Flocculation, self-similarity and the rheology of aqueous clay suspensions

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A dense cohesive sediment suspension, sometimes referred to as fluid mud, is a thixotropic fluid with a true yield stress. Its time-dependent rheological behaviour can be described by the structural kinetics theory, based on the Worrall-Tuliani (1964) approach, in which the fluid is described as a non-ideal Bingham fluid and the yield stress is taken as a measure for the structural parameter (Toorman, 1997). For a fluid in equilibrium (exposed to a constant shear rate for a sufficiently long time), the shear stress is given by,

$$\tau = \tau_0 + \mu_\infty \dot{\gamma} + \frac{c\dot{\gamma}}{1+\beta\dot{\gamma}} \qquad (1)$$

where τ is the shear stress, τ_0 is the yield stress at equilibrium, $\dot{\gamma}$ is the shear rate, μ_∞ is the Newtonian viscosity of the suspension at high shear rate, $c = \mu_\infty - \mu_0$, μ_0 is the viscosity associated with low shear rate and β is a constant.

The Worrall-Tuliani approach requires 4 empirical values for $\tau_0, \mu_\infty, c, \beta$ (and 2 more empirical constants for identifying the transient behaviour). However, in a time varying cohesive sediment suspension these empirical values depend on the sediment concentration. This makes the Worrall-Tuliani approach unwieldy for use in sediment transport studies where sediment concentrations are constantly changing.

This paper is concerned with establishing a framework for establishing values for the constants, μ_∞ , c and β, which is valid for the whole range of sediment concentrations from the yield-point to Newtonian flow. The shear stress equation is re-constructed from first principles using an approach, based on floc fractal theory, put forward by Mills and Snabre (1988). This results in a Casson-like rheology equation which defines the value of μ_∞ in terms of the sediment concentration. Structural kinetics theory (Moore, 1959) is then added to this equation, giving an equation similar to Equation (1). The form of c and β are derived by comparison of the resulting equation with the results of Coussot (1995). Application of the self-similar relationships found by Coussot indicates that $c \sim c(\tau_0, \mu_\infty)$ while β remains an empirical constant. The resulting rheological equation for equilibrium conditions is as follows

$$\tau^{1/2} = \tau_0^{1/2} + \left(1 + \frac{\alpha(\tau_0/\mu_\infty)^m}{1+\beta\dot{\gamma}^m}\right)^{1/2} (\mu_\infty \dot{\gamma})^{1/2} \qquad (2)$$

where α and m are empirical constants. This equation contains the same number of empirical unknowns $(\tau_0, \alpha, \beta, m)$ as Equation 1 but of these only τ_0 is a function of the sediment concentration.

An example of the comparison of Equation 2 against data derived by Coussot (1995) for kaolinite in freshwater is shown in Fig. 1. Additional comparisons with other data sets are presented in the paper.

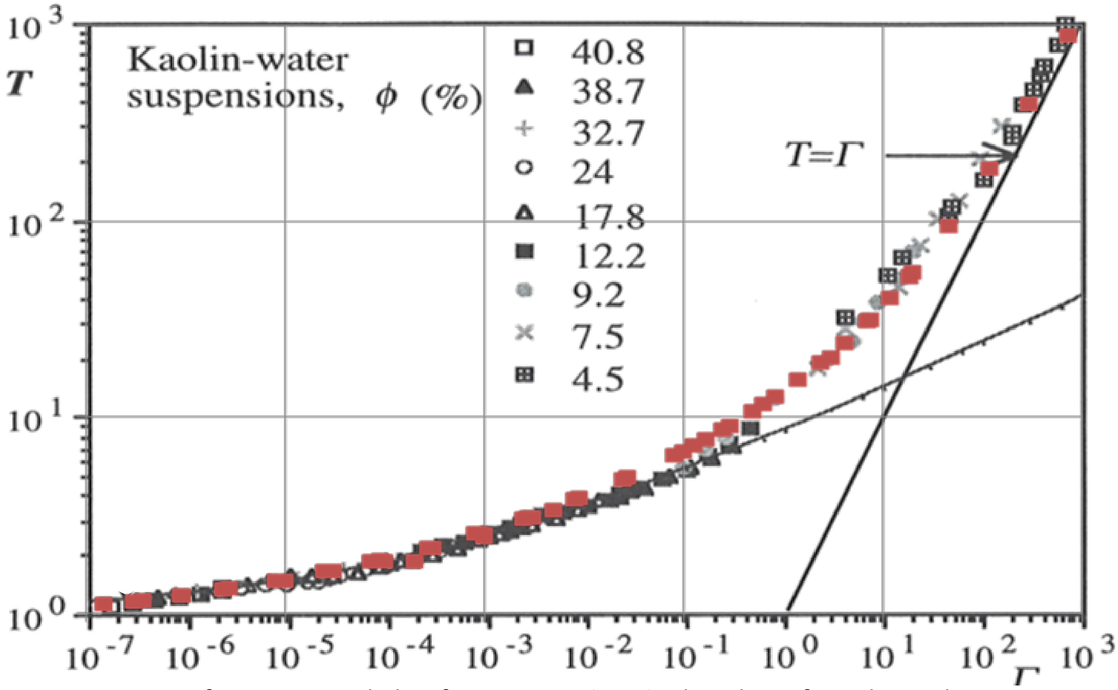

Fig. 1. Comparison of Equation 2 with data from Coussot (1995). The values of T and Γ on the axes are given by $T = \left(\frac{\tau}{\tau_0}\right)$. $\Gamma = \left(\frac{\dot{\gamma}\mu_\infty}{\tau_0}\right)$.

References

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