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Some Applications of the Kalman Filter in Fisheries Research

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Summary

The Kalman filter is a tool for estimating the parameters of a relationship that is evolving over time. Two situations in fisheries research where its application might be appropriate are described. First, the relationship between the concentration of a contaminant in fish tissue and fish length, as a surrogate for age, may vary because of the changing availability of the contaminant in the environment. Second, the weight-at-age relationship of a fish stock may vary because of changes in the environment, including a reduction in the quantity and/or quality of prey species. Simple Kalman-filter models are developed for, and applied to, data representing these two situations. Since the Kalman filter is an empirical Bayes procedure, the data from all available years are used to obtain estimates for individual years that are less affected by limitations of the data such as outliers and missing or poorly distributed values of the independent variable; temporal trends may become more apparent. For the two situations considered, potentially meaningful trends are identified. The pricise causes of these trends are, however, outside the scope the the present paper.

Introduction

Relationships between some variable of interest and one or more concomitant variables which, for example, may be used for prediction of the variable of interest or its adjustment to a common level, are often assumed to be fixed over time. There are, however, situations where this does not seem to be the case. Two examples from fisheries research are the weight at age of species of fish from a specified region, and the relationship between the concentration of contaminants in fish tissue and a measure of fish size.

In particular, it has been observed that the weights at age of northern cod appear to have been decreasing in recent years (Baird et al. 1992). The causes of this are not fully understood but might include, *inter alios*, a reduction in the quantity and/or quality of prey species, a period of adverse environmental conditions and a reduction in the area of favourable habitat. The objective here is to clarify the nature of the temporal trend in the weight-age relationship; although such clarification, in itself, makes no pretence towards isolating the cause(s) of the trend, it can, perhaps, be viewed as a step in that direction.

As illustration of the second situation, ICES Cooperative Research Report No. 162 (1989) presents the regressions of the logarithm of the concentration of various contaminants (e.g. mercury, lead, zinc, PCB) in fish muscle on fish length, by year, for several species of fish. In general, these regressions show marked inter-year variability. It may be hypothesized that changes in the concentration - length relationship reflect a changing availability of the particular contaminant in the environment. Consider the following, perhaps overly simplistic scenario. Suppose that, through reglatory mechanisms, the availability of the contaminant is reduced over time. In the year or so following the reduction, one might expect to see a positive relationship between the concentration of the contaminant in fish tisse due to higher concentrations in the older and, therefore, larger fish, that were exposed to the contaminant before its reduction was initiated. Over the next few years, as the older fish are removed from the population through fishing or natural mortality, one might expect to see this relationship become progressively weaker until it disappears entirely. Conversely, the introduction of a contaminant into the environment might give rise to a negative relationship since the younger, and hence smaller, fish may then have disproportionately high concentrations. In actuality, these scenarios would be complicated by differential rates up uptake and excretion. With some knowledge of these rates and given scenarios, one may construct models of the relationship between concentration and fish age, or size. Although it may not be possible to associate observed patterns of change uniquely with hypothesized scenarios, it should be possible to gain some insight into which scenarios are feasible for the observed pattern and which are not. The identification of any scenario is beyond the scope of the present paper. The purpose here is to determine if, indeed, temporal trends in the concentration - length relationship exist and, if so, in what direction and magnitude.

In both of the above situations, it would seem reasonable to assume that the relationship in one year is, in some way, connected to the relationship in the previous year(s). The Kalman filter (Kalman 1960) provides a mechanism for modeling situations where the parameters of a relationship evolve over time. We here describe the development and application of Kalman-filter models to data representative of these two situations. For completeness a brief outline of Kalman-filter methodology is first presented.

The Kalman Filter: an Outline

In the notation of van Deusen (1989) the generic form of the Kalman filter is as follows:

The relationship between the vector of observations at time t, Y_t, and the parameters, α_t, is assumed to be

$$Y_t = F_t\alpha_t + v_t$$

where Y_t is a vector of length n_t (the sample size at time t), α_t is a $p \times 1$ vector of state parameters, F_t is a fixed $n_t \times p$ matrix, and v_t is an $n_t \times 1$ vector of residuals with zero expectation and variance matrix V_t. This is referred to as the observation or measurement equation.

The state parameters are variables that evolve over time, thus

$$\alpha_t = G_t\alpha_{t-1} + w_t$$

where G_t is a fixed $p \times p$ matrix and w_t is a $p \times 1$ vector of residuals with zero expectation and variance matrix W_t. This latter is referred to as the transition or system equation.

Let a_t denote the optimal estimator of α_t as based on all information up to and including Y_t and let $Var(a_t - \alpha_t) = P_t$. The prediction equation for α_t and the associated variance matrix, conditional on α_{t-1} and P_{t-1} are

$$a_{t/t-1} = G_t a_{t-1}$$

$$P_{t/t-1} = G_t P_{t-1} G_t' + W_t$$

When Y_t becomes available, the updating equations for the estimation of α_t and the associated variance matrix are

$$a_t = a_{t/t-1} + P_{t/t-1} F_t' H_t^{-1} E_t$$

$$P_t = P_{t/t-1} - P_{t/t-1}F_t'H_t^{-1}F_tP_{t/t-1}$$

where

$$E_t = Y_t - F_t a_{t/t-1}$$

and

$$H_t = F_tP_{t/t-1}F_t' + V_t.$$

(The setting of starting values a_0 and P_0 will be discussed below).

At time t, a_t is the optimal linear estimate given all previous information, but only the estimate at time T contains all available information. The optimal solution for any time t given all available information is referred to as smoothing or signal extraction. The optimal smoothed estimate of the state parameters, and the associated variance matrix, are then given by

$$a_{t/T} = a_t + P_t^*(a_{t+1/T} - G_ta_t)$$

$$P_{t/T} = P_t + P_t^*(P_{t+1/T} - P_{t+1/t})P_t^{*\prime}$$

where

$$P_t^* = P_tG_{t+1}'P_{t+1/t}^{-1}$$

with $a_{T/T} = a_t$ and $P_{T/T} = P_T$ as starting values.

The above requires knowledge of V_t and W_t. Maximum likelihood has been suggested as a means for estimating the unknown parameters. Under the assumption of normality, the log likelihood of a sample is given by

$$L = -\frac{1}{2}\sum_t(\log|H_t| + E_t'H_t^{-1}E_t).$$

Duncan and Horn (1972) observe that, under normality, a_t is the posterior mean for α_t given Y_t and is thus the Bayes estimator under squared error loss; they refer to Sage (1968 pp. 265-275) for a complete discussion. (See also e.g. Harrison and Stevens 1976, Meinhold and Singpurwalla 1983, Gruber 1985).

As starting values, following Harvey (1984), van Deusen (1989) takes $\alpha_0 = 0$ and $Var(\alpha_0) = kI$ where k is a large number. Visser and Molenaar (1988) state that "If no information is available in advance ... this uncertainty can be accounted for by choosing $a_{0/0}$ arbitrarily and $P_{0/0}$ very large". They go on to note that "The first few, say N_s iteration steps then serve as a transient period, in which the filter itself constructs reliable starting values for $t > N_s$". Duncan and Horn (1972) take $\alpha_{1/0} = \mu_1$, assumed known, and $P_{1/0} = W_1$ [our notation]. Meinhold and Singpurwalla (1983) indicate that "the recursive procedure is started off at time zero by choosing a_0 and Σ_0 [$= Var(\alpha_0|Y_0)$] to be our best guesses about the mean and variance of α_0, respectively". If T is small, i.e. if the series under study is of relatively short length, the approach advocated by van Deusen or Visser and Molenaar would seem to be inappropriate, since N_s might well represent a significant proportion of the series length, T. Careful choice of a_0 would then appear preferable to arbitrary selection. P_0 should then reflect the degree of uncertainty in this initial value. Duncan and Horn's (1972) choice of W_1 for $P_{1/0}$ would imply $P_0 = 0$, i.e. no uncertainty, which agrees with their assumption that $a_{1/0}$ is known. (Note that, since the G_t are known, a_0 known implies $a_{1/0} = G_1a_0$ known).

Implementation

CONTAMINANT CONCENTRATION AND FISH LENGTH

Data

The data are drawn from the ICES Cooperative Monitoring Programme. For implementation to be successful, series of reasonable length are required. None of the available series extend back beyond 1978 with the most recent available data being for, 1990. Accordingly, the maximum length of the available series is 13 years. Thirteen years is, perhaps, minimal for the application of the Kalman filter. The shortest series considered by Harvey (1989) in his definitive text comprises 26 years. Notwithstanding, if seemingly meaningful results are obtained, continued application of the method as the data base is extended would appear justified.

For illustration here the data have been limited to the concentrations of mercury and zinc in the muscle of cod and plaice from ICES statistical rectangle 31F2. The number of sampled fish of each species in any one year varies but is generally of the order of 20-25. The range of lengths of the sampled fish, however, varies considerably from year to year. There are years where all sampled fish were relatively small, years where the lengths were spread over a relatively large range, and years where most fish were small but there were also a few large individuals. In the first-mentioned situation the regression of concentration on length is poorly defined, while in the last-mentioned the regression may be strongly influenced by an atypical concentration in one of the larger fish. Consequently, any temporal trend in the individual regressions is likely to be obscured.

Very few observations were recorded as below detection limit for these four data sets; to avoid complications, these have been omitted with minimal effect.

Conventionally the logarithms of the concentrations have been regressed of fish length. Plots of the data, however, indicate little, if any need for logarithmic transformation. To determine if there would be any effect on the inferences being made, for some data sets analyses have been carried out on both the raw and transformed data. The inferences were unaffected. Accordingly, only the results based on the raw (untransformed) data will be presented.

Model

The following is suggested as the specific Kalman filter model for this these data.

$$Y_t = \alpha_{1t} + \alpha_{2t} x_t + v_t$$

where Y_t is an $n_t \times 1$ vector of concentrations or their logarithms, as appropriate, x_t is an $n_t \times 1$ vector of corresponding fish lengths and

$$V_t = \sigma_v^2 I_{n_t}$$

The simplest assumption that can be made for the state parameters is

$$\alpha_{1,t} = \alpha_{1,t-1} + \epsilon_{1,t}$$

$$\alpha_{2,t} = \alpha_{2,t-1} + \epsilon_{2,t}$$

with

$$Var(\epsilon_{jt}) = \sigma_j^2,\ Cov(\epsilon_{jt}, \epsilon_{j't}) = \rho\sigma_j\sigma_{j'}, j \neq j'$$

Thus

$$F = [j_{n_t}\ x_t]$$

where j_{n_t} represents an $n_t \times 1$ vector of 1's and 0's, respectively. Further

$$\alpha_t = [\alpha_{1t}\ \alpha_{2t}]'$$

whence, for this simple model,

$$G_t = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = I_2$$

and

$$W_t = I_2 \begin{bmatrix} \sigma_1^2 & \rho\sigma_1\sigma_2 \\ \rho\sigma_1\sigma_2 & \sigma_2^2 \end{bmatrix} I_2$$

Given V_t, W_t, a_0 and P_0 it is then a straighforward matter to obtain the a_t, $a_{t/T}$, etc. including the H_t, E_t and the log-likelihood, L. The procedure can be easily programmed in any matrix language.

V_t and W_t may then be estimated such that L is maximized. Clearly, even in the relatively simple example outlined above with only four parameters, σ_v^2, σ_1^2, σ_2^2 and ρ, construction and analytical solution of the maximum likelihood equations is intractable. Van Deusen (1989) suggests that the log-likelihood be maximized by either a grid search or the method of scoring (Engle and Watson 1981). A simple search procedure was here found to be adequate. This involved manipulating each parameter in turn while holding the other three parameters constant. Convergence was relatively slow so that, while the method was satisfactory for these few illustrative data sets, it would not be suitable for use on a routine basis.

The Kalman filter was initiated with α_{10} and α_{20} equal to the estimated intercept and slope, respectively, of the 1978 regression. Since, with the possible exception of zinc in plaice, the 1978 regressions were relatively well defined (for one thing, the lengths covered the full range) this should have little effect on the estimates. In other words, whatever is taken for a_0, the estimates for 1978 should be close to the regular least-squares fit.

As noted above, P_0 can be regarded as reflecting the degree of uncertainty in the choice of a_0. The choice of $P_0 = kI$, with k a large finite number is analogous to using a diffuse or noninformative prior within a Bayesian framework (Harvey 1984). Here we have chosen to make a rough guess at the components of W_t and inflate these for substitution as P_0. The choice of P_0 would normally be reflected in the approximate confidence limits of the a_t; the greater the uncertainty assumed for a_0, the wider the confidence limits, although this effect diminishes over time. Again, since the 1978 regression is relatively well defined, the diminution of the effect of the choice of P_0 would be expected to be rapid.

For computational convenience, lengths (originally in millemeters) were converterd to centimeters and concentrations multiplied by 10^6.

Results

The estimates of a_t, $a_{t/T}$, their variances and covariances (i.e. the elements of P_t and $P_{t/T}$) are given in the Appendix. A problem arises for zinc in both cod and plaice in the the maximum likelihood estimate of ρ turns out to be -1.00, thus making $P_{t+1/T}$ singular. As a way of getting around this problem, in these two cases, ρ was taken as 0.999999.

To reiterate, $a_{t/t-1}$ provides a forecast of the parameters at year t based on all the information available up to and including year $t-1$; a_t is an update of that projection that takes into account the data made available in year t. On the other hand, $a_{t/T}$ revises the parameter estimates in year t on the basis of the information from all years.

There are no radical differences between the initial estimates, a_t, and the smoothed estimates $a_{t/T}$. In accordance with the nomenclature, the relationship of the latter on year appears to be slightly smoother. Since more information is used, the variances $P_{11,t/T}$ and $P_{22,t/T}$ are always less than $P_{11,t}$ and $P_{22,t}$ (with equality when $t = T$). Accordingly, the confidence intervals for the $a_{2,t/T}$ are shorter than those for the $a_{2,t}$. The amount of the reduction depends as well on how well the regression in a particular year was defined. Thus, for 1978 mercury and cod the linear correlation was fairly strong so that the reduction was relatively small although for $a_{t/T}$ there is more additional information for this year than any other.

The $a_{1,t}$ and $a_{1,t/T}$ are strongly negatively correlated with the $a_{2,t}$ and $a_{2,t/T}$, respectively. Accordingly, plots of the estimates against year have been restricted to the $a_{2,t/T}$; these are given in Figs. 1-4.

For mercury in cod, apart from some fluctuation, these slopes appear to decrease from 1978 until 1987 after which they clearly increase. Mercury in plaice shows asomewhat similar tendency, although there is a suggestion of an initial increase in the slopes from 1978 until 1980 and, subsequent to 1986, they appear to be fairly stable. Zinc, in cod, shows a rapid decrease in the slopes from 1978 until 1981. Subsequently the slopes appear relatively stable but with minimum values again in 1986-87. In all three cases the initial slopes are positive and, with the exception of mercury in cod, the later values close to zero. On the other hand, apart from a substantially negative slope in 1978, the slopes for zinc in plaice are slightly negative with a hint of an increasing trend.

By definition, under the least-squares criterion, the Kalman-filter fits, based on either prior and current information or on all the information, cannot be as good as the individual least-squares fits. It is of interest, therefore, to examine by how much the residual sums of squares are increased by use of the Kalman filter. Accordingly, the residual mean squares from the smoothed Kalman filter and from the ordinary least squares fits are given in the Appendix. In addition to such as the average and maximum percentage increases one may count the number of years for which the increase is less than, say, 5%.

With the exception of zinc in cod, the decrease in precision brought about by the Kalman filter would seem, for the most part, to be rather minor.

In those cases where the least-squares fit is relatively well defined, that is in those years where a relatively large range of fish lengths have been sampled, the difference between the various fits is small and inconsequential. Larger differences appear when the range of lengths is small or there exist concentrations that are atypical with respect to fish length, i.e. potential outliers. It would appear that for zinc in cod, where the linear relationships are less well defined, the model, so to speak, overrides the data.

WEIGHT AT AGE

Data

The relevant data are available in NAFO SCR Doc. 92/18. Tabulated are the weights at age (from ages 2 through 12) from the autumn research surveys in NAFO Divisions 2J, 3K and 3L for the years 1978 through 1991 (1981-1991 for Division 3L). The 1992 data are also available.

For the purpose of this analysis, we estimate the growth curve for each cohort (within each division). Thus 12 year old fish in 1978 represent the 1966 year class or cohort. Likewise 2 year old fish in 1992 represent the 1990 cohort. Accordingly, there are data from 25 successive cohorts (22 cohorts for Division 3L) although all ages from 2 to 12 appear in only 4 cohorts (2 for division 3L). (The number of cohorts for 2J and 3K is not 5 because of the absence of older ages in the 1992 samples).

There are, therefore, cohorts represented by only one age, either 2 or 12. It is, of course, not possible to fit a growth curve to the weight at a single age. Since the Kalman filter is an empirical Bayesian procedure, the information contained in these single points can be utilized. For example, it can be determined whether the weight is or is not consistent with the relationship, or trend in relationship, developed from the other cohorts, and, in fact, it does provide information, admittedly very little, to support or contradict the existence of a trend or change.

Model

The first step in constructing an appropriate Kalman-filter model for these data is the determination of an appropriate form for the growth curve or weight-age relationship. Let y denote the weight and x the age. After a certain amount of trial and error, $y = a(x-\delta)^b$ appeared as the most satisfactory for all divisions, although different values of δ were needed for each division to ensure reasonable tracking of the data at the low end. On the other hand, it appeared that the same value of δ could be used for all cohorts within a division. The values of δ employed were 0.75, 1.25 and 1.5 for Divisions 3L, 2J and 3K, respectively.

The Kalman filter requires the relationship between the dependent and independent variables to be linear. (Nonlinear relationships are sometimes handled via the development of linear approximations, in particular, Taylor series expansions, Harvey 1989). The above model is linearized simply by taking logarithms, i.e.

$$\log(y) = \log(a) + b\log(x-\delta)$$

Since, clearly, weights at age 12 are considerably more variable than weights at age 2, this transformation also has the merit that it should go towards stabilizing the residual variance.

On the other hand, the logarithmic transformation cannot completely stabilize the variance. This is because the weight-at-age data are, in effect, means of very different sample sizes. In actuality, the weights are not obtained directly but via a age-length key and an assumed stable length-weight relationship. This makes it difficult to determine what is the appropriate sample size of the so-calculated "average weights". The are, of course, relatively few age 12 fish sampled with, in most years, ages 4-5 being the most common. NAFO SCR Doc. 92/18 tables the mean number per tow by (estimated) age (with the 1992 survey data also available). For the want of anything better, it seems reasonable to suppose that the effective sample sizes for the "average weights" would be proportional to these average numbers per tow. (The question of whether there has been a change in condition factor is not addressed here, but a lowering of condition factor would, therefore, cause the actual weights to be less than those estimated. This should be kept in mind when interpreting the results).

In matrix notation, the model for a cohort is then

$$E(\log(\underline{y})) = A\underline{p}$$

$$Var(\underline{y}) = V\sigma^2$$

where

$$A = [\underline{j}, \underline{x-\delta}]$$

$$\underline{p} = [\log(a), b]'$$

and

$$V = [1/n_i]$$

with n_i the effective sample size for the weight of the i^{th} age.

The general linear model estimates of $\underline{p}$ are then given by

$$A'V^{-1}A\underline{p} = A'V^{-1}\log(\underline{y})$$

This is equivalent to regressing $y_i^* = \sqrt{n_i}\log(y_i)$ on $\sqrt{n_i}$ and $x_i^* = \sqrt{n_i}\log(x_i - \delta)$. The observation equations for the Kalman filter analysis are then taken as

$$y_t^* = a_t^*\sqrt{n_i} + b_t x_t^* + v_t$$

with $a_t^* = \log(a_t)$ and $V_t = \sigma_v^2 I_{n_t}$ where n_t denotes the number of weights at age available for the t^{th} cohort.

The simplest assumption that can be made for the state parameters is that they form a random walk, i.e.

$$a_t^* = a_{t-1}^* + \epsilon_{1,t}$$

$$b_t = b_{t-1} + \epsilon_{2,t}$$

with

$$Var(\epsilon_{j,t}) = \sigma_j^2,\ Cov(\epsilon_{1,t}, \epsilon_{2,t}) = \rho\sigma_1\sigma_2.$$

The matrices G_t and W_t then take the same form as for the Concetration-Length example above.

Initial estimates a_0^* and b_0 were obtained by regressing y^* on x^* and $\sqrt{n_i}$ jointly for the 4 cohorts for which weights at all ages from 2 to 12 were available (2 cohorts for Division 3L). These were thought of as what would be good estimates if there were no temporal trend in the relationship. Since P_0 should reflect the degree of uncertainly in these initial values, the estimated variance-covariance matrix of these regression estimates was used. The values of a_t^*, b_t and P_t are, of course, affected by this choice although the effect diminishes as t increases. Since, in this study, we are no so much interested in forecasting as in examining the nature of any trend in the relationship, our focus is on the $a_{t/T}^*$, $b_{t/t}$ and $P_{t/T}$, and these are negligibly, if at all, affected by the initial values.

The V_t and W_t are then determined by maximizing the log likelihood, L. The search technique employed in the Concentration-Length example was again used.

Results

The values of $a_{t/T}^*$ and $b_{t/T}$ are plotted, along with approximate 95% confidence limits, in Figs. 5.1 through 7.2. It will be seen that the confidence intervals are very wide for the early and later cohorts but quite narrow in midrange. This is as would be expected since there is very is very little information for these early and late cohorts.

Since a^* and b are the parameters of a nonlinear relationship, it is not immediately clear how they should be interpreted. Recall that the assumed growth curve is $y = a(x - \delta)^b$. The growth rate at age x is then given by

$$dy/dx = ab(x - \delta)^{b-1}$$

Plots of the estimated weight and growth rate at age 5 are given in Figs. 5 and 9, respectively. The parallel behaviour of the plots for Divisions 2J and 3K is remarkable. Both show a steady decline since the 1971 cohort with a slight recovery for the 1980/81 to 1983 cohorts, followed by a steep decline. The growth rate for Division 3L does not show such a marked decline although a parallel recovery in the early 1980's is noticeable.

These trends are presented individually for each division in Figs. 6-8 for weight and Figs. 10-12 for growth rate, with their approximate 95% confidence limits. To obtain these limits the first-order approximation of $Var(ab(x - \delta)^{b-1}$ was used, namely

$$Var(f(a,b)) = \left(\frac{\partial f}{\partial a}\right)^2 \sigma_a^2 + 2\left(\frac{\partial f}{\partial a}\right)\left(\frac{\partial f}{\partial b}\right)\sigma_{ab} + \left(\frac{\partial f}{\partial b}\right)^2 \sigma_b^2$$

where $f(a,b) = a(x-\delta)^b$ for weight and $f(a,b) = ab(x-\delta)^{b-1}$ for growth rate. The analysis provides estimates of $\sigma^2_{a^*}$ and σ_{a^*b} rather than σ^2_a and σ_{ab}. However, since $a = \exp(a^*)$, σ^2_a and σ_{ab} were estimated as $\exp(2a^*)\sigma^2_{a^*} = a^2\sigma^2_{a^*}$ and $\exp(a^*)\sigma_{a^*b} = a\sigma_{a^*b}$, respectively. Then

$$Var(a(x-\delta)^b) \approx (a(x-\delta)^b)^2[\sigma^2_{a^*} + 2b(x-\delta)^{-1}\sigma_{a^*b} + b^2(x-\delta)^{-2}\sigma^2_b],$$

$$Var(ab(x-\delta)^{b-1}) \approx (ab(x-\delta)^{b-1})^2[\sigma^2_{a^*} + 2(b-1)(x-\delta)^{-1}\sigma_{a^*b} + (b-1)^2(x-\delta)^{-2}\sigma^2_b]\cdot$$

The approximate 95% limits were taken as the estimate ± the square root of this variance.

These confidence intervals show the estimated weights and growth rates (for 5 year old fish) to be highly conjectural prior to approximately the 1972 cohort (and somewhat later for Divison 3L) although for Divisons 2J and 3K the steady decline in weight and growth rate since the cohorts of the early 1970's seems well established, as does the slight recovery for the early 1980's cohorts. It is questionable whether there has been a decline in weight or growth rate in Division 3L and one might be prepared to disregard the increase with early 1980's cohorts were it not coincident with the same phenomenon in Divisions 2J and 3K. In this respect, it should be noted that the data for the three divisions are independent, as were the analyses.

Estimates of weight and growth rate with approximate confidence limits can likewise be obtained for any age.

Discussion

With respect to the concentration-length analyses, there is, *a priori*, no reason why the trends for different metals should be the same. Although similar trends for the same metal in different fish species might be expected, the rates of uptake and excretion will generally differ between species and, further, if the species do not follow the same migratory pattern, their exposure to the metal will differ. Notwithstanding, it is interesting to see an indication of a parallel trend for mercury in cod and plaice, suggestive, until 1986/87, of a reduced availability. More research on the linking of scenarios of changing availability and expected trends is clearly needed.

While the results of these few examples cannot be regarded as definitive, they seem to endorse the potential of the Kalman filter to model, in a meaningful way, the evolution of these relationships. It will be interesting to see whether the trends that have been identified continue as the data for 1991 and 1992 become available. Something not yet attempted with these data, but also of interest, would be to examine the reliability of one-year-ahead prediction, in particular, forecasting the 1991 regressions from the 1978-1990 data.

In the weight-at-age example, one may question how it is possible to present weights or growth rates for 5 year-old stock for cohorts prior to 1973 or subsequent to 1987. Consider an extreme case for which we have data, age 12 in 1978. On the basis of the model and the pattern generated by the available data we have an estimate of what the growth rate would have been at age 5 to attain the weight observed at age 12. Likewise, on the basis of the growth weight observed for the 1990 cohort at age 2 we can project what its growth rate would be expected to be in 1995, but, of course, with a wide confidence interval.

The Kalman-filter models employed in these two examples are, essentially, the same. Both assume the evolution of the parameters to be a simple random walk. In the concentration-length example, the temporal trends are obscured by the presence of potential outliers and that fact that there are commonly years where the lengths of the sampled fish are a poor representation of the lengths in the population, possibly as a consequence of cluster sampling. On the other hand, in the weigh-at-age example, the ages are regularly spaced but with, inherent to the situation, the younger ages systematically and progressively unavailable as one goes back in time and the older ages likewise unavailable as one goes forwards in time.

As an empirical Bayes procedure, the Kalman-filter utilizes the full data set to eliminate, or at least reduce, inconsistencies in a temporal sequence of regressions consequent on these limitations of the data. One might argue, therefore, that greater confidence should be placed in individual year relationships that are more consistent and use all the available information, than in those that are more dependent on the vagaries of a particular year's data. The Kalman filter provides a means of achieving such harmonization although, as with any method, its performance depends of the viability of the model assumptions. In the present examples, these assumptions do not appear to be stringent and the procedure should be, therefore, fairly robust.

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APPENDIX

Table 1.
Parameter Estimates for Mercury in Cod

Year	$a_{1,t}$	$a_{2,t}$	$a_{1,t/T}$	$a_{2,t/T}$
1978	-0.06563	0.006257	-0.06662	0.006176
1979	-0.05942	0.005752	-0.03525	0.004764
1980	-0.04248	0.004902	0.01795	0.003123
1981	-0.00732	0.002507	-0.04432	0.003201
1982	-0.13008	0.004798	-0.12638	0.004717
1983	-0.07054	0.003393	0.05484	0.003116
1984	0.06345	0.001004	-0.05443	0.001194
1985	-0.08021	0.003680	-0.06918	0.003469
1986	0.00821	0.002016	0.02598	0.001586
1987	0.11863	-0.000290	0.09741	0.000225
1988	0.01626	0.001640	0.00043	0.002017
1989	0.01917	0.002042	-0.02051	0.002762
1989	-0.21135	0.006098	-0.21135	0.006098

Table 2.
Parameter Estimates for Mercury in Plaice

Year	$a_{1,t}$	$a_{2,t}$	$a_{1,t/T}$	$a_{2,t/T}$
1978	-0.3248	0.01805	-0.2778	0.01705
1979	-0.5069	0.02543	-0.4195	0.02324
1980	-0.6942	0.03359	-0.5410	0.02878
1981	-0.1649	0.01454	-0.1691	0.01446
1982	-0.1131	0.01185	-0.0925	0.01127
1983	-0.0126	0.00890	-0.0835	0.01107
1984	-0.2459	0.01709	-0.2267	0.01622
1985	-0.1270	0.01258	-0.1148	0.01167
1986	0.1123	0.00267	0.0664	0.00406
1987	-0.1984	0.01434	-0.1652	0.01301
1988	0.0298	0.00546	-0.0137	0.00696
1989	-0.1713	0.01311	-0.1640	0.01275
1989	-0.0413	0.00779	-0.0413	0.00779

Table 3.
Paremeter Estimates for Zinc in Cod

Year	$a_{1,t}$	$a_{2,t}$	$a_{1,t/T}$	$a_{2,t/T}$
1978	2.2673	0.08535	2.7557	0.08067
1979	3.1761	0.05495	3.4187	0.04010
1980	3.7478	0.01996	3.8095	0.01618
1981	4.0016	0.00442	4.0030	0.00434
1982	4.0274	0.00284	4.0158	0.00355
1983	3.8403	0.01429	3.8364	0.01434
1984	3.7363	0.02066	3.7931	0.01719
1985	4.0378	0.00221	4.0459	0.00171
1986	4.1435	-0.00426	4.1362	-0.00382
1987	4.2075	-0.00818	4.1003	-0.00161
1988	3.5730	0.03065	3.5884	0.02971
1989	3.6835	0.02389	3.7067	0.02247
1990	3.9530	0.00740	3.9530	0.00740

Table 4.
Paremeter Estimates for Zinc in Plaice

Year	$a_{1,t}$	$a_{2,t}$	$a_{1,t/T}$	$a_{2,t/T}$
1978	58.68	-1.0824	54.43	-0.9976
1979	13.02	-0.1756	13.35	-0.1782
1980	17.37	-0.2585	17.61	-0.2632
1981	20.78	-0.3264	20.04	-0.3118
1982	9.35	-0.0983	9.67	-0.1049
1983	16.61	-0.2432	16.55	-0.2417
1984	15.04	-0.2119	14.92	-0.2095
1985	12.98	-0.1709	12.74	-0.1660
1986	7.03	-0.0522	7.28	-0.0571
1987	9.67	-0.1049	9.90	-0.1093
1988	15.14	-0.2139	14.80	-0.2071
1989	7.38	-0.0592	7.35	-0.0585
1990	6.85	-0.0487	6.85	-0.0647

Table 5
Variance of a_t and $a_{t/T}$ for Mercury in Cod

Year	$\sigma^2_{1,t}$ $\times 10^{-4}$	ρ_t	$\sigma^2_{2,t}$ $\times 10^{-7}$	$\sigma^2_{1,t/T}$ $\times 10^{-4}$	$\rho_{t/T}$	$\sigma^2_{2,t/T}$ $\times 10^{-7}$
1978	9.96	-0.93	3.35	9.07	-0.95	3.10
1979	7.55	-0.92	8.32	6.18	-0.94	5.99
1980	32.84	-0.99	21.63	20.89	-0.97	12.72
1981	47.00	-0.97	17.06	33.13	-0.98	12.06
1982	8.54	-0.97	2.43	7.83	-0.97	2.28
1983	14.74	-0.96	5.30	13.01	-0.97	4.72
1984	7.94	-0.95	4.50	7.26	-0.95	3.90
1985	12.32	-0.95	4.37	11.18	-0.94	4.02
1986	24.53	-0.99	14.06	18.97	-0.99	10.97
1987	24.15	-0.98	14.95	17.95	-0.99	10.59
1988	33.46	-0.98	15.15	25.44	-0.99	11.32
1989	21.64	-0.99	7.92	18.34	-0.97	6.70
1990	14.23	-0.96	3.66	14.23	-0.96	3.66

Table 6
Variance of a_t and $a_{t/T}$ for Mercury in Plaice

Year	$\sigma^2_{1,t}$ $\times 10^{-2}$	ρ_t	$\sigma^2_{2,t}$ $\times 10^{-5}$	$\sigma^2_{1,t/T}$ $\times 10^{-2}$	$\rho_{t/T}$	$\sigma^2_{2,t/T}$ $\times 10^{-5}$
1978	1.087	-0.983	1.16	0.833	-0.983	0.94
1979	1.819	-0.987	2.05	1.238	-0.989	1.47
1980	1.515	-0.984	1.57	0.983	-0.982	1.12
1981	1.883	-0.988	2.14	1.198	-0.989	1.47
1982	1.330	-0.984	1.53	0.931	-0.984	1.15
1983	1.153	-0.984	1.44	0.892	-0.989	1.16
1984	1.179	-0.982	1.53	0.930	-0.985	1.25
1985	1.183	-0.984	1.51	0.928	-0.988	1.21
1986	0.790	-0.978	0.75	0.544	-0.974	0.58
1987	1.257	-0.984	1.59	0.981	-0.988	1.28
1988	1.099	-0.983	1.38	0.933	-0.982	1.18
1989	1.109	-0.984	1.16	0.963	-0.983	1.02
1990	1.159	-0.982	1.14	1.159	-0.982	1.14

Table 7

Variance of a_t and $a_{t/T}$ for Zinc in Cod

Year	$\sigma^2_{1,t}$ $\times 10^{-2}$	$\sigma^2_{2,t}$ $\times 10^{-4}$	$\sigma^2_{1,t/T}$ $\times 10^{-2}$	$\sigma^2_{2,t/T}$ $\times 10^{-4}$
1978	1.269	0.475	1.187	0.445
1979	6.830	2.558	4.614	1.729
1980	3.508	1.314	2.726	1.021
1981	1.237	0.463	1.119	0.419
1982	0.711	0.266	0.672	0.252
1983	1.005	0.376	0.935	0.350
1984	2.471	0.925	2.049	0.767
1985	0.995	0.373	0.926	0.347
1986	2.219	0.831	1.902	0.712
1987	2.204	0.863	1.944	0.728
1988	1.427	0.535	1.277	0.478
1989	1.036	0.388	0.953	0.357
1990	0.630	0.236	0.630	0.236

Table 8

Variance of a_t and $a_{t/T}$ for Zinc in Plaice

Year	$\sigma^2_{1,t}$	$\sigma^2_{2,t}$ $\times 10^{-2}$	$\sigma^2_{1,t/T}$	$\sigma^2_{2,t/T}$ $\times 10^{-2}$
1978	18.199	0.7242	16.608	0.6610
1979	13.787	0.5487	12.871	0.5124
1980	17.051	0.6786	15.640	0.6227
1981	12.476	0.4965	11.689	0.4654
1982	8.417	0.3350	8.053	0.3208
1983	8.070	0.3212	7.737	0.3082
1984	9.573	0.3810	9.103	0.3626
1985	8.033	0.3197	7.716	0.3075
1986	16.631	0.6618	15.265	0.6082
1987	7.934	0.3158	7.612	0.3034
1988	8.122	0.3232	7.794	0.3108
1989	13.362	0.5317	12.506	0.4985
1990	14.482	0.5763	14.501	0.5780

Table 9

Increase (%) of Residual S.S. of Smoothed Kalman Filter over Individual Least Squares Fits

Year	Mercury Cod	Mercury Plaice	Zinc cod	Zinc Plaice
1978	0.4	1.3	0.2	0.4
1979	3.4	0.8	33.9	88.8
1980	1.2	4.1	34.3	4.8
1981	2.7	17.7	37.5	10.8
1982	0.6	1.1	109.4	4.0
1983	1.0	4.9	15.4	4.7
1984	4.6	10.0	38.6	0.0
1985	2.0	2.9	37.2	13.3
1986	0.8	5.7	15.5	11.3
1987	5.0	19.2	79.5	2.5
1988	3.1	6.4	34.5	3.8
1989	10.6	5.8	1.4	10.2
1990	1.7	0.8	0.8	10.4
Av.	2.8	6.2	33.7	12.7

Fig. 1
Smoothed Estimates of Slope with Approximate 95% Confidence Limits
Mercury in Cod

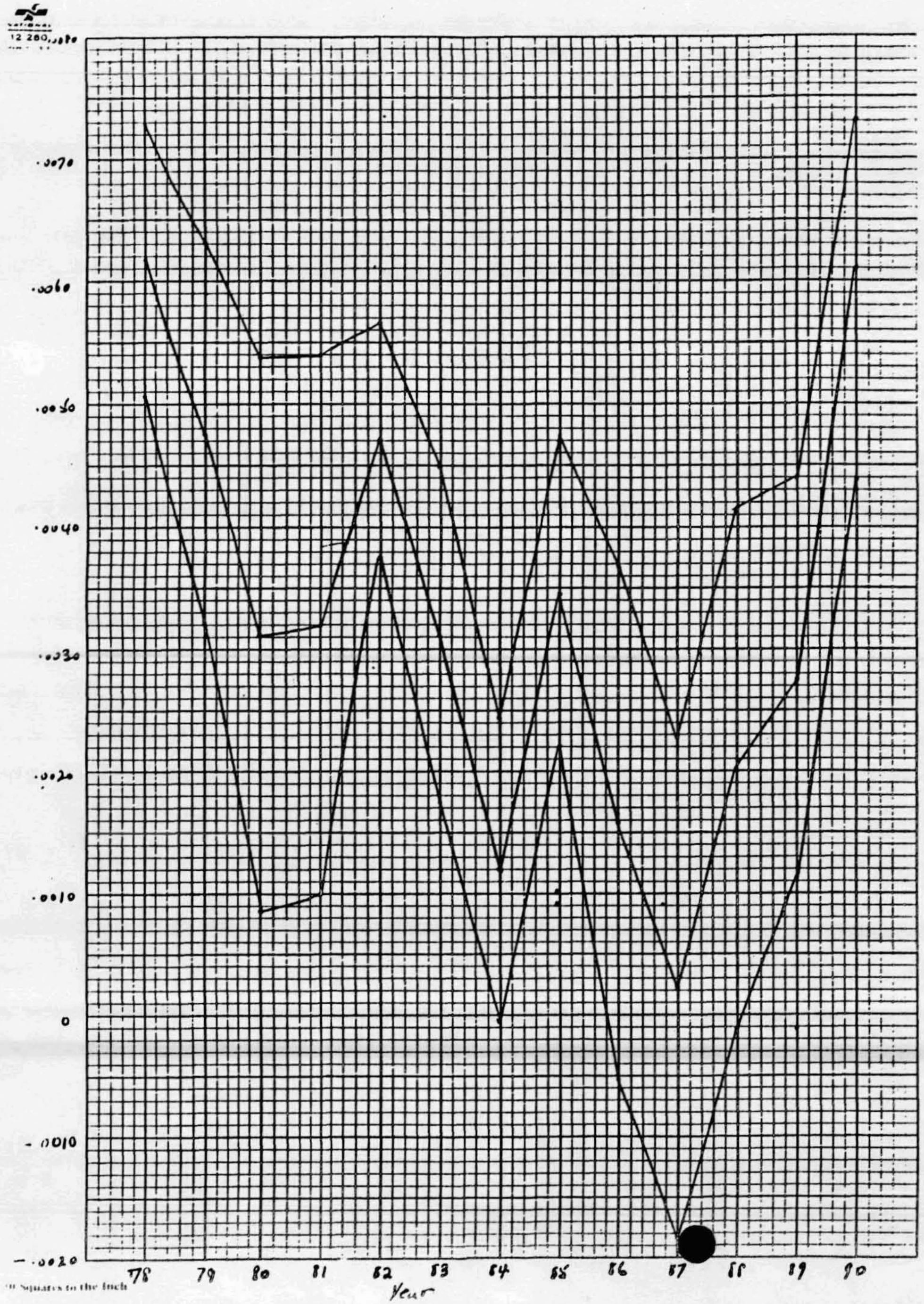


Fig. 2
Smoothed Estimates of Slope with Approximate 95% Confidence Limits
Mercury in Plaice

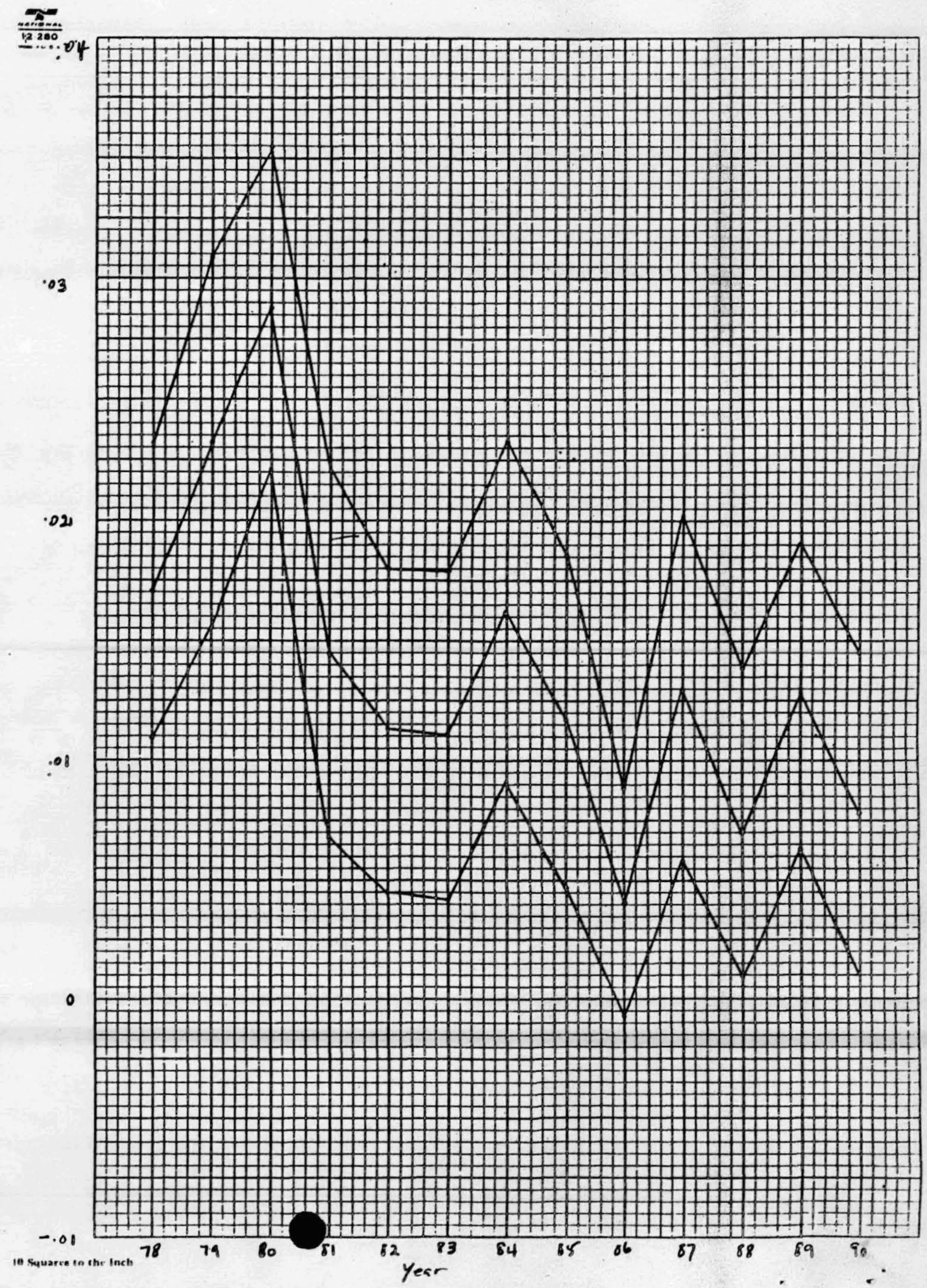


Fig. 3

Smoothed Estimates of Slope with Approximate 95% Confidence Limits
Zinc in Cod

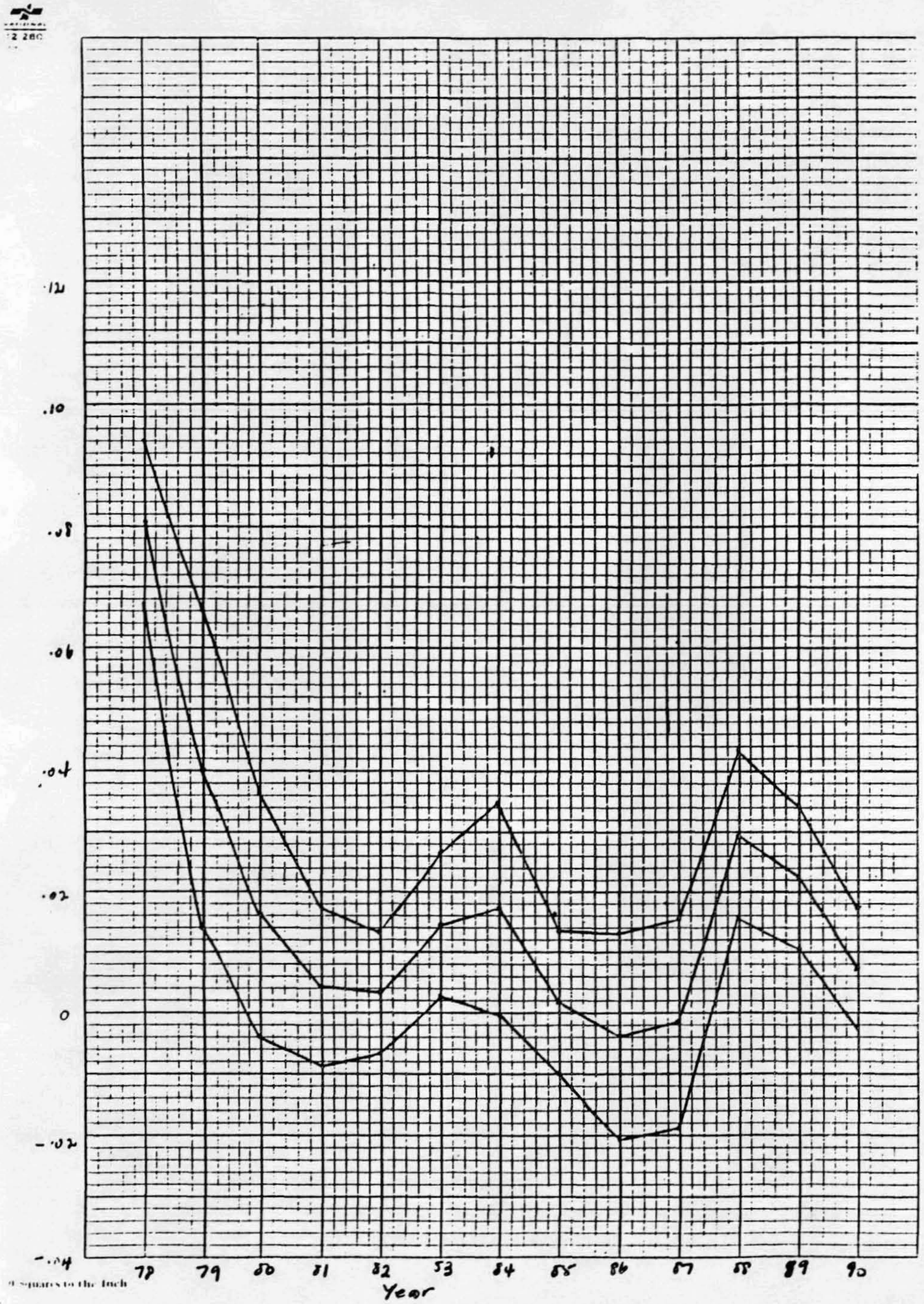


Fig. 4

Smoothed Estimates of Slope with Approximate 95% Confidence Limits
Zinc in Plaice

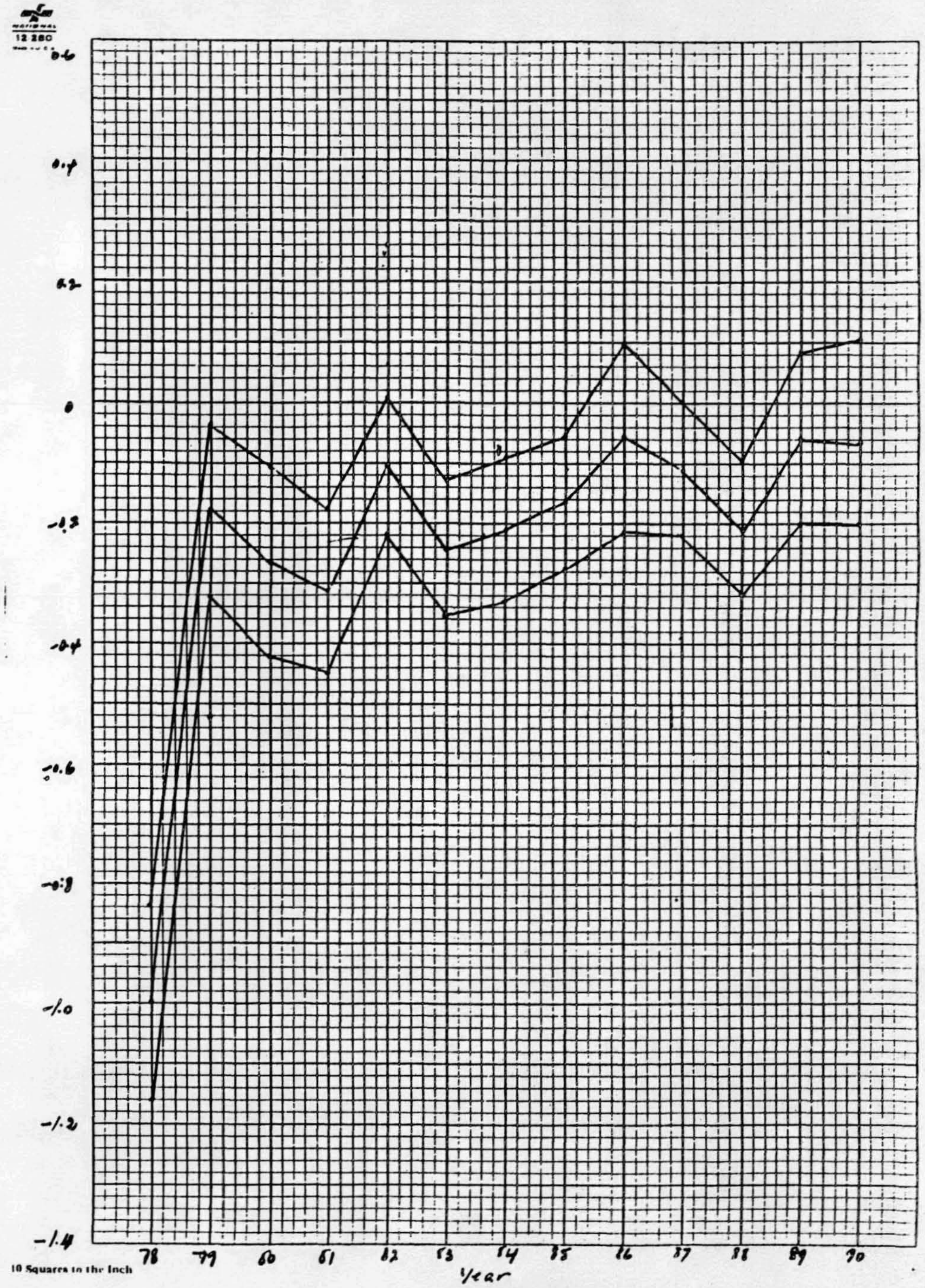


Fig. 5
Estimates of Weight at Age 5

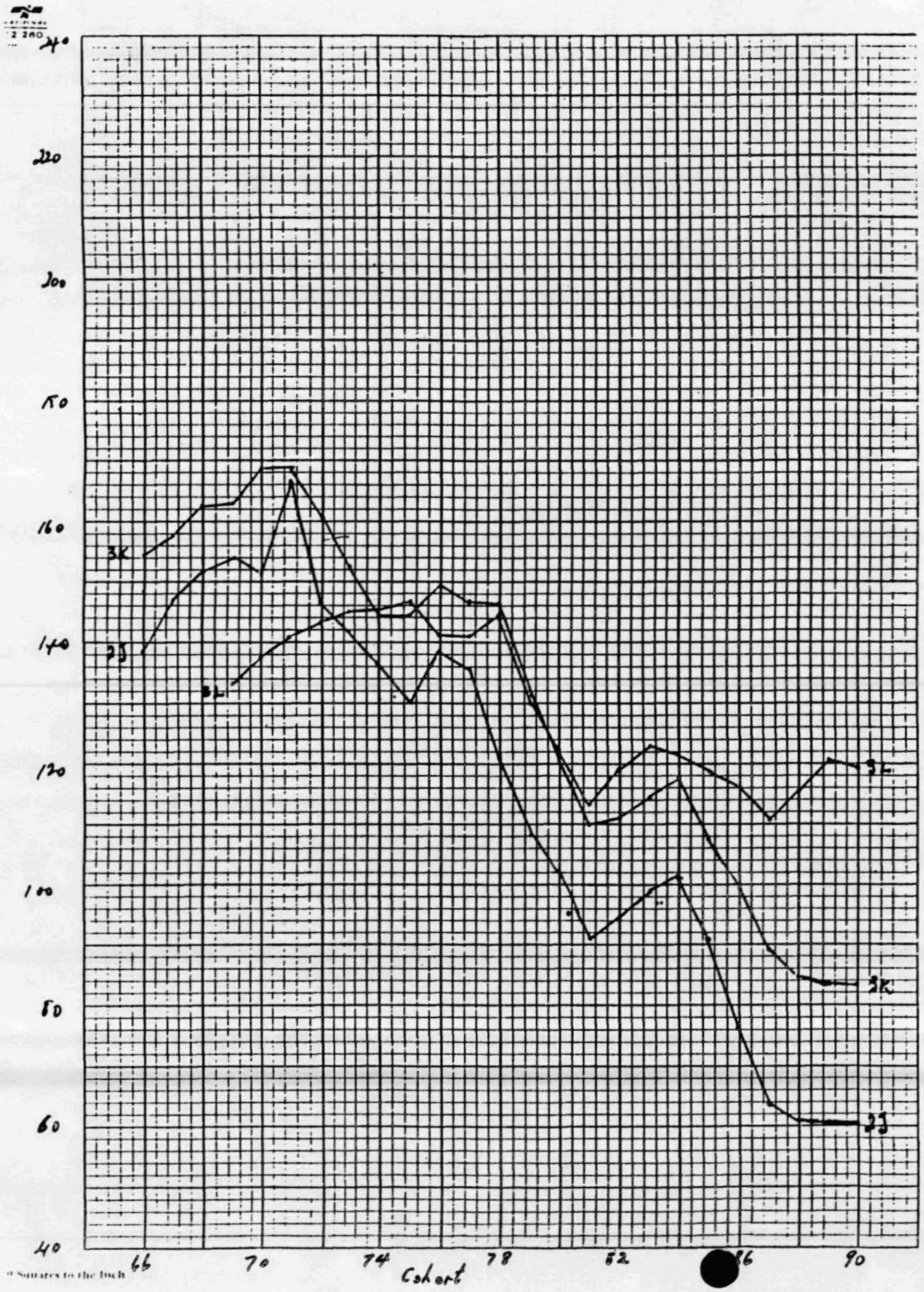
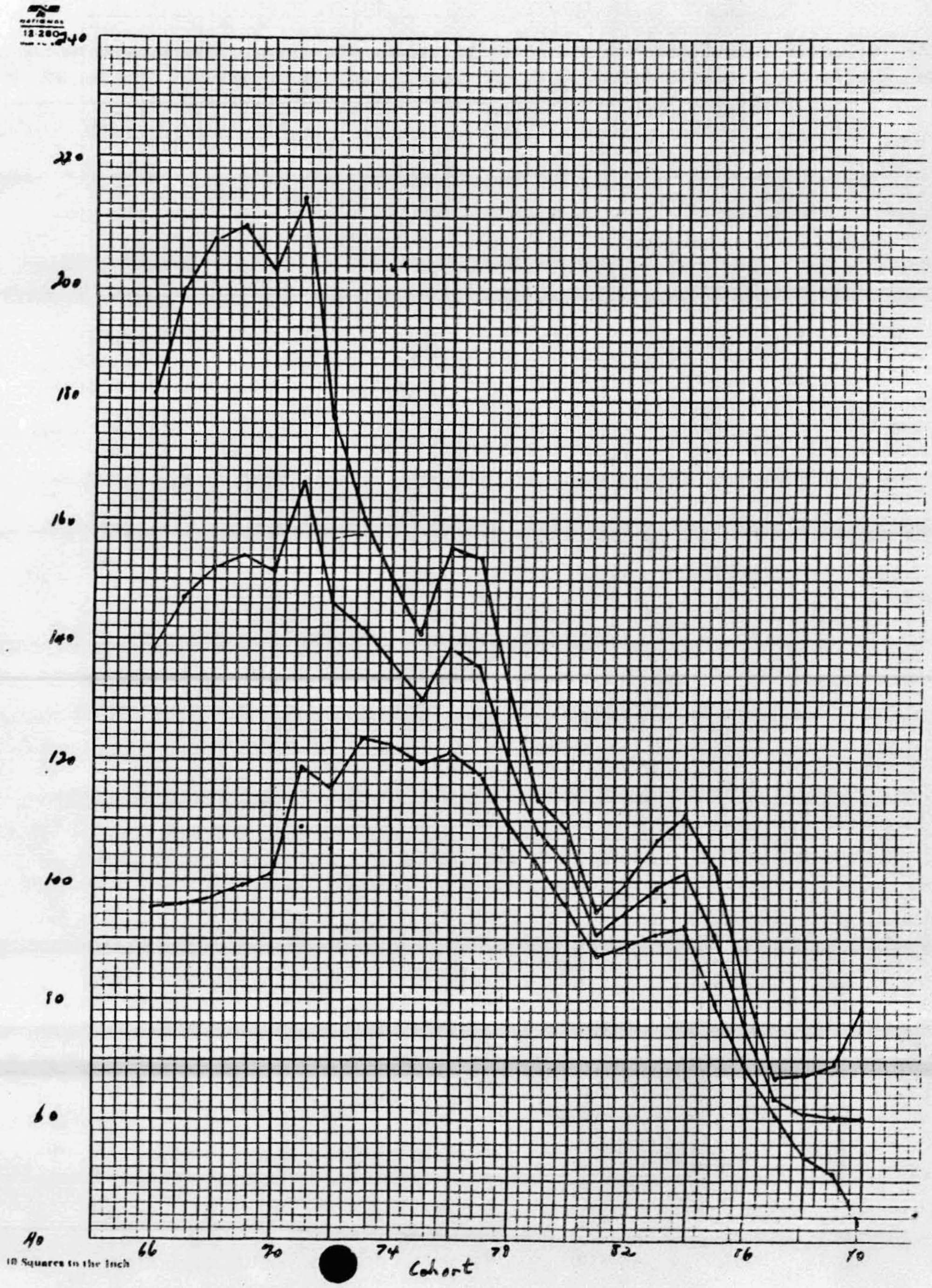


Fig. 6
Estimates of Weight at Age 5 with 95% Confidence Limits
Division 2J



Estimates of Weight at Age 5 with 95% Confidence Limits
Division 3K

Estimates of Weight at Age 5 with 95% Confidence Limits
Division 3L

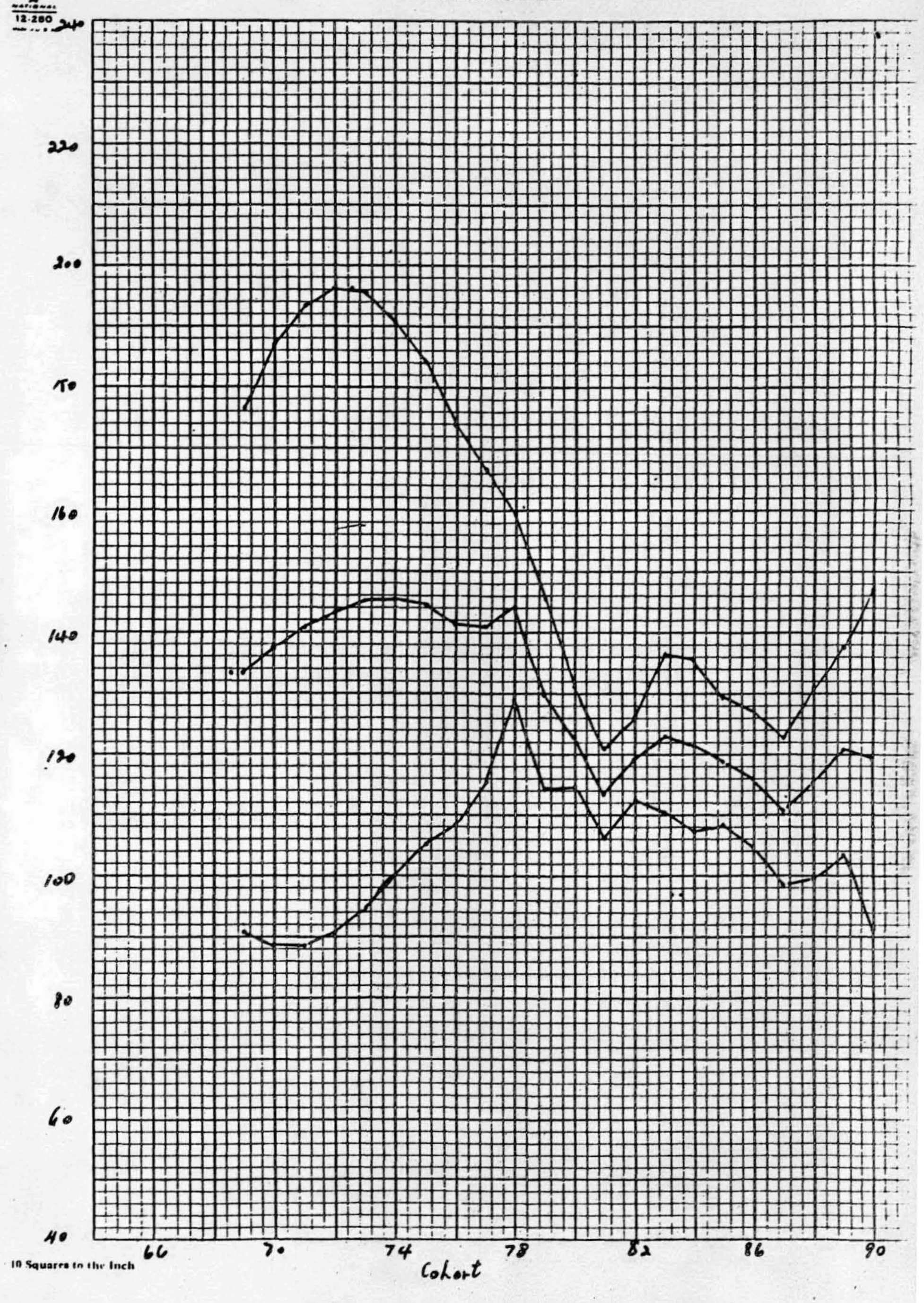


Fig. 9
Estimates of Growth Rate at Age 5

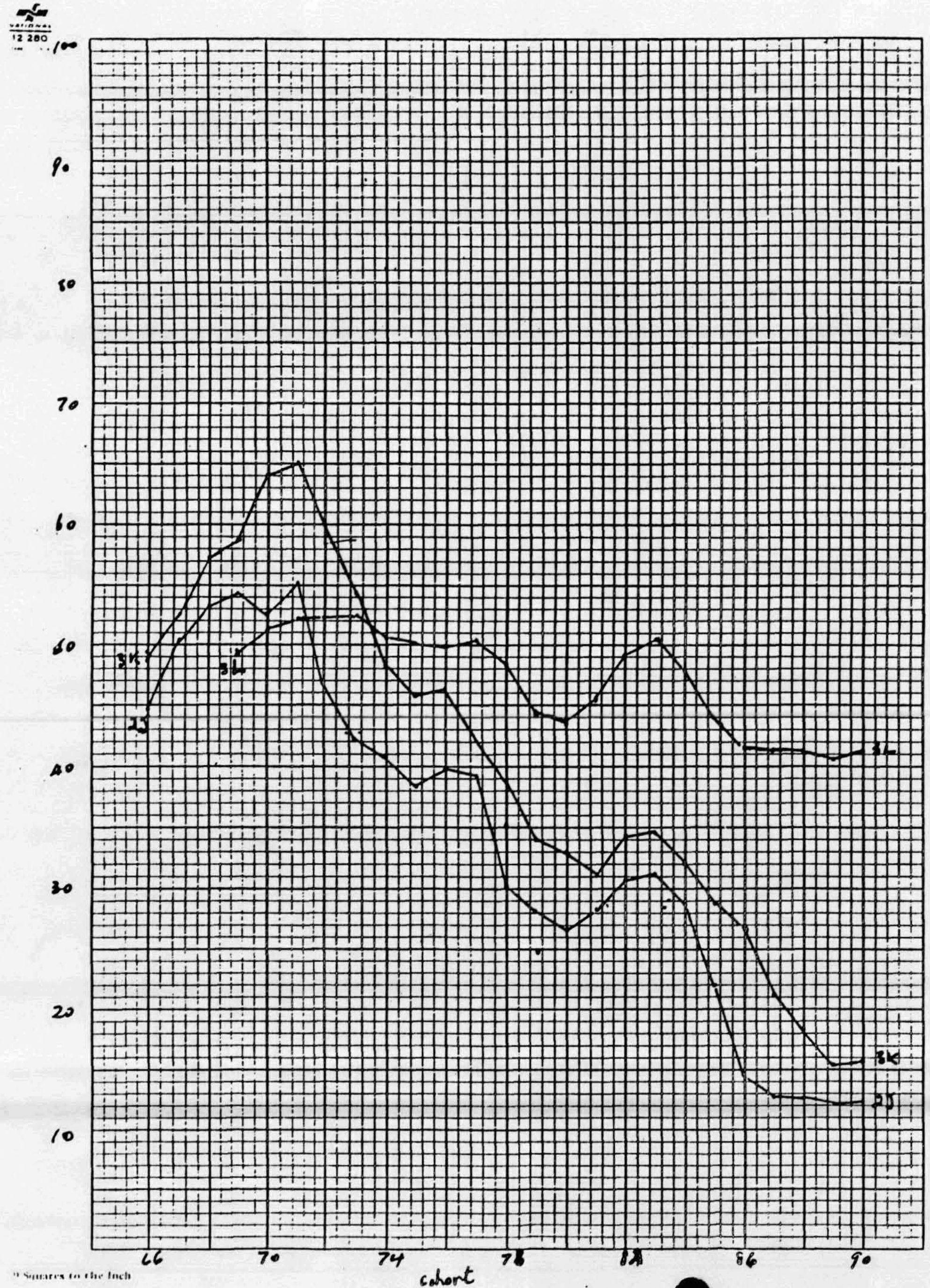


Fig. 10
Estimates of Growth Rate at Age 5 with 95% Confidence Limits
Division 2J

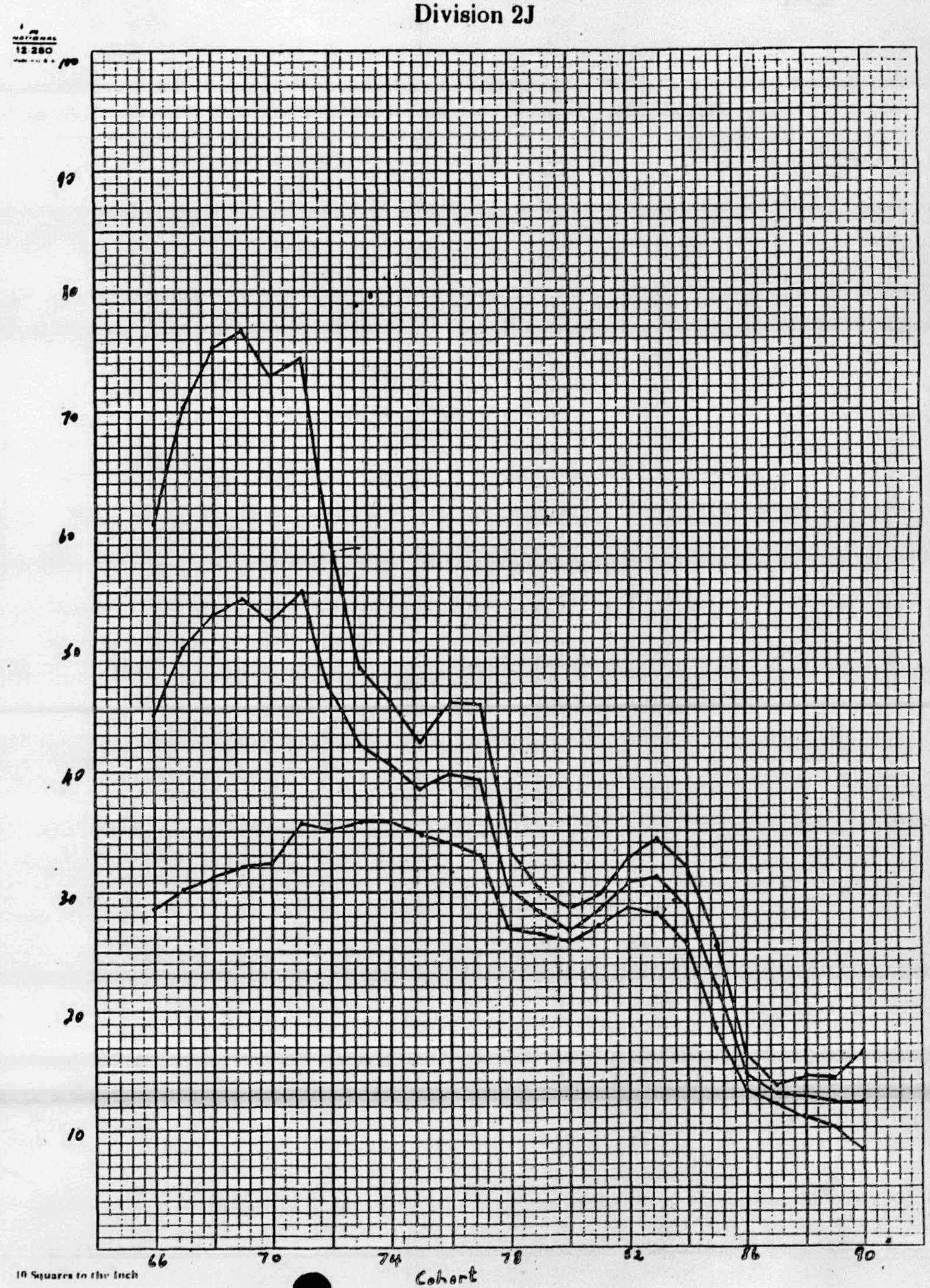


Fig. 11

Estimates of Growth Rate at Age 5 with 95% Confidence Limits

Division 3K

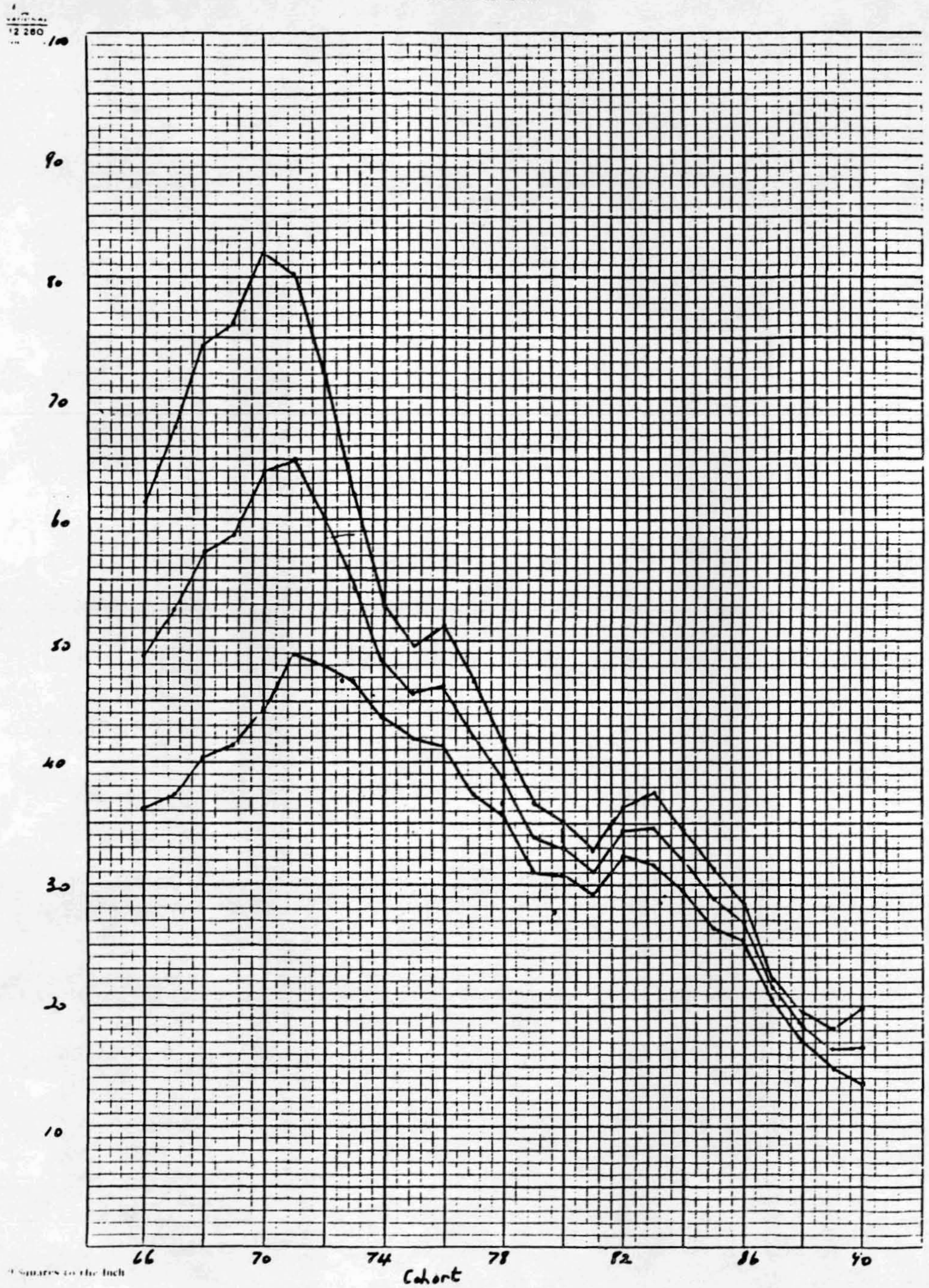


Fig. 12

Estimates of Growth Rate at Age 5 with 95% Confidence Limits

Division 3L