations demonstrate the more rapid change of H_2 with fishing effort compared to H_1 . For example, if F were reduced by 30% ($F/F_a = 0.7$), then H_1 and H_2 would reduce by 15% and 32% respectively.

Some examples of hypothetical exploitation patterns are shown in Table 4 and repeated graphically in Figure 2. In each case the fishing mortalities are such that H_2 is minimised for the stated number of year-classes in the fishery. Furthermore, F on the youngest age has been set equal to F_{∞} , the largest value for which the solution of equations (13) and (20) is possible in terms of real fishing mortalities. This condition results in F for the oldest age being infinite. Such a solution may not be realisable in practice, but it does remove the problem of the plus-group!

TABLE 4: Exploitation patterns which minimise H_2 for k year-classes in the North Sea cod fishery. In each case, F on the recruiting year class is F_{∞} , the largest initial F for which a solution is possible

	k=4	k=5	k=6	k=7	k=8	k=9	k=10	k=11
Age	F	F	F	F	F	F	F	F
1	0.26	0.20	0.17	0.14	0.12	0.11	0.09	0.08
2	0.74	0.56	0.45	0.37	0.31	0.26	0.23	0.19
3	1.24	0.79	0.58	0.46	0.37	0.30	0.25	0.21
4	∞	1.18	0.72	0.51	0.39	0.30	0.24	0.20
5		∞	1.09	0.64	0.44	0.33	0.25	0.20
6			∞	1.04	0.60	0.41	0.30	0.23
7				∞	1.03	0.59	0.40	0.29
9					∞	1.00	0.57	0.38
9						∞	1.01	0.57
10							∞	0.98
11								∞
H_1	0.509	0.459	0.421	0.394	0.366	0.349	0.331	0.318
H_2	0.316	0.239	0.189	0.154	0.129	0.110	0.095	0.084

4. DISCUSSION

As already shown by MacLennan and Shepherd (1988), the variation in the long-term measure H_1 is only moderate, and halving fishing mortality from 1 to 0.5 would only decrease variability (increase stability) by about 30%. The short-term measure H_2 is by contrast more nearly proportional to fishing mortality, and halving the fishing mortality would give an increase of stability of about 60%.

The shape of the exploitation pattern which minimises H_2 depends on the number of year-classes in the fishery (Fig. 2). When k is small, F increase steadily from the youngest to oldest ages. When k is large, there is a sharp change between ages 1 and 2, then a plateau extending over most of the middle ages. This suggests that in a heavily exploited fishery, gears which have a "knife-edged" selection characteristic are not the best choice if the reduction of short-term catch variability is considered to be an important objective.

At low levels of F , we find that H_2 is less than H_1 , as expected because the auto-correlation of the stock size is high, and year-to-year changes are less than the long-term changes. By contrast, when F exceeds about 0.5, H_2 is greater than H_1 , because the differencing associated with the short-term measure overwhelms the (reduced) auto-correlation of the stock size. This change-over is interesting, because practical experience suggests that instability of stocks (and yield) becomes troublesome for fishing mortality levels greater than 0.5, and this may be the explanation.

It is suggested that the short-term measure of variability presented here is a better indicator of the problem as perceived by fishermen, compared to the simple variance-based indicators used previously. Since H_2 is roughly proportional to fishing mortality over the range up to 1.5 per year, this conclusion provides a further powerful incentive for aiming at low fishing mortalities, preferably not exceeding 0.5 per year. The analysis presented here has

used a simple representation of an exploited stock which ignores any stock-recruitment relationship, but this should certainly be adequate for elucidating the gross behaviour of variability as a function of the level of exploitation. More detailed analysis of finer features of the dependence will require the use of more complex theory such as the transfer function model of Horwood and Shepherd (1981). The measure of short-term variability proposed here should, however, still be useful.

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