On two-phase/mixture modelling of sediment transport and turbulence modulation due to fluid-particle interactions

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A fundamental issue in modelling sediment transport is to understand the complex mechanism that involves fluid-particle interactions. When the suspension concentration becomes significantly high near the bottom, interactions between particles also play an important role in turbulence modulation as well as affecting the suspension capacity in the above water column. These insights have been provided by using two-phase flow theory to analyse the four-way coupling effects in the high concentrated benthic layer, which cannot be done with traditional sediment transport models. In the work of Toorman and Bi (2013), the fundamental equations have been reconstructed from two-phase flow/mixture theory by expressing mixture theory equations in terms of fluid variables, combined with a drift closure. Following the previous work, the relative particle movement is solved separately with a force balance equation for completing the equations. In addition, a modified low-Reynolds k-ε turbulence model has been developed by Toorman (2011). For application to the two-phase flow/mixture theory, an extended version is proposed as turbulence closure.

Fundamental equations of two-phase/mixture theory

Unlike the two-fluid model, the mixture theory usually chooses volume- and/or weight- averaged variables for the mixture, which is the sum of carrier and dispersed phase. The additional complexity regarding momentum exchanges between phases temporarily can be circumvented in this way (but returns at a later stage). By employing ensemble-averaged fluid velocity U_i and solids velocity V_i, related to Reynolds-averaged properties proposed by Toorman (2008):

$$U_i = \overline{u_i} + \frac{-\overline{u_i'\phi'}}{1-\overline{\phi}} = \overline{u_i} + U_{Di} \quad \text{and} \quad V_i = \overline{v_i} + \frac{\overline{v_i'\phi'}}{\overline{\phi}} = \overline{v_i} + V_{Di} \tag{1}$$

where: $\overline{u_i}$ and $\overline{v_i}$ = the respective Reynolds-averaged fluid and solids velocities, U_{Di} and V_{Di} = the respective fluid and solids drift velocities, $\overline{\phi}$ = the Reynolds-averaged solids volume fraction, $-\overline{u_i'\phi'}$ and $\overline{v_i'\phi'}$ = the respective Reynolds averaged fluid and solids turbulent flux. Substitution of (1) into the Reynolds-averaged equations leads to the following basic equations (where the overbar is dropped). The suspension continuity equation reads:

$$\frac{\partial\left(U_j + W_j\phi\right)}{\partial x_j} = 0 \tag{2}$$

where: x_j = the location coordinate and $W_i = V_i - U_i$ = the (ensemble-averaged) slip velocity. The exact suspension momentum equation becomes:

$$\begin{aligned}\rho\left(\frac{\partial U_i}{\partial t} + U_j\frac{\partial U_i}{\partial x_j}\right) &= -\frac{\partial p}{\partial x_i} + \rho g \delta_{iz} + \frac{\partial\left(\sigma_\mu + \sigma_{Tij} + \sigma_{Dij}\right)}{\partial x_j} \\ &\quad - \rho_s\phi\left(\frac{\partial W_i}{\partial t} + U_j\frac{\partial W_i}{\partial x_j} + W_j\frac{\partial\left(U_i + W_i\right)}{\partial x_j}\right)\end{aligned} \tag{3}$$

with: $\rho = \rho_s\phi + \rho_w(1-\phi)$ = the suspension bulk density, ρ_s and ρ_w = the particle and fluid density, p = pressure, g = gravity constant, σ_μ = the viscous stress, σ_T = the turbulent Reynolds stress:

$$\sigma_{Tij} = \rho_s\phi\left(-\overline{v'_i v'_j}\right) + \rho_w(1-\phi)\left(-\overline{u'_i u'_j}\right) \approx -\rho\overline{u'_i u'_j} \approx \rho\nu_t\left(\frac{\partial U_i}{\partial x_j} + \frac{\partial U_j}{\partial x_i}\right) \tag{4}$$

and σ_D = the drift diffusion stress:

$$\sigma_{Dij} = \rho_s \phi (V_{Di} V_{Dj}) + \rho_w (1-\phi)(U_{Di} U_{Dj})$$
$$\approx \left(\frac{\nu_t}{Sc}\right)^2 \left(\rho_s \phi \frac{\partial \ln \phi}{\partial x_i} \frac{\partial \ln \phi}{\partial x_j} + \rho_w (1-\phi) \frac{\partial \ln(1-\phi)}{\partial x_i} \frac{\partial \ln(1-\phi)}{\partial x_j} \right) \tag{5}$$

using the Boussinesq approximation. The slip velocity W can be obtained either from a simplified particle force balance (e.g. Kim *et al.*, 1998) or the suspension momentum balance equation.

Low-Reynolds turbulence modelling

It is observed that in benthic suspension layers high suspended sediment concentrations above the saturation limit are found (resulting in “fluid mud” or sheet flow conditions; Toorman, 2002) and turbulence is no longer fully developed (because the four-way particle-particle-fluid interactions consume much energy). Therefore, it is necessary to determine the eddy viscosity in the above equations from low-Reynolds turbulence models (Patel *et al.*, 1985). In order to reduce the necessary high grid resolution for this type of turbulence closure a two-layer approach (Rodi, 1991) is applied where a low-Reynolds *k-ε* model for the outer layer is combined with a low-Reynolds mixing-length model for the bottom layer (Toorman, 2011). Besides semi-empirical damping functions, the *k-ε* model also contains extra terms originating from the extra terms in the momentum equation (eq.3). Since the exact *k-ε* model for suspensions (Elgobashi and Abou-Arab, 1983) cannot be solved, simplified models (e.g. Chauchat and Guillou, 2008) are investigated and new closures are proposed based on the analysis of experimental data in combination with dimensional analysis and theoretical considerations. The model is first evaluated with the simulation of flume experiments for sand suspensions. Next, the model is combined with a non-Newtonian rheological closure for the viscosity of fluid mud (based on Toorman, 1997) to simulate (steady and unsteady) shear flow over a muddy bottom.

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