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THE USE OF CONDITIONAL AND VIRTUAL COEFFICIENTS OF MORTALITY FOR RETROSPECTIVE ASSESMENT OF THE ABUNDANCE OF YEAR CLASSES OF FISH

by

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Abstract

The expression of fish mortality by the simplest and most realistic way - by a fraction or percentage - is hampered by difficulties of separate estimation of interactional coefficients of annual fishing and natural mortalities. Acting concurrently and being constantly competitive these two factors depress each other. The analysis of this decrease and defenition of real coefficients are carried out in this paper. By means of these coefficients it was made an attempt to restorate in the number of year classes without traditional assumption of their exponential abundance decrease. "Start" fishing mortality is determinated for the most representative neighbouring mean age groups in which like values of annual coefficients of mortalities were accepted as equal.

Analyzing the systems of representing fish mortality Ricker (Ricker,1975) suggested that it would be most realistic and simple if it was expressed as the virtual mortality coefficient in terms of percentage or portions of the abundance of year class which died by the beginning of the year or were caught per annum.

This approach, basically correct, was, however, assumed to hold little promise due to difficulties associatied with assessing those forms of mortality expressed in this way. The fishing mortality and mortality caused by other factors nonassociated directly with the fishery, acting at the same time and being constantly competitive, depress each other.

Until recently, only the summary value of this decrease was estimated without differentiating each rate of mortality and extent of their mutual interaction with respect to the estimates of the one or the other factor. Apart from the difficulties associated with estimating portions of each factor, the development of this approach seemed to be impeded by the fact that when F.I.Baranov (Baranov,1918) published his book the fishing mortality and natural mortality were represented by relative mortality rates (instantaneous coefficients) which are not subject to interrelation and as if they exist independently from each other.

This approach realized by Fry (Fry,1949) as VPA and modified by Gulland (Gulland,1966) and Pope (Pope,1972) is attracting more and more followers and devotees.

Various modifications of the VPA suitable for computations allow for resolving many applied problems of fishery science in a fairly operative manner. But it must be admitted that even they suffer from some drawbacks and do not meet high modern requirements in many cases. There is still often an unbalance between the actual decrease in the abundance of year classes and exponential mortality, which is the basis of most traditional methods of computation, resulted in serious errors in the final conclusions, therefore some other possible means for assessing fish stocks should be found.

Interrelations of natural mortality coefficients and fishing mortality coefficients

The interrelation of coefficients of natural mortality and fishing mortality was analyzed in the previous paper (Borisov, 1988) where both factors worked at the same time. Nevertheless, some main positions of the analysis should be discussed again to substantiate and logically state the method of assessing the abundance of year classes of fish, presented here.

From the standpoint of the law of probabilities instantaneous coefficients of mortality are considered as probabilities of two independent events. This is justified by the fact that one of them indicates the mortality rate caused by fishing (F) and the other shows the mortality rate by natural causes (M) which is not associated with the former. So the probability (P) of their sum is equal to the sum of their probabilities : $P(F+M) = P(F) + P(M)$. In case both factors are expressed as portions from the initial abundance, the two events proceeding at the same time should be considered as interrelated since the probability of taking fish P(m) affects that of their natural mortality $P(n)$ and vice versa. The probability of the sum of such two event $P(A)$, as is known, is calculated by the following formula : $P(A)= P(k) + P(m) - P(mn)$. It is this dependence that W.E. Ricker (Ricker, 1975) used for finding the total annual mortality

$$A = m+n-mn \qquad (1)$$

Here the product mn is the size of the total summary dacrease in the conditional coefficients of mortality, i.e.

$$mn = \Delta m+\Delta n \qquad (2)$$

The cause of the reduction in m is an inevitable natural elimination among the specimens which would be among those to be caught per annum, whereas n is reduced because candidates for natural elimination are annually taken by the fishery on an equal footing with other specimens. The reduction in n depends upon the fishing intensity, i.e. Δn depends on m. In its turn, the reduction in m (or Δm) is directly associated with the impact of natural mortality on the fishable part of the population or Δm depends on n. So, we can write

$$\frac{m}{n} = \frac{\Delta n}{\Delta m} \quad \text{or} \quad m\Delta m = n\Delta n \quad \text{or} \quad \Delta m = \frac{n\Delta n}{m} \qquad (3)$$

Keeping in mind that

$$\Delta m + \Delta n = mn \quad \text{or} \quad \Delta m = mn - \Delta n \qquad (4)$$

Equating (3) and (4) we find

$$\Delta n = \frac{m^2 n}{m+n}$$ and in the same manner we have

$$\Delta m = \frac{n^2 m}{m+n}$$

Now the virtual (but not conditional) fishing mortality (m^r) existing under the simultaneous impact of natural causes of mortality, can be expressed as

$$m^r = m - \frac{n^2 m}{m+n} \qquad (5)$$

and the virtual natural mortality (n^r) as

$$n^r = n - \frac{m^2 n}{m+n} \qquad (6)$$

The determination of the "start" value of the conditional coefficient of fishing mortality

The total annual mortality $A = m+n-nm$ from the age i to the age $i+I$ reduces the initial abundance of the year class N_i by $N_i(n+m-nm)$, i.e. $N_{i+I} = N_i - N_i(n+m-nm)$ or

$$N_{i+I} = N_i(I-m)(I-n) \qquad (7)$$

Let us note in passing that the product $(I-n)(I-m)$ is nothing but the survival coefficient S. This is seen from a simple rearranging: $S = I-A = I-(m+n-mn) = (I-n) - m(I-n)(I-m)$. 214

The abundance of the year class N_i at age i represents a sum of three components: the number of specimens C_i caught per annum; the number of specimens died of natural causes per annum E_i which can be represented with the help of (6) as

$$E_i = N_i\left(n_i - \frac{n_i m_i^2}{n_i + m_i}\right);$$

of the remainder of this year class by the beginning of the next year N_{i+I}. Then, based on the mentioned above

$$N_i = C_i + N_i\left(n_i - \frac{n_i m_i^2}{n_i + m_i}\right) + N_{i+I} \qquad (8)$$

Rearranging (8) we have:

$$N_i = \frac{(n_i + m_i)(N_{i+I} + C_i)}{n_i m_i^2 + (n_i + m_i)(I-n_i)}$$

and substituting N_{i+I} for the corresponding (7) here, we have:

$$N_i = \frac{C_i(n_i + m_i)}{m_i \quad (n_i + m_i - n_i^{\ 2})} \qquad (9)$$

In order to find N_i, we have to know n_i and m_i. If conditional coefficients of natural mortality can be roughly obtained in some indirect manner(through relations of natural mortality with maximum age or age at maturity; physiological indicators of emaciation, weakness and ageing ; data on unfished populations and so on), the determination of conditional coefficients of fishing mortality existing only in theory, but needed for calculations, can be done analytically through other empirically accessible values.

Our numerous attempts to find conditional coefficients of fishing mortality from the above mentioned expressions were useless until assumptions of the equalities $n_i = n_{i+I}$ and $m_i = m_{i+I}$ were made. The error made in the assumption is insignificant if two neighbouring mean age groups are taken. In fact, the coefficients of natural mortality in ages corresponding to age at maturity of fish are almost equal forming a plateau on a parabolic curve showing their changes with age (Tyurin, I972). On the other hand, the catchability of fish from two neighbouring mean age groups is equal or almost equal, hence it seems to be reasonable to state that their coefficients of fishing mortality are also equal or almost equal. However, the last note will be valid under one condition - if the fishing efforts were equal for two neighbouring years. It should be noted that the year class analyzed at the age i was fished in year j; next year j+I the year class is at the age i+I, but the fishing effort f_{i+I} will differ from the previous f_i , therefore the catch C_{i+I} should be brought into line with the fishing efforts exercised in year j, i.e. C_{i+I} should be increased (or decreased) by so many times again as f_{i+I} was more (or less) than the last f_i. This procedure provides an answer what kind of the catch C_{i+I} would be on condition the fishing efforts are equal in year j and year j+I .

Hence, accepting the equalities $n_i = n_{i+I} = n$ and $m_i = m_{i+I} = m$ for fish from the same year class and using (9) the absolute numerical strengths of two neighbouring mean age groups can be written as

$$N_i = \frac{C_i(n+m)}{m(n+m-n^2)} ; \qquad N_{i+I} = \frac{C_{i+I} \quad (n+m)}{m\ (n+m-n^2)} .$$

By substituting them into (7),we have:

$$\frac{C_i(n+m)}{m(n+m-n^2)} = C_i + \frac{C_i(n+m)\ [n(n+m)-nm^2]}{m(n+m-n^2)\ (n+m)} + \frac{C_{i+I}(n+m)}{m(n+m-n^2)} .$$

which after transformations and solution for m will give us:

$$m = I - \frac{C_{i+I}}{C_i\ (I-n)} \qquad (I0)$$

The same result can be achieved in a simpler logical way. Rearranging the catches of fish from the same year class taken for two neighbouring years to equal fishing efforts we transform them into indices of abundance, i.e. C_i will be more than C_{i+I}^{cor} by so many times as N_i is more than N_{i+I}. Owing to this, we can write $C_{i+I} = C_i(I-n)\ (I-m)$ by the analogy with $N_{i+I} = N_i(I-n)(I-m)$ and from here

$$m = I - \frac{C_{i+I}}{C_i(I-n)} .$$

So, m found in this way will be a "start" value of the conditional coefficient of fishing mortality, on the basis of which and using conditional coefficients of natural mortality differentiated by ages and catches of this year class by years we shall obtain its abundance at all fishable ages.

The use of conditional and virtual coefficients of mortality in calculations of the abundance of a year class

The procedure of finding the numerical strength of a year class by each age group consists of two main stages differing by the way of calculations. We shall find the abundance of the year class for ages older than age i at the first stage and younger than age i at the second stage.

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The first stage. By substituting the start value of m_i found into (9) N_i will be calculated, and using N_i in Formula (7) we have N_{i+I}. The characteristics found allow for finding n_i^r and, thus, E_i, i.e. the number of fish died of other than fishing causes. Having the value N_{i+I} and knowing the virtual (without correction) value C_{i+I} the virtual coefficient of fishing mortality will be found:

$$m^r_{i+I} = \frac{C_{i+I}}{N_{i+I}}$$

In order to find the abundance $N_{i+2} = N_{i+I}\,(I-m_{i+I})\,(I-n_{i+I})$ we must calculate m_{i+I} from Equation (5):

$$m^r_{i+I} = m_{i+I} - \frac{m_{i+I}n^2_{i+I}}{n_{i+I} + m_{i+I}}$$

where from

$$m_{i+I} = \frac{\sqrt{(n_{i+I} - n^2_{i+I} - m^2_{i+I}) + 4n_{i+I}m^r_{i+I}} - (n_{i+I} - n^2_{i+I} - m^r_{I+I})}{2} \qquad (II)$$

Knowing m_{i+I}, we shall find N_{i+2} using formula (7) and knowing C_{i+2} we have m^r_{i+2}, hence we can find m_{i+2} from (II). Then using N_{i+2}, n_{i+2} and m_{i+2} we have N_{i+3} and so on till the last age group taking part in the fishery. The validity of the parameters found for each age can be checked by the obligatory convergence obtained for a given age group abundance and its three components:

$$N_i = C_i + E_i + N_{i+I} .$$

<u>The second stage</u>. While finding the abundance of a year class younger than the start age some difficulty arises only in association with finding the conditional coefficient of fishing mortality. We shall use Equation (7) for age i-I to obtain it:

$$N_i = N_{i-I}(I-n_{i-I})(I-m_{i-I}).$$

Here the unknown multiplier N_{i-I} can be substituted by (9) for i-I ,hence

$$N_i = \frac{C_{i-I}(n_{i-I}+m_{i-I})(I-n_{i-I})(I-m_{i-I})}{m_{i-I}(n_{i-I}+m_{i-I}-n^2_{i-I})}$$

With the aim of making it more compact the indices can be eliminated assuming that $N_i=N$; $C_{i-I}=C$; $n_{i-I}=n$; $m_{i-I}=m$.

Then, after removing the brackets and multiplying the left part by the denominator of the right part, Expression (I2) will take on the form:

$$Nmn-Nmn^2+Nm^2 = Cn-Cnm-Cn^2+Cn^2m+Cm+Cm^2-Cmn+Cm^2n,$$

which can be written in the form of a quadratic equation referring to the unknown m:

$$m^2\left(\frac{N}{I-n} + C\right) + m\left[Nn - C(I-n)\right] -Cn = 0.$$

For the sake of simplicity, write „b" for the coefficient of „m" and „a" for that of „m^2", we have:

$$m = \frac{\sqrt{b^2 + 4aCn} - b}{2a} \qquad (I3)$$

It should be remembered that the indices were above eliminated, so in this case $m = m_{i-I}$ and correspondingly

$$a = \frac{N_i}{I-n_{i-I}} + C_{i-I}; \quad b = N_i\, n_{i-I} - C_{i-I}(I-n_{i-I}).$$

Now, when m_{i-I} is found we obtain the virtual coefficient of fishing mortality m^r_{i-I} from (5) and using the catch of this year class at this age C_{i-I} the absolute abundance is found:

$$N_{i-I} = \frac{C_{i-I}}{m^r_{i-I}} .$$

The values found allow us to estimate the number to fish died of natural causes E_{i-I}. Using the formula (6) we have

$$n^r_{i-I} = n_{i-I} - \frac{m_{i-I}\, n_{i-I}}{m_{i-I} + n_{i-I}}, \quad \text{and then } E_{i-I} = N_{i-I}\, n^r_{i-I} .$$

The sum $N_i + C_{i-I} + E_{i-I} = N_{i-I}$ can be used for checking the validity of N_{i-I} calculated.

In case beside age group i-I, age group i-2 also took part in the fishery, we shall again use (I3) assuming that $m = m_{i-2}$;

$$a = N_{i-I} : (I-n_{i-2}) + C_{i-2}; \quad b = N_{i-I}\, n_{i-2} - C_{i-2}\,(I-n_{i-2}).$$

Then we shall find m_{i-2}, then N_{i-2} and E_{i-2}.

Illustration of the calculation according to the scheme suggested

One of the year classes of navaga from the Pechora River is used as an example. The data available on the annual catch (in specimens), the age composition of catches, fishing efforts and conditional coefficients of natural mortality (Borisov, Zalesskikh, I980) allow for making a retrospective restoration of the abundance of year classes of the population according to the scheme suggested. The results of the calculations are shown in Tables I and 2.

The data in Columns 2 and 7 are known. On the basis of them all calculations are made and the rest columns filled up. The sequence of calculations was the following:

I. The conditional coefficients of natural mortality of navaga are the same at ages 3 and 4, namely $n_3 = n_4 = 0.28$. Let's assume that the conditional coefficients of fishing mortality of these age groups are also equal, namely $m_3 = m_4$. On this assumption, using the formula (IO) we have;

$$m_3 = m_4 = I - \frac{C_4}{C_3 (I - n_3)} = I - \frac{3816}{7066\ (I - 0.28)} = 0.25$$

2. The equalities $m_3 = m_4$ and $n_3 = n_4$ suggest also equalities $m_3^r = m_4^r$ and $n_3^r = n_4^r$ which according to (5) and (6) gives:

$$m_3^r = m_4^r = m_3 - \frac{m_3\ n_3^2}{m_3 + n_3} = 0.25 - \frac{0.25 \times 0.28^2}{0.53} = 0.2I3;$$

$$n_3^r = n_4^r = n_3 - \frac{n_3\ m_3^2}{m_3 + n_3} = 0.28 - \frac{0.28 \times 0.25^2}{0.53} = 0.247.$$

3. The sum m_3^r and n_3^r gives the coefficient of total mortality $A_3 = A_4 = 0.46$. The same result can be obtained by the formula (I) : $A_3 = A_4 = m_3 + n_3 - m_3 n_3 = 0.25 + 0.28 - 0.25 \times 0.28 = 0.46$

4. Knowing m_3^r and m_4^r we have $N_3 = C_3 : m_3^r = 7066 : 0.2I3 = 33I75$; $N_4 = C_4 : m_4^r = 38I6 : 0.2I3 = I79I5$.

5. Using n_3^r and n_4^r we have also $E_3 = N_3 n_3^r = 33I75 \times 0.247 = 8I94$:

Table I

Results of calculations of the abundance of the year class on condition mortality and fishing efforts are equal at two neighbouring years ($n_3=n_4;m_3=m_4$) and ($f_3=f_4$)

Index								
Age	n	m	n^r	m^r	A	C	E	N
I	2	3	4	5	6	7	8	9
I	0.43	0.02	0.429	0.0I2	0.44I	I07I	32288	89250
2	0.30	0.05	0.298	0.037	0.335	I853	I4863	4989I
3	0.28	0.25	0.247	0.2I3	0.460	7066	8I94	33I75
4	0.28	0.25	0.247	0.2I3	0.460	38I6	4425	I79I5
5	0.3I	0.45	0.227	0.393	0.620	3803	2I95	9674
6	0.36	0.20	0.334	0.I54	0.488	566	I228	3676
7	0.43	0.48	0.320	0.382	0.702	7I9	602	I882
8	0.50	0.33	0.435	0.232	0.666	I30	244	56I
9	0.63	0.I4	0.6I4	0.069	0.683	I3	II5	I87
I0	0.75	0.I0	0.74I	0.034	0.775	2	44	59

Table 2

Results of calculations of the abundance of the year class when $n_3=n_4$ and fishing efforts f_4 in year j are I.5 time more intensive than f_3 in year j-I.

Age	n	m	n^r	m^r	A	C	E	N
I	2	3	4	5	6	7	8	9
I	0.43	0.02	0.429	0.0I2	0.44I	I07I	38288	89250
2	0.30	0.05	0.298	0.037	0.335	I846	I4868	4989I
3	0.28	0.25	0.247	0.2I3	0.460	7067	8I95	33I77
4	0.28	0.36	0.224	0.3I9	0.543	5724	4008	I79I5
5	0.3I	0.45	0.227	0.393	0.620	32I6	I857	8I83
6	0.36	0.20	0.334	0.I54	0.488	479	I039	3II0
7	0.43	0.48	0.320	0.382	0.720	608	509	I592
8	0.50	0.33	0.435	0.232	0.667	II0	207	475
9	0.63	0.I4	0.6I4	0.069	0.683	II	97	I58
I0	0.75	0.I0	0.74I	0.034	0.775	2	37	50

$E_4 = N_4 n_4 = 17915 \times 0.247 = 4425.$

Hence the columns for ages 3 and 4 are filled in the table. Moreover, the parameters determined for age group 4 allow for assessing the abundance and virtual fishing mortality in the following age group 5.

6. Based on the formula (7) we have

$N_5 = N_4 (1-m_4)(1-n_4) = 17915 (1-0.25) \times (1-0.28) = 9674.$

The same result can be obtained in another way:

$N_5 = N_4 - C_4 - E_4 = 17915 - 3816 - 4425 = 9674.$

Taking the catch from age group 5 into account, we have

$$m_5^r = C_5 : N_5 = 3803:9674 = 0.393.$$

7. Now, having n_5 and m_5^r, there is a possibility of obtaining the conditional coefficient of fishing mortality for this age :

$$m_5 = \frac{\sqrt{(0.31-0.31^2-0.393)^2 + 4 \times 0.31 \times 0.393} - (0.31 - 0.31^2 - 0.393)}{2} = 0.45$$

then taking the formula (6) we have the virtual coefficient of natural mortality: $n_5^r = 0.31 - \frac{0.31 \times 0.45^2}{0.76} = 0.227$ and thus the number of specimens of this year class died of natural causes at age 5 will be $E_5 = N_5 n_5^r = 9674 \times 0.227 = 2196$ and the coefficient of annual mortality will be : $A_5 = n_5 + m_5 - n_5 m_5$ or $A_5 = n_5^r + m_5^r = 0.227 + 0.393 = 0.62$. So, all columns in Table 2 are filled for age 5. The procedure is repeated for the next age group.

By analogy with Paragraph 6 we have $N_6 = N_5(1-m_5)(1-n_5) = N_5(1-A_5) = 9674 (1-0.62) = 3676$; $m_6^r = C_6/N_6 = 566:3676 = 0.154$.

The repetition of the procedures stated in Paragraph 7 will give $m_6 = 0.2$; $n_6^r = 0.334$; $E_6 = 1228$; $A_6 = 0.488$.

The calculations for other older age groups are made according to the same procedure. Let's estimate the same parameters for ages younger than the "start" one.

8. Using (I3) we find when

$$a = \frac{N_3}{I-n_2} + C_2 = 33I75:(I-0.3) + I853 = 49245.857$$

$$b = N_3 n_2 - C_2(I-n_2) = 8655.4$$

the conditional coefficient of fishing mortality for age 2:

$$m_2 = \frac{\sqrt{b^2+4aC_2n_2} - b}{2a} = \frac{\sqrt{749I5949.I6 + I09503087.62} - 8655.4}{9849I.7I4} = 0.05$$

9. The index found allows us to obtain from (5) and (6)

$$m_2^r = 0.05 - \frac{0.05 \times 0.3^2}{0.635} = 0.037;$$

$$n_2^r = 0.3 - \frac{0.3 \times 0.05^2}{0.35} = 0.298; \quad A_2 = m_2^r + n_2^r = 0.335;$$

and then $N_2 = C_2/m_2^r = I853:0.037 = 4989I$; $E_2 = N_2 n_2^r = 49$.

In order to obtain m_I, by analogy with Paragraph 8, we find $a = N_2(I-n_I) + C_I = 88\ 599$ and $b = N_2 n_I - C_I(I-n_I) = 20842$ which in (II) will give $m_I = 0.02$.

By repeating the procedure of Paragraph 9 we can find also $m_I^r = 0.0I2$; $n_I^r = 0.429$; $A_I = 0.44I$; $N_I = 89250$ and $E_I = 38288$.

The example is simplified as it assumes the same fishing intensity in the "start" year and following year. It is quite realistic, but such a coincidence occurs not so often.

The calculations shown in Table 2 illustrate a case when the year class at age 4 was fished I.5 times more intensively (by the amount of efforts) than the year class at age 3.

If the state of the population is such that the annual catch is proportional to the fishing efforts made, the equality $m_3 = m_4$ can be reached on the account of reducing the virtual catch C_4 by I.5 time. The operation makes it possible to use the formula (IO) to obtain $m_3 = m_4$, where instead of the virtual $C_4 = 5724$ the corrected $C_4^{cor} = 38I6$ must be used, i.e. the value reduced by I.5 time.

Having gained that the conditions $n_3 = n_4$ and $m_3 = m_4^{cor}$ are met, we find under Paragraph I that

$$m_3 = m_4^{cor} = I - \frac{C_4^{cor}}{C_3(I-n_3)} = I - 38I6 : 7076(I-0.28) = 0.25.$$

Using the formulae (5) and (6) we have $m_3^r = 0.2I3$ and $n_3^r = 0.247$. Naturally, they prove to be the same as in the first example (Table I) since the same initial data were used. Similar considerations refer to N_3, E_3, A_3, N_4 to be found under Paragraphs 3-6.

Further, having N_4 and the virtual catch C_4 we have already had a virtual coefficient of fishing mortality for age group 4:

$$m_4^r = C_4 / N_4 = 5724 : I79I5 = 0.3I95.$$

Now with known n_4 and m_4^r the genuine non-corrected value m_4 can be obtained using the formula (II):

$$m_4 = \frac{\sqrt{(0.28-0.28^2-0.3I95)^2 + 4 \times 0.28 \times 0.3I95} - (0.28 - 0.28^2 - 0.3I95)}{2} =$$

$$= 0.224.$$

Further calculations with respect to age 4 are made in full agreement with the procedure mentioned in Paragraph 7. The procedure is repeated in the same sequence in calculations in regard to the following age groups including the last.

The calculations under Paragraphs 8 and 9 will give necessary parameters for ages younger than the "start" age.

In essence, the data of Table 2 are analogous to those of Table I. Some differences stem from the increase in the coefficient m_4^r, which correspondingly involves changes in m_4, n_4^r, A_4, E_4. The increased catch of fish at age 4 affected the absolute abundance of the year class at the following ages and the number of specimens caught and died of natural causes despite the preservation of the former coefficients n_{5-I0} and m_{5-I0}.

Table 3

Calculation of the abundance of the same year class as was estimated in Table 3 using the VPA

Age	Index				
	n	M = - ln(I-n)	C	F	N
I	0.43	0.5620	I07I	0.0I62	86,906
2	0.30	0.3570	I846	0.0460	48,755
3	0.28	0.3285	7067	0.2909	32,584
4	0.28	0.3285	5724	0.4748	I7,542
5	0.3I	0.37I0	32I6	0.6544	7,856
6	0.36	0.4460	479	0.2343	2,8I8
7	0.43	0.5620	608	0.772I	I,427
8	0.50	0.6930	II0	0.4995	376
9	0.63	0.9940	II	0.I622	II4
I0	0.75	I.3860	2	0.I060	36

Table 3 shows the results of VRA calculations. To make the comparison more precise the same initial data as in Table 2 are used: instantaneous coefficients of natural mortality variable with ages are obtained on the basis of the relation $M_i = -\ln(I-n_i)$; the difference between Z_{IO} and M_{IO} is taken as a "start" instantaneous coefficient of fishing mortality (F_{IO}), i.e. $F_{IO} = Z_{IO} - M_{IO}$ where $Z_{IO} = -\ln(I-A_{IO})$. As is seen, on condition the initial data are equivalent, the numerical strengths of the year class by the same ages prove to be in comparative agreement.

Discussion

According to the traditional method of estimation of fish mortality based on the assumption that the mortality of year classes is exponential, it is believed that the decrease in the abundance (dN) in each size (or age) group is proportional to the abundance of the group available in a short period of time (dt).

This approach corresponds to the classical example from applied mathematics on radioactive decay of atoms. The radioactive decay law is in full agreement with a simplified scheme of assessing fish mortality: The relation of the number of decayed (died, caught) atoms (fish) per unit time to the total number of remained (survived) atoms (fish) at the present moment is a constant (Z) depending only on the type (species) of atoms (fish), i.e. $dN/Ndt = -Z$.

The appearing analogy of the processes allowed us to accept that the initial abundance of fish (N_o) decreased from age(t_o) to age(t) along the exponential curve, which was the result of fishing mortality and some other causes (M): $N(t) = N_o e^{-(F+M)t}$. In this case the mechanism of interrelation between the fishing and natural mortality remains concealed as these two forms of mortality are expressed by instantaneous indices (rates) which are independent from each other. The expression of mortality in terms of rates is mathematically convenient. Although the technique of calculations is simplified the convenience conceals a fairly significant inconsistency (beside the assumed exponential mortality which is not always substantiated): the interrelated processes of fishing and natural mortality running at the same time are studied on the basis of the characteristics expressed in terms of rates independent from each other. Hence, it seems to be more reasonable if coefficients of annual mortality could be expressed in fractions (as a percentage) of the initial abundance with regard to the interaction resulted from the simultaneous action of the two factors of mortality.

An example of practical realization of such an approach is considered in the present paper. It seems that it has some advantage over the VPA and its modifications.

Firstly, there is no necessity of putting data on the abundance of year classes in Procrustes' bed of the exponential curve, moreover there is often no weighty reason in favour of the assumption that mortality of the species studied occurs according to the exponential law during their life span or per annum.

Secondly, the "centrifugal" method suggested (the calculation starts from the central group of the age series toward younger and older age groups) specifies that the start value of the coefficient of fishing mortality is estimated by the middle age group which is often encountered in catches and, hence, catch statistics is fairly representative for this age group. This secure more reliable calculations in contrast to the traditional start from the oldest group where errors are unavoidable due to the difficulties encountered in age detemination of older groups, their infrequent occurrence and to the fact that sometimes representatives of several older age groups are pooled into the last one.

Thirdly, it offers possibilities of obtaining a start conditional coefficient of fishing mortality using most representative data on catch per effort for middle age groups, therefore catch per effort can be put into the formula (IO) instead of catches. The availability of relatively reliable data on catch per effort for a year class for two neighbouring years makes it possible to find the annual mortality A, which opens an additional possibility of estimating the start m. For this purpose, by solving the expression (I) $A = m+n-mn = m(I-n) + n$ referring to m when A is found, we have $m = \frac{A+n}{I-n}$.

Fourthly, the "centrifugal" method makes us observe more stringent requirements for initial information (the age composition, fishing efforts, catch statistics) , so that the conditions $N_i > N_{i+I}$; $C_i > C^{cor}_{I+I}$; $N_i - C_i - E_i = N_i(I-n_i)(I-m_i) = N_i - N_i(n^r_I + m^r_i) = N_{i+I}$. The last equalities secure the check of the validity of calculations and their correspondence to the reality at any intermediate stage, as well.

Particular emphasis and care should be placed on the moment of correction of catches for the start ages depending on the amount of efforts f_j (in year j) against efforts f_{j+I} (in year j+I). It is evident that the correlation between the annual fishing effort and annual catch is linear only on a certain interval. Further, the increase in fishing efforts will not give increase in the catch at some stage, or even it can result in the decline in the catch affecting the proportionality of the relation between these values. Thus, the correction of catches, if necessary, can be mantained according to the curvilinear dependence between efforts and catches to be found previously. The formalization of the described scheme for the computer does not seem to create difficulties at all.

Conclusion

Other merits and demerits of the "centrifugal" procedure of restoration of the numerical strengths of year classes with regard to the interaction of fishing and natural mortality will be revealed with use of the procedure to the study of particular populations and specific features of the fishery for them. It is likely that the procedure would be only slightly suitable or inadequate at all for some commercial species of fish, wheras for the others especially for those where there is available fairly reliable catch statistics, data on fishing efforts,catch per effort, information on natural mortality differentiated by ages, it can be more suitable than VPA and its modifications. The traditional fish stock assessment methods based on the assumption of exponential mortality of year classes cannot pretend to be universal. As is known, the sphere of their application is rather limited,e.g. the results of calculations of populations with a short span of life are,as a rule, unsatisfactory. In view of this, the approbation of new approaches is promising to extend our possibilities in the study of fish population dynamics and fish stock assessment.

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