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COMPARISON OF EQUAL-AREA CYLINDRICAL AND CIRCULAR PISTON TRANSDUCERS

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ABSTRACT

Design of sondes for in situ measurement of zooplankton or other scatterers requires choosing among alternative transducer geometries. This contribution addresses the problem of choosing between cylindrical and circular piston transducers by comparing the performance of the two according to the principle that the acoustically active areas be equal. Computations are performed with the actual dimensions of six fabricated cylindrical transducers, whose beam patterns have been measured by the manufacturer at a total of eleven frequencies spanning the range 27-710 kHz. Nominal power levels assigned to the cylindrical transducers are also used for both transducer types. Comparison of theoretically computed beam patterns with measurement gives confidence in the radiation model, which is used to compute the directivity index and on-axis sensitivity loss due to curvature of the cylindrical transducers, referred to as the curvature loss. Under identical conditions of excitation, isotropic ambient noise, and detection threshold of 20 dB, the active sonar equation is exercised to estimate the maximum detection range of both single targets and multiple targets distributed throughout the sampling volume. In every single case, the performance of the equal-area circular piston is superior to that of the corresponding cylindrical transducer. This is directly attributable to differences in directivity index and curvature loss. Other, pragmatic considerations argue for the choice of the circular piston transducer over the cylindrical transducer. Three problems requiring future treatment are identified.

RESUME: COMPARAISON DE TRANSDUCTEURS-PISTON DE FORMES CIRCULAIRE ET CYLINDRIQUE A SURFACE EGALE

La conception de sondes pour la mesure in-situ du zooplancton ou d'autres diffuseurs exige un choix parmi différentes géométries de transducteurs. Cette note concerne le problème du choix entre des transducteurs-piston circulaire et cylindrique en comparant les performances des deux avec le principe de surfaces actives équivalentes. Des calculs sont effectués pour les dimensions de six transducteurs cylindriques existants, dont les diagrammes d'emission ont été mesurés par le constructeur pour un total de onze fréquences couvrant la gamme 27-710 kHz. Les niveaux de puissance nominaux prévus pour les transducteurs cylindriques sont aussi utilisés pour les deux types. La comparaison des diagrammes de directivité calculés théoriquement avec les mesures permet de valider le modèle de rayonnement qui est utilisé pour calculer l'index de directivité et la perte de sensibilité sur l'axe due à la courbure du transducteur cylindrique, désignée comme la "perte de courbure". Avec des conditions identiques d'excitation, un niveau de bruit isotropique équivalent et un seuil de détection de 20 dB, l'equation du sonar actif est utilisée pour estimer la portée maximum de détection dans le cas de cibles uniques et multiples distribuées dans le volume d'échantillonnage. Dans chaque cas simple, les performances du piston circulaire sont supérieures à celle du piston cylindrique correspondant. Ceci est directement imputable aux differences dans les index de directivité et les pertes de courbure. Sur un autre plan, des considérations d'ordre pragmatique plaide pour le choix du transducteur circulaire plutôt que le cylindrique. Les autres problèmes demandant des études futures sont identifiés.

INTRODUCTION

For some years there has been discussion on the use of cylindrical transducers on a sonde for the in situ measurement of zooplankton or other scatterers. An alternative is that of circular piston transducers, as used, for example, in Holliday's renowned Multifrequency Acoustic Profiling System (MAPS) (Holliday et al. 1989).

Here the ordinary active sonar equation is exercised in an ambient-noise-limited environment to compare the performance of cylindrical and circular piston transducers. An important analysis principle is that the acoustically active areas of corresponding transducers at the same frequency be equal.

Another analysis principle is use of the dimensions of cylindrical transducers as fabricated, for which the manufacturer's beam pattern measurements are available, as well as use of nominal power levels attached or assigned to the transducers. This removes some of the abstractness of the *a priori* argument compared to the partly *a fortiori* argument advanced here. This is additionally valuable for resolving somewhat a technical matter concerned with the acoustic boundary condition on the cylindrical transducer, which is generally unknown and difficult to know, but is clearly different from that which usually applies on the planar circular piston transducer.

TRANSDUCER GEOMETRIES AND DIMENSIONS

The idealized form of the cylindrical transducer is a right circular cylinder of length ℓ and outer dimension 2a. The acoustically active area is thus $2\pi a\ell$.

The circular piston transducer is assumed to be a planar circular surface of radius s set in an infinite baffle. By assumption of equal area with the respective cylindrical transducer, $\pi s^2=2\pi a\ell$ or $s=(2a\ell)^{\frac{1}{2}}$.

Six cylindrical transducers were fabricated by International Transducer Corporation, Santa Barbara, California, no later than 1988, with ITC Model No. 8151, Serial No. 001. Because of their planned sequential alignment on a common axis, for use on a sonde, the transducers were referred to as "sections" by the manufacturer. The transducer associated with Section A was intended to be driven at each of two frequencies, 27 and 38 kHz. Section B was to be driven at 70 and 88 kHz, C at 107, 120, and 150 kHz, D at 200 and 250 kHz, E at 375 kHz, and F at 710 kHz.

Maximum overall dimensions of the six transducers, as read off ITC engineering drawing no. 017110, dated 15 April 1988, are presented in Table 1. Computed radii of corresponding equal-area circular pistons are also shown, as are nominal electrical power levels assigned to the cylindrical transducers and assumed for the corresponding circular piston transducers too.

Table 1. Transducer dimensions and nominal electrical power levels.

Section	Frequencies (kHz)	Cylinder Length (mm)	Cylinder Diameter (mm)	Cylinder Area (mm^2)	Piston Radius (mm)	Power (w)
A	27,38	204.5	34.8	22351	84.3	600
B	70,88	88.8	24.2	6761	46.4	400
C	107,120,150	58.0	13.2	2400	27.6	200
D	200,250	33.6	34.3	3619	33.9	150
E	375	25.3	25.5	2028	25.4	50
F	710	11.5	14.0	506	12.7	20

BEAM PATTERNS

The beam pattern is defined in terms of the transmitting or receiving amplitude of the transducers. In the farfield of the transducer when transmitting or for a farfield source when receiving, the amplitude can be expressed thus:

$$f = A^{-1} \int_A \exp(i\underline{k}\cdot\underline{r})\, dS \quad , \qquad (1)$$

where A is the acoustically active area of the transducer, $\underline{k}$ is the wavevector, and $\underline{r}$ is the position vector of the area element dS on the transducer surface. The integration is performed over the entire transducer surface.

Cylindrical transducer

It can be instructive to develop the beam pattern of a cylindrical transducer in stages for two reasons: the boundary condition on the transducer is unknown, and the literature appears incomplete on this subject. Here the boundary conditions are explicitly given. According to the geometry defined above, $\underline{k}\cdot\underline{r}=k(a \sin\theta \cos\psi + z \cos\theta)$, where θ is the polar angle describing the direction of $\underline{k}$ relative to the physical axis of the cylinder, ψ is the azimuthal coordinate relative to the azimuth of evaluation, namely $\psi=0$, and z marks the distance along the axis.

Case i. Acoustically transparent cylinder

The transducer is assumed to be acoustically sensitive, without otherwise affecting the propagation of incoming or outgoing waves. According to the geometry defined above,

$$f = (2\pi a \ell)^{-1} \int_{-\ell/2}^{\ell/2} \int_0^{2\pi} \exp(i\underline{k}\cdot\underline{r})\, a\, d\psi\, dz \quad . \qquad (2)$$

Substituting for $\underline{k}\cdot\underline{r}$, integrating, and normalizing by the factor $J_o(ka)$,

$$f = \frac{\sin(\frac{k\ell}{2}\cos\theta)}{\frac{k\ell}{2}\cos\theta} \; \frac{J_o(ka\sin\theta)}{J_o(ka)} \quad , \tag{3}$$

where $J_o(\zeta)$ is the ordinary Bessel function of order 0. This amplitude is normalized, as its value in the broadside direction $\theta=\pi/2$ is unity, and the beam pattern is thus given by the expression $b=|f|^2$.

Case ii. Acoustically opaque cylinder

The first case is clearly unrealizable. A much more realistic case, if still an idealization, is that of an acoustically opaque cylinder, by which a state of perfect internal baffling is understood. This is represented mathematically through the individual element directivity function: $\cos\psi$ for $-\pi/2<\psi<\pi/2$ and 0 for $|\psi|\gtrsim\pi/2$. Equation (1) is thus generalized:

$$f = A^{-1} \int_{-\ell/2}^{\ell/2} \int_{-\pi/2}^{\pi/2} \exp(i\underline{k}\cdot\underline{r}) \cos\psi \; a\, d\psi\, dz \quad , \tag{4}$$

where A is now the effective, acoustically active area of the transducer, namely

$$A = \int_{-\ell/2}^{\ell/2} \int_{-\pi/2}^{\pi/2} \cos\psi \; a\, d\psi\, dz = 2a\ell \quad . \tag{5}$$

In order to perform the integration in equation (4), it is useful to note the respective definitions of Anger's and Weber's functions (Abramowitz 1965):

$$\mathbf{J}_\nu(z) = \pi^{-1} \int_0^\pi \cos(\nu u - z\sin u)\, du \quad , \tag{6a}$$

$$\mathbf{E}_\nu(z) = \pi^{-1} \int_0^\pi \sin(\nu u - z\sin u)\, du \quad . \tag{6b}$$

The result of the integration is thus

$$f = \frac{\sin(\frac{k\ell}{2}\cos\theta)}{\frac{k\ell}{2}\cos\theta} \frac{\pi}{2} i[\mathbf{J}_1(ka\sin\theta) - i\mathbf{E}_1(ka\sin\theta)] \quad . \tag{7}$$

This can be reduced further, for the function $\mathbf{J}_1(\zeta)$ is just the ordinary Bessel function of order 1, $J_1(\zeta)$, and $\mathbf{E}_1(\zeta)=2/\pi - H_1(\zeta)$, where $H_1(\zeta)$ is the Struve function of order 1.

Equation (7) is quite interesting in the present context. In the long-wavelength limit, $f(\pi/2)=1$. At shorter or finite wavelengths, $f(\pi/2)<1$. This can be understood in terms of Fresnel zones (Neubauer 1963, Born and Wolf 1970): because of the curvature of the transducer surface, the number of zones increases with decreasing wavelength, or increasing frequency. Because of the destructive effect of an increasing number of zones, the sensitivity of the transducer decreases with increasing frequency. This loss of sensitivity is measured by $f(\pi/2)$. It is connected with the so-called diffraction constant (Henriquez 1964, Bobber 1965, Milosič 1993).

In computing the beam pattern of the acoustically opaque cylinder, therefore, the lack of normalization in f is to be remembered. The beam pattern is consequently

$$b = \left| \frac{\sin(\frac{k\ell}{2}\cos\theta)}{\frac{k\ell}{2}\cos\theta} \right|^2 \frac{J_1^2(ka\sin\theta) + E_1^2(ka\sin\theta)}{J_1^2(ka) + E_1^2(ka)} \quad . \tag{8}$$

Another candidate boundary condition, not considered here, is that of the rigid but infinite cylinder, as described by Morse (1948). Notwithstanding description of radiation by an arbitrary azimuthal distribution of the normal component of surface velocity, it is not clear how this can be applied to the problem of transmission and reception by a cylindrical transducer of finite length.

The complicated nature of the boundary condition is illustrated by Ho (1994) for the case of an elastic cylindrical shell. The distribution of total surface pressure in azimuth is clearly non-uniform for the chosen wavenumber-radius product ka=3.05. Fundamental physical reasons for the complexity of this surface distribution, hence boundary condition too, are elaborated by Ho (1993).

<u>Circular piston transducer</u>

For the planar circular piston transducer of radius s in an infinite perfect baffle, the farfield beam pattern is just

$$b = \left| \frac{2J_1(ks\sin\theta)}{ks\sin\theta} \right|^2 \quad . \tag{9}$$

Here θ is the polar angle describing the field direction relative to the acoustic axis, which is coincident with the physical axis.

BEAM PATTERN PARAMETERS

Several quantities are useful for describing the beam patterns of cylindrical transducers. In the longitudinal plane, including the transducer axis, these are the (1) half-width $\Delta\theta$ of the main lobe, measured from the axis to the angle at which $10\log b = -3$ dB, (2) angle θ_1 between cylinder axis and first sidelobe, or its complement $\pi/2-\theta_1$, and (3) beam pattern level at the angle θ_1, $B_1 = 10\log b(\theta_1)$. In the transverse plane, perpendicular to the transducer axis, a useful quantity is the total variation in the beam pattern with respect to the azimuthal angle, $\Delta B = 10\log\{\mathrm{Max}\, b_\psi(\pi/2)\} - 10\log\{\mathrm{Min}\, b_\psi(\pi/2)\}$.

For the cylindrical transducers fabricated by International Transducer Corporation, the beam patterns were measured by the manufacturer in August 1988 over 360 deg in both the longitudinal and transverse planes. By symmetry, each of the first three enumerated quantities is determined by four values. The four beam pattern parameters are presented in Table 2. Included with the values derived from the beam pattern plots are the corresponding results of theoretical computation based on the measured, maximum overall cylindrical transducer dimensions given in Table 1. In fact, each of transducer sections A, B, and C, covering the frequency range 27-150 kHz, is a stacked array of identical cylindrical elements. The spacing between adjacent elements is small and is consequently neglected. In the computations, the sound speed is assumed to be 1481.8 m/s (Mackenzie 1981), as the ITC measurements were made in a fresh water tank at 20°C, at depth 1.5 m. By symmetry, the theoretical value for ΔB in the transverse plane is zero, hence is not shown.

Table 2. Beam pattern parameters of six cylindrical transducers at eleven frequencies. The measured half-beamwidth $\Delta\theta$ and angle $\pi/2-\theta_1$ of first sidelobe relative to the broadside plane are each averages of the four corresponding values. The range of measured values of the first sidelobe level B_1 is given. In the single case of the transducer at 710 kHz, θ_1 and B_1 could not be determined because of the irregular shape of the beam pattern. The assumed sound speed in the computations is 1481.8 m/s.

Frequency	$\Delta\theta$ (deg)		$\pi/2-\theta_1$ (deg)		B_1 (dB)		ΔB (dB)
(kHz)	Meas.	Comp.	Meas.	Comp.	Measured range	Comp.	Meas.
27	6.6	6.8	22	22.6	[-15.2,-11.5]	-13.2	1.6
38	4.8	4.8	16	15.8	[-13.7,-10.9]	-13.1	2.0
70	6.9	6.1	22	20.0	[-15.7,-13.0]	-12.9	6.5
88	5.8	4.8	18	15.8	[-16.8,-13.3]	-12.8	4.5
107	4.6	6.1	21	20.0	[-19.7,-14.0]	-13.0	7.7
120	5.0	5.4	16	17.8	[-15.5,-10.3]	-13.0	6.8
150	4.1	4.3	13	14.2	[-18.2,-17.0]	-13.0	6.3
200	6.5	5.5	24	18.2	[-8.2,-5.4]	-14.2	5.0
250	5.4	4.5	23	14.8	[-27.0,-15.0]	-12.2	7.0
375	5.8	3.9	20	12.9	[-17.4,-14.5]	-13.6	7.3
710	5.3	4.5	-	14.9	-	-14.1	13.0

SONAR MODEL

Two performance measures are chosen for comparison of the equal-area cylindrical and circular piston transducers. These are the maximum detection ranges for single targets and multiple targets distributed throughout the sampling volume. The maximum detection range is computed by means of the active sonar equation with ambient-noise-limited conditions, assuming a constant detection threshold of 20 dB. Details are given here.

Source level The usual equation for source level (Clay and Medwin 1977) requires generalization to non-planar transducers for which there is generally a loss in sensitivity on the acoustic axis. This affects, firstly, the target echo level through the incident signal level, i.e., transmission process; and, secondly, both target echo level and ambient-noise level through the reception process, but in equal proportions. Thus it is convenient to incorporate the described loss due to transducer curvature, CL, directly in the expression for source level SL, hence

$$SL = 10 \log P + DI_T - CL + 170.8 \quad , \tag{10}$$

where P is the transmitted acoustic power in watts, and DI_T is the transmitting directivity index. The acoustic power is related to the nominal electrical power P_{el} by the basic expression

$$P = \eta P_{el} \quad , \tag{11}$$

where η is the conversion efficiency of the transducer, assumed to be 0.6 for the particular transducers. The transmitting directivity index is defined by the expression

$$DI_T = 10 \log \frac{4\pi}{\int b \, d\Omega} \quad , \tag{12}$$

where the integration of the transmit beam pattern b is performed over all 4π sr. Numerical integration has been employed to compute the integral in equation (12) for both transducer types. Computed values for the baffled circular piston transducer of radius s agree well with values computed according to the ordinary narrow-beam approximation (Urick 1983), namely $DI_T = 20 \log (ks)$. Nominal values of the several quantities in equation (10) are presented in Table 3. Here the sea temperature is assumed to be 5°C, salinity 35 ppt, and depth 0 m, hence the sound speed is 1470.6 m/s (Mackenzie 1981). Since the areas of cylindrical and circular piston transducers are equal, the acoustic power is common to both. It is observed that CL=0 for the circular piston transducer.

Transmission loss For one-way propagation over the range r, this is $TL = 20 \log r + \alpha r$, where α is the absorption coefficient, given by Francois and Garrison (1982). For two-way propagation, the transmission loss is just double the one-way loss, or $2TL = 40 \log r + 2\alpha r$. In determining the absorption coefficient, the temperature, salinity, and depth take the same values as in computation of the source level, respectively 5°C, 35 ppt, and 0 m, hence with sound speed 1470.6 m/s, and the pH is assumed to be 7.7. Values of α used in the performance computations are shown in Table 4.

Table 3. Acoustic power P, transmitting directivity index DI_T in decibels, transducer curvature loss CL in decibels, and source level SL in decibels re 1 μPa at 1 m, for corresponding cylindrical and equal-area baffled circular piston transducers.

Frequency (kHz)	P(W)	Cylinder DI_T	Cylinder CL	Cylinder SL	Piston DI_T	Piston SL
27	360	8.8	0.9	204.3	19.8	216.1
38	360	10.3	1.7	205.0	22.8	219.1
70	240	9.3	2.7	201.2	22.9	217.5
88	240	10.2	4.1	200.8	24.8	219.4
107	120	9.3	1.9	199.0	22.0	213.6
120	120	9.8	2.4	199.0	23.1	214.7
150	120	10.7	3.6	198.7	25.0	216.6
200	90	9.6	9.7	190.2	29.3	219.6
250	90	10.5	10.7	190.2	31.2	221.5
375	30	11.1	11.2	185.5	32.2	217.8
710	12	10.4	11.3	180.7	31.7	213.3

Echo level The two mentioned cases are distinguished. For a single target, with target strength TS, the echo level is

$$EL = SL - 2TL + TS \quad . \tag{13}$$

For multiple targets distributed throughout the sampling volume V, with mean volume backscattering strength S_v,

$$EL = SL - 2TL + S_v + 10 \log V \quad . \tag{14}$$

The sampling volume is assumed to take its nominal value,

$$V = \frac{c\tau}{2} r^2 \psi \quad , \tag{15}$$

where τ is the pulse duration, assumed to be 0.1 ms, and ψ is the equivalent beam angle,

$$\psi = \int b^2 d\Omega \quad . \tag{16}$$

This has been computed by numerical integration for both transducer types. The results for the circular piston transducer agree closely with the simple narrow-beam approximation (Clay and Medwin 1977), $\psi=5.78/(ks)^2$, and a derived expression, $\Psi=10 \log \psi=-DI_T + 7.6$. Values assumed for Ψ in the performance computations are presented in Table 4.

Table 4. Additional parameter values assumed in the performance computations, namely absorption coefficient α, equivalent beam angle Ψ in decibels, noise spectral level SPL in decibels re 1-Hz band, and noise band level NL in decibels.

Frequency (kHz)	α (dB/km)	Ψ(dB) Cyl.	Ψ(dB) Piston	Sea state 0 SPL	Sea state 0 NL	Sea state 6 SPL	Sea state 6 NL
27	6.4	0.5	-12.1	22.8	57.2	47.0	81.3
38	10.7	-1.0	-15.1	22.4	58.2	44.5	80.3
70	21.6	0.0	-15.2	25.3	63.7	40.1	78.6
88	26.1	-1.0	-17.2	27.1	66.5	38.6	78.0
107	29.9	0.0	-14.4	28.7	69.0	37.5	77.8
120	32.2	-0.5	-15.4	29.6	70.4	36.9	77.7
150	37.2	-1.4	-17.4	31.5	73.3	36.2	77.9
200	45.6	-0.3	-21.6	34.0	77.0	36.2	79.2
250	55.1	-1.3	-23.6	36.0	79.9	37.1	81.0
375	86.1	-1.8	-24.6	39.5	85.2	39.8	85.5
710	226.6	-1.2	-24.1	45.0	93.5	45.0	93.6

Noise level The ambient noise is assumed to be isotropic and with a level specified by three different sources. The Knudsen curves describe the noise spectral level SPL due to wave action. It is a function of transducer frequency f in Hertz and sea state number n_{ss} (Bartberger 1965):

$$SPL_{amb} = 46 + 30 \log (n_{ss}+1) - 17 \log (f/1000) \quad . \tag{17}$$

The noise spectral level for thermal noise is described by Mellen (1952):

$$SPL_{th} = -15 + 20 \log (f/1000) \quad . \tag{18}$$

It is reasonable to assume a receiver electronic noise level that is equivalent to the thermal noise level. If S denotes the corresponding antilogarithm, then the spectral noise level due to all three sources is

$$SPL_{tot} = 10 \log (S_{amb}+2S_{th}) \quad . \tag{19}$$

The noise band level NL is just

$$NL = SPL + 10 \log BW \quad , \tag{20}$$

where BW is the receiver bandwidth in Hertz. In all of the present computations, the receiver bandwidth is assumed to be 10% of the transmit frequency, hence BW=0.1f. The noise spectral levels and noise band levels assumed in the computations are stated in Table 4.

Sonar equation The several quantities are combined in the ordinary, active sonar equation, given ambient-noise-limited conditions. This is (Urick 1983)

$$EL - (NL - DI_R) = DT \quad , \qquad (21)$$

where DI_R is the receiving directivity index, and DT is the detection threshold. Here, $DI_R = DI_T$. The detection threshold is assumed uniformly to be 20 dB. That is, the signal-to-noise ratio is chosen to be 20 dB, in order to ensure unambiguous signal detection. The single unknown in the sonar equation where EL is given by equation (13), assuming a parametric value for TS, is the range r. This is the sought maximum range for single-target detection. The corresponding unknown in the sonar equation where EL is given by equation (14), assuming a parametric value for S_v, is the maximum range for multiple-target detection.

RESULTS AND DISCUSSION

Results of the described performance computations are presented in Tables 5 and 6. Table 5 pertains to single targets according to equations (13) and (21), and Table 6, to multiple targets according to equations (14) and (21). A range of values of target strength and mean volume backscattering strength are examined, in both cases from -120 to -60 dB. Maximum detection ranges less than 0.05 m are not shown, but in fact ranges less than the Rayleigh distance, which is the effective transducer area divided by the acoustic wavelength (Clay and Medwin 1977), are uncertain for lying within the transducer nearfield.

In every single instance, the performance of the baffled planar circular piston transducer is superior to that of the respective equal-area cylindrical transducer. This is to be expected from two considerations. (1) The directivity index of the equal-area circular piston transducer is considerably greater than that of the corresponding cylindrical transducer. According to Table 3, the difference is in the approximate range 10-20 dB, but the directivity index appears twice in the ambient-noise-limited sonar equation: in the term for source level, as DI_T, and in the term for discrimination against isotropic ambient noise, as DI_R. (2) Because of the curvature of the cylindrical transducer, it suffers an on-axis loss in sensitivity to which the circular piston transducer is exempt. This is described by the term CL in equation (10), with numerical values in Table 3.

Computation of both the directivity index and the sensitivity loss for the cylindrical transducer, according to the present theoretical method, depends on the acoustic boundary condition. As observed in the Introduction, this is indeed problematical. However, by reference to the beam pattern measurements on the six fabricated cylindrical transducers, confidence is gained in the particular assumed boundary condition, the acoustic opacity described in the section on beam patterns. The respective values of beam pattern parameters in Table 2 support this use.

Table 5. Maximum single-target detection range in meters for equal-area cylindrical and circular piston transducers, with dimensions in Table 1, assuming ambient-noise-limited conditions and detection threshold of 20 dB.

Frequency	TS	Sea state 0		Sea state 6	
(kHz)	(dB)	Cylinder	Piston	Cylinder	Piston
27	-120	2.5	9.2	0.6	2.3
27	-110	4.4	16.3	1.1	4.1
27	-100	7.9	28.7	2.0	7.3
27	-90	13.9	50.3	3.5	12.9
27	-80	24.5	87.1	6.2	22.8
27	-70	43.0	148.0	11.0	40.0
27	-60	74.8	245.2	19.4	69.6
38	-120	2.7	12.2	0.8	3.5
38	-110	4.7	21.4	1.3	6.1
38	-100	8.4	37.4	2.4	10.8
38	-90	14.8	64.3	4.2	19.0
38	-80	25.9	108.3	7.4	33.3
38	-70	45.0	176.9	13.1	57.4
38	-60	77.0	277.8	23.0	97.3
70	-120	1.5	8.1	0.6	3.5
70	-110	2.6	14.2	1.1	6.1
70	-100	4.6	24.6	2.0	10.8
70	-90	8.2	41.9	3.5	18.8
70	-80	14.3	69.6	6.2	32.4
70	-70	24.7	111.4	10.9	54.5
70	-60	42.1	170.9	18.9	88.9
88	-120	1.3	8.5	0.7	4.5
88	-110	2.3	14.9	1.2	7.8
88	-100	4.0	25.6	2.1	13.7
88	-90	7.1	43.3	3.7	23.7
88	-80	12.5	70.8	6.6	40.1
88	-70	21.6	111.5	11.5	65.9
88	-60	36.7	167.6	19.9	104.5
107	-120	1.0	4.6	0.6	2.8
107	-110	1.7	8.0	1.0	4.9
107	-100	3.0	13.9	1.8	8.6
107	-90	5.3	23.9	3.2	14.9
107	-80	9.3	40.3	5.7	25.5
107	-70	16.2	65.6	9.9	42.8
107	-60	27.6	102.7	17.2	69.4

Table 5. (Cont.)

Frequency (kHz)	TS (dB)	Sea state 0 Cylinder	Sea state 0 Piston	Sea state 6 Cylinder	Sea state 6 Piston
120	-120	0.9	4.8	0.6	3.1
120	-110	1.6	8.3	1.1	5.5
120	-100	2.9	14.5	1.9	9.7
120	-90	5.0	24.8	3.3	16.8
120	-80	8.8	41.5	5.9	28.6
120	-70	15.3	67.1	10.3	47.4
120	-60	26.2	104.1	17.7	75.9
150	-120	0.8	5.0	0.6	3.8
150	-110	1.4	8.7	1.1	6.8
150	-100	2.5	15.1	1.9	11.8
150	-90	4.4	25.7	3.4	20.2
150	-80	7.7	42.5	6.0	33.8
150	-70	13.4	67.8	10.4	55.0
150	-60	22.9	103.5	18.0	85.7
200	-120	0.4	6.1	0.3	5.4
200	-110	0.7	10.6	0.6	9.4
200	-100	1.2	18.0	1.0	16.1
200	-90	2.1	30.1	1.8	27.0
200	-80	3.6	48.6	3.2	43.9
200	-70	6.4	75.2	5.7	68.6
200	-60	11.1	110.8	9.8	102.2
250	-120	0.3	6.3	0.3	6.0
250	-110	0.6	11.0	0.6	10.3
250	-100	1.0	18.6	1.0	17.5
250	-90	1.8	30.6	1.7	29.0
250	-80	3.2	48.6	3.0	46.2
250	-70	5.7	73.6	5.3	70.5
250	-60	9.8	106.4	9.3	102.4
375	-120	0.2	4.0	0.2	3.9
375	-110	0.3	6.9	0.3	6.8
375	-100	0.6	11.7	0.6	11.6
375	-90	1.1	19.4	1.1	19.1
375	-80	1.9	30.7	1.9	30.3
375	-70	3.3	46.7	3.3	46.1
375	-60	5.8	67.5	5.7	66.8
710	-120	0.1	1.8	0.1	1.8
710	-110	0.2	3.2	0.2	3.2
710	-100	0.3	5.3	0.3	5.3
710	-90	0.5	8.7	0.5	8.7
710	-80	0.9	13.6	0.8	13.5
710	-70	1.5	20.3	1.5	20.2
710	-60	2.6	28.9	2.6	28.8

Table 6. Maximum multiple-target detection range in meters for equal-area cylindrical and circular piston transducers, with dimensions in Table 1, assuming ambient-noise-limited conditions and detection threshold of 20 dB.

Frequency (kHz)	S_V (dB)	Sea state 0		Sea state 6	
		Cylinder	Piston	Cylinder	Piston
27	-120	1.8	5.7	0.1	0.4
27	-110	5.6	17.9	0.4	1.1
27	-100	17.5	53.6	1.1	3.6
27	-90	52.5	147.7	3.5	11.2
27	-80	144.9	348.1	11.0	34.4
27	-70	342.9	678.5	33.6	98.9
27	-60	670.7	>1000.0	97.0	250.5
38	-120	1.7	7.2	0.1	0.6
38	-110	5.4	21.9	0.4	1.8
38	-100	16.6	62.6	1.4	5.7
38	-90	48.6	156.8	4.3	17.4
38	-80	126.6	326.4	13.2	50.6
38	-70	276.7	568.3	39.0	131.2
38	-60	502.0	864.9	104.9	284.3
70	-120	0.6	3.2	0.1	0.6
70	-110	1.9	9.7	0.3	1.8
70	-100	5.8	28.0	1.1	5.6
70	-90	17.2	71.4	3.3	16.9
70	-80	47.0	151.6	10.1	46.1
70	-70	109.0	268.2	29.2	107.4
70	-60	209.3	412.9	73.9	207.0
88	-120	0.4	2.8	0.1	0.8
88	-110	1.3	8.6	0.3	2.4
88	-100	4.0	24.8	1.1	7.3
88	-90	11.9	62.4	3.3	21.3
88	-80	33.2	130.9	10.2	55.0
88	-70	79.6	229.4	28.7	118.6
88	-60	157.6	350.6	70.7	213.0
107	-120	0.3	1.1	-	0.4
107	-110	0.8	3.4	0.3	1.3
107	-100	2.5	10.3	0.9	3.9
107	-90	7.5	28.7	2.8	11.7
107	-80	21.6	68.8	8.6	32.1
107	-70	54.4	136.5	24.3	75.4
107	-60	114.1	228.7	60.1	146.4

Table 6. (Cont.)

Frequency (kHz)	S_v (dB)	Sea state 0 Cylinder	Sea state 0 Piston	Sea state 6 Cylinder	Sea state 6 Piston
120	-120	0.2	1.1	-	0.5
120	-110	0.7	3.3	0.3	1.5
120	-100	2.1	10.0	0.9	4.5
120	-90	6.4	27.8	2.8	13.3
120	-80	18.6	66.1	8.6	35.7
120	-70	47.4	130.0	24.3	80.8
120	-60	100.8	216.5	59.2	151.4
150	-120	0.2	0.9	-	0.6
150	-110	0.5	2.9	.3	1.7
150	-100	1.5	8.8	.9	5.3
150	-90	4.5	24.4	2.7	15.5
150	-80	13.2	57.9	8.1	39.8
150	-70	34.6	113.6	22.6	85.3
150	-60	76.5	188.7	54.4	152.1
200	-120	-	0.9	-	0.7
200	-110	0.1	2.7	-	2.1
200	-100	0.4	8.1	0.3	6.4
200	-90	1.1	22.2	0.9	18.0
200	-80	3.5	51.6	2.7	43.6
200	-70	10.3	99.1	8.1	87.2
200	-60	27.2	162.0	22.2	147.0
250	-120	-	0.8	-	0.7
250	-110	-	2.4	-	2.1
250	-100	0.3	7.1	.2	6.4
250	-90	0.8	19.3	.7	17.5
250	-80	2.5	44.5	2.2	41.0
250	-70	7.4	84.6	6.6	79.5
250	-60	19.9	137.2	18.0	130.9
375	-120	-	0.3	-	0.3
375	-110	-	0.9	-	0.8
375	-100	-	2.6	-	2.5
375	-90	0.3	7.6	.3	7.3
375	-80	0.8	19.0	.8	18.6
375	-70	2.5	39.8	2.4	39.1
375	-60	7.1	69.7	6.9	68.7
710	-120	-	-	-	-
710	-110	-	0.2	-	0.2
710	-100	-	0.6	-	0.6
710	-90	-	1.8	-	1.8
710	-80	0.2	4.9	0.2	4.9
710	-70	0.6	11.2	0.5	11.1
710	-60	1.6	21.1	1.6	21.0

The directivity index and sensitivity loss factor may be combined in two useful measures of performance. The first, due to Urick (1983), is useful for characterizing transducer performance against single targets,

$$PF_1 = SL - (NL - DI_R) \quad . \tag{22}$$

The second measure, by simple extension, is useful for characterizing transducer performance against multiple targets distributed throughout the sampling volume,

$$PF_2 = SL + \Psi - (NL - DI_R) \quad . \tag{23}$$

The first measure is computed for each transducer type and each of the sea states in Table 7. Included in the table is the acoustic intensity I, derived by dividing the acoustic power in Table 3 by the respective total transducer area.

Table 7. Performance figure PF_1 for equal-area cylindrical and circular piston transducers, and acoustic intensity I on the transducer surface.

Frequency	Sea state 0		Sea state 6		I
(kHz)	Cylinder	Piston	Cylinder	Piston	(W/cm^2)
27	155.9	178.7	131.8	154.6	1.6
38	157.1	183.7	135.0	161.6	1.6
70	146.8	176.7	131.9	161.8	3.5
88	144.5	177.7	133.0	166.2	3.5
107	139.3	166.6	130.5	157.8	5.0
120	138.4	167.4	131.1	160.1	5.0
150	136.1	168.3	131.5	163.7	5.0
200	122.8	171.9	120.6	169.7	2.5
250	120.8	172.8	119.7	171.7	2.5
375	111.4	164.8	111.1	164.5	1.5
710	97.6	151.5	97.5	151.4	2.4

Reference to the cavitation threshold (Urick 1983) indicates that this increases rapidly with frequency. The acoustic intensity for the cylindrical transducers shows the expected dependence up to about 150 kHz. At higher frequencies, the intensity is far less than the estimated average and far less than is ordinarily used in the design of planar, resonant transducers, for example, the circular piston transducer considered in this work. In general, such a transducer is mechanically more robust than a cylindrical transducer and can tolerate being driven at a higher electrical power level than is used with the cylindrical transducers above 150 kHz. Thus, for the circular piston transducers at these higher frequencies, the performance figures in Table 7 and the maximum detection ranges in Tables 5 and 6 are underestimates.

Cylindrical transducers undoubtedly have application in underwater acoustics, including fisheries acoustics. A current example is in observation of food pellets in a fish-farming pen (Juell et al. 1993). For measurement of dispersed or locally inhomogeneous aggregations of plankton and other weak scatterers, the conventional circular piston with equal area clearly gives superior performance.

There are pragmatic considerations too for choosing between different transducer types. Here, the circular piston transducer is also the better choice, for it is cheaper and easier to fabricate than is the corresponding equal-area cylindrical transducer. Inspection of the measured beam patterns in the transverse plane of the cylindrical transducers, describing the azimuthal dependence, are particularly revealing of practical difficulties in fabrication, for the range of variation with azimuth exceeds 3 dB for five of the six transducers, at nine of the eleven frequencies. In principle, it should be zero.

A further advantage of the circular piston transducer is standardization. Given the complexity of the general process of underwater acoustic mensuration, avoidance of special devices is a general rule, with particular force for multiple-frequency sondes.

Three matters not addressed here but deserving of future treatment are identified. (1) The precise range, number, and spacing of transducer frequencies require optimization for the scatterer species and sizes of interest. This naturally depends on the scattering properties of target organisms. (2) Given specification of a particular set of transducer frequencies, the radii of the transducers must be chosen apropos of their arrangement on a sonde. A major aim of this might be to ensure the greatest possible coincidence or overlap of sampling volumes. (3) Performance of the ultimately chosen set of transducers should be calculated on the basis of a generally closer approach to the cavitation limit than has been assumed at all frequencies in the present analysis. Clearly, performance will be enhanced under ambient-noise-limited conditions by driving transducers at higher power levels, consistent with avoiding cavitation, especially at the highest frequencies, where absorption is a major cause of attenuation.

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