Mud flocculation model considering the effects of distribution of fractal dimensions and yield strength

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Introduction

Flocculation of cohesive sediment particles has a strong influence on settling, deposition and other sediment transport processes. The aggregation and disaggregation rates of flocs depend on many factors including the fluid shear, turbulence and the properties of flocs such as primary particle size, floc size, density and shape (fractal dimension) not all of which are independent.

Based on the constant fractal dimension assumption Winterwerp (1998) developed a flocculation model by linearly combining aggregation and break-up processes. This assumption was challenged by Martinis and Risovic (1998) who proposed that the value of fractal dimension decreases from 3 to 1.2 as the floc size increases. The values of the fractal dimension are also affected by variable flow conditions and aggregation mechanisms (Dyer and Manning, 1999; Khelifa and Hill, 2006; Maggi *et al.*, 2007; Vahedi and Gorczyca, 2011). Son and Hsu (2009) improved the existing flocculation model by taking into account the variable fractal dimension suggested by Khelifa and Hill (2006) and also considering variable yield strength of flocs.

In many field measurements a range of settling velocities are measured for one floc size. This has led Vahedi and Gorczyca (2012) to propose a normal distribution of fractal dimensions and as a consequence a floc of given size can also have a probability distribution of yield strength. The purpose of this study is to improve the flocculation model by taking into consideration the distributions of both fractal dimensions and yield strength.

The formulation

Normal distribution of fractal dimensions and yield stress

Here we follow the study by Vahedi and Gorczyca (2012) and the normal distribution of fractal dimensions for one floc size is given as:

$$P(nf)_D = \frac{1}{\sqrt{2\pi}\sigma_D}\exp\left(-\frac{(nf-\mu_D)^2}{2\sigma_D^2}\right) \tag{1}$$

where nf is the fractal dimension, D is the floc size, μ_D and σ_D are the mean and standard deviation of fractal dimensions for floc with size D respectively. For a normal distribution the probability $P(\mu-4\sigma < x < \mu+4\sigma)$ is much more than 99.9% thus for simplicity we assume all the fractal dimensions of one fixed floc size fall within the interval $[\mu-4\sigma, \mu+4\sigma]$.

The yield stress of floc is the function of fractal dimension. According to Son and Hsu (2009) it can be estimated based on the number of primary particles in the plane crossing the centre of a floc as:

$$\tau_y = B_1\left(\frac{D}{d}\right)^{2nf/3} D^{-2} \tag{2}$$

where $B_1 = (\pi/6)^{-2/3} F_{c,p}$ and $F_{c,p}$ is the cohesive force of primary particles.

The complete model

Both flocculation models of Son and Hsu (2009) and Winterwerp (1998) assume that the break-up rate of flocs is proportional to the number (n) of flocs suspended in the water. But in reality only those flocs with yield stress smaller than the turbulent shear stress contribute to the break-up rate. In this study n is replaced as $n_e = nP(D)$, $P(D)$ is the probability of yield strength smaller than turbulent shear with floc size of D. $P(D)$ is calculated using equation (3):

$$P(D) = \int_{\mu_D-4\sigma_D}^{nf_{max}(D)} \frac{1}{\sqrt{2\pi}\sigma_D}\exp\left(-\frac{(\mu_D-\sigma_D)^2}{2\sigma_D^2}\right) \tag{3}$$

where $nf_{max}(D)$ is the maximum fractal dimension allowing yield stress smaller than turbulent shear stress for a floc with size D.

The flocculation model by Son and Hsu (2009) can thus be modified as:

$$\frac{dD}{dt} = \frac{Gd^{\beta}}{\beta \ln(D/d)+1}\left[\frac{k_A'}{3}\frac{c}{\rho_s}d^{nf-3}D^{-nf+4-\beta}\right.$$

$$\left. -\frac{k_B'P(D)}{3}(\frac{\mu G}{B_1})^q d^{-p+(2q/3)nf}D^{1-\beta+(2q/3)(3-nf)}(D-d)^p\right] \qquad (4)$$

where $k_A' = (3e_c\pi e_d)/2f_s$ and $k_B' = ae_b$. This model is denoted as Model A, model originally proposed by Son and Hsu (2009) is denoted as Model B.

Model results
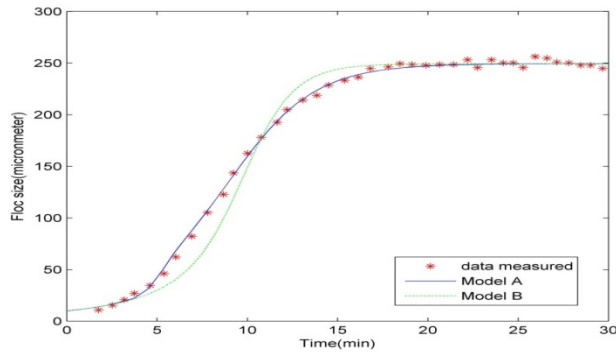The typical model results are shown in Fig. 1 and 2.

Fig. 1. Experimental data of Spicer (1998) and simulation results
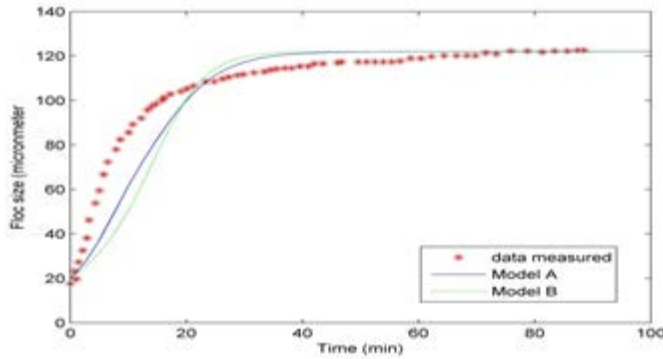of flocculation models.

Fig. 2. Experimental results of Biggs and Lant (2000) and simulation
results of flocculation models.

Conclusions
It can be concluded that the results obtained by considering variable yield stress and variable fractal dimensions in the flocculation model are encouraging. In both simulations (Fig. 1.and 2), model A shows much improved predictions.

References
Dyer K.R. and A.J. Manning. 1999. Observation of the size, settling velocity and effective density of flocs, and their fractal dimensions. Journal of Sea Research 41:87-95.
Khelifa A. and P.S. Hill. 2006. Models for effective density and settling velocity of flocs. Journal of Hydraulic Research 44:390-401.
Maggi F., F. Mietta and J.C. Winterwerp. 2007. Effect of variable fractal dimension on the floc size distribution of suspended cohesive sediment. Journal of Hydrology 343:43-55.

Martinis M. and D. Risovic. 1998. Fractal analysis of suspended particles in seawater. FIZIKA B-ZAGREB- 7:65-72.

Son M. and T.J. Hsu. 2009. The effect of variable yield strength and variable fractal dimension on flocculation of cohesive sediment. Water Res. 43:3582-92.

Vahedi A. and B. Gorczyca. 2011. Application of fractal dimensions to study the structure of flocs formed in lime softening process. Water Res. 45:545-56.

Vahedi A. and B. Gorczyca. 2012. Predicting the settling velocity of flocs formed in water treatment using multiple fractal dimensions. Water Res. 46:4188-94.

Winterwerp J.C. 1998. A simple model for turbulence induced flocculation of cohesive sediment. Journal of Hydraulic Research 36:309-326.