A conceptual one-dimensional flocculation model for floc size distributions of suspended kaolinite in a cylindrical tank

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Introduction

Cohesive sediments transport is essential to managing coastal water quality and dredging operation, and to understanding coastal ecology, chemical fluxes and geological record. Nevertheless, the transport and fate of cohesive sediments are poorly predicted mainly due to the aggregation and breakage of fine, cohesive aggregates. A primary challenge to develop a flocculation model is that the time variation of Floc Size Distribution (FSD) is controlled by a partial differential equation that also contains the integration of the FSD itself. In this study, a conceptual one-dimensional flocculation model, i.e. the Population Balance Equation (PBE), is solved by using Quadrature Method Of Moments (QMOM). Model results of FSDs are verified by a laboratory experiment in a cylindrical tank placed at Virginia Institute of Marine Science (VIMS). The turbulence in the tank is generated by a marine bilge pump and measured by a 5MHz ADVOcean. The FSDs at selected places are statistically obtained by an underwater camera system and processed by using MATLAB® image processing toolbox. The results have shown that the FSDs can be efficiently and reasonably displayed by the quadrature nodes (i.e. the characteristic sizes) and corresponding weights (i.e. the characteristic number densities) in this PBE-QMOM flocculation model.

Flocculation model

The general transport equation (i.e. the PBE) that includes the kinetics of aggregation and breakage of flocs with size *L* is given in Eq.1 (Prat and Ducoste, 2006).

$$\frac{\partial n(L;t)}{\partial t}+\left(U_s - W_{s,s}\right)\frac{\partial n(L;t)}{\partial s}-\frac{\partial}{\partial s}\left(\frac{\nu_t}{\sigma_t}\frac{\partial n(L;t)}{\partial s}\right)$$
$$=\frac{L^2}{2}\int_0^L\left[\frac{\beta\left((L^3-\lambda^3)^{1/3},\lambda\right)\cdot\alpha\left((L^3-\lambda^3)^{1/3},\lambda\right)}{\left(L^3-\lambda^3\right)^{2/3}}\cdot n\left((L^3-\lambda^3)^{1/3};t\right)\cdot n(\lambda;t)\right]d\lambda$$
$$-n(L;t)\int_0^\infty \beta(L,\lambda)\alpha(L,\lambda)n(\lambda;t)d\lambda+\int_L^\infty a(\lambda)\cdot b(L\,|\,\lambda)\cdot n(\lambda;t)d\lambda - a(L)\cdot n(L;t) \qquad (1)$$

where n (L; t) is the number density function defined on the basis of floc size L and time t, s denotes the direction along the streamline, U_s is the flow velocity along s, $W_{s,s}$ is projection of settling velocity in the direction of s, ν_t is eddy viscosity, σ_t is the turbulent Prandtl-Schmidt number, β is the collision frequency function, α is collision efficiency function, a is breakup frequency function, and b is fragmentation distribution function. The flocculation source and sink terms on the right hand side of Eq. 1 include: (1) the birth of flocs with size L due to aggregation of smaller particles, (2) the death of flocs with size L due to aggregation with other particles, (3) the birth of flocs with size L due to fragmentation of bigger particles, and (4) the death of flocs with size L due to breakup into smaller particles. The left hand side terms include, from left-to-right, an unsteady term, an advection and settling term, and a diffusion term, respectively. Among all the available methods for solving PBE, Shen and Maa (submitted) enhanced the classical QMOM approach (McGraw, 1997) to reasonably monitor the FSDs using a maximum of eight size classes in their flocculation box model. In this study, that box model is extended into a conceptual 1-D model to demonstrate the ability of simulating the time variation of FSDs in a tank.

Experimental setup

A kaolinite suspension in a cylindrical tank (diameter = 0.75m, height = 1.5m) is agitated with a 3700GPH submersible pump, mounted at the bottom with an upward outlet at the centre (Fig. 1). The submersible pump is started at a pre-determined power setting to mix the suspension during the first several minutes, obtaining a steady initial condition. Turbulence eddy diffusion coefficients and time-averaged current velocities are measured with a 5MHz SonTek ADVOcean, and the suspended sediment concentrations are measured by an OBS. These measured values are used as inputs for the flocculation model (i.e. Eq. 1). The evolution of kaolinite floc size will be observed by using an underwater camera system at selected locations. This camera system consists of a Sony Alpha NEX-7 camera, a Nikon Macro Lens, a Kenko extension tube, LED light sources, and a control circuit board assembled in a waterproof house. This combination has an image size of 23.5 x

15.6mm (6000 x 4000 pixels), and thus a native 3.9μm/pixel. With a subject to image ratio 1:2 selected, an effective resolution of 1.9μm/pixel can be obtained for a sufficiently large subject window of 12 x 8mm. The images from the underwater camera system are analysed using MATLAB image processing toolbox. Several operations are applied to the raw images prior to statistical analysis, including contrast stretching, grayscale to binary conversion, background noise removing, dilation and erosion, and on-border incomplete objects removing. This underwater camera system, as well as the image processing program, is sufficient to find the time variation of FSDs with minimum particle size around 4μm (i.e. using 2 x 2 pixels).

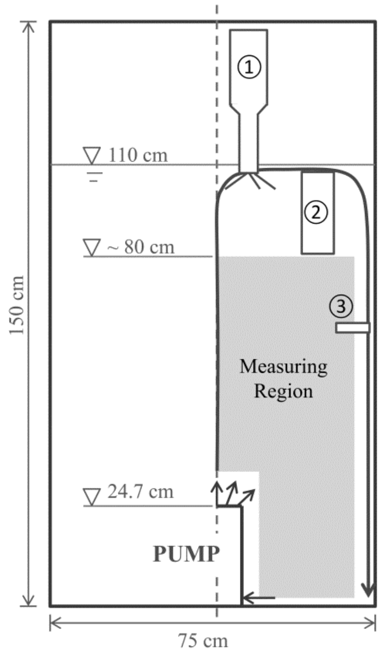

Fig. 1. Experimental setup (not to scale):
① ADV;
② Underwater Camera System;
③ OBS.

Results and conclusions

This cylindrical tank experiment provides a valuable dataset dedicated to verify the One-Dimensional (1-D) Floc Size Distribution (FSD) under selected flow and sediment conditions. Although this is a conceptual 1-D model, i.e. along the streamline, it can be applied to the typical vertical 1-D simulation. The only difference is to couple with a vertical 1-D hydrodynamics model (also submitted to this conference in another abstract by Maa *et al.*) for flow field and turbulence information required in Eq. 1. While demonstrating the evolution of FSDs in a laboratory tank, this underwater camera system can also be utilized for in-situ measurements.

References

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